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Boundary Layer Leakage Model

Jan Prins
Danfoss

BOUNDARY LAYER LEAKAGE MODEL (Outline and Results)

Jan Prins

Danfoss A/S, Central Compressor R&D
6430 Nordborg, Denmark
prins@danfoss.com

ABSTRACT

This paper presents a model for gas leakage that takes the development of the velocity profile into account. It includes the physical ideas behind the model, a brief discussion of the mathematical approach and a comparison with experimental data.

1. INTRODUCTION

Leakage is one of the major sources of energy and capacity losses in positive displacement compressors. It is therefore desirable to have a reliable theory of leakage flow. Unfortunately, such a theory is not available.

In earlier work, together with Infante-Ferreira (1998), a selection of leakage models was validated by making a comparison with experimental data. None of those models showed general validity. Some work for one experiment but fail for another. All of these models were posed ad hoc; assumptions were made in an intuitive manner without regarding the physical properties of leakage flows.

Further research, again with Infante-Ferreira (2003) and extended since, aimed to identify those features that affect the flow most. The results suggest that a key role is played by the development of the velocity profile, a feature that was neglected in all models that were found in literature.

The leakage model that is presented in this paper tries to include the development of the velocity profile, together with a number of other features that are expected for leakage flows (valid for the experiments that are used for validation later on):

- Continuum theory is probably valid.
- The distribution of the gap height is the only relevant geometric feature.
- Turbulence is unlikely.
- Compressibility cannot be ignored.

Because the model relies heavily on boundary layer theory, it was given the name *boundary layer leakage model*, or, *BLLM* for short.

This paper will mainly focus on the physical ideas behind the *BLLM*, only giving hints to the actual model equations. It limits itself to pure gas flows and to converging-diverging leakage path geometries that are long in comparison to the gap height. Finally, it ignores the possibility of moving walls.

2. VELOCITY PROFILE DEVELOPMENT

Consider the gas flow through a converging-diverging path as shown in figure 1. The flow accelerates rapidly in the converging part. This is known to flatten the velocity profile. This suggests to model the profile by a flat part in the center of the flow and and only thin viscous boundary layers at the walls. At the wall itself, the velocity must be zero (the so called no-slip condition).

As the acceleration is progressive, the flattening of the profile is expected to continue towards the throat. The boundary layer may still grow in the inlet, but is expected to slink to a minimum near the throat.

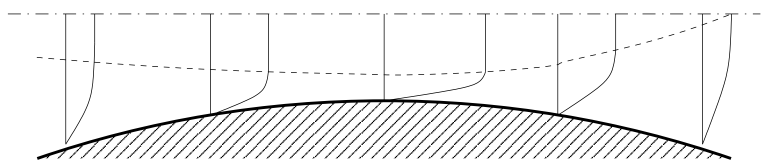

Figure 1: Sketch of the expected velocity profile development in the lower half of a symmetric path. It shows both the profile and the boundary layer thickness.

When the flow is fully subsonic, it will decelerate in the diverging part of the path. As a consequence, the boundary layer grows quickly. Obviously, its growth is limited by the gap height. When the boundary layer thickness reaches the symmetry line, it reached its maximum and the flow is fully viscous.

3. OUTFLOW PHENOMENA

The natural boundary conditions are the pressure and temperature at the inlet plus the pressure at the outlet. How the outlet condition should be interpreted is not obvious. The *BLLM* takes three options into account:

- The flow chokes when the outlet pressure is sufficiently low. In that case, it has no effect on the mass flow.
- When the outlet pressure is too high for choking, the flow remains fully subsonic. In the simplest model, it decelerates in the diverging part of the flow. It comes to rest at the outlet pressure. Thus the outlet pressure must be interpreted as the stagnation pressure.
- Another situation occurs when the subsonic flow separates from the wall. It then continues as a free jet (see figure 2). Between the wall and the jet appears a recirculation zone. The pressure difference between the jet and the recirculation or within the recirculation is expected to be small. Thus the outlet condition must be interpreted as the pressure in the separation point.

These three options can be implemented in a computer code by using a shooting method. Instead of the outlet boundary condition, a guess is made for the velocity at the inlet. With only inlet boundary conditions, the flow can be solved in down-stream direction.

The solver is terminated as soon as one of the three options is encountered. When the flow chokes, the inlet velocity was guessed too high. When the solution is fully subsonic and the obtained pressure is lower than the outlet boundary condition, the velocity was also too high. Thus a new guess can be made for the inlet velocity. By repeating this process, the inlet velocity can be determined with any required accuracy.

