Geometrical Optimisation of a Rotary Compressor for Refrigerators

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GEOMETRICAL OPTIMISATION OF A ROTARY COMPRESSOR FOR REFRIGERATORS

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ABSTRACT

The rolling piston has been widely used in refrigeration and air-conditioning industries. It is a geometrically simple machine with only four basic parts: a cylinder, roller, a vane and a vane spring. Its performance depends on the balance amongst heat transfer, fluid flow and mechanical aspects of the machine. In this study, effort has been focused on improving the coefficient of performance of the compressor (COP) by improving its mechanical performance. A lumped mathematical model for the compressor has been formulated to predict the overall performance of the machine. The model accounts for geometrical dimensions, operating conditions, valve dynamics, thermodynamics aspects, flow and mechanical losses. The prediction of the model has been compared with measured results and good agreement has been obtained. The model has been linked with an optimisation algorithm to search for a combination of some 12 design variables together with 20 sets of design constraints for an optimum compressor performance in terms of reduced mechanical losses and hence improved the COP of the machine. A constrained multi-variable direct search optimisation technique has been employed. Theoretical studies suggest that a total reduction of mechanical losses of 70% is possible and this brings about an overall improvement in the COP of the compressor of 14%.

1. INTRODUCTION

The rolling piston compressor has been widely used in the refrigeration and air conditioning industries since early 1980s. It attracted a lot of attention from the industries and research centres in 1980-1990s. More recent comprehensive theoretical and experimental studies are available in the literature [1-11].

Fig. 1 shows a pump and electrical motor assembly for a typical rolling piston compressor used in refrigerators. In its basic form, the pump assembly consists of 4 main parts: a stator, a roller piston, a vane and a vane-spring. During operation, the roller rotates and two working chambers formed by the stator inner wall, the roller and the vane. During the operation, these chambers change in size and hence results in compression and expansion of the working fluid which forms a complete compressor working cycle. A typical cycle takes two motor revolutions to complete. The design of the compressors is constantly reviewed into for further performance improvement. With the advancement in computer technology, computerized design optimization procedures have been used to speed up the design process and improve performance. The paper presents an 12-variable design optimisation aims to increase the COP of a refrigeration rolling piston compressor, under preset operational conditions and some 20 sets of design constraints.

2. MATHEMATICAL MODEL

In the following sections, a brief account of the mathematical model will be presented. A more comprehensive treatment of the can be found in refs. [1,8].

2.1 GEOMETRICAL MODEL

The volume of the working chamber [9], taking into consideration of the vane thickness, can be shown as:

\[ V(\theta) = \frac{1}{2} R^2 \left\{ (l-a)(R-a) - \frac{a-2}{2} \sin \theta - a^2 \sin^2 \left( \frac{1}{n} \sin \theta \right) - d(1-a)\ln \theta \sqrt{\sqrt{c(\frac{1}{n})^2 \sin \theta}} \right\} \]
\[-\frac{t_{s} h_{s} R_{s}}{2} \{1 - (1 - a)\cos \theta - \sqrt{[1 - a]^{2} \cos^{2} \theta + 2a - 1}\}\]

(1)

2.2 Thermodynamic processes

The inter-relationships among temperature, pressure and mass are given by equations (2), (3) and (4).

\[
\frac{dV}{dt} = \frac{1}{\gamma} \left[ \frac{dQ}{dT} + \sum \frac{dm}{dT} \left( h - h_{i} \right) - \sum \frac{dm}{dT} \left( \rho_{s} - \rho_{i} \right) \right] + \frac{\left( \frac{\partial p}{\partial T} \right)_{\rho_{i}}}{V_{s}} \frac{dV}{dt} + \frac{\left( \frac{\partial p}{\partial V} \right)_{T}}{V_{s}} \frac{dV}{dt} + \sum \frac{dm}{dt} - \sum \frac{dm}{dt} = \frac{dm}{dt}
\]

(2)

\[
\frac{dp}{dt} = \left( \frac{\partial p}{\partial T} \right)_{\rho_{i}} \frac{dT}{dt} + \left( \frac{\partial p}{\partial V} \right)_{T} \frac{dV}{dt}
\]

(3)

\[
\sum \frac{dm}{dt} - \sum \frac{dm}{dt} = \frac{dm}{dt}
\]

(4)

The significance of heat transfer, \( Q \), on compressor performance, has been debated by several researchers [9-11]. In this project however, heat transfer effect is excluded to reduce computational time. The accuracy of the prediction has been verified with measured results and good accuracy is obtained, as shown Table 1.

