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ABSTRACT

A method of identification of a dynamic process, which is governed by a certain form of exponential function of time and contaminated with noise, is presented. The approach that is used consists of numerical filtering of experimental data, numerical derivative of filtered numerical data, numerical filtering of the derivative and natural logarithm of derivative of filtered experimental data, and linear curve fit to the logarithm of   derivative of assumed exponential function. All the steps are carried out in the EXCEL spreadsheet (EXCEL is a trade mark of Microsoft Corporation).

1. INTRODUCTION

Many phenomena that appear in physics and engineering may be described by an exponential function of time. In this case, a prototype of a newly designed compressor was built with several temperature sensitive plastic parts. Because of the concern of unrepairable damage, the system could be tested for a limited time only, and the test had to be stopped before those parts reached the steady state temperature. It was assumed the temperature raise could be described by a nonlinear exponential function of time. The purpose of the analysis was to find two constants that govern the process, maximum temperature the system can reach and the time constant of the system.

The known commercially available software such as spreadsheets does not offer any method of fitting experimental data to an exponential function and producing required constants directly. The expansion of the above function into a truncated infinite power series, although mathematically justifiable and practically attainable, does not produce required constants either. The method that has been developed is easy to implement. It consists of non-recursive filtering, numerical derivation of experimental data and a linear curve-fit to the derivative of the above function.

2. THE GOVERNING EQUATIONS

The equation that governs dynamic process such as the rise of temperature of a system may have the form

$$T(t) = T_m \left(1 - e^{\frac{-(t-t_0)}{\tau}} \right) + RND(t) \tag{1}$$

Where

$T(t)$ is temperature that approximates experimental data [^{0}C]

t is time [s]

T_m is unknown maximum temperature the system can reach at the time $t = \infty$ [^{0}C]

t_0 is time shift, usually equal to zero, [s]

τ is unknown time constant of the system [s]

e is the base of natural logarithm

$RND(t)$ is a random noise [^{0}C]

2. 1 Linearization of Governing Equation

It is impossible to fit directly any experimental data to equation (1). The only way of linearization of equation (1) is to take its derivative first and then to take a natural logarithm of the derivative of (1). The derivative of equation (1) is

$$\frac{dT(t)}{dt} = T'(t) = T_m \cdot \frac{1}{\otimes} \cdot e^{\frac{-(t-t_0)}{\otimes}} \tag{2}$$

The natural logarithm of equation (2) is

$$\ln\left(T'(t)\right) = \ln\left(T_m\right) - \ln\left(\frac{1}{\tau}\right) - \frac{t - t_0}{\tau} \tag{3}$$

If we designate

$$y = \ln\left(T'(t)\right) \tag{4}$$

$$b = -\frac{1}{\tau} \tag{5}$$

$$x = t - t_0 \tag{6}$$

$$a = \ln\left(T_m\right) - \ln\left(\frac{1}{\tau}\right). \tag{7}$$

Because T_m and τ are constant, the sum of their natural logarithms has to be a constant as well. Thus, the equation (3) can be rewritten as

$$y = a + b \cdot x. \tag{8}$$

The logarithm of the time derivative of experimental data can be fitted to this linear equation.

3. PREPARING EXPERIMENTAL DATA FOR CURVE FIT

First step in the preparation of experimental data is the reduction of noise. Especially, when the derivative of experimental data has to be used, as it is in our case, the noise has to be filtered out first.

3. 1 Filtering Data

The literature on Signal Processing offers many types of more or less sophisticated numerical filters. In our case, we cannot use recursive or feedback filters that impose a time or phase shift on the signal, which is not desirable. The non-recursive filters such as symmetrical moving average filters are the oldest ones, but their advantage is in that they do not impose any time or phase shift on the filtered signal. The simplest moving average filter has the form

$$y_i = \frac{1}{2 \cdot n + 1} \cdot \sum_{j=i-n}^{j=i+n} x_j \tag{9}$$

Where
y_i is average value of i-th sample
x_j is current value of row data point
i   is index
j   index

It is easy to implement this type of filter in the spreadsheet. Let us assume the time data is in the column A, the raw temperature data is in column B and that it starts at the row 3. The half-width of the filter is chosen to be n = 32. The filtered data will be in the column C. The filtering can start at the row 3+32 = 35, and it has to stop at the row N-32, where N is the last row of the raw data. The only thing we need to do is to type into cell $C35: =AVERAGE($B3,$B67), and drag down to the row N-32.

