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Yang Zhu

*University of Texas at Dallas, United States of America, zhuyang19881218@163.com*

Yaoyu Li

*University of Texas at Dallas, United States of America, yxl115230@utdallas.edu*

Liujia Dong

*University of Texas at Dallas, United States of America, lxd122030@utdallas.edu*

Timothy I. Salsbury

*Johnson Controls, Inc., timothy.i.salsbury@jci.co*

John M. House

*Johnson Controls, Inc., john.m.house@jci.com*

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# Distributed Extremum Seeking Control for a Variable Refrigerant Flow System

Yang Zhu<sup>1</sup>, Yaoyu Li<sup>1\*</sup>, Liuja Dong<sup>1</sup>, Timothy I. Salsbury<sup>2</sup>, John M. House<sup>2</sup>

<sup>1</sup> Department of Mechanical Engineering, The University of Texas at Dallas, Richardson, TX 75080, USA.  
(yxz152631@utdallas.edu, yaoyu.li@utdallas.edu, lxd122030@utdallas.edu)

<sup>2</sup> Building Efficiency Group, Johnson Controls, Inc., 507 E Michigan St., Milwaukee, WI 53201, USA.  
(Timothy.I.Salsbury@jci.com, John.M.House@jci.com)

\* Corresponding Author

## ABSTRACT

The variable refrigerant flow (VRF) system features the regulation of refrigerant flow to achieve individualized heating and cooling for multi-zone operation. However, such flexibility in configuration presents large variations in system characteristics and thermal loads. Performance of model based control and optimization would thus be limited. The extremum seeking control (ESC) has recently been considered as a model-free control solution of maximizing the efficiency of multi-split VRF system. In this paper, a saddle-point dynamics based distributed ESC strategy is applied to optimize the indoor unit (IDU) superheat setpoint for minimizing the IDU fan power provided the satisfaction of thermal comfort. The ODU runs a stand-alone single-input ESC implemented with the suction pressure setpoint as control input and the ODU power output as the feedback. The proposed distributed ESC strategy is evaluated for the all-cooling operation mode of a VRF system that consists of one outdoor unit (ODU) and four IDU's. Simulation study is performed with a Modelica based dynamic simulation model of the VRF system using the distributed ESC and the centralized multi-input ESC in Dong et al. (2016). Simulation results show that the distributed ESC strategy achieves comparable performance as the centralized ESC in terms of steady-state and transient performance.

## 1. INTRODUCTION

The variable refrigerant flow (VRF) technology has found its wide application for temperature regulation for residential, commercial, and industrial buildings. The working medium (refrigerant) is conditioned by a single outdoor unit (ODU), and then circulated within the building to multiple indoor units (IDU) (Park et al., 2001). By making use of variable capacity compressors and electronic expansion valves (EEV), the VRF systems are able to control the refrigerant flow to the multiple evaporators of IDUs, thus enabling each IDU to achieve individualized zoning control of different capacities. The VRF systems can be configured to provide simultaneous heating and cooling operation for different zones via the so-called mode change unit (MCU). The MCU is a valve array to regulate the refrigerant flows through the IDUs to achieve five possible operation modes including cooling-only, heating-only, cooling-dominated, heating-dominated, and heat recovery (Shi et al., 2003, Xia et al., 2004, Hai et al., 2006). The multi-split nature and flexibility in configuration make the VRF systems more challenging for controls. Aynur et al. (2006) presents a field study on both individual and master control methods for a multisplit VRF system in an actual building to evaluate performance characteristics. Joo et al. (2011) studies the performance characteristics of a simultaneous cooling and heating multi-heat pump with four IDUs at partial load conditions.

