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Compressor Rigid-Body Vibration Measurement

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ABSTRACT

A practical method is proposed to determine compressor rigid-body vibration. The method is based on measured linear accelerations to calculate linear and angular velocities of a rigid-body. Potential errors of the proposed method are discussed. A test process is also presented for validating the method.

INTRODUCTION

Compressors are one of main vibration or noise sources in residential air conditioning (AC) or heat pump (HP) outdoor units. Compressor vibrations transmitting to other components in the units through refrigerant pipelines and compressor grommets could cause noise problems or system failures. Knowledge about compressor vibration is very beneficial for system designs in terms of reducing noise and improving reliability. For a rotary type compressor, rotational motions could be dominant. Overall vibration at any point on the compressor outside surface is due mainly to rotational motions. Therefore, besides translational vibrations, it is necessary to measure rotational vibrations as well in order to fully quantify compressor motions. Since there is no simple device available to measure rotational vibration directly, it is desirable to develop a methodology that can be easily implemented in practice to determine compressor vibrations including both linear and rotational motions.

In this paper, an algorithm based on rigid-body kinematics is proposed to determine rotational vibrations from linear acceleration measurements. The paper is constituted in two parts. In the first part, basics are introduced as to principle, formulation and limitation of the algorithm. In the second part, an experimental validation on the algorithm is presented. Also discussed are some possible practical applications with the use of the algorithm.

1. FUNDAMENTALS

1.1 Rigid-body Assumption

When a compressor is in operation, compressor shell deformations at any locations may be negligible compared to overall motions. This is because the shell is relatively stiff and the compressor is usually linked to other structures with flexible grommets and pipes. As a result, it is reasonable to treat a vibrating compressor simply as a rigid-body. It should be noted that this approach is sufficient at low frequencies but may not be accurate at high frequencies.

1.2 Velocity Relationship

Assume vibration at an arbitrary point on the compressor, *p*, is interested and its location is specified as in the following equation:

$$\vec{R}_p(t) = \vec{R}_o(t) + \vec{r}_p(t)\,; \tag{1}$$

where, refer to Figure 1, $\vec{r}_p(t)$ is the instantaneous location vector for point *p* in the mobile Cartesian coordinate system *o*-***ijk***; $\vec{R}_p(t)$ and $\vec{R}_o(t)$ are the instantaneous location vectors for points *p* and *o* in the fixed Cartesian coordinate system *O*-***IJK***, respectively.

The coordinate system *o*-***ijk*** is glued to the compressor and its origin is positioned at a reference point inside the compressor. It is a good practice to choose reference point close to compressor c.g. The coordinate system *o*-***ijk*** is also referred to as the compressor coordinate system. The origin and orientation of the compressor coordinate system vary constantly with the compressor due to vibration. Since no deformation is considered, the distance between point *o* and point *p* does not change with the time, i.e., the following equation holds:

$$\frac{d\|\vec{r}_p(t)\|}{dt}=0. \tag{2}$$

The instantaneous velocities at points *p* and *o*, $\vec{v}_p(t)$ and $\vec{v}_o(t)$, can be obtained by taking derivatives on the location vectors $\vec{R}_p(t)$ and $\vec{R}_o(t)$ with respect to the time, respectively, as shown in the following equations:

$$\vec{v}_p(t)=\frac{d\vec{R}_p(t)}{dt},\ \vec{v}_o(t)=\frac{d\vec{R}_o(t)}{dt}. \tag{3a, 3b}$$

Take derivatives on both sides of equation (1) with respect to the time and then combine with equation (3), the velocity at point *p* can be related to the velocity at point *o* as shown in the following equation:

$$\vec{v}_p(t)=\vec{v}_o(t)+\frac{d\vec{r}_p(t)}{dt}. \tag{4}$$

The second term on the right side of equation (4) results from the rotation of the compressor and is equal to the vector product between the instantaneous compressor angular velocity, $\vec{w}(t)$, and the location vector, $\vec{r}_p(t)$, i.e.,

$$\frac{d\vec{r}_p(t)}{dt}=\vec{w}(t)\times\vec{r}_p(t). \tag{5}$$

The derivation of equation (5) can be found in reference books on rigid-body kinematics. It is necessary to point out that the compressor angular velocity, $\vec{w}(t)$, does not change if the reference point is placed at any other locations. Substitute equation (5) into equation (4), the velocity at point *p*, $\vec{v}_p(t)$, can be computed in terms of $\vec{v}_o(t)$ and $\vec{w}(t)$ as follows:

