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Asad Awan

Ronaldo A. Ferreira

Suresh Jagannathan
Purdue University, suresh@cs.purdue.edu

Ananth Y. Grama
Purdue University, ayg@cs.purdue.edu

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UNSTRUCTURED PEER-TO-PEER NETWORKS
FOR SHARING PROCESSOR CYCLES

Asad Awan
Ronaldo Ferreira
Suresh Jagannathan
Ananth Grama

Department of Computer Sciences
Purdue University
West Lafayette, IN  47907

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Asad Awan Ronaldo Ferreira Suresh Jagannathan Ananth Grama
Department of Computer Sciences Purdue University West Lafayette, IN 47907 Email: {awan,rf,suresh,ayg}@cs.purdue.edu

Abstract

Motivated by the needs and success of projects such as SETI@home and genome@home, we propose an architecture for a sustainable large-scale peer-to-peer environment for distributed cycle sharing among Internet hosts. Such networks are characterized by highly dynamic state due to high arrival and departure rates. This makes it difficult to build and maintain structured networks and to use state-based resource allocation techniques. We build our system to work in an environment similar to current file-sharing networks such as Gnutella and Freenet. In doing so, we are able to leverage vast network resources while providing resilience to random failures, low network overhead, and an open architecture for resource brokering.

This paper describes the underlying analytical and algorithmic substrates based on randomization for job distribution, replication, monitoring, validation, and aggregation. It also describes a rendezvous service that allows oblivious resource sharing and communication between participating hosts. We support our claims of robustness and scalability analytically with high probabilistic guarantees. Our algorithms do not introduce any state dependencies, and hence are resilient to dynamic node arrivals, departures, and failures. We support all analytical claims with a detailed simulation-based evaluation of our distributed framework.

1 Introduction

The use of a large number of unreliable hosts over a wide area network to solve compute-intensive problems has been pioneered by projects such as SETI@home [25], genome@home [9], and distributed.net [7] among others. While key advantages such as increased performance, reliability, and scalability motivate the use of decentralized peer-to-peer (P2P) systems instead of traditionally used client-server models, we consider the broader goal of an open environment for cycle sharing as a major driver for a move towards a P2P solution for harnessing idle CPU cycles of Internet hosts. Such an environment would allow any participant to submit tasks, in contrast to the inflexible nature of a client-server model. Furthermore, an open P2P system provides an incentive for peers to contribute their resources, expecting cycles in return, as opposed to the altruistic basis for current systems.

In this paper we propose and evaluate an unstructured P2P architecture for distributed cycle sharing among Internet hosts. The dynamic nature of P2P networks resulting from high arrival and departure rates motivates our choice of an unstructured (stateless) model. A majority of Internet-deployed examples of successful, resilient, large-scale, and massively-distributed systems rely on such unstructured topologies (e.g., Gnutella [11], Freenet, etc. for file sharing). Our design decision trades off the overheads of building and maintaining a structured overlay (with associated guarantees on network path lengths) for a less expensive model (with probabilistic guarantees on network delays). The use of efficient randomized algorithms in our network affords simplicity and scalability. Our system capitalizes on the large number of participating nodes to achieve robustness through redundancy. The underlying approach is novel and carries its share of complexities, solutions to which form the key contributions of this paper. The design goals of our system are:

1. Low job makespans by ensuring load balance.
2. Resilience to node failures and frequent node arrivals and departures.
3. Validation of computed output by redundant distributed computations.
5. An accounting framework for the resources contributed by various nodes.
The key substrate supporting our design is efficient uniform random sampling using random walks. Uniform sampling in networks provides the basis for a variety of randomized algorithms and are of interest on their own as well. In context of our system, uniform sampling allows us to design randomized algorithms for load balancing, applying redundancy to support fault tolerance, and building a probabilistic rendezvous service for monitoring task progress and contributions of participating nodes.