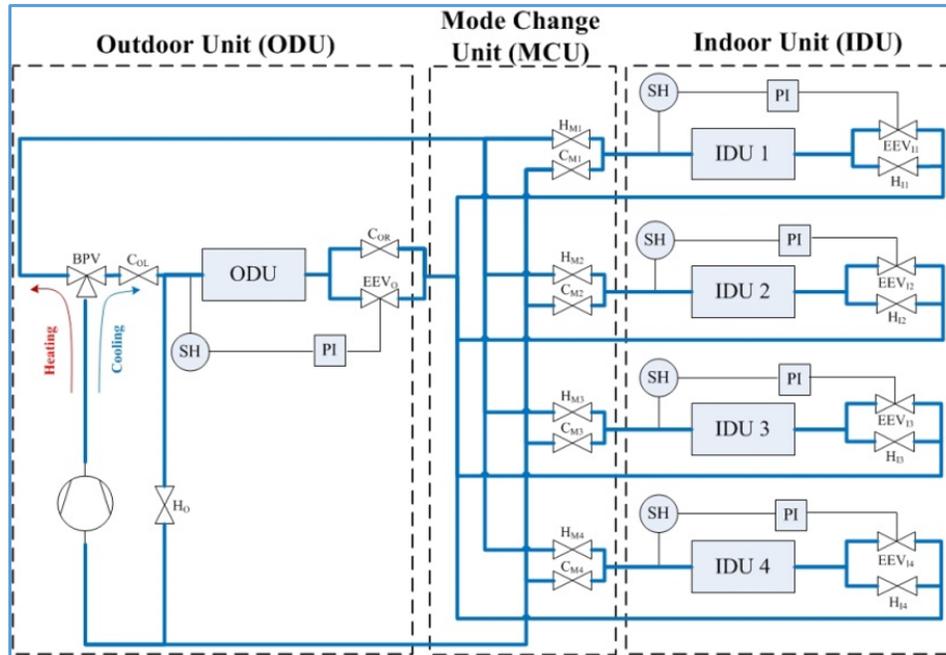
Although the VRF systems offer advantages such as elimination of duct loss of air distribution, design and installation flexibility, compactness and reduced maintenance cost (Aynur et al., 2010), it is usually difficult/expensive to obtain plant models required by model based control strategies due to large variation of ambient and load conditions, complexity of inherent physical processes, and loop interactions among multiple subsystems. Therefore, model-free control strategy has become a more beneficial solution to consider. Recently, the extremum seeking control (ESC) (Ariyur and Krstic, 2003) has emerged as a model-free control strategy for real-time optimization. ESC is a dynamic real-time optimization technique that extracts online estimate of gradient using dither-demodulation schemes, which has experienced a dramatic growth since the first rigorous stability analysis by Krstic and Wang (2000). Tan et al.

(2006) extends the proof of local stability of traditional ESC to a semi-global stability framework. Later, Lie bracket approximation of extremum seeking systems was investigated (Durr, 2013). ESC has found remarkable applications in autonomous vehicles and mobile robots, internal combustion engines, flow control, and process control. The heating, ventilation and air conditioning (HVAC) systems feature large-scale, nonlinear, and distributed nature, with hard-to-model dynamics subject to significant disturbance and process variation. These challenges make model-free real-time optimization solutions like ESC a viable solution to optimizing energy efficiency in practical operation. ESC has been applied to various HVAC systems, e.g. air-side economizer (Li et al., 2010), chilled water plant (Li et al., 2013), and air source heat pump (Xiao et al., 2014, Dong et al., 2015). Koeln et al. (2014) presents an optimal subcooling in vapor compression systems via ESC. Jain et al. (2014) proposes a hybrid control scheme for VRF system, combining an outer-loop ESC with model predictive control. More recently, Dong et al. (2016) presents a centralized multi-input ESC for various modes of operation for a multi-functional VRF system.

For large-scale complex engineering systems, it is desirable to apply distributed/decentralized control strategies so that the system operation can be more robust and computational load can be handled by individual agents. Therefore, for real-time optimization of large-scale systems, various distributed ESC schemes have been investigated (Frihauf et al., 2012, Kvaternik and Pavel, 2012, Xu and Soh, 2013, Menon and Baras, 2014, Dougherty and Guay, 2014, Poveda and Quijano, 2013), which are essentially based on the measurements of the local cost and constraint functions while neither explicit expressions on them nor their gradients are required. Frihauf et al. (2012) gave a first attempt in applying an ESC scheme to seek the Nash equilibrium in non-cooperative games. However, Nash solution is usually not Pareto-optimal or socially optimal solution. Kvaternik and Pavel (2012) proposed an analytic framework based on an extremum seeking control approach to solve the distributed optimization problem in order to locate a socially optimal solution. This method is limited to an interconnection structure of a numerical gradient estimation and a numerical optimization with continuous-time system dynamics. Xu and Soh (2013) proposed a distributed extremum seeking method by using push-sum protocol. The proposed method requires the distributed optimization problem to have a global potential function. By using an average consensus protocol, Menon and Baras (2014) demonstrated a local convergence result to the solution of a class of unconstrained distributed optimization problems. Similarly, the average consensus protocol was utilized in Dougherty and Guay (2014) in the extremum seeking control loop to solve constrained optimization problems with strongly convex social objective functions and constraints. Distributed resource allocation for multi-agent systems was investigated in Poveda and Quijano (2013), where local replicator dynamics and Shahshahani gradient were utilized in the extremum seeker design. The resource allocation was formulated as a distributed optimization problem with decoupled objective functions subject to quasi-static dynamics and a linear simplex constraint. Based on the above statement, these distributed ESC schemes have various kinds of constraints on the structures of the plants that they focus on, they do not cover general distributed optimization problems.

Distributed optimization for multi-agent systems is investigated in Ye and Hu (2016), where the saddle point dynamics were utilized in the extremum seeker design. The distributed ESC in Ye and Hu (2016) was formulated as a distributed optimization problem with decoupled objective functions subject to several constraint functions. A general model-free convex optimization problem by ESC is first investigated that lays the foundation for the analysis of distributed optimization problems. This formulation may cover a class of more general distributed optimization problems, as a result, it will be the main theoretical algorithm taken into account for the optimization of the VRF systems in later chapter.

