ANALYSIS OF REGIONAL TRAVEL PATTERNS FOR A MEDIUM-SIZED COMMUNITY

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ABSTRACT

Tittemore, Lawrence Haylett. MSCE, Purdue University, August 1965. Analysis of Regional Travel Patterns for a Medium-Sized Community. Major Professor: J. C. Oppenlander

The purpose of this study was to determine the factors that generate highway travel between the central city of a region and its surrounding zone of influence. The central city, Fort Wayne, Indiana, depends on the people in the smaller communities to sustain its regional facilities. In the same manner, these smaller cities depend upon Fort Wayne for various needs. It is this interdependence that produces and attracts the traffic movements that were analyzed.

The two types of traffic flow analyzed in this investigation were trips attracted to and trips produced by the central city. The basic form of the regression equations was the gravity model; that is, trip production or attraction is directly proportional to the product of a given mass function of the two cities and inversely proportional to some power of the distance between the communities. Internal and external competition factors were introduced into the models to describe more completely the variations in regional trip generation and distribution.

The separation of trips into specific trip categories (work, shopping, social-recreational, and all-purpose) and travel type (produced and attracted) gave better estimation models. In addition, the division of the total study region into core and fringe areas demonstrated that the internal competition within a city had a negative effect on trip generation throughout the study region. However, the external competition of other cities was only significant in reducing trip generation for communities in the fringe area. A total of 24 statistical models was developed to describe the various arrangements of trip purpose, travel type, and area designation.

INTRODUCTION

In recent years many types of transportation studies have analyzed origin-destination data related to individual urban areas. These studies accomplish their purpose by reporting present urban transportation deficiencies. However, with the increased distance in travel that has accompanied improvements in both the means of travel and the highway system, the sphere of influence of the city is no longer confined to its geographic boundaries. Unfortunately, traffic studies have not kept pace with the broader aspect of regional travel. Perhaps this fact is due to a lack of knowledge concerning the significant factors influencing travel generation. When those factors underlying present travel motivation are better understood, more reliable estimates of future travel will be possible.

The traffic within an urban area flows from zones of production to zones of attraction. For example, each morning people travel from residential zones to zones of commercial and industrial activity. Each evening the flow is reversed with people returning to their homes. The community statistics which have been used to quantify this flow of traffic include population, employment, work-force size, wholesale and retail sales, car registration, and recreational facilities. This investigation makes use of this same concept of flow between zones with the exception that the zone is not part of the city but is the city itself. (See Figure 1)

Population and car registration are two statistics that have been generally selected to explain the magnitudes of regional traffic movements. There has been little attempt, however, to evaluate regional trip generation with analytical methods to discover the real factors that underlie the generation of vehicular trips. The relative importance of an urban community as a traffic generator and attractor is shown by its potential to attract trips from and to generate trips to primary, secondary, and tertiary communities.

The purpose of this research investigation was to study the intercommunity traffic linkages between Fort Wayne, Indiana, and surrounding communities in northeastern Indiana, northwestern Ohio, and southern Michigan. Estimation models were developed to predict both trip total and trip purpose interchanges between the various classes of urban centers.

Fort Wayne is linked to surrounding communities by the highway network. Over this network of roads, various communities of the region supply Fort Wayne with the people and goods which are necessary to maintain its position as a city of regional importance. These communities, in turn, depend on Fort Wayne for employment, educational, and

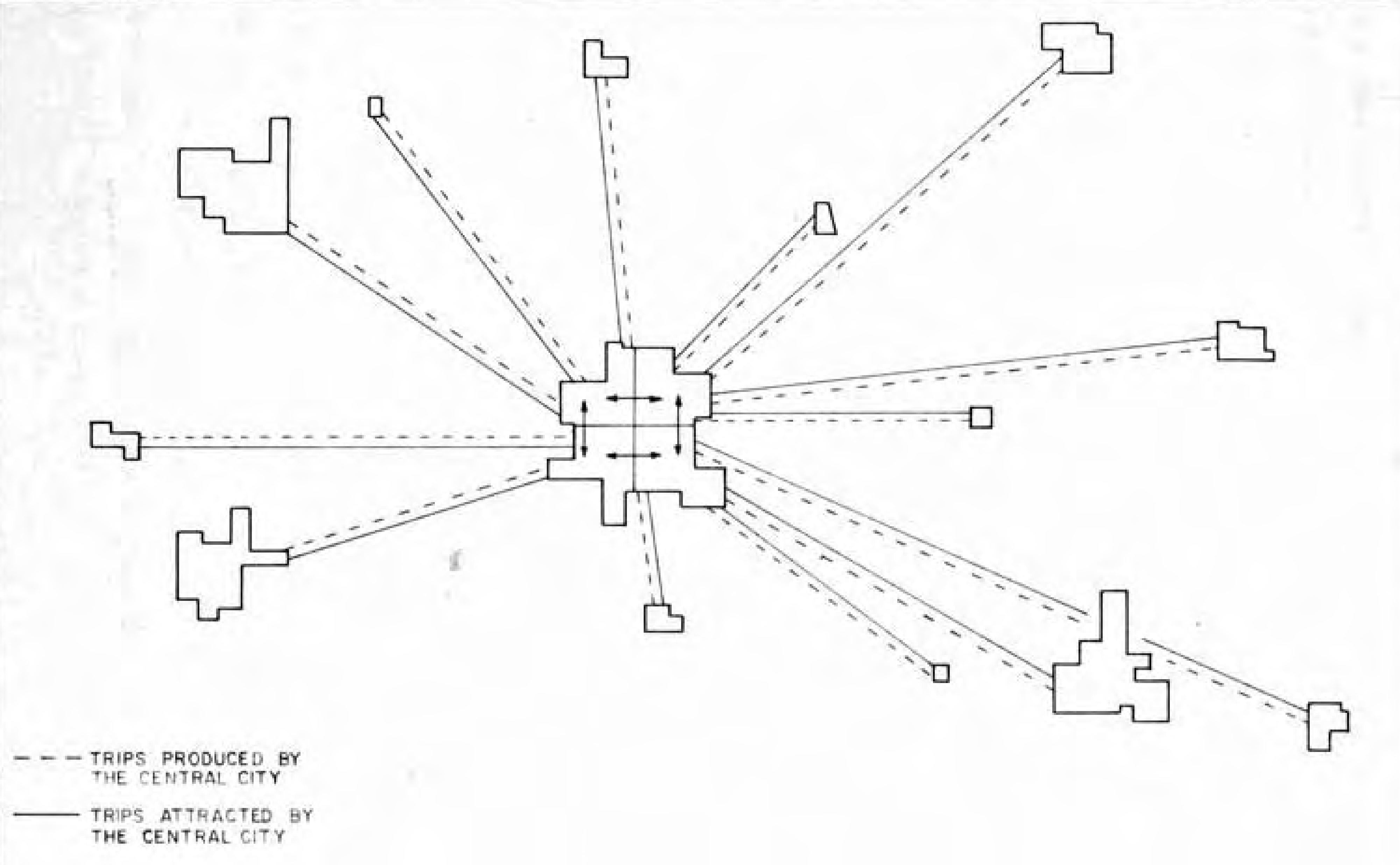

FIGURE I TRIP INTERCHANGES WITHIN A REGION religious opportunities and various recreational, shopping, and medical facilities. This interdependence produces the traffic movements that were analyzed. The daily movements between Fort Wayne and the surrounding communities were obtained as part of a comprehensive urban origin-destination study. These trip totals were evaluated to determine those factors which significantly influence the magnitude and direction of regional movements. The quantitative importance of these factors was also ascertained. This fact permits the prediction of regional traffic flow in other locations possessing similar characteristics.

The development and evaluation of mathematical models is most useful in highway planning. These models afford an opportunity to gain further understanding of the regional significance of highway linkages between central cities and outlying communities. Once this relationship is established for one region, it should be verified in other areas.

After models have been developed for various regions within the United States, regional travel patterns can be estimated on a nationwide basis. These estimates can then be used to determine the location of additional highways of interstate standard where they will best serve the desire for regional travel. Also, on a smaller scale, these traffic models provide a means of choosing between alternate locations for intercity highway routes to satisfy the demand for travel between cities.

Highway linkage findings offer quantitative information by which to determine the need for improving existing rural highways. These improvements are accomplished by the redesign and reconstruction of existing routes or the location of new facilities. Other applications are found in highway and route classification, determination of priorities, community analysis and planning, and the development of highway impact studies.

Mathematical models represent an economic and efficient manner of obtaining planning information needed for assigning functional uses to traffic facilities, defining adminstrative responsibilities, and determining financial policies in the construction and maintenance of highway systems.

Up to this point, the discussion has been based entirely upon the benefits to traffic engineering. However, the discipline of land-use planning will also gain a new tool. Highway facilities promote the development and growth of various forms of land-use. Therefore, regional planning benefits from a means of estimating the potential use that regional facilities will receive. Planners gain a method of measuring the dependence of a community on outside sources for the satisfaction of the needs of its residents. Both public and private interests will be able to appraise their competitive status for existing services or proposed new ones.

Studies designed to determine the impact of actual or proposed changes in local or regional facilities, interstate or primary highways, industrial development, shopping centers, centralized schools, etc., can make use of traffic models. Traffic patterns are flexible and bend with environmental and land-use changes. Therefore, these patterns serve to indicate the effects caused by a change in regional land-use.

REVIEW OF LITERATURE

During the past decade, several methods of forecasting travel patterns have met with varying degrees of acceptance. Although there is no clear agreement as to which method is best, the so-called gravity concept appears to have sufficient potential outside the urban engineering field to attract periodic attention.

Gravity Model Concept

The first known explicit statement of the gravity concept as applied to social behavior was proposed by H. C. Carey in the early 1800's. In this work, the fundamental law of gravitation was applied in explaining the social interaction between people.(4) The principle difficulty with this analogy is that man and molecules are basically different in nature. Man can make decisions on which to base his actions while the individual molecule is presumably not capable of rational thinking. The possession of this trait does not mean that the interactions of a large number of people cannot be described mathematically, but it must be remembered that people behave differently in groups than they do individually.

The gravity concept lay dormant until the late 1920's when E. C. Young used a modified form to measure migration. His proposal was that the number of people migrating to one destination from several sources was directly proportional to the force of attraction of the destination and inversely proportional to the square of the separation between the source and the destination.(48).

At approximately the same time, W. J. Reilly postulated his "Law of Retail Gravitation." This law states that a city attracts retail trade from individuals in the hinterlands in direct proportion to the population of the city and in inverse proportion to the square of the distance between the individual and the city. Between any two cities, which are competing for retail trade, there exists a point of equilibrium where their competitive influence is equal.(28) This point can be located by using the following relationship:

$$\frac{P_i}{(D_{ix})^2} = \frac{P_j}{(D_{jx})^2} \tag{1}$$

where P_i = the population of city i,

P_j = the population of city j,

x = the point of equilibrium on the line joining i and j,

D_{ix} = the distance from city i to point x, and

D_{jx} = the distance from city j to point x.

During the early 1940's, both J. Q. Stewart and G. K. Zipf developed similar hypotheses. They stated that the interaction between two population centers varies directly with the product of the masses of the two centers and inversely as the square of the distance between them.(35, 36,37,49,50,51,52) A mathematical representation of this relationship is:

$$Y_{ij} = k \frac{P_i P_j}{(D_{ij})^2} \tag{2}$$

where P_i = the population of city i,

P_j = the population of city j,

D_{ij} = the distance between city i and j,

k = a constant needed to correct for the difference in dimensions, and

Y_{ij} = the number of interchanges between city i and city j.

Effects of Population and Distance

The nature of the gravity concept lends itself to graphical representation. The total potential can be calculated at a series of points and plotted on a map of the study area. On these maps, areas of differing potential are apparent, and interrelationships between areas are visibly evident.

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There is general agreement among investigators that the attractive power of a city is proportional to its size.(5,6, 12,28,30) However, there is not complete agreement as to how the effect of distance should be represented.

