Design Methodology for a High-Frequency Transformer in an Isolating DC-DC Converter

Veda Samhitha Duppalli

Purdue University

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DESIGN METHODOLOGY FOR A HIGH-FREQUENCY TRANSFORMER IN AN ISOLATING DC-DC CONVERTER

A Dissertation

Submitted to the Faculty

of

Purdue University

by

Veda Samhitha Duppalli

In Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy

May 2018

Purdue University

West Lafayette, Indiana
THE PURDUE UNIVERSITY GRADUATE SCHOOL
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To my grandparents Chintapalli Ranga Reddy and Saroja, 
and my mom, Chintapally Varalaxmi.
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ABSTRACT

Veda Samhitha Duppalli Ph.D., Purdue University, May 2018. Design Methodology for a High-Frequency Transformer in an Isolating DC-DC Converter. Major Professor: Scott D. Sudhoff.

The objective of this work is to propose a design methodology for a high-frequency transformer in the context of an isolating DC-DC converter. The design challenges relating to high-frequency operation include transformer parasitics, high-frequency loss mechanisms, and their coupled effect on the performance of the transformer and DC-DC converter. These issues are addressed in this work by incorporating high-frequency effects into the relevant magnetic, electrical, and thermal analyses needed for multi-objective optimization based design.

First, an accurate and computationally-efficient method for the transformer magnetic analysis is proposed. Leakage inductance is calculated using a procedure based on the Biot-Savart law and the method of mirror images. The method is validated for a prototype high-frequency transformer. Another key issue addressed is high-frequency transformer loss estimation. In this regard, the winding loss due to proximity effect is analyzed. A simplified time-domain model of the DC-DC converter is set forth to determine the transformer currents necessary for loss estimation. Next, analytical methods are used to estimate the parasitic capacitances in a transformer. The high-frequency transformer common-mode and differential mode impedances are derived. A transformer thermal analysis is set forth to estimate the temperature rise in the windings and to include the thermal effect on conductor material parameters. Finally, these analyses are coupled using a multi-objective optimization in order to create a new comprehensive and automated high-frequency transformer design paradigm.
1. INTRODUCTION

1.1 Objective and Motivation

Magnetic components in power converters include inductors and transformers. With the advancements in power semiconductor technology, there has been an increase in the operating frequency of power converters. The goal of this increase is to reduce the size of magnetic components and hence achieve higher power densities. To this end, conventional silicon IGBTs have facilitated tens of kilohertz switching frequencies. With the performance advances in SiC and GaN switching devices, however, operating frequencies have risen to 100 kHz or higher. Although there is significant size reduction in the magnetic component with high switching frequencies, it does not eliminate the need for these components.

Transformers are commonly used for isolation and to facilitate large changes in voltage levels in DC-DC converters. The transformer is necessary in case of medium to high power DC-DC converters to provide galvanic isolation. An advantage of high-frequency operation is that the size of the isolating transformer is reduced. However, there are several key issues to be addressed when designing the high-frequency transformer.

The first key issue is to accurately estimate the transformer leakage inductance. The leakage inductance impacts the operation of the DC-DC converter by effecting the transformer currents and consequently the transformer loses and also causes reactive voltage drop. Also, the transformer leakage inductance may be included in the converter operation as part of efforts to implement soft switching [1–7].

The second key issue is related to the transformer losses. Because of the higher operational frequency, skin effect and proximity effect losses can become significant. Skin effect losses are associated with the current distribution within the conductor.
which, at high frequencies, tends to become concentrated at the outside of the conductor resulting in increased loss. Proximity effect loss is associated with eddy currents in each conductor caused by the exposure of each conductor to a time-varying magnetic field. For given current waveforms, skin effect losses are readily calculated [8]. However, calculation of proximity effect losses is more involved, and requires knowledge of the magnetic field distribution within each winding bundle [9–12].

The third key issue is that the transformer loss calculations require detailed knowledge of the current waveforms. This is true for both skin effect and proximity effect losses. In general, this requires an integrated electro-magnetic analysis of the power converter. This could be accomplished, for example, by a time domain circuit simulation which includes a magnetic analysis as part of the state model. An alternate approach is to decouple the magnet and electrical analysis, which allows for a high-speed waveform level simulation to calculate the current waveforms. The former approach is more general and more accurate; the latter approach is more computationally expedient, at least in terms of a design model.

The fourth key issue is the parasitic capacitance associated with the transformer windings that become significant at high-frequency operation. The capacitances associated with the transformer windings may result in the common-mode current conduction [13]. Also, the parasitic capacitance along with inductances may cause ringing [14] or unwanted overshoots [15,16] in the transformer currents, causing additional transformer loss.

Another key issue is to account for the thermal dependency of the transformer material properties. The transformer loss result in temperature rise that may be estimated using thermal analysis similar to [17].

The objective of this work is to propose an automated design methodology of a high-frequency transformer which addresses all of these key issues.

Research on the design of high-frequency transformer has mainly included treatment of one or two of these issues while partial or no treatment of others. The design methodology presented in [18] uses simplified design equations to address the
high-frequency losses and temperature rise in the windings while maximizing the high-frequency transformer efficiency. The transformer leakage inductance or parasitic capacitances were not considered while analyzing the transformer performance. In [19], the design methodology is again based on simplified design equations but with a slightly different objective function, where transformer volume and power loss are integrated into a single objective. The design analysis in this case also did not include the high-frequency transformer parasitics. In [20], the high-frequency transformer design with an application in induction heating system is presented but lacks thorough analysis of the transformer parasitics. In [21], the high-frequency transformer with a shell-type topology is designed to be utilized in Dual Active Bridge (DAB) converter, but the design process did not include optimization. Additionally, these four works included traditional method of maximizing the core window utilization to minimize the core size. However, with the high-frequency transformer parasitics not included, it is unclear if core window area can be used as a characteristic parameter to determine the core size.

A comprehensive design methodology for a shell-type transformer in the context of a DAB converter is presented in [22]. Therein, an optimization process with a wider range of parameters is considered, incorporating high-frequency loss analysis, isolation requirements, and thermal analysis. The outlined design process, however, did not include the transformer parasitic capacitances. Also, a one-dimensional magnetic analysis was used to derive the leakage inductance of the shell-type transformer.

The research presented in [23] focuses on core material characteristics along with transformer parasitics to achieve high-power density resonant converter. Here again, one-dimensional electrical and magnetic analyses are used to calculate the winding AC losses and leakage inductance in a shell type topology. This work illustrated high-frequency transformer design as an application in a parallel resonant converter (PRC), where design equations based on detailed analysis are used to design a core-type transformer. The applicability of 1D analyses for a core-type transformer is not
justified. Also, the design methodology included the selection of minimum size for the core based on design equations, without performing rigorous optimization.

The design aspects of the high-frequency transformer as an application in the resonant converter are explored in [24]. Therein, a core-type transformer is illustrated for MW-class resonant converter with emphasis on mechanical and electrical stresses. In this example, the transformer is designed using simplified design equations that lack a detailed analysis.

Recently, there has been an increased interest in optimizing the high-frequency transformer in the context of the DC-DC converter [25–28], rather than a standalone device. This is because, with the significant improvements in wide-bandgap semiconductor technology, the focus now is to achieve the soft switching requirements of the converter by integrating the high-frequency transformer characteristics into the converter operation. All of these designs are either based on design equations, lack detailed magnetic, electric, and thermal analyses or absent of rigorous design methodology. Besides, although high-frequency transformers are used in isolating unidirectional DC-DC converter, there has been minimal attention paid to the high-frequency transformer design in the context of a unidirectional converter. In case of applications where bidirectional power flow is not required, unidirectional converters, for example the Isolating Converter Module (ICM) [29], offer simple and cost-effective solution compared to a DAB or resonating converters that utilize additional switches to implement soft switching. Similar to DAB or resonating converters, successful operation of unidirectional converter depends on the transformer parasitics. Therefore, the focus of this research is to propose a design methodology for the high-frequency transformer in the context of a unidirectional converter. Moreover, depending on the application, the proposed design methodology can be readily modified to include a different converter topology.
1.2 System Description

In this research, a high-frequency core-type transformer is designed in the context of an isolating DC-DC converter. The topology of an example isolating DC-DC converter considered for this work and geometry of the transformer are described below.

1.2.1 DC-DC Converter

A single-phase DC-DC converter topology, which will be referred to herein as an Isolating Converter Module (ICM) [29], is shown in Figure 1.1. The ICM offers simple implementation of a unidirectional DC-DC conversion with a high-frequency transformer to achieve galvanic isolation. It has LC filters on the input and output side to filter the high-frequency ripple caused due to switching. The H-bridge inverter is used to regulate the output voltage. A high-frequency transformer that connects the inverter output on the primary side to a full bridge diode rectifier on the secondary side is the focus of this research. A parallel RC snubber is connected across the rectifier output to suppress the ringing in the voltage waveform after rectification. A DC link inductor is connected in series to the rectifier output to reduce the output current ripple.

Fig. 1.1. Isolating converter module
The primary function of the ICM is to provide isolation and regulate the DC output voltage using the inverter switching. A dual loop (PI based) control is utilized to obtain the desired duty cycle for the H-bridge inverter switching.

The ICM operation along with a time domain simulation model will be described in Chapter 4.

1.2.2 Core-Type Transformer

Depending on the core structure and windings arrangement, different transformer architectures are possible. Two broad categories are core-type and shell type. Due to its advantage of lower leakage inductance and better thermal parameters, a core-type architecture is chosen over shell type architecture in this work.

The core type transformer analyzed in this research is shown in Figure 1.2. There in, the dark (gray) region is the transformer core, the lightly shaded (yellow) region is the secondary winding, and the moderately shaded (orange) region is the primary winding. The lower voltage winding is wound closer to the core. In this work, the secondary winding is assumed to be of lower voltage. There are two coils for each of primary and secondary windings, resulting in two sets of four coil regions. In Figure 1.2, the four distinct coil regions are the primary coil in the interior region denoted by $p_i$, secondary coil in the interior region denoted by $s_i$, primary coil in the exterior region denoted by $p_e$ and secondary coil in the exterior region denoted by $s_e$.

Fig. 1.2. Core type transformer cross section
The top view of transformer end leg with primary and secondary winding is shown in Figure 1.3. Note the region shown in brown color is an inert material used to provide necessary bend radius for the winding conductors.

![Transformer top view](image)

Fig. 1.3. Transformer top view

1.3 Design Approach

A high-frequency transformer design to be utilized in a DC-DC converter needs to fulfill a variety of specifications relating to its geometry and magnetic, electrical, and thermal performances. A sequential design approach addresses these specifications by using design equations based on detailed analysis. This step often involves several approximations and requires prior experience. The designs obtained are then validated using numerical analysis or physical experiment. Based on the validation results, several revisions may be required to obtain designs that satisfy all the specifications. As a result, this approach can be inefficient, especially when multiple objectives are of interest. In the end, even when all the specifications are met, this approach yields working designs and not necessarily the best possible design.

An alternate approach to efficiently optimize competing objectives such as mass and loss, is by using multi-objective optimization, similar to the work in [30]. Through this approach, the magnetic, electrical, and thermal analyses are incorporated into a fitness function which is based on metrics of interest such as mass, loss, as well as the satisfaction of the various constraints. The output of the multi-objective opti-
mization is a set of non-dominated designs referred to as the Pareto-optimal front. The advantage of this approach is that it can be semi-automated, it requires little design experience, and that the correlation between the high-frequency transformer attributes such as leakage inductance and parasitic capacitance, loss, and mass can be observed.

1.4 Thesis Organization

The transformer magnetic analysis is addressed in Chapter 2. A new method to calculate the frequency dependent leakage inductance of the transformer is outlined in this chapter. The estimation of transformer losses is presented in Chapter 3. The time-domain analysis of the DC-DC converter is set forth in Chapter 4. The transformer parasitic capacitances are derived using analytical methods in Chapter 5. The transformer thermal analysis is given in Chapter 6. The high-frequency transformer design methodology utilizing magnetic, electric, and thermal analyses is presented in Chapter 7. In Chapter 8, the contributions of the proposed high-frequency transformer design methodology is illustrated by comparing four different case studies. A summary and suggestions for future work are described in Chapter 9.
2. MAGNETIC ANALYSIS

In this chapter, a magnetic analysis is set forth for the core-type transformer described in Section 1.2.2. This involves establishing methods to estimate parameters to model transformer magnetic behavior and calculate the core flux density. The proposed method for transformer magnetic analysis is accurate, computationally efficient and is suitable for optimization based transformer design.

The core-type transformer cross-section along with the variables used to denote dimensions of the core and the windings are illustrated in Figure 2.1. The four distinct coil regions are the primary coil in the interior region denoted by $p_i$, secondary coil in the interior region denoted by $s_i$, primary coil in the exterior region denoted by $p_e$ and secondary coil in the exterior region denoted by $s_e$.

![Fig. 2.1. Core type transformer cross section with dimensions](image)

The top view of core-type transformer end leg with primary and secondary windings is as shown in Figure 2.2. As shown in the Figure 2.2(a), the transformer core end-leg has corners with rounding radius, $r_c$ and width, $w_{ecc}$. In this case, the core end-leg corners may be used to provide sufficient bend radius for the conductors. All the variables used to denote transformer geometry and winding conductors are listed along with their descriptions in Table 2.1.
Fig. 2.2. Top view of the transformer core end-leg with rounded corners

Table 2.1.: Transformer dimensions listing

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
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<tbody>
<tr>
<td>$w_{cs}$</td>
<td>Width of the core slot</td>
</tr>
<tr>
<td>$d_{cs}$</td>
<td>Depth of the core slot</td>
</tr>
<tr>
<td>$w_c$</td>
<td>Width of the core</td>
</tr>
<tr>
<td>$d_c$</td>
<td>Depth of the core</td>
</tr>
<tr>
<td>$l_c$</td>
<td>Length of the core</td>
</tr>
<tr>
<td>$w_{ce}$</td>
<td>Width of the core end leg</td>
</tr>
<tr>
<td>$w_{cb}$</td>
<td>Width of core/base top leg</td>
</tr>
<tr>
<td>$c_{sc}$</td>
<td>Horizontal clearance of secondary winding to core</td>
</tr>
<tr>
<td>$c_{ps}$</td>
<td>Horizontal clearance between primary and secondary windings</td>
</tr>
<tr>
<td>$c_{pp}$</td>
<td>Horizontal clearance between primary winding coils</td>
</tr>
<tr>
<td>$c_{pv}$</td>
<td>Vertical clearance between primary winding and core</td>
</tr>
<tr>
<td>$c_{sv}$</td>
<td>Vertical clearance between secondary winding and core</td>
</tr>
<tr>
<td>$N_{pep}$</td>
<td>Number of primary coils in parallel</td>
</tr>
<tr>
<td>$N_{pl}$</td>
<td>Number of layers in primary coil</td>
</tr>
<tr>
<td>$N_{ptl}$</td>
<td>Number of turns in each primary coil layer</td>
</tr>
<tr>
<td>$\sigma_{cp}$</td>
<td>Conductivity of primary material</td>
</tr>
<tr>
<td>$a_{pc}$</td>
<td>Area of primary coil conductor</td>
</tr>
<tr>
<td>$r_{pc}$</td>
<td>Radius of primary coil conductor</td>
</tr>
<tr>
<td>$t_{pins}$</td>
<td>Thickness of primary conductor insulation</td>
</tr>
<tr>
<td>$d_{pw}$</td>
<td>Depth of primary winding coil</td>
</tr>
<tr>
<td>$w_{pw}$</td>
<td>Width of primary winding coil</td>
</tr>
<tr>
<td>$r_{pi}$</td>
<td>Inner radius of primary winding</td>
</tr>
<tr>
<td>$r_{po}$</td>
<td>Outer radius of primary winding</td>
</tr>
<tr>
<td>$l_{pc}$</td>
<td>Straight length of primary winding</td>
</tr>
<tr>
<td>$k_{spf}$</td>
<td>Secondary winding packing factor</td>
</tr>
</tbody>
</table>

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Table 2.1.: continued

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{scp}$</td>
<td>Number of secondary coils in parallel</td>
</tr>
<tr>
<td>$N_{sl}$</td>
<td>Number of layers in secondary coil</td>
</tr>
<tr>
<td>$N_{stl}$</td>
<td>Number of turns in each secondary coil layer</td>
</tr>
<tr>
<td>$\sigma_{cs}$</td>
<td>Conductivity of secondary material</td>
</tr>
<tr>
<td>$a_{sc}$</td>
<td>Area of secondary coil conductor</td>
</tr>
<tr>
<td>$r_{sc}$</td>
<td>Radius of secondary coil conductor</td>
</tr>
<tr>
<td>$t_{sins}$</td>
<td>Thickness of secondary conductor insulation</td>
</tr>
<tr>
<td>$d_{sw}$</td>
<td>Depth of secondary winding coil</td>
</tr>
<tr>
<td>$w_{sw}$</td>
<td>Width of secondary winding coil</td>
</tr>
<tr>
<td>$r_{si}$</td>
<td>Inner radius of secondary winding</td>
</tr>
<tr>
<td>$r_{so}$</td>
<td>Outer radius of secondary winding</td>
</tr>
<tr>
<td>$l_{sc}$</td>
<td>Straight length of secondary winding</td>
</tr>
<tr>
<td>$k_{spf}$</td>
<td>Secondary winding packing factor</td>
</tr>
<tr>
<td>$r_{w}$</td>
<td>Radius of semi-circular inert material used for support</td>
</tr>
</tbody>
</table>

The magnetic analysis approach is different for the core interior and exterior regions. Hence, it will be convenient to define the coil mean turn length in both regions. The dotted line shown in windings in Figure 2.2(b), indicate the mean turn length. The point where coil center-dotted line crosses the core edge differentiates the coil in the interior region to the exterior region. The angle at this point is given by

$$\alpha_x = \arcsin \left( \frac{2r_c}{r_{xi} + r_{xo}} \right)$$

(2.1)

The average turn length of the exterior and interior coil may be expressed as

$$l_{xi} = l_c + \alpha_x (r_{xi} + r_{xo})$$

(2.2)

$$l_{xe} = l_c + 2w_{csc} + (\pi - \alpha_x) (r_{xi} + r_{xo})$$

(2.3)

where the winding inner radius, $r_{xi}$ and outer radius, $r_{xo}$ are as illustrated in Figure 2.2(b).

In many cases, the transformer end leg may have sharp corners, in which case, an inert material may be used to provide necessary bend radius for the winding
conductors as shown in Figure 2.3. The proximity of the windings to the core is different in this case as compared to the geometry shown in Figure 2.2. The distance between the secondary winding and the core along the core length $l_c$ is different from the clearance along the core end-leg width, $w_{ce}$. The magnetic analysis outlined in this chapter addresses the resulting end-winding effect by accounting for the varying distance between the core end-leg and windings. For this reason, the transformer exterior region is segregated into four distinct regions as shown in Figure 2.4. The coil exterior region 1, $R_{e1}$, has constant distance, $c_{sc}$, between the core end leg and secondary coil (along length, $l_c$). The coil exterior region 2, $R_{e2}$, has constant distance $c_{sc} + r_w$ between the core end leg and secondary coil (along width, $w_{ce}$). The coil exterior regions 3, $R_{e3}$, and 4, $R_{e4}$, have distance between the core end-leg and coil varying from $c_{sc}$ to $c_{sc} + r_w$ (along curvature). The difference between $R_{e3}$ and $R_{e4}$ is their proximity to core inner region. The lengths of these coil regions are listed in Table 2.2.

The proposed magnetic analysis uses (2.2)-(2.1) or lengths in Table 2.2 to include the 3-Dimensional (3D) geometry of the transformer.

As depicted in Figure 2.1, each winding has two coils present. These coils are connected either in series or parallel depending on the voltage/current specification of the transformer. Assigning $N_{xcs}$ a value 1 in case of parallel connection of the coils.
Fig. 2.4. Top view of the transformer core end-leg with exterior region partitions

Table 2.2.
Core-type transformer winding interior and exterior regions lengths

<table>
<thead>
<tr>
<th>Winding</th>
<th>Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interior region</td>
<td>$l_{xi} = l_c$</td>
</tr>
<tr>
<td>Exterior region 1</td>
<td>$l_{xe1} = l_c$</td>
</tr>
<tr>
<td>Exterior region 2</td>
<td>$l_{xe2} = w_{ce}$</td>
</tr>
<tr>
<td>Exterior region 3</td>
<td>$l_{xe3} = \pi (r_{xi} + r_{xo})/4$</td>
</tr>
<tr>
<td>Exterior region 4</td>
<td>$l_{xe4} = \pi (r_{xi} + r_{xo})/4$</td>
</tr>
</tbody>
</table>

and 2 in case of series connection, the relation between the number of turns in each winding, $N_x$ and the number of turns in each coil,$N_{xcl}$ is given by

$$N_x = N_{xcs} N_{xcl}$$ (2.4)

Accordingly, the conductor current in each coil, $i_{xcl}$ is given by

$$i_{xcl} = \frac{i_x}{N_{xep}}$$ (2.5)

The conductor arrangement within a coil of width $w_{xw}$ and depth $d_{xw}$ is illustrated in Figure 7.2. Therein, the parallel strands in a conductor, denoted by $N_{xpr}$, are arranged as a single strand width rectangle, with length parallel to the core depth.
Number of turns in each coil is denoted by $N_{xcl}$. The number of layers and the number of turns per layer in a coil are respectively denoted by $N_{xl}$ and $N_{xtl}$. The radius and area of each strand are denoted by $r_{xs}$ and $a_{xs}$ respectively.

![Fig. 2.5. Coil cross-section depicted conductor arrangement](image)

The magnetic analysis of the transformer begins with describing Magnetic Equivalent Circuit (MEC) in Section 2.1. The magnetic field analysis proposed for estimating leakage inductance is presented in Section 2.2, followed by study of frequency dependency of leakage inductance in Section 2.3. The validation of proposed magnetic analysis is presented in Section 2.4.

### 2.1 Magnetic Equivalent Circuit

The MEC analysis is useful in estimating quantities such as the magnetizing current and the flux densities in the core to determine core loss. An MEC for a core type transformer is shown in Figure 2.6. Therein, each of the four coils of the core type transformer is represented by a magneto motive force (MMF) source, $N_{xcl}i_{xcl}$. The leakage flux in the windings is represented using two quantities, primary leakage permeance, $R_{pd}$ and secondary leakage permeance, $R_{sl}$. Compared to the transformer MEC described in [31], the leakage permeances in Figure 2.3 are simplified into just two quantities. The simplification assumed here is reasonable in that the effect of leakage inductance on flux density in the core is modest. The core end-leg reluctance
$R_{el}(\Phi)$ and base-leg reluctance $R_{bl}(\Phi)$ are defined as a function of flux $\Phi$ through the core

\[ R_{el}(\Phi) = \frac{l_{el}}{A_{el}\mu_B(\Phi)} \tag{2.6} \]
\[ R_{bl}(\Phi) = \frac{l_{bl}}{A_{bl}\mu_B(\Phi)} \tag{2.7} \]

where the length and area of cross section of the end leg and base leg are given by

\[ l_{el} = d_{cs} + w_{eb} \tag{2.8} \]
\[ l_{bl} = w_{cs} + w_{ce} \tag{2.9} \]
\[ A_{el} = \begin{cases} w_{cc}l_{cc} + 2w_{ccc}r_c + \pi r_c^2 & \text{Figure 2.2} \\ w_{ce}l_c & \text{Figure 2.3} \end{cases} \tag{2.10} \]
\[ A_{bl} = w_{eb}l_c \tag{2.11} \]

Magnetic permeability of the core, $\mu_B(\Phi)$ in (2.6) and (2.7) is defined as a non-linear function of flux, $\Phi$ in order to model magnetic saturation.

![Diagram](image)

Fig. 2.6. MEC of core-type transformer
Using symmetry, the MEC in Figure 2.6 can be simplified into the circuit shown in Figure 2.7. The MMF drops are equated to the source MMF, given by

\[ N_{pcl}i_{pcl} + N_{scl}i_{scl} = \Phi (R_{bl} (\Phi) + R_{el} (\Phi)) \]  

(2.12)

To solve for \( \Phi \), an error function \( f(\Phi) \) is defined as

\[ f(\Phi) = \Phi (R_{bl} (\Phi) + R_{el} (\Phi)) - N_{pcl}i_{pcl} + N_{scl}i_{scl} \]  

(2.13)

The derivative of \( f(\Phi) \) with respect to \( \Phi \) may be expressed as

\[ f' (\Phi) = R_{bl} (\Phi) + R_{el} (\Phi) - \Phi \left( \frac{l_{el}}{A_{el}\mu_{B}^2(\Phi)} + \frac{l_{bl}}{A_{bl}\mu_{B}^2(\Phi)} \right) \frac{d\mu_{B}(\Phi)}{d\Phi} \]  

(2.14)

The Newton-Raphson method is used to solve \( f(\Phi) = 0 \) using (2.14) and non-linear definition of \( \mu_{B}(\Phi) \).

Once the flux through the core is determined, the flux density in base and end legs are calculated using

\[ B_{el} = \frac{\Phi}{A_{el}} \]  

(2.15)

\[ B_{bl} = \frac{\Phi}{A_{bl}} \]  

(2.16)
Flux densities in the core are later used to estimate core loss using Modified Steinmetz Equation (MSE). The transformer MEC shown in Figure 2.7 is validated using a prototype transformer in Section 2.4.4.

One of the magnetic parameters used to the model transformer is magnetizing inductance. Using the core branch reluctances defined in (2.6) and (2.7), the magnetizing inductance as seen from primary when the transformer is operating in the linear region is given by

\[ L_{m0} = \frac{N_p^2}{2 (R_{el}(\Phi = 0) + R_{bl}(\Phi = 0))} \]  \hspace{1cm} (2.17)

2.2 Leakage Inductance Calculation using Magneto-Static Analysis

In this section, a new method to estimate leakage inductance of a core-type transformer is proposed that is a mixture of analytical and numerical analysis. The approach provides an accurate estimate of the leakage inductance while reducing the time and effort required.

A common way to model the transformer electrical behavior is by using T-equivalent circuit, as shown in Figure 2.8. Here, the primary and secondary winding resistances are denoted by \( r_p \) and \( r_s \) respectively, and the leakage inductances of the windings is denoted by \( L_{lp} \) and \( L_{ls} \). In Figure 2.8, the transformer secondary side quantities are referred to the primary side and are denoted by a prime. The referred secondary voltage and secondary current are defined as

\[ v_s' = \frac{N_p}{N_s} v_s \]  \hspace{1cm} (2.18)

\[ i_s' = \frac{N_s}{N_p} i_s \]  \hspace{1cm} (2.19)

The secondary winding resistance and secondary leakage inductance are referred to primary side as

\[ r_s' = \left( \frac{N_p}{N_s} \right)^2 r_s \]  \hspace{1cm} (2.20)
\[ L'_{ts} = \left( \frac{N_p}{N_s} \right)^2 L_{ts} \] (2.21)

The magnetizing inductance as seen from primary is denoted by \( L_m \). The current through magnetizing inductance, \( i_m \) is given in terms of the primary current, \( i_p \) and secondary current, \( i_s \) as

\[ i_m = i_p + i'_s \] (2.22)

From the transformer T-equivalent circuit, the energy stored in the transformer is given by

\[ E = \frac{1}{2} (L_{tp} + L_m) i_p^2 + L_m i_p i'_s + \frac{1}{2} (L'_{ts} + L_m) i'_s^2 \] (2.23)

When the transformer winding currents are such that the magnetizing current is zero, that is

\[ i'_s = -i_p \] (2.24)

energy of the transformer is stored in leakage inductances alone. In this case, (2.23) reduces to

\[ E = \frac{1}{2} (L_{tp} + L'_{ts}) i_p^2 \] (2.25)

Alternately, field energy of a magnetic component under magnetically linear conditions can be calculated using the expression [31]

\[ E = \frac{1}{2} \mu \int \int H^2 dv \] (2.26)
where \( H \) denotes magnetic field intensity and \( \mu \) denotes magnetic permeability in the region \( V \). Based on the knowledge of flux distribution in and around the transformer for the zero-magnetizing current excitation, \( H \) can be estimated and hence used to calculate the transformer energy. The calculation of \( H \) will also prove useful in the calculation of proximity effect losses. In this work, the transformer energy for the zero magnetizing current case is calculated using (2.26) first and then equated to (2.25) in order to estimate the transformer total leakage inductance as referred to the primary side, expressed as \( L_{lp} + L'_{ls} \).

To calculate field energy using (2.26) the core type transformer can be divided into three regions, the core interior, the core itself, and the core exterior. The sum of the energies of the three regions gives the total energy of the transformer. The energy in the core is negligible when the magnetizing current is zero. This is because the field due to both the coils cancels out in the core. Furthermore, since the core permeability is very high, the field intensity and hence the energy is very small. Considering this fact, the magnetic field needs to be analyzed only in the core exterior and interior regions for the case of zero-magnetic current excitation. Since both the exterior and interior region consists of magnetically linear materials, (2.26) may be used to calculate the energy in these two regions.

For the purpose of illustration, the magnetic analysis for zero magnetizing case is investigated using 2-dimensional (2D) Finite Element Analysis (FEA) of a core type transformer. In particular, the FEA is performed for the case of prototype transformer using ANSYS Maxwell 2015.2. The dimensions of the prototype transformer are listed in Table 2.5. The winding current excitation is chosen such that magnetizing current is zero, thus

\[
N_p i_p = -N_s i_s \quad (2.27)
\]

The energy density plot as shown in Figure 2.9 conforms with the assumption that the core energy is negligible. The FEA analysis resulted in leakage flux paths as shown in Figure 2.10. Examining the flux paths through the coil regions closely, the fact that primary and secondary coil regions carry currents in opposite direction influences the
leakage flux paths largely resulting in the combined effect of both the windings in the resulting flux pattern.

On examining the coil regions in the interior region, the leakage flux path can be assumed to be vertical. The flux lines in the interior region are fit using simple geometry allowing analytical calculation of energy in the interior region using Amperes law. Unlike the core interior region, leakage flux paths in the core exterior region are complicated, making it difficult to establish a highly accurate analytical approach. For this reason, a numerical method is used for field analysis in the core exterior region.

In the following subsection, magnetic analysis of the interior and exterior regions is presented.

2.2.1 Interior Region Analysis

The energy in the interior region is calculated in this section using an analytical approach. The flux in the core interior region is due to the interior coil regions $s_i$ and
Fig. 2.10. Leakage flux paths as obtained using 2D FEA for zero-magnetizing current excitation

$p_i$ and is assumed to follow path P as shown in Figure 2.11. Also, note the definition of $x$. Using Ampere’s law, the field intensity, $H$ is given by

$$H = \frac{i_{enc}}{l_p}$$  \hspace{1cm} (2.28)$$

In (2.28), length of the flux path, $l_p$ in the interior region is equal to twice the depth of the slot, $2d_{cs}$. Notice the flux path through the core is neglected. This is because the permeability of the core is much higher than the air and conductor materials, therefore resulting in negligible magneto motive force (MMF) drop along the path through the core. The current enclosed by the path varies with $x$ as

$$i_{enc}(x) = \begin{cases} 
0 & 0 < x < k_0 \\
N_p i_p (x - c_{pp}/2)/w_{pw} & k_0 < x < w_{pw} + k_0 \\
N_p i_p & w_{pw} + k_0 < x < k_1 \\
N_p i_p + N_s i_s (x - k_1)/w_{sw} & k_1 < x < k_1 + w_{sw} \\
N_p i_p + N_s i_s & k_1 + w_{sw} < x < w_{cs}/2
\end{cases}$$  \hspace{1cm} (2.29)$$
where \( k_0 = \frac{c_{pp}}{2} \) and \( k_1 = w_{pw} + \frac{c_{pp}}{2} + c_{ps} \).

For a test current of magnitude, \( I_{p,t} \) through the primary winding and zero-magnetic current excitation, the field intensity, \( H \) in the interior region can be evaluated using (2.27), (2.28) and (2.29) as

\[
H(x) = \begin{cases} 
0 & 0 < x < k_0 \\
\frac{N_p I_{p,t} (x - \frac{c_{pp}}{2})}{2d cs w_{pw}} & k_0 < x < w_{pw} + k_0 \\
\frac{N_p I_{p,t}}{2d cs} & w_{pw} + k_0 < x < k_1 \\
\frac{N_p I_{p,t} (w_{sw} - x + k_1)}{2d cs w_{sw}} & k_1 < x < k_1 + w_{sw} \\
0 & k_1 + w_{sw} < x < \frac{wcs}{2} 
\end{cases} \quad (2.30)
\]

Fig. 2.11. Leakage flux path in the core interior region

By using (2.26), the energy of core interior region is

\[
E_i = \frac{1}{2} \mu_0 l_i \int_{a}^{\frac{2\pi}{2}} H^2(x) da \quad (2.31)
\]

where \( l_i \) denotes the length of the interior region, taken as mean of the lengths of interior primary and secondary coil regions expressed as

\[
l_i = \frac{(l_{pi} + l_{si})}{2} \quad (2.32)
\]
On integrating using \( da = 2d_{cs} dx \), the expression in (2.31) reduces to

\[
E_i = \frac{\mu_0 N_p^2 I_{p,t} I_i}{12d_{cs}} (w_{pw} + 3c_{ps} + w_{sw}) \tag{2.33}
\]

To set the stage for proximity effect loss calculation, the field quantities relating to the calculation of loss due to proximity effect are established here by using the same magnetic analysis. The field intensity in the winding coil regions is normalized by primary current as

\[
\hat{H} = \frac{H}{I_{p,t}} \tag{2.34}
\]

The mean squared normalized field intensity \( \langle \hat{H}^2 \rangle \) over the winding region \( r \) is defined as

\[
\langle \hat{H}^2 \rangle = \frac{1}{A_r} \int_r \hat{H}^2 da \tag{2.35}
\]

Using (2.30), (2.34) and (2.35), the normalized squared field intensities for the interior primary and secondary coil regions are derived as

\[
\langle \hat{H}^2 \rangle_i = \frac{N_p^2}{12d_{cs}^2} \tag{2.36}
\]

\[
\langle \hat{H}^2 \rangle_i = \frac{N_p^2}{12d_{cs}^2} \tag{2.37}
\]

Note that the field intensity is normalized using primary current, regardless of the primary or secondary coil regions.

In case of core exterior region, a numerical analysis based on Biot-Savart law is introduced. The numerical method proposed is outlined in the following sub-section first.

### 2.2.2 Numerical Analysis based on Biot-Savart Law

As mentioned previously, the flux paths in the exterior region are complicated. It is difficult to analytically derive the field using simple geometry for the leakage flux
paths. FEA or Boundary Element Method are alternatives that can be used instead. However, these methods are computationally expensive and problematic when analyzing $10^4 - 10^7$ designs in the context of optimization based design. Alternately, a simplified numerical analysis of the transformer is proposed by taking advantage of homogeneity in the shape of the core and windings cross sections.

Starting with Biot-Savart law, the magnetic flux density due to line current, $i$ is given by

$$B(r) = \frac{\mu_0 i}{4\pi} \int \frac{d\mathbf{l}' \times \hat{r}}{r^2}$$

(2.38)

where $d\mathbf{l}'$ is an element of length along the wire, $\mu_0$ is the magnetic permeability of air and $\mathbf{r}$ is the vector from the current source to point of interest $P$ as shown in Figure 2.12.

![Fig. 2.12. Flux density due to line current using Biot-Savart law](image)

Applying Biot-Savart law to the case of line current $i$ (denoted by point source in Figure 2.13), the magnitude of magnetic field density at a point $P$ in the 2D space outside of the conductor cross section may be expressed as

$$B(r) = \frac{\mu_0 i}{2\pi r}$$

(2.39)

As shown in Figure 2.13, the direction of the field at $P$ is tangential to the circle with radius $r$ centered around the point source. Assuming the Cartesian coordinates of
the current source as \( (x_s, y_s) \) and that of the observation point \( P \) as \( (x_o, y_o) \), the field may be expressed in terms of Cartesian components as

\[
B_x = -\frac{\mu_0 i (y_o - y_s)}{2\pi \left((x_o - x_s)^2 + (y_o - y_s)^2\right)} \\
B_y = \frac{\mu_0 i (x_o - x_s)}{2\pi \left((x_o - x_s)^2 + (y_o - y_s)^2\right)}
\]

(2.40)  

(2.41)

Using (2.40) and (2.41), it is possible to calculate the flux density at any point in the 2D plane, except at the source point. In the proposed numerical analysis, this concept is extended to find the field density due to rectangular coil across 2D space as of coil’s cross-section.

Consider the case of rectangular coil containing \( N \) conductors with each conductor carrying current \( i \). If the conductors are tightly wound, a uniform current density can be assumed across the coil cross section, as shown in Figure 2.14 (a), with \( Ni \) being the total current in the rectangular region. For the analysis using the proposed numerical method, the coil cross section is split into a rectangular grid of \( S \) conductor source points (represented by dots as shown in Figure 2.14 (b)), each carrying current \( Ni/|S| \), where \( |S| \) denotes the number of elements in \( S \). At any point of observation,
P across the 2D space, the field due to the rectangular coil is equivalent to the superposition of the fields due to each of the point sources in $S$, expressed as

$$B_{Px} = -\frac{\mu_0 N i}{|S|} \sum_{j \in S} \frac{(y_{o,P} - y_{s,j})}{(x_{o,P} - x_{s,j})^2 + (y_{o,P} - y_{s,j})^2}$$  \hspace{1cm} (2.42)$$

$$B_{Py} = \frac{\mu_0 N i}{|S|} \sum_{j \in S} \frac{(x_{o,P} - x_{s,j})}{(x_{o,P} - x_{s,j})^2 + (y_{o,P} - y_{s,j})^2}$$  \hspace{1cm} (2.43)$$

where $(x_{o,P}, y_{o,P})$ are the Cartesian coordinates of observation point P and $(x_{s,j}, y_{s,j})$ are the Cartesian coordinates of the $j^{th}$ source point. The summation in (2.42) and (2.43) is over $S$ number of points sources.

To calculate the field due to the rectangular coil across the entire 2D space, the cross-sectional region and its surrounding region is divided into another grid called points of interest (represented by crosses), as shown in Figure 2.14 (c). Note that within the coil cross section, the points of interest (crosses) are placed such that they are distinct from the current source points (dots). This way, field variation within the coil cross section can be determined without resulting in any singularities when using (2.42) and (2.43).

Fig. 2.14. Field analysis for a rectangular coil region using Biot-Savart law
The field intensity, $\mathbf{H}$ at each point of interest is calculated using the relation

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} \quad (2.44)$$

Suppose the observation points are arranged in rectangular grid of $N_r$ number of rows and $N_c$ number of columns as shown in Figure 2.15. The location of each point is denoted as $(x_{o,m}, y_{o,n})$ where $1 \leq m \leq N_r$ and $1 \leq n \leq N_c$. Accordingly, using (2.42), (2.42) and (2.47) the Cartesian components of the field intensity at point of interest P with location $(x_{o,m}, y_{o,n})$ are

$$H_{mx} = \frac{-Ni}{2\pi |S|} \sum_{j \in S} \frac{(y_{o,n} - y_{s,j})(x_{o,m} - x_{s,j})}{(x_{o,m} - x_{s,j})^2 + (y_{o,n} - y_{s,j})^2} \quad (2.45)$$

$$H_{my} = \frac{Ni}{2\pi |S|} \sum_{j \in S} \frac{(x_{o,m} - x_{s,j})(y_{o,n} - y_{s,j})}{(x_{o,m} - x_{s,j})^2 + (y_{o,n} - y_{s,j})^2} \quad (2.46)$$

Note the distinction in the source and observation points are made by using 'o' and 's' respectively in the subscript.

In order to compute the energy, the integral form of (2.26) is approximated as a weighted sum. Weights are assigned to each of the observation points in a way analogous to the 1D Trapezoidal integration rule but extended to 2D space. Depending on the location of point of interest, weights are assigned using the pseudo-code in Table 2.3.

Using (2.45), (2.46) and weights assigned to observations points in Table 2.3, the energy of the region of interest is calculated as

$$E = \frac{1}{2} \mu_0 l \sum_{1 \leq m \leq N_r} \sum_{1 \leq n \leq N_c} w_{m,n} \left( H_{mx}^2 + H_{my}^2 \right) \quad (2.47)$$

where $l$ is the length of the coil into the page and $w_{m,n}$ is the weight assigned to the point of interest at location $(x_{o,m}, y_{o,n})$. $H_{mx}$ and $H_{my}$ are the $x$ and $y$ components
Table 2.3.
Pseudo-code for assigning weights

\[
\begin{align*}
\text{if } m &= 1, n = 1 \\
    w_{1,1} &= \frac{1}{4} (x_{o,2} - x_{o,1}) (y_{o,1} - y_{o,1}) \\
\text{if } m &= 1, n = N_r \\
    w_{1,1} &= \frac{1}{4} (x_{o,2} - x_{o,1}) (y_{o,N_r} - y_{o,N_r-1}) \\
\text{if } m &= N_r, n = 1 \\
    w_{N_r,1} &= \frac{1}{4} (x_{o,N_r} - x_{o,N_r-1}) (y_{o,1} - y_{o,1}) \\
\text{if } m &= N_r, n = N_r \\
    w_{N_r,N_r} &= \frac{1}{4} (x_{o,N_r} - x_{o,N_r-1}) (y_{o,N_r} - y_{o,N_r-1}) \\
\text{if } m &= 1, 2 \leq n \leq (N_r - 1) \\
    w_{1,n} &= \frac{1}{2} (x_{o,2} - x_{o,1}) [(y_{o,n} - y_{o,n-1}) + (y_{o,n+1} - y_{o,n})] \\
\text{if } m &= N_r, 2 \leq n \leq (N_r - 1) \\
    w_{N_r,n} &= \frac{1}{2} (x_{o,N_r} - x_{o,N_r-1}) [(y_{o,n} - y_{o,n-1}) + (y_{o,n+1} - y_{o,n})] \\
\text{if } 2 \leq m \leq (N_r - 1), n = 1 \\
    w_{m,1} &= \frac{1}{2} (y_{o,2} - y_{o,1}) [(x_{o,m} - x_{o,m-1}) + (x_{o,m+1} - x_{o,m})] \\
\text{if } 2 \leq m \leq (N_r - 1), n = N_r \\
    w_{m,N_r} &= \frac{1}{2} (y_{o,N_r} - y_{o,N_r-1}) [(x_{o,m} - x_{o,m-1}) + (x_{o,m+1} - x_{o,m})] \\
\text{if } 2 \leq m \leq N_r - 1, 2 \leq n \leq N_c - 1 \\
    w_{m,n} &= [x_{o,m+1} - x_{o,m} + x_{o,m} - x_{o,m-1}] [y_{o,n+1} - y_{o,n} + y_{o,n} - y_{o,n-1}] \\
\end{align*}
\]

\[
\begin{array}{cccccccc}
    j=1 &  &  &  &  &  &  &  \\
    j=2 &  &  &  &  &  &  &  \\
    j=3 &  &  &  &  &  &  &  \\
    j=n-2 &  &  &  &  &  &  &  \\
    j=n-1 &  &  &  &  &  &  &  \\
\end{array}
\]

\[
\begin{array}{cccccccc}
    j=1 &  &  &  &  &  &  &  \\
    j=2 &  &  &  &  &  &  &  \\
    j=3 &  &  &  &  &  &  &  \\
    j=n-2 &  &  &  &  &  &  &  \\
    j=n-1 &  &  &  &  &  &  &  \\
\end{array}
\]

\[
\begin{array}{cccccccc}
    j=1 &  &  &  &  &  &  &  \\
    j=2 &  &  &  &  &  &  &  \\
    j=3 &  &  &  &  &  &  &  \\
    j=n-2 &  &  &  &  &  &  &  \\
    j=n-1 &  &  &  &  &  &  &  \\
\end{array}
\]

\[
\begin{array}{cccccccc}
    j=1 &  &  &  &  &  &  &  \\
    j=2 &  &  &  &  &  &  &  \\
    j=3 &  &  &  &  &  &  &  \\
    j=n-2 &  &  &  &  &  &  &  \\
    j=n-1 &  &  &  &  &  &  &  \\
\end{array}
\]

\[
\begin{array}{cccccccc}
    j=1 &  &  &  &  &  &  &  \\
    j=2 &  &  &  &  &  &  &  \\
    j=3 &  &  &  &  &  &  &  \\
    j=n-2 &  &  &  &  &  &  &  \\
    j=n-1 &  &  &  &  &  &  &  \\
\end{array}
\]

\[
\begin{array}{cccccccc}
    j=1 &  &  &  &  &  &  &  \\
    j=2 &  &  &  &  &  &  &  \\
    j=3 &  &  &  &  &  &  &  \\
    j=n-2 &  &  &  &  &  &  &  \\
    j=n-1 &  &  &  &  &  &  &  \\
\end{array}
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\[
\begin{array}{cccccccc}
    j=1 &  &  &  &  &  &  &  \\
    j=2 &  &  &  &  &  &  &  \\
    j=3 &  &  &  &  &  &  &  \\
    j=n-2 &  &  &  &  &  &  &  \\
    j=n-1 &  &  &  &  &  &  &  \\
\end{array}
\]

\[
\begin{array}{cccccccc}
    j=1 &  &  &  &  &  &  &  \\
    j=2 &  &  &  &  &  &  &  \\
    j=3 &  &  &  &  &  &  &  \\
    j=n-2 &  &  &  &  &  &  &  \\
    j=n-1 &  &  &  &  &  &  &  \\
\end{array}
\]

\[
\begin{array}{cccccccc}
    j=1 &  &  &  &  &  &  &  \\
    j=2 &  &  &  &  &  &  &  \\
    j=3 &  &  &  &  &  &  &  \\
    j=n-2 &  &  &  &  &  &  &  \\
    j=n-1 &  &  &  &  &  &  &  \\
\end{array}
\]

\[
\begin{array}{cccccccc}
    j=1 &  &  &  &  &  &  &  \\
    j=2 &  &  &  &  &  &  &  \\
    j=3 &  &  &  &  &  &  &  \\
    j=n-2 &  &  &  &  &  &  &  \\
    j=n-1 &  &  &  &  &  &  &  \\
\end{array}
\]

\[
\begin{array}{cccccccc}
    j=1 &  &  &  &  &  &  &  \\
    j=2 &  &  &  &  &  &  &  \\
    j=3 &  &  &  &  &  &  &  \\
    j=n-2 &  &  &  &  &  &  &  \\
    j=n-1 &  &  &  &  &  &  &  \\
\end{array}
\]

Fig. 2.15. Grid of points of interest
of the magnetic field intensity at same observation point given by (2.45) and (2.46). The summation is over all points of interest (crosses) across the region of interest.