Uniform Sampling in Unstructured Networks

Uniform sampling in a network requires randomly selecting a node, such that every node in the network has the same probability of being selected. A trivial approach to this problem would be to collect the entire set of node identifiers at each node and index randomly into this table of identifiers. This simple approach, however, does not work for our target applications because the overhead of frequently updating system state at each node (if at all possible) would be extremely high. An alternate approach to this problem relies on the notion of a random walk. Starting from an initial node, a random walk (of predetermined length) transitions through a sequence of intermediate nodes with probabilities defined for each link and ends at a destination node. The likelihood of terminating a random walk at any node determines whether the walk is a uniform sampling random walk or not. Formally, we define a uniform sampling random walk as follows:

Definition 1.1 (Uniform sampling using random walk)
A random walk of a given length samples uniformly at random from a set of nodes of a connected network if and only if the walk terminates at any node belonging to the network, with probability $1/N$, where $N$ is the number of nodes in the network.


The parameters of interest in sampling via random walk are: (i) it should provide a uniform sample irrespective of the topology of the network, and (ii) the length of the walk required to reach stationarity (mixing time of the walk) should be small. A number of researchers, over the years, have studied properties of random walks. Lovasz [15] provides an excellent survey of these techniques. The simplest random walk algorithm selects an outgoing edge at every node with equal probability, e.g., if a node has degree four, each of the edges is traversed with a probability $0.25$. It can be shown that the probability distribution associated with target nodes becomes stationary after a finite length random walk (also known as the mixing time for the corresponding Markov chain). This length can be shown to approach $O(\log n)$. These concepts are discussed in greater detail in Section 4. The main drawback of the simple random walk is that, while it reaches a stationary distribution, this distribution is not uniform for typical networks. In fact, it can be shown that the probability of terminating a random walk at a node is directly proportional to the degree of the node. In the context of conventional unstructured P2P networks, where node degrees can vary significantly, this does not correspond to an acceptable uniform sample.

Much like other typical applications of random walks, our system is sensitive to the quality of uniform sampling. Biases in sampling may result in poor performance of randomized algorithms, congestion in underlying networks, and significant load imbalances. Thus, realizing random walks that yield uniform sampling irrespective of topology is a key focus of our work. In addition to the quality of uniform sampling, an important performance parameter is the length of the random walk. Since longer random walks correspond to a higher number of network messages, it is highly desirable to minimize the length of the walk.

Technical Contributions

The paper makes the following specific contributions:

- It presents a scalable, robust, and efficient architecture for a P2P resource-sharing network.
- The basis for the proposed network is a load balancing, replication, and monitoring scheme that relies on efficient randomized algorithms. It presents a random walk based algorithm for uniform sampling in large real-world networks with low overhead. This sampling methodology provides a substrate for our randomized algorithms.
- It provides empirical results that demonstrate the efficiency of our algorithms for computing a large number of tasks on unstructured P2P networks with high node failure and churn rates. For example, we show that our randomized algorithm based P2P infrastructure achieves an efficiency of over 40% compared to an ideal parallel ensemble.

The rest of this paper is organized as follows. In Section 2, we summarize related results. In Section 3, we present an overview of our randomization-based P2P computing architecture. In Section 4, we show how uniform sampling can be achieved via random walks. We also present an algorithm that allows efficient (short length) random walks to obtain uniform sampling. In Section 5, we empirically evaluate the performance of our architecture. We show that our architecture yields high efficiencies for distributed computations. We also evaluate strategies for job replication. We derive conclusion from our work in Section 6.
3 Architectural Overview

In this section we provide a brief overview of unstructured P2P networks and describe a simple randomized job allocation scheme that achieves good load balance. We also motivate the need for redundancy, in the context of target applications and show how our protocol caters to replicating tasks in the network. A key aspect of our P2P cycle sharing environments is a decentralized rendezvous service for monitoring job progress, supporting loosely coupled inter-task communication, and aggregating completed tasks. We describe the distributed construction and the probabilistic guarantees on the performance of this service. We also show how our architecture can be leveraged to manage reputation of participating hosts in the network as a means to counter “free-rider” behaviors.

