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Acoustic Filters: Part II: The Four-Microphone Method of Experimental Verification of Acoustic Characteristics of Mufflers

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ACOUSTIC FILTERS PART II: THE FOUR-MICROPHONE METHOD OF EXPERIMENTAL VERIFICTION OF ACOUSTIC CHARACTERISTICS OF MUFFLERS

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ABSTRACT

An experimental method that complements computer modeling of transmission characteristics of mufflers is devised. It is an extension of well-established two-microphone method of experimental investigation of acoustic characteristics of materials. The four-microphone method is actually the two-microphone method that is applied two times at two different locations of an acoustic system. The method does not require use of dedicated dual-channel Digital Frequency Analyzer. If the excitation signal is random, stationary and ergodic, microphones do not have to be accurately calibrated and they do not need to be phase-matched either. Under these conditions, even in-situ measurement can be done with ordinary microphones and both measurements have not to be simultaneous.

1. INTRODUCTION

In order to make experimental method consistent with the analytical method (Eversman (1987), Bukac (2006) we have to be able to measure complex amplitudes of progressive and reflected waves at any two locations of an acoustic transmission line. The two-microphone method (ASTM standard E1050-98), which is also known as the Chung and Blaser (1980) method, uses cross-spectrum between two pressure signals to find reflection coefficient and impedance of an acoustic termination. A variety of two-microphone method that has been developed earlier by Seybert and Ross (1977) uses auto-spectral and cross-spectral density of two pressure signals measured at two locations to find auto-spectral and cross-spectral densities of amplitudes of progressive and reflected waves in the duct. Both methods require use of dual-channel spectrum analyzer, and one method can be developed from the other, Munjal (1987). The four-microphone method uses Fourier transforms of four pressure signals that are measured at two pairs of locations of an acoustic transmission line. For this reason, there is no need for a spectral analyzer. An inexpensive analog to digital converter board (ADC), personal computer (PC) and standard mathematical software such as a spreadsheet will suffice. The knowledge of Fourier transforms of two pairs of complex amplitudes of incident and reflected waves at two locations in an acoustic transmission line is sufficient to calculate any desired spectral acoustical characteristic of a muffler.

2. ACOUSTIC PRESSURE IN A DUCT

The method requires that the inlet duct, inside which the pressures are measured, has distinct change in impedance such as abrupt change in cross-section. The outlet duct, also called tail duct, may have any type of termination. The pressure distribution at any location and at any time inside a smooth duct that has constant cross-section S and finite length L is given by the solution of the wave equation. In order to be consistent with the analytical theory, we assume that the origin of the x-coordinate is at the beginning of the duct. In addition, we assume that the density of gas throughout the duct is constant, and that gas does not move. Thus, the pressure measured by a microphone that is located at a distance x from the beginning of the duct is

$$p(x,t) = p(x)\cdot e^{j\cdot\omega\cdot t} = \left[A\cdot e^{-j\cdot k\cdot x} + B\cdot e^{j\cdot k\cdot x}\right]\cdot e^{j\cdot\omega\cdot t} \tag{1}$$

Where

p is pressure [Pa]
x is coordinate along the duct [m]
t is time [s]
A is complex amplitude of incident wave [Pa]
B is complex amplitude of reflected wave [Pa]

j is imaginary unit, $\sqrt{-1}$
k is wave number, $k = \omega / c$, $[m^{-1}]$
ω is angular frequency $[s^{-1}]$
c is velocity of sound in the duct $[m.s^{-1}]$
e is base of natural logarithm

The finite Fourier transform of equation (1) is

$$F_P(\omega) = F_A(\omega) \cdot e^{-j \cdot k \cdot x} + F_B(\omega) \cdot e^{j \cdot k \cdot x} \tag{2}$$

Where
F_P is finite Fourier transform of pressure p(x,t)
F_A is finite Fourier transform of amplitude of incident wave
F_B is finite Fourier transform of amplitude of reflected wave
ω is angular frequency $[s^{-1}]$

In the following, we drop ω in equation (2) and we will use letter F only for the finite Fourier transform. Suffixes at the letter F will designate location at which the Fourier transform is calculated.