J. D. Carroll has reported a technique by which to describe the magnitude of a city's influence on surrounding communities. In testing this theory, Carroll used both long distance telephone calls and intercity auto travel between 21 major cities located in southern Michigan. The power to which distance was raised ranged from 3.36 for calls from a neighborhood center to 2.84 for calls from a major regional center. The corresponding exponent for intercity travel was 2.98. Based upon these results, Carroll concluded that the influence of a city decreases according to the cube of the distance from that city.(5)

While attempting to explain the relationship between traffic volume, population, and distance, F. C. Ikle separated composite traffic into specific trip purposes so as to make the influence of population stand out more clearly. Ikle correctly believed that population would affect one type of trip more strongly than another. Empirical testing of automobile trips between Fort Wayne, Indiana, (or passing Fort Wayne) and counties in northern Indiana produced an exponent of 2.57 for the distance factor. Similar intercity travel characteristics were evidenced in the state of Washington, where an exponent of 2.6 was determined.(14)

Until the past few years, most researchers agreed that the function representing the attraction between communities was the product of their masses, although the community characteristic used to represent this mass differed from investigation to investigation. However, recent studies have introduced transformations of the mass term in order to more fully explain the interaction between people.

Willa Mylroie applied the gravity concept to explain the relative desire for travel between cities in the state of Washington. A square root transformation of population was used to represent the mass function, while the distance function was raised to the second power. (22)

As part of a project conducted at the University of Illinois, G. W. Greenwood developed multiple linear regression and factor analysis equations for travel in the region surrounding Champaign-Urbana, Illinois. The traffic totals used in this study were classified by trip purpose in order to better understand the basic factors underlying traffic generation. Prediction models were developed for trips attracted to and trips produced by the urban center of Champaign-Urbana. The region of analysis was divided into a core and a fringe area in order to obtain homogeneity in type of trips and in community statistics. The method used for dividing the region was similar to that developed by Carroll.

The results of this investigation showed that the division of trips into specific trip categories and the region into core and fringe areas made it possible to explain the effect of the mass function on travel more completely.(12)

While investigators are in general agreement that the attractive power of any city is proportional to its size, recent studies have questioned just what form this function takes. It is also evident that the influence of distance is not uniform. Rather, the exponent is a variable which depends upon unique characteristics of the situation under study.

A more detailed discussion of the gravity concept and its applications to traffic analysis is presented in Appendix A.

DESIGN OF STUDY

This section translates the overall theoretical concepts into useful traffic prediction models. The underlying theory for the development of mathematical models is explained as it applies to regional travel. The equations developed can possibly be extended to estimate nationwide travel patterns and provide quantitative planning information for evaluating the needs of the rural highway system.

Intercity Competition Concept

The gravitational concept hypothesizes that the attractive force of interaction between two cities is produced by the masses of the two cities and that the distance between the two cities acts as a deterrent to this interaction. This relationship can be stated as:

$$Y_{1-2} = \frac{M_1 \cdot M_2}{(D_{1-2})^{\alpha}} \qquad (3)$$

where Y_{1-2} = the total vehicle trips between area 1 and area 2,

M_1 = some measure of the traffic generating potential of area 1,

M_2 = some measure of the traffic generating potential of area 2,

D_{1-2} = the distance between area 1 and area 2, and

α = an exponent.

While equation 3 is applicable to the prediction of trip interchange between pairs of cities, areas, etc., it has serious limitations as a regional model. This model yields only total interchange and conceals any difference between the two areas as to their ability to produce or attract trips. Because communities do vary considerably in this ability, provision must be made to allow for these inherent differences. The basic form also implies that the two communities are in a state of isolation,that is, they are removed from the influence of other communities. These stipulations are not practical as the basis for a regional approach to the prediction of motor vehicle interchange because many communities of varying size and activity are located within a region.

Conversion of the Basic Model to One for Regional Analysis

The basic model can be converted to a regional model by changing the subscripts to denote whether Fort Wayne acts as the producer or the attractor of vehicular trips and by eliminating Fort Wayne's mass term from the basic equation for this term appears in all models. In this investigation, all trips are defined as one-way with either the origin or destination within the city of Fort Wayne.

Trips Produced by Fort Wayne

Subscript 'a' refers to any trip attracting community in the region and the subscript 'fw' indicates Fort Wayne as the producing community. The basic model now becomes:

$$(4) \qquad Y_{fw-a} = \frac{M_{fw} \cdot M_a}{(D_{fw-a})^{\alpha}}$$

Because Fort Wayne is the producer of trips to the cities throughout the study region, the index of productiveness of Fort Wayne (M_{fw}) is a constant term and can be dropped from the model. The inclusion of this term is simply equivalent to multiplying each community index (M_a's) by the same value. All indices are affected in the same proportion, and consequently the mass of Fort Wayne has no bearing upon the development of the regional model. The produced-trips model can now be written as:

$$(5) \qquad Y_{fw-a} = \frac{M_a}{(D_{fw-a})^{\alpha}}$$

Trips Attracted by Fort Wayne

The analogy for the situation in which Fort Wayne is the destination for trips which originate in the other population centers of the region is similar to that for the produced trips. Subscript 'p' refers to any trip producing community in the region and the subscript 'fw' indicates Fort Wayne as the attracting community. The following equation describes Fort Wayne as the attracting city and some outlying city as the producer. The model is written as:

$$(6) \qquad Y_{p-fw} = \frac{M_p \cdot M_{fw}}{(D_{p-fw})^{\alpha}}$$

Because the productiveness index of Fort Wayne is a constant term, it can be deleted from the equation. The regional model for attracted trips reduces to the following form:

$$(7) \qquad Y_{p-fw} = \frac{M_p}{(D_{p-fw})^{\alpha}}$$

Models (5) and (7) were developed by tailoring the basic concept of human interaction to fit a regional approach. These models, however, are only a transformation stage with no attempt to account for competitive forces.

Theory of Competition

Competition between communities is viewed as a type of pull or force similar to that implied in the basic gravity concept. The attraction with respect to the central city is negative and tends to reduce the amount of interchange between communities. This theory requires a system including both positive attraction and negative competition before the basic model can be applied on a regional basis.

The inhabitants of the study region have three alternatives when it comes to satisfying their needs and desires:

1. They can patronize the facilities within their own community.
2. They can make regional trips to major communities other than the central city.
3. They can make regional trips to the central city, Fort Wayne.

The hypothesis is made that factors contributing to the selection of one of the above alternatives are the result of competitive forces. The relative strength of competitive forces exhibited by the various communities must be evaluated and included within the prediction models as modifying factors affecting intercommunity travel.

The hypothesis is also made that the ability of one community to attract trips is dependent upon two different types of competition. The first, internal competition, is defined as a measure of the ability of the trip producing community to satisfy the desires of its inhabitants within its own boundaries. The second is external competition, which is defined as a measure of the ability of the other communities to vie with Fort Wayne for the chance to satisfy the people of the producing community.

Internal Competition

As previously defined, internal competition is the ability of the producing community to be self-sufficient. Greenwood describes an aerial view of the central city with radial lines extending outward, which link Fort Wayne to the other communities of the region. When Fort Wayne produces trips, traffic flows outward along these radials. Each movement symbolizes a produced trip, or when summed, a set of trips describing the pattern of trip production for Fort Wayne.

Internal competition is inherent to the trip <u>producing</u> city, therefore, the internal competition factor is a constant and not required because it affects all models in the same proportion. The above reasoning is not valid when visualizing trips <u>attracted</u> by the central city. In this case the traffic flow is inward along the radials and each outlying community was a potential producer. Thus, the internal competition factor was required by virtue of the fact that it was a variable which differed with each community.(12)

The extent of internal competition can only be discussed by relating it to individual trip purposes. For example, the greater number and type of shopping facilities within a community, the greater is the ability of the community to meet the needs of its inhabitants. The effect on regional travel is that adequate internal shopping facilities reduce the trips made to outlying communities.

With the addition of internal competition, equation (5) remains unchanged for produced-trips and equation (7) for attracted trips becomes:

$$(8) \qquad Y_{p-fw} = \frac{M_p}{(D_{p-fw})^{\alpha}} + C_p$$

where C_p = a measure of the ability of the producing community to satisfy its residents' needs.

External Competition

External competition is defined as a measure of the ability of major communities within the region to vie with each other and with the central city for the opportunity to serve the people in the trip producing community. As in the case of internal competition, the external competition factor was omitted as an explicit factor when dealing with trips <u>produced</u> by the central city. The effects of this element of competition are already included within the basic variables (M_a's) of the model. As long as consideration is focused on trips produced by the central city, all outlying cities are competing with one another to attract these trips. Therefore, the effectiveness of this competition depends on the value of the relevant attraction variables within these communities and the distance of these communities from Fort Wayne. Because external competition is implicitly incorporated within the basic variables of the produced-trip models, it is redundant to include it again as an explicit component.

The logic underlying the inclusion of external competition in the models for trips <u>attracted</u> to the central city is more obvious because this situation involves multiple trip-production sources and a single attractor. Every community of the region is a possible producer of trips, while the influence exerted by each competitor in the Fort Wayne area varied from one community to another. Therefore, external competition was explicitly incorporated as a variable within the attracted-trip models.

The inclusion of internal competition in the attracted-trip models and external competition in the produced-trip models necessitated the quantification of these variables. Internal competition was represented by those community statistics which described activities directly related to trip purpose.

On the other hand, external competition presented a different problem. Theoretically, there are an unlimited number of competing cities that exert influences upon the trip generation pattern of a producing community. Based on the results of a previous study, the decision was made to use only the competitor, c, with the highest probable value of the selected competition.(12)

Once the limits for competing cities had been set, an external competition factor was evaluated in three steps:

1. The major competitors within the study region were identified.
2. The communities under the influence of a common competitor were grouped together forming the competition zone of that competitor.
3. The external competition factor was quantified.

The following criteria were used in the identification of major competitors:

A. The competitor was in the sphere of influence of the central city.

B. The competitor had a population of 25,000 or more. The selection of population size was justified because population can be considered as a composite measure of all other indices.

C. The competitor had a minimum of ten trip interchanges per day between it and Fort Wayne.

D. The competitor with the highest probable value of competition was selected from among the major competitors. For a given producer, p, the competitor, c, was selected from among the following values:

$$(9) \qquad \frac{M_{c_i}}{(D_{c-p})_i}$$

where M_c = some measure of the attractiveness of the major competitor,

D_{c-p} = the distance between the outlying community and the major competitor, and

i = 1, 2, ... n - the major competitors.

The second step involved the establishment of competition zones. Population was selected as the most appropriate community index to define competition zones. Population is a general statistic allowing one configuration of zones to be used irrespective of which particular trip purpose was analyzed. In the actual calculations, the ratio of population to distance was used to define the competition zones.

The area dominated by Fort Wayne was first delimited. The procedure developed by J. D. Carroll and described in Appendix A was employed to define the zones of competition. The procedure involved the determination of where the retail trade influences between Fort Wayne and the competing cities were equal. The boundary of this zone was a set of lines representing the location of a state of equilibrium between Fort Wayne's influence and that of each competing city.

The distance from Fort Wayne to various points on the boundary of dominant influence was obtained by solving the following relationship:

$$\frac{P_{fw}}{D_i} = \frac{P_c}{D_{i-c}} \tag{10}$$

where D_i = the distance from Fort Wayne to the balance point on the boundary of the influence zone,

D_{i-c} = the distance between the same point on the boundary and the competing city,

P_{fw} = the population of Fort Wayne, and

P_c = the population of the competing city.

However, $D_{i-c} = D - D_i$, where D is the distance from Fort Wayne to the competing city. The previous relationship can now be rewritten as:

$$\frac{P_{fw}}{D_i} = \frac{P_c}{D - D_i} \tag{11}$$

or solving for D_i,

$$D_i = \frac{D\ (P_{fw})}{P_c + P_{fw}} \tag{12}$$

By evaluating this relationship between Fort Wayne and the competing cities, the zone of primary influence was located for Fort Wayne. The procedure was repeated to establish the influence zones for all the centrally-located major competitors in the study area.

Zonal boundaries were located by application of the population to distance relationship for adjacent pairs of non-centrally-located competing cities. Zonal boundary lines were extended toward Fort Wayne by computing the highest probable values of competition relative to the non-centrally-located-competing cities for several outlying communities. The boundaries were then located so that each community was within the zone of influence of the most probable competitor.

Two distinct levels of competitive influence were now defined for Fort Wayne - the core area in which Fort Wayne exerted the principle influence and the fringe area where the influence of the competing cities predominated. This division produced two areas that were each more homogeneous than the total area.

