2008

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ABSTRACT

Conformability is related to deformation of tip seals, when they do not conform to their grooves perfectly. A comprehensive analytical tool is developed for the conformability calculation of the tip seal. The tip seal is modeled with 3D finite element beam model, and the interactions between the seal and the groove are modeled with a simplified asperity contact model. The relation between the displacements and the loads of the tip seal are governed by a group of non-linear finite element equations in closed form. Examples for the piston ring (ring-shape) and tip seal (involute-shape) are given to demonstrate the validation of the model. Given the free shape and the cross-section, the tool can be used to compute the contact force distribution between the surfaces of the tip seal and its groove. Inversely, the geometry of the free shape also can be designed by the tool to obtain the desired contact force distribution.

1. INTRODUCTION

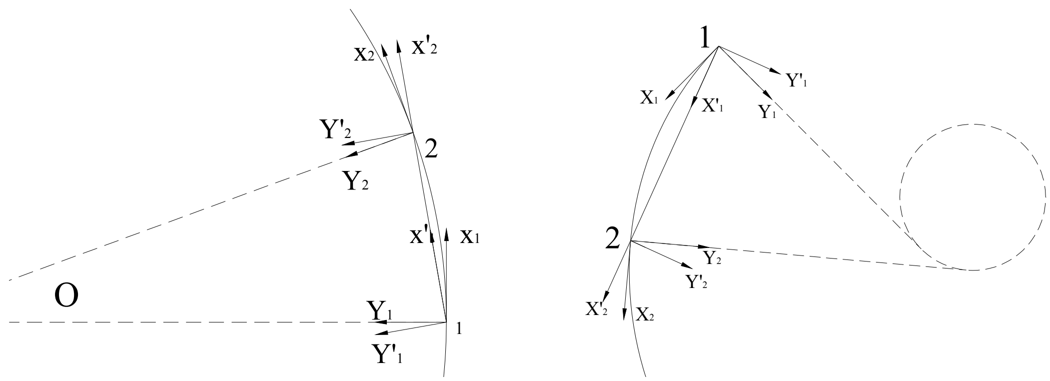
Sealing between moving surfaces widely exists in many applications, such as piston ring for reciprocating compressor and tip seal for scroll compressor. When a ring or seal is inserted into its groove, the nonconformability of their shapes can result in additional contact force on their surfaces that affects the sealing effect of the ring/seal during operation. There are three key elements namely the ring/seal shape in its free state (called free shape hereafter), the crossing-section and the contact surface profile, which play important roles in determining the contact force distribution. Useful tools are required to facilitate the calculation of the contact force and its distribution as well as the ring/seal structure design that enables expected contact force distribution. In current practice, the tools in use depend on simple empirical equations or simplified structural analysis, which are limited when the design features are beyond the applications of these tools.

Many researchers developed their tools for the conformability calculation of the piston ring-cylinder bore of the combustion engines. Sun (1991) conducted his study for ring-bore conformability, in which the ring was modeled as a curved beam under in-plane loads (elastic load, gas load and thermal load) at steady state in any distorted bore. Liu and Tian (2003, 2005) developed an FEA tool for piston ring design. In their study, 3D finite element beam method was applied for the structural response of the ring not only under static conditions but also under dynamic loading condition. A clarification of a semi-empirical approach in piston ring-cylineder bore conformability prediction was presented by Dunaevsky and Rudzitis (2007). They considered the three-dimensional deformation of piston rings with arbitrary cross-section in a cylinder.

This work aims to develop a comprehensive analytical tool to address structure related issues in the tip seal design. The tip seal is modeled with 3D finite element beam model, and the interactions between the seal and the groove are modeled with a simplified asperity contact model. In the following sections, these models will be described and examples will be given for tip seal and piston ring to illustrate the applications of the tool.