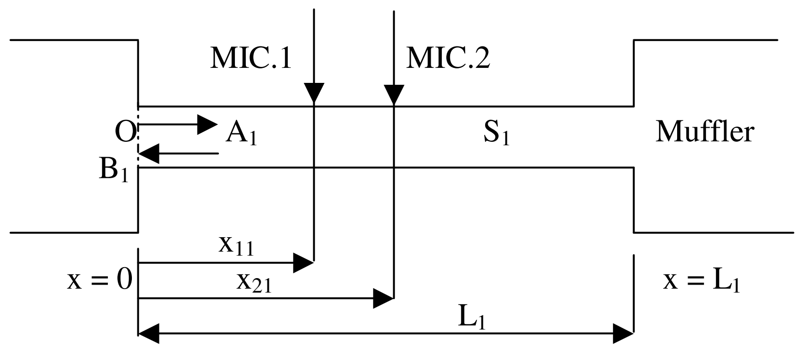

Fig. 1: Location of microphones in the inlet duct

2.1 Pressures in the Inlet Duct

The finite Fourier transforms of pressures measured at the distance x_{11} and x_{21} from the beginning of the inlet duct in Fig.1 are

$$F_{P11} = F_{A1} \cdot e^{-j \cdot k \cdot x_{11}} + F_{B1} \cdot e^{j \cdot k \cdot x_{11}} \tag{3}$$

$$F_{P21} = F_{A1} \cdot e^{-j \cdot k \cdot x_{21}} + F_{B1} \cdot e^{j \cdot k \cdot x_{21}} \tag{4}$$

Where
F_{P11} is finite Fourier transform of the pressure sensed by microphone 1 in the inlet duct (wave system 1)
F_{P21} is finite Fourier transform of the pressure sensed by microphone 2 in the inlet duct (wave system 1)
F_{A1} is finite Fourier transform of the complex amplitude of incident wave of the wave system 1
F_{B1} is finite Fourier transform of the complex amplitude of reflected wave of the wave system 1

Equations (3) and (4) can be solved for unknown finite Fourier transforms of amplitudes of incident (F_{A10}) and reflected (F_{B10}) waves at the beginning of the duct, x = 0. The solution yields

$$F_{A10} = \frac{F_{P11} \cdot e^{-j \cdot k \cdot x_{21}} - F_{P21} \cdot e^{-j \cdot k \cdot x_{11}}}{e^{j \cdot k \cdot d_1} - e^{-j \cdot k \cdot d_1}} \tag{5}$$

$$F_{B10} = \frac{F_{P21} \cdot e^{j \cdot k \cdot x_{11}} - F_{P11} \cdot e^{j \cdot k \cdot x_{21}}}{e^{j \cdot k \cdot d_1} - e^{-j \cdot k \cdot d_1}} \tag{6}$$

Where
d_1 is distance between microphones, $d_1 = x_{21} - x_{11}$, [m]

Equations (5) and (6) show the method fails at the frequency $f_{CR} = c/(2.d_1)$. The method also fails for frequencies f > f_{CR} due to under-sampling.

The finite Fourier spectra of incident wave (F_{A1L1}) and reflected wave (F_{B1L1}) at the end of inlet duct, x = L_1, which is also the inlet of the muffler under investigation are

$$F_{A1L_1} = \frac{F_{P11} \cdot e^{-j \cdot k \cdot x_{21}} - F_{P21} \cdot e^{-j \cdot k \cdot x_{11}}}{e^{j \cdot k \cdot d_1} - e^{-j \cdot k \cdot d_1}} \cdot e^{-j \cdot k \cdot L_1} \tag{7}$$

$$F_{B1L_1} = \frac{F_{P21} \cdot e^{j \cdot k \cdot x_{11}} - F_{P11} \cdot e^{j \cdot k \cdot x_{21}}}{e^{j \cdot k \cdot d_1} - e^{-j \cdot k \cdot d_1}} \cdot e^{j \cdot k \cdot L_1} \tag{8}$$

2.2 Pressures in the Outlet Duct

Finite Fourier transforms of pressures measured by two microphones at the distance x_{12} and x_{22} from the beginning of the outlet, or termination duct in Fig.2 are

$$F_{P12} = F_{A2} \cdot e^{-j \cdot k \cdot x_{12}} + F_{B2} \cdot e^{j \cdot k \cdot x_{12}} \tag{9}$$

$$F_{P22} = F_{A2} \cdot e^{-j \cdot k \cdot x_{22}} + F_{B2} \cdot e^{j \cdot k \cdot x_{22}} \tag{10}$$