Additional features are included in the numerical analysis using Biot-Savart law. In case of multiple coils, the effective field of all the coils is calculated by using the principle of superposition. The grid for points of interest is chosen as per the given system, making use of symmetry. Away from the coils where the field is weak, coarser grid is used. This reduces the computational effort when approximating infinite region and results in a stretched grid towards the outer boundary of the region of interest as shown in Figure 2.15.

The numerical method presented in here is validated for the case of air-cored coil example in Section 2.4.1. In the following subsection, the numerical method using Biot-Savart law is applied to calculate the energy in the core exterior region. Note that the numerical analysis based on Biot-Savart law is applicable only if the net current of the system is zero, translating to the assumption that the total energy of the system is predominantly present in a finite region. This is because the proposed numerical analysis does not consider any finite boundary conditions. The single coil example considered for validation in Section 2.4.1 and the core-type transformer exterior region satisfy this condition and hence the proposed numerical analysis can be applied in these two cases.

2.2.3 Exterior Region Analysis

Before applying the Biot-Savart law to analyze the transformer exterior region, the coupled system of the core and exterior windings shown in Figure 2.16(a) is first simplified. The disparity in the magnetic permeability of the core material and surrounding region is the key factor which is used to simplify the analysis. On close observation of the flux paths in the exterior region as shown in Figure 2.10 for zero magnetizing current excitation using 2D FEA, the core outer edge acts as a boundary of symmetry for the field. This observation is used to simplify the exterior region
analysis using the method of images as shown in Figure 2.16(b). The core-coils system is replaced with just the exterior coils \( (p_e \text{ and } s_e) \) and their mirror images \( (p_{ei} \text{ and } s_{ei}) \) about the core outer edge taken as the symmetric boundary line. This simplification eliminates the core and hence its dimensions from the exterior region analysis, instead replacing it with an imaginary symmetric boundary. Using 3D FEA simulation, the effect of core on the transformer leakage inductance is investigated by varying the core outer dimension, \( d_c \). The results, presented in Section 2.4, indicate that it is reasonable to use method of images in the calculation of the leakage inductance.

![Diagram](image)

**Fig. 2.16.** Exterior coil simplification using method of mirror images

The numerical analysis using Biot-Savart law can now be applied to the simplified system in Figure 2.16(b). As described in subsection 2.2.2, both the coils and their images are discretized into current source points (dots) first, followed by another set of grid with points of interest (crosses) as shown in Figure 2.17. The total current across the primary coil cross section \( (p_e) \) is taken as \( N_{pcl}I_{p,t}/N_{pcp} \), where \( I_{p,t} \) is the test current. Using the zero magnetizing current excitation condition, the total current considered across the secondary coil cross section \( (s_e) \) is \( -N_{scl}N_pI_{p,t}/(N_{scp}N_s) \). The image coils are assigned the total currents with same magnitude and direction as their counterparts.

The principle of superposition is used to calculate the resultant field at each point of interest, by taking sum of the field due to each coil,

\[
H_{ix,eff} = H_{ix,se} + H_{ix,sei} + H_{ix,pe} + H_{ix,pei} \tag{2.48}
\]
\[ H_{iy,\text{eff}} = H_{iy,\text{se}} + H_{iy,\text{sei}} + H_{iy,\text{pe}} + H_{iy,\text{pei}} \] 

(2.49)

By symmetry, it is sufficient to evaluate field at observation points across one-fourth of the total region, denoted by lightly shaded region on top right corner (cyan color) in Figure 2.17. The energy per unit length in external region is then given by

\[ e_e = \mu_0 \sum_{i \in R} w_i \left( H_{ix,\text{eff}}^2 + H_{iy,\text{eff}}^2 \right) \] 

(2.50)

where \( w_i \) is the weight assigned to point of observation according to pseudo code given Table 2.3. Observe the grid away from the coil in Figure 2.17 is stretched to an approximate infinite boundary. This reduces the computational effort.
In case of uniform spacing between the core and windings as shown in Figure 2.2, the exterior region energy, $E_e$ is given as

$$E_e = 2e_e l_e$$  \hspace{1cm} (2.51)

where the mean length of the exterior region, $l_e$ is given as

$$l_e = (l_{pe} + l_{se})/2$$  \hspace{1cm} (2.52)

In (2.51), a factor of two is used to account for two exterior regions, one on each side of the core end legs.

To address the end winding effect and account for the varying distance between core and windings as shown in Figure 2.3, an additional exterior region analysis for the extreme case of maximum distance between core and windings is carried out. This analysis includes the radius of inert material, $r_w$ plus the clearance between the core and secondary winding, $c_{sc}$ as shown in Figure 2.18.

Fig. 2.18. Exterior region with maximum distance between core and windings

The energy per unit length of exterior region with maximum distance between core and coils is denoted by $e_{e, mx}$. To account for the curvature of the windings around the inert material, and the resulting varying distance of windings to the core, a linear
variation of energy per unit length is assumed within exterior regions $R_{e3}$ and $R_{e4}$. The energy per unit length of exterior region $R_{e3}$, denoted by $e_{e3}$ is given by

$$e_{e3} = \frac{1}{4}(e_i + 2e_{e, mx}) \tag{2.53}$$

where core interior region energy per unit length is considered due to its proximity to the core interior region. The energy per unit length of exterior region $R_{e4}$, denoted by $e_{e4}$ is given by

$$e_{e4} = \frac{1}{2}(e_e + e_{e, mx}) \tag{2.54}$$

Considering the corresponding lengths of each region from Table 2.2 and the total number of such regions existing around the two core end-legs, yields a total exterior region energy of

$$E_e = 2(e_{e1}l_{e1} + 2e_{e, mx}l_{e2} + 2e_{e3}l_{e3} + 2e_{e4}l_{e4}) \tag{2.55}$$

Once the energy of the interior and exterior region are calculated using (2.33) and (2.50), the total leakage inductance of the transformer referred to primary side is given by

$$L_{lp} + L'_{ls} = \frac{2(E_i + E_e)}{I_{p,t}^2} \tag{2.56}$$

The field analysis applied across the exterior region is also used to evaluate mean squared field intensities in the exterior winding coils. In particular,

$$\langle \hat{H}^2 \rangle_{pe} = \frac{1}{d_{pu}w_{pu}l_{p,t}^2} \sum_{i \in pe} w_i \left( H_{ix,eff}^2 + H_{iy,eff}^2 \right) \tag{2.57}$$

$$\langle \hat{H}^2 \rangle_{se} = \frac{1}{d_{sv}w_{sv}l_{p,t}^2} \sum_{i \in se} w_i \left( H_{ix,eff}^2 + H_{iy,eff}^2 \right) \tag{2.58}$$

### 2.3 Frequency Dependency of Transformer Leakage Inductance

Section 2.2 presented a 2D magneto-static analysis of a core-type transformer. At high frequencies, the time varying magnetic leakage fields generate eddy-currents in
the windings that in turn affects the magnetic field distribution. As a result, the effective value of transformer leakage inductance is dependent on frequency. The frequency dependency is captured using 1D (one-dimensional analysis) magnetoquasistatic analysis in [32] and [23]. Therein, the magnetic and electric field interactions due to sinusoidal winding currents at frequency, $\omega$ are analyzed using magnetoquasistatic analysis. For these conditions, Maxwell’s equations in a linear homogeneous isotropic system are given as

$$\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}$$  \hspace{1cm} (2.59)

$$\nabla \times \mathbf{H} = \sigma \mathbf{E}$$  \hspace{1cm} (2.60)

Assuming one dimensional analysis of the fields (i.e. the electric field strength $\mathbf{E}$ has only $z$ component and the magnetic field strength $\mathbf{H}$ has only $y$ component as shown in Figure 2.19) and Cartesian coordinate reference system shown in Figure 2.19, (2.59) and (2.60) reduce to

$$\frac{dE_z}{dx} = \mu_0 \frac{\partial H_y}{\partial t}$$  \hspace{1cm} (2.61)

$$\frac{dH_y}{dx} = \sigma E_z$$  \hspace{1cm} (2.62)

Here, the field components $H_y$ and $E_z$ are function of space in $x$ direction and time, $t$. Using time-harmonic analysis, the phasor notation can be used to represent these fields as

$$H_y(x, t) = \text{Re} \left( \tilde{H}_y(x)e^{j\omega t} \right)$$  \hspace{1cm} (2.63)

$$E_z(x, t) = \text{Re} \left( \tilde{E}_z(x)e^{j\omega t} \right)$$  \hspace{1cm} (2.64)

Note, in (2.63) and (2.64) tilde is used to denote the phasor notation of the fields and the peak values are used for phasor magnitude. Using phasor analysis, (2.61) and (2.62) become

$$\frac{d\tilde{E}_z}{dx} = j\omega \mu_0 \tilde{H}_y$$  \hspace{1cm} (2.65)
Fig. 2.19. One-dimensional time harmonic analysis of winding layers in core window

\[ \frac{d\tilde{H}_y}{dx} = \sigma \tilde{E}_z \]  \hspace{1cm} (2.66)

Substituting \( \tilde{E}_z \) using (2.66) in (2.65) yields

\[ \frac{d^2\tilde{H}_y}{dx^2} = j\omega \mu_0 \sigma \tilde{H}_y \]  \hspace{1cm} (2.67)

The solution to the second order equation in (2.67) is

\[ \tilde{H}_y = H_1 e^{\gamma x} + H_2 e^{-\gamma x} \]  \hspace{1cm} (2.68)

where \( H_1 \) and \( H_2 \) are complex constants derived using boundary conditions and \( \gamma \) is propagation constant given by

\[ \gamma = \sqrt{j\omega \mu_0 \sigma} \]  \hspace{1cm} (2.69)

Also, the skin depth is defined as, \( \delta_{sd} = \sqrt{2/ (\omega \mu_0 \sigma)} \) can be used to express \( \gamma \) as

\[ \gamma = \frac{1}{\delta_{sd}} + j \frac{1}{\delta_{sd}} \]  \hspace{1cm} (2.70)
The boundary conditions necessary to evaluate complex coefficients $H_1$ and $H_2$ in (2.68) depend on the field paths. In [32] and [23], this approach was used to evaluate leakage fields in a shell type transformer, where the core window height is assumed to be much greater than the core width and that the winding layers completely fill the core window height. These assumptions favor the magnetic field through the core window to be one-dimensional, parallel to the slot depth including negligible end effects. Subsequently, the 1D analysis magnetoquasistatic analysis discussed in (2.61)-(2.67) can be applied to transformer for zero magnetic current excitation to obtain the leakage fields given by (2.68) as function of frequency. By calculating the magnetic field energy using (2.68), leakage inductance can be estimated as function of frequency.

In the core-type transformer case that is considered in this research, the assumptions for 1D magnetic fields are valid for the core interior region although the same is not true for the exterior region. The interior region has one-dimensional fields as depicted in Figure 2.11 and hence, the 1D time-harmonic analysis can be used to find the core interior region energy as a function of frequency. In case of the core exterior region, a 1D time-harmonic analysis would be a poor approximation because of the complex leakage field paths involved. To take advantage of the more accurate numerical 2D magneto-static analysis set up for the exterior region case in Section 2.2.3, a correction factor that is dependent on frequency is used to scale the exterior region energy calculated from static analysis. The ratio of the interior region energies calculated from 1D harmonic to static analysis is used as an approximate scaling factor to account for high-frequency effects on the exterior region leakage field energy.

In the core interior region, magnetic flux path is as depicted in Figure 2.11. Denoting $x_{pmi}$ and $x_{pno}$ as $x$ coordinates of start and end of $n^{th}$ primary winding layer, the magnetic field intensity satisfies following boundary conditions

$$
\tilde{H}_y (x_{pmi}) = (n - 1) H_{p0}
$$

(2.71)
\[ H_y(x_{pno}) = n H_{p0} \]  
(2.72)

where

\[ H_{p0} = \frac{N_{ptl} I_p}{N_{pcp} d_{cs}} \]  
(2.73)

Here, \( I_p \) is the peak value of the sinusoidal current through the primary winding.

Applying (2.71) and (2.72) to (2.68), it can be shown that

\[ H_1 = H_{p0} \frac{n e^{-\gamma x_{pni}} - (n - 1) e^{-\gamma x_{pno}}}{2 \sinh(\gamma t_p)} \]  
(2.74)

\[ H_2 = H_{p0} \frac{(n - 1) e^{\gamma x_{pno}} - n e^{\gamma x_{pni}}}{2 \sinh(\gamma t_p)} \]  
(2.75)

Substituting (2.74) and (2.75) in (2.68) and \( x' = x - x_{pni} \), the magnetic field intensity phasor in \( y \) direction across the width of \( n^{th} \) primary winding layer is

\[ \tilde{H}_{pn}(x') = H_{p0} \frac{n \sinh(\gamma x') + (n - 1) \sinh(\gamma t_p - \gamma x')}{\sinh(\gamma t_p)} \]  
(2.76)

where \( t_p = x_{pno} - x_{pni} \) denotes thickness of primary layer and variable \( x' \in [0 \ t_p] \).

The temporal peak magnetic energy per unit length stored in the core interior region slot with \( x_{pni} \leq |x| \leq x_{pno} \) (including two primary layers equidistant from core center) is given by

\[ W_{pn} = \mu_0 d_{cs} \int_0^{x_p} \tilde{H}_{pn}(x') \tilde{H}_{pn}^*(x') dx' \]  
(2.77)

where \( \tilde{H}_{pn}^*(x') \) denotes complex conjugate of the phasor \( \tilde{H}_{pn}(x') \) given in (2.76). Evaluating the integral yields

\[ W_{pn} = \frac{\mu_0 H_{p0}^2 d_{cs}}{4k_3(\frac{t_p}{\delta_{sd}})} \left( k_1 \left( \frac{t_p}{\delta_{sd}} \right) (n^2 + (n - 1)^2) + 4k_2 \left( \frac{t_p}{\delta_{sd}} \right) n(n - 1) \right) \]  
(2.78)

where \( k_1, k_2 \) and \( k_3 \) are functions defined as

\[ k_1(x) = \sinh(2x) - \sin(2x) \]  
(2.79)
\[ k_2(x) = \cosh(x) \sin(x) - \cos(x) \sinh(x) \] (2.80)

\[ k_3(x) = \cos(x)^2 \sinh(x) + \sin(x)^2 \cosh(x)^2 \] (2.81)

The peak energy stored in the core interior region with \( 0 \leq |x| \leq (c_{pp}/2 + w_{pw}) \) is obtained by taking summation of (2.78) over total number of primary coil layers, \( N_{pl} \)

\[ W_p = \sum_{n=1}^{N_{pl}} W_{pn} \] (2.82)

Evaluating the summation in (2.82) using (2.78) yields

\[ W_p = \frac{\mu_0 H_{ps0}^2 \delta_{sd} d_{cs}}{24 k_3 \left( \frac{t_p}{\delta_{sd}} \right)} \left( k_1 \left( \frac{t_p}{\delta_{sd}} \right) (2 N_{pl}^2 + 1) + 4 k_2 \left( \frac{t_p}{\delta_{sd}} \right) (N_{pl}^2 - 1) \right) \] (2.83)

The field intensity magnitude over the region \((c_{pp}/2 + w_{pw}) \leq |x| \leq (c_{pp}/2 + w_{pw} + c_{ps})\) is a constant given by

\[ H_{ps} = \frac{N_{pl} N_{ptl} I_p}{N_{scp} d_{cs}} \] (2.84)

and the temporal peak magnetic field energy per unit length stored in the region \((c_{pp}/2 + w_{pw}) \leq |x| \leq (c_{pp}/2 + w_{pw} + c_{ps})\) is given by

\[ W_{ps} = \mu_0 d_{cs} c_{ps} H_{ps}^2 \] (2.85)

The magnetic field phasor over the region \((c_{pp}/2 + w_{pw} + c_{ps}) \leq |x| \leq (c_{pp}/2 + w_{pw} + c_{ps} + w_{sw})\) is again of the form (2.68) except the boundary conditions for each secondary winding layer are

\[ \tilde{H}_y(x_{sin}) = H_{ps} + (n - 1) H_{s0} \] (2.86)

\[ \tilde{H}_y(x_{sno}) = H_{ps} + n H_{s0} \] (2.87)

where

\[ H_{s0} = -\frac{N_{ptl} I_p}{N_{scp} d_{cs} N_{sl}} \] (2.88)
Note, zero magnetizing current condition, \( N_{ptl}N_{pl}I_p = -N_{stl}N_{sl}I_s \) is used in (2.88), where \( I_s \) is the peak value of secondary winding current. Using the boundary conditions in (2.86) and (2.87), the magnetic field phasor over \( n^{th} \) secondary winding layer is given as

\[
\tilde{H}_{sn}(x') = \frac{(H_{ps} + nH_{s0}) \sinh(\gamma x') + (H_{ps} + (n - 1)H_{s0}) \sinh(\gamma t_p - \gamma x')}{\sinh(\gamma t_p)}
\]  \( (2.89) \)

where \( x' \) is a local variable defined as \( x' = x - x_{sni} \) and \( t_s = x_{sni} - x_{sno} \) denotes the secondary winding layer thickness.

The temporal peak magnetic energy per unit length stored in core interior region slot with \( x_{sni} \leq |x| \leq x_{sno} \) ((including two secondary layers equidistant from core center) is given by

\[
W_{sn} = \mu_0 d_{cs} \int_0^x \tilde{H}_{sn}(x') \tilde{H}_{sn}^*(x') dx
\]  \( (2.90) \)

Evaluating the expression (2.90) yields

\[
W_{sn} = \frac{\mu_0 \delta_{sd} d_{cs}}{4k_3 \frac{t_p}{\delta_{sd}}} \left( k_1 \left( \frac{t_p}{\delta_{sd}} \right) (p^2 + q^2) + 4k_2 \left( \frac{t_p}{\delta_{sd}} \right) pq \right)
\]  \( (2.91) \)

where \( p = H_{ps} + nH_{s0} \), \( q = H_{ps} + (n - 1)H_{s0} \) and functions \( k_1, k_2 \) and \( k_3 \) are same as defined in (2.79)-(2.81).

The peak energy stored in the core interior region with \( (c_{pp}/2 + w_{pw} + c_{ps}) \leq |x| \leq (c_{pp}/2 + w_{pw} + c_{ps} + w_{sw}) \) is obtained by taking summation of (2.91) over total number of secondary coil layers, \( N_{sl} \)

\[
W_s = \sum_{n=1}^{N_{sl}} W_{sn}
\]  \( (2.92) \)

Evaluating (2.92) using (2.91) yields

\[
W_s = \frac{\mu_0 \delta_{sd} d_{cs}}{8k_3 \frac{t_p}{\delta_{sd}}} \left( k_1 \left( \frac{t_s}{\delta_{sd}} \right) m_1 + 4k_2 \left( \frac{t_s}{\delta_{sd}} \right) m_2 \right)
\]  \( (2.93) \)

where

\[
m_1 = 2H_{ps}^2 + \frac{2N_{sl}^2 + 1}{3} H_{s0}^2 + 2N_{sl} H_{ps} H_{s0}
\]  \( (2.94) \)
\[
m_2 = H_{ps}^2 + \frac{N_{sl}^2 - 1}{3}H_{s0}^2 + N_{sl}H_{ps}H_{s0}
\]  \hspace{1cm} (2.95)

The peak total magnetic energy of the transformer interior region, \( W_i \), as a function of frequency, \( \omega \) is

\[
W_i(\omega) = l_i(W_p + W_{ps} + W_s)
\]  \hspace{1cm} (2.96)

As described in Section 2.2, the leakage fields in the core exterior region have complicated paths and do not satisfy the conditions necessary for a 1D magnetic analysis. For this exact reason, Biot-Savert law based 2D numerical analysis is proposed for exterior region magneto-static analysis. Likewise, using a 1D harmonic analysis would be a poor choice to compute frequency dependent time average of the transformer exterior region magnetic energy. Instead, since the winding cross-section in the transformer exterior and interior regions are exactly the same, it is assumed that the ratio of the magnetic energy stored when accounted for eddy current to that calculated from magneto-static analysis is the same at a given frequency.

For a test current of \( I_{p,t} \) through the primary winding, the interior region energy as calculated using static analysis is given by (2.33). Assuming the current through the primary is a sinusoid with peak \( I_{p,t} \), the peak energy of the interior region as of function of frequency, \( W_{i,t}(\omega) \), can be calculated using (2.73)-(2.96). The scaling factor, \( r_{cf}(\omega) \) is defined as

\[
r_{cf}(\omega) = \frac{W_{i,t}(\omega)}{E_i}
\]  \hspace{1cm} (2.97)

The exterior region energy, \( E_e \), as calculated using the numerical analysis based on Biot-Savart law is scaled by \( r_{cf}(\omega) \) to obtain the time average of exterior region magnetic energy over one cycle, \( W_{e,t} \) as

\[
W_{e,t}(\omega) = r_{cf}(\omega)E_e
\]  \hspace{1cm} (2.98)
The total leakage inductance of the transformer referred to primary side, \( L_{lk} \), as a function of frequency is given by

\[
L_{lk}(\omega) = \frac{2(W_{i,t}(\omega) + W_{c,t}(\omega))}{I_{p,t}^2}
\]  

(2.99)

Hence, by using (2.99), the frequency analysis is coupled to the magneto-static analysis presented in Section 2.2 to calculate the leakage inductance of the high-frequency transformer as a function of frequency.

The combined analysis allows accurate estimation of the leakage inductance when the high-frequency transformer is designed in the context of a DC-DC converter switching at frequency, \( f_{sw} \). This value of leakage inductance, so calculated, is used in the time-domain system model. To summarize, firstly, the magneto-static analysis is carried out to find the transformer energy for the zero-magnetizing current excitation. An analytical approach is used to calculate the core-interior region magnetic energy using (2.33). To calculate the core exterior region energy, numerical analysis based on Biot-Savart law, as described in Section 2.2.3, is performed. Depending the core geometry, the numerical analysis based on Biot-Savart law may be repeated twice, for each of the two cases of minimum and maximum clearance between the core and secondary winding. Secondly, a 1D harmonic analysis as described in Section 2.3 is used to calculate interior region energy at \( f_{sw} \) for zero magnetizing excitation and from thereon calculate the scaling factor, \( r_{cf} \), by using (2.97). To calculate exterior region energy at \( f_{sw} \), the exterior region energy calculated using static analysis is then scaled by a factor of \( r_{cf} \). The total transformer energy hence obtained is used to estimate the total leakage inductance at \( f_{sw} \) by using (2.99).

For modeling purposes, it is useful to distinguish between the primary and secondary windings individual leakage inductances. With regards to this, it is assumed
that the total leakage inductance, \( L_{lk}(\omega) \) is distributed between the transformer primary and secondary windings as given by

\[
L_{lp}(\omega) = \frac{1}{2} L_{lk}(\omega) \tag{2.100}
\]

\[
L_{ls}(\omega) = \frac{N_s^2}{2N_p^2} L_{lk}(\omega) \tag{2.101}
\]

### 2.4 Validation

In this section, the numerical method using Biot-Savart law is first used to calculate the energy of a single coil as an example. Second, the numerical method using Biot-Savart law is validated for the case of a core-type transformer exterior region analysis. Third, the proposed magneto-static based method to estimate leakage inductance is validated using experimental data and FEA. This is followed by the validation of core-type transformer MEC and harmonic magnetic analysis.

#### 2.4.1 Biot-Savart Law Method Validation

The numerical analysis using Biot-Savart law is validated using a simple case of air-core coil, as shown in Figure 2.20. The cross section of the coil results in two coil regions as shown. The energy due to the coil is evaluated using the numerical method based on Biot-Savart law and compared to 2D FEA results. The grid shown in Figure 2.20 is used for proposed numerical analysis. Away from the coil regions where the field is weak, grid is spaced farther apart compared to the region closer to the coils. This way, the computational effort is reduced when approximating infinite region. Taking the coil dimensions as listed in Table 2.4 and unit current through the coil, energy of the coil calculated by Biot-Savart law method is \( 6.76 \mu J \). The energy of the system using 2D FEA simulation in ANSYS Maxwell is \( 6.74 \mu J \), with an error of 0.3%. This example shows that the proposed method can be used for field calculation of simple coil systems with results equivalent to that of 2D FEA.
Fig. 2.20. Air-cored coil example for Biot-Savart law method validation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d$</td>
<td>Depth of the coil</td>
<td>5 mm</td>
</tr>
<tr>
<td>$w$</td>
<td>Width of the coil</td>
<td>5 mm</td>
</tr>
<tr>
<td>$c$</td>
<td>Distance between coil regions</td>
<td>5 mm</td>
</tr>
<tr>
<td>$l$</td>
<td>Thickness into page</td>
<td>5 mm</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of turns</td>
<td>30</td>
</tr>
</tbody>
</table>

### 2.4.2 Core-Type Transformer Exterior Region Static Analysis Validation

A prototype 5 kVA, 20 kHz core type transformer was constructed. It was designed for a square wave primary voltage of 750V. The prototype transformer constructed is shown in Figure 2.21(a) with the end winding geometry as shown in Figure 2.21(b). This is an extreme case of the winding geometry shown in Figure 2.3, in that the inert material used has semicircular cross section, with width, $w_{sec} = 0$. Here, the transformer core is comprised of two pairs of U-shaped ferrite cores stacked to form a rectangular core. P-type ferrite material from Mag-inc was used [33]. The dimensions of the prototype transformer core and windings are listed in Table 2.5.
Fig. 2.21. Prototype core type transformer built for 20kHz

Table 2.5. Prototype transformer dimensions

<table>
<thead>
<tr>
<th>Core Parameter</th>
<th>Value</th>
<th>Primary winding Parameter</th>
<th>Value</th>
<th>Secondary winding Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>Ferrite-Ptype</td>
<td>Material</td>
<td>Copper</td>
<td>Material</td>
<td>Copper</td>
</tr>
<tr>
<td>$l_{cc}$</td>
<td>50.8 mm</td>
<td>$w_{cs}$</td>
<td>63.4 mm</td>
<td>$d_{cs}$</td>
<td>50.8 mm</td>
</tr>
<tr>
<td>$w_{cs}$</td>
<td>50.8 mm</td>
<td>$d_{cs}$</td>
<td>114.2 mm</td>
<td>$w_{pw}$</td>
<td>4.1 mm</td>
</tr>
<tr>
<td>$d_{c}$</td>
<td>101.6 mm</td>
<td>$d_{pw}$</td>
<td>48.5 mm</td>
<td>$r_{pi}$</td>
<td>20.1 mm</td>
</tr>
<tr>
<td>$l_c$</td>
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<td>$r_{po}$</td>
<td>24.1 mm</td>
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<tr>
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<td>25.4 mm</td>
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<tr>
<td>$w_{ccc}$</td>
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<tr>
<td>$w_{ce}$</td>
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<td>$c_{sc}$</td>
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<td></td>
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</tr>
<tr>
<td>$c_{ps}$</td>
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<td>$c_{pp}$</td>
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</tr>
<tr>
<td>$c_{pv}$</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c_{sv}$</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The energy per unit length of the transformer interior and exterior regions as estimated by the proposed magnetic analysis in Section 2.2 are compared to Maxwel Ansys 2D FEA results in Table 2.6. The exterior region I is along the length of the core end leg (clearance between core and secondary coil is $c_{sc}$) and exterior region II is
along the curvature of the inert material (with clearance between core and secondary coil at maximum, \( c_{ac} + r_w \)). The cases of transformer exterior regions is of particular interest to validate the performance of numerical method using Biot-Savart law. On a DELL OPTIPLEX 7010 with Intel(R) Core (TM) i7-3770 processor, at a rated clock of 3.40GHz, running Windows 7 Enterprize, the time taken for the two exterior region analyses by the proposed method using Biot-Savart law (implemented using Matlab 2016a) is 200 ms. The time taken for two 2D FEA simulations of the transformer (simulating one fourth of the cross section) using Ansys Maxwell is 102 s.

Table 2.6. Validation of interior and exterior regions magnetic analysis

<table>
<thead>
<tr>
<th>Transformer region</th>
<th>Energy per unit length</th>
<th>Error %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed method</td>
<td>68.80</td>
<td>0.30</td>
</tr>
<tr>
<td>2D FEA</td>
<td>69.01</td>
<td></td>
</tr>
<tr>
<td>Interior</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exterior I</td>
<td>64.45</td>
<td>1.43</td>
</tr>
<tr>
<td>Exterior II</td>
<td>61.68</td>
<td>2.02</td>
</tr>
<tr>
<td></td>
<td>63.55</td>
<td></td>
</tr>
<tr>
<td></td>
<td>60.46</td>
<td></td>
</tr>
</tbody>
</table>

Next, experimental validation of the transformer total leakage inductance calculation using magneto-static analysis is presented.

### 2.4.3 Transformer Characterization and Leakage Inductance Validation for Static Analysis

The proposed leakage inductance method described in Section 2.2 is validated in this section. The prototype transformer was first characterized experimentally. The following measurement procedure refers to the T-equivalent circuit parameters shown in Figure 2.8. The winding resistances, \( r_p \) and \( r_s \) are measured using a small DC voltage applied across each winding, while measuring the resulting current and by using Ohm’s law. An Agilent E3631A DC power supply was used to apply voltage
across the windings. The currents were measured using 701930 current probe with Yokogawa DL850 oscilloscope.

As part of characterization process, a two-step procedure described is used to determine the transformer turns ratio. This is because turns ratio may not be equal to the designed value. Instead, it is chosen using experimental data. In the first-step, the primary side of the transformer is excited with a sinusoid current while the secondary winding is left open. The current through the primary winding and voltages across both the secondary winding are measured. Then, the time derivative of secondary winding flux linkage is calculated using

$$\frac{d\lambda_s}{dt} = v_s$$  \hspace{1cm} (2.102)

Since the current through the secondary winding is zero, calculated secondary flux linkage when referred to primary side, $\lambda'_s = N_p/N_s \lambda_s$, is equal to the magnetizing flux linkage, $\lambda_m$, given by

$$\lambda_m = \frac{N_p}{N_s} \lambda_s$$  \hspace{1cm} (2.103)

Note the turns ratio $N_p/N_s$ is an unknown at this point and will be determined along with data collected in the second step of this procedure.

In second step, the secondary side of the transformer is excited with sinusoid current source, keeping primary open. Like the previous case, secondary winding current and voltage across both the windings are measured. The flux linkages in the primary winding is calculated using

$$\frac{d\lambda_p}{dt} = v_p$$  \hspace{1cm} (2.104)

Finally, the turns ratio is chosen such that $\lambda'_s$ vs $i_p$ plot (in the secondary open-circuited case) matches with $\lambda_p$ vs $i'_s$ plot (with the primary open-circuited case) where the referred secondary current is obtained using the relation $i'_s = N_s/N_p i_s$. 
In order to determine the turns ratio of the prototype transformer using the two-step procedure, a constant current source California Instruments, 4500CS is used to excite the transformer windings at 1 kHz. The measurement set up used includes Yokogawa DL850 oscilloscope with an analog voltage input module 720210. Yokogawa 701930 current probe is used for measuring current and Tektronix P5200 high-voltage differential probe is used for measuring voltage. The probes may cause DC offset errors in the measured AC signals. For this reason, the mean value of the measured data (taken over a cycle) is subtracted from each data point prior to analyzing the data.

The waveforms obtained for first step (primary winding excitation) are as shown in Figure 2.22 (‘measured’). Therein, the measured data is truncated to one switching cycle and compensated for the DC offset error. Further, to remove the high-frequency noise, the waveforms are decomposed into a Fourier series and reconstructed using limited number of harmonics, as shown in Figure 2.22 (‘reconstructed’). Using (2.102),

![Graph](image)

Fig. 2.22. Transformer open-circuit test, primary excitation
the flux linkage of secondary winding, $\lambda_s$ is calculated by numerically integrating the reconstructed secondary voltage waveform over one switching cycle. The initial condition for the integration is determined by requiring the mean value of the flux linkages over a cycle to be zero.

The DC offset compensated waveforms and Fourier series reconstructed waveforms as obtained in second step are shown in Figure 2.23. The flux linkage in the primary winding is calculated numerically using reconstructed primary waveform data and (2.104). Here again, the initial conditions are determined such that mean value of $\lambda_p$ over a cycle is zero.

After the flux linkage waveforms ($\lambda_s$ from first step and $\lambda_p$ from the second step) are obtained, the turns ratio is chosen such that $\lambda'_s$ vs $i_p$ plot (in the secondary open-circuited case) matches with $\lambda_p$ vs $i'_s$ plot (with the primary open-circuited case). The turns ratio obtained by this process for matching plots shown in Figure 2.24 is $N_p/N_s = 1.312$. 

![Fig. 2.23. Transformer open-circuit test, secondary excitations](image-url)
Once the turns ratio is obtained, $L_m$ is obtained as slope of the $\lambda_m$ vs $i_p$ as shown in Figure 2.25. To determine the initial slope of the curve in Figure 2.25, anhysteretic curve of the magnetizing flux linkage is obtained using the procedure outlined in [34]. The anhysteretic curve in the linear region (for $i_p \in [-0.5, 0.5]$A) is then curve fit to line to obtain the magnetizing inductance measured from primary side. These characteristics are shown in Figure 2.25. Using this procedure, $L_m$ is measured to be 20.1 mH.

The prototype transformer total leakage inductance was measured using short circuit test. Due to the small values of leakage inductance, it is difficult to experimentally measure $L'_{ls}$ and $L_{lp}$ individually. Therefore, the transformer total leakage inductance is measured using short-circuit test, where the impedance of magnetizing inductance can be assumed to be negligible. The voltage across the primary winding and resulting current through it are measured. 

Fig. 2.24. $\lambda_m$ vs $i$ curves from primary and secondary excitation
Fig. 2.25. $\lambda_m$ vs $i_p$ curve from open circuit test with primary side excitation

A sinusoid current excitation at 5 kHz was applied across the primary winding with secondary winding shorted, using a high speed bi-polar power supply from Matsusada Precision, DOSF60. A relatively low excitation frequency was chosen to make sure the static inductance was measured. The waveforms recorded for this test after correcting for DC offset are shown in Figure 2.26. To remove noise in measured signals, the current and voltage waveforms are decomposed using Fourier series and reconstructed. For the short circuit condition, using the relation between input voltage to input current, time rate of total leakage flux linkage, $\lambda_{lkg}$, of the transformer is given by

$$\frac{d\lambda_{lkg}}{dt} = v_p - (r_p + r'_s) i_p$$  (2.105)

The DC offset in the $\lambda_{lkg}$ is compensated and plotted against the primary current in Figure 2.27.

To determine the leakage inductance from Figure 2.27, an anhysteretic curve of the leakage flux linkage is obtained using the procedure outlined in [34]. The anhysteretic
Fig. 2.26. Excitation waveforms for short-circuit test

Fig. 2.27. Leakage flux linkage vs primary current for short-circuit test
data in the region of origin is then fit to a line to obtain the total leakage inductance, as shown in Figure 2.27. The slope of the line, 28.8 µH, is the measured value of the prototype transformer total leakage inductance.

The T-equivalent circuit parameters measured using the above procedure are listed in Table 2.7.

### Table 2.7.
Prototype transformer T-equivalent circuit parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N_p/N_s)</td>
<td>1.31 µH</td>
</tr>
<tr>
<td>(r_p)</td>
<td>0.18 Ω</td>
</tr>
<tr>
<td>(r_s)</td>
<td>0.15 Ω</td>
</tr>
<tr>
<td>(L_{lp} + L'_{ls})</td>
<td>28.81 µH</td>
</tr>
<tr>
<td>(L_m)</td>
<td>20.10 mH</td>
</tr>
</tbody>
</table>

The total leakage inductance of the prototype transformer as obtained by performing 3D FEA using Ansys Maxwell magnetostatic analysis is 28.4 µH. By symmetry, only one eighth of the transformer as shown in Figure 2.28 is simulated in 3D FEA. The transformer energy obtained for zero magnetizing current excitation is used to calculate total leakage inductance using (2.56).

The total leakage inductance as estimated by the proposed analysis in Section 2.2 is 28.7 µH. This is in close agreement with the measured value of 28.8 µH and 28.4 µH as obtained using 3D FEA. With the same computational resource as previously mentioned, the time taken for 3D FEA is 320 s. The total leakage inductance of the prototype transformer as estimated by the proposed analysis is faster than 2D and 3D FEA by 510 and 1600 times, respectively.

The proposed method is of comparable accuracy to the 3D FEA. However, the field analysis of exterior coil region was simplified using method of images, which reflects that the core outer dimensions does not have any effect on the leakage inductance. This is true because leakage inductance is mostly dependent on the spacing between
the core and coils. To investigate this further, the core end-leg dimension, $w_{cb}$ as shown in Figure 2.1, is varied, keeping rest of the dimensions the same as the prototype transformer, as listed in Table 2.5. The leakage inductance as calculated using 3D FEA for different values of $w_{cb}$ are listed in Table 2.8. The variation in the $d_c$ resulted in almost no (less than 0.06%) variation in leakage inductance when evaluated using 3D FEA. This indicates the use of the method of images is reasonable for field analysis of transformer exterior region.

Table 2.8.
Core outer edge effect on leakage inductance

| $w_{cb}$ (mm) | $L_{lp} + L'_{ls}$ ($\mu$H) |
|---------------|-----------------|---|
| 76.2          | 28.37           |
| 101.6         | 28.41           |
| 152.4         | 28.34           |
2.4.4 Core-type Transformer MEC Validation

The core-type transformer MEC presented in Section 2.1 is validated in this section on the prototype transformer using an open-circuit test.

The hysteretic and anhysteretic plots of magnetizing flux linkage versus primary current, as obtained by exciting prototype transformer primary and keeping secondary open are repeated in Figure 2.29. Additionally, the core flux values for corresponding values of measured primary current and zero secondary current are calculated by solving MEC shown in Figure 2.7. The magnetizing flux linkage, \( \lambda_m \) is then obtained by using

\[
\lambda_m = N_{pcx}N_{pcf}\Phi
\]  

(2.106)

The magnetizing flux linkage obtained using MEC, \( \lambda_{m,\text{MEC}} \), is shown in Figure 2.29. It can be observed from the slope of curves in this figure, that the prototype transformer magnetizing inductance is smaller than its design value. This is because of the prototype transformer core construction. Two pairs of C-cores (altogether 4) are stacked to form a rectangular core, introducing two air-gaps (each \( g = 80\mu\text{m} \) thick) along the flux path at each of the intersections of two core pairs due to surface roughness. The corresponding air-gap reluctance, \( R_g \),

\[
R_g = \frac{g}{A_{cdl}\mu_0}
\]  

(2.107)

in included in MEC shown in 2.7, in addition to the core reluctances given by (2.6) and (2.7). Using the hence modified MEC, the air-gap rectified magnetizing flux linkage, \( \lambda^*_m \), is calculated. The revised plot of \( \lambda^*_m-i_p \), also shown in Figure 2.29, matches with the experimental data.

2.4.5 Validation of High-Frequency Magnetic Effects

A core-type transformer design with design parameters as listed in Table 2.9 is used for validation of frequency dependent leakage inductance calculation in this section.
This design is similar to the one listed in Table 2.5 except for different number of turns in each winding and the resulting coils widths, \( w_{pw} \) and \( w_{sw} \). This choice of design is to ensure the design has complete secondary and primary windings layers, so that end-field effects are avoided and the assumptions made for 1D magnetic field analysis in Section 2.3 are satisfied for the core-type transformer core interior region.

The expressions (2.76), (2.84) and (2.89) along with the definition in (2.63) are used to plot magnitude of magnetic field intensity along the horizontal line passing through the interior coils, with \( c_{pp}/2 \leq x \leq w_{cs}/2 \) as shown in Figure 2.30. The fields are calculated for zero magnetic current excitation at 20 kHz and 1A peak current through the primary winding. The analytically calculated fields using static and harmonic analysis are compared to the fields obtained using 2D FEA simulation. The decrease in the magnetic field intensity magnitude within each layer is observed to be dominant at 20 kHz as shown in Figure 2.30. The transformer geometry used to
Table 2.9.
Prototype transformer dimensions II

<table>
<thead>
<tr>
<th>Core Parameter</th>
<th>Value</th>
<th>Primary winding Parameter</th>
<th>Value</th>
<th>Secondary winding Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>Ferrite-Ptype</td>
<td>Material</td>
<td>Copper</td>
<td>Material</td>
<td>Copper</td>
</tr>
<tr>
<td>$l_{cc}$</td>
<td>50.8 mm</td>
<td>$N_{pl}$</td>
<td>4</td>
<td>$N_{st}$</td>
<td>3</td>
</tr>
<tr>
<td>$w_{cs}$</td>
<td>63.4 mm</td>
<td>$N_{ptl}$</td>
<td>14</td>
<td>$N_{stl}$</td>
<td>13</td>
</tr>
<tr>
<td>$d_{cs}$</td>
<td>50.8 mm</td>
<td>$w_{pw}$</td>
<td>5.2 mm</td>
<td>$w_{sw}$</td>
<td>4.9 mm</td>
</tr>
<tr>
<td>$w_{c}$</td>
<td>114.2 mm</td>
<td>$d_{pw}$</td>
<td>48.5 mm</td>
<td>$d_{sw}$</td>
<td>49.5 mm</td>
</tr>
<tr>
<td>$l_{c}$</td>
<td>51.0 mm</td>
<td>$r_{pi}$</td>
<td>20.1 mm</td>
<td>$r_{si}$</td>
<td>13.9 mm</td>
</tr>
<tr>
<td>$w_{cb}$</td>
<td>25.4 mm</td>
<td>$r_{po}$</td>
<td>24.1 mm</td>
<td>$r_{so}$</td>
<td>18.3 mm</td>
</tr>
<tr>
<td>$w_{csc}$</td>
<td>0 mm</td>
<td>$c_{sc}$</td>
<td>1.2 mm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w_{ce}$</td>
<td>25.4 mm</td>
<td>$c_{ps}$</td>
<td>1.8 mm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c_{ps}$</td>
<td>40.4 mm</td>
<td>$c_{pp}$</td>
<td>1.2 mm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c_{pv}$</td>
<td>1.2 mm</td>
<td>$c_{sv}$</td>
<td>0.65 mm</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

perform 2D FEA simulation in Ansys Maxwell is as shown in Figure 2.31. An eddy current solver type was used with the layers of each winding individually excited.

The overall decrease in the magnitude of $H$ field across coil regions at high frequencies translates to decrease in energy stored in the interior and exterior coil regions. The core interior region energies as calculated using static and harmonic analysis are compared to that obtained using 2D FEA simulation over a high-frequency range in Figure 2.32. As demonstrated in the Figure 2.32, the interior region energy significantly decreases with frequency. Next, the exterior region energy as a function of frequency is calculated using a scaling factor as given by (2.97)-(2.98). The core exterior region energy hence calculated is compared to that obtained using 2D FEA simulation over a high-frequency range in Figure 2.33. This further demonstrate the decrease in transformer leakage inductance at high frequencies.
Fig. 2.30. Magnetic field intensity magnitude across center of coil regions at 20 kHz for zero magnetizing current excitation.

Fig. 2.31. Core-type transformer design considered for harmonic analysis validation.
Fig. 2.32. Magnetic field energy per unit length stored in transformer interior region for zero magnetizing current excitation.

