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ABSTRACT

The Logarithmic Mean Temperature Difference (LMTD) method and the effectiveness-NTU method are the two important methods for design and analysis of heat exchangers. The derivation of these two methods relies on a critical assumption, i.e., the fluid specific heats are constant. Under special operating conditions where one fluid experiences condensation or evaporation at constant temperature, these two methods are still valid. In practice, however, the fluid temperature in heat exchangers will never remain constant during phase change because of pressure drop. Meanwhile, zeotropic refrigerant mixtures exhibit temperature variations even during a constant pressure phase change process. Therefore, both LMTD and effectiveness-NTU methods can introduce appreciable errors when applying to the cases in which refrigerant temperature change is not caused by heat transfer, rather than by pressure drop or temperature glide. This paper proposes modified LMTD method and effectiveness-NTU relations to remove the restriction of constant temperature phase change in the original approaches. The new methods account for the effects of pressure drop and temperature glide on the two-phase heat transfer process and make corresponding corrections based on simplifying assumptions. The new methods are applicable for both parallel-flow and counter-flow configurations, with phase change on one side. Rigorous error analyses indicate that the new approaches can substantially improve the thermal performance prediction for heat exchangers with large pressure drop and temperature glide.

Keywords: LMTD, Pressure drop, Temperature glide, Heat transfer, Zeotropic refrigerant, Phase change

1. INTRODUCTION

The heat exchanger is an essential component of any kind of heating, ventilation, air-conditioning and refrigeration (HVAC&R) system. To design or optimize a heat exchanger often requires a model-based approach that can accurately predict the thermal and hydraulic performance of such a device. Among various heat exchanger design methods, the classical Log Mean Temperature Difference (LMTD) approach is widely adopted as the basis of numerical heat exchanger models due to its simplicity. However, the LMTD approach is developed based on a series of strict assumptions, and concerns of the validity of this approach arise when these assumptions are violated. Several noteworthy studies have been conducted by modifying or correcting the original LMTD formulation to relax its restrictions and therefore broaden its applications. Wong *et al.* (2009) developed a so-called "Log Mean Heat Transfer Rate" method to take into account the effect of heat radiation, which is neglected in the LMTD approach, under the circumstance of low ambient convective heat coefficient and high surface emissivity. Cui *et al.* (2014) proposed a modified LMTD method to account for latent heat transfer due to water vaporization in indirect evaporative heat exchangers. Inspired by these two studies, we attempt to scrutinize the underlying assumptions of this classical method, and try to explore potential improvements and therefore make it more effective in solving complex heat transfer problems.

One of the fundamental assumptions adopted in the derivation of the LMTD method is that the fluid specific heats are constant and the fluid temperature variations only result from heat exchange. The LMTD method can be also applied under special operating conditions where phase change occurs at constant temperature, i.e., $c_p \to \infty$. In reality, however, a fluid never experiences phase change at constant temperature, due to pressure drop, or temperature glide, or both. However, temperature glide and pressure drop is inevitable during phase change of a zeotropic refrigerant mixture. The temperatures of a zeotropic refrigerant mixture decrease during condensation and increase during evaporation because of temperature glide. This behavior gives rise to significantly different temperature profiles for zeotropic refrigerant mixtures compared to pure refrigerants. Distinct from temperature glide, pressure drop always leads to temperature decrease. As a result, pressure drop diminishes temperature differences between refrigerant and fluid during condensation, whereas it negates the effect of temperature glide during evaporation. As pointed out by

Itard and Machielsen (1994), the conventional LMTD method cannot be applied to model heat exchangers or calculate COP when working with large pressure drop and temperature glide, because its vital assumptions are violated under these circumstances.

The effectiveness-NTU method is an alternative approach in heat exchangers analysis. It is convenient to use the LMTD method when the fluid inlet and outlet temperatures are known. However, if only the fluid inlet temperatures are given, the effectiveness-NTU method is more preferably adopted. Unfortunately, the effectiveness-NTU method is built upon the same assumptions that the LMTD method relies on. Therefore, neither of these two approaches can address the fundamental challenges when working with pressure drop and temperature glide. To fill in this research gap, the presented paper will propose new approaches that aim to overcome the deficiency of these two methods by taking into account the effects of pressure drop and temperature glide in heat exchanger analysis. The remainder of the paper is organized as follows. Section 2 describes the derivation of the modified LMTD approach. Section 3 presents the derivation of the modified effectiveness-NTU relations. Section 4 provides the error analyses of these two approaches. The conclusions are summarized in Section 5.

2. MODIFIED LMTD APPROACH

The LMTD method relates the total heat transfer rate to the inlet and outlet fluid temperatures, the overall heat transfer coefficient, and the overall heat transfer area.

$$q = UA\Delta T_{lm} \tag{1}$$

The LMTD is determined by

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\Delta T_1 / \Delta T_2\right)} \tag{2}$$

where subscripts 1 and 2 denote the opposite ends of the heat exchanger.

The derivation of this classical approach can be found in any heat transfer textbook and hence is not repeated here for brevity. However, it is worthwhile to point out the underlying assumptions of this approach (Incropera *et al.*, 2011): (1) there is no heat loss to the surroundings, which means that heat exchange only occurs between the hot and cold fluids; (2) the flow conditions are steady; (3) axial conduction along the tube is negligible; (4) potential and kinetic energy changes are ignored; (5) the fluid specific heats are constant (if the fluid does not undergo phase change); (6) the overall heat transfer coefficient is constant.