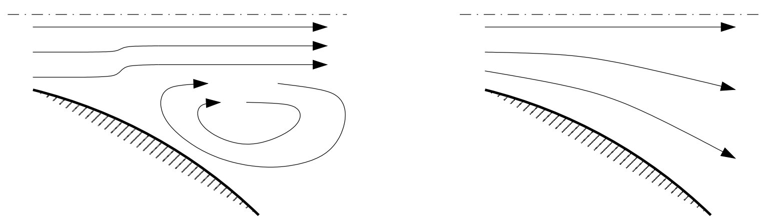

Figure 2: Outlet flow with (left) and without (right) separation.

4. SIMILARITY FUNCTION

So far, the velocity profile inside the boundary layer (see figure 3) has been left open. It plays an important role in the development of the boundary layer and in separation of the flow.

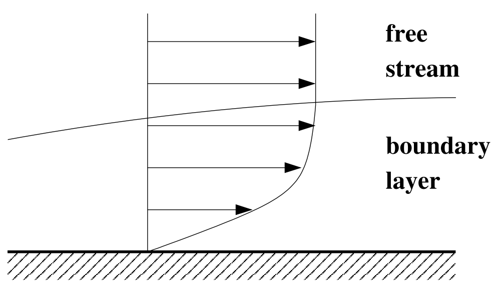

Figure 3: The velocity profile near a wall. It is flat in the free stream in the center of the path and decreases in the boundary layer to zero at the wall.

A convenient way to describe the velocity profile is to non-dimensionalize it. The result is known as the similarity function:

$$f(\zeta) = \frac{u}{u_\infty} \qquad \text{with} \qquad 0 \leq \zeta = \frac{y}{\delta} \leq 1 \tag{1}$$

Here u is the local velocity, u_∞ is the velocity on the symmetry line, y is the distance to the wall and δ the boundary layer thickness. Thus the dimensionless coordinate ζ gives the relative position within the boundary layer. It equals zero at the wall and one at the edge.

The choice for the similarity function is arbitrary, provided that it is reasonable. A convenient form that is often used is a cubic polynomial:

$$f(\zeta) = f_0 + f_1\ \zeta + f_2\ \zeta^2 + f_3\ \zeta^3 \tag{2}$$

The coefficients need to be determined by boundary conditions. Three of these are obvious:

- Free stream velocity at the wall, $f(0) = 0$.
- Continuity at the edge, $f(1) = 1$.
- Smooth transition to the central flow, $f'(1) = 0$.

As the cubic function has four coefficients, a fourth equation is needed. The *BLLM* uses a fixed value for the second derivative at the wall, $f''(0) =$ constant. Figure 4 shows the similarity function for different values of $f''(0)$. It shows some interesting features:

- For $-6 \leq f''(0) \leq 6$ the function shows no surprises.
- For $f''(0) < -6$ there is a region in which the velocity is larger than that of the central flow ($f > 1$). Its it hard to imagine this as realistic behavior.
- For $f''(0) > 6$ there appears counter flow near the wall ($f < 0$). This is the separation that was described above.

The fixed value of $f''(0)$ is therefore limited to the interval $-6 \ldots 6$.

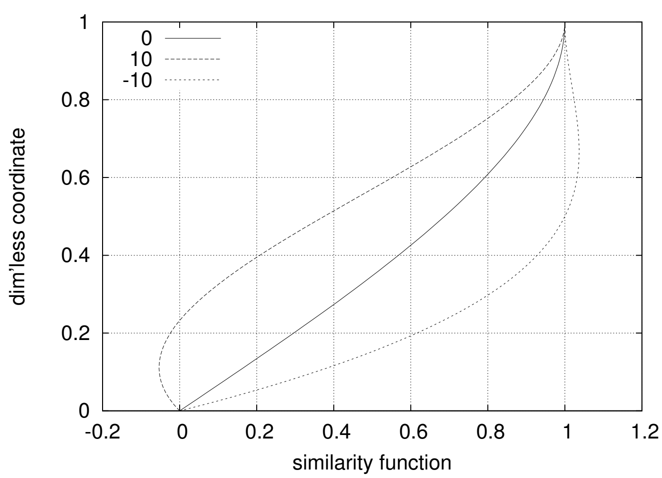

Figure 4: The cubic similarity function for three different values of $f''(0)$.

There is in fact a fourth boundary condition for the similarity function (which hasn't been included yet). It is useful to predict the point where the flow separates from the wall. This condition follows after applying the momentum equation to the wall:

$$f''(0) = \frac{\delta^2}{\mu \; u_\infty} \; \frac{dp}{dx} \tag{3}$$

Where p is the pressure and μ is the dynamic viscosity. The right hand side can be evaluated from the flow. The *BLLM* assumes separation when it reaches six, conform the cubic velocity profile.