2. MODEL DEVELOPMENT

2.1 3D Finite Element Method



(a) ring shape (b) involute shape

Figure 1 Global and local coordinate system

3D finite element method (FEM) has been proved to be a very useful tool to calculate the displacement or the load of the tip seal. First, straight beam element is utilized to discretize the curved tip seal. Each element has two nodes and six degrees of freedom (DOFs) for each node, including three linear displacements and three angular displacements. The radial, tangential and axial directions are chosen as the local coordinate system for each node. The definition of the global and local coordinate system is show in Figure 1. The tip seal has an involute-shape that is different from the ring-shape, so the geometric issues related to the stiffness matrix, the load matrix and the coordinate transformation matrix needs to be considered here. After transforming the local coordinate system ($X'_1Y'_1Z'_1$ and $X'_2Y'_2Z'_2$) into the local coordinate system ($X_1Y_1Z_1$ and $X_2Y_2Z_2$), the finite element formulation relating displacements with load is:

$$\mathbf{K}^i\mathbf{U}^i = \mathbf{P}^i + \mathbf{P}_I^i \qquad (1)$$

where, $\mathbf{K}_i$ is stiffness matrix for *i*th element in the global coordinate systems, $\mathbf{U}i$ is the displacement vector, $\mathbf{S}_i$ is the load vectors and $\mathbf{S}_I^i$ is the initial load vector. All of the finite elements formulas are assembled into the total finite element formulas as following:

$$\mathbf{KU} = \mathbf{P} + \mathbf{P}_I \qquad (2)$$

The detailed description for the FEM can be found (Wang, 2003). Given boundary conditions, Equation (2) can be solved.

2.2 Conformability Calculation

The free shape of the tip seals and the contact force distribution between the tip teal-groove are related with each other. For a tip seal with a symmetric cross-section (such as rectangle), the contact force distribution is related only to its radial displacement, since there is no twist displacement. The conformability calculation includes two stages: the first stage is to force the seal into its groove and in the second stage, the asperity contact model is applied to calculate the contact force distribution (Liu and Tian , 2003).

Stage One: Fitting the Ring to a Circular Bore with Consideration of the Large Displacement Issue

The step-by-step method that is widely used in FEA to solve large displacement problem is adopted in this study. Because the ring free shape is normally symmetry and the bore is circular, the analysis hereafter will be conducted on half of a ring.

Figure 2 illustrates the step-by-step procedure developed in this study. The solid line with nodes is the ring, and the dashed line represents a nominal circular bore. At *i*th step, the (*i*-1)th node is fixed and the radial displacement of *i*th node is calculated. These known constraints and displacements are viewed as boundary conditions and then the FEA formula is solved. Thus the displacements of *i*th to *N*th node and reaction forces for constraints are obtained and saved for next step. The stiff matrix needs to be calculated based on the updated shape. This procedure repeats throughout all of nodes. Finally, the ring coincides with the bore and the reaction forces on each node are also prepared for stage two.

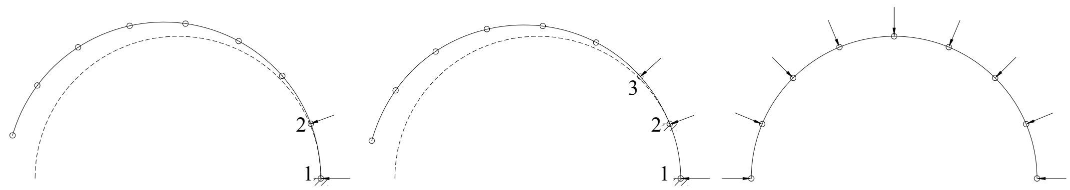


Figure 2 Step-by-step procedure of fitting a ring to a circular bore

Stage Two for Symmetric Ring: Ring/bore Asperity Contact Analysis

In stage two, all of the radial loads artificially applied in stage one are released and the interaction between the ring and the bore is investigated. Asperity contact analysis is implemented for the first time, and it proves to be more efficient and more robust. To conduct asperity contact analysis, FEA is performed again, based on the ring shape at the end of stage one and with the reaction forces set to be the initial loads. The following finite element equation that is similar to Equation (2) is applied:

$$\mathbf{KU} = \mathbf{P}(\mathbf{U}) - \mathbf{P}_I \quad (3)$$

where $\mathbf{P}(\mathbf{U})$ relates the load vector **S** with the displacement vector **U** through the asperity contact, and $\mathbf{S}_I$ is the initial load vector. For symmetric rings, the contact pressure is a function of the clearance between the ring and the bore liner. An asperity contact model is employed to calculate the contact pressure. As will be described, the asperity contact pressure is a nonlinear function of the clearance. Therefore, Equation is a set of nonlinear equations. Newton's method is used to solve these equations.

