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ABSTRACT

A numerical method is presented for computing the time varying stresses in compressor flapper valves that have a flexible backer or stop. Valve and backer geometry are described by finite element models as are commonly produced by commercial finite element software. Mass and stiffness matrices are assembled into a second order system which is then solved by a direct numerical integration in a time step fashion along with mechanism and thermodynamic models. Constraint relationships are allowed to change as the valve contacts different points on the backer and valve seat.

1. INTRODUCTION

Analysis is no substitute for testing. However, building and testing prototypes is time consuming. The purpose of a valve simulation is to shorten the development time by estimating the operating stresses quickly and eliminating poor designs early. Furthermore, going through the analysis process invariably leads to a better understanding of the basic physics of the problem.

Previously, Lenz (2000) assumed that if a backer was present, it was perfectly rigid. The primary purpose of a valve backer (also known as a stop, or retainer), is to limit the motion of the valve in order to maintain acceptable bending stress levels. The limited motion of the valve also servers to ensure timely valve closing, A valve that closes late will allow reverse flow, which reduces capacity and efficiency. Many valve backer designs are stiff enough that the rigid assumption is valid. For a number of reasons, flexible backers are becoming increasing popular. One significant benefit of a flexible backer is to act as a cylinder pressure limiting device under slugging conditions. In this role, the backer is not expected to flex significantly under normal operating conditions, however under some extreme starting conditions, when liquid is present in the return gas, the backer may flex and allow the discharge valve to open further than normal for a small number of cycles. This serves to protect other parts of the compressor, such as the suction valve and piston pin bearing.

As will be shown below, the rigid backer assumption can lead to exaggerated stress levels in some cases. This has been found to occur in situations where the valve pries up on the backer like a lever. A small motion of the backer leads to large reduction in valve stress.

This paper describes a new valve simulation that allows for a finite element model of the valve backer in addition to the finite element model of the valve. Both finite element models can be prepared using commercial finite element pre-processors.

2. MATHEMATICAL MODELS

2.1 Overview of the simulation

The mechanism and thermodynamic models were derived by Gatecliff and Griner (1980). A time step begins by incrementing the mechanism simulation by the major time step and computing a new cylinder volume. A thermodynamic calculation provides a new cylinder pressure and the force vector for the valve model is created.

The equations of motion for the valve and backer are then incremented forward in time. The finite element results are used to calculate flow areas for the thermodynamic model.

2.2 Equations of Motion

In general, the equations of motion for a structural finite element system can be written

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{C}\dot{\mathbf{U}} + \mathbf{K}\mathbf{U} = \mathbf{R} \tag{1}$$

Here $\mathbf{M}$, $\mathbf{C}$,and $\mathbf{K}$ are the global mass, damping, and stiffness matrices; $\mathbf{R}$ is the global load vector; and $\mathbf{U}$, $\dot{\mathbf{U}}$, and $\ddot{\mathbf{U}}$ are the displacement, velocity and acceleration vectors of the system. In this case the equations of motion will include finite element models for both the valve and backer.

2.3 The Valve Model

In this simulation, as in Lenz (2000), the valve is constructed of 3-noded triangular high precision plate bending elements. Each node on the valve model has the following six degrees of freedom

$$\mathbf{U}_{\text{valvenode}} = \left[w \quad \frac{\partial w}{\partial x} \quad \frac{\partial w}{\partial y} \quad \frac{\partial^2 w}{\partial x^2} \quad \frac{\partial^2 w}{\partial x \partial y} \quad \frac{\partial^2 w}{\partial y^2} \right]^T \tag{2}$$

Here w represents the displacement in the z direction. This differs from the standard shell elements in the choice of the six DOF's. The standard shell element node has three displacements and three rotations for each node. In the high precision plate-bending element used here, the two in-plane displacements and the normal rotation have been replaced by three second-order derivatives.

Element mass and stiffness matrices for the valve model are derived in Cowper, et. al. (1968). This reference also details the derivation of the element load vector due to pressure. In this case we use a unit pressure load (i.e. p=1.0). The element mass and stiffness matrices are assembled into global matrices using standard finite element procedures. Similarly, the element load vectors are assembled into a global load vector, but only for elements that are subject to cylinder pressure.