2.3 Mass flow through valve

The compressor simulation assumes that the flow through valves is a steady one-dimensional adiabatic flow. Thus, the mass flow through valves can be expressed as

\[
\frac{dm}{dt} = C \Lambda \sqrt{\frac{2(h_{h} - h_{s})}{V_{s}}}
\]

(5)

where \( C \) indicates the combined effect of non-isentropic and flow losses [8].

The area of the valve requires that the valve opening to be known at any point of time. This is obtained by solving the valve dynamic behaviour, the details of which is available ref [7].

To solve eqns. (1) to (5), it is necessary for the gas properties and their respective derivatives to be determined. In this study, R22 is used as the working fluid and its properties are taken from reference [12], from which the pressure, enthalpy and other derivatives [8] for the gas properties can be obtained.

2.4 Frictional losses

One of the important factors affecting the performance of the compressor is the friction between the rubbing parts. In the simulation model [8], there are seven areas of concern and these are:

i. Frictional loss between the eccentric and the inner surface of the roller, \( L_{er} \):

\[
L_{er} = \left( \omega - \omega_{r} \right) \frac{2 \pi \gamma}{(\omega - \omega_{r})} \frac{R_{s}}{\delta_{i}}
\]

(6)

ii. Frictional loss between the roller face and the cylinder head faces, \( L_{rc} \):

\[
L_{rc} = \omega \frac{2 \pi \gamma R_{s}^{*}}{\delta_{i}}
\]

(7)

iii. Frictional loss between eccentric face and the cylinder head faces, \( L_{ec} \):

\[
L_{ec} = \omega \frac{\pi \gamma}{(\delta_{i} + R_{s}^{*})^{2}}
\]

(8)

iv. Frictional loss between vane tip and roller, \( L_{v} \):

\[
L_{v} = F_{v} R_{s} \omega + \frac{\omega \cos \theta}{\cos \theta}
\]

(9)

v. Frictional loss between vane sides and vane slot, see Fig. 2(b), \( L_{v'} \):

\[
L_{v'} = \left( F_{v1} + F_{v2} \right) \frac{R_{s}}{\delta_{i}}
\]

(10)

vi. Frictional loss between shaft and bearing S, \( L_{sb} \):

\[
L_{sb} = \omega \frac{2 \pi \gamma l_{s} R_{s}^{3}}{\delta_{i}}
\]

(11)
vii. Frictional loss between shaft and bearing $L_{bl}$:

$$L_{bl} = \frac{2\pi \mu \omega_l l_{bl} R_s^4}{\delta_s} \quad (12)$$

The total frictional loss is obtained by summing up all the losses shown in equations (6) to (12).

The above equations show that, among other parameters, the frictional analysis is dependent significantly on the eccentric radius $R_e$, the roller radius $R_r$, the cylinder radius $R_c$ and the shaft radius, $R_s$ and the height of the cylinder, $h_b$ (note that the height of the cylinder is also same as the height of the vane, $h_b$). All these parameters should be included in the optimization study as free variables. Free variables are variables that are allowed to vary during an optimization search.