2.3 Asperity Contact Model

When the tip seal is closely enough to the groove flank, asperity contact occurs due to their surface roughness. The asperity contact results in the contact pressure distribution between their surfaces, which is modeled by using Greenwood & Tripp's theory (1971). For computation convenience, the fitting formula of Hu, et al. (1993) is used for rough surfaces with Gaussian distribution assumed:

$$p_c = \begin{cases} 0 & h/\sigma > 4 \\ K_c\left(4 - \dfrac{h}{\sigma}\right)^z & h/\sigma < 4 \end{cases} \quad (4)$$

where p_c is the contact pressure, σ is the combined surface roughness, h is the nominal clearance between a ring and its groove, K_c is a parameter dependent on asperity and material properties of the rings and the bore, and z is a correlation constant.

2.4 Free Shape Calculation

The reverse procedure of the conformability calculation is employed to calculate the ring free shape if the contact pressure distribution is specified. Before the calculation, specified radial forces are applied to all of the nodes, forces are removed one by one, starting from *N*th node all the way to 2nd node. After each removal, ring shape and reaction forces are updated and the new stiffness matrix is calculated. By the time the force applied on 2nd node is removed, the ring shape is considered to be the sought final free shape.

This procedure can be applied to an arbitrarily assigned contact pressure distribution. As long as the specified load at the beginning is self-balanced, which is physically required the calculated ring free shape can produce the desired contact force distribution.

3. RESULTS AND DISSUTIONS

The above FEA model is applied to the conformability calculation for the ring-shape piston ring and the involute-shape tip seal. The parameters of the tip seal and piston ring used in this study are listed in Table 1.

Table 1 Parameters of the piston ring and the tip seal

Parameter	Value (piston ring)	Value (tip seal)
Nominal (basic) radius, mm	43	2.5
Starting (involute) angle, rad	0	3.42π
Ending (involute) angle, rad	2π	4.42π
Radial width, mm	2.5	3.7
Axial width, mm	3.7	2.5
Young's modulus, GPa	83.3	83.3
Composite surface roughness, μm	1.6	1.6

The cross-section of the piston ring and tip seal are shown in Figure 5 and 6. The clearance between the tip seal and the groove flanks (i.e. ID and OD) depends on the radial displacement of the tip seal, so the asperity contact pressure is a function of the radial displacement essentially.

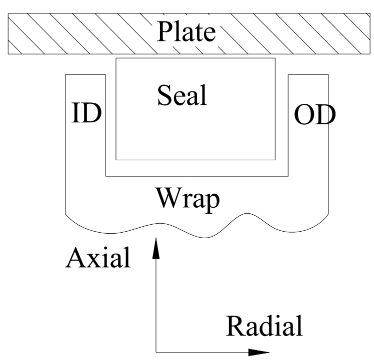


Figure 5 Cross-section of a tip seal

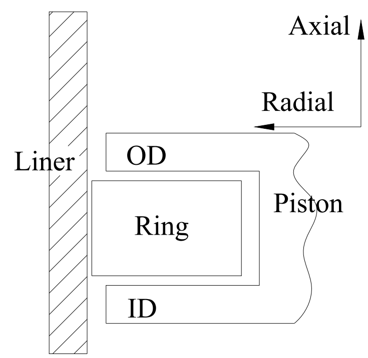


Figure 6 Cross-section of a piston ring

80, 120, and 160 are selected as grid number to test the grid convergence of the model. The radial displacement is shown in Figure 7. They shows good agreement, thus all of the finite element analysis are conducted by using 120 elements in this study. The contact force distribution along circumferential direction without the first and last node is plotted in Figure 8, when the constant radius of free ring shape are 43.5mm, 43.3mm, 42.7mm and 42.5 mm respectively. It is found that the contact force distribution of the first node are 3.37×10^8, 6.21×10^7, -6.58×10^7 and 1.77×10^8 respectively, and the contact force distribution of the last node are 3.47×10^7, 8.04×10^6, -9.12×10^6 and -4.87×10^7 respectively, which indicates that the maximum contact force distribution always appears at the both ends. However, they do not influence the whole ring because their acting area is very small. The piston ring of constant radius fitting into the circular groove generates fairly nonuniform contact force distribution. The piston ring, whose