2.4 The Backer Model

The backer model uses 10-noded parabolic tetrahedral solid elements. These elements follow the isoparametric formulation of Zienkiewicz (1977) and use 4-point numerical integration in the element mass and stiffness matrices. The backer nodes have the following three degrees of freedom

$$\mathbf{U}_{\text{backernode}} = [u \quad v \quad w]^T \tag{3}$$

where u, v, and w represent the displacement in the x, y, and z directions respectively. This element was chosen for the backer model for several reasons. A solid element is versatile enough to be useful for any backer type. Tetrahedrons are routinely produced by automatic meshing programs. And the parabolic formulation (as apposed to a linear formulation) assures reasonable accuracy. The disadvantage is that a large number of nodes are required, which can strain the in-core solution algorithm and extend run times.

As before, the element mass and stiffness matrices are assembled into global matrices. However, an additional step is required for the backer model after the global matrices are assembled. In most designs, the flexible backer is pre-deformed during assembly, providing, in effect, a preload at the contact points with the valve. This is, in general, a contact problem, because it is not known beforehand which nodes are constrained or what their displacements are. The procedure is a simplified version of the contact problem described below. The final result provides the initial boundary conditions for the backer as well as the initial displacements, which will be used in starting the subsequent dynamics problem.

2.5 Dynamic Solution of the Valve and Backer

The global mass and stiffness matrices for the valve are combined with the global matrices for the backer. The combined, but uncoupled matrices look like

$$\mathbf{K}_{uncoupled} = \begin{bmatrix} \mathbf{K}_{valve} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{backer} \end{bmatrix} \tag{4}$$

$$\mathbf{M}_{uncoupled} = \begin{bmatrix} \mathbf{M}_{valve} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{backer} \end{bmatrix} \tag{5}$$

Assuming Rayleigh damping (Bathe and Wilson, 1976), we obtain the damping matrix $\mathbf{C}$ as a linear combination of the mass and stiffness matrices

$$\mathbf{C} = x\mathbf{M} + b\mathbf{K} \tag{6}$$

We never explicitly form the matrix C in memory. Rather we insert the above relationship into the finite difference equations. The weighting factors x and b can be estimated from knowledge of the modal damping ratios and how they vary with frequency.

The simulation involves using finite difference approximations to the time derivatives in system (1). The Newmark method uses the following finite difference approximations (Bathe and Wilson, 1976)

$$\ddot{\mathbf{U}}_{\mathbf{t+\Delta t}} = a_0(\mathbf{U}_{\mathbf{t+\Delta t}} - \mathbf{U}_{\mathbf{t}}) - a_2\dot{\mathbf{U}}_{\mathbf{t}} - a_3\ddot{\mathbf{U}}_{\mathbf{t}} \tag{7}$$

$$\dot{\mathbf{U}}_{\mathbf{t+\Delta t}} = \dot{\mathbf{U}}_{\mathbf{t}} + a_6\ddot{\mathbf{U}}_{\mathbf{t}} + a_7\ddot{\mathbf{U}}_{\mathbf{t+\Delta t}} \tag{8}$$

These are substituted into system (1) and solved for the unknown displacements $\mathbf{U}_{\mathbf{t+\Delta t}}$. This results in the following system of linear of equations

$$\hat{\mathbf{K}}\mathbf{U}_{\mathbf{t+\Delta t}} = \hat{\mathbf{R}}_{\mathbf{t+\Delta t}} \tag{9}$$

where

$$\hat{\mathbf{K}} = \mathbf{K} + a_0\mathbf{M} + a_1\mathbf{C} \tag{10}$$

$$\hat{\mathbf{R}}_{\mathbf{t+\Delta t}} = \mathbf{R}_{\mathbf{t+\Delta t}} + \mathbf{M}(a_0\mathbf{U}_{\mathbf{t}} + a_2\dot{\mathbf{U}}_{\mathbf{t}} + a_3\ddot{\mathbf{U}}_{\mathbf{t}}) + \mathbf{C}(a_1\mathbf{U}_{\mathbf{t}} + a_4\dot{\mathbf{U}}_{\mathbf{t}} + a_5\ddot{\mathbf{U}}_{\mathbf{t}}) \tag{11}$$

$$a_0 = \frac{1}{a\Delta t^2}; \quad a_1 = \frac{d}{a\Delta t}; \quad a_2 = \frac{1}{a\Delta t}; \quad a_3 = \frac{1}{2a} - 1$$

$$a_4 = \frac{d}{a} - 1; \quad a_5 = \frac{\Delta t}{2}\left(\frac{d}{a} - 2\right)$$