The final step was quantification of the external competition factor. For any given competitive zone, the effect of the major competitor was incorporated into the trip-attracted model by including a quantity measuring the competing city's attractiveness modified by the distance between the trip-producing city and the competing city.

After the study region was separated into Fort Wayne's core and fringe areas, it was necessary to evaluate the influence of external competition. Within the core area, the effect would either be measured as described above or neglected entirely. This latter alternative was predicated on the minor external competition exerted by the communities within Fort Wayne's zone or primary influence. The absence of external competition necessitated the application of a second condition to the concept of highest probable value of competition. In order for a community, p, to be considered under the influence of a competitor, c, the value of $(\frac{P_c}{D_{p-c}})$ would have to be greater than $(\frac{P_{fw}}{D_{p-fw}})$. That is, models for trips attracted to Fort Wayne contained an external competition factor only when the trip-producing communities fell outside the primary influence area of Fort Wayne. In the present study, the external competition factor was included in the analysis for the core area.

The addition of external competition to models (5) and (8) yielded:

For trips produced by Fort Wayne (no change)

(5) $$Y_{fw-a} = \frac{M_a}{(D_{fw-a})^{a}}$$

For trips attracted by Fort Wayne

(13) $$Y_{p-fw} = \frac{M_p}{(D_{p-fw})^{a}} + C_p + C_c$$

where C_c = a measure of the external competition produced by Fort Wayne's major competitor.

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The addition of external competition to models (5) and (8) yielded:

For trips produced by Fort Wayne (no change)

$$Y_{fw-a} = \frac{M_a}{(D_{fw-a})^{\alpha}} \tag{5}$$

For trips attracted by Fort Wayne

$$Y_{p-fw} = \frac{M_p}{(D_{p-fw})^{\alpha}} + C_p + C_c \tag{13}$$

where C_c = a measure of the external competition produced by Fort Wayne's major competitor.

A summary of competition forces, with the reasons for inclusion or exclusion of competition factors is given in Table 1.

TABLE 1

REVIEW OF COMPETITION FORCES

I. Internal Competition.

A) Produced Trips - internal competition is a constant term and therefore omitted.

B) Attracted Trips- an explicit factor must be included to measure internal competition.

II. External Competition.

A) Produced Trips - external competion is included within the mass variables.

B) Attracted Trips- an explicit factor must be included to measure external competition.

Measures of Separation

Three measures that can be used to quantify the separation between communities are highway distance in miles, travel time in minutes, and travel cost in dollars. The total cost of travel would be an excellent way of weighting the distance factor, for travel cost reflects many considerations important to the traveler. However, travel cost is extremely difficult to assign a numeric value because of intangibles such as comfort and convenience incorporated within it. Travel time is also a measure of the friction against travel and is particularly appropriate within congested areas where it gives relative weights to alternate routes.

While distance alone does not consider all the factors that affect travel, this measure was selected for use in this study. The rural highways within the study region have approximately the same travel characteristics so that the selection of a route based solely on miles of travel was an adequate criterion. The shortest possible highway distance was used in the analysis of all travel linkages.

The Selection of Community Statistics

This section is devoted to the selection of community statistics used to measure the ability of a community to produce or attract trips. Population and employment totals are reported in persons while sales totals are in thousands of dollars. The trip-produced categories employed and the variables used to measure each category were:

1. Work trips.
   a. Population.
   b. Total employment in a community.
   c. Employment in the construction industry.
   d. Employment in the manufacture of durable goods.
   e. Employment in the manufacture of non-durable goods.

difficult to assign a numeric value because of intangibles such as comfort and convenience incorporated within it. Travel time is also a measure of the friction against travel and is particularly appropriate within congested areas, where it gives relative weights to alternate routes.

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    c. Employment in the construction industry.
    d. Employment in the manufacture of durable goods
    e. Employment in the manufacture of non-durable goods.

f. Employment in transportation, communication, and other utilities.

g. Employment in wholesale and retail trade.

h. Employment in finance, insurance, and real estate.

i. Employment in business and repair services.

j. Employment in personal services.

k. Employment in entertainment and recreational services.

l. Employment in public administration.

m. Employment in agriculture and related services.

2. Shopping trips.

a. Population.

b. Total sales in a community.

c. Wholesale sales in a community.

d. Sales by merchant wholesalers.

e. Retail sales in a community.

f. Sales in lumber, building materials, hardware, and farm equipment.

g. Sales in the general merchandise group.

h. Sales in the food group.

i. Sales by automotive dealers.

j. Sales by gas stations.

k. Sales by apparel and accessory stores.

l. Sales by furniture, home furnishings, and equipment stores.

m. Sales by eating and drinking places.

n. Sales by drug and proprietory stores.

3. Social-recreational trips.

a. Population.

b. Employment in wholesale and retail trade.

c. Employment in entertainment and recreational services.

d. Retail sales in a community.

e. Sales by eating and drinking places.

f. Acres of recreational open space in the community.

4. All-purpose trips.

a. Population.

b. Total employment in a community.

c. Employment in the construction industry.

d. Employment in the manufacture of durable goods.

e. Employment in the manufacture of non-durable goods.

f. Employment in transportation, communication, and other utilities.

g. Employment in wholesale and retail trade.

h. Employment in finance, insurance, and real estate.

i. Employment in business and repair services.

j. Employment in personal services.

k. Employment in entertainment and recreational services.

l. Employment in professional and related services.

m. Employment in public administration.

n. Employment in agriculture and related services.

o. Total sales in a community.

p. Wholesale sales in a community.

q. Sales by merchant wholesalers.

r. Retail sales in a community.

s. Sales in lumber, building materials, hardware, and farm equipment.

t. Sales in the general merchandise group.

u. Sales in the food group.

v. Sales by automotive dealers.

w. Sales by gas stations.

x. Sales by apparel and accessory stores.

y. Sales by furniture, home furnishings, and equipment stores.

z. Sales by eating and drinking places.

aa. Sales by drug and proprietory stores.

bb. School enrollment (kindergarten, elementary, secondary, and college).

cc. Number of hospital beds in a community.

dd. Acres of recreational open space.

The wide range of activities that generate the social-recreational trip make it necessary to select variables which explain the two classes of recreation - social and commercial. In this study, overall social activity was described by population size. The measurement of commercial recreation is complicated because the establishments which produce this form of recreation are heterogeneous in terms of activities, patronage, and physical size. The measure of dollar sales was considered but a serious limitation was inherent to this statistic for a large portion of the facilities did not have reportable sales. As a result, employment was the only other index available.

The characteristics of the work, shopping, and social-recreational trip are sufficiently described by their names alone. However, the all-purpose was a special category comprised of all trips between Fort Wayne and the other communities in the study region.

Certain variables employed as measures of trip-production and attraction can also be used to describe competition. External competition was evaluated by the population of the competing city. However, for internal competition the index differs with each trip category. The explicit variables used with attracted-trip models were total employment for work trips, retail sales for shopping trips, acres of recreational open-space for social-recreational trips, and population for all-purpose trips. For produced-trip models, internal competition is implicit in the measurement variables themselves as these variables also described the attractiveness of a community to its own residents.

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There is a distinction to be made concerning the dual function of community variables. The magnitude of the variables can both increase and decrease the generation of regional travel. For example, a large employment center attracts workers from the hinterland, thus producing regional travel. On the other hand, people within the community where the industry is located can also find employment, reducing the trip making potential of this community. Therefore, certain variables can have both a positive and negative effect on the generation of regional trips.

Criteria for Selecting a Model

The choosing of the regression equation with the highest correlation coefficient is not, in itself, sufficient criteria for assuming the adequacy of a model. Logical requirements exist which must be met in addition to statistical requirements.

The logical requirements selected in this investigation demand that the model pass a critical appraisal to assure the relationship represented is reasonable in terms of competition theory. Statistical requirements caution against complete reliance on correlation coefficients. Strict adherence may bias the results toward those variables or transformations that provide the "best" fit and away from those basic variables which truly influence traffic movements. This phenomenon occurs because an independent variable, which initially showed little correlation with trip data, may show significant correlation after considering its relationship to the other independent variables. The selection of the model was based both on a priori considerations of competition and on examination of inter-correlations between proposed independent variables.

Linear regression coefficients are subject to sampling variations. The setting of confidence limits for the true value of a particular regression coefficient or to determine whether the computed coefficients were possibly equal to zero is frequently desirable. The "Student t" was used to test the hypothesis that the true value of the regression coefficient is zero. The formula for the "t" test is: (47)

$$t = \frac{b - u}{T_b} \tag{13}$$

where b = the observed value of the regression coefficient,

u = the true value of the regression coefficient, and

T_b = S_b/n = the standard error of the regression coefficients.

Mathematical Models

The development of functional relationships between the dependent variables and the independent variables involved the formulation of mathematical models. The type of linear regression equations developed was:

$$Y = a + b_1 X_1 + b_2 X_2 + \ldots + b_n X_n \tag{14}$$

where Y = predicted mean dependent variable,

a = intercept,

b = regression coefficient,

X = independent variable, and

n = number of independent variables.

The multiple linear regression equation for each trip category was developed under the assumption that the sample data were randomly selected from normal populations. It was also assumed that homogeneity of variance existed for the study variables.

Summary

The basic gravity concept was not considered as applicable to the prediction of travel interchange within a region. The tailoring of this concept to fit a regional viewpoint required the separation of total trips into those trips attracted to the central city and those produced by it. This division also eliminated the need to include indices for the measurement of trip generation by the central city as such indices were constant terms.

Models were then developed on the theory that the ability of any community to attract trips from another community was governed by two types of competition. One type was internal competition--the ability of the trip producing community to fulfill the desires of its residents. The other type was external competition--the measure of the ability of other communities, excluding the attracting city, to vie for the chance to satisfy the people of the trip producing community.

The application of this theory produced the following models:

For trips produced by Fort Wayne:

$$Y_{fw-a} = \frac{M_a}{(D_{fw-a})^{\alpha}} \tag{5}$$

where M_a = a measure of the attractiveness of the outlying community,

D_{fw-a} = the distance between the community and Fort Wayne, and

α = an exponent.

For trips attracted by Fort Wayne:

(12) $$Y_{p-fw} = \frac{M_p}{(D_{p-fw})^{\alpha}} + C_p + C_c$$

where M_p = a measure of the productiveness of the outlying community,

D_{p-fw} = the distance between the community and Fort Wayne,

α = an exponent,

C_p = a measure of the ability of the producing community to satisfy its residents' needs, and

C_c = a measure of the external competition produced by Fort Wayne's major competitor.

For trips attracted by Fort Wayne:

(12) $$Y_{p-fw} = \frac{M_p}{(D_{p-fw})^{\alpha}} + C_p + C_c$$

where M_p = a measure of the productiveness of the outlying community,

D_{p-fw} = the distance between the community and Fort Wayne,

α = an exponent,

C_p = a measure of the ability of the producing community to satisfy its residents' needs, and

C_c = a measure of the external competition produced by Fort Wayne's major competitor.

PROCEDURE

This chapter describes the schema used in data collection and analysis. The city of Fort Wayne, Indiana was used as the central city with portions of Indiana, Michigan, and Ohio included in the study region.

Data Collection

Origin-destination data were collected by the Indiana State Highway Commission as part of a traffic study of Fort Wayne, Indiana. The interviews were taken during the month of June 1961. From the original field sheets, trips were broken down into two categories - those originating in Fort Wayne and those having Fort Wayne as a destination. In addition, the other trip terminal was recorded.

Almost 90 percent of the trips were found to lie within an area comprising northeastern Indiana, northwestern Ohio, and southern Michigan. This area was designated as the sphere of influence for Fort Wayne. (See Figure 2) Only those trips having both origins and destinations within this region were evaluated in this analysis of regional travel patterns. To provide reasonable values of travel interchange, a community had to have a total of ten or more interchanges

FIGURE 2. THE REGION UNDER THE SPHERE OF INFLUENCE OF FORT WAYNE, INDIANA

with Fort Wayne. The remaining total of 156 communities, 97 in the core area and 59 in the fringe area, represented about 85 percent of the trips in the origin-destination data. These cities are listed in Appendix B. The total travel data used in the regression analysis were 20,020 one-way trips of which 45 percent were work, 16 percent were shopping, and 11 percent were social-recreational. The remaining 28 percent were only included in the all-purpose category.