$$\vec{v}_p(t)=\vec{v}_o(t)+\vec{w}(t)\times\vec{r}_p(t). \tag{6}$$

Equation (6) indicates that velocities at any points, which are inside or even outside the compressor shell, can be derived from the reference velocity and the compressor angular velocity. In other words, the compressor vibration is completely characterized after the reference velocity and the compressor angular velocity are determined. Since there are three independent components in each vector, six measurements are at least needed in order to describe compressor vibration. Detailed discussions about measurement requirements are given in the following sections.

1.3 Formulation

Vector form equation (6) has to be rewritten in component form for the purpose to discuss measurements of $\vec{v}_o(t)$ and $\vec{w}(t)$. Assume that vectors $\vec{v}_p(t)$, $\vec{v}_o(t)$, $\vec{w}(t)$ and $\vec{r}_p(t)$ are expressed in the compressor coordinate system as

$$\vec{v}_p(t)=v_{px}(t)\boldsymbol{i}+v_{py}(t)\boldsymbol{j}+v_{pz}(t)\boldsymbol{k},$$
$$\vec{v}_o(t)=v_{ox}(t)\boldsymbol{i}+v_{oy}(t)\boldsymbol{j}+v_{oz}(t)\boldsymbol{k},$$
$$\vec{w}(t)=w_x(t)\boldsymbol{i}+w_y(t)\boldsymbol{j}+w_z(t)\boldsymbol{k},$$

and

$$\vec{r}_p(t)=x_p\boldsymbol{i}+y_p\boldsymbol{j}+z_p\boldsymbol{k},$$

respectively. The component form of equation (6) in the compressor coordinate system can then be expressed as shown in the following equations:

$$v_{px}(t)=v_{ox}(t)+z_pw_y(t)-y_pw_z(t), \tag{7}$$
$$v_{py}(t)=v_{oy}(t)+x_pw_z(t)-z_pw_x(t), \tag{8}$$
$$v_{pz}(t)=v_{oz}(t)+y_pw_x(t)-x_pw_y(t). \tag{9}$$

It should be noted that the location vector $\vec{r}_p(t)$ has constant components in the compressor coordinate system and its variation over the time is due simply to the change of the orientation of the base vectors, ***i***, ***j*** and ***k***.

For convenience, the direction of ***k*** is chosen to be aligned with the compressor vertical axis as shown in Figure 1. In such a case, $w_z(t)$ describes compressor torsional motions and the other two components, $w_x(t)$ and $w_y(t)$, describe compressor rocking motions in ***i*** and ***j*** directions, respectively. With equations (7) to (9), the components of the reference velocity and the angular velocity in the compressor coordinate system can be determined by measuring linear velocities at selected locations and in selected directions.

Figure 2 shows proposed four pairs of measurements to determine (v_{ox}, v_{oy}, v_{oz}) and (w_x, w_y, w_z). Each of measured quantities is designated by an over bar. Calculations for all these components are given in the following:

1. $w_z(t)$ and $v_{oy}(t)$ can be determined by measuring $\bar{v}_{1y}(t)$ at (x_1, 0, 0) and $\bar{v}_{2y}(t)$ at (x_2, 0, 0):

$$w_z(t) = \frac{\bar{v}_{1y}(t) - \bar{v}_{2y}(t)}{x_1 - x_2},\ v_{oy}(t) = \frac{x_1\bar{v}_{2y}(t) - x_2\bar{v}_{1y}(t)}{x_1 - x_2};\ x_1 \neq x_2. \qquad \text{(10a, 10b)}$$

2. $w_y(t)$ and $v_{oz}(t)$ can be determined by measuring $\bar{v}_{3z}(t)$ at (x_3, 0, 0) and $\bar{v}_{4z}(t)$ at (x_4, 0, 0):

$$w_y(t) = \frac{\bar{v}_{3z}(t) - \bar{v}_{4z}(t)}{x_3 - x_4},\ v_{oz}(t) = \frac{x_3\bar{v}_{4z}(t) - x_4\bar{v}_{3z}(t)}{x_3 - x_4};\ x_3 \neq x_4. \qquad \text{(11a, 11b)}$$

