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Little Dynamics

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Instantaneous Frequency: Another Tool of Source of Noise Identification

Hubert Bukac
Little Dynamics
21 County Road 1238
Vinemont, AL 35179-6301, USA
Tel.: (256) 775-2871, E-mail: hbukac@earthlink.net

Abstract

Instantaneous frequency was used to identify a source of noise that dominated noise spectrum of a compressor. Precise timing mark indicated top dead center position of the slider-crank mechanism. An analytical signal calculated by means of Hillbert Transform was used to calculate instantaneous frequency and the envelope of the signal. It was found, the frequency in question was associated with the discharge process.

1. Introduction

The compressor, either refrigeration or air-conditioning is always considered to be a major source of noise of any refrigeration or air-conditioning system. No matter how simple the compressor as a machine may seem to be, the mechanism of the noise generation and noise propagation is usually very complex. Human ears, as well as a measuring microphone, always perceive sound as the total acoustic pressure at the point in space where the microphone or a person's ear is located. At the particular point in space, the resulting noise is the superposition of all sources of noise. However, not all the acoustic frequencies to which the human ear is sensitive are perceived with the same feeling. Some frequencies are more annoying than the other ones. A compressor, as a cyclically operating machine, produces noise that has wide spectrum of frequencies. Usually, it is stipulated, the noise of a certain frequency that dominates the spectrum or that is most annoying is associated with a particular function of the compressor, such as the flow of the gas during the suction and discharge period, valve impacting its seat etc. The most usual way of noise reduction is the attenuation of the path through which the noise propagates. This approach always adds to the cost of the compressor or to the cost of the system. On the other hand, attenuating the noise at the location where it is generated is frequently much cheaper. Nevertheless, attenuating noise at the source may be effective only when it is known which part of the compressor generates the noise of that particular frequency and when.

Many methods of the source of noise identification are available, such as the spectral analysis by means of Fast Fourier Transform, Wavelet Analysis, Wigner Distributions, Ambiguity Function etc. The spectral analysis always produces a spectrum without any relation to the position of the mechanism of the compressor. A zoomed or gated spectral analyses, and Short Fourier Transform give better results because they produce spectra of a short signal of a finite length that is more easily correlated with the position of the mechanism. The information presented by these methods is in the frequency-amplitude domain. The Wavelet Analysis, Wigner Distribution or Ambiguity Function can provide information in both the amplitude-frequency domain as well as in the time-frequency domain. The later methods became widely used in many different applications. However, they have one common disadvantage in that, they require proper selection of wavelets or kernels by the user. In the dependence on the choice of wavelets or kernels, the results may dramatically vary. On the other hand, the instantaneous frequency and instantaneous amplitude method is a straight forward. It does not require any intervention by the user. The instantaneous frequency and instantaneous amplitude are true frequency and true amplitude contained in the time history at every instant of time. The instantaneous frequency and instantaneous amplitude are both presented in the time-frequency and time-amplitude domain respectively. Although, the instantaneous frequency seems to be a rather crude method that lacks the mathematical sophistication of other methods, it gives surprisingly good results.

2. Analytical Signal

The analytical signal $u(t)$ of a real causal signal $x(t)$ is a complex signal, which real part $v(t)$ is equal to the original real signal $x(t)$, and which imaginary part $w(t)$ is equal to the negatively taken Hillbert Transform of the real signal $x(t)$ [1],[2].