The final step was to categorize the trips by purpose; i.e., work, shopping, social recreational, and all-purpose trips. The four trip type totals made up the dependent variables used in the regression analysis.

Community statistics necessary for developing the regression equations were collected from the U.S. Census of Population; 1960, Vol. I, Characteristics of the Population (37,38,39), the U.S. Census of Business; 1958, Vol. II, Retail Trade-Area Statistics, parts 1 and 2 (40), the U.S. Census of Business; 1958, Vol. IV, Wholesale Trade-Area Statistics (41) the U.S. Census of Business; 1958, Vol. VI, Selected Service-Area Statistics (42), and various state publications. (13,18,21,36) The population census figures were taken as reported for 1960, and the figures from the business census were projected from 1958 to 1961.

When all the necessary community statistics had been entered on summary sheets, the information was punched on IBM data cards suitable for use on the IBM 7094 Computer.

Data Analysis

Preliminary Data Analysis

Before the regression and correlation analysis was performed, the study region was divided into core and fringe areas according to J. D. Carroll's "tent" method of describing urban trade areas. (5) The cities competing with Fort Wayne met the competition criteria and are summarized in Table 2. Once the competing cities were selected, the relationship,

$$(12) \qquad D_i = \frac{D(P_{fw})}{P_c + P_{fw}}$$

where D_i = the distance from Fort Wayne to any point on the boundary of the influence zone,

D = the distance from Fort Wayne to the competing city,

P_{fw} = the population of Fort Wayne, and

P_c = the population of the competing city,

was solved to obtain Fort Wayne's zone of primary influence. This relationship was also used to delimit the zones of influence for the competing cities. (See Figure 3) A more complete description of this procedure was developed in Chapter 3, Design of Study.

FIGURE 3. COMPETITION ZONES WITHIN THE STUDY REGION

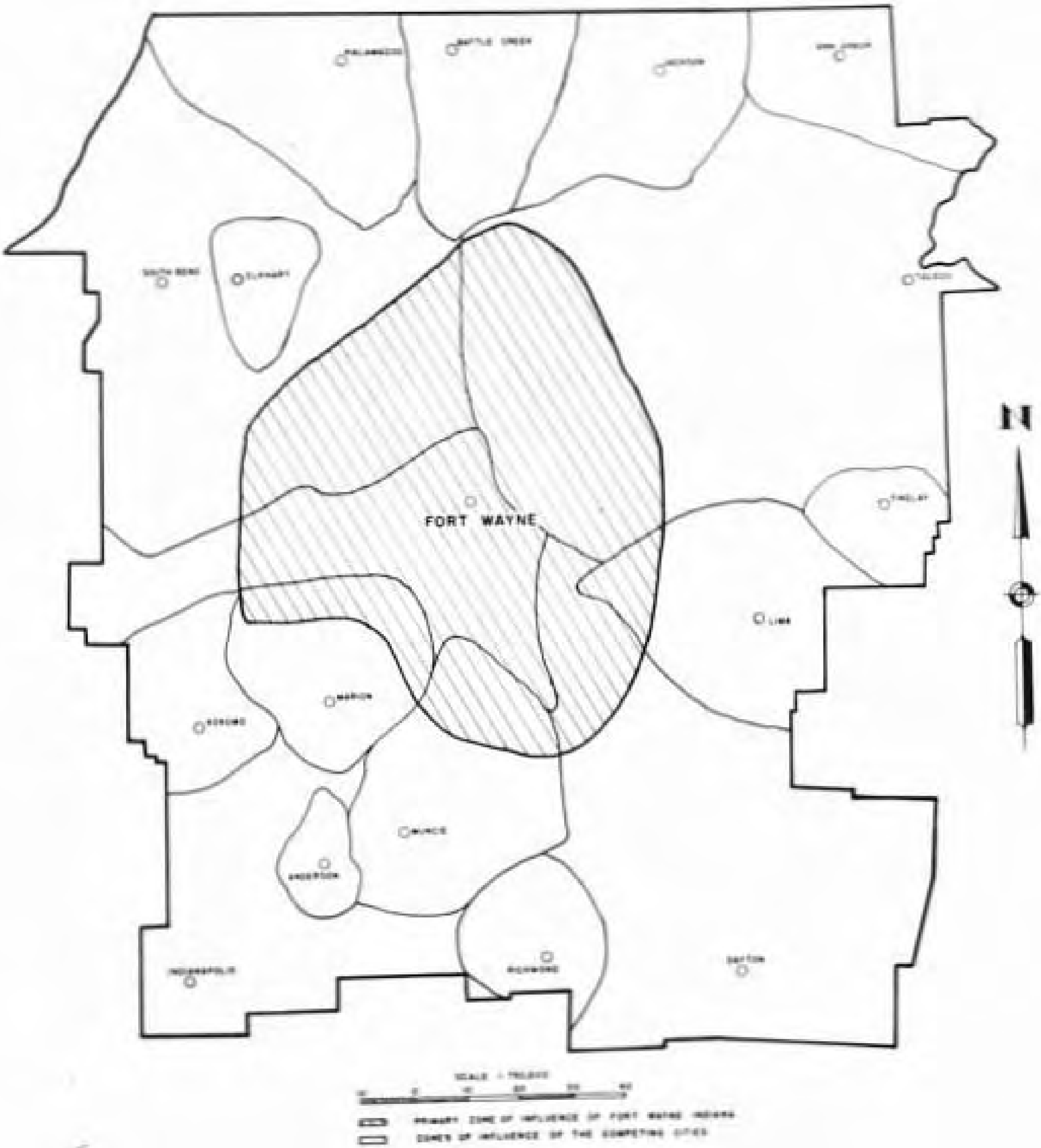

TABLE 2

MAJOR COMPETING CITIES

State	City	Population	Distance from Fort Wayne	Trips
Indiana	Anderson	165,806	79	24
	Elkhart	37,854	66	75
	Indianapolis	476,258	118	220
	Kokomo	47,197	82	28
	Marion	40,274	50	177
	Muncie	68,603	67	124
	Richmond	44,149	92	30
	South Bend-Mishawaka	165,806	90	170
Michigan	Ann Arbor	75,000	142	10
	Battle Creek	44,169	94	12
	Jackson	50,720	110	10
	Kalamazoo	82,089	113	14
Ohio	Dayton-Kettering	316,794	115	36
	Findley	30,344	90	17
	Lima	51,037	64	74
	Toledo-Maumee	330,066	108	82

Basic Procedure of Model Evolution

In the model evolution phase, traffic generation models were developed by specific trip purposes. The first decision in this development, involved the use of either the composite or the individual variable approach. For illustrative purposes, a general expression for trips produced by Fort Wayne is discussed. The composite variable form is:

$$(15) \qquad Y_p = F(X)$$

$$\text{where} \quad X = \frac{M_a}{D_{fw-a}}$$

The independent variables, which measure the attractive power of the community and the distance from Fort Wayne to the attractor, were combined into a single variable. This form required a single regression coefficient for each combination and was the one used in the development of the regression equations.

The individual variable approach differs because each variable retains its own identity. This approach takes the form:

$$(16) \qquad Y_p = F(M_a; D_{fw-a})$$

where M_a = the measures of the attractive force, and

D_{fw-a} = the distance from Fort Wayne to the attractor.

The desirable solution is that approach which measures the maximum information obtainable from each variable and which does the most efficient job as an estimation model. However, although the individual approach is intuitively the one to employ, empirical testing showed that the composite variable approach provided the better estimation model. Because an objective of this study was to develop models capable of producing efficient estimations of travel, the composite approach was chosen.

The Selection of Variables

The community variables, as listed in Chapter 3, were used to measure the attractiveness and productiveness of hinterland communities. Following a review of theoretical and empirical literature in the field of traffic linkage, the most commonly adopted mathematical transformations were found to be logarithms, roots, and powers. Therefore, the following forms for the variables were chosen:

1. For the dependent variable - linear, and
2. For the independent variables:
   a. For measures of community attractiveness and productiveness - linear and square root.
   b. For the distance variable - square root, and

first, second, third, and fourth powers.

Different combinations of these variables were tested by correlation with the trip data.

Application of Intercommunity Competition Theory

Because the produced-trip models required no explicit competition factors, only the regression coefficients for the variables were required to validate the equation. On the other hand, the models depicting attracted trips required the evaluation of the external and internal competition factors, in addition to the basic variables. Population of the competing city divided by the distance separating the competing city from the producing city was chosen as the external competition factor. The internal competition factor, however, varied with the trip category under consideration. With all-purpose trips, this factor was the population of the producing city divided by the distance from the center of the city to its periphery. For the work trip, shopping trip, and social-recreational trip categories, population was replaced by total employment, retail sales, and recreational open-space, respectively.

Multiple Linear Regression and Correlation Analysis

A build-up regression procedure was used to evaluate the composite variables selected for this investigation. (33)

The first step in the analysis was the calculation of a correlation matrix for the study variables. Certain variables which formed a common group were deleted to avoid singularities. The variable having the smallest product-moment correlation with the dependent variable was removed from each group.

At each step one variable was removed from or added to the multiple regression equation. In the case where a variable was added, this added variable made the greatest reduction in the error sum of squares. Equivalently, it was the variable which had the highest partial correlation with the dependent variable partialed on the variables which had already been added, that is, the variable which had the highest F value.

In the computational procedure an "F-to-remove-value" for each independent variable in the regression equation and an "F-to-enter-value" for each independent variable not in the equation were computed at each step. Independent variables were added or deleted in accordance with the following criteria:

1. If one or more independent variables in the regression equation had an F value less than the critical "F-to-remove-value" specified, then the variable with the smallest F value was removed;

2. If no variable was removed and one or more independent variables which were not in the regression equation passed the tolerance test, the variable with the highest F value was added; and
3. The process was terminated when no variable was added or deleted.

The values of 0.01 and 0.005 were set as F levels for inclusion and deletion of independent variables, respectively. In regard to criterion no. 2, an independent variable which was not in the regression equation passed the tolerance test if its tolerance value was equal to or greater than the minimum specified value of 0.001.

Model-Selection Criteria

Model selection was based largely on the form that produced the highest correlation with the trip data. However, the choosing of the equation with the highest correlation coefficient was not the only criterion for assuming the adequacy of the model.

The following requirements were used in the determination of variables for inclusion in the multiple linear regression equations:

1. A coefficient of determination of 0.50 or greater was desired;
2. The final form of the model contained as few variables as possible while still performing its

TABLE 6
WORK TRIP MODELS, PRODUCED AND ATTRACTED

Model No.	Trip Type	Independent Variable Form	Regional Trip Models
17	Produced Work Trip Total Area	$\frac{P_{ij}}{D_j}$	$Y_{Pw-a} = 5.45 - 107488X_2^* - 1663948X_3^* - 842908X_5^*$ $+ 3527468X_9^* + 2605298X_{10}^* + 817368X_{12}^*$ $+ 2536648X_{13}^*$
18	Attracted Work Trip Total Area	$\frac{P_{ij}}{D_j}$	$Y_{p-Pw} = 5.01 + 491638X_7^* - 3X_8^* + 17X_{11}^*$ $- 6922X_{13}^* + 2507X_{14}^* - 0.00068X_{31}^*$
19	Produced Work Trip Core Area	$\frac{P_{ij}}{D_j}$	$Y_{Pw-a} = 4.14 - 63688X_3^* - 3067X_5^* - 3308X_6^*$ $- 25708X_7^* + 18644X_9^* + 212158X_{10}^* + 3883X_{12}^*$ $+ 3638X_{13}$
20	Attracted Work Trip Core Area	$\frac{P_{ij}}{D_j}$	$Y_{p-Pw} = 3.03 + 111X_2^* - 375X_3^* - 555X_5^* - 6X_6$ $+ 60943X_7^* - 0.1X_8^* + 1X_{11}^* + 270X_{12}^*$ $- 1555X_{13}^* + 305X_{14}^* - 0.00176X_{31}^*$
21	Produced Work Trip Fringe Area	$\frac{P_{ij}}{D_j}$	$Y_{Pw-a} = 1.69 - 883712X_4^* - 715688X_5 + 1039688X_7$ $+ 3542968X_8^* + 12955538X_9^* - 12798508X_{11}^*$ $- 2204912X_{13}^*$
22	Attracted Work Trip Fringe Area	$\frac{P_{ij}}{D_j}$	$Y_{p-Pw} = 2.96 + 128812X_3 - 1360448X_4^* - 2045548X_5^*$ $+ 5246608X_8^* - 35553558X_9^* - 5091168X_{10}^*$ $- 12798838X_{11}^* - 3863968X_{13}^* - 0.00003X_{31}^*$