The hidden hypothesis of the LMTD method is that variations in fluid temperatures, if any, are solely a result of heat exchange. Clearly, the validity of the LMTD method will be compromised in the design of heat exchangers with zeotropic refrigerant mixture as working fluid, because this hypothesis can never be satisfied. Therefore, a new approach that can account for the effects of temperature glide and pressure drop is needed to relax this hypothesis.

Without loss of generality, the temperature profiles of a parallel-flow heat exchanger depicted in Fig. 1 is used to derive the new approach. In this heat exchanger, the hot stream is the condensing refrigerant while the cold stream is the secondary fluid, e.g., air or brine. Since the refrigerant is a zeotropic mixture, its temperature decreases along the heat exchanger during condensation because of temperature glide and pressure drop. To simplify the analysis, a few additional assumptions are made: (1) two-phase temperature change is caused by pressure change and temperature glide; (2) temperature change caused by glide varies linearly with quality; (3) temperature change induced by pressure loss/gain varies linearly with length; (4) two-phase temperature change caused by pressure loss/gain is approximately equal to the change in dew point for condensation, and equal to the change in bubble point for evaporation.

In general, the specific heat of a refrigerant during phase change at a constant temperature is infinity. However, if the temperature glide exists, one can define a pseudo two-phase specific heat as

$$c_{p,TP} = \frac{h_{dew} - h_{bub}}{T_{dew} - T_{bub}} \tag{3}$$

where the subscripts dew and bub represent the dew point and bubble point, respectively. Accordingly, one can define the capacitance of refrigerant and fluid as $C_r = \dot{m}_r c_{p,TP}$ and $C_f = \dot{m}_f c_{p,f}$.

The refrigerant temperature is a function of pressure and specific enthalpy $T = T(p,h)$, therefore

$$dT = \frac{\partial T}{\partial p} dp + \frac{\partial T}{\partial h} dh \tag{4}$$

In two-phase, the partial derivative of temperature with respect to specific enthalpy $\partial T/\partial h$ can be approximated as

$$\frac{\partial T}{\partial h} \approx \frac{1}{c_{p,TP}} \tag{5}$$

While the partial derivative of temperature with regards to pressure $\partial T/\partial p$ can be estimated using the Clausius-Clapeyron relation (Clausius-Clapeyron relation should not introduce appreciable errors although it is derived based on the constant temperature phase change process). Alternatively, this partial derivative can be approximated as

$$\frac{\partial T}{\partial p} \approx \frac{\Delta T_{sat}}{\Delta p_{HX}} \tag{6}$$

where Δp_{HX} is the pressure change across the heat exchanger. ΔT_{sat} is the resulting change in the dew point temperature. For condensation, $\Delta T_{sat} = T_{dew,out} - T_{dew,in}$; for evaporation, $\Delta T_{sat} = T_{bub,out} - T_{bub,in}$.
Therefore, the refrigerant temperature is calculated as

$$T_r = T_{r,in} + \Delta T_p + \Delta T_g \tag{7}$$

where ΔT_p and ΔT_g are the temperature change due to pressure change and temperature glide, respectively.
Substituting Eqs. (5) and (6) into Eq. (7) yields

$$T_r = T_{r,in} + \Delta T_{sat} \Delta p / \Delta p_{HX} + \dot{m}_r \left(h_r - h_{r,in} \right) / C_r \tag{8}$$

Assuming that refrigerant pressure drop is proportional to the distance that the refrigerant has travelled, one can have

$$\Delta p / \Delta p_{HX} \approx A / A_{HX} \tag{9}$$

Also, one can define the heat transfer rate from the 0 to A.

$$q = \dot{m}_r \left(h_{r,in} - h_r \right) \tag{10}$$

Eq. (8) can be rewritten as

$$T_r = T_{r,in} + \Delta T_{sat} A / A_{HX} - q / C_r \tag{11}$$

The temperature of the fluid can be determined as

$$T_f = \begin{cases} T_{f,in} + q / C_f & \text{for parallel flow} \\ T_{f,out} - q / C_f & \text{for counter flow} \end{cases} \tag{12}$$

Differentiating Eqs. (11) and (12) yields

$$dT_r = \frac{\Delta T_{sat}}{A_{HX}} dA - \frac{1}{C_r} dq \tag{13}$$

$$dT_f = \pm \frac{1}{C_f} dq \tag{14}$$

where the upper and lower signs are for the parallel-flow and counter-flow (Fig. 2) cases, respectively.
Subtracting Eq. (14) from Eq. (13) results in

$$d\left(\Delta T\right) = d\left(T_r - T_f\right) = -\left(\pm \frac{1}{C_f} + \frac{1}{C_r} \right) dq + \frac{\Delta T_{sat}}{A_{HX}} dA \tag{15}$$

The heat transfer rate across the differential area dA can be expressed as

$$dq = U\left(T_r - T_f\right) dA = U \Delta T dA \tag{16}$$

Substituting Eq. (16) into Eq. (15) and rearranging yields

$$\frac{d\Delta T}{-\left(\pm \frac{1}{C_f} + \frac{1}{C_r} \right) U \Delta T + \frac{\Delta T_{sat}}{A_{HX}}} = dA \tag{17}$$