3.2 Numerical Derivative
The simplest numerical derivative is

$$y'_i = \frac{y_{i+1} - y_{i-1}}{2 \cdot \Delta t} = \frac{y_{i+1} - y_{i-1}}{t_{i+1} - t_{i-1}} \tag{10}$$

Where
$\Delta t =(t_{i+1}-t_{i-1})/2$ is the sampling interval [s].

If the sampling interval Δt is in the cell \$B1 and it is a constant, we can get numerical derivative of the filtered data in column C, by typing in the cell \$D37: = (\$C38-\$C36)/(2*\$B\$1), and drag down to the row N-32-1. If the sampling interval Δt is variable, and the time data is in the column A, we type in the cell \$D37:= (\$C38-\$C36)/(\$A38-\$A36), and drag down.

4. PRACTICAL PROCEDURE

The above outline sequence of operations can be easily done in the spreadsheet. We assume the time data is in the column A and the raw temperature data is column B. Table 1 shows headings of an EXCEL spreadsheet.

Table 1: Spreadsheet headings

	A	**B**	**C**	**D**	**E**	**F**	**G**
1	Time	Tempera-ture	Filtered Tempera-ture	Derivative of Filtered Temperature	Filtered Derivative of Filtered Temperarure	Natural Logarithm	Eq.(1)
2	*	*					
3	*	*					

Fig. 1 shows raw temperature record and filtered temperature (column B in Table. 1). The half-width of moving average filter was n=128.

Fig. 2 shows the time derivative of original raw temperature and the derivative of the filtered temperature signal (columns C and D in Table 1). It is clearly seen the noise in the derivative of original unfiltered temperature signal is so large that it completely buries any useful information. Furthermore, the derivative has both positive and negative values. The natural logarithm is defined for positive values only. Therefore, the unfiltered derivative cannot be used

Fig. 3 shows the time derivative of filtered signal and the filtered derivative. The half-width of the filter was again n=128. The filtered derivative has a reasonable shape and it is positive in the range of interest. Thus, its natural logarithm can be found.

Fig. 4 shows natural logarithm of the derivative of temperature. As expected, it is almost linear in the range where it follows an exponential function. In order to find unknown constants T_m and τ, we need to find linear curve fit of that linear part.

Fig.5 shows a cutout of the linear part in the Fig. 4 and a linear curve fit. The coefficient at x in the Fig. 5 is equal to b and the second expression is equal to a in the Eq. (8).

Fig. 6 shows original unfiltered temperature signal and the analytical curve fit to equation (1).

By using Eq. (5) and Eq. (7), we find

$$\tau = -\frac{1}{b} \tag{11}$$

$$T_m = e^{\left(a + \ln\left(\frac{1}{\tau}\right)\right)} \tag{12}$$

Because the linear curve-fit is to the natural logarithm of the derivative of assumed exponential function, the constants T_m and τ are very sensitive to the accuracy of this liner curve fit. Usually, the curve fit is not perfect and some additional adjustment of constants T_m and τ is needed. In most cases, the adjustment is within $\pm 5\%$. Thus, we can write

$$\bar{\tau} = \tau \cdot c_\tau \tag{13}$$

$$\bar{T}_m = T_m \cdot c_T \tag{14}$$

Where
c_τ is coefficient that adjusts τ
c_T is coefficient that adjusts T_m

4. CONCLUSION

The procedure that is described above offers a method of identification of main parameters of a dynamic system that are governed by an exponential type of function of time. In the case of warming up of a compressor whose main parts were plastic, we were able to predict maximum attainable temperature and the time when that temperature would be reached. This significantly accelerated development of the compressor.