Fig. 2.33. Magnetic field energy per unit length stored in transformer exterior region for zero magnetizing current excitation.
3. TRANSFORMER LOSSES

Transformer losses can be categorized into core and conductor losses. In case of a high-frequency transformer, conductor losses become particularly important because the AC effects become accentuated. Thus, it is important to address high frequency effects in the windings when designing a high-frequency transformer. The goal of this chapter is to set forth a method to estimate the total loss in a high frequency transformer accurately and computationally efficiently.

The core-type transformer has four coils, two for the primary and two for the secondary as shown in Figure 3.1. The exterior primary and secondary coil regions are denoted by pe and se respectively, and the interior primary and secondary coil regions are denoted by pi and si respectively.

Similar to the magnetic analysis in Chapter 2, the physical distinction between exterior and interior region of the coils for the two cases of core end-leg geometries, is as shown in Figure 3.2. The average length of a turn in the exterior and interior regions are repeated here for convenience. For the case of rounded core end-leg corners, as shown in Figure 3.2(a), the average turn length in interior and exterior regions are

\[ l_{xi} = l_c + \alpha_x (r_{xi} + r_{xo}) \]  \hspace{1cm} (3.1)

\[ l_{xe} = l_c + 2w_{cec} + (\pi - \alpha_x) (r_{xi} + r_{xo}) \]  \hspace{1cm} (3.2)

where \( x \) in the subscript is denoted by \( p \) for primary or \( s \) for secondary coil accordingly, and angle \( \alpha \) is defined as

\[ \alpha_x = \arcsin \left( \frac{2r_c}{r_{xi} + r_{xo}} \right) \]  \hspace{1cm} (3.3)

For the case with core end-leg with inert material to provide conductor bend radius, shown in Figure 3.2(b), the lengths of each region are repeated in Table 3.1.
The proposed loss analysis uses (3.1)-(3.3) or Table 3.1 to include the 3-Dimensional (3D) geometry of the transformer.

Being consistent with the notation used in Chapter 2, the notation referring to the windings is repeated here for reference. The number of turns in each winding...
Table 3.1.
Core-type transformer winding interior and exterior regions lengths

<table>
<thead>
<tr>
<th>Winding</th>
<th>Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interior region</td>
<td>( l_{xi} = l_c )</td>
</tr>
<tr>
<td>Exterior region 1</td>
<td>( l_{xe1} = l_c )</td>
</tr>
<tr>
<td>Exterior region 2</td>
<td>( l_{xe2} = w_{cec} )</td>
</tr>
<tr>
<td>Exterior region 3</td>
<td>( l_{xe3} = \pi (r_{xi} + r_{xo}) / 4 )</td>
</tr>
<tr>
<td>Exterior region 4</td>
<td>( l_{xe4} = \pi (r_{xi} + r_{xo}) / 4 )</td>
</tr>
</tbody>
</table>

is denoted by \( N_x \), where \( x \) may be ‘p’ for primary or ‘s’ for secondary. In case of parallel connection of the two coils, \( N_{xcp} \) is assigned a value of 2, or else a value of 1 is assigned. In case of series connection of the two coils, \( N_{xcs} \) is assigned a value of 2, or else a value of 1 is assigned.

The illustration of conductor arrangement in a coil is repeated in Figure 3.3. Proximity effect loss in the windings depends on the inter spacing between the conductors. For the conductor arrangement as illustrated in 3.3, the interspacing between the conductors along the coil width, \( h_x \), is given by

\[
h_{xw} = \begin{cases} 
0, & N_{xl} = 1 \\
\frac{(w_{xw} - 2N_{xtl}r_{ps})}{N_{xtl} - 1}, & N_{xl} > 1 
\end{cases}
\]  
(3.4)

The interspacing between the conductors along the coil depth, \( v_x \), is given by

\[
v_{xw} = \begin{cases} 
0, & N_{xtl}N_{xpr} = 1 \\
\frac{(d_{xw} - 2N_{xtl}N_{xpr}r_{ps})}{N_{xtl}N_{xpr} - 1}, & N_{xtl}N_{xpr} > 1 
\end{cases}
\]  
(3.5)

Note, the spacing between the conductors in a coil is same throughout the coil exterior and interior regions.

In this chapter, analytical methods used to calculate total loss in the case of a high-frequency, core-type transformer are outlined. Section 3.1 describes core loss estimation using the Modified Steinmetz Equation (MSE). In the case of conductor
losses, losses can be categorized into DC losses and AC losses. The estimation of DC losses is presented in Section 3.2. The loss due to skin-effect is presented in Sections 3.3. Section 3.4 focuses on the estimation of loss due to proximity effect. The validation of winding loss model is presented in Section 3.5, followed by conclusions in Section 3.6.

3.1 Core Loss

AC current through the transformer windings causes alternating flux density in the core material. Alternating flux through the core results in losses, categorized as eddy current loss due to localized circulating currents and hysteresis loss due to domain wall movements. Since the transformer is designed for a high-frequency application, ferrites are generally used for the transformer core. Due to high resistivity of the ferrite material, eddy current loss is negligible making hysteresis loss dominant. Losses in ferrite materials may be calculated using Modified Steinmetz Equation (MSE) [35]. In MSE, the impact of non-sinusoidal flux densities on core loss is captured by defining the equivalent frequency term, $f_{eq}$ as

$$f_{eq} = \frac{2}{\Delta B^2 \pi^2} \int_0^T \left( \frac{dB}{dt} \right)^2 dt$$  \hspace{1cm} (3.6)
The term on the right hand side of (3.6) includes integral of square of time rate of change of flux density over one cycle period $T$ and flux density term, $\Delta B$, defined as

$$\Delta B = B_{mx} - B_{mn} \tag{3.7}$$

where $B_{mx}$ and $B_{mn}$ are the maximum and the minimum values of flux density within one cycle. Using (3.6) and the curve fit parameters $k_h$, $\alpha$ and $\beta$ as defined in case of Steinmetz equation, power loss density by MSE is calculated as

$$p_h = k_h \left( \frac{f_{eq}}{f_b} \right)^{\alpha-1} \left( \frac{\Delta B}{2B_b} \right)^{\beta} \frac{f}{f_b} \tag{3.8}$$

where $f$ is the fundamental switching frequency of the DC-DC converter, $B_b$ and $f_b$ are base flux density and base frequency respectively, used to estimate the curve fit parameters $k_h$, $\alpha$ and $\beta$ for a given magnetic material.

In order to evaluate core loss, transformer currents are estimated using time domain analysis as will be described in Chapter 4. The corresponding flux density waveforms through the transformer core legs are obtained using MEC described in Section 2.1 of Chapter 2. Using the resulting flux densities in the core end leg and base leg, the corresponding core loss densities $p_e$ and $p_{cl}$ are estimated by employing equations (3.6)-(3.8). The transformer core loss is then calculated as

$$P_{cl} = 2 \left( p_{bl} A_{bl} l_{bl} + p_{el} A_{el} l_{el} \right) \tag{3.9}$$

where $A_{bl}$ and $A_{el}$ are the cross sectional areas of base leg and end leg respectively. $l_{bl}$ and $l_{el}$ are the mean lengths of the base leg and end leg respectively. The core leg dimensions are as described in Section 2.1 of Chapter 2.
3.2 DC Resistance

The losses in the conductor can be divided into DC loss and AC losses. These losses are dependent on the currents through the winding, geometry and temperature the winding. The conductor conductivity that is used to calculate the resistances is dependent on the winding temperature, given by the relation

\[
\sigma_x = \frac{\sigma_{x0}}{1 + \alpha_x(T_{xmn} - T_0)}
\]  

(3.10)

where \(\sigma_0\) denotes the material conductivity as measured at \(T_0 = 293.15\, K\), \(\alpha_x\) denotes the thermal coefficient of conductivity and \(T_{xmn}\) denotes the mean temperature of the winding. In (3.10), \(x\) may be replaced with ‘p’ for primary and ‘s’ for secondary.

The estimation of DC loss is presented in this section.

The DC resistance of a winding with conductor length, \(l_{cd}\) and cross section area, \(a_{cd}\) is given by

\[
r_{dc} = \frac{l_{cd}}{\sigma_{cd}a_{cd}}
\]  

(3.11)

where \(\sigma_{cd}\) is the material conductivity. Using (3.11), DC resistance of the primary winding, denoted by \(r_p\) is calculated as

\[
r_p = \frac{N_{pcs}N_{pcl}l_{pt}}{N_{pcp}N_{ppr}\sigma_p a_{ps}}
\]  

(3.12)

where \(l_{pt}\) denotes the average length of a primary coil turn. The secondary winding resistance, \(r_s\) is calculated using (3.11) as

\[
r_s = \frac{N_{scs}N_{scd}l_{st}}{N_{scp}N_{spr}\sigma_s a_{ss}}
\]  

(3.13)

where \(l_{st}\) denotes the average length of a secondary coil turn. The DC conductor loss is then given by

\[
P_{dc} = P_{dcp} + P_{dcs}
\]  

(3.14)
where primary and secondary windings DC loss, denoted by $P_{dep}$ and $P_{dcs}$ receptively, are given by

$$P_{dep} = \frac{1}{2} \sum_{n=1}^{N} I_{p,i}^2 r_p$$

(3.15)

$$P_{dcs} = \frac{1}{2} \sum_{n=1}^{N} I_{s,i}^2 r_s$$

(3.16)

In (3.15) and (3.16), $I_{p,i}$ and $I_{s,i}$ are the peak magnitudes of the current harmonics through primary and secondary windings respectively.

### 3.3 Skin Effect

AC winding currents create alternating fields that pass through the conductor cross section resulting in internal eddy-currents. These eddy-currents result in increased loss.

Depending on the source of the alternating fields, losses due to AC currents are categorized into skin-effect loss and proximity effect loss. The method to estimate loss due to skin-effect is presented in this section followed by analysis of proximity effect loss in the following section.

The tendency of high frequency currents to flow through the outer layer of the conductor is called skin effect. This results in resistance greater than DC resistance because of the reduced effective cross sectional area of the conductor. The increased conductor resistance results in higher winding loss, which increases with increasing frequency. Often the conductor size at a given frequency is chosen taking skin effect into consideration using multiple parallel strands instead of a single conductor. The radius of the conductor is chosen such that it is lower than the skin depth, forcing the current to flow through the complete cross section of the conductor. Skin depth, $\delta$, is defined as function of frequency, $f$, as

$$\delta = \frac{1}{\sqrt{\pi \sigma_{cd} f}}$$

(3.17)
where $\sigma_{cd}$ and $\mu$ are conductivity and magnetic permeability of the conductor material. Even if strand radius is chosen to be smaller than skin depth at the fundamental frequency of the conductor currents, due to the presence of higher harmonics in actual operation, the increase in conductor resistance due to skin effect is inevitable.

The resistance due to skin effect along with DC resistance is calculated using Kelvin functions [8] as

$$r_{skn} = \frac{r_0 l_{cd}}{\sqrt{2\pi r_{cd}}} \frac{\text{ber}_0(\tilde{r})\text{bei}'_0(\tilde{r}) - \text{bei}_0(\tilde{r})\text{ber}'_0(\tilde{r})}{\text{ber}^2_0(\tilde{r}) + \text{bei}^2_0(\tilde{r})}$$  \hspace{1cm} (3.18)

where $l_{cd}$ and $r_{cd}$ denote the conductor length and radius respectively. The fundamental resistance, $r_0$ is expressed as

$$r_0 = \frac{1}{\sigma_{cd}\delta}$$  \hspace{1cm} (3.19)

In (3.18) $\tilde{r}$ is a unit less scaled radius defined in terms conductor radius, $r_{cd}$ as

$$\tilde{r} = \frac{\sqrt{2}r_{cd}}{\delta}$$  \hspace{1cm} (3.20)

Note the resistance, $r_{skn}$ varies with frequency. The transformer ac resistance due to skin effect combined with DC resistance is calculated for primary and secondary windings at each harmonic frequency of the winding currents. These are denoted as $r_{psk}(\omega_i)$ and $r_{ssk}(\omega_i)$ respectively, where $\omega_i$ denotes angular frequency of the $i^{th}$ current harmonic. Using (3.17)-(3.20), $r_{psk}(\omega_i)$ and $r_{ssk}(\omega_i)$ are given by

$$r_{psk}(\omega_i) = \frac{r_{0pi} N_{pcps} N_{pcI}}{\sqrt{2N_{pcp} N_{pcr}}} \frac{\text{ber}_0(\tilde{r}_{pi})\text{bei}'_0(\tilde{r}_{pi}) - \text{bei}_0(\tilde{r}_{pi})\text{ber}'_0(\tilde{r}_{pi})}{\text{ber}^2_0(\tilde{r}_{pi}) + \text{bei}^2_0(\tilde{r}_{pi})}$$  \hspace{1cm} (3.21)

$$r_{ssk}(\omega_i) = \frac{r_{0si} N_{scs} N_{scI}}{\sqrt{2N_{scp} N_{spr}}} \frac{\text{ber}_0(\tilde{r}_{si})\text{bei}'_0(\tilde{r}_{si}) - \text{bei}_0(\tilde{r}_{si})\text{ber}'_0(\tilde{r}_{si})}{\text{ber}^2_0(\tilde{r}_{si}) + \text{bei}^2_0(\tilde{r}_{si})}$$  \hspace{1cm} (3.22)

where $r_{xs}$ denotes the strand radius, and $r_{0xi}$ and $\tilde{r}_{xi}$ are fundamental radius and scaled radius at $\omega_i$ given by (3.19) and (3.20) respectively. Noting the fact that
DC resistances of the windings are included in (3.21) and (3.22), and assuming \( N \) harmonic frequencies of primary and secondary currents are present, sum of skin effect power loss and DC loss is then given by

\[
P_{dc} + P_{SE} = \frac{1}{2} \sum_{n=1}^{N} \left( I_{p,i}^2 r_{psk} (\omega_i) + I_{s,i}^2 r_{ssk} (\omega_i) \right)
\]

(3.23)

where \( I_{p,i} \) and \( I_{s,i} \) denote the peak amplitudes of primary and secondary currents at \( i^{th} \) harmonic frequency.

### 3.4 Proximity Effect

Another high frequency effect observed in the conductors is proximity effect. In the case of skin effect, the current density in a conductor is influenced by the magnetic field induced by the current flowing through the conductor. Proximity effect is due to the field generated by the current through the adjacent conductors.

Consider the case of array of round conductors as shown in Figure 3.4. Currents through the adjacent conductors create a magnetic field that results in eddy-currents in the conductor, increasing the conductor resistance and hence increased conductor losses. Proximity effect is dependent on the space between conductors and current through the conductors. These parameters are used to determine the field distribution across the conductors and hence calculate the resistance due to proximity effect.

Two analytical methods used to calculate proximity effect loss are Dowell’s method and the Ferreira method. Dowell’s method uses the approximation that round conductors are replaced with equivalent rectangular foils. The Ferreira method assumes 1-D field for the analysis which gives exact results for a single conductor case but fails to include inter spacing between conductors. The drawback with these two methods is that they are applicable to limited range of conductor diameter and frequency. Another way to calculate the losses is to use numerical methods such as Finite Ele-
ment Analysis (FEA). This yields accurate results but is computationally expensive in the context of the optimization based design.

![Diagram of round conductors with spacing and dimensions labeled](image)

**Fig. 3.4. Spacing between round conductors**

An empirical method for calculating proximity effect loss is presented in [3]. This method uses static field simulation with semi-empirical expressions to determine proximity effect loss. In the case of single conductor highlighted in Figure 3.4, assuming the magnetic field intensity across the conductor is sinusoidal with respect to time and has peak amplitude $H$, proximity effect loss per unit length of the single conductor is given by

$$p_{PE} = \frac{\hat{G}H^2}{\sigma_{cd}}$$

(3.24)

where $\hat{G}$ is the proximity loss factor and $\sigma_{cd}$ is the conductivity of the conductor material. The proximity loss factor, $\hat{G}$ is defined as a function of conductor spacing in the horizontal direction, $h$, and vertical direction, $v$, conductor diameter, $d$, and frequency of the magnetic field intensity across the coil using empirical expressions. This term will be discussed in detail in Section 3.4.1.

The magnitude of field intensity across the highlighted conductor, $H$, may be considered as a resultant of the currents through the conductors surrounding the highlighted conductor. Hence, it depends both on the coil geometry and the current.
through the conductors. Assuming sinusoid current with peak amplitude $I$ flows through each conductor in the coil, the normalized field intensity, $\hat{H}$ is defined as

$$\hat{H} = \frac{H}{I}$$  \hspace{1cm} (3.25)

By normalizing the field quantity, $\hat{H}$ can be estimated using static field analysis and hence determined by the winding geometry alone.

The proximity effect loss for all the conductors in the winding is estimated by using spatial average of the field across the coil cross section. In particular, the spatial average of the squared normalized field intensity is defined for region, $r$ as

$$\left\langle \hat{H}^2 \right\rangle = \frac{1}{S_r} \int_r \hat{H}^2 dS$$ \hspace{1cm} (3.26)

where $S_r$ denotes the area of the region $r$. Substituting $H^2$ using (3.24) and (3.26) in (3.24), the proximity effect loss per unit length in one conductor is given by

$$p_{PE} = \frac{\hat{G} \left\langle \hat{H}^2 \right\rangle I^2}{\sigma_{cd}}$$ \hspace{1cm} (3.27)

Assuming there are $N$ conductors in the coil, with average turn length denoted by $l_t$, the total proximity effect loss in the conductors present in region, $r$, $P_{PE,r}$, is given by

$$P_{PE} = \frac{Nl_t \hat{G} \left\langle \hat{H}^2 \right\rangle I^2}{\sigma_{cd}}$$ \hspace{1cm} (3.28)

The equivalent resistance of the coil due to proximity effect loss is defined as

$$r_{PE,r} = \frac{2Nl_t \hat{G} \left\langle \hat{H}^2 \right\rangle}{\sigma_{cd}}$$ \hspace{1cm} (3.29)
Note that the only frequency dependent term in the definition of \( r_{PE} \) by (3.29) is proximity loss factor, \( \hat{G} \). The proximity effect loss factor, \( \hat{G} \) is described in the next section.

### 3.4.1 Proximity Effect Loss Factor

Proximity loss factor, \( \hat{G} \) captures the effect of frequency of the current and winding geometry on the observed proximity effect. It is derived using empirical expressions [10,11], by calculating the proximity effect loss in a single conductor using FEA and comparing to (3.24). In particular, in [11] \( \hat{G} \) is expressed as weighted average of two functions, expressed as

\[
\hat{G} = (1 - w)\hat{G}_1 + w\hat{G}_2
\]

(3.30)

where \( \hat{G}_1 \) denotes the modified Dowell’s function, \( \hat{G}_2 \) denotes dual slope function and \( w \) is a weighting factor. These functions are defined in [10] as

\[
\hat{G}_1 = \frac{3\pi}{16}k^{-3}X \frac{\sinh(kX) - \sin(kX)}{\cosh(kX) - \cos(kX)}
\]

(3.31)

\[
\hat{G}_2 = \frac{\pi}{32} \frac{X}{X^{-3} + b^3}
\]

(3.32)

In (3.31) and (3.32), \( X \) is defined as

\[
X = d\sqrt{\pi\sigma_{ed}\mu}\]

(3.33)

where \( d \) is the conductor diameter.

The other parameters, weighting factor, \( w \) in (3.30) and \( k \) and \( b \) in (3.31) are derived empirically using a generic function \( f(Y, s_1, s_2, q) \), given by

\[
f(Y, s_1, s_2, q) = \frac{s_1 - s_2}{Y^{-1} + q^{-1}} + s_2
\]

(3.34)
In (3.34), $Y$ is normalized spacing between conductors, denotes either $\hat{h} = h/d$ or $\hat{v} = v/d$, where the variables $h$ and $v$ denote the spacing between the conductors as shown in Figure 3.4. For the coil conductor arrangement show in Figure 3.3, $h$ and $v$ are calculated using (3.4) and (3.5). The other parameters $s_1$, $s_2$ and $q$ are curve fit values obtained from thousands of FEA of single conductor as given in [10,11]. The expressions used for curve fitting are

$$k(\hat{v}, \hat{h}) = f(\hat{h}, f(\hat{v}, s_{1k,1}, s_{2k,1}, q_{k,1}), f(\hat{v}, s_{1k,2}, s_{2k,2}, q_{k,2}), f(\hat{v}, s_{1k,3}, s_{2k,3}, q_{k,3}))$$  \hspace{1cm} (3.35)

$$b(\hat{v}, \hat{h}) = f(\hat{v}, f(\hat{h}, s_{1b,1}, s_{2b,1}, q_{b,1}), f(\hat{h}, s_{1b,2}, s_{2b,2}, q_{b,2}), f(\hat{h}, s_{1b,3}, s_{2b,3}, q_{b,3}))$$  \hspace{1cm} (3.36)

$$w(\hat{v}, \hat{h}) = \hat{h}w_1(\hat{v}) + w_2(\hat{v})$$  \hspace{1cm} (3.37)

$$w_1(\hat{v}) = c_{11} - (u_{11} - u_{01}e^{\frac{\hat{v}}{Y_{01}}})^2$$  \hspace{1cm} (3.38)

$$w_2(\hat{v}) = c_{21} - (u_{21} - u_{02}e^{\frac{\hat{v}}{Y_{02}}})^2$$  \hspace{1cm} (3.39)

The curve fit values listed in [11] are repeated in Table 3.2-3.4.

**Table 3.2.**
Parameters for calculating $w$

<table>
<thead>
<tr>
<th>$c_{11}$</th>
<th>$u_{11}$</th>
<th>$u_{01}$</th>
<th>$q_{01}$</th>
<th>$Y_{01}$</th>
<th>$Y_{02}$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.1558</td>
<td>0.3477</td>
<td>1.0673</td>
<td>1.3839</td>
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</tr>
</tbody>
</table>

**Table 3.3.**
Parameters for calculating $k$

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<th>$s_{1k,j}$</th>
<th>$s_{2k,j}$</th>
<th>$q_{k,j}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$j = 1$</td>
<td>1.0261</td>
<td>0.8149</td>
</tr>
<tr>
<td>$j = 2$</td>
<td>0.4732</td>
<td>0.8023</td>
</tr>
<tr>
<td>$j = 3$</td>
<td>0.0930</td>
<td>0.2588</td>
</tr>
</tbody>
</table>
Table 3.4.
Parameters for calculating \( b \)

<table>
<thead>
<tr>
<th>( j )</th>
<th>( s_{1b,j} )</th>
<th>( s_{2b,j} )</th>
<th>( q_{b,j} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.0037</td>
<td>0.0432</td>
<td>-0.0661</td>
</tr>
<tr>
<td>2</td>
<td>1.8167</td>
<td>0.0074</td>
<td>0.2195</td>
</tr>
<tr>
<td>3</td>
<td>0.7053</td>
<td>0.8378</td>
<td>23.8755</td>
</tr>
</tbody>
</table>

3.4.2 Proximity Effect Loss in a Two Windings Case

Magnetic components often have multiple windings depending on its functionality. The current in one winding creates a magnetic field that influences the current in the adjacent winding. Compared to the single winding case as described in Figure 3.4, the magnetic field intensity experienced by a conductor is due to the net effect of the currents through the conductors in the same winding as well as the conductors in the adjacent windings. In this section, proximity effect loss estimation in a two winding system (with one coil each) is examined.

The empirical method of proximity effect loss calculation described in single winding case is extended to two winding case in [12]. Field intensity values were treated as scalars in [12], but going by an earlier related work by the same authors in [9] on flux density vectors, following analogy can be made in case of field intensity. The field intensity is expressed in the 2D space as

\[
\mathbf{H} = H_x \hat{x} + H_y \hat{y}
\]

(3.40)

where \( \hat{x} \) is a unit vector in horizontal direction and \( \hat{y} \) is a unit vector in vertical direction. The field intensity components along horizontal and vertical directions are denoted as \( H_x \) and \( H_y \) respectively. In the case of field generated by a time varying
sinusoid current source with peak amplitude $I$, current normalized field vector is given by

$$\mathbf{H} = \frac{H_x \mathbf{x} + H_y \mathbf{y}}{I} \quad (3.41)$$

The field matrix in [12] has terms obtained by using dot product. The dot product of two field vectors $\mathbf{H}_1 = H_{1x} \mathbf{x} + H_{1y} \mathbf{y}$ and $\mathbf{H}_2 = H_{2x} \mathbf{x} + H_{2y} \mathbf{y}$ is defined as

$$\mathbf{H}_1 \cdot \mathbf{H}_2 = H_{1x}H_{2x} + H_{1y}H_{2y} \quad (3.42)$$

The dot product of a vector, $\mathbf{H}$, by itself is denoted as $\mathbf{H}^2 = \mathbf{H} \cdot \mathbf{H}$.

Using the above notation and [12], the proximity effect loss for a two winding system (one coil each) carrying sinusoid currents with peak amplitudes $I_1$ and $I_2$ is given as

$$P_{PE} = \frac{G_1 N_1 N_{1s} l_1}{\sigma_1} \begin{bmatrix} I_1 & I_2 \end{bmatrix} \left\langle \begin{bmatrix} \mathbf{H}_1^2 & \mathbf{H}_1 \cdot \mathbf{H}_2 \\ \mathbf{H}_1 \cdot \mathbf{H}_2 & \mathbf{H}_2^2 \end{bmatrix} \right\rangle_1 \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} + \frac{G_2 N_2 N_{2s} l_2}{\sigma_2} \begin{bmatrix} I_1 & I_2 \end{bmatrix} \left\langle \begin{bmatrix} \mathbf{H}_1^2 & \mathbf{H}_1 \cdot \mathbf{H}_2 \\ \mathbf{H}_1 \cdot \mathbf{H}_2 & \mathbf{H}_2^2 \end{bmatrix} \right\rangle_2 \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad (3.43)$$

where subscript denotes the winding 1 or winding 2. $N_x$ and $N_{xs}$ are number of turns and number of parallel strands respectively in the winding $x$. $l_x$ denotes the average length of each turn in the winding $x$. The current normalized magnetic field intensity due to current $I_x$ is denoted by $\mathbf{H}_x$. Each term on the right hand side of (3.43) corresponds to the coil region of the each winding.

The normalized field intensity matrix in (3.43) represents the normalized fields due to two currents obtained using superposition principle and expressed in a matrix form. The off diagonal terms in this matrix reflect the mutual interaction of the windings 1 and 2. The operator $\langle \cdot \rangle_x$ denotes spatial average over winding region $x$, given by (3.26). The advantage of this approach is that the current normalized field terms can be estimated using static field analysis reducing the computational effort.
In [12], proximity effect loss in a gapped-transformer is calculated by using (3.43). The three normalized square field quantities on the right hand side of (3.43) are found individually using FEA. First a unit current through the primary winding was assumed to obtain \( \hat{\mathbf{H}}_1 \) across both the winding regions. Secondly, a unit current was assumed through the secondary winding to obtain \( \hat{\mathbf{H}}_2 \) across both the winding regions. Thirdly, both the windings are excited with unit currents each to obtain \( (\hat{\mathbf{H}}_1 + \hat{\mathbf{H}}_2) \) across both the winding regions. Using the fields obtained for three cases, the off diagonal elements in the squared field matrix are calculated as

\[
\hat{\mathbf{H}}_1 \cdot \hat{\mathbf{H}}_2 = \frac{(\hat{\mathbf{H}}_1 + \hat{\mathbf{H}}_2)^2 - \hat{\mathbf{H}}_1^2 - \hat{\mathbf{H}}_2^2}{2}
\]  

(3.44)

The proximity loss estimation by using individual winding excitations may result in saturation of an ungapped transformer core. In order to avoid this issue, and to avoid performing the magnetic analysis thrice, a simpler approach is suggested herein in the following section.

### 3.4.3 Transformer Proximity Effect Loss Model

In this section, the formulation of proximity effect loss estimation using (3.43) is simplified for the transformer case with two windings, one coil for each of primary and secondary. Instead of using FEA for calculating the current normalized field quantities, the proximity effect loss for the transformer is modified to use the magneto-static analysis performed in Chapter 2.

Taking the two windings to be primary and secondary of the transformer, (3.43) can be expanded as

\[
P_{PE} = K_p \left( \hat{\mathbf{H}}_{p}^2 i_p^2 + 2\hat{\mathbf{H}}_p \cdot \hat{\mathbf{H}}_s i_p i_s + \hat{\mathbf{H}}_s^2 i_s^2 \right) \\
+ K_s \left( \hat{\mathbf{H}}_{s}^2 i_s^2 + 2\hat{\mathbf{H}}_s \cdot \hat{\mathbf{H}}_p i_p i_s + \hat{\mathbf{H}}_p^2 i_p^2 \right)
\]  

(3.45)
where the $p$ denotes primary and $s$ denotes secondary. The currents through winding is denoted by $i_x$ and the geometric constant $K_x$ for a winding $x$ is defined as

$$K_x = \frac{\hat{G}_x N_x N_{xs} l_x}{\sigma_x} \quad (3.46)$$

The first term on the right hand side of (3.45) denotes the proximity effect loss in the primary winding, while the second term denotes the proximity effect loss in the secondary winding. Denoting the number of primary and secondary turns as $N_p$ and $N_s$ respectively, the secondary winding current $i_s$ can be expressed in terms of primary winding current $i_p$ and magnetizing current, $i_m$ as

$$i_s = \frac{N_p}{N_s} (i_p + i_m) \quad (3.47)$$

Expanding the first term on the right hand side (RHS) of (3.45) in terms of primary and magnetizing currents, loss due to proximity effect in the primary winding, $P_{PEp}$, is given by

$$P_{PEp} = K_p \left< \hat{H}_p^2 - 2 \frac{N_p}{N_s} \hat{H}_p \cdot \hat{H}_s + \frac{N_p^2}{N_s^2} \hat{H}_s^2 \right>_{p} i_p^2$$

$$+ 2K_p i_p i_m \left< \frac{N_p}{N_s} \hat{H}_p \cdot \hat{H}_s - \frac{N_p^2}{N_s^2} \hat{H}_s^2 \right>_{p} + K_p i_m^2 \left< \frac{N_p^2}{N_s^2} \hat{H}_s^2 \right>_{p} \quad (3.48)$$

During steady state conditions, the transformer magnetizing current is much smaller in magnitude than the primary current. As a result, only the first term (highlighted by *) of the three terms on the RHS of (3.48) is included in the primary winding power loss calculation. Note that the mutual effect of the two windings is still taken into effect.

attributing to $\left< \hat{H}_s \right>_{p}$ term in the first term (highlighted by *).

Also, careful observation of the normalized mean squared field term in first term of (3.48) reveals that the net normalized field in the winding region can be obtained by assuming unit current through primary winding and $-\frac{N_p}{N_s}$ through the secondary
winding, which is the zero magnetizing current excitation condition. The net field thus obtained is denoted by \( \hat{H}_{ps} \) as

\[
\hat{H}_{ps} = \hat{H}_p - \frac{N_p}{N_s} \hat{H}_s
\]  

(3.49)

Equivalently the normalized squared field intensity of primary winding region is given by

\[
\left\langle \hat{H}_{ps}^2 \right\rangle_p = \left\langle \hat{H}_p^2 - 2 \frac{N_p}{N_s} \hat{H}_p \cdot \hat{H}_s \right\rangle + \frac{N_p^2}{N_s^2} \left\langle \hat{H}_s^2 \right\rangle_p
\]

(3.50)

Similar approximation can be made in the case of secondary winding region. Modifying (3.45) accordingly, proximity effect loss in the transformer, \( P_{PE} \) is given by

\[
P_{PE} = \left( K_p \left\langle \hat{H}_{ps}^2 \right\rangle_p + K_s \left\langle \hat{H}_{ps}^2 \right\rangle_s \right) i_p^2
\]

(3.51)

Assuming sinusoid primary current with peak amplitude, \( I_p \), proximity effect loss, \( P_{PE} \) can be expressed in the form of \( I^2R \) as

\[
P_{PE} = \frac{1}{2} R_{PE} I_p^2
\]

(3.52)

where \( R_{PE} \) is the total resistance in the transformer windings due to proximity effect expressed as

\[
R_{PE} = 2 \left( K_p \left\langle \hat{H}_{ps}^2 \right\rangle_p + K_s \left\langle \hat{H}_{ps}^2 \right\rangle_s \right)
\]

(3.53)

The resistance due to proximity effect loss, \( R_{pe} \) by (3.53) involves determining the geometric constant \( K_x \) and spatial average of current normalized magnetic field intensities for each of the winding region. As \( \hat{H}_x \) is independent of current, the spatial average field terms in (3.53) are independent of frequency and hence can be determined using static field analysis. However, the geometric constants \( K_p \) and \( K_s \) are dependent on frequency.

The proximity loss at single frequency of the transformer currents is estimated using (3.52) and (3.53). For the case of non-sinusoidal currents with multiple harmonic
frequencies through the transformer windings, (3.52) is modified to include proximity loss estimation at multiple frequencies as

\[ P_{PE} = \frac{1}{2} \sum_i R_{PE}(w_i) I_{p,i}^2 \]

where \( w_i \) denotes primary current harmonic frequency and \( I_{p,i} \) denotes corresponding peak amplitude. The resistance due to proximity effect as a function of frequency is expressed as

\[ R_{PE}(\omega_i) = 2 \left( K_p (\omega_i) \left\langle \hat{H}_{ps}^2 \right\rangle + K_s (\omega_i) \left\langle \hat{H}_{ps}^2 \right\rangle \right) \] (3.55)

Compared to [12], by using the zero-magnetic current excitation condition, the repetition of field analysis for three different winding excitation conditions is avoided. The simplified model for proximity loss estimation is now applied to the core-type transformer case in the next section.

### 3.4.4 Proximity Effect Loss in Core-Type Transformer

To apply the simplified proximity effect loss model to the core-type transformer, (3.55) is modified to include the two coils of each winding and the distinction between interior and exterior coil regions, as shown in Figures 3.1 and 3.2. The magnetic analysis performed in Chapter 2 to estimate leakage inductance is sufficient to estimate the normalized field intensities in the individual coil regions. The spatial average of the interior coil region normalized field terms, \( \left\langle \hat{H}_{ps}^2 \right\rangle_{pI} \) and \( \left\langle \hat{H}_{ps}^2 \right\rangle_{sI} \), are estimated using analytical approach. The spatial averages of the normalized fields in the exterior coil regions, \( \left\langle \hat{H}_{ps}^2 \right\rangle_{xe} \) in case of Figure 3.2(a) or \( \left\langle \hat{H}_{ps}^2 \right\rangle_{xe} \), \( i \in [1, 2] \) in case of Figure 3.2(b) are estimated using the numerical method using Biot-Savert law. Here, the subscript \( x \) may be ‘p’ for primary or ‘s’ for secondary. Note each of these field quantities are normalized with respect to total primary winding current, whereas the actual fields are due to local coil-coil field interactions. Therefore, with respect to two coils of the winding connected in parallel or series, the power loss in both the coils is
always same. Alternately this may be noted as same total length of the conductor in the winding, $2l_{xl}N_{xcl}$, irrespective of whether the two coils are connected in series or parallel.

Applying (3.53) for the case of the core-type transformer coil regions shown in Figure 3.2(a), the resistance due to proximity effect loss of the primary winding, $R_{PEp}$ and that of secondary winding referred to primary side, $R'_{PEs}$ are given by

$$R_{PEp} = 4 \left( K_{pi} \langle \hat{H}^2_{ps} \rangle \right)_{pi} + K_{pe} \langle \hat{H}^2_{ps} \rangle \right)_{pe}$$

(3.56)

$$R'_{PEs} = 4 \left( K_{si} \langle \hat{H}^2_{ps} \rangle \right)_{si} + K_{se} \langle \hat{H}^2_{ps} \rangle \right)_{se}$$

(3.57)

Note an additional factor of two is used in (3.56) and (3.57) to represent two coils of a winding. Using (3.46), the geometric constants for the coil interior and exterior regions are given as

$$K_{xi} = \frac{\hat{G}_x N_{xcl} N_{xps} l_{xi}}{\sigma_x}$$

(3.58)

$$K_{xe} = \frac{\hat{G}_x N_{xcl} N_{xps} l_{xe}}{\sigma_x}$$

(3.59)

where the number of turns in each coil denoted by $N_{xcl}$, number of parallel strands denoted by $N_{xpr}$ and average turn length in each coil region are given by (3.1)-(3.3).

Applying (3.53) for the case of the core-type transformer coil regions shown in Figure 3.1 and 3.2(b), the resistance due to proximity effect loss of the primary winding, $R_{PEp}$ and that of secondary winding referred to primary side, $R'_{PEs}$ are given by

$$R_{PEp} = 4 \left( K_{pi} \langle \hat{H}^2_{ps} \rangle \right)_{pi} + K_{pe1} \langle \hat{H}^2_{ps} \rangle \right)_{pe1} + 2K_{pe2} \langle \hat{H}^2_{ps} \rangle \right)_{pe2}$$

$$+ 2K_{pe3} \langle \hat{H}^2_{ps} \rangle \right)_{pe3} + 2K_{pe4} \langle \hat{H}^2_{ps} \rangle \right)_{pe4}$$

(3.60)
\[ R'_{PEs} = 4 \left( K_{si} \left\langle \hat{H}^2_{ps} \right\rangle_{si} + K_{se1} \left\langle \hat{H}^2_{ss} \right\rangle_{se1} + 2K_{se2} \left\langle \hat{H}^2_{ps} \right\rangle_{se2} + 2K_{se3} \left\langle \hat{H}^2_{ps} \right\rangle_{se3} + 2K_{se4} \left\langle \hat{H}^2_{ps} \right\rangle_{se4} \right) \]  

(3.61)

where the spatial average of the mean squared field intensity of regions \( R_{e3} \) and \( R_{e4} \) are obtained as an average of that of the adjacent regions, expressed as

\[ \left\langle \hat{H}^2_{ps} \right\rangle_{se3} = \frac{1}{2} \left( \left\langle \hat{H}^2_{ps} \right\rangle_{x1} + \left\langle \hat{H}^2_{ps} \right\rangle_{x2} \right) \]  

(3.62)

\[ \left\langle \hat{H}^2_{ps} \right\rangle_{se4} = \frac{1}{2} \left( \left\langle \hat{H}^2_{ps} \right\rangle_{x2} + \left\langle \hat{H}^2_{ps} \right\rangle_{x1} \right) \]  

(3.63)

where \( x \) may be ‘p’ for primary or ‘s’ for secondary.

Similar to (3.58) and (3.59), the geometric constants in (3.60) and (3.61) are calculated using the length of coil regions listed in Table 3.1.

The windings resistance at high-frequency using simplified proximity effect loss model is validated in the next section.

### 3.5 Validation

The prototype core-type transformer described in Section 2.4 is used for validating the transformer loss model.

Since, the prototype transformer has a ferrite core, the core loss is found to be negligible this case. Therefore, validation of high-frequency winding resistance model is focused in this section.

One approach to measure the transformer winding resistance is by using short circuit test. In this test, the transformer primary is excited with a sinusoid current signal, peak value denoted by \( I_{pk} \), while the secondary is shorted. By measuring the transformer input voltage and current waveforms, the transformer windings total
impedance can be obtained. In particular, the resistive component of windings’ total impedance as referred to primary is given by

\[ R = \frac{2P}{I_{pk}^2} \]  

(3.64)

where the input average power to the transformer, \( P \), is estimated using the measured input current and voltage waveforms as

\[ P = \frac{1}{T} \int_0^T vi \, dt \]  

(3.65)

In (3.65), \( T \) denotes the time period of source signal.

To perform the short circuit test on the prototype transformer over a high-frequency range (1 kHz - 40 kHz), a high speed bi-polar power supply from Matsusada Precision, DOSF60 is used as a sinusoid current source to excite the transformer primary, while the secondary is shorted. The measurement set up used includes Yokogawa DL850 oscilloscope with a 720210 analog voltage input module. Yokogawa 701933 current probe is used for measuring current and Tektronix P5200 high-voltage differential probe is used for measuring voltage.

The measured data is truncated to one switching cycle and DC offset is removed. The peak value of the fundamental of the measured current waveform, \( I_{pk} \), is found by using Fourier series. The measured current and voltage waveforms along with the reconstructed current waveform at 20 kHz are shown in Figure 3.5 as example.

The short circuit test is repeated for a set of frequencies in the range 1 kHz - 40 kHz. The measured resistances using (3.64) and (3.65) are compared to the transformer total winding resistance due to DC and skin, given by (3.21) and (3.22) and proximity effect given by (3.60) and (3.61) in Figure 3.6. An error of 4.7\% is observed between the measured and estimated values at 20 kHz.
Fig. 3.5. Spacing between round conductors

Fig. 3.6. Spacing between round conductors
3.6 Conclusion

The power loss density in case of core loss and resistances for conductor losses are presented for the core-type transformer in this chapter. The method used to estimate proximity effect loss in multi winding case as presented in [12] is simplified for the transformer application. Further, the simplified method uses field intensity in the windings as calculated using the magnetic analysis from Chapter 2. The simplified transformer proximity loss model is validated using a prototype transformer.

The currents through the transformer required to calculate transformer losses are estimated using time-domain analysis in Chapter 4.
4. TIME DOMAIN MODELING

Transformer losses are dependent on the winding currents, which in turn depend on the external circuit. High-frequency transformers often find application in DC-DC converters where they provide galvanic isolation. This chapter explores determination of the winding currents when a core-type transformer is used in a DC-DC converter. The DC-DC converter topology considered herein is referred to as Isolating Converter Module (ICM) as described in Section 1.2. The converter is again shown in Figure 4.1 for convenience.

![Fig. 4.1. Isolating converter module](image)

To determine the transformer currents for a given operating condition, a time-domain analysis of the ICM must be carried out. One approach to this is to numerically solve the Ordinary Differential Equations (ODEs) governing the ICM behavior. For a given operating condition of the ICM, the ODEs are solved, commonly starting with zero initial conditions until reaching steady-state. This approach is referred to as Waveform-Level Modeling (WLM). If the ICM components are modeled accurately, the steady-state transformer currents estimated by this approach include the high-frequency harmonics that are necessary for high-frequency winding loss estimation.
However, performing WLM for each operating condition is computationally expensive in the context of optimization based design. Therefore, a simplified approach is needed for determining the ICM waveforms during steady state conditions. To this end, a analytical analysis of a simplified ordered ICM circuit is conducted to obtain transformer current waveforms, including the high-frequency harmonics. The analytical approach, reduces the computation effort in estimating the winding currents while obtaining results similar to a WLM.

The symbols used to denote various system parameters of ICM are listed in Table 4.1. In this chapter, Section 4.1 describes the ICM operation and its control. Section 4.2 presents the formulation of state equations for WLM. The analytical analysis of the simplified (reduced order) ICM circuit along with semiconductor loss analysis is presented in Section 4.3. To calculate the high-frequency winding loss due to skin and proximity effects, harmonic analysis of the analytically estimated piece-wise linear current waveforms is presented in Section 4.4. The experimental validation of analytical time domain analysis is presented in Section 4.5.

End of introduction

4.1 Converter Control and Operation

The primary function of the ICM is to regulate the DC output voltage using the inverter switching. The control utilized for this purpose is shown in Figure 4.2. The first PI controller \((K_{pv}, K_{iv})\) compares the output filter capacitor voltage \(v_{of}\) to the commanded DC output voltage \(v_{out}^*\) to determine the desired DC link inductor current \(i_{dc}^*\). A slew rate limit is applied to the commanded output voltage to limit the inrush current. In the second PI controller \((K_{pi}, K_{ii})\), \(i_{dc}^*\) is compared to the measured DC link inductor current, \(i_{dc}\) to obtain desired duty cycle, \(d^*\) for the H-bridge inverter switching. A low pass filter \((\tau_{pi})\) is implemented on the proportional term of second PI controller. The two PI controllers are implemented with an anti-windup function along with set limits on the desired values of DC link inductor current and duty cycle.
Table 4.1.
ICM system parameters

<table>
<thead>
<tr>
<th>Description</th>
<th>Variable</th>
<th>Description</th>
<th>Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input filter</td>
<td></td>
<td>Output filter</td>
<td></td>
</tr>
<tr>
<td>Inductance</td>
<td>$L_{in}$</td>
<td>Inductance</td>
<td>$L_{out}$</td>
</tr>
<tr>
<td>Inductor series resistance</td>
<td>$r_{lin}$</td>
<td>Inductor series resistance</td>
<td>$r_{lout}$</td>
</tr>
<tr>
<td>Capacitance</td>
<td>$C_{in}$</td>
<td>Capacitance</td>
<td>$C_{out}$</td>
</tr>
<tr>
<td>Capacitor ESR</td>
<td>$r_{cin}$</td>
<td>Capacitor ESR</td>
<td>$r_{cout}$</td>
</tr>
<tr>
<td>Semiconductor devices</td>
<td>Transformer</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Switch On-voltage drop</td>
<td>$V_{ce}$</td>
<td>Primary resistance</td>
<td>$r_{p}$</td>
</tr>
<tr>
<td>Diode Saturation current</td>
<td>$I_{s}$</td>
<td>Primary leakage inductance</td>
<td>$L_{tp}$</td>
</tr>
<tr>
<td>Diode emission constant</td>
<td>$n_{e}$</td>
<td>Secondary resistance</td>
<td>$r_{s}$</td>
</tr>
<tr>
<td>Diode thermal voltage</td>
<td>$V_{T}$</td>
<td>Secondary leakage inductance</td>
<td>$L_{ts}$</td>
</tr>
<tr>
<td>Diode reverse-bias resistance</td>
<td>$R_{off}$</td>
<td>Magnetizing inductance</td>
<td>$L_{m}$</td>
</tr>
<tr>
<td>RC snubber</td>
<td></td>
<td>DC link inductor</td>
<td></td>
</tr>
<tr>
<td>Resistance</td>
<td>$R_{snub}$</td>
<td>Inductance</td>
<td>$L_{dc}$</td>
</tr>
<tr>
<td>Capacitance</td>
<td>$C_{snub}$</td>
<td>Inductor series resistance</td>
<td>$r_{dc}$</td>
</tr>
</tbody>
</table>

The measured signal $v_{of}$ is passed through a low pass filter ($\tau_v$) to remove the high frequency ripple and noise. The description of symbols used in Figure 4.2 are listed in Table 4.2.