$$a_6 = \Delta t(1 - d); \quad a_7 = d\Delta t$$

Parameters a and d can be varied to obtain optimum integration accuracy and stability. Newmark originally proposed as an unconditionally stable scheme the values

$$a = \tfrac{1}{4}\ ;\ d = \tfrac{1}{2}$$

The above formulation was done with unconstrained equations. Constraints are applied to the Newmark dynamic system (9) exactly the same as in a linear-statics problem. Single point constraints are used to apply fixed boundary conditions that never change, e.g. a riveted joint, as well as constraints that change as the valve moves and contact the valve plate or any non-flexible backer. The application of single point constraints is best explained using partitioned matrix notation. The system of equations (9) has unknowns in the displacement vector $\mathbf{U}$ and unknowns in the forcing vector $\mathbf{R}$. Unknowns in the forcing vector correspond to constrained degrees of freedom and are called reactions. The system (9) can be partitioned as follows

$$\begin{bmatrix} \hat{\mathbf{K}}_{\mathbf{aa}} & \hat{\mathbf{K}}_{\mathbf{ab}} \\ \hat{\mathbf{K}}_{\mathbf{ba}} & \hat{\mathbf{K}}_{\mathbf{bb}} \end{bmatrix} \begin{bmatrix} \mathbf{U}_{\mathbf{a}} \\ \mathbf{U}_{\mathbf{b}} \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{R}}_{\mathbf{a}} \\ \hat{\mathbf{R}}_{\mathbf{b}} \end{bmatrix} \tag{12}$$

where $\mathbf{U}_{\mathbf{a}}$ are the unknown displacements and $\mathbf{U}_{\mathbf{b}}$ are the known displacements at $t + \Delta t$. Likewise $\hat{\mathbf{R}}_{\mathbf{a}}$ are the known forces and $\hat{\mathbf{R}}_{\mathbf{b}}$ are the unknown reactions. Rearranging we obtain

$$\hat{\mathbf{K}}_{\mathbf{aa}}\mathbf{U}_{\mathbf{a}} = \hat{\mathbf{R}}_{\mathbf{a}} - \hat{\mathbf{K}}_{\mathbf{ab}}\mathbf{U}_{\mathbf{b}} \tag{13}$$

which can be solved for the unknown displacements. After this, the reactions at the constrained degrees-of-freedom are found from:

$$\hat{\mathbf{R}}_{\mathbf{b}} = \hat{\mathbf{K}}_{\mathbf{ba}}\mathbf{U}_{\mathbf{a}} + \hat{\mathbf{K}}_{\mathbf{bb}}\mathbf{U}_{\mathbf{b}} \tag{14}$$

These reactions are important when deciding whether or not a node should still be constrained by the valve stop or valve plate. Z-reactions are positive in the direction of valve opening. Therefore a negative reaction at a node constrained by the valve plate, or a positive reaction at a node constrained by the valve backer, indicates that the constraint is in violation and should be removed.

Following the application of single point constraints, we apply multi-point constraints. Multi-point constraints are used to describe contact between the valve nodes and backer element faces. As part of the initialization phase, the four faces on each tetrahedral element of the flexible backer model are extracted and tested against other such faces to establish a free face list. Then for each free face, the x and y-coordinates of the valve nodes are examined to find which valve nodes, if any, fall inside the projection of the free face on the XY-plane. The height of the backer face at this point is recorded for each node in a gap list along with the triangular shape function values. Each valve node will typically be associated with two backer faces, one on top of the backer, and one on the bottom of the backer. Only the gap list entry with the lowest height is retained as this is the bottom face and is the only face to consider when testing for collisions with the valve.

When a valve node is in contact with the backer element face the entry in the gap list is marked as closed. A closed gap is modeled with a multi-point constraint equation that links the z-displacements of the valve node with the z-displacements of the six nodes on the backer element face. Numerous gaps may be closed at any point in time. Each closed gap is treated as a separate multi-point constraint.

A description of the method of applying multi-point constraint equation is beyond the scope of this paper. The interested reader can find an excellent treatment of this topic in the ANSYS Theoretical Manual (1989). It should be noted however that the application of multi-point constraints adds non-zero entries into the $[\mathbf{0}]$ sub-matrices in the uncoupled matrices (4 and 5). This will increase the profile (or bandwidth) of some of the rows. Overall sparse matrix storage requirements using a skyline storage scheme may increase 30% or more. Re-numbering of the nodes to reduce storage requirements is important. Node re-numbering is currently done separately for the valve and backer finite element models. Sparse matrix storage could be reduced further by considering the valve, backer, and gap elements together in the renumbering process.