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Otnes R., Enochson L., 1972, Digital Time Series Analysis, *John Wiley & Sons*, New York, 467 p.
Smith S., 1998, The Scientists and Engineers Guide to Digital Signal Processing, *California Technical Publishing*, San Diego, CA, 650 p.
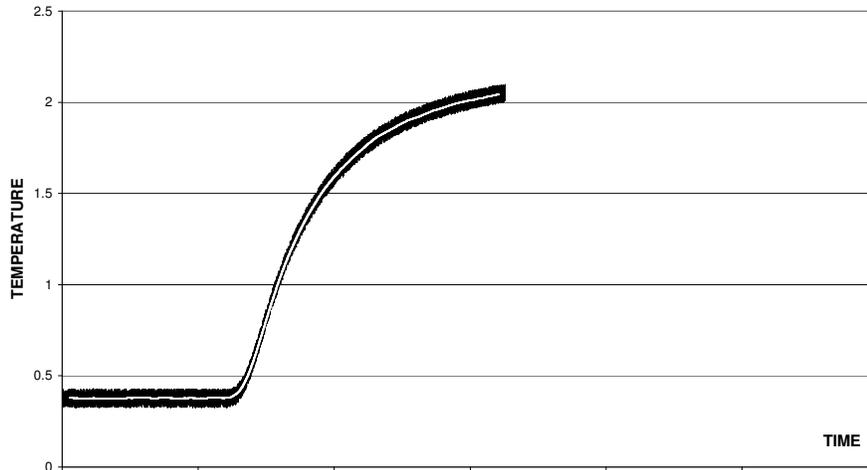
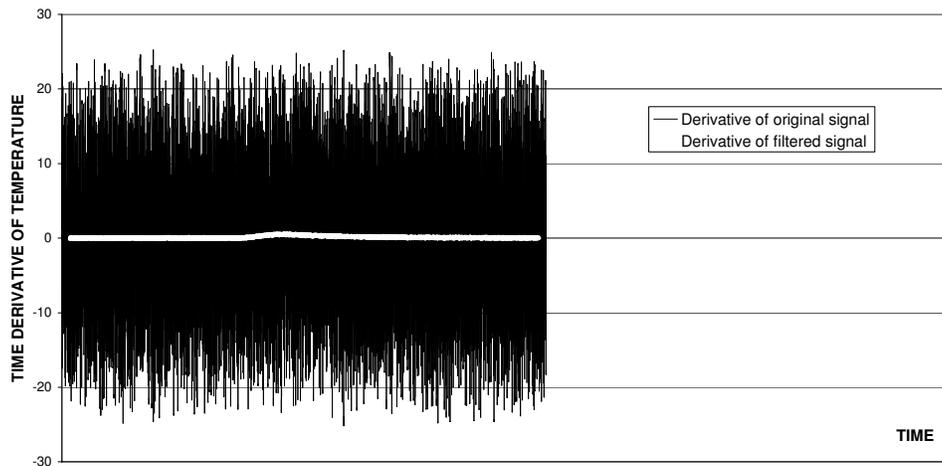Fig.1: Original raw temperature signal and filtered temperature signal

Fig. 2: Derivative of original unfiltered temperature signal and derivative of filtered temperature signal
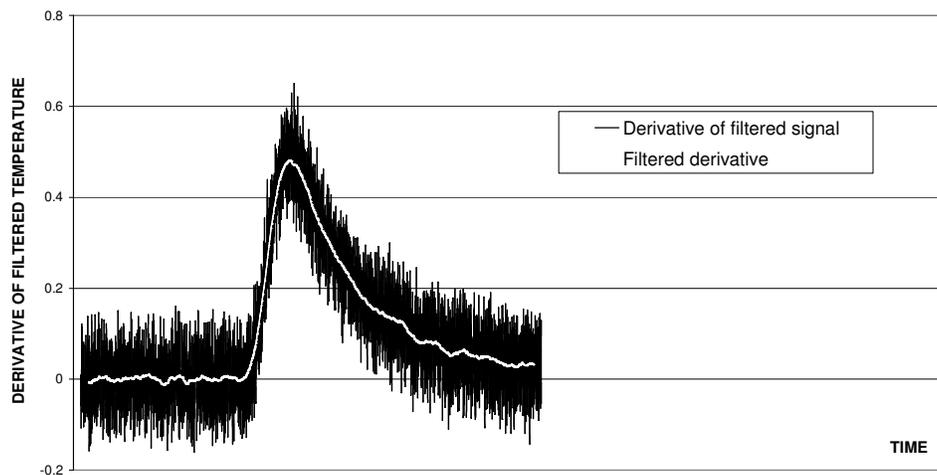