The multi-split VRF system may include up to hundreds of IDU's, which presents a large-scale complex system, for which distributed/decentralized control strategies are desirable. Elliott et al. (2013) presents a decentralized model predictive control (DMPC) for a multi-evaporator HVAC system, minimizing tracking error and energy consumption by optimizing the pressure and cooling setpoints. Jain et al. (2014) presents partially decentralized linear quadratic (LQ) control architecture for large-scale VRF systems, based on first-principle modeling and one-way communication from the individual decentralized controllers to a global controller. However, these efforts have been based on the availability of detailed plant model, which can amount to high operational cost for practical implementation. In comparison, model-free distributed ESC schemes would be naturally better solution. In this study, we have followed the saddle-point dynamics based distributed ESC (DESC) scheme proposed by Ye and Hu (2016), and performed simulation study with the Modelica based dynamic simulation model of a four-zone MFVRF system developed by Dong et al. (2016). For the all-cooling operation being considered, the four IDUs are controlled with the DESC scheme, each with the respective IDU power consumption as the feedback and the evaporator superheat (SH) temperature set-point as the input. Full information communication is assumed among all the IDU agents. The ODU is controlled with the standard ESC, with the ODU power consumption minimized by tuning the compressor suction pressure (PCS) set-point. In particular, a



**Figure 1: The schematic diagram of the multi-functional VRF system considered**

4th-order anti-notch filter is added to the output side to obtain a narrower pass-band range signal, thus we have better "isolations" for individual input channels.

The rest of the paper is organized as follows. Section 2 describes the Modelica based dynamic model of VRF system. Section 3 gives a brief description of the saddle point dynamics based distributed ESC method with its application to the VRF mode. The corresponding simulation results are presented in Section 4. Section 5 concludes the paper and discusses on possible future work.

## 2. MODELICA BASED DYNAMIC SIMULATION MODEL OF A VRF SYSTEM

In this study, the multi-functional VRF system model developed by Dong et al. (2016) is used for simulation study. The schematic diagram of the multi-functional VRF system is shown in Fig. 1. The VRF system is made up of one ODU, one MCU and four IDUs. The ODU consists of a variable speed compressor, a bypass valve (BPV), a heat exchanger (HX), an electronic expansion valve (EEV), and mode-control solenoid valves ( $C_{OL}$ ,  $C_{OR}$  and  $H_O$ ). The inlet of BPV is connected to the compressor, and the two outlets of BPV are connected to  $C_{OL}$  and the heating-mode valves in MCU respectively. The BPV can distribute the refrigerant flow to the two branches by regulating the valve opening. When the ODU HX is operated as condenser (under higher cooling demand), valves  $C_{OL}$  and  $C_{OR}$  are opened while  $H_O$  and  $EEV_O$  are closed. The BPV is fully opened to the  $C_{OL}$  side when all IDUs are operated in the cooling mode. With one IDU or more working in the heating mode, the refrigerant flow out of the compressor is split by BPV: one goes to  $C_{OL}$  and other one to the heating IDU(s). When the ODU HX works as evaporator (under higher heating demand),  $H_O$  and  $EEV_O$  are opened while  $C_{OL}$  and  $C_{OR}$  are closed. The BPV is fully opened to the heating-mode valves in the MCU. The MCU consists of pairs of mutually exclusive solenoid valves ( $H_{Mk}$  and  $C_{Mk}$ ,  $k = 1, 2, 3, 4$ ), and each pair is connected to IDU- $k$ . MCU can regulate the refrigerant flow direction for each IDU to realize cooling or heating mode. The IDU each includes one HX, one EEV ( $EEV_{Ik}$ ) and one heating-mode solenoid valve ( $H_{Ik}$ ). Thermal model of an IDU is controlled by valve actions in both MCU and IDU. For IDU- $k$  to work in the cooling mode,  $C_{Mk}$  and  $EEV_{Ik}$  are opened while  $H_{Mk}$  and  $H_{Ik}$  are closed; for IDU- $k$  to work in the heating mode,  $C_{Mk}$  and  $EEV_{Ik}$  are closed while  $H_{Mk}$  and  $H_{Ik}$  are opened.

For the four-zone illustrative case, there are five scenarios of operation, which are denoted as follows. '4C': all four IDUs are in cooling mode; '1H3C': one IDU is in heating mode, and the other three IDUs in the cooling mode; '2H2C':