The final solution of the constrained equations is done using Choleski decomposition followed by forward and backward substitution. This scheme works well with the skyline sparse matrix storage scheme because the skyline does not change during decomposition.

2.6 The Contact Problem

Following the solution of the coupled and constrained equations at a point in time, all possible contact points are checked to see if there are any violations. The z-displacement result at each valve node is checked to see if it has gone in violation of either the valve plate or the backer. Also, the z-reactions are checked for all constrained valve nodes, both single-point and multi-point. If any violations have occurred, linear interpolation is done to determine at what point in time the violation occurred. The event is added to a database of all such occurrences. After every node is checked, the database is sorted by violation time and the smallest time is selected. Because solution times will increase dramatically with the addition of the flexible backer equations, a minimum time step, equivalent to 1/100 of a major time step, is used. The dynamic solution is conceptually rolled back to that point using linear interpolation and any node that went into violation *at or before this point in time* has its constraints changed accordingly. A node with a displacement violation is constrained, and a node with a reaction violation will have its constraint removed. A coefficient of restitution of zero is assumed and velocities and accelerations of nodes with changing constraints are calculated accordingly.

3. RESULTS

Figure 1 shows a valve plate sub-assembly with discharge valve and backer from a refrigeration compressor. Figure 2 shows the FEA model of the discharge valve backer before and after assembly of the cylinder head. Figure 3 shows the resulting stress contours for the valve using a *rigid* backer at the time step where the stress is at its peak. Note the stress concentration on the right where the backer is touching the valve. Figure 4 shows the valve stresses at the same time step using the model that includes the *flexible* backer. Note that because the backer was allowed to move, the stress concentration is gone. Figure 5 shows the valve stresses using the flexible backer at the time step where the stresses are peak. Note that this occurs earlier in the cycle. The effective lift was found to be larger with the flexible backer which translates into a larger flow area. The model with flexible backer gives much improved predictions of both stresses and flow areas. These results are in keeping with actual experience with this valve.