Let $\alpha = -(\pm 1/C_f + 1/C_r)U$ and $\beta = \Delta T_{sat}/A_{HX}$, and integrating both sides of Eq. (17) gives

$$\int_{\Delta T_1}^{\Delta T} \frac{d\Delta T}{\alpha \Delta T + \beta} = \int_0^A dA \tag{18}$$

$$\Delta T = \left(\Delta T_1 + \beta / \alpha \right) \exp\left(\alpha A \right) - \beta / \alpha \tag{19}$$

Eq. (19) provides the calculation of local temperature difference between refrigerant and fluid, therefore the mean temperature difference across the entire heat exchanger is defined as

$$\Delta T_m = \frac{1}{A_{HX}} \int_0^{A_{HX}} \Delta T dA \tag{20}$$

Substituting Eq. (19) into Eq. (20) and integrating yields

$$\Delta T_m = \frac{1}{A_{HX}} \int_0^{A_{HX}} \left[(\Delta T_1 + \beta/\alpha) \exp(\alpha A) - \beta/\alpha \right] dA = \frac{1}{\alpha A_{HX}} (\Delta T_1 + \beta/\alpha) \left[\exp(\alpha A_{HX}) - 1 \right] - \beta/\alpha \tag{21}$$

According to Eq. (19), the temperature difference between refrigerant and fluid at point 2 is

$$\Delta T_2 = (\Delta T_1 + \beta/\alpha) \exp(\alpha A_{HX}) - \beta/\alpha \tag{22}$$

Thus, one can obtain

$$\alpha A_{HX} = \ln\left[(\Delta T_2 + \beta/\alpha) / (\Delta T_1 + \beta/\alpha) \right] \tag{23}$$

Substituting Eq. (23) into Eq. (21), the corrected mean temperature difference of the heat exchanger is

$$\Delta T_{lm,c} = \frac{(\Delta T_2 + \beta/\alpha) - (\Delta T_1 + \beta/\alpha)}{\ln\left[(\Delta T_2 + \beta/\alpha) / (\Delta T_1 + \beta/\alpha) \right]} - \beta/\alpha, \quad \frac{\beta}{\alpha} = -\Delta T_{sat} \Big/ \left[UA_{HX} \left(\pm \frac{1}{C_f} + \frac{1}{C_r} \right) \right] \tag{24}$$

where the upper and lower signs are for the parallel-flow and counter-flow cases, respectively. For the parallel-flow configuration, $\Delta T_1 = T_{r,in} - T_{f,in}$ and $\Delta T_2 = T_{r,out} - T_{f,out}$. For the counter-flow configuration, $\Delta T_1 = T_{r,in} - T_{f,out}$ and $\Delta T_2 = T_{r,out} - T_{f,in}$.

In general, β/α is positive for parallel-flow and negative for counter-flow. Eq. (24) applies to both condensation and evaporation. For condensation, the mean temperature difference is positive. For evaporation, the mean temperature difference is negative, since ΔT_1 and ΔT_2 are both negative. When the refrigerant (pure substance or azeotropic mixture) exhibits no or little temperature glide, the corrected LMTD is calculated using Eq. (24) with $\beta/\alpha = -\Delta T_{sat} C_f / (\pm UA_{HX})$ (because $C_r \to \infty$). When refrigerant pressure drop is negligible ($\Delta T_{sat} = 0$ and $\beta = 0$), Eq. (24) reduces to the original LMTD formulation.

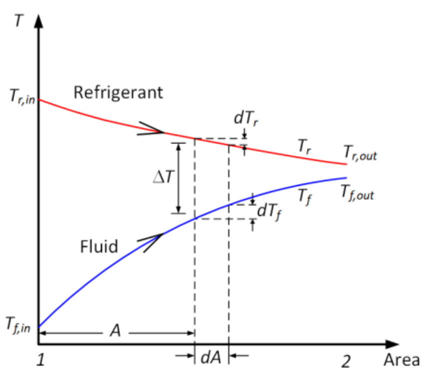

Fig. 1 Temperature profile for a parallel-flow condenser

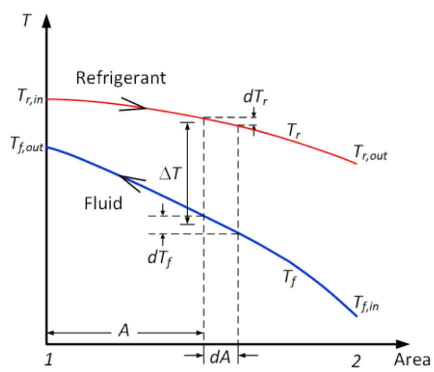

Fig. 2 Temperature profile for a counter-flow condenser

3. MODIFIED EFFECTIVENESS-NTU RELATIONS

The effectiveness of a heat exchanger, ε, is defined as the ratio of the actual heat transfer rate to the maximum possible heat transfer rate. Since this paper is tackling the case with phase change, the capacitance of the single-phase fluid is generally smaller and one can define ε as

$$\varepsilon = \frac{T_{f,out} - T_{f,in}}{T_{r,in} - T_{f,in}} \tag{25}$$

The number of transfer units is defined as

$$NTU = \frac{UA_{HX}}{C_f} \tag{26}$$

Now consider the parallel-flow heat exchanger illustrated in Fig. 1, from Eq. (22) one can obtain

$$\frac{T_{r,out} - T_{f,out} + \beta/\alpha}{T_{r,in} - T_{f,in} + \beta/\alpha} = \exp\left[-NTU (1 + C_{ratio}) \right] \tag{27}$$

where $C_{ratio} = C_f / C_r$.
Rearranging the left-side of Eq. (27) results in

$$\frac{T_{r,out} - T_{f,out} + \beta/\alpha}{T_{r,in} - T_{f,in} + \beta/\alpha} = \frac{T_{r,out} - T_{r,in} + T_{r,in} - T_{f,in} + T_{f,in} - T_{f,out} + \beta/\alpha}{T_{r,in} - T_{f,in} + \beta/\alpha} = \frac{T_{r,out} - T_{r,in} + T_{f,in} - T_{f,out}}{T_{r,in} - T_{f,in} + \beta/\alpha} + 1 \quad (28)$$