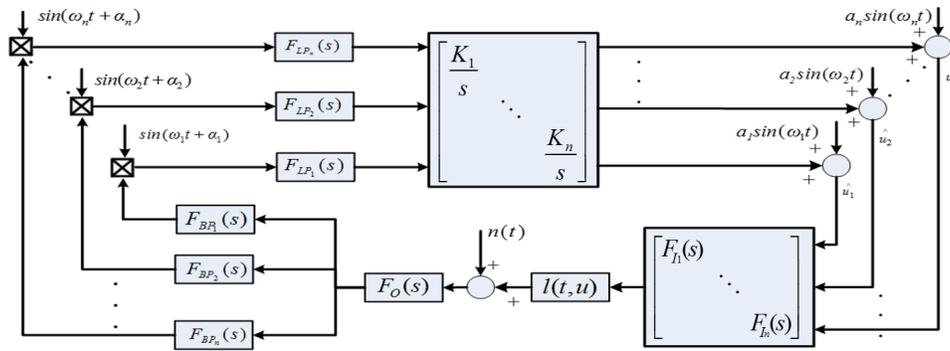


Figure 2: Block diagram of multi-variable dither-demodulation ESC

two IDUs are in heating mode and the other two in cooling mode; '3H1C': one IDU is in cooling mode and the other three in heating mode; '4H': all four IDUs are in heating mode. Three inner loop controls are applied to the VRF system. For each zone, the zone temperature is regulated by the mass flow rate of its IDU fan with a PI controller. For HXs working as evaporator, either for IDU or ODU, the superheat temperature is regulated by EEV opening with PI controller. When cooling demand dominates (i.e. for 4C, 1H3C, and some 2H2C scenarios), the compressor suction pressure ( $P_{CS}$ ) is regulated by the compressor speed with a PI controller. When the heating demands dominates (i.e. 4H, 3H1C and some 2H2C scenarios), the compressor discharge pressure ( $P_{CD}$ ) is regulated by the compressor speed with another PI controller.

To evaluate the distributed ESC strategy in the next chapter, without losing generality, we consider that the VRF system operates in the 4C mode. A Modelica based dynamic simulation model is developed for the VRF system in Fig. 1, using Dymola 2015, TIL Library 3.2 (Dassault, 2015) and TIL Media Library 3.2 (Richter, 2008).

### 3. OVERVIEW OF STANDARD AND DISTRIBUTED ESC

In this study, the distributed ESC by Ye and Hu (2016) is applied to energy efficient operation of the VRF system, and the performance is compared with the standard multi-input ESC by Dong et al. (2016). In this section, the conventional ESC is reviewed first, and then the framework of distributed ESC (Ye and Hu, 2016) is presented, which originated from the saddle-point dynamics based distributed optimization developed by Durr et al. (2013).

#### 3.1 Overview of Standard Multi-input ESC

ESC is a class of model free adaptive control algorithms, which aim to search for the optimizing input  $u_{opt}(t)$  for the generally unknown time-varying cost function  $l(t, u)$ , where  $u(t) \in \mathbb{R}^m$  is the input parameter vector. As shown in Fig. 2,  $y(t)$ , the measurement of cost function  $l(t, u)$ , is corrupted by noise  $n(t)$ .  $F_I(s)$  and  $F_O(s)$  denote LTI approximation of input and output dynamics, respectively. The dither and demodulation signals are  $d_1^T(t) = [a_1 \sin(\omega_1 t), \dots, a_m \sin(\omega_m t)]$  and  $d_2^T(t) = [\sin(\omega_1 t + \alpha_1), \dots, \sin(\omega_m t + \alpha_m)]$ , respectively, where  $\omega_i$  are the dithering frequencies,  $a_i$  are the dither amplitudes, and  $\alpha_i$  are the phase angles introduced between the respectively dither and demodulation signals.

The perturbed output, contains the gradient information in the first-harmonics term. After filtering the DC component of the dithered output by the anti-notch filter  $F_{BP}(s)$ , the resultant signal is multiplied (demodulated) by  $d_2^T$ , which shifts the gradient term to DC. Applying the low-pass filter  $F_{LP}(s)$  yields a vector-valued signal proportional to the gradient of the cost function at the input of the multivariable integrator.

#### 3.2 Distributed ESC for Unconstrained Distributed Real-time Optimization

Consider a multi-agent system with  $N$  agents and the set of agents is denoted by  $V = 1, 2, \dots, N$ . The agents can communicate with each other via an undirected and connected graph denoted by  $G = (V, E)$  with  $E \subset V \times V$  representing the edge set. The optimization objective is defined as

$$\min f(x) = \sum_{i=1}^N f_i(x) \quad (1)$$

$$s.t. g(x) \leq 0, i \in \{1, 2, \dots, N\} \quad (2)$$

where  $x = [x_1, x_2, \dots, x_n]^T \in \mathbb{R}^n$  is a vector of decision variables for the whole network and  $f_i(x) : \mathbb{R}^n \rightarrow \mathbb{R}$  is the cost function for agent  $i$ . The explicit expressions of  $f_i(x)$  and their gradients are unknown. Only measurements of the functions  $f_i(x)$ ,  $g(x)$  are available for agent  $i$ . The gradients of  $f_i(x)$ ,  $g(x)$  cannot be measured directly. The objective is to design a distributed updating law for the agents such that they can search for the solution of the optimization problem. We assume that the functions  $f_i(x)$  are strictly convex in  $x$  and sufficiently smooth, and  $g(x)$  is convex in  $x$  and sufficiently smooth. The solution to the optimization problem exists and is finite. The above optimization problem can be further reformulated as