The H-bridge inverter switching can be described as per the directed flow diagram shown in Figure 4.3. Therein, $t_{norm}$ is the real time $t$ modulo the switching period $T_{sw}$. The four states describing the H-bridge inverter switching are:

- **SC-1**: Switches S1 and S2 are on. A positive voltage is applied across the transformer primary winding.
- **SC-2**: Switches S1 and S3 are on. The voltage across the transformer primary winding is near zero.
- **SC-3**: Switches S3 and S4 are on. A negative voltage is applied across the primary winding.
- **SC-4**: Same as State-2 with the transformer primary winding is shorted.
Fig. 4.2. Double PI control for the ICM

Table 4.2.
ICM Control parameters

<table>
<thead>
<tr>
<th>Description</th>
<th>Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slew rate constant</td>
<td>$\tau_{slr}$</td>
</tr>
<tr>
<td>Slew rate maximum limit</td>
<td>$pv_{maxslr}$</td>
</tr>
<tr>
<td>Slew rate minimum limit</td>
<td>$pv_{minslr}$</td>
</tr>
<tr>
<td>Voltage filter constant</td>
<td>$\tau_v$</td>
</tr>
<tr>
<td>Current filter constant</td>
<td>$\tau_i$</td>
</tr>
<tr>
<td>Anti-windup on $i^*_d$</td>
<td>$\tau_{awi}$</td>
</tr>
<tr>
<td>Anti-windup on $d^*$</td>
<td>$\tau_{awd} t$</td>
</tr>
<tr>
<td>PI-controller proportional filter constant</td>
<td>$\tau_i$</td>
</tr>
<tr>
<td>Voltage proportional constant</td>
<td>$K_{pv}$</td>
</tr>
<tr>
<td>Voltage integral constant</td>
<td>$K_{iv}$</td>
</tr>
<tr>
<td>Current integral constant</td>
<td>$K_{ii}$</td>
</tr>
<tr>
<td>DC link current limit</td>
<td>$i_{limit}$</td>
</tr>
<tr>
<td>Low pass filter constant on rectifier output voltage</td>
<td>$\tau_f$</td>
</tr>
</tbody>
</table>

Nominally this strategy produces a square-wave output wherein the ratio of the duration of non-zero output voltage to switching signal is the duty cycle $d$. However,
includes the provision to limit the primary current $i_p$ to $i_{p,max}$. Operation of the control is illustrated in Figure 4.3. During the positive half cycle the switches are initially in SC-1 for duration with a positive voltage applied across the transformer primary. The transformer primary current increases during this interval. During normal steady-state condition, the primary current peak value is well below the limit, $i_{p,max}$ as shown in Figure 4.4(a). If, however, $i_p$ rises to the limit $i_{p,max}$ as shown in Figure 4.4(b), the switch state changes to SC-2, where the voltage applied across the primary becomes zero. The switches remain in SC-2 until the end of the positive half-cycle. During the second half cycle, a negative voltage is applied across the transformer in SC-3 for duration, again unless the current through primary winding exceeds the limit. Otherwise, the transformer voltage is set to zero by moving to SC-4. This cycle repeats for each switching period.

The transformer secondary is connected to the input of a passive full bridge diode rectifier. Depending on the polarity of the transformer secondary voltage and secondary current, diodes D1-D4 are in one of the following states:

- **RC-1**: Diodes D1 and D2 conduct. The transformer secondary current, $i_s$, is equal to rectifier output current, $i_{rec}$. The voltage across the transformer secondary is positive.
- **RC-2**: Diodes D1 and D3 commutate. The transformer secondary winding is shorted. The transformer secondary current and the rectifier output current are independent.

- **RC-3**: Diodes D3 and D4 conduct. The voltage across the transformer secondary is negative. The transformer secondary current is equal but negative of rectifier output current.

- **RC-4**: Same as the RC-2 state, except for the state preceding it.

The instantaneous ICM status can be divided into six states using a combination of inverter and rectifier states, as will be discussed in detail in Section 4.3.
• R1 Inverter switches in SC-1, with the rectifier in RC-2.
• R2 Inverter switches in SC-1, with the rectifier in RC-1.
• R3 Inverter switches in SC-2, with the rectifier in RC-1.
• R4 Inverter switches in SC-3, with the rectifier in RC-4.
• R5 Inverter switches in SC-3, with the rectifier in RC-3.
• R6 Inverter switches in SC-4, with the rectifier in RC-3.

The above states are analyzed in detail in Section 4.3.

4.2 Waveform-Level Model Simulation

In this section, the state equations used to model the ICM dynamics components are presented. Each component of the ICM is assigned a state(s) to describe its dynamic behavior. The list of the states used for modeling the converter are

- Input side filter inductor current, $i_{in}$
- Input side filter capacitor voltage, $v_{cin}$
- Transformer magnetizing current, $i_m$
- Transformer leakage flux linkage, $\lambda_{lk}$
- RC snubber capacitor voltage, $v_{csnub}$
- DC link inductor current, $i_{dc}$
- Output filter capacitor voltage, $v_{of}$
- Output filter inductor current, $i_{dc}$

The states used to model the converter control are

- Low pass filtered output voltage signal, $\hat{v}_{of}$
- Slew rate limited voltage command, $v_{out}^*$
- First PI controller integral term, $v_{ie}$
- Second PI controller proportional term, $i_{e1}$
- Second PI controller integral term, $i_{e2}$

Inputs to the ICM model include

- Converter input voltage, $v_{in}$
• Converter commanded output voltage, $v_{out}$
• Load resistance, $R_{ld}$

The input side of the converter in Figure 4.1 has an LC filter with an inductor $(L_{in}, r_{lin})$ and a capacitor $(C_{in}, r_{cin})$. The voltage across the ICM input, $v_{in}$ and current at the input of the inverter, $i_{if}$ are used as inputs to the LC filter model. The states used are the current through the input filter inductor, $i_{in}$ and voltage across the filter capacitor without the equivalent series resistance, $v_{cin}$. The corresponding state dynamic equations are

\[
\frac{di_{in}}{dt} = \frac{v_{in} - r_{lin}i_{in} - v_{cin} - r_{cin}(i_{in} - i_{if})}{L_{in}} \tag{4.1}
\]

\[
\frac{dv_{cin}}{dt} = \frac{i_{in} - i_{if}}{C_{in}} \tag{4.2}
\]

The voltage across the input filter capacitor, $v_{if}$ is obtained as an output of the LC filter model, expressed as

\[
v_{if} = v_{cin} + r_{cin}(i_{in} - i_{if}) \tag{4.3}
\]

As shown in Figure 4.1, H-bridge inverter is used for switching. The output of the inverter is connected across the transformer primary winding. The transformer primary current, $i_p$ is used as an input to the inverter model. Depending on the inverter switching state, voltage across transformer primary, $v_p$, and inverter input current, $i_{if}$, may be expressed as

\[
v_p = \begin{cases} 
  v_{if} - 2V_{ce} & SC - 1 \\
  0 & SC - 2 \text{ or } SC - 4 \\
  -v_{if} + 2V_{ce} & SC - 3 
\end{cases} \tag{4.4}
\]
\[ i_{if} = \begin{cases} \frac{i_p}{N_p} & SC = 1 \\ 0 & SC = 2 \text{ or } SC = 4 \\ -\frac{i_p}{N_p} & SC = 3 \end{cases} \] (4.5)

In (4.4), \( V_{ce} \) denotes the voltage drop across the inverter switch during the on-state.

The isolating transformer in the ICM is modeled using the T-equivalent circuit shown in Figure 4.5. The secondary quantities of the transformer are referred to the primary and are denoted by prime. In particular, the referred secondary voltage \( v'_s \), current \( i'_s \) and flux linkage \( \lambda'_s \) are defined as

\[ v'_s = \frac{N_p}{N_s} v_s \] (4.6)

\[ i'_s = \frac{N_s}{N_p} i_s \] (4.7)

\[ \lambda'_s = \frac{N_p}{N_s} \lambda_s \] (4.8)

Fig. 4.5. Transformer T-equivalent circuit

The non-linear behavior of the transformer is modeled by defining magnetizing flux linkage, \( \lambda_m \), as a function of magnetizing current, \( i_m \), using

\[ \lambda_m = \frac{a_1 i_m}{1 + (\delta i_m/b_1)} + \frac{a_2 i_m}{1 + (\delta i_m/b_2)^n} + L_{sat} i_m \] (4.9)

where \( a_i, b_i, n_i \) and \( L_{sat} \) are constants characteristic of a transformer and \( \delta i_m \) denotes signum of \( i_m \).
The primary voltage, $v_p$ and secondary voltage $v_s$ are used as inputs to the transformer model. The magnetizing current, $i_m$ and leakage flux linkage, $\lambda'_{lk}$ are used as states. These are defined as

$$i_m = i'_p - i'_s$$
$$\lambda_{lk} = \lambda_p - \lambda'_s$$

(4.10)

The corresponding dynamic equations are given by

$$\frac{di_m}{dt} = \frac{(v_p - r_p i_p)L'_ts + (v'_s - r'_s i'_s)L_lp}{L'_ts L_lp + \frac{d\lambda_{lk}}{dtm}(L'_ts + L_lp)}$$

(4.12)

$$\frac{d\lambda_{lk}}{dt} = v_p - r_p i_p - v'_s - r'_s i'_s$$

(4.13)

The term $\frac{d\lambda_{lk}}{dtm}$ in (4.12) is obtained by differentiating (4.9) with respect to $i_m$ which yields

$$\frac{d\lambda_m}{di_m} = \frac{a_1}{(1 + (\delta i_m/b_1))^2} + \frac{a_2(1 + (1 - n_2)(i_m/b_2)^{n_2})}{(1 + (\delta i_m/b_2)^{n_2})^2} + L_{sat}$$

(4.14)

Since the transformer currents are required to estimate state derivatives in (4.12) and (4.13), to model the inverter and diode-bridge rectifier, $i_p$ and $i_s$ are calculated using the transformer states as

$$i_p = \frac{\lambda_{lk} + L'_ts i_m}{l_p + L'_ts}$$

(4.15)

$$i_s = \left(\frac{N_p}{N_s}\right) \left(\frac{\lambda_{lk} + L_lp i_m}{l_p + L'_ts}\right)$$

(4.16)

The full bridge diode rectifier is modeled assuming that the diode pairs (D1, D2) and (D3, D4) are symmetric in functioning, as shown in Figure 4.6. The rectifier model uses transformer secondary current, $i_s$, and rectifier output current, $i_{rec}$, as inputs. Denoting the current flowing through the diodes D1 and D2 by $i_u$ and current through the diodes D3 and D4 by $i_t$, the diode currents are calculated using the model inputs as

$$i_u = \frac{i_s + i_{rec}}{2}$$

(4.17)
Depending on the polarity of the currents through the diode, the diodes are determined to be in forward or reverse bias condition. In the forward bias condition, the diode current-voltage relation is modeled by using Schottky diode equation

\[ i_x = I_s(e^{kv_x} - 1) \]  

(4.19)

where \( x \) is replaced with \( u \) or \( l \) accordingly. In (4.19), \( I_s \) and \( k \) are device constants. Alternately, in the reverse bias condition, a very high resistance, \( R_{off} \) is assumed to approximate diode non-conduction. Modeling the rectifier in this fashion is chosen for its simplicity. Using (4.19) and reverse bias resistance, voltage across the diodes are calculated as

\[
\begin{align*}
  v_u &= \begin{cases} 
  \ln\left(\frac{i_u+1}{k}\right), & i_u > 0 \\
  R_{off}i_u, & i_u < 0 
  \end{cases} \\
  v_l &= \begin{cases} 
  \ln\left(\frac{i_l+1}{k}\right), & i_l > 0 \\
  R_{off}i_l, & i_l < 0 
  \end{cases}
\end{align*}
\]  

(4.20)

(4.21)

The outputs of the rectifier model are the voltages across its input, \( v_s \) and output, \( v_{rec} \). These are estimated using (4.20) and (4.21) as

\[ v_s = -v_u + v_l \]  

(4.22)
\[ v_{\text{rec}} = v_u + v_t \] (4.23)

An RC snubber \((R_{\text{snub}}, C_{\text{Snub}})\) is used to mitigate voltage ringing at the output of the rectifier. The snubber model uses rectifier output voltage and DC-link inductor current, \(i_{\text{dc}}\) as inputs. Using rectifier output voltage, the current through the rectifier output, \(i_{\text{rec}}\) is calculated as an output of the RC snubber model. The current, \(i_{\text{rec}}\) is in turn fed back to the rectifier model as input. Since there are no states defined in the rectifier model, the sequence of \(v_{\text{rec}}\) and \(i_{\text{rec}}\) used as input and output of the two models consecutively results in algebraic loop. To avoid this, a low pass filter is applied on \(v_{\text{rec}}\) at the output of the rectifier model. In particular,

\[
\frac{d\hat{v}_{\text{rec}}}{dt} = \frac{v_{\text{rec}} - \hat{v}_{\text{rec}}}{\tau_f} \] (4.24)

With sufficiently small value for the filter constant \(\tau_f\), \(\hat{v}_{\text{rec}} \approx v_{\text{rec}}\)

Using the capacitor voltage \(v_{\text{csnub}}\) as a state, the dynamic equation of RC snubber model is given as

\[
\frac{dv_{\text{csnub}}}{dt} = \frac{i_{\text{rec}} - i_{\text{dc}}}{C_{\text{snub}}} \] (4.25)

In (4.25), the current through rectifier output, \(i_{\text{rec}}\) is calculated using model input \(\hat{v}_{\text{rec}}\) as

\[
i_{\text{rec}} = \frac{\hat{v}_{\text{rec}} - v_{\text{csnub}}}{R_{\text{snub}}} + i_{\text{dc}} \] (4.26)

At the output of the ICM, the DC link inductor \((L_{\text{dc}}, r_{\text{dc}})\) and LC filter are used to filter high-frequency in the output current and voltage. These three components form an inductor-capacitor network that is modeled using \(\hat{v}_{\text{rec}}\) as an input. The current through the DC-link inductor, \(i_{\text{dc}}\), filter capacitor voltage, \(v_{\text{cout}}\), and filter inductor current, \(i_{\text{ld}}\), are used as states. The corresponding dynamic equations are

\[
\frac{di_{\text{dc}}}{dt} = \frac{\hat{v}_{\text{rec}} - r_{\text{dc}}i_{\text{dc}} - v_{\text{of}}}{L_{\text{dc}}} \] (4.27)

\[
\frac{dv_{\text{cout}}}{dt} = \frac{i_{\text{dc}} - i_{\text{ld}}}{C_{\text{out}}} \] (4.28)
\[
\frac{di_{td}}{dt} = \frac{v_{of} - (r_{out} + R_{ld})i_{td}}{L_{out}}
\]  

In (4.27), \(v_{of}\) is the voltage across the output filter capacitor given by

\[
v_{of} = r_{cout}(i_{dc} - i_{td}) + v_{cout}
\]

The output filter state dynamics are presented for the resistive load, \(R_{ld}\) as a resistive load is considered in this work. In case of an inductive load, current through the load is represented as a state to model the load.

The dynamic equations used for implementing the ICM control are presented next. The control implemented is shown in Figure 4.2. It has two two PI controllers, with the first one used to estimate desired DC link inductor current, \(i_{dc}^*\) and the second one to estimate the desired duty cycle, \(d^*\). These estimated values are bounded within an upper limit (denoted with a subscript \(ul\)) and a lower limit (denoted with a subscript \(ll\)) using the function

\[
\text{bound}(x, x_{ul}, x_{ll}) = \begin{cases} 
    x_{ul}, & x > x_{ul} \\
    x, & x_{ll} < x < x_{ul} \\
    x_{ll}, & x < x_{ll}
\end{cases}
\]

The inputs to the control are \(v_{of}\) and \(i_{dc}\). These measured signals are passed through low pass filter to filter high-frequency ripple and noise, implemented using

\[
\frac{d\hat{x}}{dt} = \frac{x - \dot{x}}{\tau_x}
\]

where \(\tau_x\) denotes time constant of the low pass filter. As shown in the Figure 4.2, a slew rate limit is implemented on the commanded output voltage. Thus the slew rate limited output voltage is governed by

\[
\frac{dv_{\text{com}}^*}{dt} = \text{bound}\left(\frac{v_{\text{com}}^* - v_{\text{out}}^*}{\tau_{slr}}, v_{mxsrl}, v_{mnsrl}\right)
\]
To estimate $i_{dc}^*$, $\hat{v}_{of}$ is compared to the $v_{com}^*$ in the first PI controller. The differential equation on the integral term in the first PI controller is given by

$$\frac{dv_{ie}}{dt} = K_{iv}(v_{out}^* - \hat{v}_{of}) - u_{aw,i}$$

where signal, $u_{aw,i}$ is calculated as

$$u_{aw,i} = \frac{K_{pv}(v_{out}^* - \hat{v}_{of}) + v_{ie} - i_{dc}^*}{\tau_{aw}}$$

The output of the first PI controller is

$$i_{dc}^* = \text{bound}(K_{pv}(v_{out}^* - \hat{v}_{of}) + v_{ie}, i_{limit}, 0)$$ (4.36)

In the second PI controller, there is a low pass filter implemented on the proportional term. Representing the states on the proportional and integral terms as $i_{e1}$ and $i_{e2}$ respectively, the differential equations with respect to these two states are

$$\frac{di_{e1}}{dt} = \frac{K_{pi}i_e - i_{e1}}{\tau_{pi}}$$ (4.37)

$$\frac{di_{e2}}{dt} = K_{ii}i_e$$ (4.38)

In (4.37) and (4.38), $i_e$ is the error in the measured and desired DC link inductor current given by

$$i_e = i_{dc}^* - i_{dc} - u_{aw,d}$$ (4.39)

where anti-windup, $u_{aw,d}$ is expressed as

$$u_{aw,d} = \frac{i_{e1} + i_{e2} - d^*}{\tau_{aw}}$$ (4.40)

In (4.40), $d^*$ is the desired duty cycle given by

$$d^* = \text{bound}(i_{e1} + i_{e2}, 1, 0)$$ (4.41)
The overview of the WLM of ICM is presented as pseudo-code in Table 4.3.

Table 4.3.
Overview of ICM WLM

- **Control:** Obtain \( d^* \) using equations (4.31)-(4.41) and states \( v_{com}^*, \dot{v}_{of}, i_{dc}, v_{le}, i_{e1}, i_{e2} \)
- **Transformer I:** Obtain \( i_p \) using (4.15) and states \( i_m, \lambda_{lk} \)
- **Inverter I:** Obtain \( i_{if} \) using (4.5)
- **Input side LC filter:** Obtain \( v_{if} \) using (4.3) and states \( i_{in}, v_{cin} \)
- **Inverter II:** Obtain \( v_p \) using (4.4)
- **Transformer II:** Obtain \( i_s \) using (4.16) and states \( i_m, \lambda_{lk} \)
- **Diode rectifier:** Obtain \( v_s \) and \( v_{rec} \) using (4.22), (4.23) and state \( \dot{v}_{rec} \)
- **RC snubber:** Obtain \( i_{rec} \) using (4.26) and states \( i_{dc} \) and \( v_{csnub} \)
- **DC link inductor current:** Obtain \( i_{dc} \) (a state)
- **Output side LC filter:** Obtain \( v_{of} \) using (4.30) and states \( i_{ld}, v_{cout} \)
- **Calculate time derivatives of the states using**
  
  \( (4.1),(4.2),(4.12), (4.13), (4.24), (4.25), (4.27), (4.28) \) and \( (4.30) \)

### 4.3 ICM Reduced Order Circuit Analysis

The waveform-level model described in the previous section can be used to determine currents through the transformer for a given operating condition. However, this involves solving the system of differential equations outlined in the previous section over a time duration sufficient for the system to reach steady-state condition, commonly starting with initial values for the state variables as zero. This approach is computationally expensive when evaluating thousands of transformer designs with multiple operating conditions considered for each design. Therefore, in this work, an analytical approach is used instead. To this end, a reduced order ICM circuit is analyzed analytically to determine the steady-state conditions for a given operating condition. The steady-state conditions are then used to estimate the transformer current waveforms and semiconductor losses in ICM. This approach reduces the com-
putational effort by avoiding the need to simulate the ICM over a long period, and at the same time capture the high-frequency aspects in the transformer current waveforms.

The reduced order ICM circuit considered for analytical analysis is shown in Figure 4.7. This is a reduced order circuit in that the input and output LC filters, and RC snubber present in the ICM are not considered, as they have little effect on the steady-state condition. Also, the voltage source is taken as a variable width square-wave with duty cycle, $d$ applied across the transformer primary winding. This voltage is equivalent to the voltage at the inverter output in Figure 4.1 and also includes the voltage drop due to inverter switches.

In this approach, an alternate transformer model is used. To derive the equivalent circuit as shown in Figure 4.7, the transformer magnetic analysis is revisited here. The goal here is to model the transformer in such a way that the total leakage inductance of the transformer is used as commutating inductance for the full bridge rectifier analysis.

The flux linkage of the primary and secondary windings are given by

$$\lambda_p = L_{tp}i_p + N_p\Phi_m$$

(4.42)

$$\lambda_s = L_{ts}i_s + N_s\Phi_m$$

(4.43)

where magnetizing flux, $\Phi_m$ is defined in terms of magnetizing reluctance, $R_m$ as

$$\Phi_m = \frac{N_pl_p + N_sl_s}{R_m}$$

(4.44)

For convenience, the transformer primary voltage, primary current and primary flux linkage are referred to the secondary side using the transformation ratio $\beta$ and are denoted by superscript $r$. In particular,

$$v^r_p = \beta v_p$$

(4.45)
The magnetizing inductance referred to secondary side is defined as

\[ L'_m = \beta \frac{N_s}{N_p} L_m \]  

The flux linkage equations in terms of referred quantities and using (4.44) are

\[ \lambda^r_p = \beta^2 L_{lp} i^r_p + \beta^2 \frac{N_p^2}{R_m} i^r_p + \beta \frac{N_p N_s}{R_m} i_s \]  

\[ \lambda_s = L_{ls} i_s + \frac{N_s^2}{R_m} i_s + \beta \frac{N_p N_s}{R_m} i^r_p \]  

Replacing \( L_m \) using (4.48), equations (4.49) and (4.50) can be rewritten as

\[ \lambda^r_p = \left( \beta^2 L_{lp} + \beta^2 \frac{N_p^2}{R_m} - \beta \frac{N_p N_s}{R_m} \right) i^r_p + L'_m (i_s + i^r_p) \]  

\[ \lambda_s = \left( L_{ls} + \frac{N_s^2}{R_m} - \beta \frac{N_p N_s}{R_m} \right) i_s + L'_m (i_s + i^r_p) \]  

Note the referred leakage inductances are denoted in parenthesis in (4.51) and (4.52).
Applying the condition that the referred primary leakage inductance, $L_{lp}$ in (4.51) is zero, the transformation ratio $\beta$ is given by

$$\beta = \frac{N_s}{N_p} \frac{L_m}{L_m + l_p}$$  

(4.53)

Using the relation,

$$L_m = \frac{N_s N_p}{R_m}$$  

(4.54)

the referred secondary leakage inductance from (4.52) is expressed as

$$L_{rs} = L_{ls} + \left( \frac{N_s}{N_p} - \beta \right) \frac{N_s}{N_p} L_m$$  

(4.55)

Thus, the transformation ratio $\beta$ defined in (4.53), refers the transformer primary quantities to the secondary such that the referred primary side leakage inductance is zero. Neglecting the winding resistances, the new equivalent circuit of the transformer comprised of magnetizing inductance $L_m$ and total leakage inductance $L_{rs}$, as shown in Figure 4.7. Note that, the total leakage inductance in series with the voltage source acts as a commutating inductance, emphasizing the leakage inductance effect on the diode-rectifier operation and hence the transformer currents.

The magnitudes of input DC voltage $V_{in}$, output DC voltage $V_{out}$, and average load current, $I_{dc}$, are used to specify the ICM operating condition. These are inputs to the time-domain analysis. The input voltage, $v_p^r$ in Figure 4.7 is equivalent to the inverter output, which is a quasi-square wave with peak magnitude, $V_p^r$, calculated using (4.45) as

$$V_p^r = \beta(V_{in} - 2v_{fsw})$$  

(4.56)

where $v_{fsw}$ is the effective (to be defined shortly) on-state voltage drop due to an inverter switch. The voltage source waveform $v_p^r$ over one switching cycle is as shown in Figure 4.8.

The output voltage in Figure 4.7 is assumed to be a constant DC voltage of value $V_{out}$. The average of DC link inductor current, $i_{dc}$, in Figure 4.7 is equal to the average
load current \( I_{dc} \). In the following analysis, it will be useful to define a DC voltage, denoted by \( V_s \), in terms of output DC current and voltage as

\[
V_s = V_{out} + r_{dc}I_{dc} + 2v_{fd}
\]  

(4.57)

where \( v_{fd} \) is the effective voltage drop across the rectifier diode.

The voltage drops \( v_{fsw} \) in (4.56) and \( v_{fd} \) in (4.57) nominally represent the inverter switch and rectifier diode on-state conduction loss respectively. However, the values of these voltage drops are chosen such that they result in same total loss as of ICM switches and diodes that includes loss due to conduction, switching and reverse recovery, if any. In this sense, they are effective values. The semiconductor loss modeling using \( v_{fsw} \) and \( v_{fd} \) in ICM will be discussed in detail in Section 4.3.1.

Over one switching cycle, the inverter switches transition through four states as defined in Figure 4.3 and the diode-rectifier transitions through the four states as defined in Section 4.1. The combination of the two results in six states over a switching cycle, with detailed description as given below. The commutation period of the diodes is denoted by \( \alpha \).

- **R1**, \( t \in [0, \alpha) \)
  
  Voltage across the transformer primary is positive, with the rectifier in the RC-2 (commutation) state.

- **R2**, \( t \in [\alpha, \frac{dT_{sw}}{2}) \)
  
  Voltage across the transformer primary is positive, with the rectifier in the RC-1 state (Diodes D1 and D2 conduct).

- **R3**, \( t \in [\frac{dT_{sw}}{2}, \frac{T_{sw}}{2}) \)
  
  Voltage across the transformer primary is zero, with the rectifier in the RC-1 state (Diode D1 and D2 conduct)

- **R4**, \( t \in [\frac{T_{sw}}{2}, \alpha + \frac{T_{sw}}{2}) \)
  
  Voltage across the transformer primary is negative, with the rectifier in the state RC-4 (commutation).
- R5, \( t \in \left[ \alpha + \frac{T_{sw}}{2}, \frac{(d+1)T_{sw}}{2} \right) \)
  Voltage across the transformer primary is negative, with the rectifier in the state RC-3 (Diode D3 and D4 conduct)
- R6, \( t \in \left[ \frac{(d+1)T_{sw}}{2}, T_{sw} \right) \)
  Voltage across the transformer primary is zero, with the rectifier in the state RC-3 (Diodes D3 and D4 conduct)

As shown in Figure 4.8, the currents through the leakage inductance, \( i_s \), DC link inductor, \( i_{dc} \), and the magnetizing inductance, \( i_m^r \), are assumed to be piecewise linear in each of these regions. Also, note the half-wave symmetry of these waveforms. Due to half-wave symmetry of these waveforms, it is sufficient to analyze the ICM only for a half-switching cycle. In the following material, steady state values of duty cycle, commutation angle and current values at state transitions are derived for a given operating condition of the ICM by analytically analyzing the circuit in Figure 4.7.

Fig. 4.8. Simplified circuit piecewise-linear waveforms analysis
Magnetizing current

During the states R1 and R2, positive voltage of magnitude $V_p^r$ is applied across the magnetizing inductance. This translates to change in magnetizing current during the two states given as

$$ \Delta i_m^r = \frac{dT_{sw} V_p^r}{2L_m^r} \quad (4.58) $$

During the state R3, there is no change in magnetizing current as the voltage applied across it is zero. Hence, the magnetizing current is estimated as

$$ i_m^r = \begin{cases} 
V_p^r \left( t - \frac{dT_{sw}}{4} \right), & 0 \leq t \leq \frac{dT_{sw}}{2} \\
V_p^r dT_{sw} \left( \frac{4L_m^r}{dT_{sw}} \right), & \frac{dT_{sw}}{2} \leq t \leq \frac{T_{sw}}{2} 
\end{cases} \quad (4.59) $$

Secondary current and DC link inductor current

As the switching cycle shown in Figure 4.8 repeats, the rectifier diodes are in the state RC-3 just before the start of region R1. This translates to $i_s = -i_{dc}$ at $t = 0$. Immediately after R1, rectifier diodes transition to the state RC-1, with $i_s = i_{dc}$ at $t = \alpha$. However, during R1, diodes are commutating with currents $i_s$ and $i_{dc}$ independent of each other. Assuming $i_{dc} = I_{s0}$ at $t = 0$ and $i_{dc} = I_{s1}$ at $t = \alpha$,

$$ V_p^r = L_{ls}^r \frac{(I_{s1} + I_{s0})}{\alpha} \quad (4.60) $$

$$ -V_s = L_{dc} \frac{(I_{s1} - I_{s0})}{\alpha} \quad (4.61) $$

Note $V_s$ is as defined in (4.57). From (4.60) and (4.61),

$$ I_{s0} = \frac{\alpha}{2} \left( \frac{V_p^r}{L_{ls}} + \frac{V_s}{L_{dc}} \right) \quad (4.62) $$

$$ I_{s1} = \frac{\alpha}{2} \left( \frac{V_p^r}{L_{ls}} - \frac{V_s}{L_{dc}} \right) \quad (4.63) $$
Moving to the state R2, diodes D1 and D2 conduct and hence, \( i_s = i_{dc} \). Assuming \( i_{dc} = I_{s2} \) at \( t = \frac{dT_{sw}}{2} \),

\[
V_p^r - V_s = (L_{dc} + L_{ls}^r) \left( \frac{I_{s2} - I_{s1}}{\frac{dT_{sw}}{2} - \alpha} \right)
\]

From (4.64),

\[
I_{s2} = I_{s1} + \left( \frac{dT_{sw}}{2} - \alpha \right) \left( \frac{V_p^r - V_s}{L_{dc} + L_{ls}^r} \right)
\]

During the state R3, the voltage across transformer primary winding is zero. The diodes D1 and D2 continue to conduct with \( i_{dc} = i_s \), but with a decreasing magnitude. Due to half-wave symmetry, at \( t = T_{sw}/2 \), the magnitude of both currents is equal to \( I_{s0} \) as shown in Figure 4.8.

As the source \( v_p^r \) is half-wave symmetric, the currents in the second half cycle are same as positive half cycle except for opposite polarity.

In the above expressions for currents, commutation time, \( \alpha \) and duty cycle \( d \) are unknown. To derive these values, the observation that average value of DC link inductor current, \( i_{dc} \), is equal to the average load current, \( I_{dc} \) is used. This is expressed as

\[
I_{dc} = \frac{2}{T_{sw}} \int_0^{T_{sw}/2} i_{dc} dt
\]

By using piecewise linear approximation of current through the DC link inductor as shown in Figure 4.8, the integral in (4.66) evaluates to

\[
I_{dc} = \frac{2}{T_{sw}} \left( \left( I_{s0} + I_{s1} \right) - \frac{1}{2} \right) \alpha + \left( \frac{I_{s1} + I_{s2}}{2} \right) \left( \frac{dT_{sw}}{2} - \alpha \right) + \left( \frac{I_{s2} + I_{s0}}{2} \right) \left( \frac{1 - d}{2} \right) T_{sw}
\]

Substituting (4.62), (4.63) and (4.65) in (4.67), an algebraic equation in \( \alpha \) can be obtained. This yields

\[
\alpha = \frac{I_{dc} - ab}{c}
\]

where

\[
a = \frac{V_p^r - V_s}{L_{ls}^r + L_{dc}}
\]
\[ b = \frac{T_{sw} V_s}{4 V_p^r} \]  
\[ c = \frac{1}{2} \left( \frac{V_p^r}{I_s^r} - \frac{V_s^2}{L_{dc} V_p^r} - a \frac{V_s}{V_p^r} \right) \]  
\[ \frac{V_{out}}{r_{dc} I_{dc}} = \left( d - \frac{2 \alpha}{T_{sw}} \right) \left( \frac{V_p^r}{p} - 2 v_{fd} \right) \]

Using \( \alpha \) as calculated in (4.68), the duty cycle is estimated using the input to output voltage average value relation given as

Using the two currents, \( i_m^r \) and \( i_s \), the primary current, \( i_p \) is obtained by

\[ i_p = \beta (i_s + i_m^r) \]

Thus determining both the primary and secondary currents of the transformer during the steady state condition.

This concludes the analytical analysis of the ICM to determine the piecewise linear primary and secondary current waveforms. The ICM semiconductor loss analysis is addressed in the next section.

### 4.3.1 ICM Semiconductor Losses

The ICM semiconductor losses include losses in the inverter and the diode rectifier. These losses depend on the type of semiconductors used as well as the currents through them.

One possible ICM implementation is to use silicon Insulated-Gate Bipolar Transistors (IGBTs) for the H-bridge inverter and Silicon Carbide (SiC) diodes for the full bridge diode. The inverter side of ICM with IGBT switches \( S_1-S_4 \) and corresponding...
freewheeling diodes $D_{fwd1}-D_{fwd4}$ is as shown in Figure 4.9. The IGBT loss model includes loss due to conduction, switching and reverse recovery. The instantaneous energy loss due to turn on, turn off or reverse recovery may be calculated using

$$E_x(v_{dc}, i) = E_{x0} \left( \frac{v_{dc}}{V_0} \right) \left( \frac{i}{I_0} \right)$$

(4.74)

where $E_{x0}$, $V_0$, and $I_0$ are the nominal energy loss (turn on, turn off, or reverse recovery), nominal voltage, and nominal current ratings of the transistor respectively.

The transistor instantaneous conduction power loss, $p_{sw}(t)$ is given by

$$p_{sw}(t) = V_x i(t)$$

(4.75)

where the voltage drop, $V_x$ is replaced with $V_{ce}$ for switch forward conduction or $V_{ec}$ for freewheeling diode conduction.

The SiC diode loss is dominantly conduction loss, given by

$$p_{d}(t) = V_{fd}i(t)$$

(4.76)

where $V_{fd}$ is the nominal diode forward voltage drop.

Fig. 4.9. ICM inverter IGBT switches
To determine the semiconductor losses in ICM, the half-cycle regions \( R_1-R_3 \) are revisited with focus on the ICM semiconductors. The currents through the semiconductor devices can be derived using the transformer currents shown in Figure 4.10. Therein, region \( R_1 \) is further divided into two region \( R_{1a} \) and \( R_{1b} \), with zero-crossing of primary current at \( t = \alpha_0 \) as the boundary point. The corresponding ICM inverter switch gate signals are shown in Figure 4.11. The ICM semiconductor loss analysis over a half-cycle duration is as follows:

- **\( R_{1a} \), \( t \in [0 - \alpha_0) \)**

  Referring to Figure 4.11, at \( t = 0 \), the switch \( S_4 \) is turned off, switch \( S_1 \) is turned on and switch \( S_2 \) continues to be on. The turn off energy loss of \( S_4 \) can be obtained as

  \[
  E_{t4,off} = E_{off,0} \frac{V_{in} I_{po}}{2V_0 I_0} \tag{4.77}
  \]

  As the current rises from \( -I_{po} \) at \( t = 0 \), freewheeling diode, \( D_{fw1} \) and \( D_{fw2} \) conduct due to negative current flowing in the transformer primary until \( t = \alpha_0 \).

  The instantaneous power loss during this duration is due to conduction loss in freewheeling diodes \( D_{fw1} \) and \( D_{fw2} \), given by

  \[
  p_1(t) = 2V_{ce} i_p(t) \tag{4.78}
  \]

- **\( R_{1b} \) and \( R_2 \), \( t \in [\alpha_0 - dT_{sw}/2) \)**

  At zero-crossing of primary current \( (t = \alpha_0) \), switches \( S_1 \) and \( S_2 \) start conducting current. Due to zero-crossing of the current, there is no turn-on energy loss in the switches. The instantaneous conduction loss during this duration is given as

  \[
  p_2(t) = 2V_{ce} i_p(t) \tag{4.79}
  \]
• $R_3$, $t \in [dT_{sw}/2 - T_{sw}/2)$

As shown in Figure 4.11, at $t = dT_{sw}/2$, switch $S_2$ is turned off and $S_3$ is turned on. The turn off energy loss in switch $S_2$ is given by

$$E_{t2,off} = E_{off,0} \frac{V_{in} I_p}{2V_0 I_p}$$  \hspace{1cm} (4.80)$$

Due to positive direction of the primary current at $t = dT_{sw}/2$, freewheeling $D_{fw3}$ conducts current resulting in instantenous power loss given by

$$p_{3d}(t) = V_{ce}i_p(t)$$  \hspace{1cm} (4.81)$$

Since switch $S_1$ continues to conduct current during this duration, the instantaneous power loss in $S_1$ is given by

$$p_{3s}(t) = V_{ce}i_p(t)$$  \hspace{1cm} (4.82)$$

This concludes the transition of inverter switches during one half cycle. The second half-cycle is symmetric to first half-cycle because of the half-wave symmetry in the current waveforms. By using (4.77) and (4.80), the average switching power loss over one cycle is given by

$$P_{t,sw} = 2f_{sw}(E_{t4,off} + E_{t2,off})$$  \hspace{1cm} (4.83)$$

Using (4.79) and (4.82), the inverter total transistor average conduction loss over one cycle can be obtained as

$$P_{t,cd} = 2f_{sw} \left( \int_0^{dT_{sw}/2} p_{2} dt + \int_{dT_{sw}}^{T_{sw}/2} p_{3s} dt \right)$$  \hspace{1cm} (4.84)$$

Considering the piecewise linear current waveform shown in Figure 4.10, performing the integration in (4.84), yields

$$P_{t,cd} = 2f_{sw} V_{ce} \left( I_{p1} \left( \frac{dT_{sw}}{2} - \alpha_0 \right) + \frac{T_{sw}}{2} \left( 1 - d \right)(I_{p0} + I_{p2}) \right)$$  \hspace{1cm} (4.85)$$
Using (4.78) and (4.81), the freewheeling diode average conduction loss per cycle is

\[
P_{\text{fwd,cd}} = 2f_{\text{sw}} \left( \int_{0}^{\alpha_0} p_1 dt + \int_{T_{\text{sw}}/2}^{T_{\text{sw}}/2} p_{3d} dt \right)
\]  
(4.86)
For the piecewise linear current waveform shown in Figure 4.10, the expression in (4.86) evaluates to

\[ P_{fwd,cd} = 2f_{sw}V_{ec} \left( \alpha_0 I_{p0} + \frac{T_{sw}}{2} (1 - d)(I_{p0} + I_{p2}) \right) \]  

(4.87)

The total average loss in the inverter semiconductors is

\[ P_{inv} = P_{t,sw} + P_{t,cd} + P_{fwd,cd} \]  

(4.88)

The rectifier on the secondary side has SiC diodes. Hence, only conduction loss is considered. Neglecting the commutation duration, the rectifier power loss is given by

\[ P_{rec} = 4f_{sw} \int_0^{dT_{sw}/2} V_{fd}i_s dt \]  

(4.89)

The integration in (4.89), when evaluated using the secondary current piecewise linear waveform shown in Figure 4.10, yields

\[ P_{rec} = 2f_{sw}V_{fd} \left( \frac{dT_{sw}}{2} - \alpha \right) \left( I_{s2} + I_{s1} \right) \]  

(4.90)

As mentioned in earlier, the inverter switch and rectifier diode on-state voltage drops, denoted by \( v_{fsw} \) and \( v_{fd} \) respectively, results in same loss as of (4.88) and (4.89). Therefore,

\[ v_{fsw} = \frac{P_{inv}}{2d\bar{i}_{p,hf}} \]  

(4.91)

\[ v_{fd} = \frac{P_{rec}}{2i_{s,hf}} \]  

(4.92)

where the average primary and secondary currents over a half-cycle may be defined as

\[ \bar{i}_{p,hf} = \frac{2}{T_{sw}} \int_0^{T_{sw}/2} i_p dt \]  

(4.93)

\[ \bar{i}_{s,hf} = \frac{2}{T_{sw}} \int_0^{T_{sw}/2} i_s dt \]  

(4.94)
Note, a factor of 2 in the denominator of (4.91) and (4.92) is to account for two switches or two rectifier diodes that conduct at a time.

The inverter and rectifier semiconductor average power loss given by (4.88) and (4.89) requires the knowledge of transformer currents. Meanwhile, the simplified ICM circuit analysis described at the beginning of this section uses effective voltage drops across the semiconductor components to derive the transformer current waveforms. The issue of the coupled analysis is resolved by using an iterative approach [36] as described in Figure 4.12.

The initial estimates are $v_{fsw} = 0$, $v_{fd} = 0$, $d = 0$, and $k = 1$, where $k$ is the iteration counter. The iterative process is finished when the duty cycle error, $d_e$ is smaller than a predetermined maximum duty cycle error, $d_{emax}$ or the voltage error $v_e$ is smaller than a predetermined maximum voltage error, $v_{emax}$ and when $k$ reaches the predetermined maximum iteration times, $k_{max}$. The voltage error, $v_e$ is given by

$$v_e = \sqrt{(v_{fsw} - v_{fswnew})^2 + (v_{fd} - v_{fdnew})^2}$$  \quad (4.95)

Using the iterative approach in Figure 4.12, the analytical analysis of the reduced order ICM circuit and the semiconductor loss analysis are combined to determine the transformer currents and power semiconductor loss for a given operating condition of ICM. In the next section, this approach is implemented for a prototype ICM.

4.4 Piece-Wise Linear Waveform Harmonics Evaluation

The winding loss analysis presented in Chapter 3 utilizes transformer current harmonics. To set the stage for high-frequency winding loss evaluation, equations needed to evaluate the peak value and phase of the current harmonics are presented in this section.

Referring to the transformer current illustrated in Figure 4.10, the waveforms’ piece-wise linear approximation and their half-wave symmetry are employed to ana-
Fig. 4.12. ICM steady state analysis

alytically calculate harmonics peak and phase. To this end, Fourier series is employed. Due to half-wave symmetry, the current waveforms have only odd harmonics. The peak, $I_{pk}$ and phase, $\theta_i$ of $(2i-1)^{th}$ harmonic are given by

$$I_{pk}(\omega_i) = \sqrt{d_i^2 + b_i^2}$$

(4.96)

$$\theta_i(\omega_i) = \arctan \left( \frac{b_i}{a_i} \right)$$

(4.97)
where frequency $\omega_i = 2\pi f_i$ and

$$a_i = \frac{4f_{sw}}{\omega_i^2} \left\{ \sum_{j=1}^{3} m_j \omega_i (t_{j+1} \sin(\omega_i t_{j+1}) - t_j \sin(\omega_i t_j)) ight. \\
+ \sum_{j=1}^{3} n_j (\cos(\omega_i t_{j+1}) - \cos(\omega_i t_j)) \\
+ \left. \sum_{j=1}^{3} c_j (\sin(\omega_i t_{j+1}) - \sin(\omega_i t_j)) \right\}$$

$$b_i = \frac{4f_{sw}}{\omega_i^2} \left\{ \sum_{j=1}^{3} m_j \omega_i (t_j \cos(\omega_i t_j) - t_{j+1} \cos(\omega_i t_{j+1})) ight. \\
+ \sum_{j=1}^{3} n_j (\sin(\omega_i t_{j+1}) - \sin(\omega_i t_j)) \\
+ \left. \sum_{j=1}^{3} c_j (\cos(\omega_i t_{j+1}) - \cos(\omega_i t_j)) \right\}$$

In (4.98) and (4.99), $m_j$ and $c_j$ denotes the slope and $y$-intercept of $j^{th}$ linear curve of the piecewise linear waveform shown in Figure 4.10.