3.1 Unstructured Peer-to-Peer Networks

Unstructured P2P networks are characterized by lack of well-defined topology in the overlay and completely decentralized control. Each node in the network maintains a list of hosts it can connect to. The roles for connecting to peers and to maintain this list are usually heuristic-based and do not enforce a structured topology in the resulting network. Some nodes in the network are designated as (or choose to be) super-nodes. Super-nodes have large degrees and connect to low degree leaves and other super-nodes. The emergent, self-organized network graph has a highly skewed degree distribution with few nodes having high degrees while most nodes have only a few neighbors and are often connected to super-nodes. In such networks, the naive method for locating resources is to flood the query to a predetermined number of hops. This approach generally has high overhead in terms of network messages.

In this paper we build our system on top of an unstructured P2P network, and present novel algorithms that have low overhead and, at the same time, provide performance guarantees with high probability.\(^\text{1}\)

\(^{1}\)\text{Here, the term w.h.p implies that the probability of failure is }1−\epsilon\text{.}
3.2 Job Allocation with Redundancy

We first present a simple job allocation strategy that achieves appropriate redundancy and good load balance. Conventional unstructured P2P networks comprise of tens, even hundreds of thousands of nodes. Consequently, computational resources exist for building sufficient redundancy into the system. There are two main motivating factors for redundancy.

1. Resilience. In an open Internet environment failures (and departures) of nodes must be expected. In such an environment replication of the same task to multiple hosts is needed to account for failure of nodes.

2. Validation. We can expect that some of the nodes would return wrong results, either because of malicious or other reasons. Results from several nodes can be cross-checked to detect faults, and possibly select a correct output (e.g., simply using majority) from the available set of reported results. Indeed, several current systems such as SETI@home use similar methods.

We assume that a job, $J$, can be broken down into $n$ independent subtasks. We denote, using $N$, the number of Internet hosts that are engaged in computing the job $J$. The subtasks of $J$ can be clustered into batches ($b_1, b_2, \ldots$), each with $K$ subtasks. We discuss various considerations while choosing $K$ later. While submitting a job each batch is replicated by a factor $r \geq 1$. For example, $r = 2$ implies that two nodes will be assigned the same batch to compute. A simple randomized job submission algorithm that allows replication can be constructed as follows:

1. A host, $A$, that wants to submit subtasks of its job sets the batch size $K$ and a replication factor $r$.

2. For each batch, host $A$ selects a node uniformly at random by performing a random walk and submits a batch to it. The replication factor $r$ is also sent with the batch.

3. Each node that receives a batch decrements $r$ by one and if $r > 0$ sends a copy of the batch to another node chosen uniformly at random. The updated value of $r$ is sent with the batch. Thus, each batch is replicated at a total of $r$ nodes in the network.

Note that the number of messages sent by $A$ remains $n/K$ since redundant replication is taken care of by nodes downstream. Similarly, note that replication of each batch occurs in parallel. However, the total number of messages in the network is given by $nr/K$. In real-world situations this translates to reducing the cost of job submissions for master nodes. Any node failures during this process are handled by resubmitting jobs until $r$ replicas exist.

Several jobs can be initiated on the network concurrently. Since our job allocation protocol is based on randomization, a node may be assigned more than one batch, either of the same job or of different jobs. A node processes batches one at a time on a first come first served basis. We would like to emphasize that this randomized protocol is extremely simple, does not maintain any state information, and has a minimal overhead arising from random walks for uniform sampling.

3.2.1 Uniform Sampling and Load Balancing

Assume that a total of $m$ batches need to be assigned to the $N$ processors. If we assign batches uniformly at random to the $N$ processors, we can provide bounds on the quality of load balance achieved. Given $m$ batches, we answer the following questions:

1. What is the expected fraction of nodes that will have a batch assigned to them?

2. What is the probability that a node gets a given number of jobs to perform?

3. What is the maximum load on a node, with high probability?

The arguments presented here suggest that using $m = \log N$ provides good utilization of the network w.h.p., and at the same time yields a low probability of high load imbalance.

Lemma 3.1 Given $m$ batches the expected fraction of processors that will have a batch assigned to them is $1 - e^{-m/N}$.