### 3. COMPRESSOR OPTIMISATION

In this study, the objective function involved in the current optimisation study is highly non-linear and may not be differentiable. Therefore a direct search method introduced by M.J. Box [13] was employed. This method is a sequential technique that has proven effective in solving problems with non-linear objective functions. In general, an optimisation study can be mathematically represented by:

$$\text{Maximise: } F(x) = f(x_1, x_2, \ldots, x_N) \quad (13)$$

Subject to:

(i) $L_E(x_i) \leq E(x_i) \leq H_E(x_i), i = 1, 2, \ldots, N$ \quad (14)

(ii) $L_G(x_j) \leq G(x_j) \leq H_G(x_j), j = 1, 2, \ldots, M$ \quad (15)

where $E$ and $G$ are the explicit constraints and geometrical constraints, respectively, $L$ and $H$ represent the lower and higher limit, $I$ and $j$ are the number of explicit and geometrical constraints, respectively.

The details Box COMPLEX method will not be shown here, it can be referred to ref [13].

#### 3.1 Objective function and constraints

The objective function for this optimisation study is coefficient of performance (COP), which is effectively the overall performance figure for the compressor. This is carried out at the given operational conditions and at a fixed capacity of the compressor.

Mathematically, this is defined as:

$$\text{Maximise } f_{obj} = f(R_{SB}, R_{LB}, R_c, R_r, D_{sp}, D_{dp}, K_s, V_t, V_w, V_l, V_r) \quad (16)$$

Subject to the following constraints:

$$x_l(i) < x(i) < x_h(i), \quad i = 1, 2, \ldots, 12 \quad (17)$$

where $x(i)$ are the free variables, $x_l(i)$ and $x_h(i)$ are lower and upper limits of the free variables, respectively. The free variables used are shown in Table 3.

In this case the value of COP will be maximised during the optimisation search. In eqn. (17), $x(i), i = 1, 2, \ldots, 12$ are design variables $R_{SB}, R_{LB}, R_c, R_r, D_{sp}, D_{dp}, K_s, V_t, V_w, V_l, V_r$, respectively. The values of $x_l(i)$ and $x_h(i)$ are lower and upper limits of the design variables, call explicit constraints. This defines the region in which feasible designs are located. Table 3 tabulates the values of the constraints. The geometrical constraints are also specified to ensure that the final geometries are feasible and these are:

$$g_l(j) < g(x(j) < g_u(j), \quad j = 1, 2, \ldots, 8 \quad (18)$$

where $g(i), i = 1, 2, \ldots, 8$ are the geometrical constraint. The $g_l(j)$ and $g_u(j)$ are the lower and upper limits of the geometrical constraints. In this study, the constraint is specified to ensure that the volumetric displacement of the compressor is constant, such that the capacity of the compressor remains constant throughout the optimisation search. The constant swept volume is achieved by varying the cylinder height in accordance with the following relationship:
\[ h_b = \frac{V_{\text{swept}}}{\pi \left( D_c^2 - D_s^2 \right) / 4} \]  \hspace{1cm} (19)

where \( V_{\text{swept}} \) is the preset swept volume the compressor should have in order to fulfill the capacity requirement. The cylinder height \( h_b \) is varied according to eqn (19) during the optimisation search.

The constraints is also set such that the eccentric diameter must be conformed to eqn (20).

\[ D_e \geq (D_c - D_r + D_s) \]  \hspace{1cm} (20)

where \( D_s \) is the min of the \( D_{SB} \) or \( D_{LB} \).

Table 4 shows the geometric constraints applied. These constraints ensure that geometrical configuration of the compressor is feasible in practical situations.

4. RESULTS AND DISCUSSIONS

In this study, the operational conditions of the compressor are:

- Speed (RPM) : 3000.000 RPM
- Inlet Temp. (\( T_s \)) : 340.850 K
- Discharge Temp. (\( T_d \)) : 400.150 K
- Suction Press. (\( P_s \)) : 215.671 kPa
- Discharge Press. (\( P_d \)) : 2146.992 kPa

The optimization search was implemented on an IBM PC equipped with 2.6 GHz Pentium processor and the optimization search took 1 hours to complete.