Based on Eq. (11), the temperature difference of refrigerant between the inlet and outlet is

$$T_{r,out} - T_{r,in} = \Delta T_{sat} - \frac{1}{C_r} q_{HX} \quad (29)$$

From the energy balance, one can have

$$q_{HX} = C_f \left(T_{f,out} - T_{f,in} \right) \quad (30)$$

Substituting Eq. (30) into Eq. (29) yields

$$T_{r,out} - T_{r,in} = \Delta T_{sat} - C_{ratio} \left(T_{f,out} - T_{f,in} \right) \quad (31)$$

Therefore, Eq. (28) can be rewritten as

$$\begin{aligned} \frac{T_{r,out} - T_{f,out} + \beta/\alpha}{T_{r,in} - T_{f,in} + \beta/\alpha} &= \frac{\Delta T_{sat} - \left(1 + C_{ratio}\right)\left(T_{f,out} - T_{f,in}\right)}{T_{r,in} - T_{f,in} + \beta/\alpha} + 1 = \frac{\Delta T_{sat} - \left(1 + C_{ratio}\right)\varepsilon\left(T_{r,in} - T_{f,in}\right)}{T_{r,in} - T_{f,in} + \beta/\alpha} + 1 \\ &= \frac{\gamma - \left(1 + C_{ratio}\right)\varepsilon}{1 - \gamma / \left[NTU \left(1 + C_{ratio}\right) \right]} + 1 \end{aligned} \quad (32)$$

where $\gamma = \Delta T_{sat} / (T_{r,in} - T_{f,in})$. Generally, ΔT_{sat} is negative because of pressure drop, and $T_{r,in} - T_{f,in}$ is positive for a condenser and negative for an evaporator. Therefore, γ is negative for a condenser and positive for an evaporator.
Substituting Eq. (32) into Eq. (27) and solving for ε, one can obtain for a parallel-flow heat exchanger

$$\varepsilon = \frac{\gamma + \left[1 - \gamma / \left[NTU \left(1 + C_{ratio}\right) \right] \right] \left\{ 1 - \exp\left[-NTU \left(1 + C_{ratio}\right) \right] \right\}}{1 + C_{ratio}} \quad (33)$$

For a counter-flow heat exchanger, from Eq. (19) one can have

$$\frac{T_{r,out} - T_{f,in} + \beta/\alpha}{T_{r,in} - T_{f,out} + \beta/\alpha} = \exp\left[NTU \left(1 - C_{ratio}\right) \right] \quad (34)$$

Similarly, the left hand of Eq. (34) can be rewritten as

$$\frac{T_{r,out} - T_{f,in} + \beta/\alpha}{T_{r,in} - T_{f,out} + \beta/\alpha} = \frac{T_{r,out} - T_{r,in} + T_{r,in} - T_{f,in} + \beta/\alpha}{T_{r,in} - T_{f,in} + T_{f,in} - T_{f,out} + \beta/\alpha} = \frac{\gamma - \varepsilon C_{ratio} + 1 + \gamma / (\alpha A)}{1 - \varepsilon + \gamma / (\alpha A)} \quad (35)$$

Substituting Eq. (35) into Eq. (34) and solving for the effectiveness of the counter-flow case yields

$$\varepsilon = \frac{1 + \dfrac{\gamma}{NTU\left(1 - C_{ratio}\right)} - \left[\gamma + \dfrac{\gamma}{NTU\left(1 - C_{ratio}\right)} + 1 \right] \exp\left[-NTU \left(1 - C_{ratio}\right) \right]}{1 - C_{ratio} \exp\left[-NTU \left(1 - C_{ratio}\right) \right]} \quad (36)$$

When there is no temperature glide, Eq. (33) reduces to

$$\varepsilon = 1 - \exp\left(-NTU\right) + \gamma \left[1 - \frac{1}{NTU} + \frac{\exp\left(-NTU\right)}{NTU} \right] \quad (37)$$

and Eq. (36) reduces to

$$\varepsilon = 1 - \exp\left(-NTU\right) + \gamma \left[\frac{1}{NTU} - \exp\left(-NTU\right) - \frac{\exp\left(-NTU\right)}{NTU} \right] \quad (38)$$

When refrigerant pressure drop is neglected, Eqs. (33) and (36) will reduce to the original expressions for the parallel-flow and counter-flow cases, respectively.

