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The analysis of all Casimir force experiments using a sphere-plate geometry requires the use of the proximity-force approximation (PFA) to relate the Casimir force between a sphere and a flat plate to the Casimir energy between two parallel plates. Because it has been difficult to assess the PFA's range of applicability theoretically, we have conducted an experimental search for corrections to the PFA by measuring the Casimir force and force gradient between a gold-coated plate and five gold-coated spheres with different radii using a microelectromechanical torsion oscillator. For separations $z < 300$ nm, we find that the magnitude of the fractional deviation from the PFA in the force gradient measurement is, at the 95% confidence level, less than $0.4z/R$, where R is the radius of the sphere.

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Since the modern experimental investigation of the Casimir force began ten years ago, most measurements [1–8] have used a configuration that studies the interaction of a spherical probe with a flat substrate. Because of the difficulty of maintaining parallelism between two flat plates, only one experiment [9] has been performed using the parallel-plate configuration originally envisioned by Casimir, for which the theory has been developed to a high degree. (See Refs. [10,11] for recent reviews of the Casimir interaction.) To compare theory with experiment, the sphere-plate approach relies on the proximity-force approximation (PFA) to relate the sphere-plate force to the potential energy of two parallel plates. Specifically, according to the PFA, the Casimir force between a sphere of radius R and a flat plate separated by a distance $z \ll R$ can be written as

$$F(z) \simeq F_{\mathrm{PFA}}(z) \equiv 2\pi R \mathcal{E}^{pp}(z), \quad (1)$$

where $\mathcal{E}^{pp}(z)$ is the separation-dependent interaction energy per unit area between two parallel plates composed of the same materials as the sphere-plate system [12,13]. Since its introduction by Derjaguin, the PFA has been widely used in both physics and chemistry to characterize short-ranged forces between curved bodies [12].

Although the use of the PFA for Casimir forces has been criticized (e.g., [14]), to date no deviations from the PFA have been observed experimentally using the sphere-plate geometry. This is not by accident, since the sphere-plate experiments have been conducted in a regime ($z/R \ll 1$) where use of the PFA is believed to be safe. Unfortunately, the unusual nature of the PFA prevents one from easily calculating its leading order corrections which could be used to assess its range of applicability [15]. In the analysis of experimental results, one presently estimates that the fractional error arising from the use of the PFA is $\simeq z/R$ [3,5,8], but this estimate cannot yet be supported by a calculation from first principles. If one is to use the results of Casimir force measurements to test theories of the Casimir force, or to set improved limits on new submicron-ranged forces, it is important to have a reliable understanding of the PFA. The purpose of this Letter is to address this problem empirically. Since the principal deviations from the PFA arise from curvature, it is possible to target experiments which isolate only these effects, thus avoiding the necessity of calculating roughness and conductivity corrections which are required in ordinary Casimir force measurements.

We begin by developing a phenomenology based upon the relevant length scales involved in a sphere-plate Casimir force experiment. If the bodies are smooth and perfectly conducting, the only distance scales are the sphere-plate separation z and the sphere radius R. Therefore, in this case, curvature corrections should depend only on the geometric factor z/R, and one can then expand the curvature effects of the exact Casimir force in powers of z/R [16]:

$$F_{\mathrm{Casimir}}(z, R) = 2\pi R \mathcal{E}^{pp}_{\mathrm{Casimir}}(z)\left[1 + \beta\frac{z}{R} + \mathcal{O}\left(\frac{z^2}{R^2}\right)\right], \quad (2)$$

where β is a dimensionless parameter characterizing the lowest order deviation from the PFA, and $\mathcal{E}^{pp}_{\mathrm{Casimir}}(z)$ is the R-independent Casimir energy/area for two parallel plates.

Equation (2) is consistent with recent calculations of the Casimir force for ideal smooth, perfectly conducting sphere-plate geometries [16–22]. Shaden and Spruch used a semiclassical approach to calculate the Casimir energy for an ideal sphere-plate setup, obtaining corrections to the PFA energy [17]. (See also Refs. [18,19].) These results can be used to find corrections to the PFA force, resulting in $\beta_0 \simeq -0.087$, where $\beta \equiv \beta_0$ for the ideal geometry. Gies and Klingmüler [20,21] use a worldline numerics approach for a scalar field satisfying Dirichlet boundary conditions to obtain a Casimir energy, from which one finds $\beta_0 \simeq +0.175$. Using an optical approach, Scardicchio and Jaffe find $\beta_0 \simeq -0.10$ [16]. Although these calculations use ideal bodies, and their authors indicate that the values may have large uncertainties, they provide evidence that a phenomenology based upon Eq. (2) is valid. (This also appears to be true for PFA violations in a cylinder-plate geometry [21–23].)