Figs. 2 to 5 show the results for the optimization study. The results are normalised with respect to the parameters of the first initial complex. Fig. 2(a) shows that the objective function increases by about 14% at the end of the optimisation search. Figure 2(b) shows that the whole searching process makes a total of 338 model evaluation and out of which 211 are successful iterations. Successful iteration refers to model evaluation that yields improvement in the value of the objective function and the compressor design is fallen within all the imposed constraints. Figs. 2(c) and 2(d) shows that the S and L bearing diameters converged to the lower limits imposed. The reduction in these diameters results in 37% reduction in fictional loss at these two locations, as shown in Table 5.

Figs. 2(e) and 2(f) show that the roller and cylinder diameters reduced by more than 15% each and this has resulted in the cylinder height to increase by 30%, resulting in a much longer compressor, as shown in Fig. 2(g). Eccentric diameter is calculated from \( D_e = D_c - D_r + D_s \) and its variation is shown in Fig. 2(h). The increase in the eccentric diameter does not increase the friction between the eccentric and the cylinder end face as there is an undercut at the eccentric face. The friction in this location is in fact reduces, as shown in Table 5.

Figures 2(i) and 2(j) shows that both suction and discharge port diameters increase and cause reductions in suction and discharge losses. Figs. 2(k) to 2(n) show variations of valve dimensions. The results show that the optimum valve design is easier to open as it is thinner, narrower and longer, which has the effect of reducing valve loss. In practice, the final valve design must be checked using the fatigue analysis to prevent premature valve failure. Fig. 2(o) shows that the vane-spring gets stiffer, which has resulted in a significant increase in the valve tip loss. But as the absolute value is small, such an increase does not result in any noticeable change in compressor performance.

Table 5 summarises the overall change in compressor design variables as well as the resulted changes in compressor performance parameters.

5. CONCLUSION

The study demonstrated that a design optimization algorithm coupled with the mathematical model of the compressor is a useful tool in compressor design. It can effectively suggest a set of dimensions that produces optimum compressor performance based on the design requirement. The study suggests that, for this design, based on the criteria of maximising COP, a narrower but taller cylinder is a better design. The model suggests that more than 70% reduction in the mechanical loss is possible through a proper combination of design dimensions. This gives an overall improvement of COP of 14%, as the shaft power input reduces due to lower frictional losses and lower suction and discharge losses.

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NOMENCLATURE

\begin{align*}
\alpha & \quad R_c/R_e \\
\rho & \quad \text{specific volume, m}^3/	ext{kg} \\
\Delta & \quad \text{isentropic efficiency} \\
\dot{q} & \quad \text{angular position of rotor/crankshaft, rad} \\
\dot{\alpha} & \quad \text{angle of rolling piston centre, rad} \\
d_1 & \quad \text{clearance at roller and cylinder, m} \\
d_2 & \quad \text{clearance at eccentric and roller, m} \\
d_3 & \quad \text{clearance at eccentric and cylinder, m} \\
d_c & \quad \text{clearance at shaft and bearing S, m} \\
d_e & \quad \text{clearance at shaft and bearing L, m} \\
d_f & \quad \text{eccentric face land width, m} \\
\eta & \quad \text{viscosity of lubricating oil, Ns/m}^2 \\
\eta_{se} & \quad \text{isentropic efficiency} \\
\omega & \quad \text{angular velocity of crankshaft, rad/s} \\
\Omega & \quad \text{angular velocity of roller, rad/s} \\
\rho & \quad \text{density, kg/m}^3 \\
\alpha & \quad \text{inlet (suction) conditions} \\
o & \quad \text{outlet (discharge) conditions} \\
i & \quad \text{upstream conditions} \\
2 & \quad \text{downstream conditions} \\
c & \quad \text{cylinder chamber properties/conditions} \\
s & \quad \text{suction} \\
d & \quad \text{discharge}
\end{align*}

Subscripts

\begin{align*}
\text{i} & \quad \text{inlet (suction) conditions} \\
\text{o} & \quad \text{outlet (discharge) conditions} \\
\text{1} & \quad \text{upstream conditions} \\
\text{2} & \quad \text{downstream conditions} \\
\text{c} & \quad \text{cylinder chamber properties/conditions} \\
\text{s} & \quad \text{suction} \\
\text{d} & \quad \text{discharge}
\end{align*}

REFERENCES

Table 1 Comparison between the experimental and predicted results.  