3. $w_x(t)$ and $v_{oz}(t)$ can be determined by measuring $\bar{v}_{5z}(t)$ at (0, y_5, 0) and $\bar{v}_{6z}(t)$ at (0, y_6, 0):

$$w_x(t) = \frac{\bar{v}_{5z}(t) - \bar{v}_{6z}(t)}{y_5 - y_6},\ v_{oz}(t) = \frac{y_5\bar{v}_{6z}(t) - y_6\bar{v}_{5z}(t)}{y_5 - y_6};\ y_5 \neq y_6. \qquad \text{(12a, 12b)}$$

4. $w_x(t)$ and $v_{oz}(t)$ can be determined by measuring $\bar{v}_{7x}(t)$ at (0, y_7, 0) and $\bar{v}_{8x}(t)$ at (0, x_8, 0):

$$w_z(t) = \frac{\bar{v}_{7x}(t) - \bar{v}_{8x}(t)}{y_7 - y_8},\ v_{ox}(t) = \frac{y_7\bar{v}_{8x}(t) - y_8\bar{v}_{7x}(t)}{y_7 - y_8};\ y_7 \neq y_8. \qquad \text{(13a, 13b)}$$

The proposed algorithm suggests using eight linear velocity measurements. As a result, two components, $w_z(t)$ and $v_{oz}(t)$, are computed twice. The duplicated results may be used to check consistency of the measurements. All the measurement points in the proposed measurement arrangement are placed on the same plane that passes through the reference point and is normal to the compressor vertical axis. As mentioned previously, measurement points can be outside the compressor shell. Therefore, a plate-like adaptor may be used for easy installation of transducers and accommodation of irregular compressor shapes. To show the concept, a design of the compressor adapter is proposed in Figure 3. Four little blocks on the outside of the adapter indicate locations for multi-axial transducers or transducer mounts. Two intersected dash lines, each of which runs from one block to the opposing one, determine the origin and orientations of the compressor coordinate system. For the arrangement shown in Figure 3, there are one horizontal and one vertical measurement at each location, i.e., these relationships, $x_1 = x_3$, $x_2 = x_4$, $y_5 = y_7$ and $y_6 = y_8$, apply to equations (10) to (13) accordingly.

2. DISCUSSION

Equations (10-13) show linear relationships between the measured and calculated quantities in the time domain. Performing Fourier Transform on both sides of these equations, it can be concluded that the same relationships hold in the frequency domain as well.

Equations (10-13) require velocity inputs as in the time or frequency domain. In practice, velocities are usually converted from accelerations measured with accelerometers that can be readily mounted onto measured objects. However, errors might be involved because of approximation introduced in the conversion. It is worthwhile to understand the impact of the converted velocity on the accuracy of the calculated results. In this regard, detailed discussions are given in the following section.

2.1 Simplification

Equations (10-13) are derived based on the rigid-body theory without additional assumption. Consequently, the accuracy of the calculated results depends upon the accuracy of the velocities inserted to the equations. In practice, velocities are usually converted from accelerations measured with accelerometers that can be readily mounted onto objects. The conversion is performed in the time domain as:

$$v_{px}(t) \approx \int a_{px}(t)dt\,,\; v_{py}(t) \approx \int a_{py}(t)dt\,,\; v_{pz}(t) \approx \int a_{pz}(t)dt\,; \tag{14}$$

or in the frequency domain as:

$$v_{px}(f) \approx \frac{1}{j2pf} a_{px}(f)\,,\; v_{py}(f) \approx \frac{1}{j2pf} a_{py}(f)\,,\; v_{pz}(f) \approx \frac{1}{j2pf} a_{pz}(f)\,; \tag{15}$$

where $j = \sqrt{-1}$ and f is the frequency in Hertz. All functions in the frequency domain are complex functions in general. This conversion process is, however, not carried out in an exact fashion. Therefore, errors might be involved in the calculated velocities using converted velocities either in the time domain or in the frequency domain.