For real bodies other length scales, which characterize conductivity and roughness, enter and significantly modify the ideal Casimir force for $z \lesssim 1$ μm. For the experiments discussed below, we will consider only leading order effects so only the two dominant additional scales will be considered [11]. [As demonstrated by recent Casimir experiments (e.g., [8]), a much more detailed analysis is required if one wishes to obtain a precise comparison between theory and experiment.] If we assume as a first approximation that both bodies are composed of the same metal which can be characterized by the plasma model, the relevant length scale associated with finite conductivity corrections is the effective penetration length $\delta_p = c/\omega_p$, where c is the speed of light and ω_p is the plasma frequency of the metal [11]. For gold, which is used in our experiments, $\delta_p = 22$ nm [8]. The remaining important length scale, which is associated with corrections due to surface roughness, is the rms deviation of the surfaces from the ideal, δ_r [11]. (Here we assume both surfaces have the same δ_r.) Using atomic force microscopy, we determined that $\delta_r \lesssim 5$ nm for the spheres and substrate used in our experiments.

Conductivity and roughness effects lead to significant contributions to the Casimir force because the dimensionless ratios δ_p/z and δ_r/z can become measurably large for $z \lesssim 1$ μm. In contrast, the effects of roughness and conductivity on violations of the PFA should be significantly smaller since they enter through the dimensionless ratios δ_p/R and δ_r/R, where for nearly all experiments, $R \gg z$. (This will be made more quantitative below for our experiments.) Specifically, roughness and conductivity will modify Eq. (2) in two ways. First, the parallel-plate energy $\mathcal{E}^{pp}_{\rm Casimir}(z)$ will include these corrections as has been done in the analysis of experiments which assumes the PFA to be valid. Second, the leading order curvature-dependent corrections arising from δ_p/R and δ_r/R will lead to a z dependence of β. For example, in the simplest model which generalizes Eq. (2) by incorporating additional terms involving δ_p/R and δ_r/R, one finds to leading order $\beta(z) \simeq \beta_0 + \beta_p\delta_p/z + \beta_r\delta_r/z$, where β_0, β_p, and β_r are dimensionless parameters characterizing the lowest order deviations from the PFA due to pure curvature, finite conductivity, and surface roughness, respectively. (For a more detailed analysis of finite conductivity corrections to the PFA for the cylinder-plate geometry, see Ref. [23].) The key point, which will be used below, is to recognize that Eq. (2) is an expansion in powers of $1/R$ with coefficients (e.g., βz) that generally depend on z, roughness, and conductivity, but not on R.

Because the most precise Casimir data result from the measurement of the Casimir force gradient, rather than the Casimir force itself [7], one needs to modify the PFA-violation phenomenology for these experiments. It has become customary to use the PFA to relate the experimentally determined force gradient dF/dz to an effective pressure $P^{\rm eff}(z, R)$:

$$P^{\rm eff}(z, R) \equiv -\frac{1}{2\pi R}\frac{dF}{dz}, \qquad (3)$$

where R is the radius of the sphere used. If the PFA were exact, then $P^{\rm eff}(z, R) = P^{pp}(z)$, where $P^{pp}(z)$ is the pressure between two parallel plates composed of the same materials as the sphere and plate. Substituting Eq. (2) into Eq. (3), we obtain an expansion of $P^{\rm eff}(z, R)$ in powers of z/R, giving to leading order

$$P^{\rm eff}(z, R) = P^{pp}(z)\left[1 + \beta'\frac{z}{R} + \mathcal{O}\left(\frac{z^2}{R^2}\right)\right], \qquad (4)$$

where $\beta'(z)$ is a new dimensionless parameter that is related to β in Eq. (2) by

$$\beta'(z) = \beta\left[1 - \frac{\mathcal{E}^{pp}(z)}{zP^{pp}(z)}\right]. \qquad (5)$$

For smooth perfectly conducting parallel plates, $\mathcal{E}^{pp}_{\rm ideal}(z) = -\pi^2\hbar c/720z^3$ and $P^{pp}_{\rm ideal}(z) = -\pi^2\hbar c/240z^4$, so $\beta'(z) = \beta'_0 \equiv 2\beta_0/3$ is a constant for an ideal sphere-plate setup. For nonideal bodies, $\beta'(z)$ will include small contributions that depend on separation through Eq. (5).