* Significant at the 5-percent level

TABLE 5
SHOPPING TRIP MODELS, PRODUCED AND ATTRACTED

Model No.	Trip Type	Independent Variable Form	Regional Trip Models
23	Produced Shopping Trip Total Area	$\frac{P_i A_j}{D_{ij}}$	$Y_{fw-a} = 2.10 + 1439X_1^* + 1823X_{15}^* + 387X_{17} - 785X_{19} - 9934X_{20}^* - 10518X_{22}^* + 75636X_{23}^* - 17323X_{24} + 6401X_{25} + 77445X_{26}^* - 1900X_{27}^*$
24	Attracted Shopping Trip Total Area	$\frac{\sqrt{P_i A_j}}{D_{ij}}$	$Y_{p-fw} = 6.97 - 565036X_{22}^* + 1147673X_{23}^* + 1299193X_{24}^* - 1633908X_{27}^* - 0.00154X_{31} - 0.00337X_{32}$
25	Produced Shopping Trip Core Area	$\frac{P_i A_j}{D_{ij}}$	$Y_{fw-a} = 2.85 + 1604X_1^* + 2319X_{15}^* + 560X_{17} + 181X_{19} - 10570X_{20}^* - 21450X_{22}^* + 30034X_{23}^* - 17173X_{24}^* + 33575X_{26} - 3033X_{27}^*$
26	Attracted Shopping Trip Core Area	$\frac{P_i A_j}{D_{ij}}$	$Y_{p-fw} = 7.45 + 311X_1^* + 205X_{15} - 74X_{16}^* + 383X_{20} - 1292X_{22}^* + 778X_{23} - 2213X_{24}^* - 1047X_{25} - 341X_{27}^* - 0.00011X_{31}$
27	Produced Shopping Trip Fringe Area	$\frac{\sqrt{P_i A_j}}{D_{ij}}$	$Y_{fw-a} = -1.49 + 518X_{15}^* - 86X_{16} - 353X_{18} - 493X_{19}^* - 835X_{20}^* - 707X_{21}^* + 402X_{25}^* + 1380X_{26}^* + 175X_{27}^*$
28	Attracted Shopping Trip Fringe Area	$\frac{P_i A_j}{D_{ij}}$	$Y_{p-fw} = 0.44 + 1012X_{17}^* + 21157X_{18}^* - 49555X_{19} + 14135X_{21} - 78572X_{23}^* + 136574X_{25}^* - 35451X_{26} - 0.00001X_{32}^*$

* Significant at the 5-percent level

TABLE 8
SOCIAL-RECREATIONAL TRIP MODELS, PRODUCED AND ATTRACTED

Model No.	Trip Type	Independent Variable Form	Regional Trip Models
29	Produced Social-Recreational Trip Total Area	$\frac{\sqrt{X_i}}{D_{ij}}$	$Y_{fw-s} = 7.48 - 4773X^{*}_{1} - 778699X^{*}_{11} + 505576X^{*}_{25}$ $+ 2627X^{*}_{30}$
30	Attracted Social-Recreational Trip Total Area	$\frac{X_i}{D_{ij}}$	$Y_{s-fw} = 3.98 + 13X^{*}_{1} - 7427X^{*}_{11} + 16X_{21}$ $+ 712X^{*}_{25} + 104X^{*}_{30} - 0.05796X^{*}_{31}$ $- 0.00002X^{*}_{32}$
31	Produced Social-Recreational Trip Core Area	$\frac{\sqrt{X_i}}{D_{ij}}$	$Y_{fw-s} = 7.90 - 734X_{1} + 512X_{7} - 11368X_{11}$ $+ 1077X^{*}_{25} + 179X^{*}_{30}$
32	Attracted Social-Recretational Trip Core Area	$\frac{X_i}{D_{ij}}$	$Y_{s-fw} = 7.39 + 954X_{1} - 18909X_{7} - 92810X^{*}_{11}$ $- 19952X^{*}_{25} + 1194X^{*}_{30} + 0.00110X^{*}_{31}$
33	Produced Social-Recreational Trip Fringe Area	$\frac{\sqrt{X_i}}{D_{ij}}$	$Y_{fw-s} = -0.74 - 531250X^{*}_{1} + 812470X^{*}_{7} - 1049018X_{11}$ $+ 1376720X^{*}_{25} + 1209770X^{*}_{30}$
34	Attracted Social-Recreational Trip Fringe Area	$\frac{X_i}{D_{ij}}$	$Y_{s-fw} = 0.49 - 50X_{1} + 1260X^{*}_{7} - 4775X_{11} - 36X_{25}$ $+ 2886X^{*}_{30} - 0.02996X^{*}_{31} - 0.00001X^{*}_{32}$

* Significant at the 5-percent level

TABLE 7
ALL-PURPOSE TRIP MODELS, PRODUCED AND ATTRACTED

Model No.	Trip Type	Independent Variable Form	Regional Trip Models
55	Produced All-Purpose Trip Total Area	$\frac{\sqrt{R_i}}{B_i}$	$Y_{fw-s} = -9.47 - 135790X^*_{11} - 8815X^*_{15} + 47532X^*_{21} + 37544X^*_{23} - 5X_{25} + 9518X^*_{28} + 5605X^*_{30}$
56	Attracted All-Purpose Trip Total Area	$\frac{R_j}{B_j}$	$Y_{p-fw} = 12.29 - 72318X^*_{2} - 3705X_{3} + 7715X_{8} + 480455X^*_{10} + 8098X^*_{14} + 9935X^*_{15} - 815X_{18} - 34037X^*_{19} - 20666X^*_{20} + 5945X_{21} + 4043X^*_{25} + 276X^*_{28} - 8707X^*_{29} - 0.000011X^*_{31}$
57	Produced All-purpose Trip Core Area	$\frac{\sqrt{R_i}}{B_i}$	$Y_{fw-s} = -7.58 - 177136X^*_{11} - 6308X^*_{19} + 3940X^*_{21} + 4912X^*_{23} + 2900X_{24} + 5455X^*_{28} + 526X^*_{29}$
58	Attracted All-purpose Trip Core Area	$\frac{R_j}{B_j}$	$Y_{p-fw} = 24.53 - 483308X^*_{3} + 4669608X^*_{5} + 14393429X^*_{10} - 5011958X^*_{11} + 20081608X^*_{13} - 1376578X^*_{19} - 1094713X^*_{20} + 1023718X^*_{21} + 2713028X^*_{23} - 888772X^*_{25} - 227615X^*_{27} + 27325X^*_{28} + 2773X_{30}$
59	Produced All-purpose Trip Fringe Area	$\frac{\sqrt{R_i}}{B_i}$	$Y_{fw-s} = -5.29 + 858288X_{3} - 1649488X_{4} + 5347636X^*_{5} + 7900X^*_{16} - 3157032X^*_{20} + 1306415X^*_{21} + 3174788X^*_{25} + 3657588X^*_{26} - 1423795X^*_{28} - 3226758X^*_{29} + 3542695X^*_{30}$
60	Attracted All-purpose Trip Fringe Area	$\frac{R_j}{B_j}$	$Y_{p-fw} = -3.34 - 224465X^*_{1} - 1527558X^*_{4} + 32949048X^*_{5} - 10267688X^*_{13} + 5891279X^*_{14} + 1460081X^*_{25} + 4735008X^*_{26} - 5619233X^*_{29} + 2821648X^*_{30} - 0.000053X^*_{31}$

* Significant at the 5-percent level

TABLE 8
SUMMARY STATISTICS OF THE REGRESSION EQUATIONS

Trip Type	Independent Variable Form	Coefficient Of Correlation	Coefficient Of Determination	Standard Error of the Estimate
<u>Attracted Work Trips</u>				
18 Total Area	K/D^3	0.87	0.76	15.4
20 Core Area	K/D^2	0.91	0.83	16.3
22 Fringe Area	K/D^4	0.87	0.76	8.8
<u>Produced Work Trips</u>				
17 Total Area	K/D^4	0.87	0.76	11.9
19 Core Area	K/D^3	0.90	0.82	17.8
21 Fringe Area	K/D^4	0.88	0.77	6.0
<u>Attracted Shopping Trips</u>				
24 Total Area	$\sqrt{K}/D^4$	0.84	0.70	13.7
26 Core Area	K/D^3	0.86	0.75	15.4
28 Fringe Area	K/D^5	0.87	0.76	4.8
<u>Produced Shopping Trips</u>				
23 Total Area	K/D^4	0.74	0.54	5.9
25 Core Area	K/D^4	0.75	0.56	7.0
27 Fringe Area	$\sqrt{K}/D^2$	0.83	0.69	7.5
<u>Attracted Social-Recreational Trips</u>				
30 Total Area	K/D^3	0.82	0.67	6.0
32 Core Area	K/D^4	0.86	0.74	6.5
34 Fringe Area	K/D^3	0.83	0.69	7.1
<u>Produced Social-Recreational Trips</u>				
29 Total Area	$\sqrt{K}/D^3$	0.66	0.43	11.0
31 Core Area	$\sqrt{K}/D^2$	0.73	0.53	10.6
33 Fringe Area	$\sqrt{K}/D^3$	0.95	0.91	3.9
<u>Attracted All-Purpose Trips</u>				
36 Total Area	K/D^3	0.90	0.81	60.3
38 Core Area	K/D^4	0.92	0.85	51.2
40 Fringe Area	K/D^4	0.97	0.94	11.0
<u>Produced All-Purpose Trips</u>				
35 Total Area	$\sqrt{K}/D^2$	0.87	0.76	47.4
37 Core Area	$\sqrt{K}/D^3$	0.89	0.79	56.0
39 Fringe Area	$\sqrt{K}/D^3$	0.96	0.93	11.6

1. Trip purpose.

    a.) Work trip models.

    b.) Shopping trip models.

    c.) Social-recreational trip models.

    d.) All-purpose trip models.

2. Travel type.

    a.) Produced-trip models.

    b.) Attracted-trip models.

3. Area designation.

    a.) Total-area models.

    b.) Core-area models.

    c.) Fringe-area models.

The stratification process was accomplished in three steps. The first step was the separation of work, shopping, and social-recreational trips from the overall trip totals. A fourth set of models, the all-purpose trips, had previously been evolved using the overall totals. The next division involved separating each trip-purpose category into trips produced by Fort Wayne and trips attracted to Fort Wayne. The final refinement was the division of the total study region into the core area - the area of the study region primarily under the influence of Fort Wayne and the fringe area - the area of the study region under the influence of the competing cities.

Trip Purpose

Separate trip purpose models were developed for work, shopping, social-recreational, and all-purpose trips because the mass variables of a community generating work trips are not the same as those that generate shopping, social-recreational, or all-purpose trips. For example, the work and shopping trip models are composed primarily of employment totals and sales totals, respectively. In the social-recreational trip models, population and recreational open-space are used to describe the forces which produce social recreation. Commercial recreation forces, on the other hand, are represented by retail sales totals and employment in eating and drinking places. The all-purpose trip models differ from the specific purpose models because they predict trips in general. As a result, variables were included to describe the combined generation of work, shopping, and social-recreational travel. Employment in public administration, sales by automotive dealers, and acres of recreational open-space are some of the influencing measures that described work, shopping, and social-recreational trip generation, respectively, in the all-purpose model.

In general, the work and all-purpose trip models had higher correlations with the produced and attracted trip data than did the shopping and social-recreational trip models. The lower correlations are partially explained by the difficulty of choosing variables to represent those forces that generate shopping trips and especially social-recreational trips.

Six models were evolved for each trip category to ascertain the best method of stratifying the trip data. No two models within a specific trip category contained exactly the same composite variables. The work trip models, numbers 17 to 23, show only one constant mass variable, employment in public administration, and the form of the composite variable was dissimilar from one model to another. This pattern was also true for the other trip-purpose categories.