<table>
<thead>
<tr>
<th></th>
<th>R134a</th>
<th>R22</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normalised shaft power, $W_{calc}/W_{exp}$</td>
<td>0.895 (-10.5%)</td>
<td>0.935 (-6.5%)</td>
</tr>
<tr>
<td>Normalised refrigerant flow rate, $\dot{m}<em>{calc}/\dot{m}</em>{exp}$</td>
<td>0.986 (-1.4%)</td>
<td>1.006 (+0.6%)</td>
</tr>
<tr>
<td>Normalised cooling capacity, $C_{calc}/C_{exp}$</td>
<td>0.995 (-0.05%)</td>
<td>1.013 (-1.3%)</td>
</tr>
<tr>
<td>Normalised COP, $COP_{calc}/COP_{exp}$</td>
<td>1.11 (+11.1%)</td>
<td>1.083 (+8.3%)</td>
</tr>
</tbody>
</table>

Table 2 Mechanical analysis of the compressor  
(Running on R22; Compressor shaft input power: 180 Watt based on motor efficiency of 80.5%, mechanical efficiency is 76.3% and COP: 1.72)  

<table>
<thead>
<tr>
<th>Friction analysis results</th>
<th>Loss due to Vane side Reactions</th>
<th>0.52290 Watt</th>
<th>6.97%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Loss due to Vane Tip_Roller force</td>
<td>0.51086 Watt</td>
<td>6.81%</td>
</tr>
<tr>
<td></td>
<td>Loss due to Roller &amp; Ecc friction</td>
<td>0.71959 Watt</td>
<td>9.59%</td>
</tr>
<tr>
<td></td>
<td>Loss due to roller &amp; cyl. head faces</td>
<td>2.65651 Watt</td>
<td>35.41%</td>
</tr>
<tr>
<td></td>
<td>Loss due to Eccentric &amp; cyl. head faces</td>
<td>0.18952 Watt</td>
<td>2.53%</td>
</tr>
<tr>
<td></td>
<td>Loss due to Bearing S &amp; shaft</td>
<td>0.73922 Watt</td>
<td>9.85%</td>
</tr>
<tr>
<td></td>
<td>Loss due to Bearing L &amp; shaft</td>
<td>2.16418 Watt</td>
<td>28.84%</td>
</tr>
<tr>
<td></td>
<td>Total Friction Power</td>
<td>7.50280 Watt</td>
<td>100.00%</td>
</tr>
<tr>
<td></td>
<td>Mechanical Efficiency</td>
<td>76.33086</td>
<td></td>
</tr>
</tbody>
</table>

Table 3 Explicit constraints set in the optimization study.  
(a) Limits for design variables  

<table>
<thead>
<tr>
<th>S/N</th>
<th>Lower Limit</th>
<th>Design variables</th>
<th>Upper limit</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
<td>$D_{Sb}$ (S_bearing dia.)</td>
<td>16</td>
<td>mm</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>$D_{Lb}$ (L_bearing dia.)</td>
<td>16</td>
<td>mm</td>
</tr>
<tr>
<td>3</td>
<td>25</td>
<td>$D_{c}$ (roller dia.)</td>
<td>35</td>
<td>mm</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
<td>$D_{e}$ (cylinder dia.)</td>
<td>45</td>
<td>mm</td>
</tr>
<tr>
<td>5</td>
<td>$D_{c} - D_{r} + D_s$</td>
<td>$D_e$ (eccentric dia.)</td>
<td>$D_e$</td>
<td>mm</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>$D_{sb}$ (suction port dia.)</td>
<td>12</td>
<td>mm</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>$D_{dp}$ (Discharge port dia.)</td>
<td>4</td>
<td>mm</td>
</tr>
<tr>
<td>8</td>
<td>1000</td>
<td>$K_s$ (spring stiffness)</td>
<td>9000</td>
<td>N/m</td>
</tr>
<tr>
<td>9</td>
<td>0.2</td>
<td>$V_r$ (reed thickness)</td>
<td>0.4</td>
<td>mm</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
<td>$V_w$ (reed width)</td>
<td>5</td>
<td>mm</td>
</tr>
<tr>
<td>11</td>
<td>15</td>
<td>$V_l$ (reed length)</td>
<td>25</td>
<td>mm</td>
</tr>
<tr>
<td>12</td>
<td>4.2</td>
<td>$V_t$ (reed tip radius)</td>
<td>8</td>
<td>mm</td>
</tr>
</tbody>
</table>