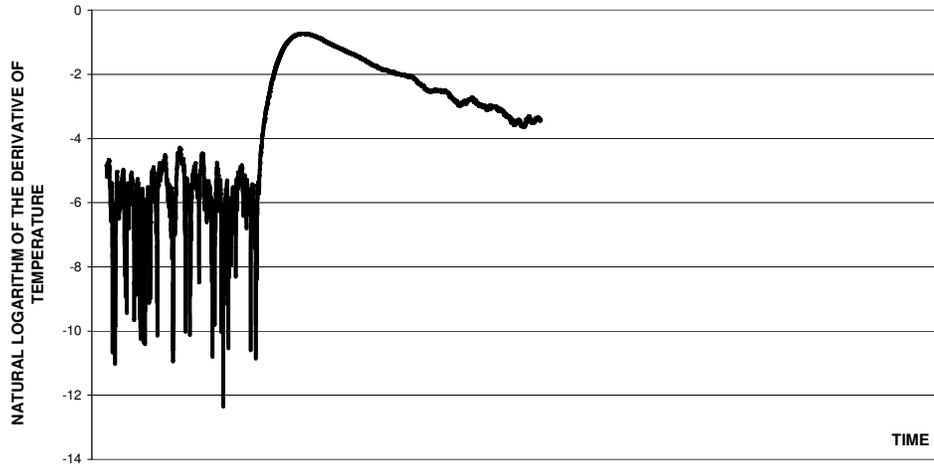Fig. 3: Derivative of filtered temperature signal and filtered derivative of filtered temperature signal

Fig. 4: Natural logarithm of filtered derivative of filtered temperature signal

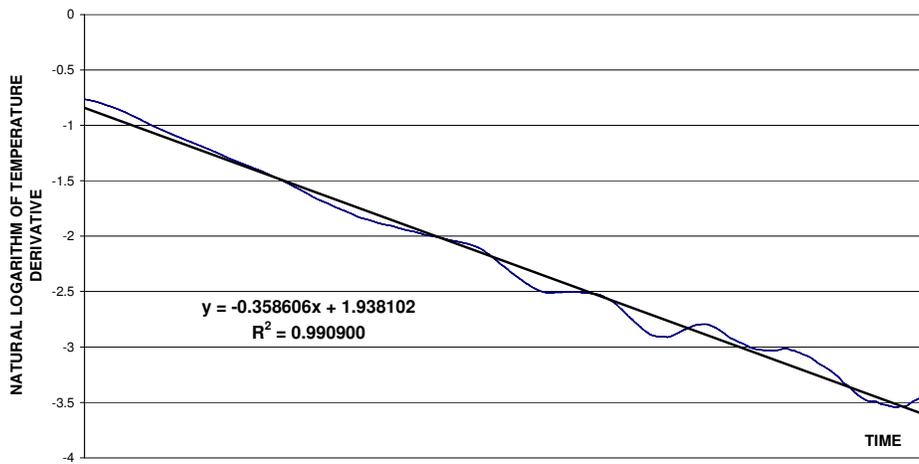y = -0.358606x + 1.938102
R^2 = 0.990900

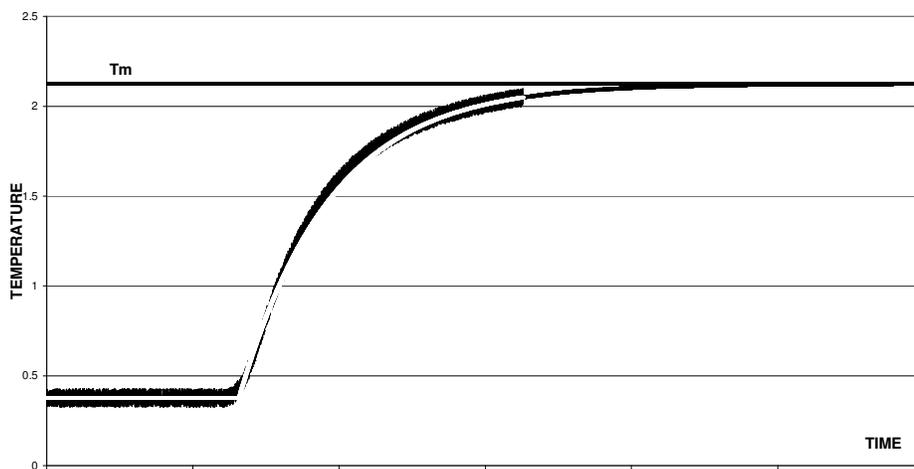Fig. 5: Cut out linear part of natural logarithm of temperature derivative

Fig. 6: Original temperature signal and analytical curve fit

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