Assume that the component form of the acceleration at point p, $\vec{a}_p(t)$, is expressed in the compressor coordinate system as follows:

$$\vec{a}_p(t) = a_{px}(t)\boldsymbol{i} + a_{py}(t)\boldsymbol{j} + a_{pz}(t)\boldsymbol{k}\,. \tag{16}$$

Because it is attached to the compressor at point p, an accelerometer measures just one of the acceleration components in the compressor coordinate system, i.e., $a_{px}(t)$, $a_{px}(t)$, $a_{px}(t)$, or their combinations depending upon transducer orientation.

The acceleration is the derivative on the velocity with respect to the time, i.e.,

$$\vec{a}_p(t) = \frac{d\vec{v}_p(t)}{dt}\,. \tag{17}$$

However, in general, there is no simple relationship between acceleration and velocity components in the compressor coordinate system. In other words, it is generally true that $a_{px}(t) \neq dv_{px}(t)\big/dt$ as well as for $\boldsymbol{j}$ and $\boldsymbol{k}$ components. The reason becomes clear after expanding the right side in equation (17) in the compressor coordinate system:

$$\frac{d\vec{v}_p(t)}{dt} = \frac{dv_{px}(t)}{dt}\boldsymbol{i} + \frac{dv_{py}(t)}{dt}\boldsymbol{j} + \frac{dv_{pz}(t)}{dt}\boldsymbol{k} + v_{px}(t)\frac{d\boldsymbol{i}}{dt} + v_{py}(t)\frac{d\boldsymbol{j}}{dt} + v_{pz}(t)\frac{d\boldsymbol{k}}{dt}\,. \tag{18}$$

If three derivative terms of the base vectors on the right side in equation (18) are negligible compared to other terms in the equation, considering equations (16) to (18), the following equations show approximate relationships between acceleration and velocity components:

$$a_{px}(t) \approx \frac{dv_{px}(t)}{dt}\,,\; a_{py}(t) \approx \frac{dv_{py}(t)}{dt}\,,\; a_{pz}(t) \approx \frac{dv_{pz}(t)}{dt}\,. \tag{19}$$

The approximate relationships shown in equations (14) and (15) are outcomes of equation (19) after integrations in the time or frequency domain.

When a compressor is simply in translational motions, there is no change in the base vector orientations and hence it is perfectly fine to use the converted velocities. This is actually a trivial case for equations (14) and (15) to be accurate. In the case of small compressor vibration, which happens to be the case to most normal compressor operations, it is assumed that compressor reference velocity and angular velocity can be calculated from the converted velocities given in equations (14) or (15) with a acceptable accuracy. A special test process has been designed to verify this assumption.

2.2 Apparatus

The goal of the test is to verify the algorithm by comparing two torsional accelerations calculated with independent ways. One is calculated from measured linear accelerations based on the proposed algorithm. The other is calculated from measured forces based on rigid-body dynamics.

A test device to simulate a compressor for torsional vibration is constructed. In the device, as shown in Figure 4, a steel cylinder with approximate size and weight of a compressor could rotate freely but with very little lateral and vertical freedom of movement. The 10-inch long cylinder is machined to 7-inch diameter. Two cylinder end surfaces are machined flat and center-drilled. Three lugs are welded into sockets at 90-degree increments on the circumference of the cylinder at the mid point. The lugs are machined to have mounting surfaces that are flat and square in relation to the cylinder. The cylinder is then supported vertically between center points in a steel frame. The dimension of the frame is 16” x 14” x 12” made with 1-inch thick steel plate and the frame is constructed in such a manor to reduce the vertical and lateral movement of the test fixture. The cylinder dimension and transducer locations with positive measurement directions are also shown in Figure 4.

The test fixture is bolted to a concrete pad with a 3/4-inch thick steel plate attached to the top of the concrete pad. The pad is supported on four adjustable legs so that the test fixture could be properly leveled. The shaker head is aligned horizontally and vertically with the force transducer mounting lug and the shaker frame is bolted to the concrete pad.

Two accelerometers are mounted on the cylinder on the two opposing lugs to measure responses of the cylinder. A force transducer is mounted on the lug positioned 90 degrees from the accelerometer locations. The shaker is connected to a force transducer through a 1/8-inch-diameter stinger to measure input forces to the cylinder.

