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Li Lin
School of Energy and Power Engineering

Xu Zhang
School of Energy and Power Engineering

Yun Li
School of Energy and Power Engineering

Guangxu Cheng
School of Energy and Power Engineering

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Li LIN, Xu ZHANG, Yun LI*, Guangxu CHENG

School of Energy and Power Engineering, Xi'an Jiaotong University, Xi'an, China
Phone: +86(29)82664928
Fax: +86(29)82664928
E-mail: lin.li@stu.xjtu.edu.cn
xu.zhang@stu.xjtu.edu.cn
yunli@mail.xjtu.edu.cn
gxcheng@mail.xjtu.edu.cn

* Corresponding Author

ABSTRACT

An analytical model based on modal series expansion method for calculating transmission loss (TL) of compressor's mufflers has been present in this paper. The approximately averaged pressure response value, obtained by calculating the weighted average pressure response value in the vicinity of the point source, is applied to derive the pressure response functions and four-pole parameters. Therefore, the numerical convergence problem of point source model can be overcome. An acoustic finite element method (FEM) model has been established for a three-dimensional rectangular muffler to verify the proposed model. The results indicate that the proposed analytical model can calculate TL of compressor's mufflers with comparatively simple geometries. Moreover, the new model is more computationally efficient than the existing surface and line source models.

1. INTRODUCTION

Pressure pulsations in the suction and discharge of compressors are undesirable because they can reduce a compressor's performance, induce pipeline vibrations, and lead to system failure. Reactive mufflers are generally used to attenuate pressure pulsations, since the small holes and porous materials of dissipative mufflers may eventually become clogged with deposits. TL is one of the most useful parameters to determine these mufflers' acoustic feature.

To obtain the mufflers' TL, modal series expansion method was first proposed by Kim and Soedel (1989) when they studied the gas pulsation in three-dimensional continuous cavities of a high speed hermetic compressor. Lai and Soedel (1996a) applied the same approach to analyze two-dimensional mufflers with uniform and non-uniform thicknesses. Gas pulsations in such mufflers have also been investigated (Lai and Soedel, 1996b). The abovementioned studies employed point source models to calculate the pressure response functions; however, these models might not converge for sound pressures at points near the point source. To overcome these convergence problems, a surface source model (Zhou and Kim, 1999) has been proposed for calculating the pressure response functions. Kadam and Kim (2007) obtained the four-pole parameters of a three-dimensional rectangular muffler using the surface source model and verified the results by experiment. In addition, Huang and Jiang (2007) used a circular line source model to calculate the four-pole parameters of two-dimensional rectangular mufflers but this model is computationally intensive.

In the present study, a revised point source model, which is more computationally efficient than the surface and line source models, is used to calculate the pressure response solutions of two-dimensional mufflers based on the method of modal expansion. The approximately averaged pressure response value, obtained by calculating the weighted average pressure response value in the vicinity of the point source, is applied to derive the pressure response functions and four-pole parameters by the new point model. The divergence problem of the original point model could be overcome. This new analytical point model is validated in comparison with the TL results obtained by three-dimensional (3-D) acoustic FEM.

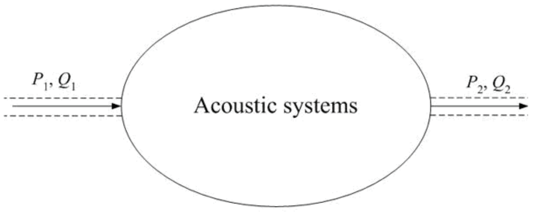

Figure 1: Schematic representation of acoustic system of a compressor's muffler.

2. TL FORMULATION BY THE NEW POINT MODEL

The transfer matrix defines the relationship between the input and output variables of an acoustic system in the frequency domain. For the acoustic system shown in Figure 1, the equations that relate the transfer matrix and the input/output variables can be expressed as

$$\begin{Bmatrix} Q_1 \\ P_1 \end{Bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{Bmatrix} Q_2 \\ P_2 \end{Bmatrix}, \tag{1}$$

A, B, C, and D are the four-pole parameters given as

$$A = \frac{f_{22}}{f_{21}},\ B = \frac{1}{f_{21}},\ C = -f_{12} + \frac{f_{11}}{f_{12}} f_{22},\ D = \frac{f_{11}}{f_{12}}, \tag{2}$$

where f_{ij} is the pressure response of the system at the point i when the system is subjected to the unit volume flow input at the point j while the other point is blocked.

In terms of the four-pole parameters, the TL of the system is defined [3] as

$$TL = 20\log 10\left\{(Z_2 / Z_1)^{1/2}\left|A(Z_2 / Z_1) + BZ_1 + C / Z_2 + D\right| / 2\right\} \tag{3}$$

where Z_1 and Z_2 are characteristic impedances of the inlet duct and outlet duct, respectively. For infinite-long ducts without reflection, $Z_1 = \rho c / S_1$ and $Z_2 = \rho c / S_2$. S_1 and S_2 are the cross area of the inlet and outlet duct, respectively. ρ and c are the density and the sound speed of the fluid, respectively.