Where t is time [s], j is imaginary unit ($j = \sqrt{-1}$), and where

$$\begin{aligned}
 v(t) &= x(t) \\
 &\text{is real part of analytical signal, and} \\
 u(t) &= v(t) - j \cdot w(t) \\
 w(t) &= H\{x(t)\} \\
 &\text{is the Hilbert transform of } x(t).
 \end{aligned} \tag{1}$$

The discrete analytical signal is found as the inverse Fourier Transform of the Fourier Transform of $x(t)$ that is multiplied by the modified Hilbert transformer H_k [1].

$$U_k = F^{-1}\left(F(x(t)) \cdot H_k\right) \tag{2}$$

The modified Digital Hilbert transformer H_k has the form (for N even) shown in Fig. 1

3. Phase of Analytical Signal

The phase angle $\varphi(t)$ of an analytical signal is defined as arc tangent of the ratio of imaginary part of analytical signal divided by the real part of analytical signal. In the case of analytical signal (1) it is

$$\varphi(t) = \text{atan}\left(\frac{w(t)}{v(t)}\right) \tag{3}$$

4. Instantaneous Frequency

The instantaneous angular frequency $\omega(t)$ is defined as the time derivative of the phase angle $\varphi(t)$, that is

$$\omega(t) = \frac{d\varphi(t)}{dt} = \dot{\varphi}(t) \tag{4}$$

Substituting (4) into (5), performing the derivative, and dividing the result by 2π , we get

$$f(t) = \frac{\dot{\varphi}(t)}{2\pi} = \frac{1}{2\pi} \cdot \frac{d}{dt} \left(\text{atan}\left(\frac{w(t)}{v(t)}\right) \right) = \frac{1}{2\pi} \cdot \frac{\dot{w}(t) \cdot v(t) - \dot{v}(t) \cdot w(t)}{v(t)^2 + w(t)^2} \tag{5}$$

Where

$$\dot{v}(t) = \frac{dv(t)}{dt} \tag{6}$$

$$\dot{w}(t) = \frac{dw(t)}{dt} \tag{7}$$

The denominator on the right side of equation (5) is known as the square of the envelope E of the signal

$$E(t) = \pm \sqrt{v(t)^2 + w(t)^2} \tag{8}$$

The instantaneous angular frequency can be calculated numerically either from the equation (4) or (5). But, in order to use equation (4) the phase angle $\varphi(t)$ has to be unwrapped because the arc tangent is defined in the range of $\pm\pi/2$ only. The use of equation (5) is preferred because it does not need any phase angle unwrapping.

5. Derivative of Analytical Signal

The derivatives of real and imaginary parts of an analytical signal can be performed by applying two different Hilbert transformers of derivation on the Fourier Transform U_k of the real signal $x(t)$. Fig. 2 and Fig. 3 show how it is done

6. Illustrative Example

In order to demonstrate computation of instantaneous frequency, an artificial signal has been generated. This signal has duration of one second. It consists of a series of four short chirps of four different frequencies and four different amplitudes. Fig. 4 and equation (9) describe the signal.

$$x(t) = \begin{cases} 0.4 \cdot \sin(2 \cdot \pi \cdot 16 \cdot t), & 0 \leq t < 0.25 \\ 0.8 \cdot \sin(2 \cdot \pi \cdot 128 \cdot t), & 0.25 \leq t < 0.5 \\ 0.6 \cdot \sin(2 \cdot \pi \cdot 64 \cdot t), & 0.5 \leq t < 0.75 \\ 0.9 \cdot \sin(2 \cdot \pi \cdot 32 \cdot t), & 0.75 \leq t < 1.0 \end{cases} \quad (9)$$