To search for PFA-violating effects using the phenomenology developed above, we have carried out a series of experiments using a microelectromechanical torsion oscillator (MTO) to measure and compare the Casimir force between five Au-coated spheres and a substrate. The experimental setup has been described previously in Refs. [7,8]. For each sphere, a thin layer (~ 5 nm) of Cr was deposited, followed by $\sim$200 nm of gold. Similar layers were used on the plates of the MTOs. Each sphere-MTO combination was calibrated using the electrostatic force between the sphere and plate as described in Refs. [7,8]. In the present Letter, we report on two different kinds of measurements. The first is a direct measurement of the Casimir force between the sphere and the plate. These measurements were performed between 160 and 750 nm in 10 nm intervals, and each data point is the average of 10 different runs. Errors in positioning between different runs are small, $\delta z < 0.2$ nm, and hence have been disregarded (i.e., data points from different runs are considered to be at the average separation of the ten runs). The second set of measurements determined the gradient of the Casimir force for $164 \leq z \leq 986$ nm in 2 nm increments. In this case, the vertical separation between the sphere and the plate was changed harmonically with time, leading to a measurement of the z derivative of the Casimir force. As described in Eq. (3), within the PFA this is equivalent to measuring the Casimir pressure for a configuration of two parallel plates. The normalized results for the $R = 148.2$ μm sphere, which are representative of the measurements from the other spheres, are shown in Fig. 1. [The normalized force (pressure) is the observed force (pressure) divided by the force (pressure) between the same bodies, assuming that the PFA is valid, and that they are smooth and perfectly conducting.]

We measured the force and force gradient for five different spheres with radii $R = 10.5$, 31.4, 52.3, 102.8, and 148.2 μm. These radii were determined with an error of 0.2 μm by a direct measurement using a scanning electron microscope, and also *in situ* through the electrostatic calibration of the system [8]. The surface of each coated sphere and MTO was characterized by means of an atomic force microscope. The roughness was always smaller than 21 nm peak-to-peak with variances $\delta_r \lesssim 5$ nm, and the surface morphology was very similar to our previous results [8]. Thus, at $z = 160$ nm, $\delta_p/R \simeq 0.14z/R$ and $\delta_r/R \simeq 0.03z/R$, indicating that the PFA violations from the finite conductivity should contribute at about the $\sim$10% level at the shortest separations relative to the dominant z/R corrections, while the PFA violations arising from surface roughness should only contribute at the percent level. At much larger separations, only the pure curvature effects will be significant (until thermal corrections become large), so $\beta(z) \simeq \beta_0$.

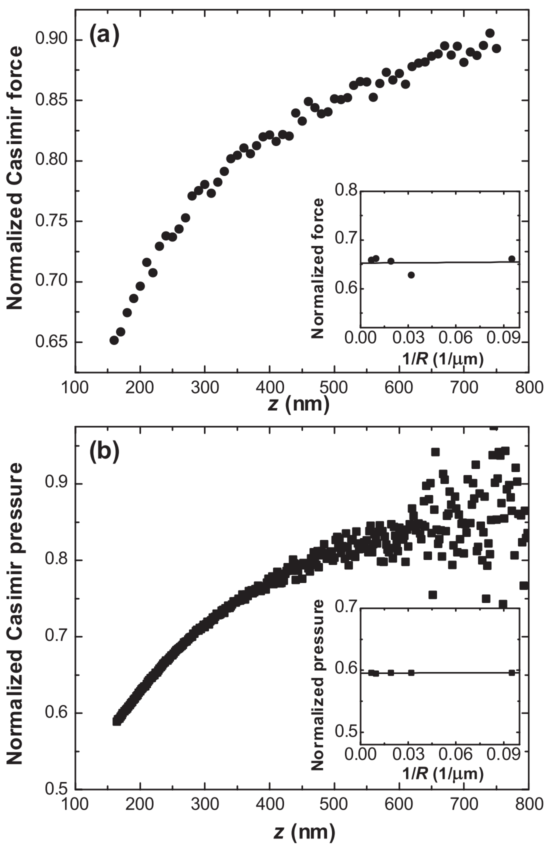

FIG. 1. Measured values of the force (a) and effective pressure (b), as defined by Eq. (3), obtained using the 148.2 μm sphere. They have been normalized to the force and pressure of idealized bodies as described in the text. The deviations from unity result from finite conductivity and roughness corrections. The inset in (a) exhibits the plot of the normalized force as a function of $1/R$ for $z = 170$ nm. The value of β is obtained from the best fit line as described in the text. The inset in (b) is a similar plot of the normalized pressure for the measurements obtained at $z = 170$ nm from which β' is obtained.