5. MATHEMATICAL MODEL

There is not enough room here to go into the details of the model equations, only a general discussion will be given. Much is plain boundary layer theory that is well described in many text books, see for example Bird *et al.* (2002), Hoogendoorn and van der Meer (1978) or the standard work on boundary layer theory by Schlichting (1979).

The equations of flow are of course the Navier-Stokes equations, complemented with the conservation laws for mass and energy. Luckily, the special conditions of leakage flows allow a more simple formulation. Because the length scales parallel and perpendicular to the flow differ orders of magnitude, it can be shown that many term in the equations will be small. These terms are removed. The result is a compressible form of boundary layer equations. These equations are two-dimensional. By integration over a cross-section perpendicular to the flow, they are one-dimensionalized.

The boundary layer y-momentum equation reduces to $\partial p/\partial y = 0$. Thus the pressure is constant over a cross-section. The same is assumed for the temperature.

The concept of the similarity function was first introduced by Von Karman. He devised an approximative method the solve the boundary layer thickness. Though the method was originally designed for incompressible flow, the same technique can be used for compressible flow.

The result of this is a set of four one-dimensional differential equations for the pressure, density, velocity and boundary layer thickness. These are simultaneously integrated in down-stream direction starting from the inlet. The inlet velocity is guessed and improved with the shooting method that is described above.

6. VALIDATION

Peveling (1988) published a series of leakage experiments. In these, air was blown through more than twenty different geometries. He distinguished three types of behavior, depending on the geometry. Figure 5 shows a rough sketch of the three geometry classes:

- Class **A**: Very short channels in which wall friction is negligible.
- Class **B**: Paths with a long region in which the height is close to minimal. The wall friction has therefore a strong influence.
- Class **C**: Paths that are long, but, with a relative short region in which the height is close to minimal. This class fits in between the other two.

The class **A** geometries do not allow the boundary layer approach and are therefore not used for validation. Peveling presents experimental results for the geometries shown in figure 6. The gap height distribution is computed along the central lines (radii 16.25 and 64 respectively). The gap height, W, is taken perpendicular to these lines.

Figure 7 shows a typical distribution of the boundary layer thickness (class **C**, gap height 0.2 mm, pressure ratio 1.4). It confirms the original idea about the development of the velocity profile: the acceleration in the inlet suppresses the boundary layer. The thickness decreases towards the throat. After the throat it grows rapidly until the boundary layer fills the entire leakage path.

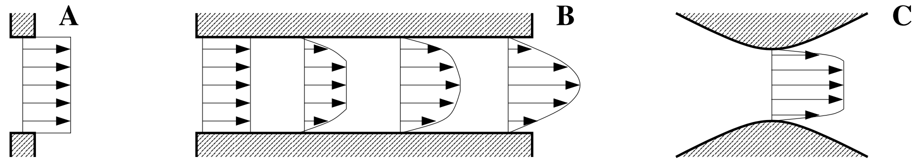

Figure 5: Sketch of Peveling's geometry classes.

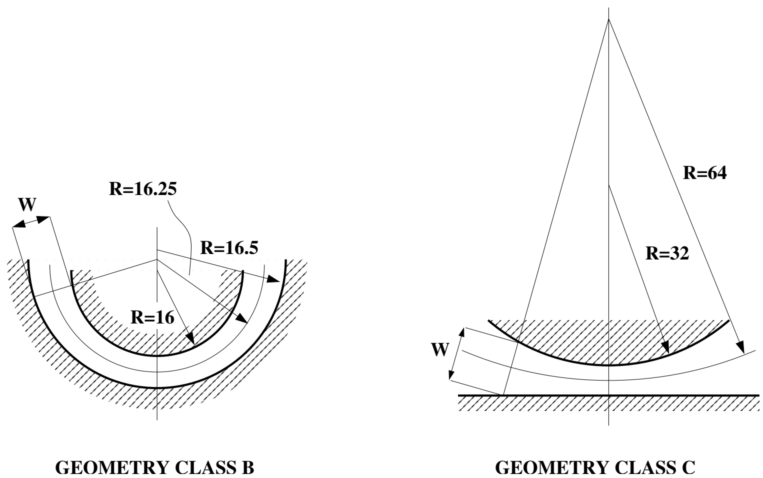

Figure 6: Geometries used for validation of the *BLLM*.