Note that $h_b$ is not treated as a free variable but is varied to keep the swept volume constant.
Table 4  Geometrical constraints set in the optimization study.

<table>
<thead>
<tr>
<th>S/N</th>
<th>Design variables</th>
<th>Upper limit</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$D_e - (D_c - D_r + D_s) = 8$ mm</td>
<td>8</td>
<td>mm</td>
</tr>
<tr>
<td>2</td>
<td>$(D_c - D_r) = 8$ mm</td>
<td>8</td>
<td>mm</td>
</tr>
<tr>
<td>3</td>
<td>$(D_c - D_e) = 8$ mm</td>
<td>8</td>
<td>mm</td>
</tr>
<tr>
<td>4</td>
<td>$(D_c - D_s) = 8$ mm</td>
<td>8</td>
<td>mm</td>
</tr>
<tr>
<td>5</td>
<td>$(D_r - D_e) = 8$ mm</td>
<td>8</td>
<td>mm</td>
</tr>
<tr>
<td>6</td>
<td>$(D_r - D_s) = 8$ mm</td>
<td>8</td>
<td>mm</td>
</tr>
<tr>
<td>7</td>
<td>$(D_e - D_s) = 8$ mm</td>
<td>8</td>
<td>mm</td>
</tr>
<tr>
<td>8</td>
<td>$V/\pi \cdot (D_e^2 - D_s^2) = 55$ mm</td>
<td>55</td>
<td>mm</td>
</tr>
</tbody>
</table>

Note: The swept volume is kept constant by varying the cylinder height.

Table 5 Changes of the dimensions and the performance parameters after optimisation

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>Dia_Bear_S</th>
<th>Dia_Bear_L</th>
<th>Roller_Dia</th>
<th>Cyl_Dia</th>
<th>Eccen_Dia</th>
<th>SP_Dia</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refrig_capacity</td>
<td>0.858</td>
<td>0.857</td>
<td>0.807</td>
<td>0.825</td>
<td>1.144</td>
<td>1.332</td>
</tr>
<tr>
<td>Ratio(OPT/INIT)</td>
<td>1.0427</td>
<td>0.041</td>
<td>0.934</td>
<td>1.297</td>
<td>0.717</td>
<td>0.989</td>
</tr>
<tr>
<td>Shaft_Bear_S</td>
<td>1.333</td>
<td>0.055</td>
<td>35.222</td>
<td>0.101</td>
<td>1.787</td>
<td>0.674</td>
</tr>
<tr>
<td>Shaft_Bear_L</td>
<td>0.632</td>
<td>0.630</td>
<td>0.326</td>
<td>1.001</td>
<td>1.120</td>
<td>1.137</td>
</tr>
<tr>
<td>Eff_Mech</td>
<td>1.014</td>
<td>1.105</td>
<td>0.383</td>
<td>0.926</td>
<td>1.616</td>
<td>0.985</td>
</tr>
<tr>
<td>Rmass_rate</td>
<td>1.013</td>
<td>1.014</td>
<td>1.333</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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Figure 2 Variations of normalized free variables during optimization search
Figure 2 Variations of normalized free variables during optimization search (cont)