$$\min \check{f}(\mathbf{x}) = \sum_{i=1}^N f_i(\mathbf{x}_i) \quad (3)$$

$$s.t. \mathbf{L}\mathbf{x} = 0, g(\mathbf{x}_i) \leq 0, i \in \{1, 2, \dots, N\} \quad (4)$$

where  $\mathbf{x}_i = [x_{i1}, x_{i2}, \dots, x_{in}]^T \in \mathbb{R}^n$  stands for agent  $i$ 's estimation on the optimal solution to above optimization problem.  $\mathbf{L} = L \otimes I_n$ ,  $\mathbf{x} = [\mathbf{x}_1^T, \mathbf{x}_2^T, \dots, \mathbf{x}_n^T]^T$ .  $L$  is the Laplacian matrix of the communication topology and  $I_n$  is a  $n \times n$  identity matrix. Inspired by the saddle point dynamics, the following extremum seeking law for agent  $i$  under no constraint is proposed to find the optimal solution to the optimization problem depicted above.

$$x_{ij} = \hat{x}_{ij} + b_{ij} \sin(\omega_{ij}t) \quad (5)$$

$$\dot{\hat{x}}_{ij} = -k_{ij} \left( f_i(\mathbf{x}_i) \sin(\omega_{ij}t) + \frac{b_{ij}}{2} \left( \sum_{k=1}^N a_{ik}(\hat{x}_{ij} - \hat{x}_{kj}) + \sum_{k=1}^N a_{ik}(z_{ij} - z_{kj}) \right) \right) \quad (6)$$

$$\dot{z}_{ij} = \theta \sum_{i=1}^N a_{ik}(\hat{x}_{ij} - \hat{x}_{kj}) \quad (7)$$

where  $a_{ik} = 1$  or  $0$  if there is or no communication between two agents  $i$  and  $k$ . The parameters  $b_{ij} > 0$  and  $\omega_{ij} > 0$  are the amplitude and frequency of the dither signals, respectively. The loop gain  $k_{ij} > 0$  and  $\theta > 0$  are designing parameters. Furthermore,  $\hat{\mathbf{x}}_i = [\hat{x}_{i1}, \hat{x}_{i2}, \dots, \hat{x}_{in}]^T \in \mathbb{R}^n$ ,  $\mathbf{x}_i = [x_{i1}, x_{i2}, \dots, x_{in}]^T \in \mathbb{R}^n$  are estimation vectors of the decision variable by use of agent  $i$ 's cost function and the  $z_{ij}$  is an intermediate variable.

### 3.3 Application of Distributed ESC to VRF Model

For the all-cooling operation, the four IDUs and the ODU are regarded as five agents. The standard ESC is independently applied to the ODU agent, while the distributed ESC is performed on the four IDU agents. In comparison with the original method developed by Ye and Hu (2016), a four-order anti-notch filter is added to the output side to obtain a narrower pass-band range signal, thus we have better "isolations" for individual input channels. The distributed ESC scheme is shown in Fig. 3. For each agent, the respective power consumption is used as feedback. The objective is to minimize the total power consumption of all agents. The compressor suction pressure ( $P_{CS}$ ) set-point is employed as the input for the ODU agent, while the evaporator superheat ( $SH$ ) temperature set-point of each IDU is used as the input for the respective IDU agent. The distributed ESC scheme assumes full information communication among all IDUs. The diagram of application of the distributed ESC to the VRF system is shown in Fig. 4.

## 4. SIMULATION RESULTS

Simulation study is performed to evaluate the control strategy based on the Modelica based dynamic simulation model developed by Dong et al. (2016) which is presented in section II. The ESC is designed under the ambient condition

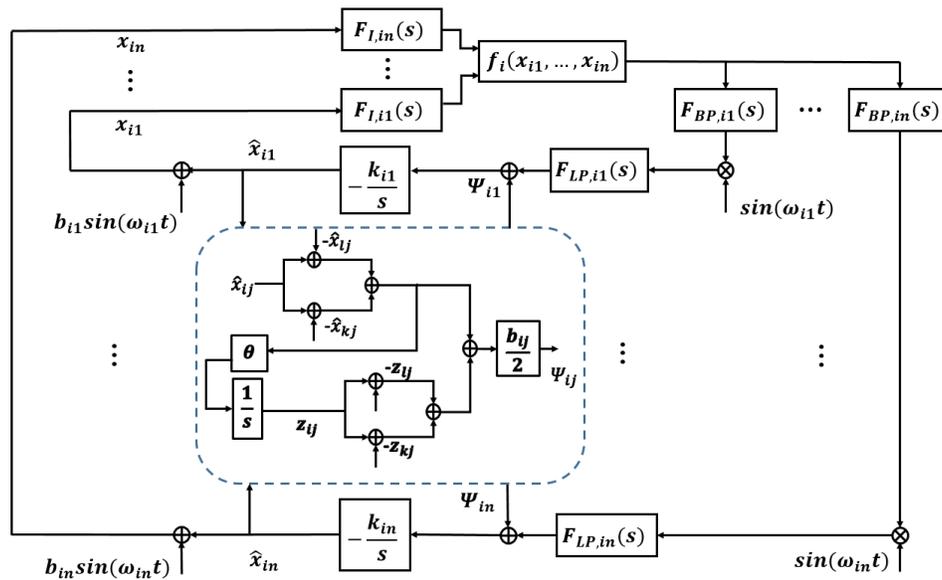


Figure 3: Distributed ESC scheme based on Ye and Hu (2016)