Where

F_{P12} is finite Fourier transform of the pressure sensed by microphone 1 in the outlet duct (wave system 2)

F_{P22} is finite Fourier transform of the pressure sensed by microphone 2 in the outlet duct (wave system 2)

F_{A2} is finite Fourier transform of the complex amplitude of incident wave of the wave system 2

F_{B2} is finite Fourier transform of the complex amplitude of reflected wave of the wave system 2

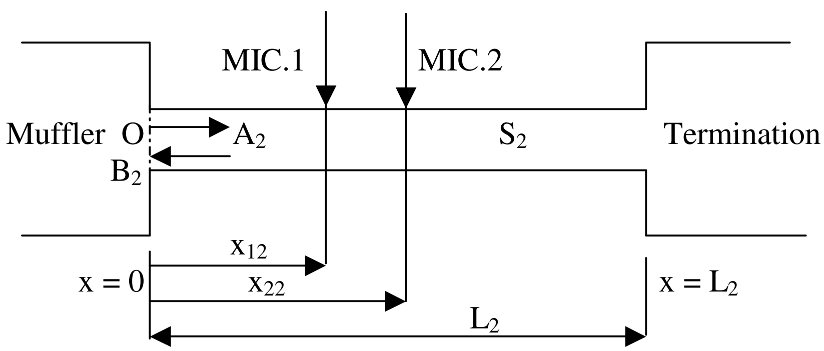

Fig. 2: Location of microphones in the outlet duct

The solution of equations (9) and (10) yields finite Fourier transforms of incident (F_{A20}) and reflected wave (F_{B20}) at the beginning of the outlet duct, x = 0, in Fig.2.

$$F_{A20} = \frac{F_{P12} \cdot e^{-j \cdot k \cdot x_{22}} - F_{P22} \cdot e^{-j \cdot k \cdot x_{12}}}{e^{j \cdot k \cdot d_2} - e^{-j \cdot k \cdot d_2}} \tag{11}$$

$$F_{B20} = \frac{F_{P22} \cdot e^{j \cdot k \cdot x_{12}} - F_{P11} \cdot e^{j \cdot k \cdot x_{22}}}{e^{j \cdot k \cdot d_2} - e^{-j \cdot k \cdot d_2}} \tag{12}$$

Where

d_2 is distance between microphones, $d_2 = x_{22} - x_{12}$, [m]

The finite Fourier transforms of incident (F_{A2L2}) and reflected wave (F_{B2L2}) at the end of outlet duct, x = 0 in Fig. 2, are

$$F_{A2L_2} = \frac{F_{P12} \cdot e^{-j \cdot k \cdot x_{22}} - F_{P22} \cdot e^{-j \cdot k \cdot x_{12}}}{e^{j \cdot k \cdot d_2} - e^{-j \cdot k \cdot d_2}} \cdot e^{-j \cdot k \cdot L_2} \tag{13}$$

$$F_{B2L_2} = \frac{F_{P22} \cdot e^{j \cdot k \cdot x_{12}} - F_{P11} \cdot e^{j \cdot k \cdot x_{22}}}{e^{j \cdot k \cdot d_2} - e^{-j \cdot k \cdot d_2}} \cdot e^{j \cdot k \cdot L_2} \tag{14}$$

Equations (7), (8), (11) and (12) suffice to calculate pressures and volume velocities at the inlet to the muffler and at the outlet from the muffler.

3. ACOUSTIC CHARACTERISTICS OF MUFFLERS

Once we know finite Fourier spectra of incident and reflected waves at the inlet to the muffler and at the outlet from the muffler, we can calculate several frequency characteristics of the muffler under investigation. Nevertheless, before we can proceed further, we need to know finite Fourier transforms of volume velocity. The volume velocity is defined as

$$V(x,t) = V(x) \cdot e^{j \cdot \omega \cdot t} = Z_0 \cdot \left(A \cdot e^{-j \cdot k \cdot x} - B \cdot e^{j \cdot k \cdot x} \right) \cdot e^{j \cdot \omega \cdot t} \tag{15}$$

Where

V is volume velocity [m.s^{-3}]

Y_0 is characteristic admittance (reciprocal of impedance) of the inlet duct [m^5.N^{-1}.s^{-1}]

$$Y_0 = \frac{S}{\rho \cdot c} \tag{16}$$

Where

ρ is density of gas [kg.m^{-3} =N.s^2.m^{-4}]