### 4.5 Experimental Validation

The transformer current waveforms obtained by analytical analysis in Section 4.3 are compared to the WLM simulation and experimental data using a ICM prototype. The components used in the ICM prototype are listed in Table 4.4. The values used for the constants in ICM control are listed in Table 4.5. The prototype 20kHz transformer described in Chapter 2 is used for isolation in the ICM prototype, whose T-equivalent parameters are presented in Table 4.6 and the $\lambda_m-i_m$ parameters used in (4.9) are listed in Table 4.7.

The transformer current waveforms are measured in the laboratory for the ICM prototype for an output load of 32Ω. These are compared to the current waveforms as obtained by WLM simulation and the analytical analysis in Figures 4.13 and 4.13.
Table 4.4.
Prototype ICM components

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<thead>
<tr>
<th>Description</th>
<th>variable/Part Number</th>
<th>Value</th>
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</thead>
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<tr>
<td>Input DC Voltage</td>
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<tr>
<td>Switching frequency</td>
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</tr>
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<td>Input filter inductance</td>
<td>$L_{in}$</td>
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<tr>
<td>Input filter inductor series resistance</td>
<td>$r_{in}$</td>
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</tr>
<tr>
<td>Input filter capacitance</td>
<td>$C_{in}$</td>
<td>210 $\mu$F</td>
</tr>
<tr>
<td>Inverter switch module</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Full bridge diode rectifier</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RC snubber resistance</td>
<td>$R_{snub}$</td>
<td>47 Ω</td>
</tr>
<tr>
<td>RC snubber capacitance</td>
<td>$C_{snub}$</td>
<td>4.7 nF</td>
</tr>
<tr>
<td>DC link inductance</td>
<td>$L_{dc}$</td>
<td>3.6 mH</td>
</tr>
<tr>
<td>DC link Inductor series resistance</td>
<td>$r_{dc}$</td>
<td>0.3 mΩ</td>
</tr>
<tr>
<td>Output filter inductance</td>
<td>$L_{out}$</td>
<td>2 μH</td>
</tr>
<tr>
<td>Output filter inductor series resistance</td>
<td>$r_{out}$</td>
<td>0.1 Ω</td>
</tr>
<tr>
<td>Output filter capacitance</td>
<td>$C_{out}$</td>
<td>210 $\mu$F</td>
</tr>
<tr>
<td>Output voltage</td>
<td>$v_{out}$</td>
<td>420 V</td>
</tr>
</tbody>
</table>

Table 4.5.
Values used for constants in ICM control

<table>
<thead>
<tr>
<th>Constant</th>
<th>Value</th>
<th>Constant</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_{slr}$</td>
<td>10 ms</td>
<td>$\tau_{v}$</td>
<td>1.6 ms</td>
</tr>
<tr>
<td>$pv_{mxstr}$</td>
<td>4200 V/s</td>
<td>$\tau_{i}$</td>
<td>1.6 ms</td>
</tr>
<tr>
<td>$pv_{mnstr}$</td>
<td>-4200 V/s</td>
<td>$i_{p,max}$</td>
<td>30 A</td>
</tr>
<tr>
<td>$K_{i_{v}}$</td>
<td>0.0829 A/(Vs)</td>
<td>$K_{ii}$</td>
<td>2.4635 (As)$^{-1}$</td>
</tr>
<tr>
<td>$K_{pv}$</td>
<td>0.0264 A/V</td>
<td>$K_{pi}$</td>
<td>0.00941 A$^{-1}$</td>
</tr>
<tr>
<td>$i_{limit}$</td>
<td>55 A</td>
<td>$\tau_{pi}$</td>
<td>15.9 ms</td>
</tr>
<tr>
<td>$\tau_{awi}$</td>
<td>10 ms</td>
<td>$\tau_{awd}$</td>
<td>10 ms</td>
</tr>
</tbody>
</table>

In the case of measured waveforms, the overshoot and steep jump are due to the effect of high switching frequency in combination with parasitics in switching devices, magnetic components and RC snubber. The component parasitics to a partially extent along with RC snubber are modeled in the case of WLM, hence capturing the
Table 4.6.
Prototype transformer T-equivalent circuit parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{N_p}{N_s}$</td>
<td>1.32 $\mu$H</td>
</tr>
<tr>
<td>$r_p$</td>
<td>100 mΩ</td>
</tr>
<tr>
<td>$r_s$</td>
<td>50 mΩ</td>
</tr>
<tr>
<td>$L_{lp} + L'_{ls}$</td>
<td>28.8 $\mu$H</td>
</tr>
<tr>
<td>$L_m$</td>
<td>20.1 mH</td>
</tr>
</tbody>
</table>

Table 4.7.
Prototype transformer $\lambda_m$-$i_m$ curve fit parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>17.3 $\mu$V/\text{s}</td>
<td>$b_1$</td>
<td>10A</td>
</tr>
<tr>
<td>$a_2$</td>
<td>17.49 m$\mu$V/\text{s}</td>
<td>$b_2$</td>
<td>2.45A</td>
</tr>
<tr>
<td>$L_{sat}$</td>
<td>2.36 mH</td>
<td>$n_2$</td>
<td>1.9978</td>
</tr>
</tbody>
</table>

Table 4.8.
Prototype ICM semiconductor loss data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{ce}$</td>
<td>IGBT switch forward voltage drop</td>
<td>2 V</td>
</tr>
<tr>
<td>$V_{ec}$</td>
<td>IGBT freewheeling diode forward voltage drop</td>
<td>2.2 V</td>
</tr>
<tr>
<td>$E_{off,0}$</td>
<td>IGBT switch turn off energy loss</td>
<td>25.2 mJ</td>
</tr>
<tr>
<td>$V_0$</td>
<td>IGBT switch nominal voltage</td>
<td>600 V</td>
</tr>
<tr>
<td>$I_0$</td>
<td>IGBT switch nominal current</td>
<td>35 A</td>
</tr>
<tr>
<td>$V_{fd}$</td>
<td>SiC diode forward voltage drop</td>
<td>1.6 V</td>
</tr>
</tbody>
</table>

overshoot in the current waveforms to a reasonable extent. The analytical analysis however does not include RC snubber and switching device parasitics, hence does not reflect the spikes in the current waveforms.
As presented in Chapter 3, the high-frequency winding losses in the transformer are estimated using the primary current harmonics. In Figure 4.15, the primary current waveforms as obtained by analytical analysis, WLM and experimental implementation are decomposed into a Fourier series. The error found in the fundamental component of primary current as compared with the experimental data is 2.7% in case of analytical analysis and 2.8% in case of WLM.

Fig. 4.13. Primary current waveform as estimated by analytical method and WLM in comparison with experimental data
Fig. 4.14. Secondary current waveform as estimated by analytical method and WLM in comparison with experimental data

Fig. 4.15. High-frequency harmonic peak values of primary current
5. PARASITIC CAPACITANCE

An aspect of high-frequency magnetics design is parasitic capacitance. Regions of conducting material separated by a dielectric will have capacitance. As magnetic components have windings with conductor material separated by insulation (dielectric), there is capacitance associated with the windings.

Consider the cross-section of a coil as shown in Figure 5.1. Notice the self-insulation of the wire and the foil insulation between the layers of the coil separates the conducting material (copper). This creates parasitic capacitive coupling between the turns of the coil. When high-frequency currents flow through the coil, parasitic capacitance across adjacent turns provides direct path for the high-frequency current components. The parasitic capacitance is often modeled as shunt reactance across the winding. This shunt reluctance along with other parasitics in the system result in currents spikes when the voltage changes rapidly due to, for example, switching of power electronics [37]. Hence, along with leakage inductance and high-frequency effects such as skin effect and proximity effect, it is important to consider the impact of parasitic capacitances.

Fig. 5.1. Parasitic capacitance in windings due to conductor self insulation and foil insulation present between layers

The parasitic capacitance in magnetic components can be observed at four levels: turn-to-turn capacitance, $C_{tt}$, layer-to-layer capacitance, $C_{ll}$, intra-winding capacitance, $C_w$, inter-winding capacitance, $C_{ww}$ and stray capacitance, $C_s$. The distributed
capacitance observed at these levels combine to generate an effective capacitance at the component level. In case of an inductor, the effective parasitic capacitance may be represented by a lumped capacitance, $C_p$, [38] in parallel to the inductance as shown in Figure 5.2.

![Fig. 5.2. Parasitic capacitance modeling in an inductor](image)

In case of a transformer, the net effect of the parasitic capacitances can be modeled as lumped capacitances across the primary winding denoted by $C_p$, the secondary winding denoted by $C_s$ and inter-winding capacitances, $C_{ps1}$ and $C_{ps2}$ as shown in Figure 5.3. Each of the windings may also have stray capacitance due on its surrounding materials such as core. In the Figure 5.3, these stray capacitances are denoted by denoted by $C_{pa}$, $C_{pb}$, $C_{sd}$ and $C_{se}$ and voltages of the windings surroundings relative to an arbitrary reference point by $V_a$, $V_b$, $V_d$ and $V_e$. These lumped parasitic capacitances are used to model conduction paths for high-frequency currents. The inter-winding capacitances provide conduction path for common mode currents which is not desirable as common mode currents can generate EMI(Electro magnetic interference) noise. Another effect is that the parasitic capacitances along with system inductances can result in resonance and result in ringing in the component voltages and currents.

This chapter presents the analytical method to calculate the transformer lumped parasitic capacitances and analyzes their effect on the overall performance of the converter. In Section 5.1, analytical methods used to calculate parasitic capacitance observed on different levels of a magnetic component are discussed. These analytical methods are then used to calculate lumped parasitic capacitances of a core-type transformer in Section 5.2. Section 5.3 analyzes the effect of transformer parasitic...
capacitances by deriving the high-frequency transformer common-mode (CM) and differential-mode (DM) equivalent circuits. The presented analytical approach and performance analysis with respect to transformer parasitic capacitance is then applied to a high-frequency transformer test case and validated using experimental measurements in Section 5.4. The conclusions are presented in Section 5.5.

5.1 Analytical Calculation of Parasitic Capacitance of Winding Components

As mentioned earlier, winding components have parasitic capacitance observed at different levels - turn-to-turn capacitance, $C_{tt}$, layer-to-layer capacitance, $C_{ll}$, intra-winding capacitance, $C_w$, inter-winding capacitance, $C_{ww}$ and stray capacitance, $C_s$. By observing the geometry and electrostatic distribution of charges across different materials present in the windings, the parasitic capacitances at these levels can be analytically calculated. Combinations of these different capacitances are then used as lumped parameters to model the winding component at high-frequency and analyze performance. Therefore, in this section, analytical methods used to calculate parasitic capacitances are described.
5.1.1 Turn-to-Turn Capacitance

The turns of a winding are separated by the self insulation of the wire along with the external insulation such as foil, paper or potting material. When high-frequency AC currents pass through the conductors, electrostatic coupling is observed between the turns through the insulation. This is significant in the case of turns that are in immediate proximity of each other. As the distance between the turns increases, the coupling effect diminishes. Because of this, only adjacent turns are considered for analysis.

Two broad categories of turns arrangement are possible, one is orthogonal arrangement as shown in Figure 5.4(a) and the other one is orthocyclic arrangement as shown in Figure 5.4(b). The amount of insulation separating the conductor material changes in these two cases. The turn-to-turn capacitance $C_{tt}$ is derived using a basic cell that is symmetric to the turns arrangement. The electrical coupling within the basic cell is investigated by looking at the electric field lines. The observed field lines are then used to determine the capacitance associated with the basic cell. In this sub-section, analytical derivation of the turn-to-turn capacitance is illustrated first for the orthocyclic case, and then for the orthogonal case.

For the orthocyclic case, winding conductors are arranged with hexagonal symmetry as shown in Figure 5.4(b). Each conductor is surrounded by six other conductors from the same layer or from adjacent layers. The exception would be the case for the winding layer placed adjacent to core or shield. This is discussed in detail in Section 5.1.6. The interaction between non-adjacent layers is assumed to be negligible. It is also assumed that the electric field lines originating from a conductor end on the surrounding six conductors and do not extend beyond the six conductors. Using hexagonal symmetry, a basic cell in the shape of rhombus ABCD [39] is used with an angle $\angle BAD = \pi/3$ at the center of the conductor as shown in Figure 5.5.

The field lines pass through two different mediums, self-insulation of conductors (shown in blue) and external insulation (shown in light gray) used between adjacent
Fig. 5.4. Geometry of the turns arrangement in a winding

Fig. 5.5. Basic cell for orthocyclic case

layers. This is illustrated in Figure 5.6. The capacitance of the basic cell depends on the conductor dimensions, amount of each insulation material and electrical permittivity of the wire self-insulation material, \( \varepsilon_{si} \), and that of external insulation, \( \varepsilon_{ei} \).
Conductor dimensions include the inner radius of the conductor by \( r_i \), outer radius by \( r_o \), and the thickness of the self-insulation of the conductor, \( \delta \), given by

\[
\delta = r_0 - r_i
\]  

(5.1)

The electric field lines are assumed to travel radially in the self-insulation region and along the shortest possible path in the external insulation region [39], as indicated in the Figure 5.6. The shortest path length in the external insulation region is along a line parallel to the line joining the centers of two conductors, given by

\[
x(\theta) = 2r_o (1 - \cos \theta)
\]  

(5.2)

![Electric field path for the turn-to-turn capacitance](image)

Fig. 5.6. Electric field path for the turn-to-turn capacitance

The capacitance included within the basic cell is derived by varying the included angle, \( \theta \) and radial distance \( r \) from the center of the conductor over the insulation regions. The differential capacitance is defined in general as

\[
dC = \frac{\varepsilon dS}{dx}
\]  

(5.3)

where \( \varepsilon \), \( dS \), and \( dx \) denote electric permittivity of the material, differential surface area, and differential length of field line. The approach followed here is to use (5.3) to find the differential capacitance of the self-insulation and external insulation regions within the basic cell and then combine the two find the effective differential capac-
ittance. In the self-insulating region, the fields lines have differential cross-sectional area, \( dS = l_t r d\theta \) and differential length as \( dx = dr \). Using the definition in (5.3), the differential capacitance in this region is given by

\[
dC_{si} = \varepsilon_{si} r \frac{d\theta}{dr} l_t
\]  

(5.4)

where \( r \) is the radial distance from the center, \( d\theta \) is the differential included angle and \( l_t \) is the turn length. The self-insulation region is present between \( r = r_i \) to \( r = r_o \). Integrating (5.4) with respect to \( dr \) over \( r \in [r_i, r_o] \) yields

\[
dC_{si} = \varepsilon_{si} \frac{l_t}{\ln \left( \frac{r_o}{r_i} \right)} d\theta
\]

(5.5)

In the external insulating region as shown in Figure 5.6, the field lines have shortest path length parallel to the line joining the center of the turns given by \( 2r_o (1 - \cos(\theta)) \) and differential cross-sectional area given by \( dS = l_t r_0 d\theta \). Using (5.3), the differential capacitance for the external insulating region is given by

\[
dC_{ei} = \varepsilon_{ei} \frac{l_t d\theta}{2 (1 - \cos(\theta))}
\]

(5.6)

The field lines indicated in Figure 5.6 pass through the self-insulation of the turn twice while passing through the external insulation in between. Hence, the effective differential capacitance is calculated as series combination of capacitance due to self-insulating material of two conductors and an external insulating material, given by

\[
dC_{tt} = \frac{dC_{si} dC_{ei}}{dC_{si} + dC_{ei}}
\]

(5.7)
Substituting (5.5) and (5.6) in (5.7), differential effective capacitance with respect to angle $\theta$ is given as

$$\frac{dC_{tt}}{d\theta} = \frac{\varepsilon_{si}\varepsilon_{ei}l_t}{2\left(\varepsilon_{si} (1 - \cos \theta) + \varepsilon_{ei} \ln \left(\frac{r_o}{r_i}\right)\right)}$$ (5.8)

Integrating (5.8) over $\theta \in [-\pi/6, \pi/6]$, yields

$$C_{tt} = \int_{-\pi/6}^{\pi/6} \frac{\varepsilon_{si}\varepsilon_{ei}l_t}{2\left(\varepsilon_{si} (1 - \cos \theta) + \varepsilon_{ei} \ln \left(\frac{r_o}{r_i}\right)\right)} d\theta$$ (5.9)

Using the indefinite integral

$$\int \frac{1}{1 - a \cos \theta} d\theta = \frac{2\tanh^{-1}\left(\frac{\sqrt{1+a} \tan \left(\frac{\theta}{2}\right)}{\sqrt{1-a}}\right)}{\sqrt{1-a^2}}$$ (5.10)

to evaluate the integral in (5.9), the turn-to-turn capacitance between two conductors of the basic cell in the orthocyclic case is obtained as

$$C_{tt} = 2\varepsilon_{ei}l_ia \frac{\tanh^{-1}\left(\frac{\sqrt{1+a} \tan \left(\frac{\theta}{2}\right)}{\sqrt{1-a}}\right)}{\sqrt{1-a^2}}$$ (5.11)

where $a$ is given by

$$a = \frac{\varepsilon_{si}}{\varepsilon_{si} + \varepsilon_{ei} \ln \left(\frac{r_o}{r_i}\right)}$$ (5.12)

A simplification and improvement of the turn-turn capacitance calculation for orthocyclic case is discussed in [39]. Note that, the differential capacitance per unit angle for self-insulating region given by (5.5) remains constant throughout the unit cell while that of external insulation given by (5.6) approaches infinity for very small values of $\theta$ and decreases to a very small value for larger values of $\theta$. This extreme variation of (5.6) indicates an unrealistic assumed field distribution which tends to underpredict $C_{tt}$ [39]. As an alternative in [39], two regions are defined within the basic cell, with each region dominated by single material. The self insulation of the
two conductors is considered in one region (for small values of $\theta$) and the external insulation in the other region (for large values of $\theta$), exclusive of each other as shown by the striped regions in Figure 5.7. Defining $\theta^*$ as the boundary separating the two regions, the expression in (5.5) is integrated over the interval $[-\theta^*, \theta^*]$. The capacitance of the self-insulation region, $C_{tt,si}$, is obtained as

$$C_{tt,si} = \frac{l_t \varepsilon_{si} \theta^*}{\ln \left( \frac{m}{r_i} \right)}$$  \hspace{1cm} (5.13)$$

Integrating (5.6) over the region $|\theta| \in [\theta^*, \frac{\pi}{6}]$, the capacitance of the external insulation region is given as

$$C_{ttg} = \varepsilon_{ei} l_t \left[ \cot \left( \frac{\theta^*}{2} \right) - \cot \left( \frac{\pi}{12} \right) \right]$$  \hspace{1cm} (5.14)$$

The capacitance of the two regions is taken to be in parallel, resulting in equivalent capacitance of the basic cell expressed as

$$C_{tt,eq} = \frac{l_t \varepsilon_{si} \theta^*}{\ln \left( \frac{m}{r_i} \right)} + \varepsilon_{ei} l_t \left[ \cot \left( \frac{\theta^*}{2} \right) - \cot \left( \frac{\pi}{12} \right) \right]$$  \hspace{1cm} (5.15)$$

In order to calculate $\theta^*$, the boundary condition that the capacitance per unit angle in the self-insulation is equal to that of external insulation is used. Equating (5.13) and (5.14),

$$\theta^* = \cos^{-1} \left( 1 - \varepsilon_{ei} \frac{\ln \left( \frac{m}{r_i} \right)}{\varepsilon_{si}} \right)$$  \hspace{1cm} (5.16)$$

The turn-to-turn capacitance given by (5.15) is shown to be more accurate than (5.11) in [39].

The approach of using differential capacitances derived for individual regions to find overall capacitance can be applied to any unit cell. In the orthogonal arrangement of winding conductors as shown in Figure 5.4(b), a rectangular shaped basic cell may be assumed [40]. Consider the specific case of orthogonal arrangement of
Fig. 5.7. Simplification to turn-to-turn capacitance calculation for othocyclic case

turns from two adjacent layers separated by foil insulation as shown in Figure 5.8. The basic cell in this case is rectangle ABCD. Similar to the orthocyclic case, the direction of the electric field is assumed to be radial in the self-insulation region of the conductor ($\varepsilon_{si}$) and along shortest path parallel to the line joining the conductor centers in the external insulation region. As illustrated in Figure 5.8, external insulation region can be made of surrounding air/potting ($\varepsilon_{ei}$) and foil insulation ($\varepsilon_f$). The minimum thickness of air or potting material is denoted by $t_p$ and and thickness of foil insulation is denoted by $h$. The effective turn-to-turn capacitance is derived using similar approach as of orthocyclic case.

Fig. 5.8. Orthogonal winding with rectangular basic cell

Within the basic cell ABCD as shown in Figure 5.8, the differential capacitance in the conductor self-insulating region is same as (5.5). Using shortest path length for the
field in potting/air as \((r_0(1-\sin \theta)+t_p)\) and foil insulation as \(h\), and differential surface area as \(dS = (l_t r_0 \sin \theta d\theta)\) in (5.3), the differential capacitance in the potting/air region is given as

\[ dC_p = \frac{\varepsilon_e l_t r_o \sin \theta d\theta}{r_0(1-\sin \theta) + t_p} \]  
(5.17)

Similarly, the differential capacitance in the foil insulation region is given as

\[ dC_f = \frac{\varepsilon_f l_t r_o \sin \theta d\theta}{h} \]  
(5.18)

There are two regions of the conductor self-insulation and potting/air regions along the electric field path indicated in Figure 5.8. Therefore, the effective differential capacitance of the basic cell is series combination of individual region capacitances given by

\[ \frac{1}{dC_{tt}} = \frac{2}{dC_{si}} + \frac{2}{dC_p} + \frac{1}{dC_f} \]  
(5.19)

Substituting (5.5), (5.17) and (5.18) in (5.19)

\[ dC_{tt} = \frac{\varepsilon_s \varepsilon_e \varepsilon_f l_t r_o \sin \theta d\theta}{k_1 + k_2 \sin \theta} \]  
(5.20)

where

\[ k_1 = \varepsilon_s (h \varepsilon_e + 2 \varepsilon_f (t_p + r_o)) \]  
(5.21)

\[ k_2 = 2 \ln(r_o/r_i) \varepsilon_e \varepsilon_f r_o - 2 \varepsilon_f \varepsilon_s r_o \]  
(5.22)

Integrating (5.20) for \(\theta \in [0 \quad \pi]\), the turn-to-core capacitance in the orthogonal case is given as

\[ C_{tt} = \frac{\varepsilon_s \varepsilon_e \varepsilon_f l_t r_o}{k_2} \left( \pi - 4k_1 \left( \frac{\tan^{-1} \left( \frac{k_1+k_2}{\sqrt{k_1^2-k_2^2}} \right) - \tan^{-1} \left( \frac{k_2}{\sqrt{k_1^2-k_2^2}} \right) }{\sqrt{k_1^2-k_2^2}} \right) \right) \]  
(5.23)

The above methods provide an analytical approach to calculate the turn-to-turn capacitance between adjacent turns. It will be shown in the next section that
turn-to-turn capacitance can be used to analytically calculate the single layer coil capacitance. It will also be the basis of the calculation of layer-to-layer capacitance.

### 5.1.2 Single Layer Capacitance

In case of single layer winding of $N_t$ turns, the lumped capacitance of the layer is effectively $(N_t - 1)$ capacitors in series, with capacitance of each being the turn-to-turn capacitance of the unit cell as shown in Figure 5.6 for orthocyclic case [39]. This is illustrated in Figure 5.9. The effective capacitance of the single layer is then given as

$$C_{0,s} = \frac{C_{tt}}{(N_t - 1)}$$

(5.24)

where $C_{tt}$ is given by (5.15).

![Fig. 5.9. Parasitic capacitance of single layer](image)

**Example:** The single-layer capacitance is measured in the case of two single layer air-core coils. Coil A has 45 turns of 18 AWG wire wound on a cylindrical wooden post with radius 54.7mm. Coil B has 15 turns of 16 AWG wound on a cylindrical post with radius 50.4mm. The capacitances in the case of these two coils are measured using Hewlett-Packard 4284A LCR meter and are compared to that estimated using (5.24) in Table 5.1. Due to very small value of coil parasitic capacitance compared to inductance, the coil turns were slit for measurement purposes. This way, the measured coil reactance is only due to parasitic capacitance as there is no continuous path along the turns for the coil to have any inductance.
Table 5.1.
Parasitic capacitance of single layer coil

<table>
<thead>
<tr>
<th>Coil</th>
<th>Analytical</th>
<th>Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>6.07 pF</td>
<td>5.27 pF</td>
</tr>
<tr>
<td>B</td>
<td>5.46 pF</td>
<td>7.45 pF</td>
</tr>
</tbody>
</table>

In case of a multi layer winding, the capacitance across a single layer is less significant than the layer-to-layer capacitance [40], as voltage across adjacent turns from single layer is small compared to the voltage across turns from adjacent layers. Therefore, the single layer capacitance can be neglected, unless the magnetic component has just one layer.

The layer-to-layer capacitance is discussed in the next sub-section.

5.1.3 Layer-to-Layer Capacitance

In case of two adjacent layers, the capacitive coupling is observed between adjacent turns that belong to the same layer as well as adjacent layers. The voltage difference between the turns is different in these two cases. The voltage difference between turns from adjacent layers is significant and higher than that between turns from same layer. Therefore, the layer-to-layer capacitance is calculated by considering voltage difference between turns from adjacent layers which varies along the length of the layers [40].

The analytical estimation of layer-to-layer capacitance uses two steps. In the first step, each layer is assumed to be an equipotential surface. That is, all the turns in a layer are assumed to be at same potential. The resulting capacitive coupling between two layers is referred to as static layer-to-layer capacitance, \( C_0 \). In the second step, the voltage variation across the length of the layers and from layer to layer is included and by using the static capacitance determined in the first step, the effective layer-
to-layer capacitance, $C_{ll}$ is determined. Further details on these two steps are as follows:

**Step I: Static layer-to-layer capacitance**

To estimate static capacitance between two layers, $C_0$, each layer is assumed as an equipotential surface. One approach to estimate $C_0$ is by using network of turn-to-turn capacitances, $C_{tt}$, with the adjacent turns belonging to adjacent layers. The basic cell to estimate $C_{tt}$ is chosen such that it captures coupling between turns from one layer to the adjacent layer. In case of orthocyclic arrangement of layers, the basic cell ABCDEF as shown in Figure 5.10 can be considered. The field lines are same as the case shown in Figure 5.5, except that two rhombi ABCD and AFED together form a basic cell in here. The turn-to-turn capacitance is then twice that obtained using (5.15).

![Basic cell for $C_0$ in orthocyclic arrangements](image)

In case of orthogonal arrangement, the rectangle ABCD as shown in Figure 5.8 captures the turn-to-turn capacitance of the turns from adjacent layers.

Once the turn-to-turn capacitance of basic cell is estimated, it is extended for all the turns in the layer as a parallel combination to obtain

$$C_0 = C_{tt}N_{sc}N_{lt}$$  \hspace{1cm} (5.25)
where $N_{sc}$ denotes number of parallel strands in the winding and $N_t$ denotes the number of turns per layer.

**Parallel plate capacitor model**

Another approach to estimate $C_0$ is to model the system of two layers as a parallel plate capacitor. Assuming each layer as a equipotential surface, the static capacitance of the two layers is given by

$$C_0 = \frac{\varepsilon_{eff} A_{ef}}{d_{eff}}$$

(5.26)

where $A_{ef}$ is the area of cross section of the layers, $d_{eff}$ is the effective distance between the two layers and $\varepsilon_{eff}$ is the effective permittivity of the insulation between the two layers.

In case of the orthogonal arrangement of turns in a tightly wound winding as shown in Figure 5.8, $d_{eff}$ is given as

$$d_{eff} = h + 2t_p + 2\delta$$

(5.27)

where the conductor self insulation thickness is denoted by $\delta$. In case of turns arrangements with center to center distance, $t_{tt}$, that is with spacing present between adjacent turns of same layer, an empirical expression for the effective thickness of the insulation material is listed in Snelling [41] as

$$d_{eff} = h + 2t_p + 2r_o - 2.3r_i + 0.26t_{tt}$$

(5.28)

Equation (5.28) was empirically derived based on experimental measurements and claimed to be accurate to 2% for values of wire diameter between $0.5t_{tt}$ and $0.9t_{tt}$ and $(h + 2r_0)$ is between $1.2t_{tt}$ and $1.8t_{tt}$ in [41].
The effective permittivity, $\varepsilon_{\text{eff}}$ is estimated using parallel combination of dielectric mediums, given by the relation

$$\frac{d_{\text{eff}}}{\varepsilon_{\text{eff}}} = \frac{h}{\varepsilon_f} + \frac{2t_p}{\varepsilon_{ei}} + \frac{2\delta}{\varepsilon_{si}}$$  \hspace{1cm} (5.29)

**Cylindrical capacitor model**

In case of cylindrical winding, static capacitance between the two winding layers may be modeled as cylindrical capacitor. Similar to the previous case, to find the layer-to-layer static capacitance, each layer is assumed to be an equipotential surface. The radial distance of the inner layer from the center of the post is denoted by $x_r$ and the effective distance between two layers as $d_{\text{eff}}$. Using cylindrical capacitor approximation, the static capacitance of the two layers is given by

$$C_{0,\text{cyl}} = \frac{\varepsilon_{\text{eff}} 2\pi l_t}{\ln \left( \frac{x_r + d_{\text{eff}}}{r_i} \right)}$$  \hspace{1cm} (5.30)

where $l_t$ is the length of each layer, $d_{\text{eff}}$ and $\varepsilon_{\text{eff}}$ are as given by (5.28) and (5.29) respectively.

**Step II: Effective layer-to-layer capacitance**

In this subsection, the voltage variation along the layer and the voltage difference between two layers is taken into consideration to determine the effective layer-to-layer capacitance, $C_{ll}$, in terms of static capacitance, $C_0$. Flux linking all turns in two successive layers is assumed to be same, resulting in linear variation in the voltage along a layer.

The voltage difference between two adjacent layers depends on the method of winding. Two ways of winding layers $L_1$ and $L_2$ are shown in Figure 5.11. After winding a layer of turns, $L_1$, it can be continued to wind layer $L_2$ either in the same direction or opposite direction. When layer $L_2$ is wound in opposite direction as shown
In case of Figure 5.11(a), it is called standard winding, where the voltage difference on connected end of the two layers is zero and voltage of \( v_{ll} \) is seen on the other end. Here \( v_{ll} \) refers to total voltage applied across two layers. Taking the connected end of the two layers as zero reference, the voltage difference along the length of the layers is given by

\[
v(x) = v_{ll} \frac{x}{l}
\]  
(5.31)

where \( l \) is the length of the layer. The second type of winding is called a flyback winding and is as shown in Figure 5.11(b). Here, layer \( L_2 \) is wound in same direction of layer \( L_1 \). In case of flyback winding, voltage difference between two successive layers is constant along the length of the layers, given by

\[
v(x) = \frac{v_{ll}}{2}
\]  
(5.32)

Depending on the winding method, the voltage distribution along the layers differs. Therefore the effective capacitance, \( C_{ll} \), is dependent on the method of winding. The energy equivalence method is used to derive effective capacitance of two adjacent layers [40]. Electrostatic energy stored in the two layers is given by

\[
W_{ea} = \frac{1}{2} C_{ll} v_{ll}^2
\]  
(5.33)
By using the voltage distribution between the two layers, electrostatic energy is given in terms of $C_0$ as

$$W_{es} = \frac{C_0}{2l} \int_0^l v^2(x)dx \quad (5.34)$$

For the case of standard winding, substituting (5.31) into (5.34) and evaluating the integral yields

$$W_{es, std} = \frac{1}{6} C_0 V_{ll}^2 \quad (5.35)$$

Comparing (5.35) and (5.33), effective layer-to-layer capacitance of standard winding, $C_{ll, std}$, is given by

$$C_{ll, std} = \frac{C_0}{3} \quad (5.36)$$

In case of flyback type winding, substituting (5.32) in (5.34) and evaluating the integral yields

$$W_{es, flb} = \frac{1}{8} C_0 V_{ll}^2 \quad (5.37)$$

Equating (5.37) to (5.34), the effective layer-to-layer capacitance of flyback winding, $C_{ll, flb}$ is given by

$$C_{ll, flb} = \frac{C_0}{4} \quad (5.38)$$

It can be observed that effective layer-to-layer capacitance is lower in case of flyback type of winding compared to standard type. However, the flyback type winding requires extra wire length and is subject to higher local electric field stresses.

5.1.4 Winding Capacitance

The analytical estimation of lumped capacitance across a multi-layer winding is described in this section. A network of layer-to-layer capacitances is analyzed to find the effective winding capacitance, $C_w$. To this end, the energy method is used.

The winding capacitance uses effective layer-to-layer capacitance described in the previous section. Depending on the layer to layer connection as standard type or
flyback type, the layer-to-layer capacitance of adjacent layers is determined. Suppose a winding consists of \( N_l \) number of layers. Denoting the voltage drop across the winding as \( v_w \) and the winding capacitance as \( C_w \), the electrostatic energy stored in the winding is given by

\[
W_{es,w} = \frac{1}{2} C_w v_w^2
\]

(5.39)

Assuming a linear voltage drop along the winding and equal number of turns in each layer, the voltage drop across each layer is \( v_w / N_l \) and the voltage difference between adjacent layers terminals is \((2v_w) / N_l\). Denoting the effective layer-to-layer capacitance of \( k^{th} \) layer with its adjacent layer as \( C_{ll,k} \), the electrostatic energy stored in the winding is given by

\[
W_{es,w} = \frac{1}{2} \sum_{k=1}^{N_l-1} C_{ll,k} \left( \frac{2v_w}{N_l} \right)^2
\]

(5.40)

Equating (5.39) and (5.40), \( C_w \) is obtained as

\[
C_w = \frac{4}{N_l^2} \sum_{k=1}^{N_l-1} C_{ll,k}
\]

(5.41)

It is useful to derive \( C_w \) in case of a standard winding with unequal number of turns per layer, as it is common to find an incomplete last layer of the winding. Denoting number of turns in \( k^{th} \) layer as \( N_{tl,k} \) and \( n_k = \min(N_{tl,k}, N_{tl,k+1}) \), the voltage drop across each layer is \((v_w n_k) / N_l\) and the voltage difference between adjacent layers is \((2v_w n_k) / N_l\). Here, \( N_l \) denotes total number of turns in the winding. The electrostatic energy stored in the winding is

\[
W_{es,w} = \frac{1}{2} \sum_{k=1}^{N_l-1} C_{ll,k} \left( \frac{2v_w n_k}{N_l} \right)^2
\]

(5.42)
Equating (5.39) and (5.42), $C_w$ is given by

$$C_w = \frac{4}{N_t^2} \sum_{k=1}^{N_t-1} n_k^2 C_{ll,k}$$  \hspace{1cm} (5.43)

### 5.1.5 Winding-Winding Capacitance

Two adjacent windings separated by insulation are shown in Figure 5.12, where $v_{x1}$ and $v_{y1}$ denote voltage potentials of first winding terminals and $v_{x2}$ and $v_{y2}$ denote voltage potentials of second winding terminals relative to an arbitrary reference point P. Since the coupling effect decreases with increasing distance between the conductors, only the layers of two windings that are adjacent to each other are considered for analyzing the electrostatic interaction between the two windings. In case of two concentric windings, the outermost layer of the inner winding and innermost layer of the outer winding are considered. In Figure 5.12, $v_1$ denotes voltage drop across outer most layer of one winding, $v_2$ denotes voltage drop across the innermost layer of adjacent winding and $v_3$ denotes the voltage between the two layers on one end.

Similar to the calculation of the layer-to-layer capacitance, the static layer-to-layer capacitance of the adjacent layers of the two windings system, $C_{0,ww}$, can be estimated using either turn-to-turn capacitance approach by (5.25), the parallel plate approximation by (5.26) or cylindrical capacitor approximation by (5.30) depending on the geometry of winding. If the winding geometry is similar to parallel plate capacitor, using (5.26), $C_{0,ww}$ is given as

$$C_{0,ww} = \frac{\varepsilon_{ww} A_{ww}}{d_{ww}}$$  \hspace{1cm} (5.44)

where $\varepsilon_{ww}$ is the effective permittivity of the inter-winding insulation, $d_{ww}$ is the interlayer distance and $A_{ww}$ is the cross sectional area common to the two windings.

The static layer-to-layer capacitance, $C_{0,ww}$ is then used to derive parasitic capacitances of the two winding system by using the energy equivalence method. From Figure 5.45, the voltage difference across the two layers varies as
Using the static capacitance of the two layers, $C_{0,ww}$, and (5.45), the electrostatic energy of the system is given by

$$W_{es} = \frac{1}{2} C_{0,ww} \left( \frac{(v_2 - v_1)^2}{3} + v_3 (v_2 - v_1) + v_3^2 \right)$$

(5.46)

By relating the voltages $v_1$, $v_2$ and $v_3$ to the corresponding windings terminal voltages $v_{x1}$, $v_{y1}$, $v_{x2}$ and $v_{y2}$ in (5.46), the effective winding-to-winding capacitance network can be derived. This electrostatic capacitor network of the two winding system is dependent on the connection between winding terminals. For example, the two windings equivalent parasitic capacitance network reduces to an inter-winding capacitance when the individual windings terminals are shorted. The inter-winding capacitance is responsible for conduction of common mode current from one winding to the other. A detailed analysis of the inter-winding capacitance for the transformer will be set forth in Section 5.2.

5.1.6 Stray Capacitance

Capacitive coupling is possible between the layers of a winding and conductive material in its surroundings. This is referred to as stray capacitance. Considering the conductivities of core materials in comparison to that of insulation and conductor
materials, the core is conductive in nature for the purposes of electrostatic field interactions. Hence, stray capacitance, denoted by $C_{cw}$, is observed across the winding layer adjacent to the core and the core. This is depicted for the case of $N_l$-layer winding adjacent to the core in Figure 5.13. Therein, the effective distance, $d_{eff}$ between the core and winding includes thickness of conductor self-insulation and thickness of bobbin or any foil insulation used between the core and bobbin. If the winding and core are enclosed in a casing, there may be additional stray capacitance due to the electric fields between the outermost layer of the winding and the casing.

As in the case of the calculation of layer-to-layer capacitance, the first step is to determine the static capacitance between the winding layer and core, $C_{0,cw}$. One approach to calculate $C_{0,cw}$ is by using the basic cell ABCD shown in Figure 5.14. Therein, the foil insulation/bobbin (permittivity $\varepsilon_f$) thickness is denoted by $h$. For the field path as shown by the continuous line in the Figure 5.14, the differential capacitance in the conductor self insulation ($dC_{si}$), air/potting ($dC_p$) and foil insulation/bobbin ($dC_f$) are given by (5.5), (5.17) and (5.18) respectively. The effective differential capacitance per unit turn between winding and core, $dC_{tc}$ is a series combination of these differential capacitances given by

$$\frac{1}{dC_{tc}} = \frac{1}{dC_{si}} + \frac{1}{dC_p} + \frac{1}{dC_f} \quad (5.47)$$
Substituting (5.5), (5.17) and (5.18) in (5.47) yields

\[ dC_{tt} = \frac{\varepsilon_{si}\varepsilon_{ei}\varepsilon_{f}l_{t}r_{o}\sin \theta d\theta}{k_{1} + k_{2}\sin \theta} \]  \hspace{1cm} (5.48)

where

\[ k_{1} = \varepsilon_{si}(h\varepsilon_{ei} + \varepsilon_{f}(t_{p} + r_{o})) \] \hspace{1cm} (5.49)

\[ k_{2} = \ln(r_{o}/r_{i})\varepsilon_{ei}\varepsilon_{f}r_{o} - \varepsilon_{f}\varepsilon_{si}r_{o} \] \hspace{1cm} (5.50)

Integrating (5.48) for \( \theta \in [0 \pi] \), the turn-to-core per unit turn capacitance, \( C_{tc} \), is given as

\[ C_{tc} = \frac{\varepsilon_{si}\varepsilon_{ei}\varepsilon_{f}l_{t}r_{o}}{k_{2}} \left( \pi - 4k_{1} \left( \tan^{-1} \left( \frac{k_{1} + k_{2}}{\sqrt{k_{1}^{2} - k_{2}^{2}}} \right) - \tan^{-1} \left( \frac{k_{2}}{\sqrt{k_{1}^{2} - k_{2}^{2}}} \right) \right) \right) \] \hspace{1cm} (5.51)

where \( k_{1} \) and \( k_{2} \) are same as given in (5.49) and (5.50).

Using (5.51), the winding layer to core static capacitance is given by

\[ C_{0,cw} = C_{tc}N_{sc}N_{ul} \] \hspace{1cm} (5.52)

where \( N_{sc} \) denotes number of parallel strands in the winding and \( N_{ul} \) denotes the number of turns per layer.

An alternate and simpler approach to calculate \( C_{0,cw} \) is to use the knowledge of core and winding layers dimensions. The static capacitance in this case can be calculated using the parallel plate approximation by (5.26) or cylindrical capacitor approximation by (5.30) according to the geometry of the core and winding.

Once the static capacitance between core and innermost layer is found, the energy method is used to calculate effective capacitance \( C_{cw} \). The winding terminal potentials, \( v_{a} \) and \( v_{b} \), defined relative to an arbitrary reference point P are as shown
in Figure 5.13. Assuming linear voltage drop along the length of the winding, the voltage drop across the layer closest to the core layer, $v_1$ is given as

$$v_1 = \frac{1}{N_l} (v_a - v_b) \quad (5.53)$$

Due to conductive nature, the core is at constant potential, $V_c$ relative to same reference point P. The voltage difference between the core and closest winding layer on one end, $v_2$, is then given by

$$v_2 = V_c - v_a \quad (5.54)$$

The voltage difference between core and winding layer along its length is given as

$$v(x) = v_2 + \frac{x}{l} v_1 \quad (5.55)$$

Using (5.54), (5.53) and (5.55), the electrostatic energy stored in winding-core system is

$$W_{es} = \frac{1}{2} C_{0,cw} \left( V_c - v_a \right)^2 + \frac{(v_a - v_b)^2}{3N_l^2} + \frac{1}{N_l} (V_c - v_a) (v_a - v_b) \right) \quad (5.56)$$

The value of core potential $V_c$ varies depending on whether the core is floating or connected to system ground. Also, there is a possibility of core potential to be center-tap of the closest winding layer potential, that is $V_c = v_a - (v_a - v_b) / (2N_l)$. 

---

Fig. 5.14. Turn-to-core parasitic capacitance
Using voltage $V_c$ relation to the winding terminal voltages, (5.56) can be used to evaluate the associated stray capacitance $C_{cw}$. This will be discussed in detail for the transformer case in Section 5.2.

At this point, methods to calculate the capacitances associated with a variety of mechanisms have been set forth. In the next section, these methods are applied to a core-type transformer so that they may be incorporated into the transformer design process.

### 5.2 Parasitic Capacitances in a Core-Type Transformer

The different electrostatic field interactions possible in a winding and their analytical analysis to determine parasitic capacitances were discussed in Section 5.1. In this section, the results are applied to a core-type high-frequency transformer. Figure 5.3 depicts the possible parasitic capacitances in a high-frequency transformer. These capacitances primarily depend on the geometry and arrangement of the core and windings in the transformer. In this section, analytical approach is used to estimate the parasitic capacitances in a core-type transformer whose geometry is shown in Figures 1.2 and 1.3. Across the core-type transformer core and windings system, electrostatic fields interact linking the tuns within primary and secondary coils, between the core and secondary coil, between secondary and primary coils and also between parts of the two primary coils present within the core window. Each of these regions are analyzed to determine the parasitic capacitances network of the core-type transformer. Note that throughout this development, the capital letter $C_{xy}$ denotes capacitance and the small letter $c_{xy}$ denotes the spatial clearance between regions $x$ and $y$.

The analytical approaches described in Section 5.1 highlight the effect of voltage variation present along the winding layers on parasitic capacitance. Consider the core-type transformer cross-sections in Figure 5.15 where winding layers are individually shown. Therein, orange and yellow rectangles represent primary and secondary
coil layers respectively, gray and black regions represent core and insulation materials respectively. The terminal voltages of each coil and their proximity to one another determine the voltage variation along the winding layers as well as their relative voltage differences. In this research, only parallel connection of winding coils is considered. It can be observed that the coil terminals can be connected in either of the two possible ways as shown in Figure 5.15 to achieve parallel connection of the winding coils.