Proof: We consider the question, how many processors do not have tasks assigned to them? The probability that a given processor did not get a task, when $m$ tasks are distributed is given by:

$$\Pr(\text{no task assigned}) = (1 - \frac{1}{N})^m \approx e^{-m/N}.$$ 

assuming that $N$ is large. We define random variable $X_i$, which is 1 if the $i^{th}$ host did not get any batch, and 0 otherwise. Then by linearity of expectation:

$$E[X] = E\left[\sum_{i=1}^{N} X_i\right] = \sum_{i=1}^{N} E[X_i] = N\left(1 - \frac{1}{N}\right)^m \approx Ne^{-m/N}.$$ 

Thus, the fraction of nodes that will get at least one batch to process is $(N - Ne^{-m/N})/N = 1 - e^{-m/N}$. □
This lemma simply implies that if $m \leq N$ at most, approximately, 65% of the nodes will be used. Similarly, if $n \geq N \log N$, with high probability all nodes will be used.

Now we find the probability that a given node gets $M$ batches to process. Given that $m$ and $N$ are large compared to $M$, this probability is given by:

$$P(m | M) = \left( \frac{1}{N} \right)^M \left( 1 - \frac{1}{N} \right)^{M-m} = e^{-\frac{m}{N} \frac{M}{N}} \frac{M^m}{M^M}$$

Similarly, the probability that a node gets at least $M$ batches is given by

$$P(M | m) = \left( \frac{1}{N} \right)^M$$

Lemma 3.2 If $m = O(N \log N)$, the probability that the maximum load on any node is more than $\Omega(\log N)$, is low.

Proof: For simplicity, we set $m = N \log N$ and maximum load to be $M = 2 \log N$. The probability that any node has load at least $e^2 \log N$ is bounded as follows:

$$P(M | m) = \frac{(\log N)^M}{N^M} \leq N \log N \left( 1 - \frac{1}{N^{\log N}} \right)^M \leq \frac{N}{N^N} \leq \frac{1}{N^N}$$

Theorem 3.1 When $m = O(N \log N)$ the load imbalance on a node diminishes.

Proof: A perfect deterministic algorithm allocates $O(\log N)$ batches to each machine, when $m = O(N \log N)$. From the previous lemma we know that the probability that the maximum load on a machine is $\Omega(\log N)$, is low. Therefore, a randomized algorithm that uses uniform sampling to distribute tasks approaches a deterministic algorithm, and yields good load balance.

3.2.2 Choosing Batch Sizes

Selecting an appropriate batch size in a distributed environment is a challenging task and depends on several variables. Krumkal and Weiss [15] show that when the running time of the subtasks are independent identically distributed (i.i.d.) random variables with mean $\mu$ and variance $\sigma^2$, then estimated completion time is given by:

$$E(T) = \frac{\mu}{p} + \frac{\mu h}{pK} + \frac{\sigma^2}{2K} \log p$$

1. It is assumed that $K/\log K$ is large, and for smaller values the error is not substantial. This expression is quiet general and holds for processing time distributions including exponential, gamma, Weibull, uniform, deterministic, and truncated normal distributions. The variance in the time required to complete the processing of a task, $\sigma^2$, depends the following parameters:
   1. The number of concurrent jobs, scheduled on the processors. This corresponds to the number of subtasks, which further reduces the set of nodes that have suitable lifetimes.
   2. The variation and non-deterministic nature of processing requirement of each subtask.
   3. The processing capability of participating hosts.

Each term in Equation 1 has important implications. The first term corresponds to the time it would take an ideal parallel system with $p$ processors to compute the $n$ subtasks.

The second term captures the benefit from aggregating jobs in terms of reducing the communication overhead.

The third term represents the overhead due to uneven finishing times of the processors. The most important implication of this result is the tradeoff between communication, which decreases as $1/K$, and the variance in processing times which increases as $\sqrt{K}$

In the above discussion there is no mention of the failure (and departure) of the nodes in the network. There is an important relationship between the size of the batch and the lifetime of a node. As a simple illustration consider the minimum processing time of all batches, $\min T(b_i)$. If $\min T(b_i)$ is greater than the expected lifetime of the nodes, the system would be reduced to using only a fraction of the nodes whose lifetime is large compared to the expected completion time of the job. The heavy load on such nodes implies that it would take much longer to complete the jobs, which further reduces the set of nodes that have suitable lifetimes.

