Analysis of Dynamic Stability of Ejector Expansion Refrigeration System

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Background

Stability is one of the basic conditions for the system operation. None of systems can work normally and reliably under unstable conditions.

Instabilities of refrigeration system

- Oscillatory motion of mixture-vapor transition point
- Instability between throttling device and evaporator
- The Lyapunov Stability Theorem
- Stability margin

Intrinsic instability

Control characteristic instability

Reflects an internal matching degree among and between structure parameters and operation parameters.
Lyapunov stability theory is usually regarded as classical theoretical foundation for stability analysis. The theory mainly involves the $V$ function method and the First Approximation Theorem. The stability analysis can be attributed to the stability criterion of zero solution of the linear differential equations.

In this study, we combine the First Approximation Theorem of Lyapunov Stability Theorem and the calculation of the stability margin to propose a method of dynamic stability to evaluate the matching degree in a system.
Analysis method

**STEP1:** Develop governing equations based on the three conservation laws.

**Continuity:**
\[
\frac{\partial \rho}{\partial t} = \frac{\dot{m}_{in} - \dot{m}_{ou}}{A\Delta z}
\]

**Momentum:**
\[
\frac{\partial \dot{m}}{\partial t} = \frac{P_{in} A_{in} - P_{ou} A_{ou}}{\Delta z} + \frac{\dot{m}_{in} u_{in} - \dot{m}_{ou} u_{ou}}{\Delta z}
\]

**Energy:**
\[
\frac{d(\dot{m}u)}{dt} = \dot{m}_{in} h_{in} - \dot{m}_{ou} h_{ou}
\]

**STEP2:** Discuss the eigenvalue structure of the linear equations.

\[
Z \cdot \dot{x} = f(x,u) \quad \dot{x} = A \cdot \delta x + B \cdot \delta u \quad A = Z^{-1} f_x
\]

**STEP3:** Calculate the stability margin and evaluate the anti-disturbance capability.

Assuming the \(j\) order eigenvalue is \(\lambda_j = -\tau_j + i\omega_j\), the logarithmic decrement is \(\delta_j = 2\pi \tau_j / \omega_j\). The stability margin is represented by the minimum logarithmic decrement, \(\delta_{\min} = \min(\delta_j)\).
Case study 1

Dynamic stability of gas cooler

\[
\begin{align*}
\dot{m}_{ge,in} & \quad T_{ge,w} & \quad \dot{m}_{ge,ou} \\
h_{ge,in} & \quad P_{gc} & \quad h_{ge,ou}
\end{align*}
\]\n
Physical model of gas cooler

Governing equations

The right side of the governing equation is dealt with by Taylor series expansion and its first order partial derivative \( f_x \) is obtained. The coefficient matrix \( A \) is

\[
A = \begin{bmatrix}
A_{11} & A_{12} & A_{13} \\
0 & 0 & 0 \\
A_{31} & A_{32} & A_{33}
\end{bmatrix}
\]

\[
\lambda_1 = 0 \quad \lambda_2 < 0, \quad \lambda_3 < 0
\]

The gas cooler is in zero solution stability

In actual thermal process of the gas cooler, when the input parameters are changed because of the small disturbance, there are variations in the output parameters, and finally it will achieve stability. The result shows the consistency between the mathematical stability and the actual stability.
Case study 2

Dynamic stability of EERS

Transcritical CO₂ EERS (Ejector Expansion Refrigeration System)

The EERS has a special equilibrium stability relationship relative to the traditional vapor compression system (VCS), and the system performance and stability are more sensitive to the operating parameter and the match of component.
Case study 2

Literatures list of component models of transcritical CO₂ EERS

<table>
<thead>
<tr>
<th>Components</th>
<th>Modeling approaches</th>
<th>literatures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressor</td>
<td>Lumped parameter</td>
<td>Sarkar et al. (2006)</td>
</tr>
<tr>
<td>Expansion valve</td>
<td>Lumped parameter</td>
<td>Ma et al. (2005)</td>
</tr>
<tr>
<td>Ejector</td>
<td>Thermodynamics method</td>
<td>Nehdi et al. (2007)</td>
</tr>
<tr>
<td>Gas cooler</td>
<td>Lumped parameter</td>
<td>Rasmussen (2002)</td>
</tr>
<tr>
<td>Separator</td>
<td>Lumped parameter</td>
<td>Eldredge et al. (2008)</td>
</tr>
</tbody>
</table>