As for the original point source model, the pressure response function of a two-dimensional (2-D) rectangular muffler can be obtained as

$$f_0(x, y, \omega) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{j\omega\rho Q c^2 p_{mn}(x_0, y_0) p_{mn}(x, y)}{hN_{mn}[(\omega_{mn}^2 - \omega^2) + 2\mathrm{j}\omega\omega_{mn}\xi_{mn}]} \tag{4}$$

where p_{mn} ω_{mn} and ξ_{mn} are the complex pressure modal amplitude, the circular eigen-frequencies and damping ratio of (m, n) mode, respectively.

However, this point model might have convergence problem at the source point (Zhou and Kim, 1999). A revised point source model has been proposed in order to overcome this problem.

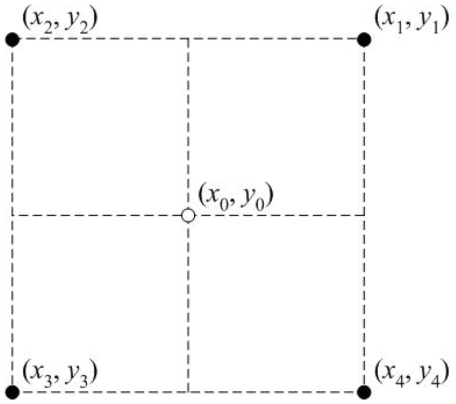

Figure 2: Revised point Sound source model.

Consider the revised point source model displayed in Figure 2. If the pressure response function is derived by using the pressure response value at the points on the source surface (not at the center point), the convergence problem at the center point can be overcome. Assuming the four corner points are chosen to formulate the pressure response function, the averaged pressure response can be obtained as

$$\overline{f_0}(x, y, \omega) = \frac{j\omega\rho Q c^2 \sum_{i=1}^{4} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \alpha_i p_{mn}(x_0, y_0) p_{mn}(x_i, y_i)}{h \sum_{i=1}^{4} \alpha_i N_{mn} \left[(\omega_{mn}^2 - \omega^2) + 2\mathrm{j}\omega\omega_{mn}\xi_{mn} \right]} \tag{5}$$

where α_i and (x_i, y_i) are the weights and the coordinates of the source corner points, respectively.

Since the pressure response value would be only divergent at the center of the source point, convergent pressure response value can be obtained by the revised point source model. Therefore, the pressure response function can be formulated accordingly and thus TL can be calculated.

3. FEM VERIFICATION

To verify the TL results of 2-D rectangular mufflers obtained by the revised point source model, a FEM model (depicted in Figure 3) was developed. A complete three-dimensional model of this muffler element was created with dimensions x = 0.1 m, y = 0.08 m, h = 0.01 m, and L = 0.006 m. The inlet and outlet pipes are located at (x, y) = (0.02, 0.03) and (0.08, 0.07), respectively. The refrigerant used by Lai and Soedel (1996a) is applied here; it has ρ = 6.04 kg/m^3 and c = 162.9 m/s. The TLs are calculated using the particle velocities and the inlet and outlet pressures obtained by FEM.

A mesh of 8385 tetrahedral elements is used to calculate the TL of the cylindrical thin muffler, which corresponding to about 20 node points per sound wavelength at the highest frequency used in this study (2000 Hz). It is confirmed that further refinement of the mesh does not give any noticeable difference in the TL results. The radiation plane wave boundary condition is applied to the inlet pipe in order to prevent pressure reflection. The impedance on the boundary at the outlet is specified to approximately simulate the anechoic termination condition. To make the pressure uniform across the section and to minimize the error involved in the averaging process, two very short pipes were added to the inlet/outlet ports of the cylindrical thin muffler's FEM model, as suggested by Zhou and Kim (1999). The TL calculation is conducted from 10 to 2000 Hz with an interval of 10 Hz. To compare the computational efficiencies, the TLs for this case were also calculated by the modal expansion method with the revised point model. Computations by both methods were performed on the same PC (CPU: Intel® Core™ 2.67 GHz). The CPU time for the modal expansion method using the square line source model for 200 frequency steps was 16 s, whereas the FEM model had a computational time of 196 s (from which the pre- and post-processing times have been excluded). Thus, the modal expansion method with the square line source model has a higher computational efficiency than that of FEM.