![Parallel connection of winding coils](image)

(a)

(b)

Fig. 5.15. Parallel connection of winding coils

The secondary coil layers for each of the two cases shown in Figure 5.15 when expanded are shown in Figures 5.16 and 5.17. In these figures, the left side of the core is referred to as side A and the right side of the core as side B. Extending this nomenclature to coils, coil present on side A of the core is referred to as coil A and coil on side B as coil B. Also, only surfaces immediately adjacent to the secondary coil are shown for the sake of clarity. Given the exact same terminal connections of coils on side A and side B as shown in Figure 5.16, asymmetry with respect to positive and negative rails is present for such a connection of parallel coils since the core is physically closer
to turns near the negative terminal. Because of the asymmetry in the proximity of positive and negative terminals, the electrostatic field interactions in this case result in a coupled transformer common-mode and differential mode impedance. In the case of parallel coils connection shown in Figure 5.15(b) however, the parallel coils combination creates a symmetry with respective to positive and negative terminals as shown in Figure 5.17. As a consequence, the electrostatic field interactions for this case result in decoupled transformer common-mode and differential mode impedance. Therefore, the core-type transformer with winding connections shown in Figure 5.15(b) is preferred and analyzed for estimating the transformer parasitic capacitances network in CM and DM configuration. In addition, note the relative voltage difference between the innermost primary layer and outermost secondary layer is different depending on the parity of the number of secondary coil layers. This is shown in Figure 5.18 for the case when the number of secondary coil layers is odd. The inherent symmetry still holds resulting in decoupled CM and DM impedances, therefore the core-type transformer case shown in Figure 5.18 is also considered in the following analysis.
Fig. 5.17. Primary coil to secondary coil and secondary coil to core subsystems along the secondary coil layers for the coil connections depicted in Figure 5.15(b)
Fig. 5.18. Parallel connection of winding coils with odd number of secondary winding layers

Before proceeding with the transformer electrostatic analysis, it is useful to define the CM (denoted by subscript \( x_c \)) and DM (denoted by subscript \( x_d \)) voltages, given by

\[
v_{xc} = \frac{1}{2}(v_{x+} + v_{x-})
\]

\[
v_{xd} = v_{x+} - v_{x-}
\]

where \( x \) is \( p \) in case of primary and \( s \) in case of secondary. The voltages \( v_{x+} \) and \( v_{x-} \) are defined relative to an arbitrary point in space and by using (5.57) and (5.58) can be written in terms of CM and DM voltages as

\[
v_{x+} = v_{xc} + \frac{1}{2}v_{xd}
\]

\[
v_{x-} = v_{xc} - \frac{1}{2}v_{xd}
\]

5.2.1 Primary and Secondary Windings Self Parasitic Capacitance

Within the coil regions, the winding turns are assumed to be wound in orthogonal arrangement as shown in Figure 5.8 and the transformer windings are wound in a standard or U-shaped winding as shown in Figure 5.11(a). For each pair of adjacent layers (varying turn length, \( l_{xkt} \)) in a winding, the turn-to-turn capacitance is cal-
culated using (5.23), followed by the static layer to layer capacitance of $k^{th}$ layer to $(k+1)^{th}$ layer given by

$$C_{0,xx} = C_{tt,xx}N_{xpr}N_{xtl}$$  \hspace{1cm} (5.61)

In (5.61), subscript $x$ refers to primary or secondary winding accordingly. Using (5.36) and (5.41), the winding parasitic capacitance is given as

$$C_{xx} = \frac{8}{3N_{zl}^2} \sum_{k=1}^{N_{zl}-1} C_{0,xx}$$  \hspace{1cm} (5.62)

which includes a factor of two since there are two parallel coils in each winding.

As the winding self capacitances reflect the equivalent shunt capacitance across the two terminals of the winding, the corresponding electrostatic energy is given by

$$W_{es,xx} = \frac{1}{2} C_{xx} v_{xd}^2$$  \hspace{1cm} (5.63)

Hence, the primary and secondary windings self-capacitances denoted by $C_{pp}$ and $C_{ss}$ respectively contribute only to the differential-mode transformer impedance, and each is represented parallel to the winding terminals.

### 5.2.2 Secondary Winding and Core Interface Parasitic Capacitance

The secondary winding coils are wound adjacent to the core and hence have electrostatic interaction with the core across the foil insulation of thickness $c_{sc}$. This results in stray capacitance, $C_{sc}$, which is calculated using winding to core static capacitance per unit turn, $C_{tt,sc}$ and the voltage gradient across the secondary layers adjacent to the core.

Assuming orthogonal arrangement of turns as shown in Figure 5.14, the foil insulation thickness as $h = c_{sc}$ and the potting layer thickness as $\alpha = 0$, the turn-to-core per unit turn capacitance, $C_{tc,sc}$, can be calculated using (5.51). Extending this to
the secondary winding innermost layer, the static winding-to-core capacitance, $C_{0,sc}$ is given by

$$C_{0,sc} = N_{spr}N_{stl}C_{tc,sc} \quad (5.64)$$

The secondary coils and core system has voltage variation as shown in Figure 5.19. Assuming a spatially linear voltage distribution, the voltage drop across each secondary layer is $v_{sd}/N_{sl}$. Since the core is adjacent to the innermost layers of secondary coils and externally not connected to any potential, the core voltage $V_c$ can be assumed to be average of the terminal voltages of secondary coil layers that are adjacent to the core. Secondary coil A has potential values of $v_{s-}$ and $(v_{s-} + v_{sd}/N_{stl})$ at its innermost layer ends. Secondary coil B has potential values of $v_{s+}$ and $(v_{s+} - v_{sd}/N_{stl})$ at its innermost layer ends. Taking an average of these four voltage potentials, the core voltage, $V_c$ is given by

$$V_c = \frac{1}{2}(v_{s+} + v_{s-}) \quad (5.65)$$

Using (5.56) and (5.65), the electrostatic energy stored in secondary coil A and core interface is

$$W_{sc,1} = \frac{N_{spr}N_{stl}C_{tt,sc}}{2} (v_{s+} - v_{s-})^2 \left(1 + \frac{1}{3}N_{sl}^2 - \frac{1}{2}N_{sl}\right) \quad (5.66)$$
Using (5.56) and (5.65), the electrostatic energy stored in secondary coil B and core system is found to be same as (5.67). Taking twice of (5.67) and substituting (5.58), the total electrostatic energy stored in secondary winding to core system is

\[
W_{sc} = \frac{1}{2} N_{spr} N_{stl} C_{tt,sc} v_{sd}^2 \left( \frac{1}{2} + \frac{2}{3N_{sl}^2} - \frac{1}{N_{sl}} \right)
\]  

(5.67)

The energy expression in (5.67) shows that the core and secondary winding electrostatic field interaction is effectively observed as shunt parasitic capacitance, \( C_{sc} \), across the secondary winding terminals, given by

\[
C_{sc} = \frac{N_{spr} N_{stl} C_{tt,sc}}{6N_{sl}^2} (3N_{sl}^2 + 4 - 6N_{sl})
\]  

(5.68)

### 5.2.3 Primary Coil to Coil Interface Parasitic Capacitance

The primary coil is not adjacent to the core, being separated by secondary coil and insulation foils as shown in Figure 1.2. Hence, primary winding does not have significant stray capacitance with the core. However, for the coil terminal connections shown in Figures 5.15(b) and 5.18, the outermost layers from the two primary coils that are at close proximity within core interior region can have electrostatic field interactions. This is specifically the case for coil connections shown in Figures 5.15(b) and 5.18, as the outermost layers of primary coils A and B has maximum possible potential difference, \( v_{pd} \) on one end as shown in Figure 5.20. Therefore, it is important to include the coil-to-coil capacitance between the parts of two primary coils that are present in the core interior region.

The cross-sectional area of two primary coils facing each other is

\[
A_{pp} = d_{pw} l_{pi}
\]  

(5.69)
where \( l_{pi} \) denotes the length of the primary coil in the interior region. The effective permittivity, \( \varepsilon_{pp} \) of the in-between region consisting of clearance (\( c_{pp} \)) filled by air (\( \varepsilon_0 \)) and conductors self insolation is given by

\[
\varepsilon_{pp} = \frac{\varepsilon_{si}\varepsilon_0 d_{eff,pp}}{\varepsilon_{si}c_{pp} + 2\varepsilon_0\delta_p}
\] (5.70)

where effective distance between the two coils is \( d_{eff,pp} = c_{pp} + 2\delta_p \). The static primary coil-to-coil capacitance is calculated as

\[
C_{0,pp} = \frac{\varepsilon_{pp}A_{pp}}{d_{eff,pp}}
\] (5.71)

To find the effective parasitic capacitance between the two primary coils, the electrostatic energy, \( W_{pp,cc} \) stored in the outermost primary layers system shown in Figure 5.20 is calculated using (5.46). Substituting the voltage drops \( v_1 = v_{pd}/N_{pl} \), \( v_2 = -v_{pd}/N_{pl} \) and \( v_3 = v_{pd} \) in (5.46) yields,

\[
W_{pp,cc} = \frac{1}{6N_{pl}^2} C_{0,pp} v_{pd}^2 \left( 3N_{pl}^2 + 4 - 6N_{pl} \right)
\] (5.72)

By (5.72), the primary coil-to-coil parasitic capacitance, \( C_{pp,cc} \) is given by

\[
C_{pp,cc} = \frac{1}{3N_{pl}^2} C_{0,pp} \left( 3N_{pl}^2 + 4 - 6N_{pl} \right)
\] (5.73)

and is effectively observed parallel to the primary winding terminals. Notice that the relative voltages across the primary coil outermost layers is as shown in Figure 5.20 irrespective of the number of primary coil layers is odd or even.
Fig. 5.20. Primary coils outermost layers in the core interior region and their voltages

5.2.4 Primary Winding to Secondary Winding Interface Parasitic Capacitance

The electrostatic field interaction between primary and secondary coils falls under the category of winding-to-winding capacitance discussed in Section 5.1.5. The effective cross sectional area, \( A_{ps} \), overlapping primary and secondary coils is

\[
A_{ps} = \frac{1}{2} (d_{pw} + d_{sw}) (l_{pi} + l_{so})
\]  

(5.74)

where \( l_{pi} \) and \( l_{so} \) are lengths of the primary coil innermost layer and secondary coil outermost layer respectively. The effective permittivity of the dielectric medium \( \varepsilon_{ps} \) between the two layers is given as

\[
\varepsilon_{ps} = \frac{\varepsilon_{si} \varepsilon_f d_{eff,ps}}{\varepsilon_{si} c_{ps} + \varepsilon_f (\delta_p + \delta_s)}
\]  

(5.75)

where \( d_{eff,ps} = c_{ps} + \delta_p + \delta_s \) is the effective distance between the windings. Using the parallel plate capacitor model, the static inter-winding capacitance, \( C_{0,ps} \), is given by

\[
C_{0,ps} = \frac{\varepsilon_{ps} A_{ps}}{d_{eff,ps}}
\]  

(5.76)
Fig. 5.21. Voltage variation across the primary coil innermost and secondary coil outermost layers in Figure 5.15(b), $N_{sl}$ is even.

Fig. 5.22. Voltage variation across the primary coil innermost and secondary coil outermost layers in Figure 5.18, $N_{sl}$ is odd.

To calculate the effective capacitance between the primary and secondary coils, consider the voltage variation along the primary coil innermost and secondary coil outermost layers shown in Figures 5.21 and 5.22. The layer terminal voltages shown in Figure 5.21 correspond to the coil connections shown in Figure 5.15(b), where the number of secondary coil layers is even. The layer terminal voltages shown in Figure 5.22 correspond to the coil connections shown in Figure 5.18, where the number of secondary coil layers is odd. Therein, the voltage drop across the primary coil innermost layer is denoted by $v_1$ and that across the secondary coil outermost layers by $v_2$. The voltage difference between the two layers on one end is denoted by $v_{3A}$ for coils on side A and by $v_{3B}$ for coils on side B. The voltages $v_1$, $v_2$, $v_{3A}$ and $v_{3B}$ for the four coil pairs shown in Figures 5.21 and 5.22 are defined in terms of CM and DM winding voltages using in Table 5.2 (5.59) and (5.60).
Using the concept of winding-winding interface electrostatic energy given by (5.46), the primary and secondary coils interface electrostatic energy on side A, denoted by $W_{psA}$, is given as

$$W_{psA} = \frac{1}{2} C_{0,ps} \left( v_{3A}^2 + v_{3A}(v_2 - v_1) + \frac{1}{3}(v_2 - v_1)^2 \right)$$

(5.77)

and that of side B, denoted by $W_{psB}$, is given as

$$W_{psB} = \frac{1}{2} C_{0,ps} \left( v_{3B}^2 + v_{3B}(v_2 - v_1) + \frac{1}{3}(v_2 - v_1)^2 \right)$$

(5.78)

The total electrostatic energy in the transformer primary-secondary winding interface, $W_{ps}$, is obtained as the sum of (5.77) and (5.78)

$$W_{ps} = W_{psA} + W_{psB}$$

(5.79)

Using the voltage expressions defined for $v_1$, $v_2$, $v_{3A}$ and $v_{3B}$ from Table 5.2 for the case when $N_{sl}$ is even, the primary winding-secondary winding interface electrostatic energy, $W_{pse}$ evaluates to

$$W_{pse} = \frac{1}{2} C_{0,ps} \left( \frac{1}{2}(v_{pc} - v_{sc})^2 + \frac{1}{2}(v_{pd} - v_{sd})^2 + \frac{2}{3}(\alpha_s v_{sd} - \alpha_p v_{pd})^2 + (\alpha_s v_{sd} - \alpha_p v_{pd})(v_{pd} - v_{sd}) \right)$$

(5.80)

where $\alpha_p = 1/N_{pl}$, $\alpha_s = 1/N_{sl}$ and ‘c’ in subscript $pse$ denotes the case when $N_{sl}$ is even.

From right hand side of (5.80), it is evident that $W_{pse}$ can be decoupled into CM and DM quantities. The primary winding-secondary winding CM electrostatic energy is defined as

$$W_{ps,cm} = \frac{1}{2} C_{ps,cm} (v_{pc} - v_{sc})^2$$

(5.81)
where CM equivalent transformer parasitic capacitance due to primary-secondary windings interface, denoted by $C_{ps,cm}$ is given by

$$C_{ps,cm} = 2C_{0,ps} \quad (5.82)$$

From (5.80), the primary winding-secondary winding DM electrostatic energy can be defined as

$$W_{pse, dm} = \frac{1}{2} C_{0,ps} \left( \frac{1}{2} (v_{pd} - v_{sd})^2 + \frac{2}{3} (\alpha_s v_{sd} - \alpha_p v_{pd})^2 + (\alpha_s v_{sd} - \alpha_p v_{pd})(v_{pd} - v_{sd}) \right) \quad (5.83)$$

The expression on the right hand side of (5.83) is convoluted and cannot be readily simplified. However, by assuming $v_{sd} = v_{pd}/n$, where $n = N_p/N_s$, the expression in (5.83) reduces to

$$W_{pse, dm} = \frac{1}{2n^2} C_{0,ps} v_{pd}^2 \left( \frac{(n-1)^2}{2} + \frac{2}{3} (\alpha_s - \alpha_p n)^2 + (\alpha_s - \alpha_p n)(n-1) \right) \quad (5.84)$$

It can be observed from (5.84) that the effective DM parasitic capacitance due to primary and secondary windings interface can be referred to across the primary terminals using capacitance $C_{pse, dm}$, given by

$$C_{pse, dm} = \frac{C_{0,ps}}{n^2} \left( \frac{(n-1)^2}{2} + \frac{2}{3} (\alpha_s - \alpha_p n)^2 + (\alpha_s - \alpha_p n)(n-1) \right) \quad (5.85)$$
Table 5.2.
Voltage drops across the primary-secondary coil layers shown in Figure 5.21 and Figure 5.22

<table>
<thead>
<tr>
<th>Voltage</th>
<th>(N_{sl} \text{ is even} )</th>
<th>(N_{sl} \text{ is odd} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(v_1)</td>
<td>(v_{pd} N_{pl} )</td>
<td>(\frac{v_{pd}}{N_{sl}})</td>
</tr>
<tr>
<td>(v_2)</td>
<td>(v_{pd} N_{pl} )</td>
<td>(-\frac{v_{pd}}{N_{sl}})</td>
</tr>
<tr>
<td>(v_{3A})</td>
<td>(v_{pc} - v_{sc} + \frac{1}{2}(v_{pd} - v_{sd}))</td>
<td>(v_{pc} - v_{sc} + \frac{1}{2}(v_{pd} - v_{sd}) + \frac{v_{pd}}{N_{sl}})</td>
</tr>
<tr>
<td>(v_{3B})</td>
<td>((v_{pc} - v_{sc}) - \frac{1}{2}(v_{pd} - v_{sd}) + \left(\frac{v_{pd}}{N_{pl}} - \frac{v_{sd}}{N_{sl}}\right))</td>
<td>((v_{pc} - v_{sc}) - \frac{1}{2}(v_{pd} - v_{sd}) + \frac{v_{pd}}{N_{sl}})</td>
</tr>
</tbody>
</table>

For the case when \(N_{sl}\) is odd, using the corresponding voltage expressions from Table 5.2 and (5.77)-(5.79), the total electrostatic energy in primary-secondary interface, \(W_{pso}\) evaluates to

\[
W_{pso} = \frac{1}{2}C_{0,ps} \left( (v_{pc} - v_{sc})^2 + \frac{1}{2}(v_{pd} - v_{sd})^2 + (\alpha_p^2 v_{pd}^2 + \alpha_s^2 v_{sd}^2) \right) - \frac{1}{3}(\alpha_s v_{sd} - \alpha_p v_{pd})^2 - (\alpha_s v_{sd} + \alpha_p v_{pd})(v_{pd} - v_{sd}) \tag{5.86}
\]

where \(\alpha_p = 1/N_{pl}, \alpha_s = -1/N_{sl}\) and 'o' subscript \(pso\) denotes \(N_{sl}\) is odd.

Similar to the case when \(N_{sl}\) is even, the right hand side of (5.86) can be separated into electrostatic energy in CM quantities given by (5.81) using capacitance, \(C_{ps,cm}\), given by (5.82) and DM electrostatic energy, \(W_{pso, dm}\), given by

\[
W_{pso, dm} = \frac{1}{2}C_{0,ps} \left( \frac{1}{2}(v_{pd} - v_{sd})^2 + (\alpha_p^2 v_{pd}^2 + \alpha_s^2 v_{sd}^2) \right) - \frac{1}{3}(\alpha_s v_{sd} - \alpha_p v_{pd})^2 - (\alpha_s v_{sd} + \alpha_p v_{pd})(v_{pd} - v_{sd}) \tag{5.87}
\]
Substituting \( v_{sd} = v_{pd}/n \) in (5.87), where \( n = N_p/N_s \), the expression on right hand side can be reduced to

\[
W_{pso, dm} = \frac{1}{2n^2} C_{0, ps} v_{pd}^2 \left( \frac{(n - 1)^2}{2} - \frac{1}{3} (\alpha_s - \alpha_p n)^2 \right.
- (\alpha_s + \alpha_p n) (n - 1) + \left( \alpha_s^2 + \alpha_p^2 n^2 \right) \left(5.88\right)
\]

From (5.88), it can be inferred that the effective DM parasitic capacitance due to primary and secondary windings interface for \( N_{sl} \) is odd case can be referred to across the primary winding terminals with capacitance, \( C_{pso, dm} \), given by

\[
C_{pso, dm} = \frac{C_{0, ps}}{n^2} \left( \frac{(n - 1)^2}{2} - \frac{1}{3} (\alpha_s - \alpha_p n)^2 \right.
- (\alpha_s + \alpha_p n) (n - 1) + \left( \alpha_s^2 + \alpha_p^2 n^2 \right) \left(5.89\right)
\]

This concludes the analysis of possible electrostatic interface regions in the core-type transformer. The high-frequency transformer performance is analyzed in next section based on the parasitic capacitances.

5.3 High-Frequency Transformer CM and DM Equivalent Circuits

In the previous section, estimation of transformer parasitic capacitances is considered. It is necessary to understand the effect of these different parasitic capacitances on the transformer performance and hence the DC-DC converter. This is undertaken in this section by deriving the common-mode (CM) and differential -mode (DM) equivalent circuit models of the high-frequency transformer using the lumped parasitic capacitances analytically derived in previous section. The analysis presented in this section is useful in deriving necessary design specifications and constraints of a high-frequency transformer.

The different lumped parasitic capacitances observed in the core-type transformer are self winding capacitances of primary and secondary, denoted by \( C_{pp} \) and \( C_{ss} \).
respectively, stray capacitance between secondary winding and core, $C_{sc}$, primary coil-to-coil parasitic capacitance, $C_{pp,cc}$ and primary-secondary windings interface parasitic capacitances, $C_{ps,cm}$ and $C_{ps,dmx}$ or $C_{ps,dme}$. Due to the selection of the transformer configurations with parallel coil connections as shown in Figures 5.15(b) and 5.18, the transformer impedance network has decoupled CM and DM parasitic impedances.

Based on electrostatic energy expression (5.81) derived in Section 5.2, only $C_{ps,cm}$ contributes to the transformer CM impedance as shown in Figure 5.23. Note that the resistive and magnetic parts of the transformer circuit contribute to the transformer DM operation alone and hence not included in Figure 5.23. Therefore, the CM equivalent circuit of a high-frequency transformer is simply a capacitance $C_{ps,cm}$, which is responsible for high-frequency conduction from primary to secondary side. For this reason, magnitude of the CM impedance, $Z_{CM}$, given by

$$Z_{CM} = 1/(2\pi f C_{ps,cm})$$

(5.90)

should be reasonably high around the transformer operating frequency, $f$ to avoid CM conduction. Beyond this, however, the edge rates can come into play. It is not so much a matter of the fundamental.

![Fig. 5.23. High-frequency transformer CM equivalent circuit, $C_{ps,cm}$](image)

The electrostatic energy expressions (5.63), (5.72), (5.84)/(5.88) indicate that the capacitances sum, $C_{11}$, given by

$$C_{11} = C_{pp} + C_{pp,cc} + C_{ps,dx}$$

(5.91)
can be modeled as lumped capacitance across the primary winding terminals as shown in Figure 5.24. The subscript \( x \) in (5.91) is \( e \) when \( N_{sl} \) is even or \( o \) when \( N_{sl} \) is odd.

The electrostatic energy expressions (5.63) and (5.67) establishes that the capacitances sum, \( C_{22} \), given by

\[
C_{22} = C_{ss} + C_{sc}
\]

(5.92)
can be modeled as lumped capacitance across the secondary winding terminals as shown in Figure 5.24. As the transformer magnetic and resistive network contributes to DM operation, the equivalent circuit in DM is as shown in Figure 5.24.

\[\begin{array}{c}
\text{Fig. 5.24. High-frequency transformer DM equivalent circuit}
\end{array}\]

To analyze the DM equivalent circuit in Figure 5.24, the lumped parameters on secondary side are referred to the primary side as shown in Figure 5.25. Therein, the referred secondary side capacitance, \( C'_{22} \) is given by

\[
C'_{22} = \frac{N_s^2}{N_p^2} C_{22}
\]

(5.93)

The transformer DM equivalent circuit in Figure 5.25 has several impedances whose value could be significant depending on the frequency. At low frequencies (which refers to the operating frequency range, since harmonics are at even higher frequencies), impedance due to the series resistance and leakage inductance is small compared to the magnetizing inductance and shunt capacitors. This results in resonance due to shunt LC in the frequency range of operation due to magnetizing
inductance, $L_m$ and DM capacitance, $C_{DM} = C_{11} + C_{22}$, with resonance frequency given by

$$f_1 = \frac{1}{2\pi\sqrt{L_mC_{DM}}} \hspace{1cm} (5.94)$$

To avoid transformer voltage/current ringing, the resonance frequency $f_1$ given by (5.94) should be much higher than the operating frequency.

In the next section, the experimental validation of parasitic capacitances is presented.

### 5.4 Validation

The analytical approach to calculate parasitic capacitances of a high-frequency core-type transformer is presented in Section 5.2. The transformer CM and DM equivalent circuits are presented in Section 5.3. The analysis set forth in these two sections is validated in this section using the core-type test transformer (Table 2.5) designed for 20kHz operation. The impedance measurements presented herein are measured using Keysight E4990A impedance analyzer.

In the transformer CM configuration, the windings positive and negative terminals are shorted on both primary and secondary sides. In this configuration, the equivalent impedance of the transformer is measured using the setup shown in Figure 5.26. Therein, the terminals 1 and 2 represent transformer primary winding terminals and the terminals 3 and 4 represent secondary winding terminals. The magnitude and
phase of the transformer CM impedance as measured using impedance analyzer are shown in Figure 5.27. The shunt capacitance \( C_p \) and shunt resistance \( R_p \) measured using an impedance analyzer over a frequency range of above 20k Hz is shown in Figure 5.28 and averages about 447 pF. The analytically estimated CM capacitance using (5.82) is 267 pF. This represents an an error of 40% compared to the measured value.

Fig. 5.26. Measurement set up for transformer CM impedance

Fig. 5.27. Test transformer CCM measurement
To measure the DM impedance, the measurement setup shown in Figure 5.29 is used. The magnitude and phase of the transformer impedance is as shown in Figure 5.30. The transformer first resonance frequency is at 37 kHz. At frequencies greater than this resonance frequency, the transformer impedance is capacitive in nature. The measured shunt capacitance and resistance plots are shown in Figure 5.31. The transformer capacitance is in the order of nF, with an average of 1.2 nF and reaches a maximum value of 2 nF. The analytically estimated value for $C_{DM}$ varies from 0.37-0.7 nF corresponding to the windings no-potting to ideal potting case. The test transformer windings are potted. However, because the secondary and primary winding layers overlap it is difficult to achieve 100 % potting.

5.5 Summary

In this chapter, general analytical methods to estimate parasitic capacitances in a winding component are surveyed first. These methods are then applied to determine...
Fig. 5.29. Measurement set up for transformer DM impedance

Fig. 5.30. Impedance plot obtained for test transformer DM measurement

the lumped parasitic capacitances of a core-type transformer. The high-frequency transformer performance over a wide frequency range is analyzed by deriving DM and CM equivalent circuits. A case study using the 20 kHz transformer is presented that validates the analytical analysis on parasitic capacitances.
Fig. 5.31. Impedance plot obtained for test transformer DM measurement
6. THERMAL EQUIVALENT MODELING

In this chapter, the thermal analysis of a core-type transformer is set forth using a Thermal-equivalent-circuit (TEC) approach [31]. The transformer thermal analysis is intended to estimate the temperature rise due to the losses in the core and windings as well as the peak temperature locations while considering the temperature effects on the material properties. This is useful in the transformer design process to limit the transformer loss and the resulting temperature rise to a value compatible with the wire insulation, bobbin, and magnetic materials.

6.1 Core-Type Transformer Thermal Modeling

The thermal equivalent modeling approach described in [42] is used herein. Due to symmetry, it is sufficient to analyze only one-eighth of the core-type transformer as shown in dotted lines in Figures 6.1 and 6.2. For thermal analysis, the core and winding regions are divided into block elements A-N. Regions A-D are core regions with homogeneous material and cuboidal shape. Regions E-N are the winding coil regions with non-homogeneous material and cylindrical shape. Coil regions G and E are underneath core regions A-C as depicted in Figure 6.2 and hence not visible in the top view shown in Figure 6.1.

Before presenting the transformer thermal network model based on Figures 6.1 and 6.2, the non-homogeneous nature of the winding regions, E-N and cylindrical shape of winding corner regions I,K,L, and N are first addressed.
6.2 Coil Homogenization

Winding coils are non-uniform regions consisting of conducting material, wire insulation, and surrounding air or potting material filling the space between the conductors, as depicted in Figure 6.3(a). A detailed representation of these regions is very involved. Instead, an equivalent homogenized coil region is derived in [31], wherein an effective material with equivalent thermal conductivities is used to model the winding region. One strategy for homogenization is to fix each material’s cross sectional area to be the same as the original coil and to impose the aspect ratio of each material to be equal to the coil cross-section aspect ratio [43], as depicted in Figure 6.3(b). The motivation for this is to cause the thermal conductivities in the $x$- and $y$- directions of the homogenized coil to be the same as the original coil.
The cross-sectional areas of conductor, $a_{\text{cond}}$, insulation, $a_i$, and air/potting, $a_a$, are expressed in terms of number of conductors, $N$, conductor radius, $r_c$, conductor insulation thickness, $t_i$, and coil dimensions $w, d$ as

$$ a_{\text{cond}} = N \pi r_c^2 \quad (6.1) $$

$$ a_i = N \pi ((r_c + t_i)^2 - r_c^2) \quad (6.2) $$

$$ a_a = w h - a_{\text{cond}} - a_i \quad (6.3) $$

The coil aspect ratio, denoted by $\xi$ is

$$ \xi = \frac{w}{h} \quad (6.4) $$

Using the coil aspect ratio in (6.4) imposed on the dimensions of the effective materials (represented in Figure 6.3(b)), the effective height and width of conductor material are given as

$$ h_c = \sqrt{\frac{\xi}{\text{cond}} \xi} \quad (6.5) $$

$$ w_c = \sqrt{\frac{\xi}{\text{cond}} \xi} \quad (6.6) $$

Assuming $w_i/h_i = \xi$ and using $(w_c + w_i)(h_c + h_i) = a_i + a_c$, the effective height and width of insulation material are

$$ h_i = \sqrt{h_c^2 + \frac{a_i}{\xi}} - h_c \quad (6.7) $$

$$ w_i = h_i \xi \quad (6.8) $$

Finally, the effective height and width of air/potting are

$$ h_a = h - h_c - h_i \quad (6.9) $$

$$ w_a = w - w_c - w_i \quad (6.10) $$
The effective regions depicted in Figure 6.3(b) are homogenized resulting in coil region as shown in Figure 6.3(c) [31]. The thermal conductivity of the homogenized coil region, $k_{xyh}$ is same in $x$- and $y$- directions given by

$$k_{xyh} = \frac{1}{\frac{1}{k_c} + \frac{h_i}{k_i h_c} + \frac{h_a}{k_a h_c}} + \frac{1}{\frac{h_c + h_i}{k_i h_i} + \frac{h_a}{k_a h_i}} + \frac{1}{h}$$

(6.11)

In (6.11), $k_c$, $k_i$ and $k_a$ denote the thermal conductivities of conductor, insulation and air respectively.

In $z$- direction (along the conductive material), the coil has good thermal conductivity, $k_{zh}$ defined as

$$k_{zh} = \frac{a_c k_c + a_i k_i + a_a k_a}{wd}$$

(6.12)

Using (6.11) and (6.12) as thermal conductivities of homogenized winding region, a non-isotropic three dimensional cuboidal thermal block in [31] can be used to model winding regions E-N.

The rectangularization of winding corner region is considered next.
6.3 Rectangularization of Rounded Corners

Transformer windings must be wound with sufficient bend radius along the core end-leg corners. The winding corner regions take the shape of prisms with circular arc cross-section as shown in Figure 6.4(a). These regions cannot be modeled as cylindrical thermal blocks in [31] as the heat flow in the tangential direction is not negligible. Instead, these corner regions can be modeled using cuboidal thermal blocks by deriving equivalent rectangular cross-section of the circular arcs as shown in Figure 6.4.

The cylindrical reference axes (radial and tangential) as shown in Figure 6.4 are transformed to the Cartesian reference axis shown in Figure 6.4(b). Preserving the effective surface area in each direction, the length of the rectangular region is given as

\[ l_e = \frac{\pi ((r_i + w)^2 - r_i^2)}{4w} \]

(6.13)

Fig. 6.4. Winding corner element thermal modeling

Using length, \( l_e \) given by (6.13) and the original circular-arc coil width, \( w \) as rectangular cross-sectional dimensions, the winding corner regions can be modeled using cuboidal thermal block.

The thermal equivalent circuit of the core-type transformer using the rectangularization and homogenization of winding coil regions is presented in the next section.
6.4 Transformer Thermal Equivalent Circuit

The thermal network of one-eighth geometry of core-type transformer using the nodal representation [31] is shown in Figure 6.5. As depicted in Figures 6.1 and 6.2, blocks A-D correspond to core regions and blocks E-N correspond to winding regions. Each block represents a three-dimensional thermal equivalent circuit of a cuboidal region [31]. Using (6.13), the winding corner regions I,K,L and N are rectangularized resulting in equivalent lengths of secondary and primary winding corner regions, denoted by \( l_{s,rec} \) and \( l_{p,rec} \) respectively, given by

\[
\begin{align*}
  l_{s,rec} &= \frac{\pi (r_{so}^2 - r_{si}^2)}{4w_{sw}} & (6.14) \\
  l_{p,rec} &= \frac{\pi (r_{po}^2 - r_{ps}^2)}{4w_{pw}} & (6.15)
\end{align*}
\]

The A-N blocks \( x, y, z \) dimension are listed in Table 6.1.

<table>
<thead>
<tr>
<th>Cuboid</th>
<th>( x )</th>
<th>( y )</th>
<th>( z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>( w_s/2 - (c_{sc} + w_{sw} + c_{ps}/2) )</td>
<td>( l_c/2 )</td>
<td>( w_{cb} )</td>
</tr>
<tr>
<td>B</td>
<td>( c_{sc} + w_{sw} + c_{ps}/2 )</td>
<td>( l_c/2 )</td>
<td>( w_{cb} )</td>
</tr>
<tr>
<td>C</td>
<td>( w_{ce} )</td>
<td>( l_c/2 )</td>
<td>( w_{cb} )</td>
</tr>
<tr>
<td>D</td>
<td>( w_{ce} )</td>
<td>( l_c/2 )</td>
<td>( d_{sw}/2 )</td>
</tr>
<tr>
<td>E</td>
<td>( w_{sw} )</td>
<td>( l_c/2 )</td>
<td>( d_{sw}/2 )</td>
</tr>
<tr>
<td>I</td>
<td>( l_{s,rec} )</td>
<td>( w_{sw} )</td>
<td>( d_{sw}/2 )</td>
</tr>
<tr>
<td>J</td>
<td>( w_{cecc} )</td>
<td>( w_{sw} )</td>
<td>( d_{sw}/2 )</td>
</tr>
<tr>
<td>K</td>
<td>( l_{s,rec} )</td>
<td>( w_{sw} )</td>
<td>( d_{sw}/2 )</td>
</tr>
<tr>
<td>F</td>
<td>( w_{sw} )</td>
<td>( l_c/2 )</td>
<td>( d_{sw}/2 )</td>
</tr>
<tr>
<td>G</td>
<td>( w_{pw} )</td>
<td>( l_c/2 )</td>
<td>( d_{pw}/2 )</td>
</tr>
<tr>
<td>L</td>
<td>( l_{p,rec} )</td>
<td>( w_{pw} )</td>
<td>( d_{pw}/2 )</td>
</tr>
<tr>
<td>M</td>
<td>( w_{cecc} )</td>
<td>( w_{pw} )</td>
<td>( d_{pw}/2 )</td>
</tr>
<tr>
<td>N</td>
<td>( l_{p,rec} )</td>
<td>( w_{pw} )</td>
<td>( d_{pw}/2 )</td>
</tr>
<tr>
<td>H</td>
<td>( w_{pw} )</td>
<td>( l_c/2 )</td>
<td>( d_{pw}/2 )</td>
</tr>
</tbody>
</table>
Addressing the heat flow across regions, the symmetric surfaces of thermal blocks do not have any heat flow across them and hence are denoted as open-circuit in Figure 6.5. The open-circuit nodes are listed in Table 6.2.

Fig. 6.5. Transformer thermal equivalent circuit

Other common nodes observed in TEC blocks in Figure 6.5 are surfaces exposed to ambient. A thermal resistance is used to model the heat transfer across the core
Table 6.2.
Open circuit nodes due to symmetry

<table>
<thead>
<tr>
<th>Cuboid</th>
<th>Nodes list</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0x, ly</td>
</tr>
<tr>
<td>B</td>
<td>ly</td>
</tr>
<tr>
<td>C</td>
<td>ly</td>
</tr>
<tr>
<td>D</td>
<td>0y, 0z, ly</td>
</tr>
<tr>
<td>E,F,G,H,I,J,K</td>
<td>0z, ly</td>
</tr>
<tr>
<td>L,M,N</td>
<td>0z</td>
</tr>
</tbody>
</table>

or winding surface to the ambient. The core/winding surface to ambient thermal resistances are listed in Table 6.3. Therein,

\[
k_1 = \frac{c_{pp}}{2} + w_{pw} + \frac{c_{ps}}{2} \quad (6.16)
\]

\[
k_2 = \frac{c_{ps}}{2} + w_{sw} + c_{sc} \quad (6.17)
\]

The variables, \(h_{ca}\) and \(h_{wa}\) denote heat transfer coefficients to air from core and winding surfaces respectively. In case of potted windings, the heat transfer coefficient from wire to air, \(h_{wa}\) is replaced with heat transfer coefficient from potted wire to air, \(h_{pa}\) in thermal resistance equations listed in Table 6.3.

Another possible scenario for heat flow is across the surfaces of two different materials when separated by air or insulation. In the core-type transformer, heat flow is possible through the insulation material which create horizontal clearance between two regions, such as winding-core interface shown in Figure 6.6(a) or between primary to secondary windings as shown in Figure 6.7(a). In the case of vertical clearances between core and windings in the core-interior region, heat flow is possible through the air filled in the vertical clearances.

The effective interface surface areas assumed for the heat flow across possible interfaces are listed in Table 6.4. Note that when the interface surface areas of the
Table 6.3.
Thermal resistances to ambient

| Elements A-C | | Elements D-H | | Elements I-K | | Elements L-N |
|--------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $R_{A0ya} = \frac{1}{h_{ca}w_ca k_1}$ | $R_{Alza} = \frac{2}{h_{ca}l_c k_1}$ | $R_{G0za} = \frac{4}{h_{wa}d_{pw}l_c}$ | $R_{Flza} = \frac{2}{h_{wul}w_{sw}}$ | $R_{flza} = \frac{1}{h_{wa}l_{,rec}w_{sw}}$ | | $R_{L0ya} = \frac{2}{h_{wa}d_{pw}l_{,rec}}$ |
| $R_{B0ya} = \frac{1}{h_{ca}w_ca k_2}$ | $R_{Blza} = \frac{2}{h_{ca}l_c k_2}$ | $R_{H1za} = \frac{4}{h_{wa}d_{pw}l_c}$ | $R_{Hlza} = \frac{2}{h_{wul}w_{pw}}$ | $R_{Jlza} = \frac{1}{h_{wa}l_{,rec}w_{sw}}$ | | $R_{M0ya} = \frac{2}{h_{wa}w_{ce}d_{pw}}$ |
| $R_{C0ya} = \frac{1}{h_{ca}w_{ce}w_{cb}}$ | $R_{Clza} = \frac{2}{h_{ca}l_c w_{ce}}$ | | | | | $R_{N0ya} = \frac{2}{h_{wa}d_{pw}l_{,rec}}$ |

Fig. 6.6. Winding to core interface
two adjacent regions are not identical, average of the two cross-sectional areas facing each other is taken. The contact resistance between primary and secondary windings or the winding to core interface is assumed to be determined by the material filling the clearance between the interface surfaces. The clearance between the core and windings in the vertical direction (along z axis direction shown in Figure 6.2) contains air. Therefore, the heat transfer coefficient between the core to primary winding in the z axis direction, \( h_{cpv} \), is given by

\[
h_{cpv} = \frac{k_a}{c_{pv}}
\]  

(6.18)
and that of the core to secondary interface in the vertical direction \( h_{csv} \) is given by

\[
h_{csv} = \frac{k_a}{c_{sv}} \tag{6.19}
\]

The primary-secondary interface in the horizontal direction (along \( x \) axis direction shown in Figure 6.2), has clearance of thickness \( c_{ps} \) filled by insulation material (conductivity, \( k_{in} \)). Therefore, heat transfer coefficient between primary and secondary winding in the horizontal direction is

\[
h_{ps} = \frac{k_{in}}{c_{ps}} \tag{6.20}
\]

Similarly, the heat transfer coefficient between core-secondary interface in horizontal direction is

\[
h_{sc} = \frac{k_{in}}{c_{sc}} \tag{6.21}
\]

The round conductors used in the winding ideally create larger effective surface area for the heat flow compared to a flat surface and contains air-gaps partially filling up the interface as shown in Figures 6.6(a) and 6.7(a). Because the blocks representing winding regions E-N are homogenized using the thermal conductivities given by (6.11) and (6.12), the possible air trapped between the windings or core to winding interfaces and increased surface area due to round conductors is accounted through coil homogenization as discussed in Section 6.2.

The thermal resistance, \( R_{ED} \), connecting \( lx \) node of block E to \( 0x \) node of block D is given by

\[
R_{ED} = \frac{1}{S_{ED}h_{sc}} \tag{6.22}
\]

By geometric symmetry, \( R_{DF} = R_{ED} \).

Similarly, the primary to secondary windings interface thermal resistance, \( R_{GE} \) connecting \( lx \) node of block G to \( 0x \) node of block E is given by

\[
R_{GE} = \frac{1}{S_{GE}h_{ps}} \tag{6.23}
\]
By symmetry in the geometry, $R_{FH} = R_{GE}$.

The primary to secondary windings interface thermal resistances, $R_{IL}$ (connecting 0y node of block I to 0y node of block L) and $R_{JM}$ (connecting 0y node of block J to 0y node of block M) are respectively given by

$$R_{IL} = \frac{1}{S_{IL} h_{ps}}$$

(6.24)

$$R_{JM} = \frac{1}{S_{JM} h_{ps}}$$

(6.25)

The A-block of core to G-block of primary winding has core to winding interface with the heat transfer coefficient given by (6.18). The heat flow across this interface is modeled using a thermal resistance, $R_{AG}$, connecting 0z node of block A to 1z of block G, given by

$$R_{AG} = \frac{1}{S_{AG} h_{pv}}$$

(6.26)

Similarly, the interface between block B of core and block E of secondary winding is modeled using thermal resistance, $R_{BE}$, connecting 0z node of block B to 1z of block E, given by

$$R_{BE} = \frac{1}{S_{BE} h_{sv}}$$

(6.27)

The TEC in Figure 6.5 is solved using the thermal resistances as defined in Table 6.3 and (6.23)-(6.27) based on the method outlined in [42]. The temperature rise in the different parts of the transformer depend on the core and windings losses. The power loss in each element is taken to be proportional to its volume. In particular, loss in the core elements A-D is

$$P_{c,X} = \frac{V_X}{8(V_A + V_B + V_C + V_D)} P_c$$

(6.28)

where $P_{c,X}$ denotes loss in element $X$, $X \in [A, B, C, D]$ and $P_c$ denotes the total core loss in the transformer. As described in Chapter 3, core loss is calculated using Modified Steinmetz Equation (MSE).
In case of the transformer windings, the total loss includes loss due to DC resistance, skin and proximity effects. The loss assigned to the primary coil elements E-N, $P_{p,X}$, is given by

$$P_{p,X} = \frac{V_X P_{lp}}{8(V_G + V_L + V_M + V_N + V_H)}$$  \hspace{1cm} (6.29)

where $X \in [G, H, L, M, N]$, $P_{lp}$ denotes total loss in the primary winding.

Similarly, the loss, $P_{s,X}$ in the case of secondary coil element X, is given as

$$P_{s,X} = \frac{V_X P_{ls}}{8(V_E + V_I + V_J + V_K + V_F)}$$  \hspace{1cm} (6.30)

where $X \in [E, F, I, J, K]$, $P_{ls}$ denotes total loss in the secondary winding.

Additionally, the windings losses are dependent on the temperature because of the conductor material properties. Therefore, an iterative approach as illustrated in [42], is used to integrate the relation between the windings temperature and the conductor material resistivity and solve for nodal temperatures in Figure 6.5.

This concludes the set up for transformer TEC.

Once the temperatures of the nodes in Figure 6.5 are solved using the approach outlined in [42] and (6.28)-(6.30), the peak and mean winding temperatures are of particular interest when designing the transformer. The mean primary winding temperature, $T_{pmn}$, is defined as the volumetric average of the mean temperatures of primary winding elements in the set $P = [G, L, M, N, H]$. In particular,

$$T_{pmn} = \frac{\sum_{x \in P} T_{x,mn} V_x}{\sum_{x \in P} V_x}$$  \hspace{1cm} (6.31)

where $T_{x,mn}$ and $V_x$ respectively denote the mean temperature and volume of block x.
Similarly, the mean secondary winding temperature, \( T_{smn} \), is defined as the volumetric average of the mean temperatures of secondary winding elements in the set \( S = [E, I, J, K, F] \). In particular,

\[
T_{smn} = \frac{\sum_{x \in S} T_{x,mn} V_x}{\sum_{x \in S} V_x}
\]  

(6.32)

The peak primary winding temperature, \( T_{ppk} \) is defined as

\[
T_{ppk} = \max_{x \in P} T_{x,pk}
\]  

(6.33)

The peak secondary winding temperature, \( T_{psk} \) is defined as

\[
T_{psk} = \max_{x \in S} T_{x,pk}
\]  

(6.34)

While designing the high-frequency transformer, the peak and mean winding temperatures given by (6.4)-(6.34) are limited to a value compatible with conductor material properties. The inclusion of the thermal analysis in the transformer design process will be discussed in detail in Chapter 7.
7. MULTI-OBJECTIVE OPTIMIZATION BASED DESIGN

In this research, the design methodology is set forth for a high-frequency core-type transformer in the context of an isolating DC-DC converter. The chosen core-type transformer geometry along with the illustration of coil cross-section are repeated in Figures 7.1 and 7.2 for reference. The DC-DC converter topology is repeated in Figure 7.3.