4. RESULTS AND DISCUSSIONS

When considering the effects of pressure drop and temperature glide, from Eqs. (11) and (12) the local temperature difference between refrigerant and fluid at $A = A_x$ for the parallel-flow case

$$\Delta T_x = T_r - T_f = \frac{\Delta T_{sat}}{A_{HX}} A_x + \alpha q_x \tag{39}$$

where $q_x = C_f(T_{f,x} - T_{f,in})$ is the heat transfer rate from $A = 0$ to $A = A_x$.

If neglecting the effects of pressure drop and temperature glide in the calculation, the original LMTD will treat the two-phase refrigerant as single-phase with inlet and outlet temperatures of $T_{r,in}$ and $T_{r,out}$, respectively. Therefore, the capacitance of this phony "single-phase" refrigerant will be

$$C_{r,SP} = \frac{C_f\left(T_{f,out} - T_{f,in}\right)}{\left(T_{r,in} - T_{r,out}\right)} \tag{40}$$

and the uncorrected local temperature difference will be

$$\Delta T_{x,un} = -\left(\frac{1}{C_{r,SP}} + \frac{1}{C_f}\right) q_x \tag{41}$$

Subtracting Eq. (41) from Eq. (39) and rearrangement yields

$$\Delta T_{x,error} = \Delta T_x - \Delta T_{x,un} = \left(\frac{A_x}{A_{HX}} - \frac{T_{f,x} - T_{f,in}}{T_{f,out} - T_{f,in}}\right)\Delta T_{sat} \tag{42}$$

Following the same analysis, one can obtain the expression for the counter-flow case

$$\Delta T_{x,error} = \Delta T_x - \Delta T_{x,un} = \left(\frac{A_x}{A_{HX}} - \frac{T_{f,out} - T_{f,x}}{T_{f,out} - T_{f,in}}\right)\Delta T_{sat} \tag{43}$$

Before calculating the errors in Eqs. (42) and (43), we need to examine the curvature of the temperature profiles of both refrigerant and fluid. Differentiating Eqs. (11) and (12) twice results in

$$\frac{d^2 T_r}{dA^2} = -\frac{1}{C_r}\frac{d^2 q}{dA^2} = -\frac{U}{C_r}\frac{d\Delta T}{dA} \tag{44}$$

$$\frac{d^2 T_f}{dA^2} = \pm\frac{1}{C_f}\frac{d^2 q}{dA^2} = \pm\frac{U}{C_f}\frac{d\Delta T}{dA} \tag{45}$$

where the upper and lower signs are for the parallel and counter-flow cases, respectively.

From Eq. (17), one can easily obtain $d\Delta T/dA$

$$\frac{d\Delta T}{dA} = \alpha \Delta T + \frac{\Delta T_{sat}}{A_{HX}} \tag{46}$$

Substituting Eqs. (46) into Eq. (45) gives

$$\frac{d^2 T_f}{dA^2} = \begin{cases} \dfrac{U}{C_f}\left[-\left(1 + \dfrac{C_f}{C_r}\right)\dfrac{U\Delta T}{C_f} + \dfrac{\Delta T_{sat}}{A_{HX}}\right], \text{ for parallel flow} \\ \dfrac{U}{C_f}\left[\left(\dfrac{C_f}{C_r} - 1\right)\dfrac{U\Delta T}{C_f} - \dfrac{\Delta T_{sat}}{A_{HX}}\right], \text{ for counter flow} \end{cases} \tag{47}$$

It is not difficult to find out that d^2T_f/dA^2 is negative and d^2T_r/dA^2 is positive for the parallel-flow case in Fig. 1. It indicates that the temperature profile of the cold stream, i.e., the fluid, is concave, whereas the temperature profile of the hot stream, i.e., the refrigerant, is convex, as illustrated in Fig. 3.

For the counter-flow case, the curvature of the temperature profiles depends on the capacitance ratio of two streams. Especially, when refrigerant pressure drop is neglected,

$$\frac{d^2 T_f}{dA^2} = \begin{cases} < 0, \text{ if } C_f / C_r < 1 \Rightarrow T_f \text{ is concave} \\ = 0, \text{ if } C_f / C_r = 1 \Rightarrow T_f \text{ is linear} \\ > 0, \text{ if } C_f / C_r > 1 \Rightarrow T_f \text{ is convex} \end{cases} \tag{48}$$

In general, the capacitance ratio is very small, i.e., $C_f/C_r << 1$, when the refrigerant is in two-phase. Therefore, d^2T_f/dA^2 and d^2T_r/dA^2 are still negative even if refrigerant pressure drop is taken into account, indicating that the temperature profiles of both refrigerant and fluid are concave, as illustrated in Fig. 4.

Evidently for the parallel-flow case, one can obtain

$$\frac{T_{f,x}-T_{f,in}}{T_{f,out}-T_{f,in}}=\frac{A'}{A_{HX}}>\frac{A_x}{A_{HX}} \tag{49}$$

Similarly, for the counter-flow case

$$\frac{T_{f,out}-T_{f,x}}{T_{f,out}-T_{f,in}}=\frac{A'}{A_{HX}}<\frac{A_x}{A_{HX}} \tag{50}$$

Therefore, from Eqs. (42) and (43) one can have

$$\Delta T_{x,error}\begin{cases}>0 & \text{for parallel-flow}\\ <0 & \text{for counter-flow}\end{cases} \tag{51}$$

Eq. (51) indicates that the local temperature difference between refrigerant and fluid when considering pressure drop and temperature glide is always larger than that when neglecting pressure drop and temperature glide for the parallel-flow case, and smaller for the counter-flow case. Therefore, the uncorrected LMTD method will under predict the performance of a parallel-flow heat exchanger, and over predict the performance of a counter-flow heat exchanger.