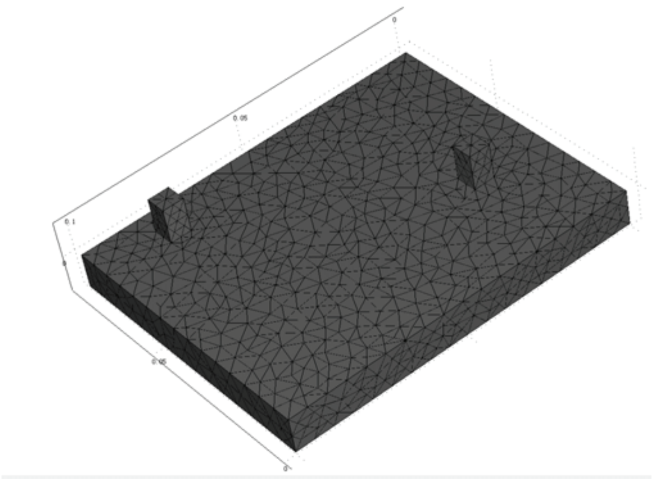

Figure 3: FEM model of a 2-D rectangular muffler.

Figure 4 shows that the TLs of a 2-D rectangular muffler calculated by the revised point model agree well with those obtained by the FEM model, especially at low frequencies. Small deviations are found from the TL results at frequencies above 1000 Hz because the analytical model of the modal expansion method for the muffler is two-dimensional in the present study, whereas that of the FEM method is inherently three-dimensional. Another reason for such deviations could be truncation errors, since the modal expansion method adopts a finite number of modes in simulating the contributions of an infinite number of modes.

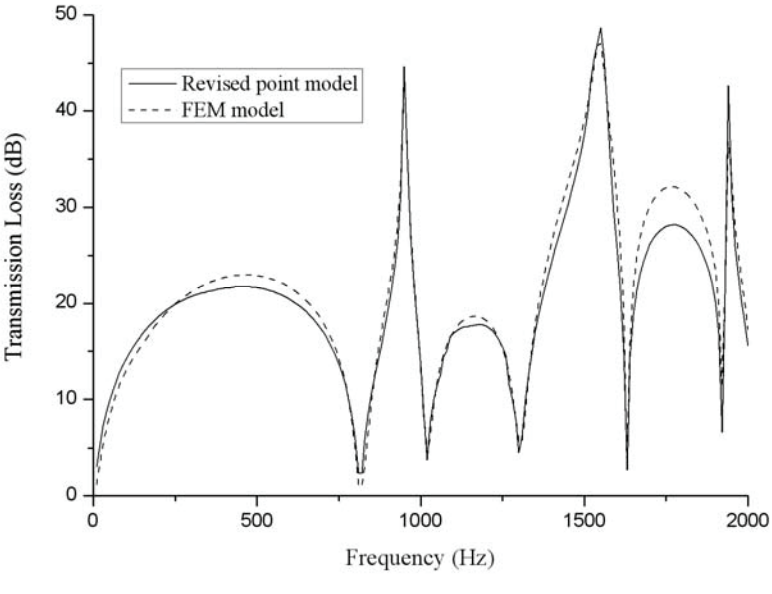

Figure 4: TL comparison of revised point model and FEM model

4. COMPARISONS

4.1 Comparison of revised point model and surface model

To prevent convergence problems, another approach is to implement sound source with dimension equal to or one less than that of the muffler element. Therefore, the square surface source model (Zhou and Kim, 1999) can be used to calculate TLs of two-dimensional mufflers. However, the surface model might produce a highly non-uniform pressure distribution, which may lead to dubious TL results. To compare the performance of the proposed model with that of the surface model, the TLs of a two-dimensional rectangular muffler are calculated using both models.

The dimension of the muffler is as the same as that used in section 3, but the inlet and outlet pipes are located at (x, y) = (0.02, 0.04) and (0.08, 0.04), respectively. The Figure 5 demonstrates that the TL results obtained by the two models have almost the same accuracies at most frequencies. However, there is a noticeable difference in the TL results in Figure 5(a) at frequencies of 810 Hz, 1640 Hz and 1720 Hz. the TL results at those frequencies were reexamined by FEM to investigate the accuracy of the two models. By comparing the TL results obtained by both models with that obtained by FEM, it is found that the revised point model is more accurate than the surface source model.

