Modeling of Initially Subcooled Flashing Vortex Flow in the Nozzle for Possible Applications in the Control of Ejector Cooling Cycles

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Challenges with Ejector Cooling Cycle

- Different working conditions/capacities favor different ejector geometry
  \[ \text{Elbel and Hrnjak (2008); Elbel (2011);} \]

- Slightly different geometry might result in significant difference in system COP under the same conditions
  \[ \text{Sumeru et al. (2012); Sarkar (2012);} \]

- Ejector motive nozzle throat diameter (nozzle restrictiveness) is one of the key points that can significantly affect COP
  \[ \text{COP changed by more than 40 \%} \]

<table>
<thead>
<tr>
<th>Condition</th>
<th>Condition 1</th>
<th>Condition 2</th>
<th>Condition 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{\text{indoor}} ) (dry/wet bulb), °C</td>
<td>26.7/19.4</td>
<td>26.7/19.4</td>
<td>26.8/19.5</td>
</tr>
<tr>
<td>( T_{\text{outdoor}} ) (dry/wet bulb), °C</td>
<td>35.0/19.5</td>
<td>30.6/16.8</td>
<td>27.8/14.9</td>
</tr>
<tr>
<td>( p_{\text{cond}} ), MPa</td>
<td>2.4</td>
<td>2.0</td>
<td>1.9</td>
</tr>
</tbody>
</table>

Test Conditions

R410A ejector air conditioning system COP with different motive nozzles under three different conditions

\[ \text{Hu et al. (2014)} \]
Adjustable Ejector (Previous Approach)

- Ejectors in parallel
- Ejector with adjustable needle
New Solution: Vortex Ejector

Utilizing an adjustable vortex at the motive inlet to control the flow expanded in the motive nozzle (no change in geometry; same effect as changing nozzle throat diameter)

Conventional ejector  Vortex ejector

Vortex ejector cooling cycle
Vortex Nozzle geometry

- Nozzle inlet diameter (mm): 15.0
- Nozzle throat diameter (mm): 1.03
- Nozzle outlet diameter (mm): 1.7
- Nozzle convergent part length (mm): 9.9
- Nozzle divergent part length (mm): 40.0
- Tangential inlet inner diameter (mm): 2.0
- Vortex decay distance (mm): 138.0

3D printed prototype

Convergent-divergent nozzle (resin)

Axial inlet

Tee (brass)

Sleeve (resin)

Tangential inlet

Section A-A

Vortex nozzle geometry
Share of Tangential Kinetic Energy in the Available Pressure Potential Decreases the Mass Flow Rate

Works for both single-phase and two-phase
Choked Mass Flow Rate with Different Inlet Vortex Strengths at Constant Inlet Pressure

Inlet subcooling = 0.5 °C

Mass flow rate can be reduced by 35% with vortex under the same inlet and outlet conditions (large control range).

Nozzle restrictiveness on the flow is changed by vortex; the stronger the vortex is, the larger the restrictiveness is.
• Bubble nucleation
• Two modeling approaches
• Governing equations
• Solution procedure
• Comparison of the modeling results with the experimental results

Note: In this paper, only the convergent part of the nozzle is considered. It is assumed that the choked mass flow rate through the nozzle is only determined by the convergent part.
Bubble Nucleation

Nozzle flow

Superheated liquid

Wall nucleation (cavity defects on the surface)

Nucleation in the bulk of the liquid (impurity; fluctuation)
Assumption:

- There is an evaporation wave at the nozzle throat.
- The bubble generation in the upstream of the evaporation wave is neglected.
- The metastable pressure in the upstream of the evaporation wave keeps constant for the same nozzle inlet pressure and subcooling.
Assumption:
Bubble nucleation during the depressurization in the nozzle all occurs at the nozzle wall.

Shin and Jones (1993)
The continuity equation for each phase is
\[ \frac{\partial \rho_k}{\partial t} + \nabla \cdot (\rho_k \mathbf{v}_k) = 0 \]  \hspace{1cm} (1)

The momentum equation for each phase is
\[ \frac{\partial \rho_k \mathbf{v}_k}{\partial t} + \nabla \cdot (\rho_k \mathbf{v}_k \mathbf{v}_k) = -\nabla p_k + \nabla \cdot \mathbf{\varepsilon}_k \]  \hspace{1cm} (2)

where \( \mathbf{\varepsilon}_k \) is the viscous stress.

The balance of energy can be written as
\[ \frac{\partial \rho_k (u_k + \frac{v_k^2}{2})}{\partial t} + \nabla \cdot \left[ \rho_k \left( u_k + \frac{v_k^2}{2} \right) \mathbf{v}_k \right] = -\nabla \cdot q_k + \nabla \cdot (\sigma_k \cdot \mathbf{v}_k) \]  \hspace{1cm} (3)

where \( q_k \) and \( \sigma_k \) represent the heat flux and the surface stress tensor, respectively.