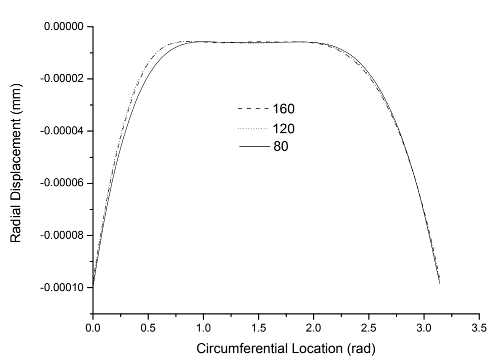


Figure 7 Radial displacements under under different grid number

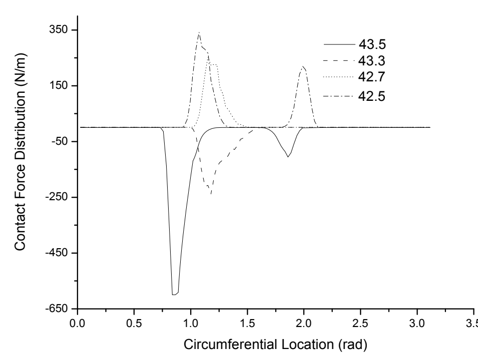


Figure 8 Contact force distribution under different constant radius

radius is larger than 43mm, has negative contact force distribution and the piston ring whose radius is smaller than 43mm has positive contact force distribution.

When uniform tension distribution of 200N/m, 100N/m, -100N/m and -200N/m are specified, the corresponding free shape of the piston ring are shown in Figure 9. The radius along the circumferential direction is not constant to achieve uniform tension distribution. When the tension distribution is positive, the radius increases from the nominal radius to the peak radius and then decreases a little and the behavior is opposite when the tension distribution is negative. Figure 10 illustrates the contact force distribution without the first and the last node for the tip seal of constant radius 34.5mm, 33,5mm, 31.5mm and 29.5mm. Because the differences between the seal free shape and the groove shape is significant, the displacement is relatively large, which leads to much larger contact force distribution. Several approaches are suggested to reduce the contact force distribution. The tip seal should be made of small Young's modulus, or designed with smaller dimension of the cross-section or divided into several parts that are inserted into the groove one by one.

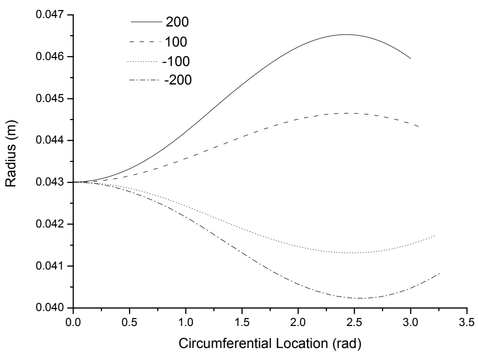


Figure 9 Free shape for different uniform tension distribution

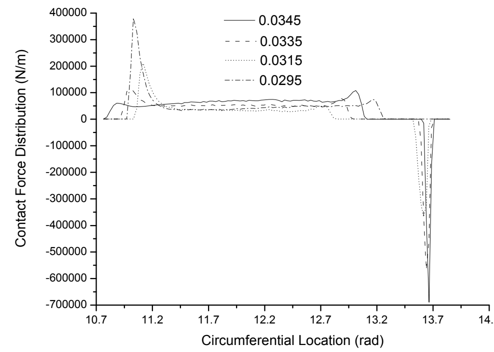


Figure 10 Contact for distribution under different constant radius

4. CONCLUSIONS

In this study, a FEA model for conformability calculation of the involute-shape tip seal is presented. The model is capable of predicting the contact force distribution according to the seal free shape and its groove shape as well as the seal free shape according to the specified contact force distribution.

Through numerical examples, it is concluded that the interaction between the seal free shape with constant radius and the circular groove generates ununiform contact force distribution while the maximum contact force distribution always appears in both ending of the seal. To achieve a uniform tension distribution of the piston ring, the tip seal should be carefully design. Attention should be drawn to the large contact force distribution, when the ring-shape tip seal is fitted into the involute-shape groove.

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ACKNOWLEDGE

The study is supported by Programme for Changjiang Scholars and Innovative Research Team in University (No. IRT0746).