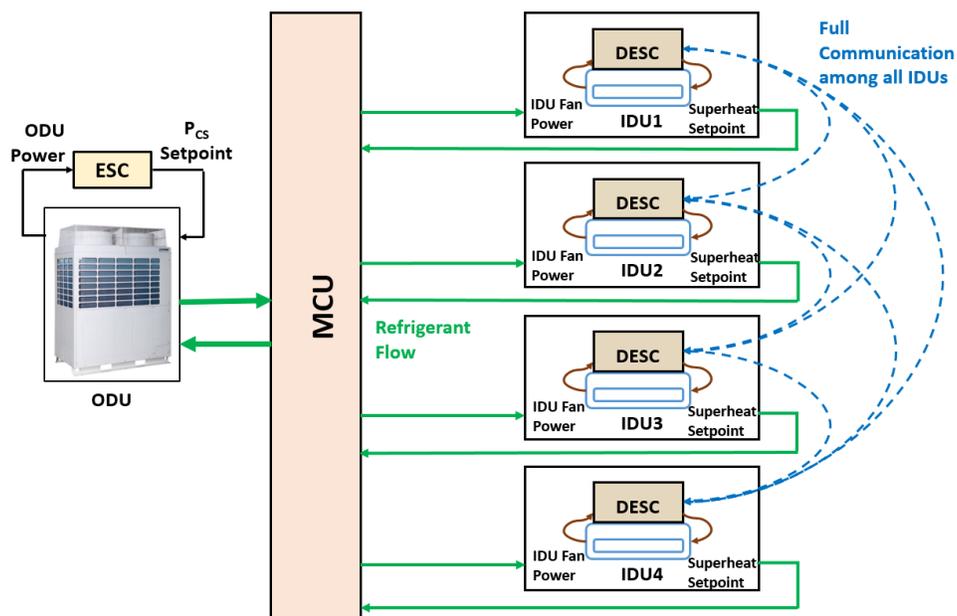


Figure 4: Proposed VRF Control Strategy: Single-input ESC for ODU and Distributed ESC for IDU's

of 35°C and 40 RH, respectively. The initial temperature of all four IDUs zone is 29°C, and the zone temperature set-point is 27°C. The heat gains for IDU1 through IDU4 are 3500W, 3000W, 2800W and 2600W, respectively. The input dynamics for compressor pressure and IDU superheat channels are estimated as follows,

$$\hat{F}_{I,PCS}(s) = \frac{0.025^2}{s^2 + 2 \times 0.2 \times 0.025s + 0.025^2} \quad (8)$$

$$\hat{F}_{I,IDU1,SH}(s) = \frac{0.022^2}{s^2 + 2 \times 0.22 \times 0.022s + 0.022^2} \quad (9)$$

$$\hat{F}_{I,IDU2,SH}(s) = \frac{0.028^2}{s^2 + 2 \times 0.3 \times 0.028s + 0.028^2} \quad (10)$$

$$\hat{F}_{I,IDU3,SH}(s) = \frac{0.026^2}{s^2 + 2 \times 0.25 \times 0.026s + 0.026^2} \quad (11)$$

$$\hat{F}_{I,IDU4,SH}(s) = \frac{0.024^2}{s^2 + 2 \times 0.2 \times 0.024s + 0.024^2} \quad (12)$$

The design parameters by independent ESC for the ODU agent are such that, the dither frequency  $\omega = 0.007rad/s$ , the dither amplitude  $b = 0.1bar$ , the loop gain  $k = 0.3$  with  $\theta = 1$ . Then all distributed ESC design parameters for each IDU agent are listed in Tables 1-4, respectively. The frequency of dither signals should satisfy that  $\omega_i \neq \omega_j$ ,  $2\omega_i \neq \omega_j$ ,  $\omega_i \neq \omega_j + \omega_k$ , for any  $i \neq j \neq k$ .