The interfacial mass balance between the liquid and vapor phases is
\[ \sum_{k=1}^{2} \dot{m}_k = 0 \]  \hspace{1cm} (4)
Governing Equations

Bubble Departure (In Approach 2 Considering Only Nozzle Wall Nucleation)
Bubble nucleation and departure are assumed to take place at where the liquid superheat is larger than zero. The departure radius of a bubble is given by (Shin and Jones, 1993)

\[ R_{\text{depart}} = \sqrt{\frac{\mu_{\text{liquid}}}{\tau_{\text{wall}}}} \sqrt{\frac{4\sigma R_c}{C_D \rho_{\text{liquid}}}} \]  

(5)

where the drag coefficient \( C_D \) is assumed to be 0.5, \( R_c \) is the minimum cavity size and is approximated as

\[ R_c \approx \frac{2\sigma T_{\text{sat}}}{\rho_{\text{vapor}} h_{fg}(T_{\text{liquid}} - T_{\text{sat}})} \]  

(6)

The frequency of bubble departure per unit area is assumed to be \( C_{\text{depart}}(T_{\text{liquid}} - T_{\text{sat}})^3 \) (Shin and Jones, 1993), where \( C_{\text{depart}} \) is a constant.
Governing Equations

- Single Bubble Motion (In Approach 2 Considering Only Nozzle Wall Nucleation)
- Bubble Growth (In Approach 2 Considering Only Nozzle Wall Nucleation)
- Boundary Conditions

The flow at the nozzle inlet is subcooled liquid. There is no bubble mass flow rate entering the nozzle through the inlet.

\[ v_r(r_i, \theta) = -\frac{m_{\text{total}}}{2\pi r_i^2 \rho_{\text{liquid}} (1-\cos \theta_o)} \quad (7) \]
\[ v_\theta(r, \theta_o) = 0 \quad (8) \]
\[ v_\phi(r_i, \theta) = v_\phi(r_i, \theta_o) \frac{\sin \theta}{\sin \theta_o} \quad (9) \]
\[ p(r_i, \theta_o) = p_i \quad (10) \]
\[ p(r_o, 0) = p_o \quad (11) \]
\[ T(r_i, \theta) = T_i \quad (12) \]
Liquid Flow Field Assumption

It is assumed that the vapor mass flow rate in the nozzle compared with that of liquid is negligible. Therefore, $\dot{m}_{\text{total}} \approx \dot{m}_{\text{liquid}}$.

The liquid velocity field in the whole computational domain of the convergent nozzle is assumed to be

\[
v_r(r, \theta) = v_r(r)
\]

\[
v_\theta(r, \theta) = 0
\]

\[
v_\phi(r, \theta) = v_\phi(r, \theta_o) \frac{\sin \theta}{\sin \theta_o}
\]
Wall Shear Stress Modeling
The wall shear stress is modeled as:
\[
\tau_{\text{wall}}(r) = \frac{1}{8} \lambda \rho_{\text{liquid}} [v_r^2(r, \theta_o) + v_{\phi}^2(r, \theta_o)]
\]
where \( \lambda \) is the Darcy–Weisbach friction factor which is modeled as a function of surface roughness \( \epsilon \) and Reynolds number \( Re \):
\[
\frac{1}{\sqrt{\lambda}} = -2 \log_{10} \left( \frac{\epsilon}{3.7D} + \frac{5.74}{Re^{0.5}} \right) \quad \text{(Swamee–Jain equation)} \quad (17)
\]
\[
Re = \rho_{\text{liquid}} \sqrt{v_r^2(r, \theta_o) + v_{\phi}^2(r, \theta_o)D_N/\mu_{\text{liquid}}}
\]
where the nozzle diameter \( D_N = 2R_N = 2r \sin(\theta_o) \).
\[
\tau_\phi(r) = -\frac{v_{\phi}(r,\theta_o)}{\sqrt{v_r^2(r,\theta_o)+v_{\phi}^2(r,\theta_o)}} \tau_{\text{wall}}(r)
\]
\[
\tau_r(r) = -\frac{v_r(r,\theta_o)}{\sqrt{v_r^2(r,\theta_o)+v_{\phi}^2(r,\theta_o)}} \tau_{\text{wall}}(r)
\]
Solution Procedures

Numerical Methods
• The governing equations have been discretized based on the finite volume method.
• The bubble motion and growth is approximated by Euler method. The contributions of vapor in the mass and momentum equations are regarded as negligible.
• The momentum equations are discretized by using first order upwind differencing. The shear stress from the velocity gradient in the radial direction is not considered.
Comparison of the Modeling Results (Approach 1) with the Experimental Results for Choked Flow

![Graph depicting the comparison between modeling results and experimental results for choked flow. The graph shows the total mass flow rate (g s⁻¹) on the y-axis and vortex strength on the x-axis. The graph includes lines for different roughness levels (0 mm, 0.01 mm, 0.1 mm, 0.2 mm, and 0.4 mm) and a red square marker for the experimental results (Inlet 925 kPa, 36 °C (Choked)).]
Modeling Results (Approach 2)

![Graph showing Total Mass Flow Rate vs Outlet Pressure with Vortex Strength legend]
Experimental Results

Outlet Pressure (kPa) vs Total Mass Flow Rate (g s\(^{-1}\))

- **Inlet 925 kPa 36 ºC No Vortex**
- **Inlet 925 kPa 36 ºC Max Vortex**
Comparison of the Modeling Results (Approach 2) with the Experimental Results for Choked Flow

![Graph comparing total mass flow rate to vortex strength](image)

- **Experimental**: Inlet 925 kPa, 36 °C (Choked)
- **Modeling**: Outlet 200 kPa
- **Modeling**: Outlet 300 kPa
- **Modeling**: Outlet 400 kPa
Summary and Conclusions

• The bubble nucleation may not all occur at the nozzle wall at high degree of metastability. Nucleation in the bulk of the liquid might be dominant and should possibly be taken into consideration in the modeling.
• The change in total mass flow rate is smaller for the same inlet vortex strength with larger surface roughness.
• The discrepancies between the modeling and experimental results might be due to
  o oversimplification of the flow velocity profile and inappropriate turbulent wall shear stress model
  o influence of vortex strength and depressurization rate on the maximum achievable degree of metastability
  o decay of vortex strength as the vortex flow travels from the vortex nozzle tangential inlet to the starting point of the convergent part of the nozzle
Thank you for your attention!
Any questions?

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