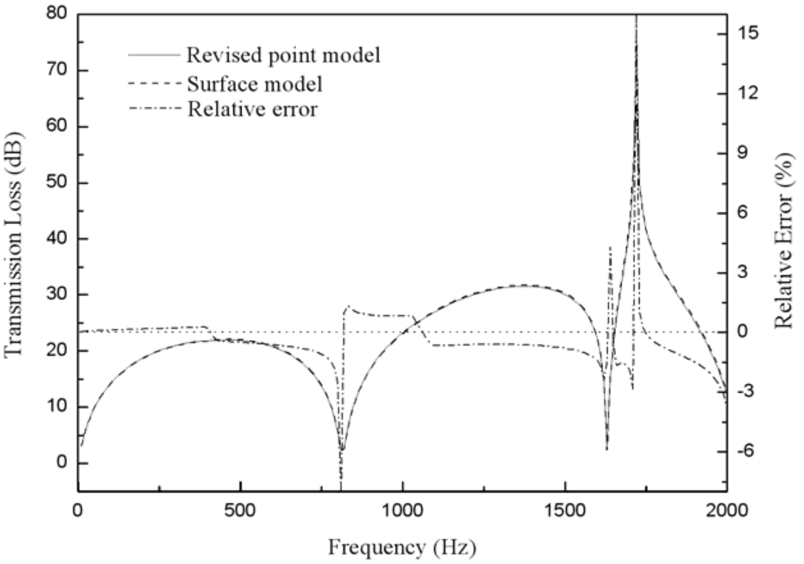

Figure 5: TL comparison of revised point model and square surface model

As for computation time, the averaged CPU time of four sets of inlet/outlet locations was 63 s for the surface model and 16 s for the revised point model. Both models used the same number of modes (40,000). The difference in their computational times can be explained by the different mathematical models adopted. It takes longer computational time for the surface model because the pressure over the whole square surface needs to be calculated to obtain the average value, whereas the proposed point model only calculate the averaged pressure on the corner points. Therefore, the revised point model is more convenient than the surface model.

4.2 Comparison of revised point model and circular line model

Huang and Jiang (2007) suggested using circular line model to obtain the pressure response functions. They found that the pressure responses determined by the circular line source model are more accurate than other models those determined by the square line source model due to the pressure distribution being much more uniform in the circular liner model. This finding can be explained by the perfect symmetry of the circular line model.

To compare the performance of revised point model and circular line model, the TLs of a 2-D rectangular muffler are calculated using both models on the same PC. Figure 6 compares the TL results obtained using the circular and square line source models. TLs of mufflers with the same dimension used in section 3 but with different inlet/outlet locations ((0.05, 0.03)/ (0.05, 0.07)) were examined. There are no obvious differences in the TL results obtained by these two models over the frequency range used in the calculation. Although the revised point source model uses the approximate pressure response value, it has almost the same accuracy as that of the circular line model. As for the computational time, the CPU time for the revised point model was 16 s, whereas that for the circular line source model was 362 s with same number of modes. The difference in the computational times can be explained by the different mathematical models they employ. The circular line source model requires more computational time because the numerical integration must be implemented for each mode when calculating the pressure response functions. Therefore, the revised point model is more computationally efficient than the circular line model. It is sufficiently accurate for calculating TLs.

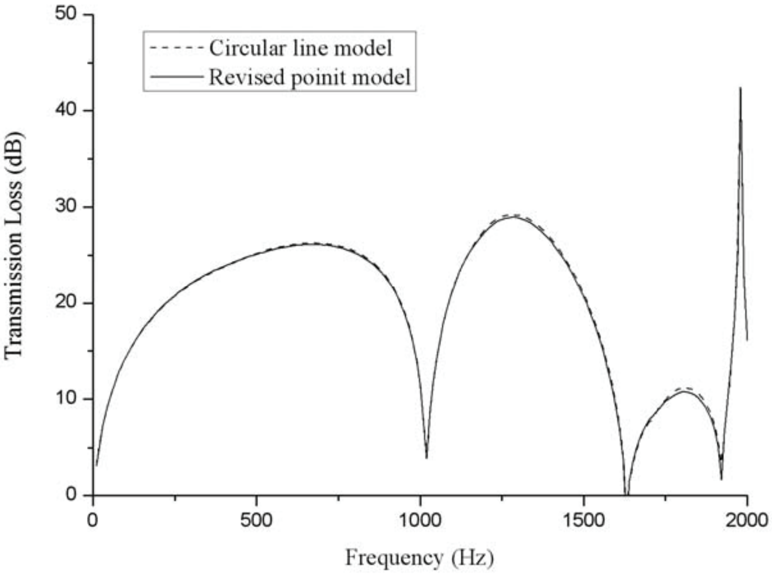

Figure 6: TL comparison of revised point model and circular line model

5. CONCLUSIONS

The modal expansion method with a revised point source model has been applied in the present study to calculate the TLs of two-dimensional thin mufflers. This new model does not suffer from the convergence problems of the original point source model. The pressure response functions of 2-D rectangular mufflers were obtained using the square line source model. The four-pole parameters were obtained and the TLs were calculated. The results obtained for a 2-D rectangular muffler using the proposed model were validated by comparison with the results obtained using FEM. The revised point model was more computationally efficient than FEM. This new model is sufficiently accurate in most cases and has a higher computational efficiency than the circular line model because of its simpler mathematical formula. The proposed point model has a higher computational efficiency and accuracy than the conventional square surface model.

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