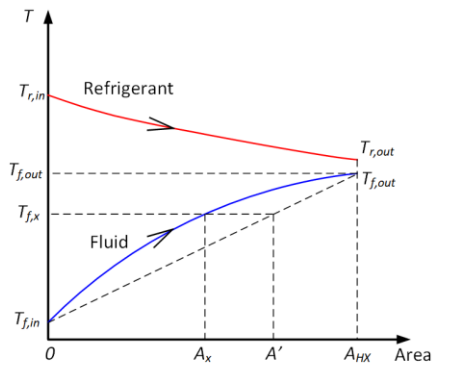

Fig. 3 Curvature of temperature profile for a parallel-flow condenser

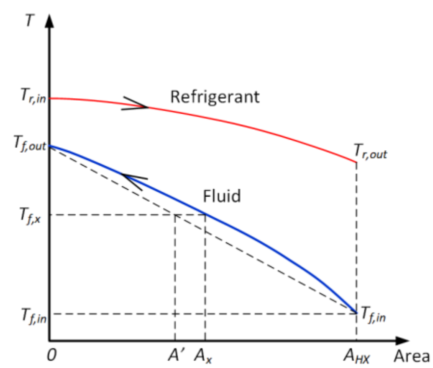

Fig. 4 Curvature of temperature profile for a counter-flow condenser

In the derivation of the modified LMTD method, it is assumed that the temperature glide varies linearly with quality. This assumption significantly simplifies the analysis without introducing substantial errors and can be justified by the two-phase temperature variations of R407C with different pressures and quality, as shown in Fig. 5.

To quantify the errors introduced by neglecting pressure drop and temperature glides in the calculation, Eq. (24) is approximated using the 2nd order Taylor expansion. Let f denote the uncorrected LMTD, $f = f(\Delta T_1,\Delta T_2)=(\Delta T_1-\Delta T_2)/\ln(\Delta T_1/\Delta T_2)$, then the error between the corrected and uncorrected LMTD is

$$\begin{aligned}\Delta T_{lm,error} &= \Delta T_{lm,c}-\Delta T_{lm} = f\left(\Delta T_1+\frac{\beta}{\alpha},\Delta T_2+\frac{\beta}{\alpha}\right)-\frac{\beta}{\alpha}-f\left(\Delta T_1,\Delta T_2\right)\\ &\approx \frac{\beta}{\alpha}\left(\frac{f^2}{\Delta T_1\Delta T_2}-1\right)+\left(\frac{\beta}{\alpha}\right)^2\frac{1}{\Delta T_2}\left(\frac{f}{\Delta T_1}-\frac{1}{2}-\frac{1}{2}\frac{\Delta T_2}{\Delta T_1}\right)\frac{f^2}{\Delta T_1\Delta T_2}\end{aligned} \tag{52}$$

It can be found that the error is largely dependent on $\Delta T_1/\Delta T_2$ (because both $f^2/(\Delta T_1\Delta T_2)$ and $f/\Delta T_1$ are only a function of $\Delta T_1/\Delta T_2$). Fig. 6 shows the relative errors between the corrected and uncorrected LMTD with $\Delta T_2 = 1$ K. In the figure, the solid and dashed lines represent the errors of the parallel-flow and counter-flow configurations, respectively. Without accounting for pressure drop and temperature glide, the uncorrected LMTD under predicts the performance of parallel-flow heat exchangers, and over predicts the performance of counter-flow heat exchangers. Meanwhile, the errors between the corrected and uncorrected LMTD increase rapidly as $\Delta T_1/\Delta T_2$ increases. It appears that the counter-flow configuration is more susceptible to the change in $\Delta T_1/\Delta T_2$. When $\Delta T_1/\Delta T_2 = 10$, the errors between the corrected and uncorrected LMTD for the counter-flow configuration can be as large as more than 10%, whereas the errors for the parallel-flow configuration is about 5%. According to Eq. (24), an increase in pressure drop results in an increase in β, whereas an increase in temperature glide results in a decrease in α. Therefore, an increase in either pressure drop, or temperature glide, or both will lead to an increase in the errors, which corroborates that the conventional LMTD method can introduce substantial deviations when working with large pressure drop and temperature glide.

The variations in the effectiveness of the parallel and counter flow cases with C_{ratio} = 0.02 are shown in Fig. 7 and 8, respectively. It can be observed that neglecting pressure drop and temperature glide can result in overestimate of the effectiveness for condenser. When neglecting pressure drop and temperature glide, refrigerant temperature remains constant. In reality, however, both pressure drop and temperature glide will decrease refrigerant temperature during condensation. Therefore, heat transfer will be enhanced under the falsely elevated temperature difference between the refrigerant and fluid when neglecting pressure drop and temperature glide, resulting in over prediction of heat exchanger effectiveness.