The finite Fourier transform of volume velocity is

$$F_V(\omega) = Y_0 \cdot \left(F_A(\omega) \cdot e^{-j \cdot k \cdot x} - F_B(\omega) \cdot e^{j \cdot k \cdot x} \right) \tag{17}$$

Where

F_V is finite Fourier transform of volume velocity

3. 1. Transmission Loss

If the cross-section of the inlet and outlet ducts is not equal, the transmission loss in terms of incident wave components is

$$TL = 10 \cdot \log_{10} \left| \frac{S_1 \cdot F_{A1L_1} \cdot \overline{F}_{A1L_1}}{S_2 \cdot F_{A20} \cdot \overline{F}_{A20}} \right| \tag{18}$$

If the inlet and outlet ducts have the same cross-section, the transmission loss can be expressed as

$$TL = 20 \cdot \log_{10} \left| \frac{F_{A1L_1}}{F_{A20}} \right| \tag{19}$$

3.2. Input Impedance

The finite Fourier transform of the input impedance of a muffler is the ratio of pressure and volume velocity at the inlet to the muffler.

The finite Fourier transform of input impedance of the muffler is equal to the ratio of finite Fourier transforms pressure and volume velocity at the inlet to the muffler

$$Z_{11} = \frac{F_{P1L_1}}{F_{V1L_1}} = \frac{F_{A1L_1} + F_{B1L_1}}{Y_{01} \cdot \left(F_{A1L_1} - F_{B1L_1} \right)} \tag{20}$$

Where

Z_{11} is input impedance [N.s.m^{-5}]

Y_{01} is characteristic admittance of inlet tube [m^5.N^{-1}.s^{-1}]

Because the Fourier transform is a complex valued quantity, the magnitude of input impedance will be equal to square root of the product of impedance and its complex conjugate.

$$S_{Z11} = \sqrt{Z_{11} \cdot \overline{Z}_{11}} \tag{21}$$

Where

S_{Z11} is magnitude of Z_{11} [N.s.m^{-5}]

$\overline{Z}_{11}$ is complex conjugate of Z_{11}

3.3. Transfer Impedance

The transfer impedance that is also known as cross impedance can be expressed in two ways. The first one is the forward cross impedance that is equal to the ratio of inlet pressure to the outlet velocity

$$Z_{12} = \frac{F_{A1L_1} + F_{B1L_1}}{Y_{20} \cdot (F_{A20} - F_{B20})} \tag{22}$$

Where

Z_{12} is forward transfer impedance [N.s.m^{-5}]

Y_{20} is characteristic admittance of outlet duct [m^5.N^{-1}.s^{-1}]

The second one is the backward transfer impedance that is defined as the ratio of outlet pressure to the inlet velocity

$$Z_{21} = \frac{F_{A20} + F_{B20}}{Y_{20} \cdot \left(F_{A1L_1} - F_{B1L_1}\right)} \tag{23}$$

3. 4. The Load Impedance

In the case, the muffler under investigation is somewhere at the middle of a large acoustical system, the rest of the acoustical system is the acoustical load of the muffler. The load impedance is actually the input impedance at the inlet to the output duct in Fig. 2. Thus, the load impedance is the ratio of the pressure to the velocity at the inlet to the outlet duct

$$Z_{22} = \frac{F_{A20} + F_{B20}}{Y_{20} \cdot (F_{A20} - F_{B20})} \tag{24}$$

3.5. Reflection Coefficient

The reflection coefficient at the input of a muffler is defined as the ratio of the reflected pressure wave to that of the incident wave

$$R = |R| \cdot e^{j \cdot \theta} = \frac{F_{A1L_1}}{F_{B1L_1}} \tag{25}$$

Where

R is reflection coefficient

θ is phase angle [rad]

Reflection coefficient can also be defined in terms of impedance, equation (20)

$$R = \frac{Z_{11} - 1}{Z_{11} + 1} \tag{26}$$

3.6. Pressure Ratio

The acoustic pressure ratio is the ratio of output pressure to the input pressure of the muffler

$$P_R = \frac{F_{A20} + F_{B20}}{F_{A1L_1} + F_{B1L_1}} \tag{27}$$

Where

P_R is pressure ratio

Alternatively, the pressure ratio of input pressure to the muffler to the output pressure from the muffler is the reciprocal of equation (27)

3. 7. Volume Velocity Ratio

The volume velocity ratio is the ratio of the volume velocity at the outlet from the muffler to that at the inlet to the muffler

$$V_R = \frac{F_{A20} - F_{B20}}{F_{A1L_1} - F_{B1L_1}} \tag{28}$$

Where

V_R is volume velocity ratio

Alternatively, the pressure ratio of input pressure to the muffler to the output pressure from the muffler is the reciprocal of equation (27).