To search for violations of the PFA due to curvature from these data, one could compare the measured forces and pressures from a particular sphere-plate setup with those predicted by a theory which assumes that the PFA is valid, but includes all appropriate roughness and conductivity corrections. Any discrepancies could then be compared with those predicted by Eqs. (2) or (4). However, an alternative approach which is less susceptible to systematic errors is suggested by the fact that the leading order violations of the PFA are unique in their dependence on $1/R$. If one plots the observed normalized force at a fixed z as a function of $1/R$ for the five different spheres and fits a line to the results, one can obtain β by dividing the slope of the best fit line by its intercept multiplied by z. An analogous calculation can be used for the pressure data, where one obtains β'. Representative plots for the normalized force and the normalized pressure at $z = 170$ nm are shown in the insets of Fig. 1, and all the results for β and β' are displayed as functions of z in Fig. 2. The uncertainties shown are statistical, which dominate for the force case. For the pressure measurements, the largest systematic error arises from the uncertainty in the sphere radii, leading to an estimated total error of β' of ± 0.4 for $z < 300$ nm.

As seen in Fig. 2, β and β' obtained from our force and pressure measurements, respectively, are consistent with zero within experimental uncertainties, and exhibit no apparent z dependence, indicating no evidence of a violation of the PFA, or of finite conductivity and roughness corrections to the PFA. The values of β from the force measurements provide weaker constraints on β than those obtained for β' from the force gradient measurements, and the latter provide new and very useful constraints on the errors associated with the use of the PFA in these types of experiments. As noted earlier, in the analysis of previous experiments it has been assumed that the relative errors arising from the PFA are $\simeq z/R$ (i.e., $\beta \sim \beta' \simeq 1$) [3,5,8], but our results show that this overestimates the effect for $z \lesssim 400$ nm. For $z \lesssim 300$, we find $|\beta'(z)| < 0.4$ at the 95% confidence level. If we assume that $\beta'(z) \simeq 2\beta/3$, as in the ideal case, we obtain $|\beta| \lesssim 0.6$ for $z \lesssim 300$, which is compatible with all of the theoretical predictions discussed earlier.

In summary, it is important for a better understanding of the Casimir force and its future applications to accurately characterize deviations from the PFA for the sphere-plate geometry. Because the PFA is extensively used in chemistry as well as in physics [12], it is clearly essential to establish the limits of its validity. We have carried out an experimental approach which specifically searches for deviations $\mathcal{O}(z/R)$ arising from curvature characterized by parameters β and β'. Our results are consistent with recent theoretical predictions and provide clear boundaries for the

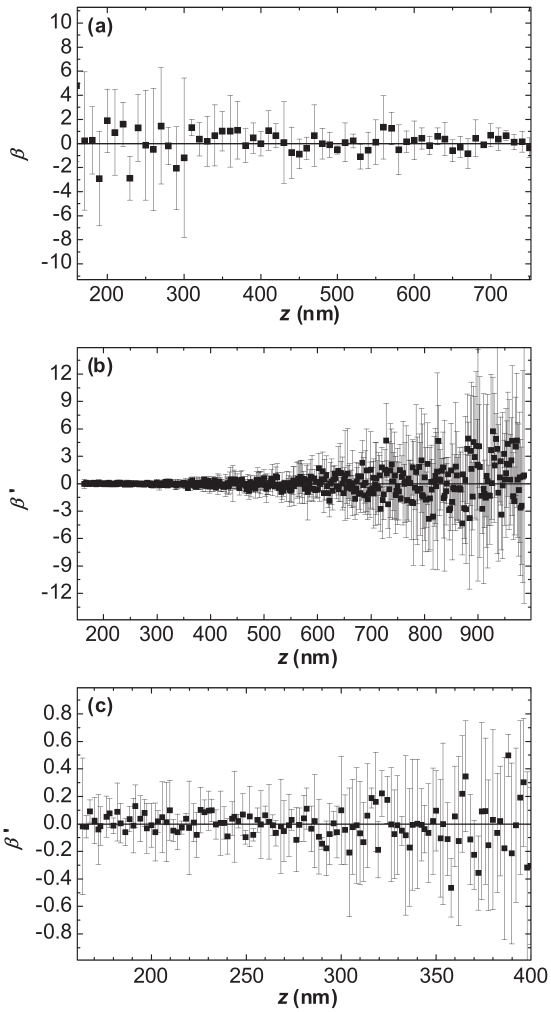

FIG. 2. (a) β vs z determined from the force measurements. (b) β' values obtained from the force gradient measurements for the entire range of separations. (c) Same as (b), but for 164 nm $< z <$ 400 nm. In all cases the error bars denote statistical 95% confidence level intervals.

use of the PFA in experiments by showing that the previously conjectured estimate of the error, $\beta \sim \beta' \simeq 1$, was conservative. Furthermore, the approach developed here can be adapted to search for PFA violation in the cylinder-plate geometry, as in the proposed experiment by Brown-Hayes *et al.* [24]. In the future, it may also be possible to observe PFA-violating composition effects noted recently by Noguez and Román-Velázquez [25] that would reveal additional novel aspects of the Casimir force.

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