While population has been used to describe the attractive and productive forces of a community in many previous studies, an inspection of the 24 estimation models reveals that the composite variables which contain population X_1, is present in only 13 situations. When variables which are more closely associated with the forces generating travel, such as employment and sales totals, are used, population no longer remains significant as a measure of travel potential. Eight instances include the six social-recreational models, numbers 29 to 34, and the two all-purpose models for the fringe area, numbers 39 and 40. Population does serve, however, as a measure of social recreation - the visiting of friends and relatives in models 29 to 34 and the all-purpose models for the fringe area, numbers 39 and 40. Thus, population does play a significant role in the generation of trips from a community for a limited number of trip categories.

Travel Type

The forces which generate produced and attracted trips are fundamentally different. This finding is evidenced in the different forms of the composite variables in the produced and attracted shopping, social-recreational, and all-purpose models, equations 23 through 40. For example, the core-area social-recreational produced trip model has the variable form $\sqrt{M}/D^2$, while the corresponding model for attracted trips has the form M/D^4. This pattern of differences is evident in the travel type pairs for other trip purposes.

The change in the composite variable form in the produced or attracted models was not the only difference observed. In one equation a specific variable serves as a generator of trips, and in another equation this same variable acts as a competition factor. This dual role was observed in variable X_{18}, which is negative (competition factor) in equation 27 and positive (trip generator) in equation 28, and in variable X_{27}, which is positive (trip generator) in equation 27 and negative (competition factor) in equation 28.

The theory of competition was included in the models of this investigation. Internal competition is defined as the ability of a community to satisfy the wants and needs of its residents within its own boundaries. On the other hand, external competition is defined as the ability of major cities within the region to vie with each other and with the central city for the opportunity to serve the people in the trip producing community. Internal and external competition factors, represented by variables X_{31} and X_{32}, respectively, were included in attracted trip models. The reason for not including these explicit factors in the produced-trip models was that competition is incorporated within the variables of the basic models and no further factors were necessary. The inclusion of these explicit factors made it possible to study the precise role of intercommunity competition in regional traffic interchange.

Internal competition was significant at the 5-percent level in eight of the twelve attracted trip models. The four equations in which this factor was not significant were total-area work trips, total- and fringe-area shopping trips, and core-area all-purpose trips, models 18, 24, 28, and 38, respectively.

External competition was not expected to be significant in the four core area models which left only the four total region and four fringe area models to consider. Five of these eight models did contain the explicit external competition factor. All-purpose models, numbers 36 and 40, lacked the explicit external competition factor which suggests that population alone is not capable of describing external competition in the all-purpose category.

Area Designation

The final step of dividing the total study region into core and fringe areas was necessary because trips originating in either the core or the fringe are basically different. The core and fringe trip models provided better estimations of travel interchange than did the models developed for the total area alone. This separation of the study region improved predictions for both trips produced by Fort Wayne and trips attracted to Fort Wayne. For example, the produced all-purpose trip models have correlation coefficients of 0.87, 0.89, and 0.96 for the total, core, and fringe areas, respectively. The coefficients of correlation increased because the core and fringe division created two sub-groups of trips, each of which was more homogeneous than the original larger group.

The creation of these two areas revealed that a variable may affect travel differently as the distance between origin and destination changes. The composite variable X_{13} is positive (trip generator) in equation 38 for attracted core-area all-purpose trips and negative (competition factor) in equation 40 for attracted fringe area all-purpose trips. In the same equations, composite variable X_{25} is negative (competition factor) for the core area and positive (trip generator) in the fringe area. This comparison demonstrates that the same variables may act as a trip generator in one model and as an internal competition factor in another.

Internal competition was evident in both the core and fringe areas. Explicit internal competition factors were included in core area models 20, 26, and 32 and fringe area models 22, 34, and 40. A city located in either the core or fringe area of the central city is capable of satisfying some needs of its residents.

External competition, on the other hand, was not expected to be significant within the core area. An inspection of these models reveals the absence of this factor from all the core models, equations 20, 26, 32, and 38. Communities located in the core area are primarily under the influence of Fort Wayne, and no other city exerts as much influence in this area.

The explicit factor for external competition appears in two of the four fringe area attracted-trip models. The all-purpose model for the fringe area, model 40, and the work model for the fringe area, model 22, did not contain this external competition factor. Again, population can not always be used as a substitute for a specific purpose variable in a specific purpose model.

Although the effect of distance on travel generation differed between trip purpose and travel type, a pattern was not obvious until models were developed for the core and fringe areas. In general, the work and shopping trip models had higher exponents for distance in the fringe area than in the core. The exponent of distance remained constant in both core and fringe areas for the social-recreational and all-purpose models. Evidently, the increase in distance that a person must travel to make a work or shopping trip from or to the fringe area has a greater influence on travel. Normally, work and shopping needs can be satisfied relatively near a person's home and to ignore these facilities one is heavily penalized by the influence of distance. However, for social-recreational and all-purpose trips, distance does not more severely deter travel in fringe area trips. This result seems reasonable because these two trip categories, especially social-recreational, contain very specific destinations regardless of the distance that must be traveled.

Comparative Results

The results of this study partially verify an investigation by G. W. Greenwood at the University of Illinois. (12) This previous study developed models for eight trip categories but reported only all-purpose models for the total, core, and fringe areas. The produced-trip models contained the composite variable form of $\sqrt{M}/D^2$. The corresponding models in the present study display the same form for the total and core areas, but the fringe area model has $\sqrt{M}/D^3$ as its composite variable form. In comparing the attracted-trip models, the mass variables were the same for the total and core areas in both studies, but the exponent of distance differed in all three equations.

Greenwood also included explicit internal and external competition factors in his attracted-trip models. This study confirms his conclusions that external competition was not significant in the core area and that internal competition was significant in the total, core, and fringe areas.

However, while Greenwood used population as the basic variable to predict travel generation, the results of the present study suggest that population is less important as the measure of the generating force than those variables closely associated to trip purpose.

SUMMARY OF RESULTS AND CONCLUSIONS

The results of this investigation are given as mathematical models which represent the relationships between community statistics and regional trip generation. The following list summarizes results that are strictly valid for the region surrounding Fort Wayne, Indiana. However, these models serve as the basis for developing similar generation models in other parts of the country.

1. Multiple linear regression equations, composed of composite variables of a gravitational form, gave satisfactory results in this investigation. The magnitude of the coefficients of determination ranged from 0.43 to 0.94. Thus, the type and form of the independent variables accounted for a large percentage of the variation in the regional travel patterns.
2. Produced and attracted trips are fundamentally different in nature and require separate equations for their explanation.
3. Internal competition is significant for the total, core, and fringe areas.
4. External competition is significant for the total and fringe areas.

5. The influence of mass and distance on the generation of regional travel varies with trip purpose (work, shopping, social-recreational, and all-purpose), with travel types (produced and attracted), and with core and fringe areas.

SUGGESTIONS FOR FURTHER RESEARCH

As the conduct of this research study progressed, it became readily apparent that various phases in the area of traffic generation and distribution required further comprehensive evaluation. The following topics are suggested for continued study.

1. The multiple linear regression equations developed in this investigation should be verified for other similar and dissimilar regions.
2. These equations should be checked with origin-destination data collected for the Fort Wayne area at some future date to ascertain the validity of these equations over time.
3. An attempt should be made to refine the method by which competition zones were delimited with a different configuration for each specific trip purpose.
4. The internal and external competition factors for attracted trip models might be expanded to include more than one variable. Therefore, the importance of these competition factors in the generation of regional trips may be more realistically evaluated.

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APPENDICES

APPENDIX A

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DETAILED LITERATURE SURVEY

During the past decade, social scientists, planners and engineers have given increased attention to several methods of forecasting travel patterns. While no general agreement exists as to which method is best, much interest has been focused on the gravity concept of human interaction. Theorists are attempting to discover the fundamental relationships that explain the structure of urban and metropolitan areas. Meanwhile, practical planners are faced with the necessity of quantifying these theories to provide specific answers to everyday problems. Both groups find promise in the gravity approach. This concept provides the basis for developing theories of urban structure and, affords a starting point for solving specific problems involving market analysis, population and migration forecasts, traffic-flow patterns, and land allocation. (6)

The gravity concept postulates that this interaction between two areas is created by the masses of these areas, and that this interaction is resisted by the space separating the two areas. In other words, the interaction between any

two communities varies directly as some function of the size of the populations and inversely as some function of the distance between them. Mathematically this relationship is stated as:

$$I_{ij} = \frac{f(P_i P_j)}{f(D_{ij})} \tag{41}$$

where I_{ij} = the interaction between center i and center j,

$P_i P_j$ = the population of areas i and j, respectively, and

D_{ij} = the distance between center i and center j.

When the hypothesis is stated in this manner, G. A. P. Carrothers explains that:

1. To produce interaction at all, individuals must be in communication, either directly or indirectly, with one another.
2. All persons are considered to generate the same amount of individual influence.
3. The probable frequency of interaction that is generated by an individual in a given location is inversely proportional to the difficulty of reaching or communicating with that location.
4. The resistance against transportation or communication is directly proportional to the intervening

two communities varies directly as some function of the size of the populations and inversely as some function of the distance between them. Mathematically this relationship is stated as:

$$(41) \qquad I_{ij} = \frac{f(P_i P_j)}{f(D_{ij})}$$

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4. The resistance against transportation or communication is directly proportional to the intervening

physical separation between the individual and the given location. (6)

The first formulation of the gravity concept as applied to social interaction was proposed by H. C. Carey in the early 1800's. He stated that the fundamental physical law for the attraction of molecules was also the basis for social attraction.

> Man, the molecule of society, is the subject of Social Science...The great law of Molecular Gravitation is the indispensible condition of the existence of the being known as man...The greater the number collected in a given space, the greater is the attractive force that is there exerted ... Gravitation is here, as everywhere, in the direct ratio of the mass, and the inverse one of distance. (4)

The basic difficulty with this analogy is the different nature of the two elements of comparison. Man can make decisions on which to base his actions, while the individual molecule does not have the power of rational thinking. This trait does not mean that the interactions of a large number of human beings cannot be described mathematically, but it does indicate that there is a point where the individual's power to make decisions critically affects the results. This point must be determined before this concept can be practically applied. (6)

Interest in this concept did not reappear until 1885, when E. G. Ravenstein used a modified form to explain the mechanics of migration. Migration tends toward cities of large population, and movement decreases as the distance between the source of migration and the center of absorption increases. (27) This relationship was expressed as:

$$(42) \qquad {}_iM_j = \frac{f(P_i)}{D_{ij}}$$

where ${}_iM_j$ = migration from the source j to the center of absorption i,

$f(P_i)$ = some function of the population i, and

D_{ij} = distance between the source j and the center i.

No further developments were made until the late 1920's when E. C. Young used the concept to explain migration. The relative volume of migration to a given destination from several sources varied directly with the force of attraction of the destination and inversely with the square of the distance between source and destination. (48) This relationship was represented as:

$$(43) \qquad {}_iM_j = k\,\frac{Z_i}{D_{ij}^{\,2}}$$

where Z_i = the force of attraction of the destination i, and

k = a constant of proportionality.

During this same period, W. J. Reilly postulated his "Law of Retail Gravitation," which approached the gravity concept in a different manner. This law states that a city attracts retail trade from individuals in its surrounding territory in direct proportion to the population size of the retail center and in inverse proportion to the square of the distance between the individual and the center. Between any two cities competing for retail trade there is a point of equilibrium where their competitve influence is equal. (28) This last statement can be mathematically described as:

$$(1) \qquad \frac{P_i}{{d_{xi}}^2} = \frac{P_j}{{d_{xj}}^2}$$

where P_i, P_j = the populations of cities i and j respectively,

x = the point of equilibrium on the line joining i and j,

d_{xi} = the distance from city i to point x,

d_{xj} = the distance from city j to point j, and

D_{ij} = $d_{xi} + d_{xj}$

Formulation of the Concepts

During the early 1940's, the gravity concept was generalized by both J. Q Stewart and G. K. Zipf. (34,35,36, 48,49,50,51,52) This generalization was expressed as:

$$(44) \qquad F_{ij} = \frac{P_i P_j}{{D_{ij}}^2}$$

where F_{ij} = the force of interaction between concentrations i and j.