![Core-type transformer configuration diagram]

**Fig. 7.1.** Core-type transformer configuration

The core-type transformer high-frequency magnetic, electrical, and thermal analyses are presented in detail in the Chapters 2-6. In this chapter, an optimization based design methodology is set forth utilizing these analyses. A multi-objective optimization based design process for a low-frequency grid connected transformer is discussed.
Fig. 7.2. Coil cross-section depicting conductor arrangement

Fig. 7.3. Isolating converter module

in detail in [31]. A similar philosophy is taken herein, though the process is much more involved because of high-frequency effects and the converter. The design development begins by setting forth the design space and the design constraints. These are enumerated in Sections 7.1 and 7.2 respectively. Section 7.3 defines the design metrics that are of interest and outlines the detailed analyses used to evaluate the design fitness in the form of a fitness function, followed by conclusions in Section 7.4.

Before proceeding, to present the transformer design optimization methodology in an organized fashion and to facilitate implementation, it is helpful to define the set of parameters of interest in the form of vectors or structures. The description of the parameters and variables are given in Tables 7.1 and 7.2.
## Table 7.1.: Core and clearances dimensions listing

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_c$</td>
<td>Material of the core</td>
</tr>
<tr>
<td>$w_{cs}$</td>
<td>Width of the core slot</td>
</tr>
<tr>
<td>$d_{cs}$</td>
<td>Depth of the core slot</td>
</tr>
<tr>
<td>$w_c$</td>
<td>Width of the core</td>
</tr>
<tr>
<td>$d_c$</td>
<td>Depth of the core</td>
</tr>
<tr>
<td>$l_c$</td>
<td>Length of the core</td>
</tr>
<tr>
<td>$w_{cb}$</td>
<td>Width of core/base top leg</td>
</tr>
<tr>
<td>$w_{ce}$</td>
<td>Width of core end leg</td>
</tr>
<tr>
<td>$r_w$</td>
<td>Radius of semi-circular inert material used for support</td>
</tr>
<tr>
<td>$w_{ce}$</td>
<td>Width of the inert material</td>
</tr>
<tr>
<td>$\rho_c$</td>
<td>Mass density of core</td>
</tr>
<tr>
<td>$\mu_r$</td>
<td>Parameter of $\mu_B(B)$</td>
</tr>
<tr>
<td>$\alpha_\mu$</td>
<td>Parameter of $\mu_B(B)$</td>
</tr>
<tr>
<td>$\beta_\mu$</td>
<td>Parameter of $\mu_B(B)$</td>
</tr>
<tr>
<td>$\gamma_\mu$</td>
<td>Parameter of $\mu_B(B)$</td>
</tr>
<tr>
<td>$k_h$</td>
<td>MSE loss parameter</td>
</tr>
<tr>
<td>$k_c$</td>
<td>MSE loss parameter</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>MSE loss parameter</td>
</tr>
<tr>
<td>$\beta$</td>
<td>MSE loss parameter</td>
</tr>
<tr>
<td>$A_{cbl}$</td>
<td>Area of cross-section of the core base leg</td>
</tr>
<tr>
<td>$l_{cbl}$</td>
<td>Length of the core base leg</td>
</tr>
<tr>
<td>$A_{cel}$</td>
<td>Area of cross-section of the core end leg</td>
</tr>
<tr>
<td>$l_{cel}$</td>
<td>Length of the core end leg</td>
</tr>
<tr>
<td>$V_c$</td>
<td>Volume of the core</td>
</tr>
<tr>
<td>$M_c$</td>
<td>Mass of the core</td>
</tr>
<tr>
<td>$c_{sc}$</td>
<td>Horizontal clearance of secondary winding to core</td>
</tr>
<tr>
<td>$c_{ps}$</td>
<td>Horizontal clearance between primary and secondary windings</td>
</tr>
<tr>
<td>$c_{pp}$</td>
<td>Horizontal clearance between primary winding coils</td>
</tr>
<tr>
<td>$c_{pv}$</td>
<td>Vertical clearance between primary winding and core</td>
</tr>
<tr>
<td>$c_{sv}$</td>
<td>Vertical clearance between secondary winding and core</td>
</tr>
</tbody>
</table>

## Table 7.2.: Core and clearances dimensions listing

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_p$</td>
<td>Material of the primary winding</td>
</tr>
<tr>
<td>$\rho_{pc}$</td>
<td>Mass density of primary</td>
</tr>
<tr>
<td>$N_{pcp}$</td>
<td>Number of primary coils in parallel</td>
</tr>
</tbody>
</table>

*continued on next page*
Table 7.2: continued

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{pl}$</td>
<td>Number of layers in primary coil</td>
</tr>
<tr>
<td>$N_{ptl}$</td>
<td>Number of turns in each primary coil layer</td>
</tr>
<tr>
<td>$\sigma_{cp}$</td>
<td>Conductivity of primary material</td>
</tr>
<tr>
<td>$a_{pc}$</td>
<td>Area of primary coil conductor</td>
</tr>
<tr>
<td>$a^*_{pc}$</td>
<td>Desired area of primary coil conductor</td>
</tr>
<tr>
<td>$N_{ppr}$</td>
<td>Desired number of primary coil strands in parallel</td>
</tr>
<tr>
<td>$R^*_{pdw}$</td>
<td>Desired aspect ratio of primary coil</td>
</tr>
<tr>
<td>$N_{pcl}$</td>
<td>Desired number of turns in primary coil</td>
</tr>
<tr>
<td>$r_{pc}$</td>
<td>Radius of primary coil conductor</td>
</tr>
<tr>
<td>$a_{ps}$</td>
<td>Area of primary coil strand</td>
</tr>
<tr>
<td>$r_{ps}$</td>
<td>Radius of primary coil strand</td>
</tr>
<tr>
<td>$t_{pins}$</td>
<td>Thickness of primary conductor insulation</td>
</tr>
<tr>
<td>$d_{pw}$</td>
<td>Depth of primary winding coil</td>
</tr>
<tr>
<td>$w_{pw}$</td>
<td>Width of primary winding coil</td>
</tr>
<tr>
<td>$r_{pi}$</td>
<td>Inner radius of primary winding</td>
</tr>
<tr>
<td>$r_{po}$</td>
<td>Outer radius of primary winding</td>
</tr>
<tr>
<td>$l_{pcl}$</td>
<td>Straight length of primary winding</td>
</tr>
<tr>
<td>$l_{pc3}$</td>
<td>Average length of primary coil in Region 3</td>
</tr>
<tr>
<td>$l_{pt}$</td>
<td>Average turn length of primary coil</td>
</tr>
<tr>
<td>$w_{pcl}$</td>
<td>Straight width of primary winding</td>
</tr>
<tr>
<td>$k_{ppf}$</td>
<td>Primary winding packing factor</td>
</tr>
<tr>
<td>$V_{pcl}$</td>
<td>Primary coil volume</td>
</tr>
<tr>
<td>$M_p$</td>
<td>Mass of the primary winding</td>
</tr>
<tr>
<td>$k_{pbnd}$</td>
<td>Primary strand bend radius ratio</td>
</tr>
<tr>
<td>$m_s$</td>
<td>Material of the secondary</td>
</tr>
<tr>
<td>$\rho_{sc}$</td>
<td>Mass density of secondary</td>
</tr>
<tr>
<td>$N_{scp}$</td>
<td>Number of secondary coils in parallel</td>
</tr>
<tr>
<td>$N_{st}$</td>
<td>Number of layers in secondary coil</td>
</tr>
<tr>
<td>$N_{stl}$</td>
<td>Number of turns in each secondary coil layer</td>
</tr>
<tr>
<td>$\sigma_{cs}$</td>
<td>Conductivity of secondary material</td>
</tr>
<tr>
<td>$a_{sc}$</td>
<td>Area of secondary coil conductor</td>
</tr>
<tr>
<td>$a^*_{sc}$</td>
<td>Desired area of secondary coil conductor</td>
</tr>
<tr>
<td>$N_{spr}$</td>
<td>Desired number of secondary coil strands in parallel</td>
</tr>
<tr>
<td>$R^*_{Nps}$</td>
<td>Desired primary to secondary turns ratio</td>
</tr>
<tr>
<td>$R^*_{sdw}$</td>
<td>Desired aspect ratio of secondary coil</td>
</tr>
<tr>
<td>$N_{sc}$</td>
<td>Desired number of turns in secondary coil</td>
</tr>
<tr>
<td>$r_{sc}$</td>
<td>Radius of secondary coil conductor</td>
</tr>
<tr>
<td>$a_{ss}$</td>
<td>Area of secondary coil strand</td>
</tr>
<tr>
<td>$r_{ss}$</td>
<td>Radius of primary coil strand</td>
</tr>
<tr>
<td>$t_{sins}$</td>
<td>Thickness of secondary conductor insulation</td>
</tr>
</tbody>
</table>

continued on next page
Table 7.2.: continued

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_{sw}$</td>
<td>Depth of secondary winding coil</td>
</tr>
<tr>
<td>$w_{sw}$</td>
<td>Width of secondary winding coil</td>
</tr>
<tr>
<td>$r_{si}$</td>
<td>Inner radius of secondary winding</td>
</tr>
<tr>
<td>$r_{so}$</td>
<td>Outer radius of secondary winding</td>
</tr>
<tr>
<td>$l_{sc}$</td>
<td>Straight length of secondary winding</td>
</tr>
<tr>
<td>$l_{sc3}$</td>
<td>Average length of secondary coil in Region 3</td>
</tr>
<tr>
<td>$l_{st}$</td>
<td>Average turn length of secondary coil</td>
</tr>
<tr>
<td>$k_{spf}$</td>
<td>Secondary winding packing factor</td>
</tr>
<tr>
<td>$V_{scd}$</td>
<td>Secondary coil volume</td>
</tr>
<tr>
<td>$M_s$</td>
<td>Mass of the secondary winding</td>
</tr>
<tr>
<td>$k_{sbnd}$</td>
<td>Secondary strand bend radius ratio</td>
</tr>
</tbody>
</table>

The transformer performance is evaluated by analyzing the ICM circuit shown in Figure 7.3. Therefore, it is useful to describe the ICM device ratings and its components’ (other than transformer) specifications in the form of a vector as

$$D_s = [V_{in} V_{out} f_{sw} D_{inv}^T V_{fd} L_{dc} r_{dc}]^T$$  \hspace{1cm} (7.1)

where $V_{fd}$ denotes the rectifier diode forward voltage drop and $D_{inv}$ denotes the vector with the inverter power-semiconductor parameters given by

$$D_{inv} = [V_{ce} V_{ce} E_{off} V_0 I_0]^T$$  \hspace{1cm} (7.2)

The definitions of the parameters in (7.1) and (7.2) are listed in Table 7.3. In the case of transformer, a set of independent and dependent parameters for each of core, clearances, primary and secondary windings may be defined in the form of vectors or structures. The independent and dependent parameters are used together to describe the complete transformer design.
Table 7.3.
Prototype ICM semiconductor loss data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{out}$</td>
<td>Output voltage</td>
</tr>
<tr>
<td>$V_{in}$</td>
<td>Input DC Voltage</td>
</tr>
<tr>
<td>$f_{sw}$</td>
<td>Switching frequency</td>
</tr>
<tr>
<td>$L_{dc}$</td>
<td>DC link inductance</td>
</tr>
<tr>
<td>$r_{dc}$</td>
<td>DC link Inductor series resistance</td>
</tr>
<tr>
<td>$V_{ce}$</td>
<td>IGBT switch forward voltage drop</td>
</tr>
<tr>
<td>$V_{ec}$</td>
<td>IGBT freewheeling diode forward voltage drop</td>
</tr>
<tr>
<td>$E_{of f,0}$</td>
<td>IGBT switch turn off energy loss</td>
</tr>
<tr>
<td>$V_0$</td>
<td>IGBT switch nominal voltage</td>
</tr>
<tr>
<td>$I_0$</td>
<td>IGBT switch nominal current</td>
</tr>
<tr>
<td>$V_{fd}$</td>
<td>SiC diode forward voltage drop</td>
</tr>
</tbody>
</table>

The independent parameters in case of the core, clearances, primary and secondary windings may be defined as

$$C_I = [m_c l_c r_w w_{ecc} w_{eb}]^T$$ \hspace{1cm} (7.3)

$$G_I = [c_{pp} c_{sc} c_{ps}]^T$$ \hspace{1cm} (7.4)

$$P_I = [m_p N_{ptl} N_{pl} N_{ppr} a_{ps} N_{pcp} N_{pcs}]^T$$ \hspace{1cm} (7.5)

$$S_I = [m_s N_{stl} N_{st} N_{spr} a_{ss} N_{scp} N_{scs}]^T$$ \hspace{1cm} (7.6)

The corresponding set of dependent variables defined in the case of the core, clearances, primary and secondary winding, respectively are

$$C_D = [P_c^T d_{cs} w_{cs} d_c w_c l_c w_{cc} A_{ccl} l_{ch} A_{cel} l_{cel} V_c M_c]^T$$ \hspace{1cm} (7.7)

$$G_D = [c_{pv} c_{sv} c_{ps}]^T$$ \hspace{1cm} (7.8)

$$P_D = [P_p^T a_{pc} r_{ps} d_{pw} w_{pw} r_{pi} r_{po} l_{pcl} w_{pct} l_{pc3} k_{ppf} V_{pcl} M_x k_{phmd}]^T$$ \hspace{1cm} (7.9)
\( S_D = [P_s^T \ a_{sc} \ r_{ss} \ d_{sw} \ w_{sw} \ r_{si} \ r_{so} \ l_{sccl} \ w_{sccl} \ l_{sc3} \ k_{xpf} \ V_{sccl} \ M_{x} \ k_{xbnd}]^T \) (7.10)

where the core material properties are incorporated into a vector form, \( P_c \), as

\[ P_c = [\rho_c \ \mu_r \ \alpha \mu \ \beta \mu \ \gamma \mu \ \kappa_h \ \kappa_e \ \alpha \ \beta]^T \] (7.11)

Similarly, the primary and secondary winding material properties are described as

\[ P_p = [\rho_{pc} \ \sigma_{pc} \ \alpha_{pc}]^T \] (7.12)

\[ P_s = [\rho_{sc} \ \sigma_{sc} \ \alpha_{sc}]^T \] (7.13)

Further, the complete vectors for each of the core, clearances, the primary winding, and secondary winding may be defined as

\[ C = [C_D^T \ C_D^T]^T \] (7.14)

\[ C = [C_I^T \ C_D^T]^T \] (7.15)

\[ P = [P_I^T \ P_D^T]^T \] (7.16)

\[ S = [S_I^T \ S_D^T]^T \] (7.17)

The geometric relations for calculating the dependent parameters of windings and core are given by

\[ a_{xc} = a_{xs}N_{xpr} \] (7.18)

\[ r_{xs} = \sqrt{\frac{d_{xs}}{\pi}} \] (7.19)

\[ r_{xbnd} = 2r_{xs}k_{xbnd} \] (7.20)

\[ N_x = N_{xcl}N_{xcs} \] (7.21)

\[ d_{xw} = 2N_{xlt}N_{xpr}k_{xb}(r_{xs} + t_{xins}) \] (7.22)

\[ w_{xw} = 2N_{xl}k_{xb}(r_{xs} + t_{xins}) \] (7.23)
\[ k_{xpf} = \frac{N_{xcl}a_{xc}}{d_{xw}w_{xw}} \] (7.24)

\[ l_{xcl} = l_c \] (7.25)

\[ w_{xcl} = w_{cc} \] (7.26)

\[ r_{xi} = \begin{cases} r_w + c_{sc} & x = 's' \\ r_{xo} + c_{ps} & x = 'p' \end{cases} \] (7.27)

\[ r_{xo} = r_{xi} + w_{xw} \] (7.28)

\[ l_{xcl} = l_{cc} \] (7.29)

\[ w_{xcl} = w_{cc} \] (7.30)

\[ l_{xe} = \frac{1}{4} \pi (r_{xi} + r_{xo}) \] (7.31)

\[ l_{xt} = 2(l_{xcl} + w_{xcl} + 2l_{xe}) \] (7.32)

\[ V_{xcl} = d_{xw} \left( \pi (r_{xo}^2 - r_{xi}^2) + 2w_{pw}(l_{xcl} + w_{xcl}) \right) \] (7.33)

\[ M_x = 2\rho_{xw}V_{xcl}k_{xpf} \] (7.34)

\[ d_{cs} = d_{sw} + c_{sv} \] (7.35)

\[ c_{pv} = \frac{1}{2}(d_{cs} - d_{pw}) \] (7.36)

\[ w_{cs} = c_{pp} + 2(w_{sw} + c_{sc} + w_{pw} + c_{ps}) \] (7.37)

\[ w_{ce} = w_{cc} + 2r_w \] (7.38)

\[ d_c = d_{cs} + 2w_{cb} \] (7.39)

\[ w_c = w_{cs} + 2w_{ec} \] (7.40)

\[ A_{cbl} = w_{cbl}c \] (7.41)

\[ l_{cbl} = d_{cs} + w_{ce} \] (7.42)

\[ A_{cel} = l_cw_{ce} \] (7.43)
\[ l_{cel} = d_{cs} + w_{cb} \quad (7.44) \]
\[ V_c = 2(A_{cl} w_c + A_{cel} d_{cs}) \quad (7.45) \]
\[ M_c = \rho_c V_c \quad (7.46) \]

In addition, the calculations relevant to the overall transformer dimensions include its total length, \( l_T \), total width, \( l_w \), total depth and total electromagnetic mass, \( M_T \). These are given by

\[ l_T = l_c + 2(c_{sc} + w_{sw} + c_{ps} + w_{pw}) \quad (7.47) \]
\[ w_T = w_c + 2(c_{sc} + w_{sw} + c_{ps} + w_{pw}) \quad (7.48) \]
\[ M_T = M_c + M_p + M_s \quad (7.49) \]

The total depth of the transformer is equal to the core depth (see Figure 7.1(a)).

Starting with the winding parameters, (7.18)-(7.34) are implemented first for the case of secondary winding, and then followed by the primary winding case. Next, the core dimensions are evaluated using (7.35)-(7.49). These are followed by the overall transformer dimensions given by (7.47)-(7.49). Alternately, the assignment of the geometric and material properties may be expressed in a functional form as

\[ T = F_{geom}([T^T_I \quad G^T_j \quad P^T_i \quad S^T_j])^T) \quad (7.50) \]

where \( T \) denotes a vector (or structure of structures) that aggregates the transformer description, given by,

\[ T = [C^T \quad G^T \quad P^T \quad S^T \quad l_T \quad w_T \quad M_T]^T \quad (7.51) \]
7.1 Design Space

The design space of an optimization problem is a set of free parameters that define the design. In case of the high-frequency core-type transformer herein, the design space is selected as

$$\Theta = [m_c \ l_c \ r_w \ w_cec \ w_{ch} \ m_p \ a_{pc}^* \ N_{ppr}^* \ N_{ptl}^* \ R_{pdw}^* \ m_s \ a_{sc}^* \ N_{spr}^* \ R_{Nps}^* \ R_{sdw}^* \ c_{pp} \ c_{ps} \ c_{ps}]^T$$  \hspace{1cm} (7.52)

In (7.52), the first five parameters define the transformer core, followed by five parameters for each of primary and secondary. The last three parameters define the clearances in the horizontal direction. The clearances are particularly included in the design space as they are critical to the high-frequency transformer parasitic capacitance and leakage inductance values. The remaining independent variables are included in a vector, $D_{fp}$,

$$D_{fp} = [c_{sv} \ N_{pcp} \ N_{pcs} \ k_{pb} \ t_{pins} \ N_{scp} \ N_{scs} \ k_{sb} \ t_{sins}]^T$$  \hspace{1cm} (7.53)

and are defined as part of design specifications.

In (7.52), parameters denoted with an asterisk indicate that they are desired values, based on which feasible values will be determined. The winding parameters, $N_{ppr}$ and $N_{spr}$ obey

$$N_{xyz} = \text{round}(N_{xyz}^*)$$  \hspace{1cm} (7.54)

where the function ‘round’, rounds the arguments to the nearest integer.

The primary parameters, $N_{pcl}$, $N_{ptl}$ and $N_{pl}$ are calculated using the following sequence

$$N_{ptl} = \text{ceil} \left( \sqrt{\frac{N_{pcl}^* R_{pdw}^*}{N_{ppr}^*}} \right)$$  \hspace{1cm} (7.55)

$$N_{pl} = \text{ceil} \left( \frac{N_{ptl}^*}{N_{ptl}} \right)$$  \hspace{1cm} (7.56)

$$N_{pcl} = N_{ptl} N_{pl}$$  \hspace{1cm} (7.57)
where the function ‘ceil’, rounds the argument to the nearest integer greater than or equal to itself.

The secondary parameters, \( N_{sc1} \), \( N_{sti} \) and \( N_{sl} \) are calculated using the following sequence

\[
N_{sc1}^* = \frac{N_{pc1} N_{pcs}}{R_{N_{pc1}N_{scs}}} \tag{7.58}
\]

\[
N_{sti} = \text{ceil} \left( \frac{N_{sc1}^* R_{sdw}^*}{N_{spr}} \right) \tag{7.59}
\]

\[
N_{sl} = \text{ceil} \left( \frac{N_{sc1}^*}{N_{sti}} \right) \tag{7.60}
\]

\[
N_{sc1} = N_{sti} N_{sl} \tag{7.61}
\]

Note, (7.55)-(7.61) are defined so as to enforce a complete final layer for both the primary and secondary coils.

The sizes of the winding strands that are used in parallel to make up the conductor is calculated conforming to the standard sizes as

\[
a_{xs} = \text{round}_{\text{AWG}} \left( \frac{k_{xs}}{N_{xpr}} \right) \tag{7.62}
\]

where \( \text{round}_{\text{AWG}}() \) denotes a function that rounds the argument to closest available standard AWG wire gauge [44].

### 7.2 Design Constraints

A set of constraints are implemented during the high-frequency transformer design process using ‘greater than or equal to’ (gte) or ‘lesser than or equal to’ (lte) function as described in [45]. The definitions of these functions are repeated here for convenience,

\[
gte(x, x_{mn}) = \begin{cases} 
1, & x \geq x_{mn} \\
\frac{1}{x_{mn} - x}, & x < x_{mn} 
\end{cases} \tag{7.63}
\]
The status of a constraint is assigned a value of one if the constraint is satisfied, and less than one if the constraint is not satisfied. The first set of constraints are geometrical. These include constraints on the transformer total length, width and depth, expressed as

\[
c_1 = \text{lte}(l_T, l_{T\text{max}})
\]

\[
c_2 = \text{lte}(w_T, w_{T\text{max}})
\]

\[
c_3 = \text{lte}(d_c, d_{T\text{max}})
\]

where \(l_{T\text{max}}, w_{T\text{max}}\) and \(d_{T\text{max}}\) are the maximum allowed length, width and depth, respectively.

The transformer total mass, denoted by \(M_T\) is restricted to be below a maximum allowed value, \(M_{T\text{max}}\), using a constraint

\[
c_4 = \text{lte}(M_T, M_{T\text{max}})
\]

The primary and secondary conductors each require a minimum bend radius around the core end-leg. An inert material, of thickness, \(r_w\), as shown in Figure 7.1(b) is used for this purpose. In particular, the bend radius ratio of primary, \(k_{p\text{bnd}}\), and secondary strands, \(k_{s\text{bnd}}\), defined as

\[
k_{p\text{bnd}} = \frac{2r_{ps}}{r_{ps}}
\]

\[
k_{s\text{bnd}} = \frac{2r_{ss}}{r_{si}}
\]
are required to be greater than a minimum required value, denoted by \( k_{bndmn} \). This constraint is implemented using

\[
c_5 = \text{gte}(k_{pbnd}, k_{bndmn})
\]  
(7.71)

\[
c_6 = \text{gte}(k_{sbnd}, k_{bndmn})
\]  
(7.72)

To stack the primary winding on top of secondary, the depth of the secondary winding should be larger than that of the primary winding. To meet this requirement, constraint \( c_7 \) is defined as

\[
c_7 = \text{gte}(d_{sw}, d_{pw})
\]  
(7.73)

The packing factor of the primary and secondary windings is ensured to be practically achievable. Therefore, constraints \( c_8 \) and \( c_9 \) are implemented as

\[
c_8 = \text{lte}(k_{ppf}, k_{pfmx})
\]  
(7.74)

\[
c_9 = \text{lte}(k_{spf}, k_{pfmx})
\]  
(7.75)

where \( k_{pfmx} \) denotes maximum allowed packing factor.

The remaining constraints are related to the performance of the transformer and the DC-DC converter or successful numerical convergence of the performance analysis. The transformer magnetizing inductance for a heavily saturated case is evaluated using iterative approach. Constraint \( c_{10} \) is associated with the iterative solver convergence used to calculate magnetizing inductance and implemented as

\[
c_{10} = \text{lte}(c_{lm}, C_{lm,mx})
\]  
(7.76)

where \( c_{lm} \) denotes number of iterations utilized and \( C_{lm,mx} \) denotes the maximum limit.

The common-mode impedance of the high-frequency transformer is responsible for high-frequency conduction from the primary side to the secondary side. There-
fore, a constraint is used to attain a required minimum transformer CM impedance magnitude, denoted by $|Z_{CM}|$,

$$c_{11} = \text{gte}(|Z_{CM}|, Z_{CMmn}) \quad (7.77)$$

In (7.77), $Z_{CMmn}$ denotes the minimum required CM impedance magnitude, set according to the converter switching frequency. Also, $Z_{CM}$ is calculated using (5.90) at the converter switching frequency, $f_{sw}$.

The transformer DM parasitic capacitances can cause ringing in the differential mode voltage and currents. Therefore, the resonance frequency, $f_{res}$, associated with shunt DM capacitance $C_{DM}$ and magnetizing inductance, $L_m$ is larger than a minimum value, $f_{res,mn}$. In particular,

$$c_{12} = \text{gte}(f_{res}, f_{res,mn}) \quad (7.78)$$

Next, the constraints relating to operating point analysis are presented. Following constraints are repeated for each of the operating points and therefore are represented with incremental subscript using $k$ to denote the $k^{th}$ operating point.

The time domain analysis uses an iterative approach to solve for transformer currents and the total semiconductor loss, $P_{sc,k}$. Therefore, constraint $c_{13+8(k-1)}$ is based on the convergence of time-domain analysis. In particular,

$$c_{13+8(k-1)} = \text{lte}(c_k, C_{mx}) \quad (7.79)$$

where $c_k$ denotes the number of iterations used and $C_{mx}$ denotes the maximum number of allowed iterations.

The semiconductor loss is restricted to be below a maximum value, $P_{scmx}$, using

$$c_{14+8(k-1)} = \text{lte}(P_{sc,k}, P_{scmx}) \quad (7.80)$$
The safe functioning of power semiconductors in the ICM is ensured by implementing constraint \( c_{15+8(k-1)} \). This constraint limits the peak value of the primary current. It is defined as

\[
c_{15+8(k-1)} = \max(i_p, I_{pk, mx})
\]  

(7.81)

where \( i_p \) denotes vector of primary current waveform data as defined in (7.106) and \( I_{pk, mx} \) denotes the maximum allowed peak value of the transformer primary current.

The transformer MEC shown in Figure 2.7 is derived based on the assumption that the core flux dominates the leakage fluxes. To make sure this assumption holds true, a magnetizing flux ratio vector, \( r_\Phi \), is evaluated. This vector contains the ratio of magnetizing flux linkage to the maximum of the total flux linkage of the individual windings calculated over one cycle duration, with the \( n^{th} \) element given by

\[
r_{\Phi n} = \frac{L_m i_m(n)}{\max(L_m i_m(n) + L_{ip} i_{p}(n), L_m i_m(n) + L'_{ip} i_s(n))}
\]  

(7.82)

In (7.82), vector of magnetizing current data, \( i_m \) is given by,

\[
i_m = i_p + i'_s
\]  

(7.83)

where prime is used to denote quantities referred to primary side and \( i_s \) denotes secondary current waveform data as defined in (7.107). The MEC for the design is reasonably accurate if flux ratio values in \( r_\Phi \) are close to one. This condition is implemented in the design process as a constraint using

\[
c_{16+8(k-1)} = \min(r_\Phi, r_{\Phi mn})
\]  

(7.84)

The constraint \( c_{17+8(k-1)} \) is based on the convergence of MEC used for core loss calculation.
Constraint $c_{18+8(k-1)}$ is implemented to check the convergence of thermal analysis. The TEC analysis depends on the material properties such as conductor conductivities and transformer dimensions. Constraint $c_{19+8(k-1)}$ is used to ensure the TEC analysis results obtained are feasible, and is implemented as

$$c_{19+8(k-1)} = \text{lte}(\min(T_{TEC,k}), T_a)$$  \hspace{1cm} (7.85)$$

where $T_{TEC,k}$ denotes temperature of the nodes in TEC shown in Figure 6.5 and $T_a$ denotes ambient temperature.

To limit the peak temperature in the transformer, constraint $c_{20+8(k-1)}$ is implemented as

$$c_{20+8(k-1)} = \text{lte}(\max(T_{pk,k}), T_{max})$$  \hspace{1cm} (7.86)$$

where $T_{pk,k} = [T_{pk,c}, T_{pk,p}, T_{pk,s}]$ denotes a vector consisting of peak temperatures of core, $T_{pk,c}$ primary, $T_{pk,p}$ and secondary coils, $T_{pk,s}$ given by (6.33)- (6.34) and $T_{max}$ is maximum allowed peak temperature.

This concludes the list of constraints implemented for each operating point. Beyond the operating point analysis, a constraint, $c_{21}$, is implemented on the transformer losses as

$$c_{21+8(N_{op}-1)} = \text{lte}(\max(P_{txf,1}, ..., P_{txf,N_{op}}), P_{txf,max})$$  \hspace{1cm} (7.87)$$

where $P_{txf,k}, k \in [1, 2, ..., N_{op}]$ denotes the transformer loss for $k^{th}$ ICM operating point and $P_{txf,max}$ denotes the maximum allowed transformer loss.

Once the system losses are calculated for each operating point, the aggregate loss, $P_l$ is given by

$$P_l = \sum_{k=1}^{N_{op}} W_{id}(k) (P_{sc,k} + P_{txf,k})$$  \hspace{1cm} (7.88)$$
where $W_{ld}$ denotes the vector of weights assigned to the operating point and $P_{txf,k}$ denotes the total transformer loss when ICM is operating at $k^{th}$ operating point. To limit the losses in the system, a constraint is implemented on $P_l$ as

$$c_{22+8(N_{op}-1)} = \text{lte}(P_l, P_{lmax})$$

(7.89)

This concludes the list of constraints.

The limits used to evaluate constraints are defined as part of design specifications, which may be organized as a structure of the form

$$D_{ds} = [l_{T_{mx}a} w_{T_{mx}a} d_{T_{mx}a} \alpha_{T_{mx}a} M_{T_{mx}a} k_{bndmn} k_{pfx} \chi_{mx} r_{\phi_{mx}} \ldots$$

$$Z_{CM_{mn}} f_{res,mn} I_{pk,mx} T_{mx} P_{scmx} P_{txf,mx} P_{lmax} C_{mx} C_{l,mx} \ldots$$

$$N_{harm} ]$$

(7.90)

Finally, the complete list of specifications can be incorporated into single structure as

$$D = [D_{fp} D_{ds} D_{s}]^T$$

(7.91)

where $D_{fp}$ and $D_s$ are as defined in (7.53) and (7.1) respectively.

In the next section, the design metrics used in this research are discussed.

### 7.3 Design Metrics and Fitness Function

The primary design metrics include the transformer electromagnetic mass, $M_T$, given by (7.49) and the aggregate loss, $P_l$, as given by (7.88). Note, the aggregate loss consists of both transformer and semiconductor losses. The fitness of each design is evaluated based on the number of constraints satisfied. If all the constraints are met, the fitness is defined as

$$f = \left[ \frac{1}{M_T} \frac{1}{P_l} \right]^T$$

(7.92)
If all the constraints are not met, the fitness is calculated based on the percentage of constraints satisfied at the point in execution where the constraints are tested. Computations are organized so that computationally expedient constraints are tested before computationally intensive ones to facilitate computational expediency. The pseudo-code to check the constraints is described in Table 7.4. Therein, $\epsilon$ denotes a small number ($\sim 10^{-16}$), $N_c$ denotes total number of constraints implemented in the fitness evaluation, $C_I$ denotes number of constraints that are evaluated and $C_S$ denotes the sum of the evaluated constraints’ values.

Table 7.4.
Pseudo-code for check of constraints satisfied against imposed

```
update $C_S$
update $C_I$
if ($C_S < C_I$)
    $f = \epsilon \left(\frac{C_S - N_c}{N_c}\right)$
    return
end
```

The sequence of different analyses performed to evaluate the fitness function is presented in the form a Pseudo-code in Table 7.5. At each stage, a set of constraints are evaluated and checked using Table 7.4, before advancing to the next stage.

Table 7.5.: Pseudo-code for calculation of fitness function

1. **Calculate transformer geometry and constraints**
   - initialize number of constraints to $N_c = 16 + 8k$ (1)
   - assign fields of $C_I$, $G_I$, $P_I$ and $S_I$ based on $\Theta$ and $D$ (2)
   - find $N_{ppr}$, $N_{pt}$, $N_{pl}$ and $N_{pcl}$ based on (7.54)-(7.57) (3)
   - find $N_{spr}$, $N_{sl}$, $N_{sl}$ and $N_{scl}$ based on (7.54),(7.60)-(7.57) (4)
   - calculate $C_D$, $G_D$, $P_D$, and $S_D$, using (7.7)-(7.10) (5)
   - create $T$ using (7.51) (6)
   - evaluate $c_1$-$c_9$ using (7.65) -(7.75) (7)
   - test constraints using Table 7.4 (8)

continued on next page
Table 7.5.: continued

2. **Determine transformer magnetic parameters**
   - determine leakage inductance using (7.93) (9)
   - determine $L_{m0}$ using (2.17) and $L_{m, wc s}$ using Figure 7.4 (10)
   - evaluate $c_{10}$ using (7.76) (11)
   - test constraints using Table 7.4 (12)

3. **Determine transformer parasitic capacitances**
   - determine parasitic capacitances using (7.102) (13)
   - determine $Z_{CM}$ and $f_{DM}$ using (7.102) (14)
   - evaluate $c_{11}-c_{12}$ using (7.77)-(7.78) (15)
   - test constraints using Table 7.4 (16)

4. **Analyze operating points**
   - $k = 1$ (17)
   - $P_l = 0$ (18)
   - while ($k \leq N_{op}$) and ($C_S = C_I$) (19)
     - determine transformer currents and $P_{sc}$ using (7.104) (20)
     - determine transformer current harmonics using (7.110) (21)
     - Calculate $c_{13+8(k−1)}-c_{16+8(k−1)}$ using (7.79)-(7.84) (22)
     - Update $C_S$ and $C_I$ (23)
     - if ($C_S = C_I$) (24)
       - Calculate core loss using (7.112) (25)
       - Calculate $c_{17+8(k−1)}$ using (7.112) (26)
       - Update $C_S$ and $C_I$ (27)
     - if ($C_S = C_I$) (28)
       - Perform thermal analysis as shown in Figure 7.5 (29)
       - Calculate $c_{18+8(k−1)}$ and $c_{20+8(k−1)}$ using (7.85)-(7.86) (30)
       - Update constraints $C_S$ and $C_I$ (31)
       - $P_l = P_l + P_{l,k}W_{ld}(k)$ (32)
       - $k = k + 1$ (33)
     - end (34)
   - end (35)
   - test constraints using Table 7.4 (36)

5. **Transformer loss and aggregate loss constraints**
   - compute $c_{21+8(N_{op}−1)}$ and $c_{22+8(N_{op}−1)}$ using (7.87) and (7.89) (38)
   - test constraints using Table 7.4 (39)

6. **Compute fitness using (7.92)**
   - return (40)

The first stage in the fitness evaluation includes establishing the transformer geometry using (7.50) and imposition of the geometry related constraints.
The second stage addresses the magnetic analysis of the core-type transformer, as described in Chapter 2. The hybrid model based on the magneto-static and quasi-static analyses set up for the transformer zero-magnetizing current excitation case is utilized in the design process to calculate the transformer leakage inductance at the operating frequency, $f_{sw}$. The mean squared field intensities across the primary and secondary coil cross sections are also obtained during this stage. These field quantities are useful in estimating the loss due to proximity effect in the windings. Functionally, the process of obtaining winding leakage inductances and mean squared field intensities can be represented as

$$[L_{lkp}, L_{lks}, H_{MSF}] = F_{lkg}(T, D) \quad (7.93)$$

where

$$H_{MSF} = \left[ \langle \hat{H}_{ps}^2 \rangle_{pi} ; \langle \hat{H}_{ps}^2 \rangle_{si} ; \langle \hat{H}_{ps}^2 \rangle_{p1} ; \langle \hat{H}_{ps}^2 \rangle_{s1} ; \langle \hat{H}_{ps}^2 \rangle_{p2} ; \langle \hat{H}_{ps}^2 \rangle_{s2} \right] \quad (7.94)$$

In (7.94), subscript ‘$xi$’ denotes interior coil region. The subscript, ‘$xe1$’ and ‘$xe2$’ denotes exterior coil regions at minimum and maximum clearances from the core.

Another magnetic parameter used to the model transformer is magnetizing inductance. The magnetizing inductance for a worst-case scenario with respect to core saturation, $L_{m,wcs}$, is also considered during the transformer design process. To this end it is assumed that, the transformer primary is connected to the maximum input voltage, $V_{in}$, with duty cycle, $d = 1$, while the secondary is open circuited. The process used to calculate $L_{m,wcs}$ is as follows.

Neglecting the winding resistance and assuming 100% duty cycle, the primary winding flux linkage peak value for this condition is given by

$$\lambda_{p,wcs} = \frac{V_{in}}{2f_{sw}} \quad (7.95)$$
Using MEC shown in Figure 2.6, the primary flux linkage in terms of the core reluctances and the primary current, $i_{p,wcs}$ is given by,

$$\lambda_{p,wcs} = L_{lp}i_{p,wcs} + \frac{N_p^2}{2(R_{el}(\Phi_{wcs}) + R_{bl}(\Phi_{wcs}))}i_{p,wcs}$$

Equating (7.95) and (7.96), $i_{p,wcs}$ is given by

$$i_{p,wcs} = \frac{V_in}{2f_{sw}(L_{lp} + L_{m,woc})}$$

where

$$L_{m,wcs} = \frac{N_p^2}{2(R_{el}(\Phi_{wcs}) + R_{bl}(\Phi_{wcs}))}$$

In (7.98), the flux through the core, $\Phi_{wcs}$, is given by

$$\Phi_{wcs} = \frac{1}{N_p}(\lambda_p - L_{lp}i_{p,wcs})$$

Solving for the magnetizing inductance using (7.97)-(7.99) requires an iterative approach such as the one shown in Figure 7.4. This is because of the dependency of reluctances on core flux. Similar to MEC approach described in Section 2.1, the non-linear definition of magnetic permeability, $\mu_B(\Phi)$ is used to calculate the reluctances in (7.98).

In Figure 7.4, the initialization for the iterative approach includes setting $\Phi_{wcs} = 0$ and the iteration counter to $c_{lm} = 1$. The iteration process is finished when the core flux error, $\Phi_{wcs,e}$ is smaller than a predetermined maximum value, $\Phi_{wcs,emx}$, and when $c_{lm}$ reaches a predetermined maximum iteration times, $C_{lm,mx}$. The core flux error, $\Phi_{wcs,e}$ is given by

$$\Phi_{wcs,e} = |\Phi_{wcs} - \Phi_{wcs,new}|$$

where $|$ denotes absolute value of its argument.

The transformer magnetizing inductance for the worst-case, $L_{m,wcs}$ is used to model transformer while performing time domain analysis of the DC-DC converter.
Additionally, the transformer magnetizing inductance for linear operation, $L_{m0}$, is used to estimate the CM impedance, $Z_{CM}$. The process of evaluating $L_{m,wcs}$ using iterative approach in Figure 7.4 and $L_{m0}$ using (2.17) can be represented in the function form as

$$[L_{m0}, L_{m,wcs}, c_{lm}] = F_{Lm}(T, D) \quad (7.101)$$

The third stage in implementing the fitness function is related to the transformer parasitic capacitances. The electrostatic analysis used in calculating the CM and DM parasitic capacitances is set forth in Chapter 5. The quantities of particular interest obtained using this analysis are CM transformer impedance given by (5.90) and the
self-resonance frequency of the transformer by (5.94). The analysis of transformer parasitic capacitances is functionally expressed as

$$[Z_{CM}, f_{DM}] = F_C(T, D, L_{m0}) \quad (7.102)$$

Note, the magnetizing inductance in the linear region, $L_{m0}$ is used in calculating $f_{DM}$.

The fourth stage in the fitness evaluation is operating point analysis. During the stage, the high-frequency transformer performance in the context of the ICM is analyzed for different ICM operating conditions.

The total number of operating points is denoted by $N_{op}$ and the input to the operating point analysis are operating point conditions described in the form a structure (or vector), $O$, as

$$O = [R_{ld} \ W_{ld}]^T \quad (7.103)$$

where $R_{ld}$ denotes load resistance vector and $W_{ld}$ denotes vector of weight assigned to operating points.

Each operating point analysis is accomplished in four steps.

**Step 1**: The first step is to estimate the transformer winding currents and total power semiconductor loss, $P_{sc,k}$, using the ICM analytical time-domain analysis as described in Chapter 4. A functional form to summarize the time domain analysis is

$$[T_{TDM,k}, P_{sc,k}, c_k] = F_{TDM}(T, D, L_{ik}, L_{m,wes}, O(k)) \quad (7.104)$$

where $P_{sc,k}$ is obtained as sum of inverter semiconductor loss given by (4.88) and diode-rectifier semiconductor loss given by (4.89) and $c_k$ denotes the flag based on the convergence of iterative process described in Figure 4.12. The structure (or vector) $T_{TDM,k}$ includes transformer current waveforms data, $i_p$ and $i_s$ and time, $t$ over half-cycle cycle duration. This is mathematically expressed as

$$T_{TDM,k} = [i_p \ i_s \ t]_k \quad (7.105)$$
where
\[ i_p = [-I_{p0} I_{p1} I_{p2} I_{p0}] \quad (7.106) \]
\[ i_s = [-I_{s0} I_{s1} I_{s2} I_{s0}] \quad (7.107) \]
\[ t = [0 \alpha \frac{dT_{sw}}{2} \frac{T_{sw}}{2}] \quad (7.108) \]

The data represented in the vectors in (7.106)-(7.108) correspond to the transition points of the piecewise linear waveforms illustrated in Figure 4.10.

**Step 2:** The second step in operating point analysis is to calculate transformer current harmonics using \( T_{TDM,k} \) given by (7.105)-(7.108). Employing the current waveforms' piece-wise linear approximation and their half-wave symmetry, harmonics are analytically calculated using Fourier series.

Due to half-wave symmetry, the current waveforms have only odd harmonics. Using vector notation, the list of harmonic frequencies are represented as
\[ f_{harm} = [f_{sw} 3f_{sw} \ldots 2(N_{harm} - 1)f_{sw}] \quad (7.109) \]

where \( N_{harm} \) denotes the number of harmonics. The primary and secondary current harmonic peak and phase values are calculated using (4.96)-(4.99) at each of the frequencies listed in \( f_{harm} \). This is summarized in the vector form as
\[ [I_{ppk}, \theta_p, I_{spk}, \theta_s, f_{harm}]_k = F_{FS}(T_{TDK,k}, D) \quad (7.110) \]

Note, the number of harmonics considered in this step is specified as part of \( D \). The currents harmonic data from here on is represented using a single vector, given by
\[ T_{harm,k} = [I_{ppk}, \theta_p, I_{spk}, \theta_s, f_{harm}]_k \quad (7.111) \]
**Step 3:** The transformer core loss is calculated using MEC analysis and Modified Steinmetz Equation (MSE) described in Chapters 2 and 3 respectively. This is represented using function notation as

\[
[P_{c,k}, c_c] = F_d(T, D, T_{TDM,k})
\]  

(7.112)

where \(P_{c,k}\) denotes the total core loss and \(c_c\) is an overall indicator based on MEC convergence used in this step.

**Step 4:** The transformer winding losses depend on the winding temperatures because of the conductor material properties. Temperature of the windings in turn depends on the transformer losses. To solve this case of interdependency, an iterative approach as shown in Figure 7.5 is used.