During evaporation, however, pressure drop and temperature glide impose opposite influences on heat transfer. Explicitly, pressure drop will decrease refrigerant temperature, which elevates the temperature difference between refrigerant and fluid, whereas temperature glide will increase refrigerant temperature, which diminishes the temperature difference between refrigerant and fluid. When the capacitance ratio is small, either due to small temperature glide or large refrigerant flow rate, accounting for pressure drop and temperature glide will result in an increase in the heat exchanger effectiveness, as shown in Fig. 9 and 10. This is because pressure drop induced temperature decrease can offset the temperature increase caused by temperature glide when the capacitance ratio is small, resulting in a decrease in refrigerant temperature and larger temperature difference between refrigerant and fluid. Therefore, when the evaporator reaches a certain size, its effectiveness can be greater than unity, which means that fluid outlet temperature can be lower than refrigerant inlet temperature. This is understandable for the parallel-flow case, because refrigerant outlet temperature could be lower than its inlet temperature if pressure drop is large. When fluid temperature approaches refrigerant temperature at the outlet of the evaporator, it will be possibly lower than refrigerant inlet temperature. When the capacitance ratio is large, pressure drop induced temperature decrease cannot offset the temperature increase caused by temperature glide, resulting in an increase in refrigerant temperature and smaller temperature difference between refrigerant and fluid. Therefore, the heat exchanger effectiveness is well below that of neglecting pressure drop and temperature glide (Fig. 11).

Comparing Fig. 10 against Fig. 12, it can be found that for the counter-flow case the effectiveness also decreases when the capacitance ratio increases. The deviations of the effectiveness between neglecting and considering pressure drop and temperature glide are not significant because fluid outlet temperature is always bounded by refrigerant inlet temperature, which remains fixed. The reason why the effectiveness is greater than unity for a counter-flow evaporator with large refrigerant pressure drop is because fluid temperature can cross with refrigerant temperature (Fig. 14), which means that refrigerant will reject heat to the fluid first near the inlet of the evaporator, then absorb heat from the fluid in the remaining portion of the evaporator. This does not violate the second law of thermodynamics because the change in refrigerant temperature is primarily caused by pressure drop instead of heat transfer. Under this circumstance, fluid outlet temperature could be lower than refrigerant inlet temperature. Of course, when gradually increasing the size of the evaporator, fluid outlet temperature will eventually approach refrigerant inlet temperature, resulting in the effectiveness decreasing to unity (one can check Eq. (36) assuming NTU $\rightarrow \infty$). It is worthwhile to mention that the LMTD method is invalid when temperature crossing occurs in the heat exchanger.

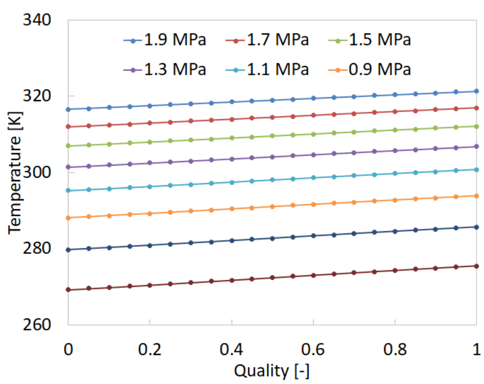

Fig. 5 Two-phase temperature of R407C

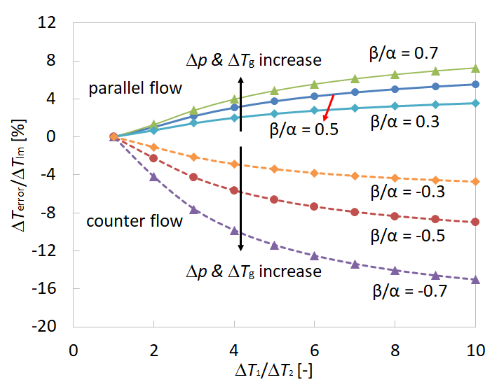

Fig. 6 Variations in LMTD error

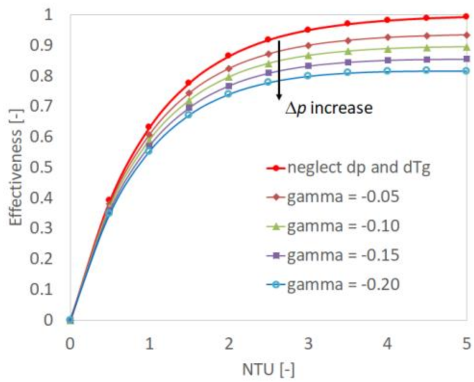

Fig. 7 ε for parallel-flow condenser (C_{ratio} = 0.02)

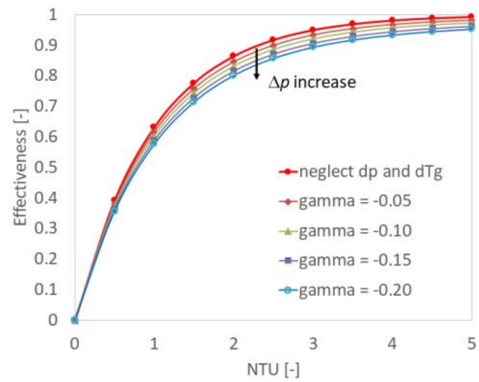

Fig. 8 ε for counter-flow condenser (C_{ratio} = 0.02)

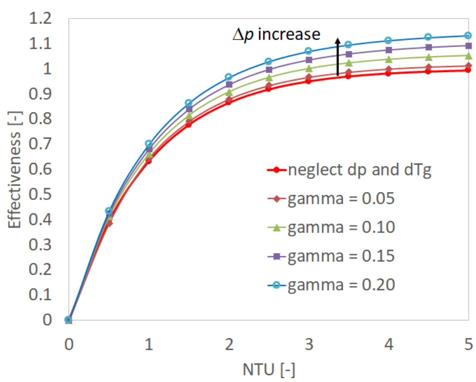

Fig. 9 ε for parallel-flow evaporator (C_{ratio} = 0.02)