2.3 Comparison

Assume the two accelerometers are mounted at (d_a, 0, 0) and (-d_a, 0, 0) and their measurements are designated as $\bar{a}_1(f)$ and -$\bar{a}_2(f)$; where d_a is the distance from the accelerometers to the cylinder center vertical axis. The torsional velocity, $w_z(f)$, can then be calculated from the two measured accelerations with equations (10) and (15) as:

$$w_z(f) \approx \frac{\bar{a}_1(f)+\bar{a}_2(f)}{2d_a \cdot j2pf}. \qquad (20)$$

On the other hand, the shaker force is designed to generate pure torsional vibration and hence the torsional velocity can also be calculated as:

$$w_z(f) = \frac{\bar{L}_z(f)}{J_z \cdot j2pf}; \qquad (21)$$

where J_z is the moment of inertial of the cylinder about its center vertical axis and $\bar{L}_z(w)$ is the torsional moment due to the shaker force, i.e.,

$$\bar{L}_z(f) = \bar{F}(f) \cdot d_f; \qquad (22)$$

where d_f is the distance between the load cell and the cylinder center vertical axis and $\bar{F}(w)$ is the shaker force measured with the load cell. Refer to Figure 4, d_a, d_f and J_z are equal to 4 1/16”, 3 7/8” and 1,462 $(\text{lbm}\cdot\text{in})^2$, respectively.

Broadband random signals are fed into the shaker. The two acceleration responses and the one input force are measured simultaneously. The force is chosen as the reference signal. Magnitudes and phases are recorded for all the signals. Figure 5 shows one acceleration spectrum from 40 Hz to 400 Hz with the resolution of half Hz. The hump around 170 Hz could be caused by forces not measured with the load cell as indicated by the coherence plot shown in Figure 6. The two acceleration measurements are then normalized by dividing them with the force magnitude at each frequency. Because of linear vibration, this normalization will not change outcomes but make the comparison easier. The calculated torsional velocity spectra based on normalized accelerations and one-pound-unit shake force are shown in Figure 7. Also plotted are two curves with plus or minus twenty-percent deviation from the force-based torsional velocity spectrum. Except for the region around 170 Hz, two results are generally consistent. Therefore, the proposed algorithm with the converted velocities can be used to determine compressor rigid-body motions as long as its vibration is small.

3. APPLICATION

The calculated linear and angular velocities could be used to estimate resultant forces acting on the compressor. If the measurements are taken when the compressor is connected to flexible and light hoses and placed on a soft base, the estimated forces result mainly from compressor internal forces like unbalance force, gas pulsation in a typical frequency range of interest. It is not entirely unreasonable to assume that compressor internal forces do not change considerably so long as the compressor operates at a similar condition even with different pipe or grommet configurations. Consequently, the estimated forces could be used to predict pipe stresses for different pipe designs and grommet layouts with the help of Finite Element Method (FEM). After establishment of a reliable database of compressor vibration and grommet dynamics, the FEM becomes a valuable tool to help achieve better pipe designs within a shorter period of time. This numerical approach not only saves material costs and lab resources but also grasps comprehensive pipe vibration behaviors.

4. CONCLUSION

A simple and practical method is proposed to determine compressor rigid-body vibration. It has been demonstrated that the method provides accurate estimates when the compressor vibration is small. It is also discussed how measurements could be setup to determine compressor internal forces.

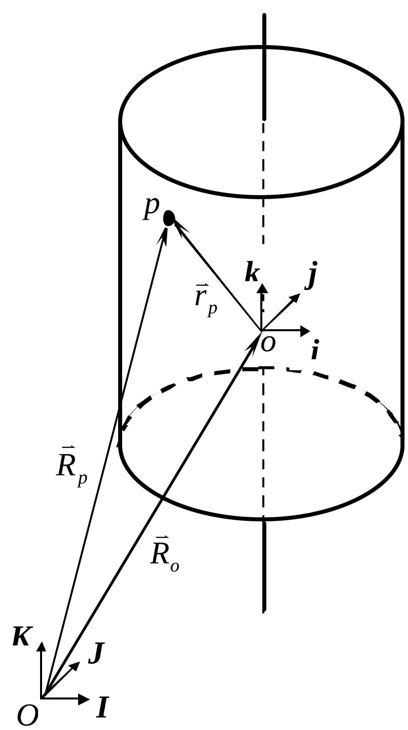

Figure 1 – Compressor and Coordinate Systems
O-***IJK***: fixed coordinate system; *o*-***ijk***: mobile coordinate system glued to compressor