The energy of interaction between the two centers, E_{ij}, resulting from this force is:

$$E_{ij} = k \frac{P_i P_j}{D_{ij}} \tag{45}$$

where E_{ij} = the energy of interaction between i and j, and

k = a constant of proportionality, equivalent to the gravitational constant from physics.

The interaction energy between any two centers of population increases as the product of the populations increases, and decreases as the distance between the two centers increases. Therefore, the total energy of interaction for a given region, i, is the sum of the interaction energy of region i with each of the n other regions into which a given universe is divided. (6) This is stated as:

$$E_i = k \sum_{j=1}^{n} \frac{P_i P_j}{D_{ij}} \tag{46}$$

Zipf, Stewart, and others have tested the gravity concept empirically by measuring the interaction energy between city pairs with such characteristics as telephone calls, newspaper circulation, and bus passenger movements. Stewart extended the physical analogy to include the potential of a population to produce or attract trips. A location, i, the potential influence or possibility of interaction generated by the mass of an area j, increases as the mass of j increases and decreases as the distance between i and j increases. This statement is written as:

$$(47) \qquad {}_iV_j = k \frac{P_j}{D_{ij}}$$

where ${}_iV_j$ = the potential at the location i of the population of area j.

The total possibility of interaction at location i, due to an individual at location i and the populations of all other areas in the study region, can then be calculated as:

$$(48) \qquad {}_iV = k \sum_{j=1}^{n} \frac{P_j}{D_{ij}}$$

where ${}_iV$ = the total population potential at location i, and

n = the number of regions into which the study area is divided.

The total potential of interaction at a given point is calculated by including a measure of the population potential of that point to account for its internal cohesive force. In practice, the distance of an area from itself, D_{ii}, is taken as the average distance from the center of the area to its periphery. (6)

Mapping the Gravity Concept

The gravity concept can be visually demonstrated by mapping. These maps show contours of equal potential in the same manner that topographical maps represent lines of equal elevation. The total potential can be calculated at a series of points and plotted on a map of a study area. Areas of differing potential are recognized and interrelationships among areas are visualized.

In an article written for Traffic Quarterly, J. D. Carroll developed a different method for describing the magnitude of the influence of a city on its hinterland. Carroll made three simplifying assumptions when measuring the magnitude of this influence on persons and organizations outside the boundaries of a community:

1. That the terrain of the region was flat,
2. That the influence is proportional to the city size, and
3. That the rate of decline of the influence of the city with distance was constant.

The influence of the central city is directly proportional to the size of the city and inversely proportional to the distance over which the influence acts. This statement is represented as:

$$UI_a = K \frac{P_a}{(f)D} \qquad (49)$$

where UI_a = the attractive power of city A,

P_a = the mass of city A,

D = a variable measure of distance,

(f) = and unknown rate at which increments of distance modify the urban area's influence, and

K = a constant to correct for differences in dimensions.

This equation describes only the effect of an isolated city (A) on any point in the surrounding area. With the introduction of a second city (B), there is a point where their influence gradients intercept. See Figure 4. Beyond this point, each city continues to exert an influence into the physical limits of the other city. Any number of cities can be considered within a given region, and by plotting the influence exerted by each community, it is possible to divide the entire region up into segments. A vertical axis was erected at each urban center and scaled in proportion to its population. Each pole supported a surface similar to a tent which represented the influence distribution of the city on the surrounding region. The tents are made up of those elements described by the equation: (6)

(50) $$ {}_jU_i = k \frac{P_j}{D_{ij}^{\,a}} $$

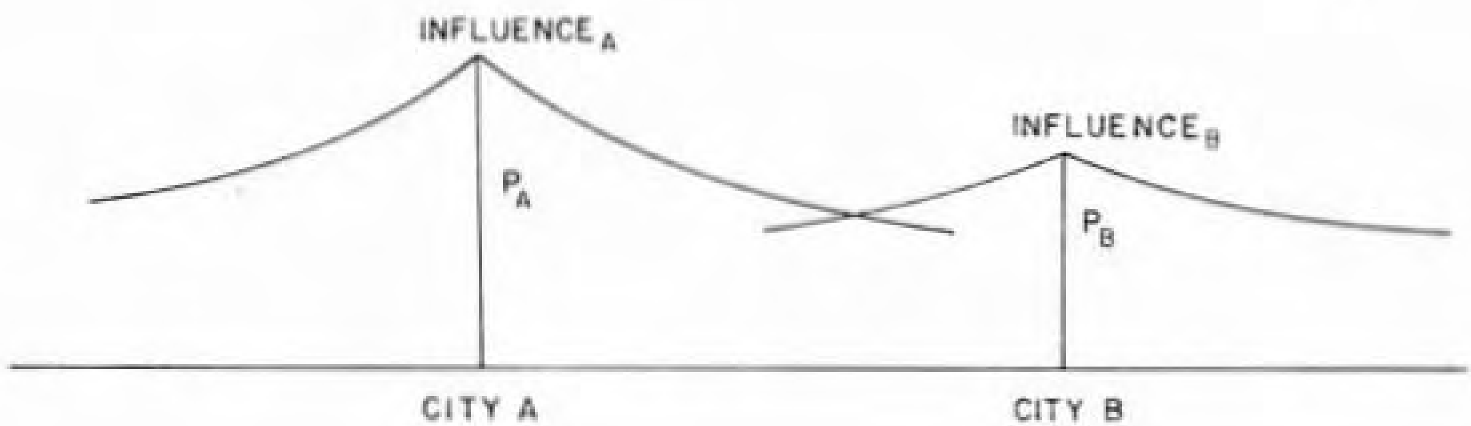

FIGURE 4 THEORETICAL INTERACTION BETWEEN CITY A AND CITY B

where ${}_{j}U_{i}$ = the urban influence of center i upon any point j, and

a = a constant exponent.

In a plan view, the lines of intersection between tents constitute a map showing the influence area of each city.

In the next step where urban influence is defined, the city is visualized as performing a pyramid of functions. The number and variety of functions are proportional to the size of the city. All communities, no matter how small, perform some functions for the hinterland population as well as the residents.

The smaller communities perform a small pyramid of functions. At the base of the pyramid are the most common functions, such as recreation and visiting. The next level may include grocery and drug shopping, and the most specialized single function that the city performs is located at the top. As the size of the city increases, the pyramid expands in breadth and width. As a city performs an increasing number of functions, the probability that similar functions are not provided in the adjacent region becomes greater. People and organizations in the hinterland come to the larger city to make use of these functions. Therefore, urban influence is operationally defined as the extent to which the people and institutions of the region make use of the urban facilities.

The outer limit of the predominant influence of a city is the point at which its magnitude of influence on the hinterland equals the magnitude of influence of a competing city. To be strictly correct, this point is located by considering those functions carried on in both cities.

The first empirical testing of this theory was done in Michigan. The data were long distance telephone calls made from fifty small communities to Flint, Detroit, Lansing, Saginaw, and Bay City. A preliminary analysis revealed that the distance from any small community to each of the five cities influenced the number of calls made. After various attempts, the best fit to the data was:

(51) $$\text{Calls per 1000 pop.} = \frac{a}{D^b}$$

were a = a constant,

D = the distance, and

b = an exponent.

or in general notation:

$$\log X = \log a - b \log Y$$

In this analysis, the exponent b ranged from 2.3 to 3.3. Also, a visual inspection of the intersections for the regression lines suggested that the volume of calls per person was a function of the size of the city called. The number of calls between any pair of cities was inferred to be directly proportional to the product of the two populations and inversely proportional to the distance to some power greater than two. This statement, as represented by equation 52, appears rational because the probability of a call occurring is proportional to the number of callers times the number of callees. Mathematically this may be expressed as:

$$\frac{\text{Calls}}{P_i \, P_j} = \frac{a}{D^b} \tag{52}$$

where P_i = population of the city from which the call is made, and

P_j = population of the city to which the call is made.

This equation made it possible to correlate telephone calls between 244 pairs of cities because calls between any pair of cities are put on a comparable basis. The results gave a correlation coefficient of -0.89 and an exponent of 2.62. When Lansing was excluded (for it is the state capitol and its influence exceeds that measured by the resident population), the correlation coefficient was -0.92 with an exponent of 2.85.

A second test was made with intercity travel data supplied by the Michigan State Highway Department. When the destination was controlled, the number of trips to that city from any other city was predicted by equation 52 in which trips per unit auto was substituted for calls per unit of population. Curves for those cities with similar attraction at a given distance were grouped to form composite curves. The exponents for these curves ranged from 3.36 for a neighborhood center to 2.84 for a major regional center.

J. D. Carroll also studied the frequency of auto travel per unit of residental population to Detroit from one hundred communities within a 300-mile radius. The results were a correlation coefficient of -0.92 and an exponent of 2.98.

While these three studies have added to the understanding of the effect of distance on travel, empirically determined boundaries only represent an approximate solution. The assumption of flat terrain and equal accessibility from all directions is not completely true. Natural and man-made features must be considered in order to refine the first approximation. Carroll does warn, however, that the equations are only valid for the range of distance investigated in these studies. The exponent may change at undetermined distances because a different mode of transportation may be substituted for the automobile.

F. C. Ikle approached the relationship between traffic volume, population, and distance by separating composite traffic into specific trip purposes. Specific trip purposes were used because a person is not equally involved in each trip type; that is, some housewives may never make work trips. The following trip classifications were employed:

population size of two cities jointly affects the frequency of type-1 trips. Each trip involves a relationship between a pair of people - one in the city of origin and one in the city of destination. As the number of possible pairs increases, the more likely or more frequently does the relationship lead to a trip. The number of all possible pairs between two cities of populations P_1 and P_2 equals the product of the two populations. Therefore, the combined influence of the two cities on the frequency of travel is the product of the populations.

The effect of distance, on the other hand, can not be determined without the use of empirical data. However, distance does affect the frequency of travel in two ways. As the distance between the origin and destination increases the time and cost of the trip becomes greater. Also, the greater the distance between the cities the less likely there is to be an actual relationship between a potential pair of people.

The first effect of distance leads to an inverse relationship with the number of trips decreasing as the distance increases. Because a person has only a certain amount of time and money available for traveling, he can take either a few long trips or a series of short trips.

The second effect depends upon the social relationships that previously existed between the two cities. These relationships are related to factors that initiate or motivate travel. Unfortunately, very little is known about the genesis of interpersonal relationships.

In practical work several simplifying assumptions are made concerning the effect of distance upon trip frequency. Generally, these assumptions disregard previous relationships and state that the distance function is an inverse one with the number of trips between the two cities being proportional to the product of the populations. This relationship is written as:

$$(53) \qquad H_{ij} = k \frac{P_i P_j}{D_{ij}}$$

where P_i = the population of city i,

P_j = the population of city j,

H_{ij} = the number of trips between cities i and j for a given time period,

D_{ij} = the distance between the cities i and j, and

k = constant needed to adjust the difference in dimensions.

Because statistical relationships are involved, a large number of city pairs are required to test the theory empirically.

Perhaps the most thorough formulation of the relationship among population, distance, and frequency of interaction was introduced by S. C. Dodd. Dodd made provision for the distance to enter the problem in a more complicated way. (8,9,10) Using Dodd's formulation, J. Cavanaugh carried out many correlations with empirical data. However, these correlations do not allow the distance factor to assume anything but an indirect function. (7) Cavanaugh plotted logarithms of the H_{ij} values against the logarithms of $\frac{P_i P_j}{D_{ij}}$. The exponent, q, was determined so that:

$$(54) \qquad H_{ij} = k \left(\frac{P_i P_j}{D_{ij}}\right)^q = k \frac{(P_i P_j)^q}{(D_{ij})^q}$$

Ikle found this expression difficult to intepret for the expression $(P_i P_j)^q$ has no meaning in terms of paired relationships. Instead an exponent b, was added to the distance measure, as follows:

$$(55) \qquad H_{ij} = k \frac{P_i P_j}{(D_{ij})^b}$$

This exponent is determined by reformulating the previous equation:

$$(56) \qquad \frac{H_{ij}}{P_i P_j} = k \frac{1}{(D_{ij})^b}$$

$$\text{or } \log \frac{H_{ij}}{P_i P_j} = \log k - b \log D_{ij}$$

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This transformed relationship is the equation of a straight line, and b can be readily found by the least square method.(14)

The effects of population size on trip generation have been developed for type-1 trips, but the arguments applied to this category do not apply to the other trip types. Ikle stated that the number of trip origins for trip types 2 and 3 is directly proportional to the population size of the origin. The number of destinations is only approximately proportional to the population size when the whole city is made the area of destination. As the population of the total city increases the number of destinations for trip types 2 and 3 are augmented because there are more work places, shops, schools, churches, etc. The product of the two population masses best describes the generating forces for these categories of trips as well as type-1.