**Table 1: DESC Design Parameters of Agent 1 for 4C Mode**

ESC Input	IDU1 SH by IDU1 Fan Power, $x_{11}$	IDU1 SH by IDU2 Fan Power, $x_{21}$	IDU1 SH by IDU3 Fan Power, $x_{31}$	IDU1 SH by IDU4 Fan Power, $x_{41}$
Dither Freq. $\omega$	0.00210 (rad/s)	0.00201 (rad/s)	0.00190 (rad/s)	0.00231 (rad/s)
Dither Amp. $b$	0.1°C	0.1°C	0.1°C	0.1°C
Loop Gain $k$	40	50	40	30

**Table 2: DESC Design Parameters of Agent 2 for 4C Mode**

ESC Input	IDU2 SH by IDU1 Fan Power, $x_{12}$	IDU2 SH by IDU2 Fan Power, $x_{22}$	IDU2 SH by IDU3 Fan Power, $x_{32}$	IDU2 SH by IDU4 Fan Power, $x_{42}$
Dither Freq. $\omega$	0.00215 (rad/s)	0.00211 (rad/s)	0.00250 (rad/s)	0.00230 (rad/s)
Dither Amp. $b$	0.1°C	0.1°C	0.1°C	0.1°C
Loop Gain $k$	40	50	40	30

**Table 3: DESC Design Parameters of Agent 3 for 4C Mode**

ESC Input	IDU3 SH by IDU1 Fan Power, $x_{13}$	IDU3 SH by IDU2 Fan Power, $x_{23}$	IDU3 SH by IDU3 Fan Power, $x_{33}$	IDU3 SH by IDU4 Fan Power, $x_{43}$
Dither Freq. $\omega$	0.00225 (rad/s)	0.00235 (rad/s)	0.00233 (rad/s)	0.00213 (rad/s)
Dither Amp. $b$	0.1°C	0.1°C	0.1°C	0.1°C
Loop Gain $k$	40	30	50	50

Based on the ESC controller designed in section III, all-cooling, i.e., 4C mode is simulated. On the one hand, to evaluate the performance of the proposed ESC controller, the actual optimum for the VRF system is found by applying

Table 4: DESC Design Parameters of Agent 4 for 4C Mode

ESC Input	IDU4 SH by IDU1 Fan Power, $x_{14}$	IDU4 SH by IDU2 Fan Power, $x_{24}$	IDU4 SH by IDU3 Fan Power, $x_{34}$	IDU4 SH by IDU4 Fan Power, $x_{44}$
Dither Freq. $\omega$	0.00223 (rad/s)	0.00251 (rad/s)	0.00221 (rad/s)	0.00245 (rad/s)
Dither Amp. $b$	0.1°C	0.1°C	0.1°C	0.1°C
Loop Gain $k$	40	50	60	70