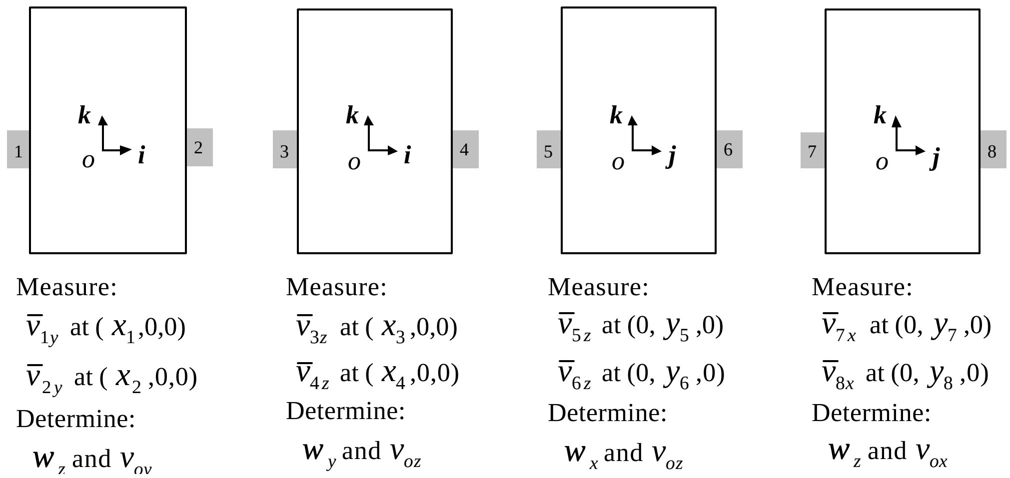

Figure 2 – Measurement Locations and Directions
Four pairs of measurements are used to determine (w_x w_y w_z) and (v_{ox} v_{oy} v_{oy})

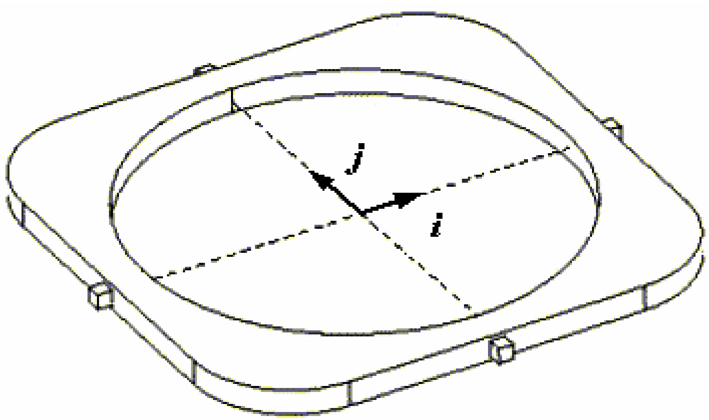

Figure 3 – Compressor Adapter for Transducer Installation

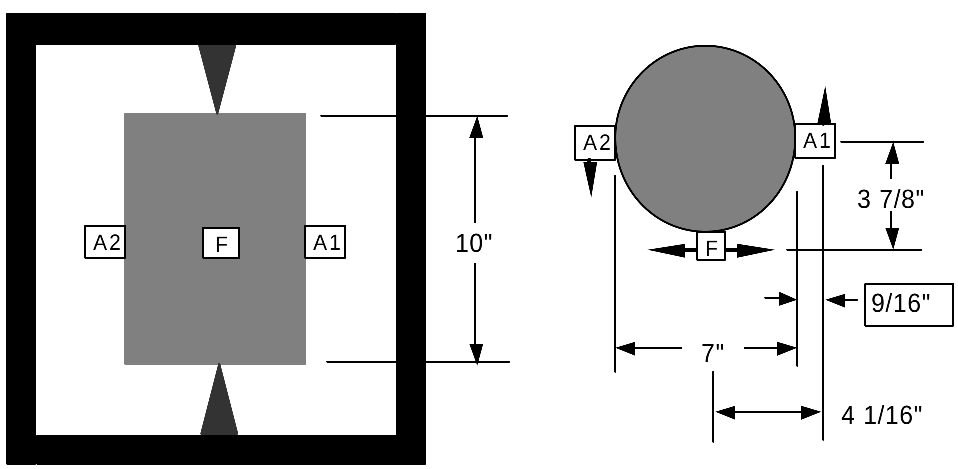

Figure 4 – Schematic Diagram of Test Setup
Two accelerometers and one load cell are used.