Furthermore, being able to use the long lived nodes would require a job allocation scheme that maintains state information. This can be prohibitively expensive in large dynamic networks.

Our use of a randomized approach avoids these overheads, but at the same time is susceptible to failure under the conditions discussed. Thus, a batch size should be small enough, so that the required time for processing the batch is comparable to the lifetime of the nodes. Lifetime of nodes is also important in the context of developing a replication strategy as discussed in Section 3.2.3.

In summary, the size of a batch, $K$, should be:

1. Large enough so that the network overhead is reduced.
2. Small enough so that total job completion is minimally affected by variation in processing times taken by hosts, $\sigma$, and
3. Small enough so that computation time required for a given host is comparable to the hosts' lifetime.

Furthermore, the results from Section 3.2.1 suggest that if the total number of tasks to be computed in the network (from all jobs) is $n'$, then the number of batches, $n'/N$,
should be $N \log N$ for good load balance and network utili-
ization. We propose $K = \log^2 N$ as an ideal compromise for
aggregating tasks. This allows computation of a very large
number of tasks in the network, $N^* = \log^2 N$. It results in
good network utilization since the number of batches is
sufficient. For example, for a network of 100,000 hosts,
around 150 million tasks can be executing, which would
achieve high utilization, while the load imbalance would
be bounded. As $K$ grows faster than $\log N$, Equation 1
can approximate (with low error) the total running time for
$n'$ tasks. Plugging in the values shows that this value of
$K$ gives a low network overhead as

$$\alpha \sqrt{K \log p} = \sqrt{\log N}.$$

### 3.2.3 Multi-Step Replication

We revisit the replication strategy keeping in view the fact
that increased replication at submission time implies in-
creased time to completion of the jobs. Increased makespan
of a job implies that more nodes would leave the network
during that time, conditioned on the distribution of lifetime
of the nodes in the network.

Let, the number of nodes leaving the network, over unit
time be $y$. We denote the time for completion of $n$ tasks by
$T(n)$. As defined earlier, $r$ denotes the number of replicas of
a given task in the network. Recall that the protocol given
earlier in this section performs replication at job submit-
tion time. Then, $n_r = \gamma T(n)$ gives the number of nodes
leaving the network during the makespan of the job. The
level of replication to deterministically counter the failures
would require $r = n_r$. This in turn asserts, the stability con-
dition: $\gamma = n_r / T(n) \leq 1 / T(n)$, i.e., at most one node
failure over the job makespan. This stability requirement is
degenerate.

Thus, a job submission protocol should use a multi-step
replication strategy instead of replication-at-initiation. This
algorithm is well suited for a high node failure rate environ-
ments. The protocol works as follows:

1. A host, $A$, that wishes to submit subtasks of its job, sets
   the batch size, $K$. The replication factor $r$ is set to one.

2. For each batch, host $A$ selects a node uniformly at ran-
   dom, by performing a random walk, and submits a batch to it.

3. Host $A$ also calculates $T$, which is the time it would
take an ideal parallel ensemble to complete the job,
as $T = n_r / \alpha$. It then waits for a time $\epsilon T$, where $\epsilon$ is
the expected efficiency of our distributed system with-
out failures. This can be estimated to be roughly 40%
using Kruskal's equation for realistic internet environ-
ments.

4. After waiting for this time period, it collects the jobs
   that have been computed using the algorithm presented
   in Section 3.3.2.

5. Host $A$ determines the jobs that did not run to termi-
   nates and resubmits them with $r = 2$. If $n_f$ jobs were
   missing $T$ is calculated again as $\max(\gamma / n_f, 1 / \mu)$. The
   waiting and re-submission, with $r$ increments at each
   step, continues until results for all tasks are re-
   trieved.

In Section 5, we show using simulation that this multi-
step protocol in fact executes tasks much faster than the
replication-at-initiation strategy when this strategy uses
more than 3 replicas. However, the key advantage is that
this algorithm achieves 100% job completion, which the
other algorithm cannot achieve, inherently.