Time t in equation (9) is the time when the signal is sampled

$$t = T \cdot \frac{i}{N}, \quad i = 0, 1, 2, \dots, N-1 \quad (10)$$

The spectrum of the signal is shown in Fig. 5. It was calculated by applying an FFT on the signal in Fig. 4. Flat window (no window) has been used. The amplitudes and frequencies are displayed correctly as expected. Nevertheless, there is no way to find out the time when each of those four frequencies occurred. Application of computational procedures described schematically in Fig. 1, Fig. 2 and Fig. 3 and in equation (5) yield instantaneous frequency shown in Fig. 6, and the instantaneous envelope of the signal shown in Fig. 7. The instantaneous frequency in Fig. 6 clearly shows the signal has frequency 16 Hz from 0 sec to 0.25 sec, frequency 128 Hz from 0.25 sec to 0.5 sec, frequency 64 Hz from 0.5 sec to 0.75 sec, and frequency 32 Hz from 0.75 to 1.0 sec. The instantaneous envelope in Fig. 7 shows the signal has dimensionless amplitude of 0.4 from 0 sec to 0.25 sec, amplitude 0.8 from 0.25 sec to 0.5 sec, amplitude 0.6 from 0.5 sec to 0.75 sec, and amplitude 0.9 from 0.75 sec to 1.0 sec. The results in Fig. 6 and in Fig. 7 exhibit overshoots around the correct levels at the time of sudden change in frequency. This is known as the Gibbs' effect. In the case of the signal in Fig. 4, the ideal frequency and the ideal envelope should be smooth stepwise functions. While the application of a window would lower side-lobes of the spectrum in Fig. 5, it would also deform instantaneous frequency and the envelope of the signal. This is the main reason why the Hilbert analysis of instantaneous frequency and envelope of the signal uses flat window. A post-filtering can be used to both, the instantaneous frequency and the instantaneous envelope of the signal.

7. Filtering Instantaneous Frequency and Envelope of the Signal

If the removal of the Gibbs' effect is desired a filter can smooth the instantaneous frequency, and the instantaneous envelope of the signal. The main reason for the calculation of instantaneous frequency is to correlate frequencies contained in the signal with the physical phenomena that generate that signal. For this reason, any filter that imposes a variable, frequency dependent, phase or time delay is not acceptable. If necessary, a Finite Impulse Response (FIR) filter can be used. The FIR filters is a class of filters that has constant phase delay, and therefore serve well to our purpose. An example of such FIR filter is running average filter defined by

$$X_k = \frac{1}{2M+1} \sum_{i=k-M}^{i=k+M} X_i \quad (11)$$

Where M is number of averaged values symmetrical on both sides of sample X_k .

This filter does not impose any time or phase delay. Such a filter can be, if necessary, applied several times on previously filtered series. Fig. 8 and Fig. 9 show instantaneous frequency and envelope of the sample signal from Fig. 4 that is filtered by the filter in equation (11). The disadvantage of filtering is the loss of first M and last M samples of instantaneous frequency, or of the envelope, because filtering starts at the sample $M+1$ and ends at the sample $N-1-M$. This can always be fixed by shifting timing mark so that the most important frequencies are at the center of the diagram.

8. Other Effects Affecting Source of Noise Identification

In the process of identification of the source of noise, or the source of vibration, we have to take into consideration some other effects. Most important is the time delay caused by finite velocity of noise, or vibration, propagation. When identifying the source of vibration we do not need to worry about the time delay caused by the finite speed of propagation of deformation waves in the metal parts. The velocity of wave propagation in the metal parts is high, and distances are usually short. On the other hand, the distance of microphone from the source can introduce a substantial delay. This time delay causes disagreement between the timing mark and the noise signal. The time shift t is

$$t = \frac{s}{c} \quad (12)$$

Where s is distance between the source, the compressor, and the microphone, and c is velocity of sound in air.

During the time t the noise travels from the source to the microphone the compressor may turn of an angle

$$\varphi^\circ = \frac{\omega \cdot t}{c} \cdot s \quad (13)$$

Where φ° is the angle of rotation of the crankshaft of the compressor in degrees.

9. Improving Instantaneous Frequency Analysis

Application of band pass filters that have constant, not frequency dependant, time or phase delay can improve the accuracy of the source of noise identification, and the instantaneous frequency analysis. In order to avoid undesirable time shift, the same filter must filter both the noise signal and the timing mark signal.

10. Conclusion

Fig. 10 shows results of noise identification of a single cylinder refrigeration compressor. The compressor had high noise level at 1250 Hz third-octave band that dominated its noise spectrum. A precise timing mark and instantaneous frequency analysis identified the discharge process as the primary cause of the noise of compressor. A simple redesign of discharge valve decreased noise of the compressor of about 3dB with no harm to the capacity and energy efficiency rating of the compressor. The instantaneous frequency analysis proved to be a useful tool in the source of noise identification of refrigeration compressors.