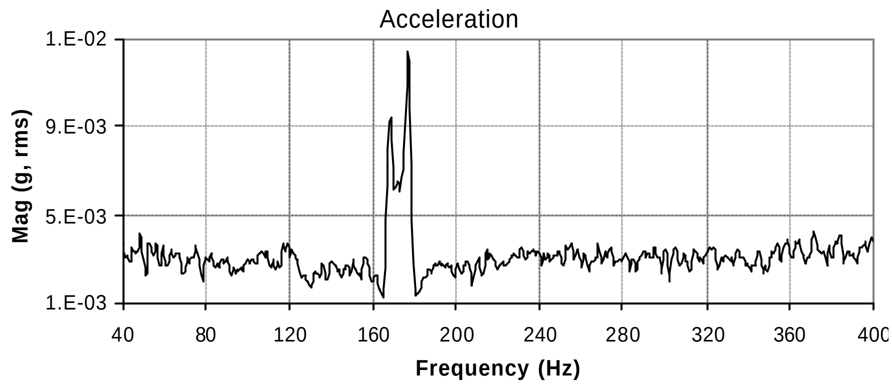

Figure 5 – Measured Acceleration Spectrum

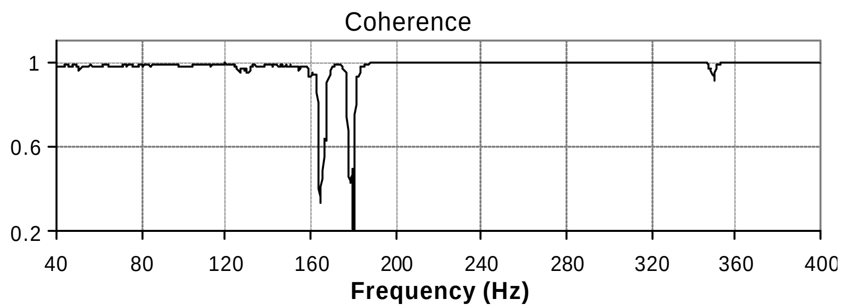

Figure 6 – Coherence between Force and Acceleration

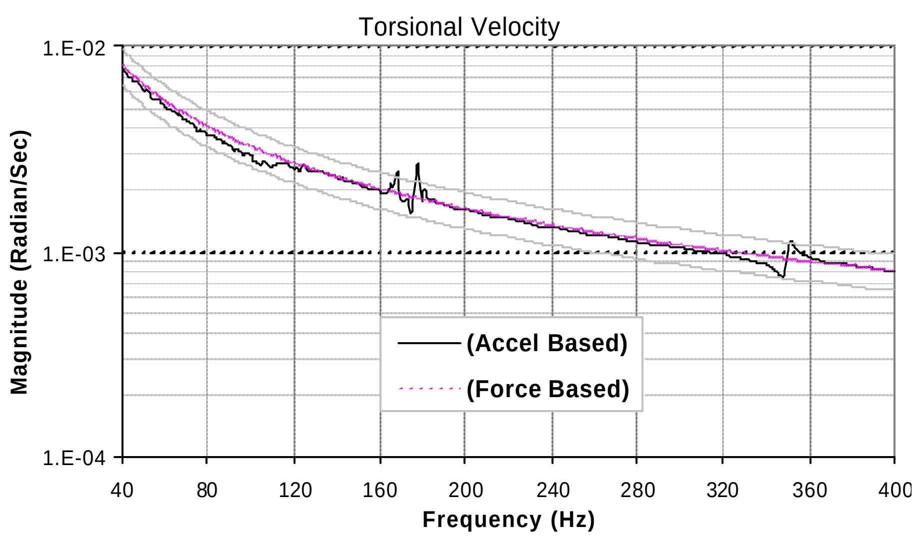

Figure 7 – Comparison of Torsional Ve locity
Dark color: acceleration based; Light color: force based.