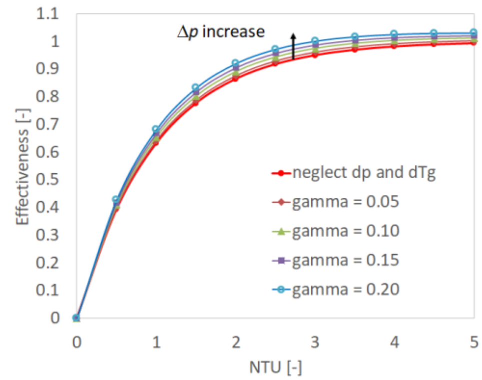

Fig. 10 ε for counter-flow evaporator (C_{ratio} = 0.02)

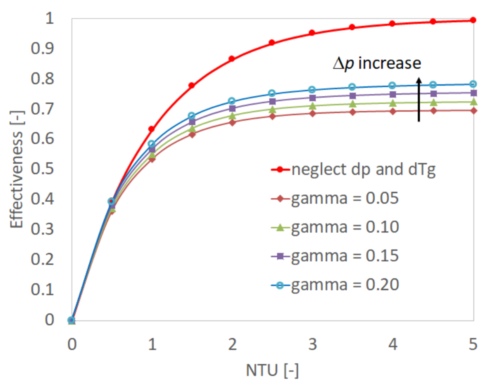

Fig. 11 ε for parallel-flow evaporator (C_{ratio} = 0.5)

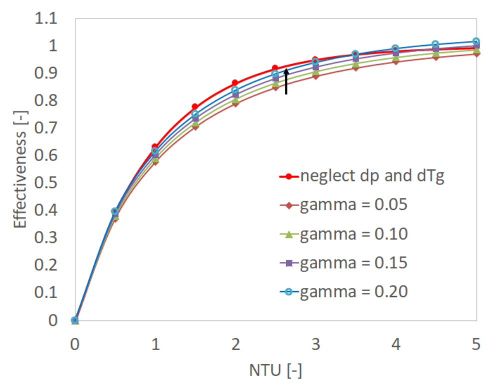

Fig. 12 ε for counter-flow evaporator (C_{ratio} = 0.5)

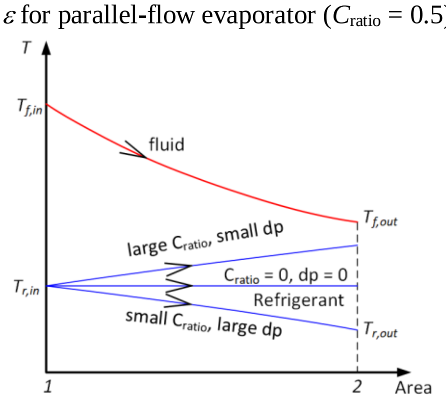

Fig. 13 Temp. profile for a parallel-flow evaporator

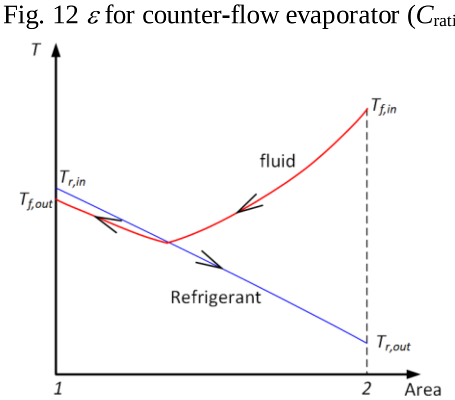

Fig. 14 Temp. crossing in a counter-flow evaporator

5. CONCLUSIONS

The applicability of the LMTD method and effectiveness-NTU relations has been investigated in the two-phase heat transfer with pressure drop and temperature glide. It has been demonstrated that the conventional heat exchanger design methods could introduce substantial errors when working with large pressure drop and temperature glide. Modified LMTD method and effectiveness-NTU relations have been developed to take into account of these effects in the calculation. The proposed methods are applicable for both parallel-flow and counter-flow configurations, with phase change on one side. Rigorous error analyses have indicated that the original LMTD method under predicts the performance of a parallel-flow heat exchanger, whereas over predicts the performance of a counter-flow heat exchanger. Meanwhile, the original effectiveness-NTU relations over estimates the heat exchanger effectiveness for condenser, while under estimates the heat exchanger effectiveness for evaporator. Therefore, the proposed modified LMTD method and effectiveness-NTU relations are recommended working with large pressure drop and temperature glide.

NOMENCLATURE

Symbols		***Subscripts***	
A	area	bub	bubble point
C	capacitance	c	corrected value
c_p	specific heat	dew	dew point
h	specific enthalpy	f	fluid
$\dot{m}$	mass flow rate	g	temperature glide related
NTU	number of transfer units	HX	heat exchanger
p	pressure	in	inlet
q	heat transfer rate	lm	log mean
T	temperature	out	outlet
U	overall heat transfer coefficient	p	pressure related
ε	effectiveness	r	refrigerant
Δ	difference	sat	saturation
		SP	single-phase
		TP	two-phase
		un	uncorrected value

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