Figure1: Valve plate sub-assembly showing discharge valve and backer

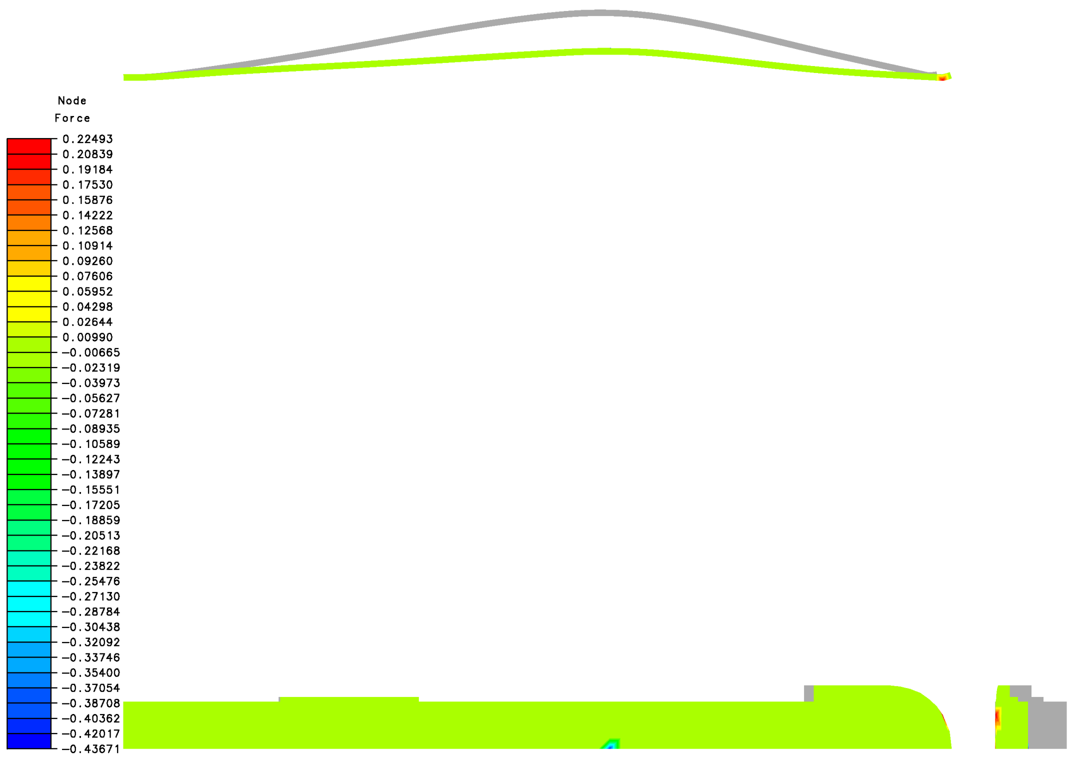

Figure 2: Discharge valve backer before and after deflection by assembly of cylinder head. Color contours show reaction forces.

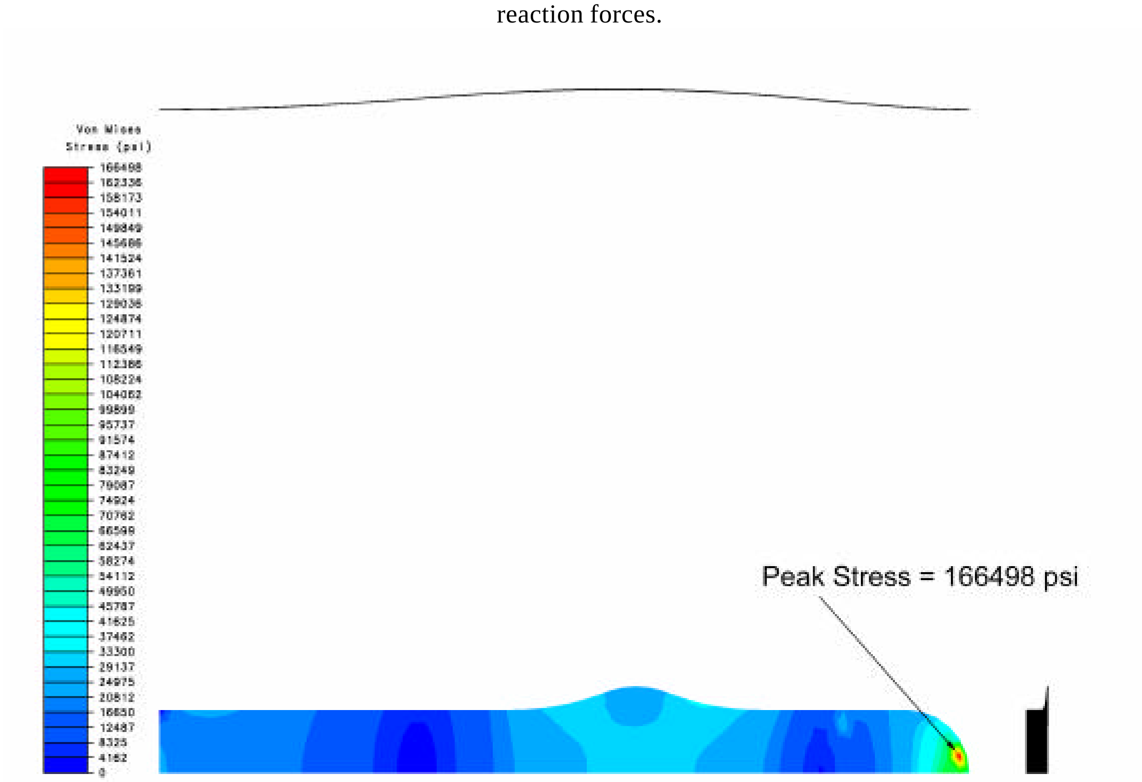

Figure 3: Discharge valve stresses with rigid backer at crank angle = 350.5 degrees (peak stress time)