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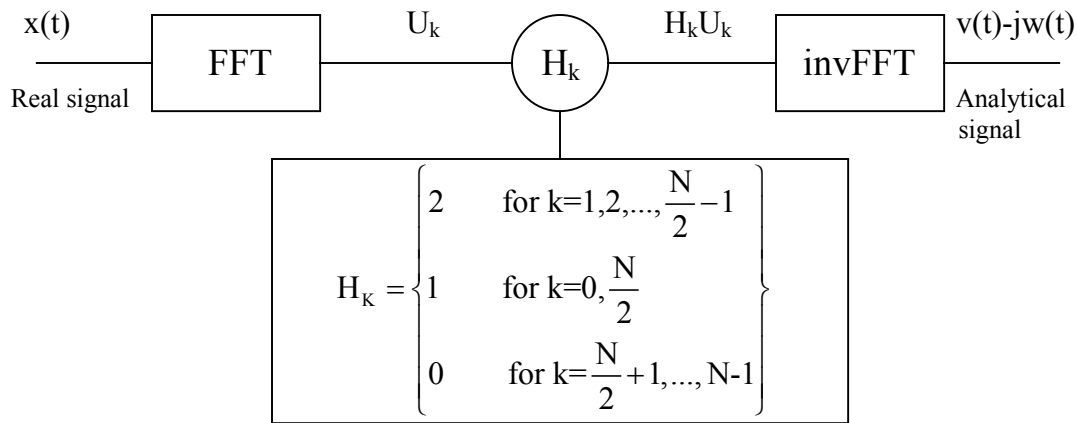