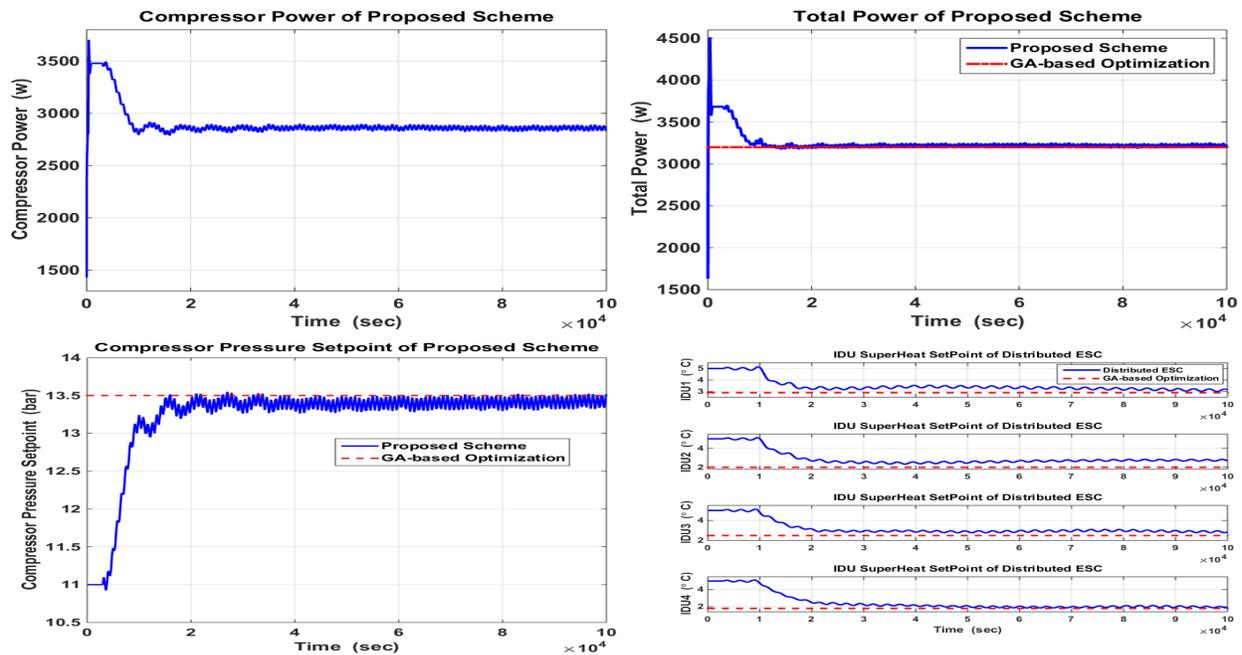
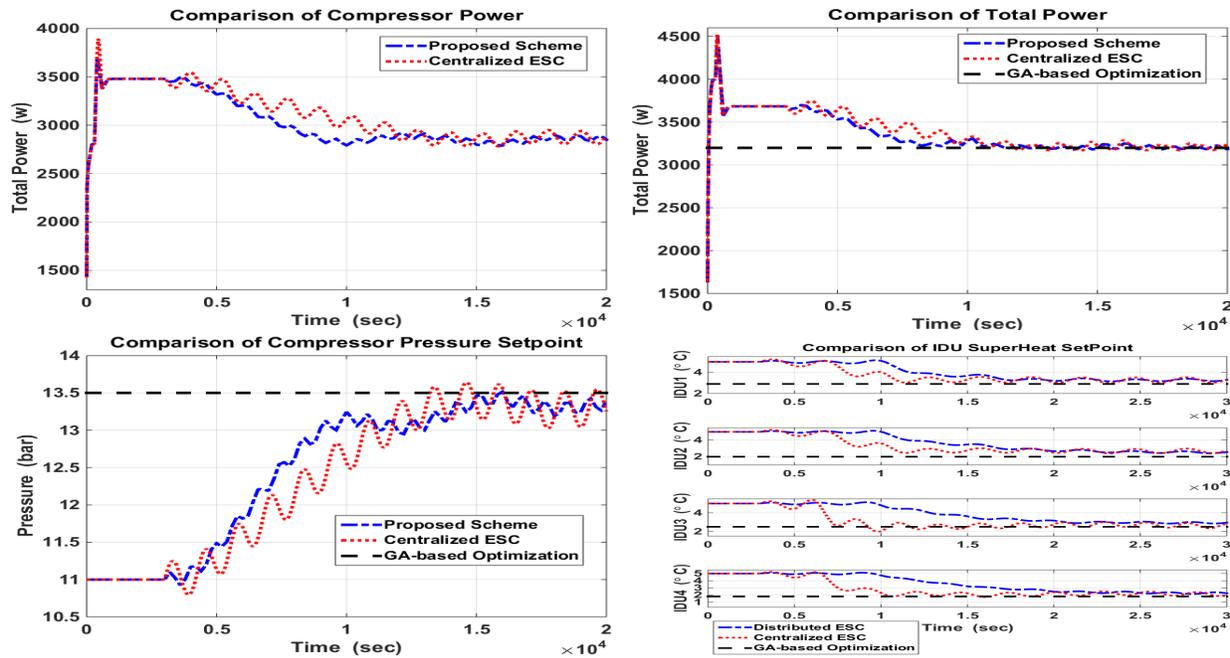


Figure 5: Total Power, Pressure and Superheat Setpoint of Proposed Scheme

a simulation based optimization procedure available in the Dymola Optimization Library. For the 4C mode, the genetic algorithm (GA) method finds the globally minimum total power at 3198 W, with  $P_{CS}$  at 13.5 bar and the SH values for IDU1 through IDU4 at 3°C, 2°C, 2.5°C and 1.8°C, respectively. On the other hand, simulation results of the distributed ESC scheme are shown in Fig. 5. ESC converges to an average total power of around 3200 W in steady state, with PCS around 13.4 bar, and the SH values of IDU1 through IDU4 at 3.2°C, 2.3°C, 2.2°C, and 1.9°C, respectively. The total power consumption of VRF 4C model was decreased from around 3700 W to around 3200 W, i.e. by 13.51. The steady-state error in comparison between the GA and the DESC method is smaller than 0.5. We also give a simulation comparison between the distributed ESC of Ye and Hu (2016) and the centralized ESC of Dong et al. (2016) in Fig. 6. It is apparent that the steady-state error in comparison between the centralized ESC and the DESC method is also small.

## 5. CONCLUSIONS

In this paper, a model-free distributed ESC strategy proposed by Ye and Hu (2016) is applied to optimize the IDU SH setpoint for the all-cooling mode of a four-zone VRF system. Full communication among the IDU agents is assumed. For each IDU agent, the feedback is the IDU fan power only, and the manipulative input is its evaporator SH setpoint. The ODU operation is optimized by a stand-alone ESC, for which the feedback is the ODU power, and the manipulative input is the compressor suction pressure setpoint. Simulation study is performed with a Modelica based dynamic simulation model of the VRF system, and the simulation results are compared with those obtained by the standard multi-input ESC and those by simulation based optimization using a genetic algorithm procedure provided



**Figure 6: Total Power, Pressure and Superheat Setpoint for Comparison**

by Dymola. It reveals that the distributed ESC can achieve comparable performance to the multi-input ESC scheme, and the steady-state performance is very close to that of GA optimization results. Work under way is to include the ODU as an additional agent so as to develop a complete distributed ESC solution to the VRF system operation. Also, the improvement of transient performance will be conducted.

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