![Flow chart](chart.png)

**Fig. 7.5.** Flow chart describing thermal-loss analysis
As part of initialization, the primary and secondary winding mean temperatures, denoted by $T_{pmn}$ and $T_{smn}$ respectively, are assumed to be at ambient temperature, $T_a$ and the iteration counter, $n$, is initialized to $n = 1$. The iteration process is finished when the absolute error, $T_{mne}$ reaches a value smaller than predetermined maximum absolute temperature error, $T_{mne,mx}$, and when $n$ reaches a predetermined maximum iteration times, $N_{T_{mx}}$. The temperature error, $T_{mne}$ is given by

$$T_{mne} = \max(|T_{pmn} - T_{pmn,new}|, |T_{smn} - T_{smn,new}|)$$

(7.113)

where $|x|$ denotes absolute value of $x$.

The temperature dependency of the winding material conductivities is incorporated into calculating resistances by utilizing the primary and secondary winding mean temperatures, denoted by $T_{pmn}$ and $T_{smn}$ respectively. The temperature dependent winding resistance calculation may be represented in the function form as

$$[R_p, R_s] = F_R(T, D, H_{MSF}, T_{pmn}, T_{smn}, f_{harm})$$

(7.114)

where $f_{harm}$ denotes vector of harmonic frequencies. The vectors, $R_p$ and $R_s$ denote the primary and secondary winding resistances at corresponding harmonic frequencies that includes DC resistance and resistance due to skin and proximity effects. Note, the mean squared field intensities, $H_{MSF}$, as calculated using magnetic analysis are used to calculate resistance due to proximity effect in windings. During each iteration, the winding resistances are updated according to the temperature profile.

Using the updated winding resistances, the transformer primary and secondary winding losses, denoted by $P_{ip}$ and $P_{ls}$, respectively, are calculated using the analysis described in Chapter 3. This is expressed in the functional form as

$$[P_{ip}, P_{ls}] = F_{PE}(T, D, T_{harm,k})$$

(7.115)
Based on the thermal analysis described in Chapter 6 and using the transformer losses given by (7.115), the temperature profile of the transformer is determined. This may be expressed in the form of a function as

\[
[T_{TEC}, T_{Tp,k}, T_{pmn, new}, T_{smn, new}] = F_{TEC}(T, D, P_{c,k}, P_{lp}, P_{ls})
\] (7.116)

Note, core loss, \( P_{c,k} \), is independent of temperature and remains constant during this step.

This sequence of evaluations (7.114)-(7.116), completes one iteration in the iterative approach described by Figure 7.5.

Once the convergence requirement on thermal analysis is met, the transformer loss, \( P_{txf,k} \), is calculated using

\[
P_{txf,k} = P_{cl,k} + P_{lp} + P_{ls}
\] (7.117)

Within the operating point analysis, the steps 3 and 4 are implemented on a conditional basis as described in the Pseudo-code in Table 7.5. The function evaluation after time domain analysis (Steps 1 and 2) proceeds with core loss calculation (Step-3) only when the constraints relating to time domain analysis are satisfied. The thermal analysis (Step-4) is implemented only when the constraints relating to time domain analysis and core loss calculation are satisfied. In the case when these constraints are not satisfied, the algorithm exits the operating point analysis and proceeds to testing constraints using Table 7.4.

Once the operating point analysis is implemented for all the operating conditions, and when the constraints, \( c_{13+8(k-1)} - c_{20+8(k-1)} \) are met for all \( k \leq N_{op} \), the aggregate loss, \( P_i \), is calculated using (7.88). The fitness function then proceeds to evaluate the constraints on transformer loss, constraint \( c_{21} \) and aggregate loss, constraint \( c_{22} \). In the case these are satisfied, the fitness function concludes by evaluating the design fitness as given by (7.92).
7.4 Conclusion

In this chapter, a paradigm for a high-frequency core-type transformer design optimization has been formulated through the definition of a fitness function for multi-objective design optimization. In Chapter 8, a case study is presented based on the proposed design methodology.
8. CASE STUDY

The design methodology presented in Chapter 7 incorporates magnetic, electrical and thermal analysis (discussed in Chapters 2-6) into a high-frequency transformer design optimization. In this chapter, the proposed design methodology and impact of these different analyses are illustrated using design case studies. These case studies are implemented for the high-frequency core-type transformer in the context of the ICM shown in Figure 7.3.

The transformer electrical, thermal and magnetic analyses presented in this research are interdependent. The design methodology proposed in Chapter 7 incorporates this interdependency in a multi-objective optimization. Therein, the time domain analysis is used to determine the transformer currents as driven by the transformer magnetic parameters. The transformer currents are responsible for the winding loses. The winding loss due to proximity effect further depends on the leakage fields across the coil cross-sections. The transformer geometry, especially the clearances in the horizontal direction, determine the high-frequency transformer parasitic capacitances, leakage inductance and in turn the winding loses. Also, the transformer thermal performance is heavily dependent on geometry and losses.

The outcome of all these effects that are in play is convoluted and difficult to comprehend from a single case study that illustrates the complete design methodology. Therefore, four case studies are performed, starting from a relatively less complex design optimization and incorporating one additional analysis at a time for each case study. The optimized designs obtained by implementing the four case studies are compared to highlight the impact of the different physical effect that is addressed in this research.

The first case study (Study-1) utilizes magnetic and time domain analyses. The transformer loss includes core loss and DC resistive loss in the windings, while assum-
ing a fixed preset temperature profile for the transformer. In the second case study (Study-2), the transformer thermal analysis is included in the design methodology in addition to what is implemented in Study 1. In the third case study (Study-3), the AC loss in windings due to skin and proximity effects is included in addition to the thermal analysis. Finally, the complete design methodology utilizing the parasitic capacitance constraints is illustrated in the fourth case study (Study-4). It is important to note that the first three case studies are implemented only for comparison purposes and do not hold any practical significance.

In summary, as compared to the complete design methodology, the following aspects are implemented in each case study:

- Study-1 does not incorporate thermal analysis, parasitic capacitances, or AC winding losses due to skin and proximity effects.
- Study-2 does not include parasitic capacitances or AC winding losses due to skin and proximity effects, but does incorporate thermal analysis.
- Study-3 does not include parasitic capacitances, but does incorporate thermal analysis and AC winding losses due to skin and proximity effects.
- Study-4 illustrates the complete design methodology as presented in Chapter 7.

The same design specifications are used for all four case studies. The design specifications relating to the prototype ICM ratings are as listed in Tables 4.4 and 4.8. Note that the ICM switches at 20 kHz. The other design specifications are listed in Table 8.1. Therein, two ICM operating points, half- and full load conditions, are considered for the design analysis and are specified by using load vector, \( \mathbf{R}_{ld} \) in Table 8.1, where full load resistance, \( R_{ld} = 32\Omega \). The corresponding weight vector, \( \mathbf{W}_{ld} \) is used to calculate the aggregate loss, \( P_l \). The last twelve entries in this table relate to the algorithm parameters. The two parameters \( N_{nac} \) and \( N_{nar} \) are used to set the rectangular grid while implementing the numerical method of calculating the static leakage inductance based on Biot-Savart law. The values of these parameters are on the accuracy and computational effort involved in estimating the total leakage inductance.
The fixed design parameters constituting the elements of $D_{fp}$ are listed in Table 8.2. Only the first nine elements are fixed parameters as mentioned in (7.53). The remaining are the specifications relating to the choice of materials used for foil insulation, wire insulation, and potting.

To achieve a meaningful comparison between the case studies, the same design space was used for all the case studies. The domain of design space, $\Theta$, is summarized in Table 8.3. Therein, the range of each design space parameter and its gene type are listed. For the case studies presented, the primary and secondary winding conductor material (denoted by index $m_p$ and $m_s$ in (7.52)) is set to correspond to copper and is hence not included in the design space listed in Table 8.3. The minimum allowed value of the clearances, $c_{ps}$ and $c_{sc}$, are set to the insulation thickness that is required at the transformer operating voltage. The material properties used during case study implementation are listed in Tables A.1-C.1 in Appendix.

The four case studies are implemented using the multi-objective environment in GOSET [46] to minimize transformer mass and aggregate loss. Each case study is repeated thrice to check for convergence. A population size of 2000 and generations of magnitude 2000 are used each run.

Table 8.1.: Transformer design specifications

<table>
<thead>
<tr>
<th>Sym.</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{T_{mx}}$</td>
<td>Maximum allowed electromagnet mass</td>
<td>5 kg</td>
</tr>
<tr>
<td>$l_{T_{mx}}$</td>
<td>Maximum allowed total length</td>
<td>1 m</td>
</tr>
<tr>
<td>$w_{T_{mx}}$</td>
<td>Maximum allowed total width</td>
<td>1 m</td>
</tr>
<tr>
<td>$d_{T_{mx}}$</td>
<td>Maximum allowed total depth</td>
<td>1 m</td>
</tr>
<tr>
<td>$k_{b_{bmdms}}$</td>
<td>Minimum required bend radius ratio</td>
<td>4</td>
</tr>
<tr>
<td>$k_{p_{f_{mx}}}$</td>
<td>Maximum possible packing factor</td>
<td>0.95</td>
</tr>
<tr>
<td>$P_{t_{f_{mx}}}$</td>
<td>Maximum allowed transformer loss</td>
<td>200 W</td>
</tr>
<tr>
<td>$Z_{C_{M_{mn}}}$</td>
<td>Minimum required CM impedance</td>
<td>5 kΩ</td>
</tr>
<tr>
<td>$f_{f_{res}}$</td>
<td>Minimum required DM resonance frequency</td>
<td>100 kHz</td>
</tr>
<tr>
<td>$V_{ld}$</td>
<td>Operating point load voltage</td>
<td>420V</td>
</tr>
<tr>
<td>$Z_{ld}$</td>
<td>Operating point load impedance vector</td>
<td>$R_{ld}[1 2]$</td>
</tr>
<tr>
<td>$W_{ld}$</td>
<td>Operating point loss weights</td>
<td>[0.7 0.3]</td>
</tr>
<tr>
<td>$r_{\Phi_{mn}}$</td>
<td>Minimum required magnetizing flux ratio</td>
<td>0.9</td>
</tr>
</tbody>
</table>

continued on next page
Table 8.1.: continued

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{Smx}$</td>
<td>Maximum allowed semiconductor loss</td>
<td>500 W</td>
</tr>
<tr>
<td>$P_{lmxa}$</td>
<td>Maximum allowed aggregate loss</td>
<td>700 W</td>
</tr>
<tr>
<td>$I_{pk,mx}$</td>
<td>Maximum allowed peak current</td>
<td>30 A</td>
</tr>
<tr>
<td>$T_a$</td>
<td>Ambient temperature</td>
<td>20 °C</td>
</tr>
<tr>
<td>$T_{mxa}$</td>
<td>Maximum allowed temperature of windings</td>
<td>200 °C</td>
</tr>
<tr>
<td>$N_{fhar}$</td>
<td>Number of harmonics considered</td>
<td>25</td>
</tr>
<tr>
<td>$N_{nac}$</td>
<td>Min. no. of grid points per unit area (coils)</td>
<td>$7 \times 10^8$</td>
</tr>
<tr>
<td>$N_{nar}$</td>
<td>Min. no. of grid points per unit area (ext.regions)</td>
<td>$4 \times 10^7$</td>
</tr>
<tr>
<td>$N_c$</td>
<td>Number of data points on one cycle</td>
<td>$10^3$</td>
</tr>
<tr>
<td>$C_{lm,max}$</td>
<td>Maximum number of iterations,(7.77)</td>
<td>20</td>
</tr>
<tr>
<td>$C_{mx}$</td>
<td>Maximum number of iterations, (7.79)</td>
<td>20</td>
</tr>
<tr>
<td>$N_{Tmx}$</td>
<td>Maximum number of iterations, thermal analysis</td>
<td>20</td>
</tr>
<tr>
<td>$e_{dmx}$</td>
<td>Maximum error in duty cycle</td>
<td>$10^{-4}$</td>
</tr>
<tr>
<td>$e_{vmx}$</td>
<td>Maximum error in voltage</td>
<td>$10^{-4}$</td>
</tr>
<tr>
<td>$e_{fd}$</td>
<td>Error tolerance while solving flux densities</td>
<td>$10^{-8}$ T</td>
</tr>
<tr>
<td>$\Phi_{wce,emx}$</td>
<td>Error tolerance while solving $L_{wce}$</td>
<td>$10^{-8}$ Wb</td>
</tr>
<tr>
<td>$T_{nne,mx}$</td>
<td>Maximum error in temperature</td>
<td>0.1 K</td>
</tr>
<tr>
<td>$J_{rms,mx}$</td>
<td>Maximum allowed RMS current density</td>
<td>7.6 MA/m²</td>
</tr>
</tbody>
</table>

The design methodology and design fronts obtained for each of these four studies are presented in Sections 8.1-8.4. A comparison of these four case studies is presented in Section 8.5. Finally, a design from Study-4 is used to validate the magnetic analysis, which is presented in Section 8.6.

8.1 Case Study - 1

This case study includes the magnetic analysis as presented in Chapter 2 and time domain analysis as presented in Chapter 4. The transformer loss is comprised of the winding loss due to DC resistance and core loss, while assuming the windings and core are at ambient temperature.

The fitness function algorithm to implement this case study starts with stages 1 and 2 of the pseudo code presented in Table 7.5, followed by stages 3, 4 and 5 as
Table 8.2.
Transformer fixed parameters

<table>
<thead>
<tr>
<th>Sym.</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N_{pcp})</td>
<td>Number of primary coils in parallel</td>
<td>2</td>
</tr>
<tr>
<td>(N_{scp})</td>
<td>Number of secondary coils in parallel</td>
<td>2</td>
</tr>
<tr>
<td>(N_{pcs})</td>
<td>Number of primary coils in series</td>
<td>1</td>
</tr>
<tr>
<td>(N_{scs})</td>
<td>Number of secondary coils in series</td>
<td>1</td>
</tr>
<tr>
<td>(t_{inp})</td>
<td>Wire insulation thickness</td>
<td>50 µm</td>
</tr>
<tr>
<td>(t_{ins})</td>
<td>Wire insulation thickness</td>
<td>50 µm</td>
</tr>
<tr>
<td>(k_{pb})</td>
<td>Primary coil build factor</td>
<td>1.02</td>
</tr>
<tr>
<td>(k_{sb})</td>
<td>Secondary coil build factor</td>
<td>1.02</td>
</tr>
<tr>
<td>(c_{sv})</td>
<td>Core to secondary winding clearance</td>
<td>2 m</td>
</tr>
<tr>
<td>(\varepsilon_{r_{si}})</td>
<td>Wire insulation relative permittivity</td>
<td>3.54</td>
</tr>
<tr>
<td>(\varepsilon_{rp})</td>
<td>Potting material relative permittivity</td>
<td>4</td>
</tr>
<tr>
<td>(\varepsilon_{rf})</td>
<td>Foil insulation relative permittivity</td>
<td>2.7</td>
</tr>
<tr>
<td>(h_{ca})</td>
<td>Core to air heat transfer coefficient</td>
<td>18 W/(m²K)</td>
</tr>
<tr>
<td>(h_{wp})</td>
<td>Potted winding to air heat transfer coefficient</td>
<td>23.4 W/(m²K)</td>
</tr>
<tr>
<td>(k_{in})</td>
<td>Foil insulation conductivity</td>
<td>0.139 W/K</td>
</tr>
</tbody>
</table>

described in Table 8.4. The corresponding constraints implemented during stages 3, 4 and 5 are listed in Table 8.5. Therein, seven constraints are listed for each operating point analysis. The constraints \(c_{15+7(k-1)}\) and \(c_{16+7(k-1)}\) are used to limit the windings RMS current densities to be below a maximum allowed value, denoted by \(J_{\text{rms,\(mx\)}}\). For the currents waveforms shown in Figure 4.11, the RMS current densities are calculated using

\[
J_{\text{rms,\(x\)}} = \sqrt{\frac{\sqrt{(3I_0^2 + 2I_1^2 + 2I_2^2)}}{N_{scp}a_{xc}}}
\]  

(8.1)

where ‘\(x\)’ denotes ‘\(p\)’ for primary and ‘\(s\)’ for secondary. In (8.1), the numerator is the root mean squared value of the analytically calculated current waveform data and the denominator is the primary conductor total cross sectional area.
Furthermore, the windings’ DC resistances are calculated (in line (25)) at ambient temperature, $T_a$. This may be represented in the function form as

$$[r_p, r_s, P_{dc}] = F_{dc}(T, D, T_{pmn}, T_{smn}, T_{harm,k}) \quad (8.2)$$

where primary and secondary winding DC resistances, denoted by $r_p$ and $r_s$ respectively are as given by (3.12) and (3.13). The winding mean temperatures are assigned as $T_{pmn} = T_a$ and $T_{smn} = T_a$. The corresponding DC resistive loss in the primary, $P_{dcp}$, and secondary , $P_{dcs}$, windings are calculated using (3.15) and (3.16) respectively.

Consequently, in this case study, the total transformer loss, $P_{txf,k}$ is given by

$$P_{txf,k} = P_{cl,k} + P_{dcp} + P_{dcs} \quad (8.3)$$
Table 8.4.
Case study 1 - Pseudo-code for stages 3, 4 and 5 of the fitness evaluation

3. **Analyze operating points**

   \[ k = 1 \]  \hspace{1cm} (13)
   \[ P_l = 0 \]  \hspace{1cm} (14)

   while \((k \leq N_{op})\) and \((C_s = C_I)\)  \hspace{1cm} (15)
   
   determine transformer currents and \(P_{sc}\) using (7.104)  \hspace{1cm} (16)
   determine transformer current harmonics using (7.110)  \hspace{1cm} (17)
   
   Calculate \(c_{11+7(k−1)} - c_{16+7(k−1)}\)  \hspace{1cm} (18)
   Update \(C_S\) and \(C_I\)  \hspace{1cm} (19)
   if \((C_S = C_I)\)  \hspace{1cm} (20)
       
       Calculate core loss using (7.112)  \hspace{1cm} (21)
       Calculate \(c_{17+7(k−1)}\)  \hspace{1cm} (22)
       Update \(C_s\) and \(C_I\)  \hspace{1cm} (23)
       if \((C_S = C_I)\)  \hspace{1cm} (24)
           
           Calculate the winding loss using (8.2)  \hspace{1cm} (25)
           \(P_l = P_l + P_{l,k}W_{ld}(k)\)  \hspace{1cm} (26)
           \(k = k + 1\)  \hspace{1cm} (27)
       end  \hspace{1cm} (28)
   end  \hspace{1cm} (29)
end  \hspace{1cm} (30)

   test constraints using Table 7.4  \hspace{1cm} (31)

4. **Transformer loss and aggregate loss constraints**

   compute \(c_{21+8(N_{op}−1)}\) and \(c_{22+8(N_{op}−1)}\) using (7.87) and (7.89)  \hspace{1cm} (32)
   test constraints using Table 7.4  \hspace{1cm} (33)

5. **Compute fitness using (7.92)**

   return  \hspace{1cm} (34)

   return  \hspace{1cm} (35)

The multi-objective optimization using GOSET [46] yields the Pareto-optimal fronts shown in Figure 8.1 for three runs. It can be observed that optimization has converged.

The gene distribution sorted using first objective \((1/M)\) for the optimized designs obtained in Run-1 is as shown in Figure 8.2. Therein, note the genes assigned to clearances (genes 14, 15 and 16) are at minimum allowed values.
Table 8.5.
Case study 1 - Constraints used during stages 3, 4 and 5 of fitness evaluation

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_{11+7(k-1)} = \text{lte}(c_k, C_{mx}) )</td>
<td>TDM iterative solver convergence</td>
</tr>
<tr>
<td>( c_{12+7(k-1)} = \text{lte}(P_{sc,k}, P_{scmx}) )</td>
<td>Semiconductor loss</td>
</tr>
<tr>
<td>( c_{13+7(k-1)} = \text{lte}(\max(i_p), I_{pk, mx}) )</td>
<td>Primary peak current</td>
</tr>
<tr>
<td>( c_{14+7(k-1)} = \text{gte}(\min(r_\Phi), r_{\Phi mn}) )</td>
<td>Flux ratio limit</td>
</tr>
<tr>
<td>( c_{15+7(k-1)} = \text{lte}(J_{rmsp}, J_{rms, mx}) )</td>
<td>Primary current density limit</td>
</tr>
<tr>
<td>( c_{16+7(k-1)} = \text{lte}(J_{rmss}, J_{rms, mx}) )</td>
<td>Primary current density limit</td>
</tr>
<tr>
<td>( c_{17+7(k-1)} = \text{lte}(c_{MEC}, C_{MEC,mx}) )</td>
<td>MEC convergence</td>
</tr>
<tr>
<td>( c_{18+7(Nop-1)} = \text{lte}(\max(P_{txf,1}, ..., P_{txf,Nop}), P_{txf,mx}) )</td>
<td>Transformer loss limit</td>
</tr>
<tr>
<td>( c_{19+7(Nop-1)} = \text{lte}(P_I, P_{Imax}) )</td>
<td>Aggregate loss limit</td>
</tr>
</tbody>
</table>

Fig. 8.1. Case study 1 - Pareto-optimal front

8.2 Case Study - 2

The difference between case studies 1 and 2 is that the transformer thermal analysis is included in case study 2.
The pseudo-code used to implement this case study is same as described in Table 7.5, except for two changes. The stage 3 (relating to parasitic capacitances) as described in Table 7.5 is not implemented. And, when implementing operating point analysis (line 29), only DC resistance is considered. Subsequently, the loss considered in the windings is due to DC resistance only.

The winding mean temperatures $T_{pmn,new}$ and $T_{smn,new}$ are obtained by solving TEC, expressed as

$$[T_{TEC}, T_{Tp,k}, T_{pmn,new}, T_{smn,new}] = F_{TEC}(T, D, P_{c,k}, P_{dep}, P_{dcs}) \quad (8.4)$$

In (8.4), note only DC resistance loss is considered. Also, the windings loss includes only DC resistance loss, as calculated using (8.2). Hence, the thermal analysis is implemented using (8.2) and (8.4) in an iterative approach as described in Figure 7.5.

The multi-objective optimization for this case study when implemented using GOSET [46] yields the Pareto-optimal fronts as shown in Figure 8.3.
The gene distribution for the optimized designs obtained in Run-1 of this case study is as shown in Figure 8.4. Therein, note the genes assigned to clearances (genes 14, 15 and 16) tend toward their minimum allowed values.

![Graph](image)

Fig. 8.3. Case study 2 - Pareto-optimal front

### 8.3 Case Study - 3

The implementation of Study-3 is same as Study-2 except for inclusion of winding AC losses due to skin and proximity effects. The difference between this case study and the complete design methodology presented in Chapter 7 (same as implemented in case study 4) is that parasitic capacitances are not included in Study-3. Therefore, this case study can be implemented using the pseudo-code presented in Table 7.5, except that the stage 2 (lines 13, 14, 15) is omitted.

The multi-objective optimization for this case study when implemented using GOSET [46] yields the Pareto-optimal fronts as shown in Figure 8.5. Therein, the design fronts from three runs match, ensuring the optimization convergence.
The gene distribution for the optimized designs obtained in Run-1 of this case study is as shown in Figure 8.6. Similar to Study-1 and Study-2, the genes assigned to clearances (genes 14, 15 and 16) are at minimum allowed values.

8.4 Case Study - 4

Study - 4 illustrates the complete design methodology as described in Chapter 7. The fitness function is implemented using the algorithm exactly as presented in Table 7.5.

The multi-objective optimization for this case study when implemented using GOSET [46] yields the Pareto-optimal fronts as shown in Figure 8.7. Therein, the design fronts from three runs match, ensuring the optimization convergence.

The gene distribution for the optimized designs obtained in Run-1 of this case study is as shown in Figure 8.8. unlike the case studies-1, 2, and 3, the genes assigned to clearances, $c_{sc}$ and $c_{ps}$ (15 and 16) increase to values above the minimum allowed
Fig. 8.5. Case study 3 - Pareto-optimal front

Fig. 8.6. Case study 3 - Gene distribution when sorted by $1/M_T$ objective
value. The clearance, \( c_{pp} \), however, converges to the minimum value. The variation of the three clearances with respect to mass is as shown in Figure 8.9.

![Case study 4](image)

Fig. 8.7. Case study 4 -Pareto-optimal front

The behavior of the clearances \( c_{sc} \) and \( c_{ps} \) observed in Figure 8.9 is due to the inclusion of constraints on parasitic capacitances. The constraint, \( c_{12} \) is based on the transformer natural resonance frequency, \( f_{res} \). The two parameters, the transformer magnetizing inductance, \( L_{m0} \) and DM parasitic capacitance, \( C_{DM} \) drive the resonance frequency, \( f_{res} \) and hence, compete with each other. This observation is illustrated in Figure 8.10.

Another constraint, \( c_{11} \) is based on CM parasitic capacitance. This constraint is implemented to obtain a minimum CM transformer impedance magnitude, \( Z_{CMmn} \), when operating at converter switching frequency, \( f_{sw} \). This is to ensure a low CM current conduction from primary to secondary. The variation of CM parasitic capacitance, \( C_{CM} \) with mass is shown in Figure 8.11. Also shown in the figure is the plot of transformer total leakage inductance, \( L_{lk} \) versus mass. There in, a correlation can be
Fig. 8.8. Case study 4 - Gene distribution when sorted by $1/M_T$ objective

Fig. 8.9. Case study 4 - Transformer horizontal clearances vs Mass
observed between $C_{CM}$ and $L_{lk}$, even though there is no direct constraint tying these two quantities. The correlation is due to the dependency of each of these parameters on the horizontal clearance between primary and secondary coils, $c_{ps}$.

An interesting observation found in this case study is that all the optimized designs have single layer windings as shown Figure 8.12. In the next section, this observation is explored by taking a closer look at the designs from the four case studies and their comparison with each other.

### 8.5 Comparison of Case Studies 1-4

The Pareto-optimal fronts from four case studies when plotted in single plot are as shown in Figure 8.13(a). The magnified version of the same plots are shown in Figure 8.13(b).
It can be observed that the optimized transformer designs from case study - 4 have higher mass and higher aggregate loss compared to the optimized designs from other case studies. This is expected because of additional constraints and more realistic analysis. For example, successful operation of the transformer design should account the rise in temperature windings. This condition is not ensured in case of designs obtained in Study-1, as a fixed temperature profile is assumed.

In addition to comparing the Pareto-optimal fronts, it is interesting to compare individual designs from the studies. The designs from four case studies are compared based on three measures as shown in Figure 8.14.

### 8.5.1 Comparison based on Same Loss

The first measure of comparison is based on aggregate loss. Four designs that have same aggregate loss are chosen, each one from the four case studies. An aggregate
Fig. 8.12. Case study 4 - Primary and secondary windings data

Fig. 8.13. Pareto-optimal fronts comparison for case study 1-4

loss of 138 W is chosen to select the designs. The geometric profiles of the selected designs are as shown in Figures 8.15 - 8.18.
Fig. 8.14. Selection of designs for comparison based on (1) Same loss (green) (2) Same mass (blue) (3) Maximum trade off between two objectives (black)

Fig. 8.15. Design 717 from Case Study-1 for same loss comparison

To quantify the difference in the design geometries, it is useful to define the transformer aspect ratio, $\alpha_T$, as

$$\alpha_T = \frac{\max(l_T, w_T, d_c)}{\min(l_T, w_T, d_c)}$$  \hspace{1cm} (8.5)
Fig. 8.16. Design 906 from Case Study-2 for same loss comparison

Fig. 8.17. Design 720 from Case Study-3 for same loss comparison

Fig. 8.18. Design 240 from Case Study-4 for same loss comparison
The aspect ratio and few other parameters of interest with respect to the four selected designs are listed in Table 8.6. Therein, the design number represents the position of that particular design when all the designs in the front are sorted in the order of increasing aggregate loss. The design name is used to represent the case study from which the design is obtained. Note, the parameter values that are highlighted in red are not included in the study. Between designs 1A and 2A, it can be observed that the transformer aspect ratio increases when thermal analysis is included. This is to increase the available surface area for heat dissipation. For the same reason, the number of layers in the windings, denoted by \( N_{pl} \) and \( N_{sl} \), also reduce when thermal analysis is included.

The increase in aspect ratio and reduction in number of layers is even more pronounced when AC winding losses due to skin and proximity effects are included in Study-3.

Finally, when constraints relating to parasitic capacitances are included in Study-4, significant values are chosen for the horizontal clearances \( c_{ps} \) and \( c_{sc} \). As a result of increase in the distance between the core and windings, the leakage fields increase. This is reflected in higher value of leakage inductance of Design 4A as compared to that of Design 3A. Additional effect of increased leakage fields is observed in the loss due to proximity effect. Because of increase in windings loss, the designs obtained Study 4 have higher tendency to reduce the number winding layers in order to dissipate the additional heat.

Comparing the values of \( f_{res} \) of Designs 1A, 2A and 3A to 4A, it can be observed that the requirement for the natural frequency to be above the switching frequency (20 kHz) cannot be attained unless implemented as a design constraint. This reflects the contribution of estimating the parasitic capacitances during the transformer stage, which otherwise would result in ringing in the converter voltages.
8.5.2 Comparison based on Same Mass

The second comparison is among the designs that have same mass. A mass value of 3.5 kg is used to select the designs. The position of these designs along the Pareto-optimal fronts is depicted by blue line in Figure 8.14.

The geometric profiles of the selected designs are as shown in Figures 8.19 - 8.22. The observations made in the case of ‘same loss’ comparison can be verified using the parameter values listed in Table 8.7. The transformer aspect ratio is higher for Design 2B compared to that of Design 1B. This is further increased in case of Design 3B. In case of Design 4B, aspect ratio decreases because of significant increase in horizontal clearances, $c_{ps}$ and $c_{sc}$. Compared to Design 3B, Design 4B has higher leakage inductance and higher proximity effect loss.

8.5.3 Comparison based on Same Weighted Fitness

The trade off between mass and loss varies along the Pareto-optimal front shown in Figure 8.14. In most applications, the designs which offer a balance between the two objectives are selected. These designs are positioned along the knee of the fronts.
Fig. 8.20. Design 36 from Case Study-2 for same mass comparison

Fig. 8.21. Design 78 from Case Study-3 for same mass comparison

Fig. 8.22. Design 57 from Case Study-4 for same mass comparison
To compare the designs with similar trade off between the two objectives, the designs with same weighted fitness, $F$, are selected from four case studies. The weighted fitness, $F$ is defined as

$$F = w\frac{f_1}{\max(F_1)} + (1 - w)\frac{f_2}{\max(F_2)}$$  \hspace{1cm} (8.6)

where the two objectives in case of the transformer design optimization are $f_1 = 1/M_T$ and $f_2 = 1/P_l$. In (8.6), the denominator uses a maximum function, ‘max’ to scale the objectives with respect the maximum fitness values from Study-4.

Using the objective values of the design front from Case study-4 and by varying $w$ in the range from zero to one, the weighted fitness was found to be most sensitive for $w = 0.21$. Four designs are selected from four case study results such that the selected design has maximum weighted fitness for $w = 0.21$. The selected designs obtained using this approach are highlighted by black curve in Figure 8.14.

The geometric profiles of these designs are as shown in Figures 8.23- 8.26. The parameters of interest are listed in Table 8.8.
Fig. 8.24. Design 901 from Case Study-2 for same weighted fitness comparison

Fig. 8.25. Design 938 from Case Study-3 for same weighted fitness comparison

Fig. 8.26. Design 534 from Case Study-4 for same weighted fitness comparison
8.6 Validation of Leakage Inductance Estimation

The transformer leakage inductance estimation using the magnetic analyses set forth in Chapters 2 is validated for the case of Design 534 selected from case study-4. The complete design parameters of Design 534 are listed in Table 8.9.

Ansys Maxwell is used to perform an FEA magnetic analysis. Similar to the validation of prototype transformer as presented in Section 2.4, only one eighth of the transformer is used for FEA simulation. The transformer model used for 3D FEA is as shown in Figure 8.27. The total leakage inductance of the Design 5C as obtained by performing 3D FEA using Ansys Maxwell magnetostatic analysis is 29.62 $\mu$H. The transformer energy obtained for zero magnetizing current excitation is used to calculate total leakage inductance using (2.56). The total leakage inductance as estimated by the proposed magneto-static analysis in Section 2.2 is 29.58 $\mu$H.

The total leakage inductance of the Design 534 at 20 kHz as estimated by the proposed frequency dependent magnetic analysis is 29.09 $\mu$H. Using 3D FEA Eddy-current solver in Ansys Maxwell, the total leakage inductance at 20 kHz is estimated as 29.62 $\mu$H.

Fig. 8.27. 3D model of Design 5C that is used for FEA simulation
Table 8.6.
Comparison of designs from Case studies 1-4 with same aggregate loss

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Windings data

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Full load loss (W)

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Windings mean temp. (°C)

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Magnetic Parameters

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Horizontal clearances (mm)

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Parasitic capacitance related

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Table 8.7.
Comparison of designs from Case studies 1-4 with same mass

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Windings data

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Full load loss (W)

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Windings RMS current dens. (MA/m²)

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Windings mean temp. (ºC)

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Magnetic Parameters

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Horizontal clearances (mm)

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Parasitic capacitance related

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Table 8.8.
Comparison of designs from Case studies 1-4 with same weighted fitness

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Windings data

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<td>3</td>
</tr>
<tr>
<td>Primary (AWG)</td>
<td>20</td>
<td>20</td>
<td>23</td>
<td>20</td>
</tr>
<tr>
<td>Secondary (AWG)</td>
<td>24</td>
<td>21</td>
<td>24</td>
<td>25</td>
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Full load loss (W)

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<tbody>
<tr>
<td>DC loss</td>
<td>22.5</td>
<td>22.3</td>
<td>17.2</td>
<td>22.1</td>
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<tr>
<td>Skin effect</td>
<td>0.80</td>
<td>1.24</td>
<td>0.36</td>
<td>0.6</td>
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<tr>
<td>Proximity effect</td>
<td>137.7</td>
<td>41.2</td>
<td>13.3</td>
<td>6.2</td>
</tr>
<tr>
<td>Core</td>
<td>5.1</td>
<td>5.7</td>
<td>8.6</td>
<td>15.4</td>
</tr>
<tr>
<td>Txf. tot.</td>
<td>27.6</td>
<td>34.2</td>
<td>44.2</td>
<td>48.6</td>
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<tr>
<td>Semi Cond.</td>
<td>129.0</td>
<td>127.1</td>
<td>126.9</td>
<td>128.7</td>
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<tr>
<td>Tot. loss</td>
<td>156.6</td>
<td>161.3</td>
<td>171.1</td>
<td>177.3</td>
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</table>

Windings RMS current dens. (MA/m²)

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<tbody>
<tr>
<td>$J_{prms}$</td>
<td>6.9</td>
<td>6.6</td>
<td>6.6</td>
<td>6.8</td>
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<tr>
<td>$J_{srms}$</td>
<td>7.3</td>
<td>7.2</td>
<td>7.3</td>
<td>12.2</td>
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</table>

Windings mean temp. (°C)

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</thead>
<tbody>
<tr>
<td>$T_{mp}$</td>
<td>20</td>
<td>135</td>
<td>149</td>
<td>107</td>
</tr>
<tr>
<td>$T_{ms}$</td>
<td>20</td>
<td>144</td>
<td>156</td>
<td>133</td>
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</tbody>
</table>

Magnetic Parameters

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</thead>
<tbody>
<tr>
<td>$L_{ik}$ (µH)</td>
<td>69.4</td>
<td>17.6</td>
<td>8.5</td>
<td>29.1</td>
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<tr>
<td>$L_{m0}$ (mH)</td>
<td>677</td>
<td>536</td>
<td>339</td>
<td>96.7</td>
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<tr>
<td>$L_{m WCS}$ (mH)</td>
<td>205</td>
<td>187</td>
<td>124</td>
<td>50.4</td>
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Horizontal clearances (mm)

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</tr>
</thead>
<tbody>
<tr>
<td>$c_{pp}$</td>
<td>5.00</td>
<td>5.00</td>
<td>5.00</td>
<td>5.00</td>
</tr>
<tr>
<td>$c_{ps}$</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>4.88</td>
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<tr>
<td>$c_{sc}$</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>1.14</td>
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</table>

Parasitic capacitance related

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</thead>
<tbody>
<tr>
<td>$Z_{CM}$ (kΩ)</td>
<td>17</td>
<td>9.8</td>
<td>8.7</td>
<td>97</td>
</tr>
<tr>
<td>$f_{res}$</td>
<td>10</td>
<td>8</td>
<td>9</td>
<td>100</td>
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Table 8.9.
Prototype transformer dimensions

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<thead>
<tr>
<th>Core Parameter</th>
<th>Value</th>
<th>Primary winding Parameter</th>
<th>Value</th>
<th>Secondary winding Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Material</td>
<td>MN8CX</td>
<td>Material</td>
<td>Copper</td>
<td>Material</td>
<td>Copper</td>
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<tr>
<td>l_c</td>
<td>48.1 mm</td>
<td>AWG</td>
<td>20</td>
<td>AWG</td>
<td>25</td>
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<tr>
<td>w_cs</td>
<td>20.0 mm</td>
<td>Npcl</td>
<td>55</td>
<td>Nscp</td>
<td>32</td>
</tr>
<tr>
<td>d_cs</td>
<td>58.3 mm</td>
<td>Npcp</td>
<td>2</td>
<td>Nscp</td>
<td>2</td>
</tr>
<tr>
<td>w_c</td>
<td>58.8 mm</td>
<td>Npl</td>
<td>1</td>
<td>Nsl</td>
<td>1</td>
</tr>
<tr>
<td>d_c</td>
<td>97.4 mm</td>
<td>wpw</td>
<td>0.93 mm</td>
<td>w_sw</td>
<td>0.56 mm</td>
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<tr>
<td>r_w</td>
<td>0.7 mm</td>
<td>dpw</td>
<td>51.2 mm</td>
<td>dw</td>
<td>54.3 mm</td>
</tr>
<tr>
<td>w_cw</td>
<td>19.5 mm</td>
<td>rpi</td>
<td>7.3 mm</td>
<td>rsi</td>
<td>1.8 mm</td>
</tr>
<tr>
<td>w_cec</td>
<td>0 mm</td>
<td>rpo</td>
<td>8.2 mm</td>
<td>rso</td>
<td>2.4 mm</td>
</tr>
<tr>
<td>w_ce</td>
<td>18.0 mm</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c_sc</td>
<td>1.1 mm</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c_ps</td>
<td>4.9 mm</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c_pp</td>
<td>5 mm</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c_pv</td>
<td>3.6 mm</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c_sv</td>
<td>2 mm</td>
<td></td>
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9. SUMMARY AND FUTURE WORK

9.1 Summary of Contributions

This section summarizes the work presented in this dissertation. First, a multi-objective optimization-based approach is set forth for designing a high-frequency core-type transformer in the context of an isolating DC-DC converter.

To support the design methodology, magnetic analysis of a core-type transformer is presented. A computationally efficient method to estimate the transformer leakage inductance as a function of frequency is set forth. The proposed method consists of two stages. First, static-field analysis, utilizing an analytical approach for the transformer interior region and numerical method based on Biot-Savart law for the transformer exterior region is performed. Second, a 1D quasi-static harmonic analysis is used to calculate the interior region energy at operating frequency. A scaling factor, defined as interior region energy ratio from static to operating frequency condition, is used to estimate the exterior region energy. From this, the transformer leakage inductance at the operating frequency is calculated. The proposed magnetic field analysis is validated in the case of a prototype high-frequency core-type transformer. The results demonstrate the accuracy and computational efficiency of this method.

Analytical methods to calculate the transformer loss are discussed. The Modified Steinmetz Equation is used for estimating the core loss. The necessary flux density waveforms are obtained by using the transformer MEC augmented by an analytical time domain analysis of DC-DC converter. The calculation of AC resistive loss including skin effect is based on an established method. The estimation of AC loss due to proximity effect is more involved. To this end, simplification of the work presented in [12] is proposed for calculating proximity effect loss. This simplification utilizes the static magnetic field analysis set forth for leakage inductance estimation
to calculate the normalized magnetic field intensities in the windings. The method used to estimate winding resistance at high-frequencies is verified using a prototype transformer.

The transformer currents necessary for loss estimation are obtained by performing time-domain analysis of the ICM DC-DC converter. Analytical analysis of the simplified ICM circuit is performed to calculate the transformer currents for a given operating condition. The proposed analysis captures the high-frequency harmonics in the transformer current waveforms while including the power semiconductor loses in ICM. The time domain analysis is validated by comparing transformer current waveforms as obtained by the analytical approach to waveform-level modeling (WLM) of ICM and experimental results. The comparison demonstrates that the accuracy of analytical approach is equivalent to that of WLM in estimating high-frequency harmonics in transformer currents.

A review of analytical methods to calculate winding parasitic capacitances is presented. This is used to derive the parasitic capacitances observed in a high-frequency transformer. Also, the transformer is analyzed in view of these capacitances to set relevant constraints in order to improve its high-frequency behavior. The winding capacitances contributing to transformer differential-mode impedance are responsible for the natural frequency of the transformer. The transformer inter-winding capacitance is contributing to transformer common-mode impedance is responsible for high-frequency conduction from the primary side to secondary side.

Thermal analysis is set forth to estimate the temperature rise in the windings and to include the thermal effect on conductor material parameters. The heat flow analysis set up in [45] is used to develop thermal equivalent circuit (TEC) of the transformer. Using symmetry, only one-eighth of the transformer is considered for thermal analysis.

Finally, these analyses are coupled together using a multi-objective optimization to design high-frequency core-type transformer, similar to the work in [30]. The transformer mass and loss are minimized. The organization of magnetic, electrical,
and thermal analyses required to build a fitness function is presented. Metrics used to evaluate the transformer performance based on these analyses, and the desired design constraints for these performance metrics are discussed.

The output of the optimization is a set of non-dominated designs referred to as the Pareto-optimal front. A case study is carried out using the proposed design methodology.

9.2 Future Work

This section suggests direction for further development of the work presented in this dissertation.

First, a prototype of a transformer designed using the proposed design methodology needs to be constructed and tested. The different analyses used herein can then be further validated.

The high-frequency transformer design methodology proposed in this work includes prediction of leakage inductance and parasitic capacitances. The proposed design methodology can be modified for other isolating DC-DC converter topologies, such as dual active bridge and resonant converters, where these parameters are critical to the converter operation. This would allow for a rigorous comparison between competing converter topologies. To this end, the time domains analysis needs to be modified according to the converter topology and operation. This change is simpler in the case of dual active bridge converter because of the availability of additional control. The minimum leakage inductance required to achieve desirable converter control can be set as a design specification [22]. In case of the resonant converter, the multiple operating point analysis included in the proposed design methodology can be modified to include a range of operating frequencies. Accordingly, the leakage inductance as estimated using the proposed magnetic analysis can be used to model the transformer in accordance with the selected operating frequencies.
REFERENCES
REFERENCES


[46] “Genetic optimization system engineering tool (goset) for use with matlab,” S. D. Sudhoff, Ed. School of Electrical and Computer Engineering, Purdue University, West Lafayette, IN,: with United States Naval Academy, Annapolis, MD, 205.
### A. FERRITE MATERIAL PROPERTIES

Table A.1.
Ferrite Material Properties

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<thead>
<tr>
<th>#</th>
<th>Type</th>
<th>ρ, (kg/m$^3$)</th>
<th>$B_{lm}$ (T)</th>
<th>$k$, (W/m·k)</th>
<th>$c$, (J/kg·K)</th>
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<td>4612</td>
<td>0.42</td>
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<td>750</td>
</tr>
<tr>
<td>3</td>
<td>MN67</td>
<td>4795</td>
<td>0.46</td>
<td>4.25</td>
<td>750</td>
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<tr>
<td>4</td>
<td>MN80C</td>
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<td>4.25</td>
<td>750</td>
</tr>
<tr>
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<td>3C90</td>
<td>4743</td>
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<td>P-Type</td>
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<td>0.47</td>
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# B. MODIFIED STEINMETZ EQUATION PARAMETERS

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<th>$\beta$</th>
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Table B.1.
Modified Steinmetz Equation Parameters of Ferrite Material
C. ADDITIONAL MATERIAL DATA FOR TEC

Table C.1.
Conductor Material Properties

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<thead>
<tr>
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<th>$k$, (W/m*K)</th>
<th>$c$, (J/kg*K)</th>
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<tr>
<td>Air at 300K</td>
<td>1.16</td>
<td>0.03</td>
<td>1007</td>
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<td>Magnet wire insulation</td>
<td>1400</td>
<td>0.4</td>
<td>1500</td>
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<tr>
<td>Copper</td>
<td>8890</td>
<td>385</td>
<td>390</td>
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</tbody>
</table>
VITA

Veda Samhitha Duppalli received her Bachelor of Engineering (Honours) in Electrical and Electronics Engineering from Birla Institute of Technology and Science, Pilani, India in 2013. She received Doctor of Philosophy degree from Purdue University, West Lafayette, IN, USA in 2018. Her main research interests include magnetics design for high-frequency applications, power electronics and renewable energy technologies.