3. 8. Acoustic Power

We can also define acoustic power at the inlet to the muffler and at the outlet of the muffler. The acoustic power is the product of pressure and volume velocity. Because the finite Fourier transforms of pressure and volume velocity are complex valued quantities, the acoustic pressure is

$$N = \sqrt{F_P \cdot F_V^* \cdot F_P^* \cdot F_V} \tag{29}$$

Where

N is spectrum of acoustic power [W]

F_P is finite Fourier transform of pressure

F_V is finite Fourier transform of volume velocity

$*$ indicates complex conjugate

Thus, we can find acoustic power at inlet and the outlet of the muffler by substituting relevant Fourier transforms of pressures and volume velocities into equation (29).

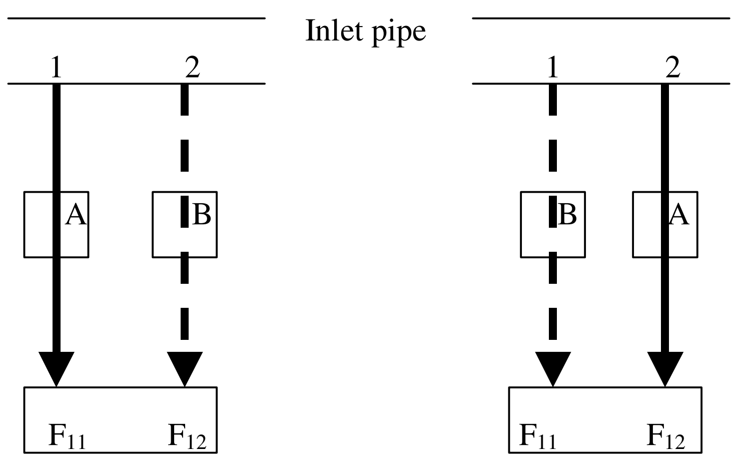

Fig. 3: Relocation of microphones

4. MEASUREMENT PROCEDURE

If the signal, the acoustic pressure, is random, stationary and ergodic, such as pseudo-white noise, Chung and Blaser (1980) and Chung (1978) found that the spectral averaging could fully eliminate phase mismatch and calibration error of microphones. The application of their approach in the four-microphone method is simple. First of all the measurement of acoustic pressures sensed by two closely spaced microphones (Fig. 1) have to be done simultaneously. The Fourier spectra of pressure signals from each microphone are averaged and stored each one in its own array. Then the microphones are physically relocated. It means that the microphone at position 1 is moved to position 2 and connected to the second channel of ADC, and microphone 2 is moved to position 1 and connected to the first channel of ADC. Then the same number of spectra is averaged. Fig. 3 illustrates how the microphones are relocated. Thus the averaged Fourier spectra of microphone 1 are always in array F_{11}, and the averaged Fourier spectra of microphone 2 are always in array F_{12}. We have to emphasize that microphones have to be relocated together with their pre-amplifiers A and B.

The measurements of Fourier spectra in the outlet duct are carried out the same way. Although, one can use four-channel ADC, the measurements in the inlet and the outlet duct do not need to be performed simultaneously.

Fig. 4 shows comparison of an analytically obtained characteristic of a muffler with the experimental one.

5. CONCLUSIONS

The four-microphone method, which is a matter of fact modified two-microphone method that is applied two times, at the inlet and at the outlet of a muffler, simultaneously or consequently, proved to be a useful tool in experimental verification of acoustical characteristics of mufflers.
Although, one can always use a dual-channel digital spectral analyzer to obtain Fourier spectra of acoustic pressure at two closely spaced locations, the use of an analyzer is not necessary. An inexpensive analog to digital converter and a PC computer suffice.

Because of the random excitation, one does not need expensive precisely calibrated and phase-matched pair of microphones. Two cheapest electret microphones that have approximately same sensitivity are sufficient.