### 3.3 Rendezvous Service

The rendezvous service provides a communication fab-
ric between nodes that are oblivious to each other. The key
used for communication is a resource identifier or a resource
query string, rather than a node address. This is analogous
to the directory service in a client-server architecture. How-
ever, unlike its client-server model based counterpart, ren-
dezvous service does not have a centralized repository and
the peers need not know or register with any predefined
server node. The required information is maintained in a
completely distributed fashion among the peers.

#### 3.3.1 Construction

Nodes become a part of the rendezvous service by creating a
"rendezvous service set" (RS-set). The RS-set of each node
contains pointers to $\sqrt{\log N}$ peers selected uniformly at
random. The creation of the RS-set at each node occurs in a
distributed fashion among the peers of a given node. Each
node is responsible for maintaining $\sqrt{\log N}$ live peers in its
RS-set, when there are node failures. The provider of a resource publishes
its resource identifier to its RS-set. Similarly, the node look-
ing for this resource sends a query to its own RS-set. The query
string, rather than a node address. This is analogous
to the directory service in a client-server architecture. How-
ever, unlike its client-server model based counterpart, ren-
dezvous service does not have a centralized repository and
the peers need not know or register with any predefined
server node. The required information is maintained in a
completely distributed fashion among the peers.

#### 3.3.2 Algorithm

Theorem 3.2 Any two RS-sets of size $\sqrt{\log N}$ nodes, in-
tersect w.h.p.

**Proof:** Since the RS-sets contain nodes selected uniformly
at random, the probability that a given node of one RS-set is
not in the other RS-set is given by $1 - \sqrt{\log N} / N$. Thus,
the probability that none of $\sqrt{\log N}$ nodes of one RS-set
are in the other RS-set is $(1 - \sqrt{\log N} / N)^{\sqrt{\log N}} \approx 1 / 2$.

The probability that at least one node in the two sets inter-
sects is $1 - \frac{1}{2}$. 

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Note that this service has very low overhead. The overhead of creating the RS-set is amortized over the life of the node. Similarly, only $\sqrt{NF/N}$ messages are required for each query. We can compare this with flooding, where the number of network messages increase exponentially with the number of hops. In the rest of this section, we show how this service is used in our architecture.

3.3.2 Monitoring and Merging Jobs

When a node completes the execution of a batch, it informs its RS-set using the job identifier. With high probability, one of the nodes in the RS-set of this node also belongs to the RS-set of the owner of the job. Such an intersecting node receives the results. Each of the nodes in the RS-set of job owners maintain independent bit vectors, where the jobs received by them are marked. The owner requests these vectors from its RS-set, once in a given interval. It may also download the results for the completed tasks. Alternatively, the owner may ask its RS-set nodes to submit completed jobs to it once a certain number of new results are available. If replication factor $r > 1$ is used in submitting jobs, the owner would get multiple copies of the same job. This information can be used to verify that the results match, and then provide a validation mechanism.

3.3.3 Reputation Monitor

The RS-set of the master receives results computed by all participating nodes. Some free-riders may not compute the tasks given to them, or similarly, some malicious users might return wrong results. The node's RS-set maintains information about the results submitted by its owner. Similarly, the owner of the job publishes the information (ID) about the nodes that reported inaccurate results. If node $x$ wants to query the reputation of node $y$, it simply inquires its RS-set. The overlap between the RS-sets reveals the required information. Using this information nodes may reject jobs coming from free-riders and malicious users, hence discouraging such activities. This system is robust to collaborative malicious activity since a large number of nodes (2 RS-sets) keep this information. Such systems have been proposed in the context of conventional file-sharing P2P networks and have been shown to handle free-riders in a scalable manner.

3.3.4 Decoupled Communication

A node might need to search for information without knowing which node has the required information. For example, a node processing a task may need to know the result of some preceding task. Such cases may arise if the subtasks of a job are not completely independent. Similarly, several other examples of resource location can be cited. In these scenarios, a node needs to send a query about the requested resource to its RS-set and with high probability it will intersect with the RS-set of the provider of the resource.