Fig. 1: Computation of analytical signal

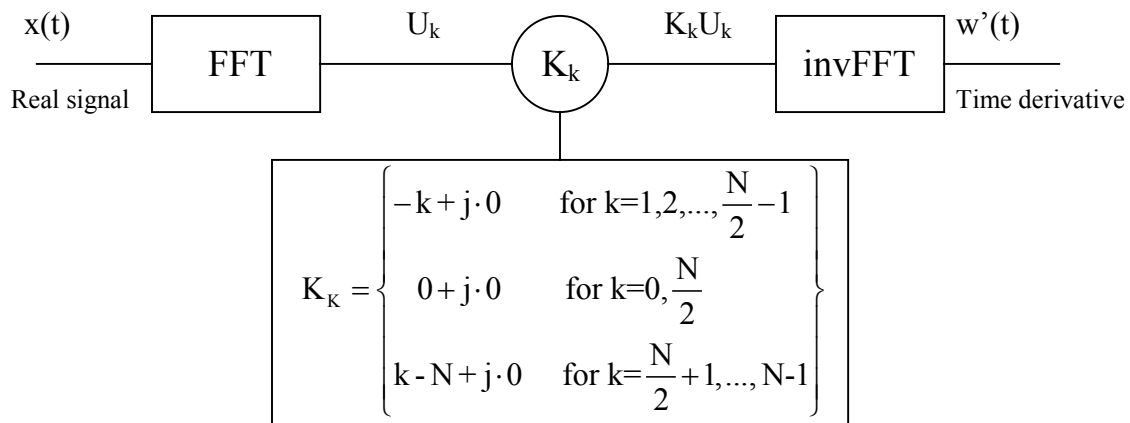


Fig. 2: Computation of time derivative of imaginary part of analytical signal

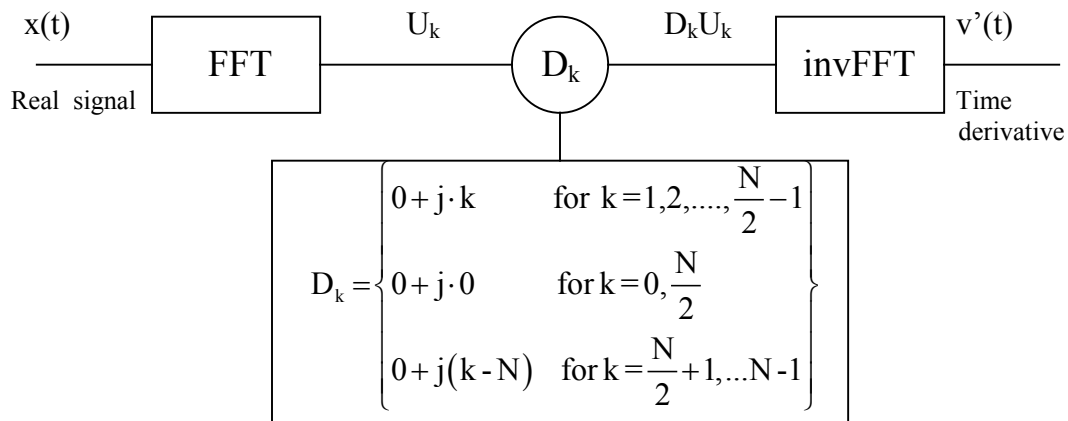


Fig. 3: Computation of time derivative of real part of analytical signal