The three simulations are run with the same conditions, but, with different initial values of the boundary layer thickness. This initial difference disappears fairly quick in down-stream direction. The effect of the initial value on the on the mass flow rate is limited to a few percent.

Figures 8 and 9 show the comparison between the *BLLM* and the experiments. The mass flow is scaled with the theoretical value of choking isentropic flow. The trends are obviously correct, but, the mass flow rate is generally overestimated by the *BLLM*. The errors are within 25% and 12% for the class **B** and **C** geometries respectively.

7. SUMMARY AND CONCLUSIONS

Insights obtained from earlier research have let to concept of a leakage model that focuses on the development of the velocity profile. It pictures a flow with a flat profile in the center and boundary layers at the walls. Furthermore, it introduces the possibility of separation in the treatment of the outlet boundary condition.

The model reproduces Peveling's experiments within 12% and 25%, for class **B** and **C** geometries, which is not satisfactory, but, good in comparison to other models (see Prins and Infante-Ferreira, 1998).

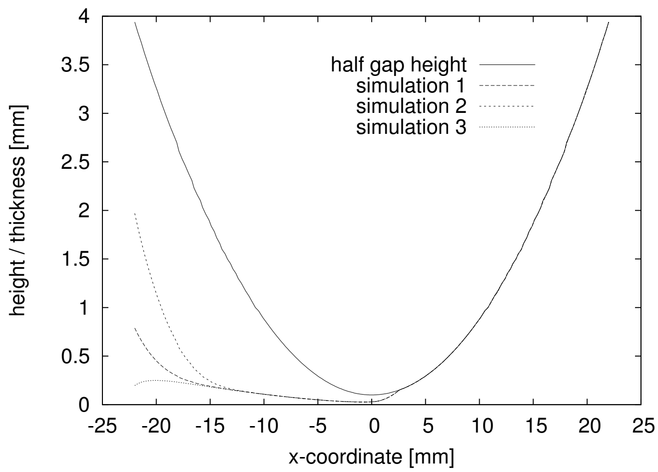

Figure 7: Typical development of the boundary layer thickness.

There is hope for improvement by including the fourth boundary condition for the similarity function. This makes the shape of the velocity profile inside the boundary layer adaptive to the local conditions of the flow. It may be expected that it refines the effect of wall friction.

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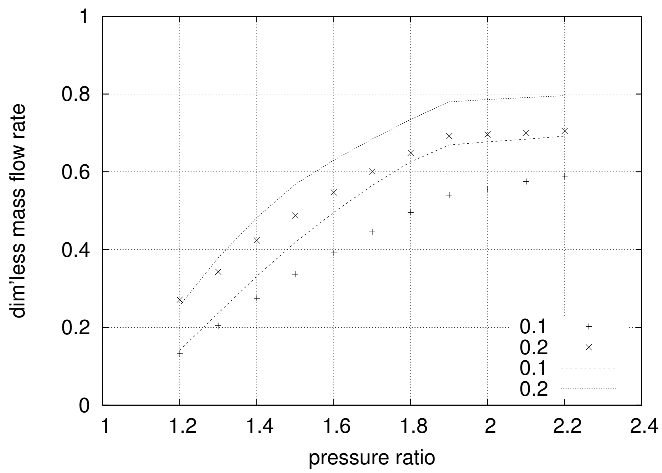

Figure 8: Comparison between the *BLLM* (lines) and experiments (points) for the class **B** geometry. The index gives the gap height in [mm].

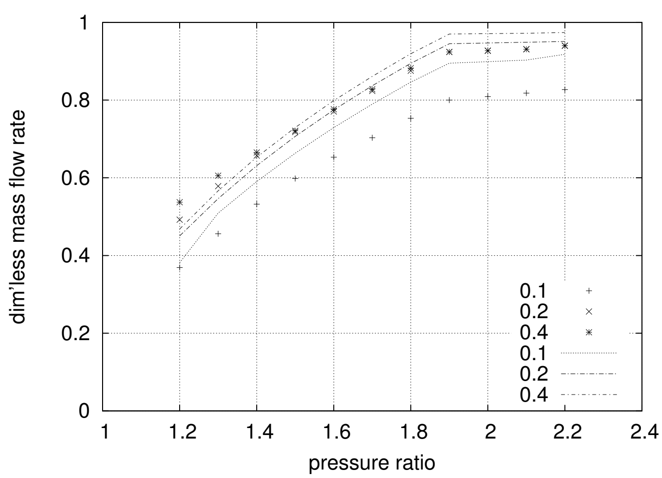

Figure 9: Comparison between the *BLLM* (lines) and experiments (points) for the class **C** geometry. The index gives the gap height in [mm].