4 Uniform Sampling With Random Walks

In this section we introduce random walks and show that how they can be used to perform random sampling. If the underlying network does not have a regular degree distribution, i.e., if few nodes are connected to many, and many nodes are connected to a few nodes, then a random walk, with transitions from a node to its randomly chosen neighbors, does not yield a uniform sample. We revisit Kruskal's equation and argue that a skewed-sampling results in a bad load balance and a long job makespan. We show how the transition of the walk from one neighbor to another must be modified to achieve a uniform sampling, and give an algorithm that computes the required transition probabilities for random walks.

4.1 Sampling With Random Walks

Random walks can be abstracted as Markov chains defined over a state space and a given state transition matrix. The network nodes form the state space and the probability of moving from a node to its neighbor govern the transitions. Using a Markov chain model, we show in this section, that (1) a random walk of a given minimum length on a connected aperiodic graph (which represents the network) reaches a stationary sample distribution, and (2) a simple random walk cannot achieve uniform sampling unless each node in the network has an identical number of connections. We also discuss various parameters that determine the length of the random walk required to achieve a stationary sample distribution.

Let $G(V,E)$ be a simple connected undirected graph representing a distributed system with $|V| = N$ nodes and $|E| = e$ links. The degree, or number of links, of a node $i$, $1 \leq i \leq N$, is given by $d_i$. The set of neighbors of a node $i$ is given by $F(i)$, where edge $(i,j) \in E, \forall j \in F(i)$. The $N \times N$ adjacency matrix of $G$ is given by $A = [a_{ij}]$, where $1 \leq i,j \leq N, a_{ij} = 1$ if the edge $(i,j) \in E$, and 0 otherwise. The corresponding $N \times N$ transition probability matrix, given by $P = [p_{ij}]$, is the probability of moving from node $i$ to node $j$ in one hop. $P$ is a row-stochastic matrix, i.e., $\Sigma_j p_{ij} = 1$.

For a simple random walk the transition from node $i$ to its neighbor is governed by the transition probability matrix $P$, where $\forall j \in F(i), p_{ij} = 1/d_i$ and 0 otherwise. The
Equation (2) simply implies that a random walk is memoryless, i.e., during a random walk the probability of transition from node \( i \) to node \( j \) in one step depends only on node \( i \). Thus, a random walk can be conveniently modeled as a Markov chain, more specifically a homogeneous Markov chain, since the right hand side of Equation (2) is independent of \( i \). Such a Markov chain has the following properties: it is irreducible if the graph \( G \) is connected and is aperiodic if \( G \) is aperiodic. A graph \( G \) is aperiodic if the greatest common divisor of the length of all cycles in the graph is 1. In particular, an undirected aperiodic graph cannot be bipartite, which is a reasonable assumption for real networks in which connections are established randomly.

4.1.1 Convergence to Random Sampling

It is well known that an irreducible and aperiodic Markov chain has a stationary distribution \( \pi = \pi P \), and \( \pi^T = \pi^T P \) follows where \( P \) implies \( t \)-step transitions). It is easy to show (221, page 132) that \( \pi_i \), the component corresponding to node \( i, 1 \leq i \leq n \), is \( d_i/2e \). From \( \pi^T = \pi^T P \), we see that \( \pi \) is a left eigenvector of \( P \) with eigenvalue \( 1 \). The right eigenvector for eigenvalue 1 is \( 1 \) (vector of all ones), since \( P^1 = 1 \). It follows that \( \pi = \pi^T 1 \). This implies that a very long walk converges to the stationary distribution \( \pi \) irrespective of the initial distribution, i.e., the starting point of the walk.

The above results indicate that a long enough random walk converges to a random sample irrespective of where the walk started. Thus, a random walk is a good candidate for random sampling in a network. However, we also know that the resulting sample distribution is dependent on the degree of the node: \( \pi_i = d_i/2e \). Thus, a result that the random sample is uniform \( \pi_{uni} = (1/N)1 \) only if the graph \( G \) is regular (i.e., the degrees of all nodes are equal). Since typical large scale, real-world, unstructured networks tend to have non-uniform degree distributions (e.g., power-law degree distribution of unstructured P2P networks [23]), uniform sampling in practical scenarios poses a significant challenge.