Certain limitations must be recognized in applying this relationship of traffic, population, and distance. This epxression is valid for a mode of travel only over a specific range of distance. Obviously, certain forms of transportation are not used for very short or very long trips. For this reason the effect of distance upon the frequency of trips can not be monotonically decreasing. (14) The second limitation is the so-called "saturation" of a city's travel desire. Ikle visualized a system of cities A, B, C, and D with various populations P_a, P_b, P_c, and P_d and located at various distances D_a, D_b, D_c, D_d from a central city Z. The total number of trips taken by the people of city Z to each of the four cities was expressed as:

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$$H_z = P_z P_a \frac{1}{f(D_{az})} + P_z P_b \frac{1}{f(D_{bz})} + P_z P_c \frac{1}{f(D_{cz})} + P_z P_d \frac{1}{f(D_{dz})} \tag{57}$$

where $f(D_{iz})$ = an unknown function of distance.

Because there are P_z people in city Z, the average number of trips taken by a resident of city Z is:

$$\frac{H_z}{P_z} = P_a \frac{1}{f(D_{az})} + P_b \frac{1}{f(D_{bz})} + P_c \frac{1}{f(D_{cz})} + P_d \frac{1}{f(D_{dz})} \tag{58}$$

If cities A, B, C, and D are now brought closer to city Z and/or their populations increased, the average number of trips taken by a resident of city Z would theoretically increase. However, an upper limit exists to the number of trips one person can take within a given time period. When this limit on travel desire has been reached, the relationship among population size, distance, and travel is no longer applicable. This saturation phenomenon was first observed by J. Q. Stewart in a study of the frequency of trips to the 1939 New York World's Fair. (34)

T. G. Anderson proposed that the exponent of distance was a variable and was inversely related to population size.

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If two urban centers a and b are of unequal population, $P_a > P_b$, at an equal distance from a third center c, $D_{ca} = D_{cb}$, the potential exerted by the smaller center b is reduced more by the intervening distance than the attraction exerted by the larger center. The relationship is stated as:

$$(59) \qquad {}_cV_a = k \frac{P_a}{(D_{ca})^{\alpha}}$$

where $\alpha = f(1/P_a)$

${}_cV_a$ = the potential exerted at location c by population a.

G. A. P. Carrothers developed another variation in which the exponent was a variable function and related inversely to distance instead of population. The friction per unit distance, against which interaction acts is disproportionally greater for short distances than the friction per unit distance caused by longer trips. The difficulty of movement within an urban area is generally greater than for a trip of the same distance in the less densely populated area between two cities. An extra unit of distance added to a long trip is less important than when added to a short movement. Thus, the exponent of equation (59) becomes $\alpha = f(1/D_{ca})$. (6)

Both the density of interaction and the resistance to interchange increase with the size of the urban center. Therefore, the exponent is more logically expected to vary directly with population size instead of in an inverse manner. This influence of community size can more properly be considered by modifying the population factor directly rather than adjusting the distance factor.

Modification of the Population Factor

The concepts and formulas presented in the preceding sections consider only the total numbers of people in predicting travel interchange. Because the traveling habits of people differ widely, various human attributes must be considered to describe more completely regional travel patterns. Both occupation and income level affect trip generation and distribution. (32) For this reason the populations, P_i and P_j, should be weighted according to social and economic characteristics which influence travel. An example of this weighting is the use of car registration in the place of population. Travel by private car depends upon car ownership, and traffic estimates are improved by taking the number of registered cars instead of the population of a community. This change amounts to weighting the population factors by the percentage of car owners. J. Q. Stewart noted that an area frequently has an attraction either greater or less than that predicted by use of total population. Population size did not necessarily exert the same influence in all circumstances. This fact is contrary to the assumption made in the original formulation where unity is assigned to the molecular weights. Stewart assigned values other than one to the molecular weights to account for the unique character of a population group. When molecular weights are added equation (53) becomes:

$$E_{ij} = k \frac{\mu_i \cdot P_i \cdot \mu_j \cdot P_j}{D_{ij}} \tag{60}$$

where μ_i = molecular weight of an individual in city i, and

μ_j = molecular weight of an individual in city j.

Stewart interpreted these "molecular weights" as measures of an individual's capacity for sociological interaction. The values of 1.0, 0.8, and 2.0 were assigned to the populations of the North, Deep South, and Far West Regions of the United States. These weights checked reasonably well with empirical data, such as the flow of bank checks into New York City from the various areas of the country. (34,36)

This formulation also corresponds to Dodd's "interaction hypothesis" where specified variables other than population and distance were introduced into the original formulation, to account for differentials in sex, income, education, etc. The basic energy equation becomes:

$$E_{ij} = k \frac{\left(\Sigma \phi_i\right) P_i \left(\Sigma \psi_j\right) P_j}{D_{ij}} \tag{61}$$

where $\Sigma \phi_i$ = weighting factors for the population i, and

$\Sigma \psi_j$ = weighting factors for the population j.

However, the application of simple indices to the population factor may not fully account for the differences in influence with changing circumstances. T. G. Anderson suggested the possibility of raising the numerator of the basic equation to some power other than one. (1) However, this formulation still implies that populations of different composition have equal unit influence. If the individual populations were raised to some equal or different power, as suggested by Carrothers, the following energy equation takes the form:

$$E_{ij} = k \frac{\Sigma \phi_i \; P_i^{\beta} \cdot \Sigma \psi_j \; P_j^{a}}{D_{ij}^{a}} \tag{62}$$

The adjustments just discussed permit the introduction of different key variables in place of population and distance. The development of a recent concept relates the number of persons traveling a given distance directly to the number of opportunities at that distance and indirectly to the number of intervening opportunities. (38) In other words, the function of distance is not necessarily continuous as evidenced in the following formula:

$$\frac{\Delta y}{\Delta s} = \frac{a \, \Delta x}{x \, \Delta s} \tag{63}$$

where Δy = the number of persons moving from the origin to a circular band of width Δs,

x = the cumulated number of opportunities between the origin and destination Δs,

Δx = the number of opportunities within a band of width Δs, and

a = a constant.

This technique is limited by the difficulties encountered measuring opportunities which intervene between the origin and the selected destination.

In the development of models for projecting national and regional products, W. Isard and G. Freutel considered income to be the critical variable, and substituted regional (or national) income for the population factor. Also, the cost of travel replaced distance as a measure of resistance to interchange and regional (or national) income was substituted. (16) The basic equation now becomes:

(64) $$_iW = \sum_{j=1}^{n} K_{ij} \frac{Y_j}{(D_{ij})^a}$$

where $_iW$ = the income potential at point i,

Y_j = the income of region j,

a = a constant exponent, and

K_{ij} = a parameter which differs for one pair of regions to another pair and which is some function of the transport cost between each pair of regions.

The substitution of travel cost for distance had been suggested earlier by Carroll and Anderson in the form of a time - cost measure of distance. (1,5) Where the time of communication is a critical factor, such as intra-metropolitan interaction, this measure is particularly pertinent.

A. Voorhees applied the gravitational principle to the analysis of traffic distribution. Distance was measured by travel time for each mode. Trip types were classified according to the nature of the destination (for example, shopping area) and the influence of the destination was measured in terms characteristic of the destination (for example, floor area devoted to the sale of apparel). (46)

The potential concept was extended by C. D. Harris to include a measure of "market potential" which is defined

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The potential concept was extended by C. D. Harris to include a measure of "market potential" which is defined as "the summation of markets accessible to a point divided by their distances to that point." This market potential quantified the markets in terms of retail sales and distance in terms of transport costs. Generalized formulas for estimating transport costs between any two points in the United States were calculated and measures of market potential were obtained at selected points. However, Dunn points out that Harris' procedure constituted raising the distance factor of equation 53 to a variable exponent because transport costs are a product of distance and rate and rate is a function of distance. (11)

In another study Harris calculated a "manufacturing potential" and a "farm potential" for the United States. Maps of these potential measures, which are comparable to the population potential maps developed by Stewart, were plotted for various sections of the United States.

A later generalization of the potential concept used various community statistics (total employment, retail sales, etc.) as the numerator. An adjustment which was included in the distance exponent decreased the energy consumed in transversing distance as transportation facilities were improved. This formulation is stated as:

$$(65) \qquad {}_iH = k \frac{\sum_{j=1}^{n} x_j}{(D_{ij})^{\alpha}}$$

where $_{i}H$ = the potential at point i created by the resource x in region j,

x_j = a measure of a given resource of region j, and

a = a constant exponent.

Recent Studies

Some recent studies have used the square root of the population, $\sqrt{P_i \; P_j}$, instead of the linear product. However, this transformation can not be logically justified if the deductive arguments proposed by Ikle are accepted.

W. Mylroie applied the gravity concept to measure the relative desire for travel between cities in the State of Washington. The desire for travel was then used to classify the highways of the state. In this study the following four formulations of travel factors were considered:

$\frac{(\text{Pop } 1)\ (\text{Pop } 2)}{D}$, $\frac{(\text{Pop } 1)\ (\text{Pop } 2)}{D^2}$, $\frac{\sqrt{(\text{Pop } 1)\ (\text{Pop } 2)}}{D}$, and

$\frac{\sqrt{(\text{Pop } 1)\ (\text{Pop } 2)}}{D^2}$. These factors were correlated with the minimum annual average daily traffic on the road connecting the centers of attraction. Because the regression equations obtained were in the form of logarithms, the line of regression was not a least squares fit to the original data. However, the discrepancy was not large. The factor

$\frac{\sqrt{(\text{Pop 1})\ (\text{Pop 2})}}{D^2}$ gave the best correlation, but the study was interested in emphasizing the importance of travel to the state. Therefore, this factor was multiplied by the distance between the two centers.

Mylroie indicated that the inter-city-travel-desire-factor was not intended to supplant traffic counts as a complete measure of highway usage. It can be used to predict the desire for travel on a proposed road if this road connects two population centers. Projected populations can be used for the towns of the state to permit the prediction of the future minimum traffic flow for any road. (22)

As part of a research project carried out at the University of Illinois, G. W. Greenwood developed regional traffic models of a gravitational form. These models were for both specific purposes (work, shopping, recreation, etc.) and "all-purpose" or total trips between Champaign-Urbana, Illinois, and the communities in a fourteen county area surrounding this urban center. Different models were developed for attracted and produced trips with internal and external competition factors incorporated into the models. Internal competition was defined as the ability of the producing community to fulfill the desires of its inhabitants within its own boundaries. External competition was defined as the abilities of the other major communities to compete with Champaign-Urbana for the opportunity to satisfy the wants of the people in the trip producing community.

In the actual formulation of the equations, eight different forms of population and distance were tested; $\frac{\sqrt{P}}{\sqrt{D}}$, $\frac{\sqrt{P}}{D}$, $\frac{\sqrt{P}}{D^2}$, $\frac{\sqrt{P}}{D^3}$, $\frac{P}{\sqrt{D}}$, $\frac{P}{D}$, $\frac{P}{D^2}$, $\frac{P}{D^3}$. The results obtained for all-purpose trip models were:

1. Produced trips, total area

(66) $$Y_{pt} = -41.1 + 2867.2 \frac{\sqrt{P_a}}{(D_{cu-a})^2} + 1572.3 \frac{\sqrt{AC_a}}{(D_{cu-a})^2} + 2305.9 \frac{\sqrt{E_a}}{(D_{cu-a})^2}$$

2. Attracted trips, total area

(67) $$Y_{at} = -31.7 + 4973.5 \frac{P_p}{(D_{p-cu})^2}$$

3. Attracted trips, core area

(68) $$Y_{ia} = 18.8 + 146.8 \frac{P_p}{(D_{p-cu})^2} - 0.00002S_p$$

4. Attracted trips, fringe area

(69) $$Y_{ao} = -13.5 + 5913.0 \frac{\sqrt{P_p}}{(D_{p-cu})^2}$$

where Y_{at} = trips attracted to Champaign-Urbana from the entire region,