Governing equation before linearization

\[
\begin{bmatrix}
Z_{EE} & Z_{EGC} & Z_{ES&E} & Z_{EJ} \\
Z_{EG} & Z_{GGC} & Z_{GS&E} & Z_{ESJ} \\
\vdots & \vdots & \vdots & \vdots \\
Z_{EJ} & Z_{EJ} & Z_{EJ} & Z_{EJ}
\end{bmatrix}
\begin{bmatrix}
\dot{X}_E \\
\dot{X}_{EG} \\
\vdots \\
\dot{X}_{EJ}
\end{bmatrix} =
\begin{bmatrix}
F_{EE}(X, E_{EV}, E_{VG}) \\
F_{EG}(X, G_{EV}, G_{VG}) \\
\vdots \\
F_{EE}(X, E_{EV}, E_{VG})
\end{bmatrix}
\begin{bmatrix}
F_{VXS} & F_{VXE} \\
F_{GXS} & F_{GXE} \\
\vdots & \vdots \\
F_{EXS} & F_{EXE}
\end{bmatrix}
\begin{bmatrix}
X_E \\
X_{GC} \\
\vdots \\
X_{EJ}
\end{bmatrix}
\]

where \(Z_E, Z_{GC}, Z_{SE}, Z_{EJ}, X_{Ev}, X_{GC}, X_{SE}\) and \(X_{EJ}\) are Z matrix and state vector matrix of evaporator, gas cooler, separator and ejector, respectively. F is The coefficient matrix.
The initial operation parameters of EERS

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Unit</th>
<th>Values</th>
<th>Parameters</th>
<th>Unit</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressor</td>
<td></td>
<td></td>
<td>flow rate</td>
<td>kg·s⁻¹</td>
<td>0.956</td>
</tr>
<tr>
<td>inlet pressure</td>
<td>MPa</td>
<td>4.29</td>
<td>inlet temperature of cooling water</td>
<td>°C</td>
<td>25.0</td>
</tr>
<tr>
<td>outlet pressure</td>
<td>MPa</td>
<td>9.50</td>
<td>outlet temperature of cooling water</td>
<td>°C</td>
<td>55.0</td>
</tr>
<tr>
<td>displacement</td>
<td>m³·h⁻¹</td>
<td>1.46</td>
<td>flow rate of cooling water</td>
<td>kg·s⁻¹</td>
<td>0.056</td>
</tr>
<tr>
<td>rotate speed</td>
<td>r·min⁻¹</td>
<td>1450</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expansion valve</td>
<td></td>
<td></td>
<td>Evaporator</td>
<td></td>
<td></td>
</tr>
<tr>
<td>outlet pressure</td>
<td>MPa</td>
<td>3.90</td>
<td>flow rate</td>
<td>kg·s⁻¹</td>
<td>0.557</td>
</tr>
<tr>
<td>area</td>
<td>mm²</td>
<td>1.7485</td>
<td>superheat</td>
<td>°C</td>
<td>5.0</td>
</tr>
<tr>
<td>Ejector</td>
<td></td>
<td></td>
<td>inlet temperature of chilled water</td>
<td>°C</td>
<td>20.0</td>
</tr>
<tr>
<td>primary temperature</td>
<td>°C</td>
<td>36.0</td>
<td>outlet temperature of chilled water</td>
<td>°C</td>
<td>10.0</td>
</tr>
<tr>
<td>area ratio</td>
<td></td>
<td>10.0</td>
<td>flow rate of chilled water</td>
<td>kg·s⁻¹</td>
<td>0.127</td>
</tr>
<tr>
<td>entrainment ratio</td>
<td></td>
<td>0.583</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gas cooler</td>
<td></td>
<td></td>
<td>Separator</td>
<td></td>
<td></td>
</tr>
<tr>
<td>high</td>
<td></td>
<td></td>
<td>diameter</td>
<td>m</td>
<td>0.1</td>
</tr>
<tr>
<td>Individual components is stability</td>
<td></td>
<td></td>
<td>high</td>
<td>m</td>
<td>0.6</td>
</tr>
</tbody>
</table>

There are two zero value and a positive value, which means the system instability. The results show the stability of individual components cannot guarantee the stability of the whole system.
Case study 2

Analysis of EERS under different working conditions

The maximal eigenvalue $\lambda_{\text{max}}$ with the change of gas cooler pressure at different gas cooler outlet temperature were obtained.

The minimum logarithmic with the change of gas cooler pressure were investigated.
A dynamic stability analysis method is put forward for the refrigeration system based on the First Approximation Theory of Lyapunov Stability Theorem and the evaluation of stability margin.

The instability of a refrigeration system is divided into the intrinsic instability and the control characteristic instability.

The case study of stability analysis of gas cooler confirms the consistency between the mathematical stability and the actual one.

The dynamic stability analysis on a transcritical CO₂ ejector expansion refrigeration system (EERS) is conducted and the present results show that, even each component of the system is in the stable state, it cannot guarantee the dynamic stability of the whole system.
Question 1: In this paper, we put forward the concept of intrinsic stability, which refers to the harmonious matching among structure parameters and operation parameters. While the Lyapunov Stability Theorem is directed against motion stability under the small disturbance. What is the relationship between the matching degree and the disturbance?

Response: In this paper, the governing equations were established based on the common conservation laws, and the Lyapunov Stability Theorem is applied to linearize the governing equations. Moreover, this approach has been used to the development of controller of refrigeration in previous literature[1].


Response: The linearization of the governing equations is core step, moreover, the higher order terms are omitted and only leave the first order term, which is the main error sources. In addition, during the ejector model, the determination of the input parameters would also result in the error.
Thank you for your attention!