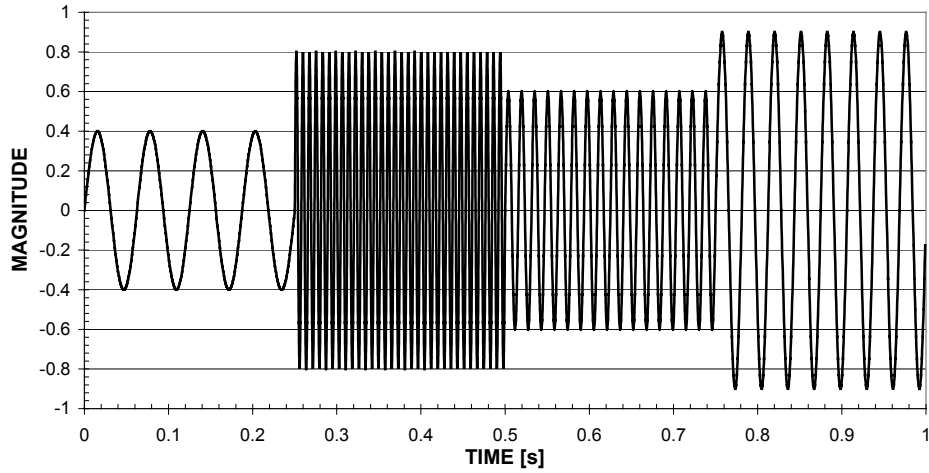


Fig. 4: Simulated signal

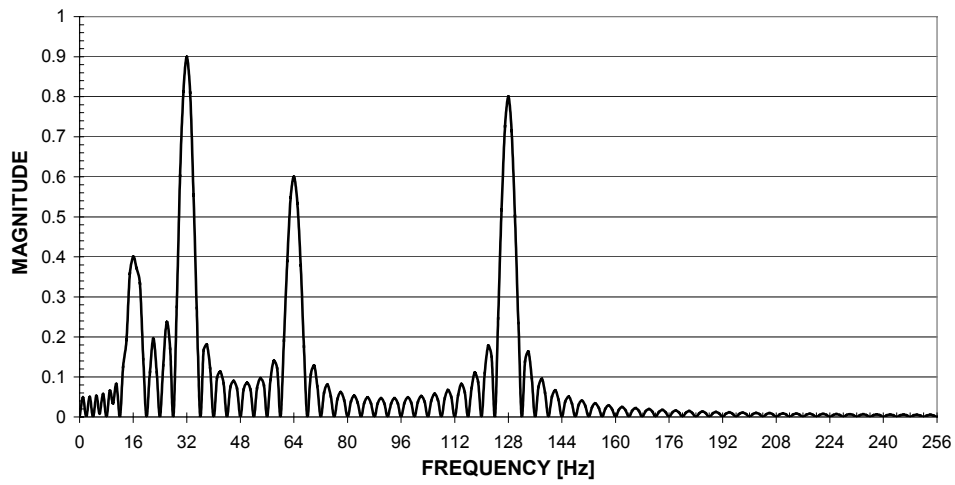


Fig. 5: Spectrum of simulated signal

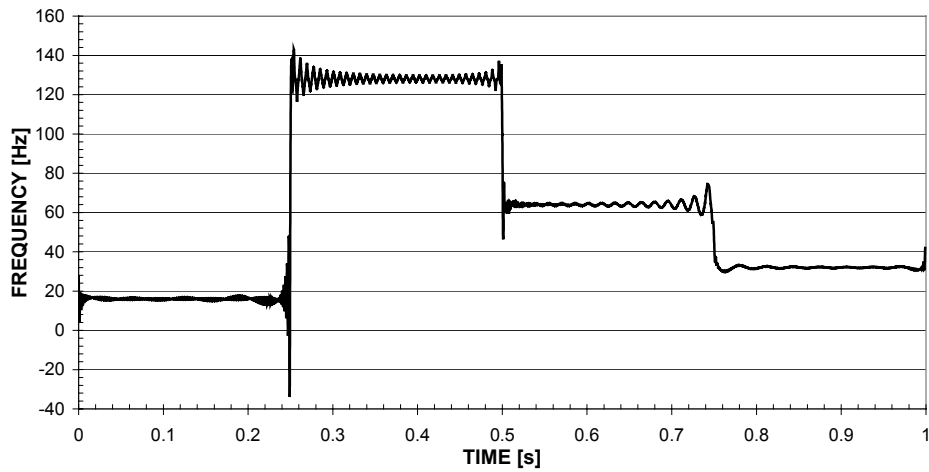


Fig. 6: Instantaneous frequency of simulated signal

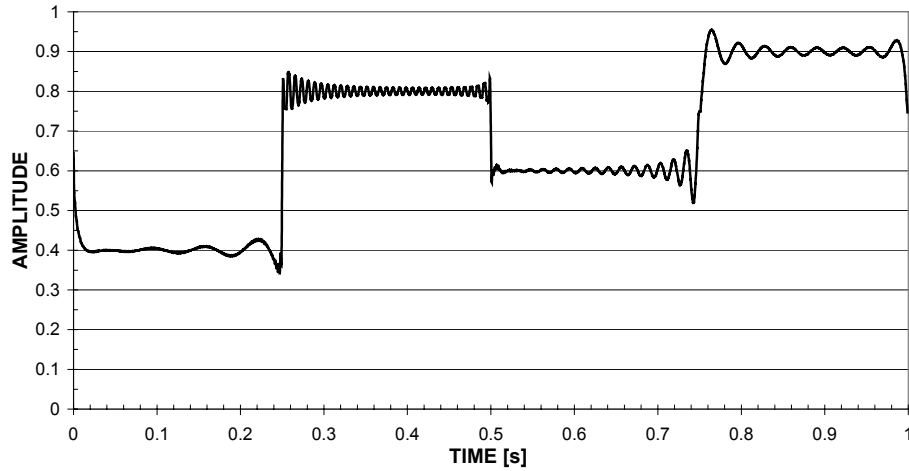


Fig. 7: Envelope of simulated signal