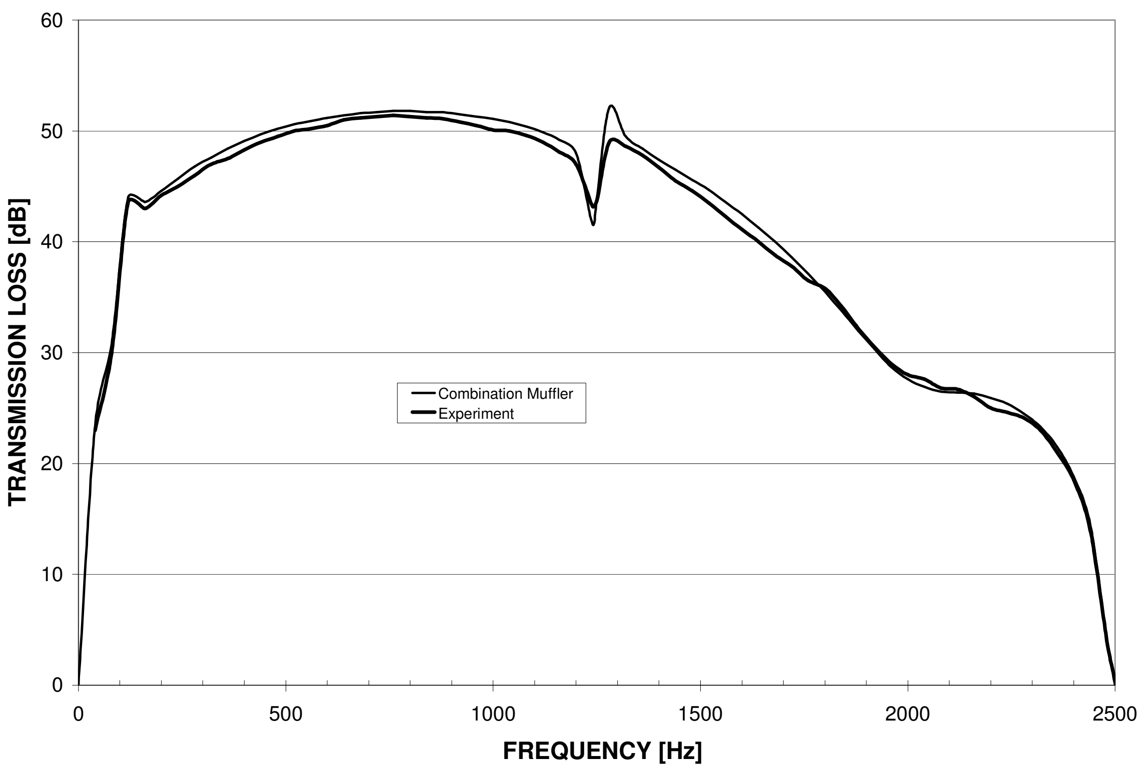

Fig. 4.: Comparison of analytical and experimental transmission loss of a combination muffler

REFERENCES

Test Method for Impedance and Absorption of Acoustical Materials Using a Tube, Two Microphones, and Digital Frequency Analysis System, *ASTM E1050-98 Standard*,

Baher H., 1990, Analog and digital signal processing*, John Willey & Sons*, New York, pp. 474.

Bukac H., 2006, Acoustic Filters Part I: Design and Analysis, *International Compressor engineering Conference at Purdue, July 17-20, 2006*.

Chung J. Y., Blaser D. A., 1980, Transfer function method of measuring in-duct acoustic properties, I: Theory, II:Experiment, *Journal of Acoustical Society of America*., 68(3), pp 907-913, 914-921.

Chung J. Y., 1978, Cross-spectral method of measuring acoustic intensity without error caused by instrument phase mismatch, *Journal of Acoustical Society of America*, 62, pp 388-395

Eversman W., 1987, A systematic procedure for the analysis of multiply branched acoustic transmission lines, *Transactions of the ASME*, Vol. 109, p. 168-177.

Munjal M., L., 1987, Acoustics of ducts and mufflers with application to exhaust and ventilation system design, *John Willey & Sons*, New York, pp. 328.

Seybert A., F., RossD., F., 1977, Experimental determination of acoustic properties using two-microphone random – excitation technique, *Journal of Acoustical Society of America*, 61(5), p 1362-1370.

Smith S., W., 1999, The scientists and engineer's guide to digital signal processing, *Analog Devices*, pp 650.