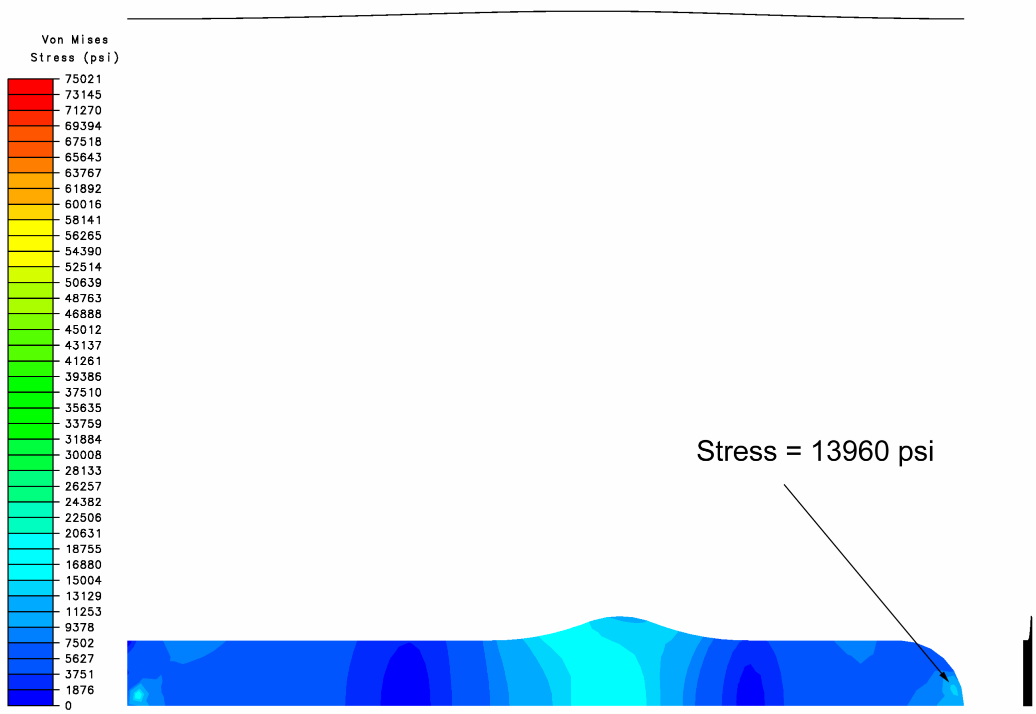

Figure 4: Discharge valve stresses with flexible backer at crank angle = 350.5 degrees

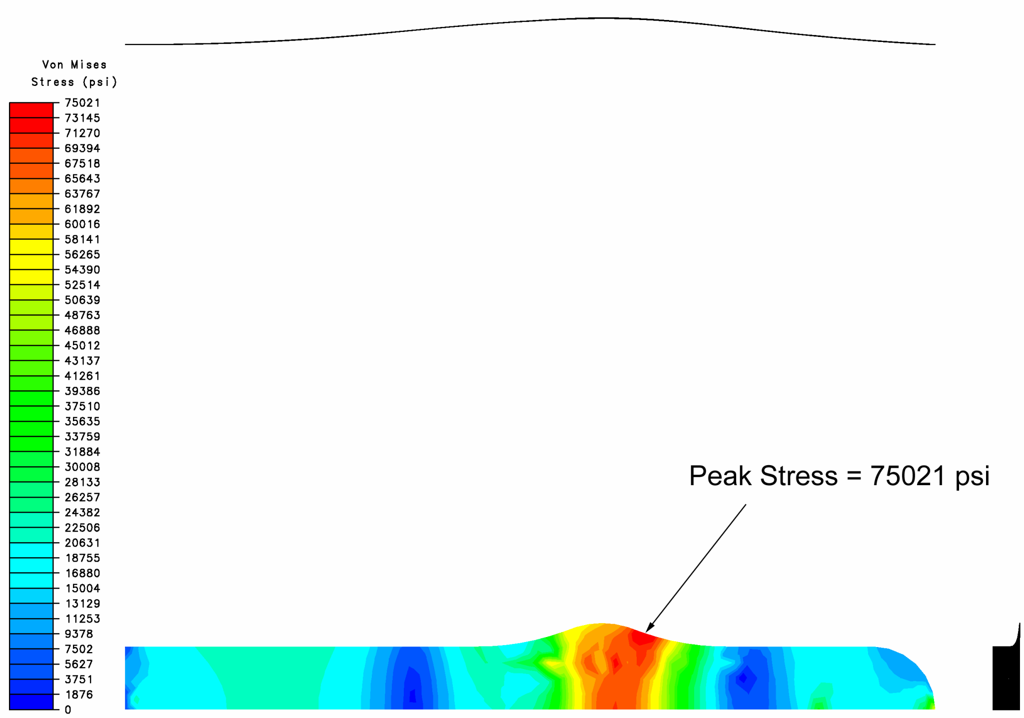

Figure 5: Discharge valve stresses with flexible backer at crank angle = 336.0 (peak stress time)

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