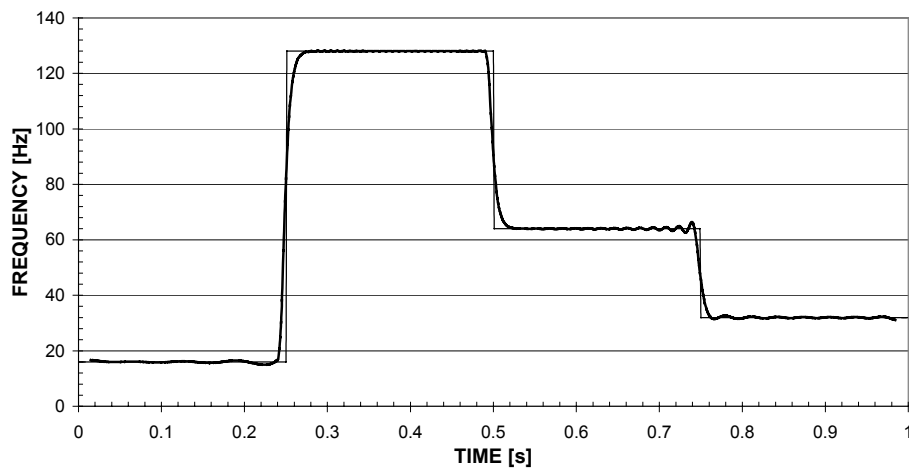


Fig. 8: Filtered instantaneous frequency

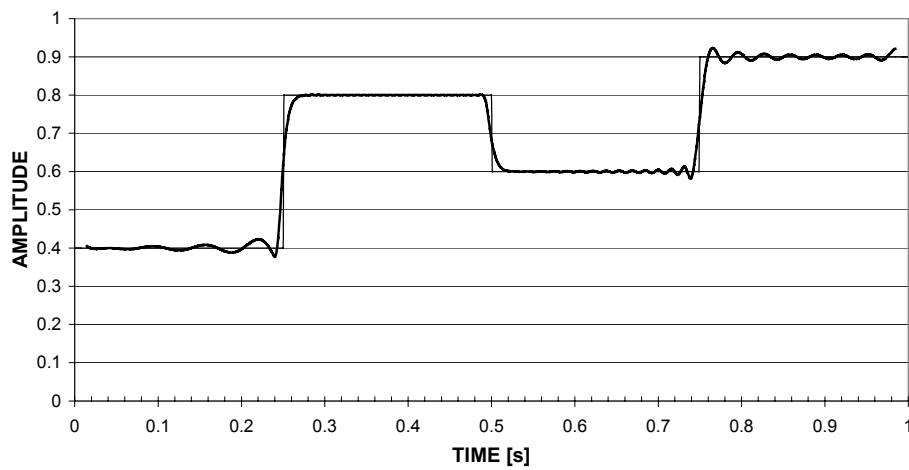


Fig. 9: Filtered envelope of simulated signal

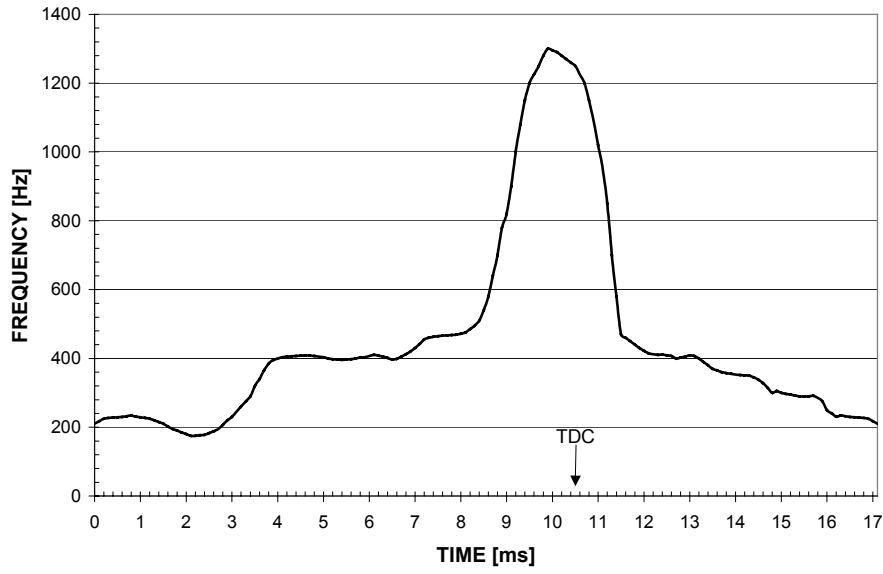


Fig. 10: Instantaneous frequency of a single-cylinder compressor