4.1.2 Length of Walk for Random Sampling

The sample distribution at step \( k \) of the walk depends on \( P^k \), which in turn depends on the eigenstructure of \( P \). From the Perron-Frobenius theorem, we have \( P^k = \lambda_1 u_1^k \), for some \( k \), \( u_1 \) is the right eigenvector corresponding to eigenvalue \( \lambda_1 \) and \( u_1 \) is the left eigenvector, and \( \lambda_1 \) is the algebraic multiplicity of \( \lambda_1 \) (see, [3] Chapter 6). Rewriting the above equation, we get \( P^k = \lambda_1^k u_1^k \). These results simply imply that

\[
P^k = \lambda_1^k u_1^k = \lambda_1^k (\pi^T 1) = \lambda_1^k \pi^T.
\]

As \( \lambda_1 < 1 \), when \( k \) is large, \( \lambda_1^k \approx 0 \). Therefore, the smaller the second largest eigenvalue modulus (SLEM), the faster the convergence to stationary distribution. As a result, a walk of smaller length is required for random sampling. The length of the required walk, or the mixing time, is often approximated as \( O(\log(N)/\epsilon) \), however the exact factors involved depend on the construction (and thus the SEM) of the transition probability matrix.

4.2 Uniform Sampling in Nonuniform Networks

As mentioned in Section 4.1.1, a random walk of a given minimum length converges to a stationary distribution \( \pi \). If the stationary distribution \( \pi_{uni} \) is such that \( \pi_{uni} = (1/N)1 \), the random walk will terminate at any node in the network with equal probability (c.f. Definition 1.3). If the stationary distribution is \( \pi_i = d_i/2e \), we pick the high degree nodes with a much higher probability. This implies that such nodes will have a high number of tasks to compute. The variance in the node sampling requires the variance of the processing time of the nodes, as the processing resource of the node will be divided over the tasks assigned to it. If the resulting variance of the running time from the mean processing time of the nodes is \( \epsilon^2 \), then the Kruskal's equation would be written as

\[
E(T) = \frac{\mu}{p} + \frac{nh}{pK} + \sqrt{\epsilon^2 + \frac{2E\log p}{\tau}}.
\]

This directly impacts the expected running times of jobs. Note that variance of the degree of the nodes may be very high if the underlying graph follows power-law degree distribution. In Section 5.2.1, we experimentally demonstrate that using simple random walks for sampling in nonuniform networks yields a poor load balance, and consequently a large job turnover time.

4.2.1 Modifying Transition Probabilities

To achieve a uniform stationary distribution in an irregular graph, we need to modify its transition probability matrix.
Let \( P \) be a probability transition matrix of a Markov chain, then 
\[ P_{\text{column}} = \frac{1}{N} \text{row}P, \]
which is the same as 
\[ \frac{1}{N} \text{row} P = P_{\text{column}}, \]
which is the same as 
\[ \frac{1}{N} \text{row} P = \frac{1}{N} \text{row} P. \]
This means that the sum of each column vector of \( P \) is 1, i.e., \( P \) is column stochastic. A probability transition matrix which is column stochastic in addition to being row stochastic is called doubly stochastic. Note that symmetric transition probability matrix are doubly stochastic. Thus, if we create a matrix with \( p_{ij} = p_{ji} \) we will achieve a uniform stationary distribution, and hence a random walk using these transition probabilities will yield a uniform sample.

Two well known algorithms, maximum-degree algorithm (MD) [2] and Metropolis-Hastings algorithm [20, 13], yield a symmetric transition probability matrix. However, these algorithms need a long walk to reach stationarity, if the graph has a highly skewed degree distribution. In our previous work [2], we present a detailed discussion and experimental evaluation of these algorithms, and suggested a new algorithm for building a transition matrix. Here, we re-produce the algorithm, however, the details and evaluation are omitted.

### 4.2.2 Random Weight Distribution Algorithm

In this section, we present our distributed algorithm, referred to as the Random Weight Distribution (RWD) algorithm. RWD is a completely decentralized algorithm that sets up transition probabilities in a connected network to enable efficient uniform sampling via random walks.

The algorithm proceeds as follows. In the initialization phase each node, locally, sets transitions probability as:

\[ p_{ij}^{(\text{i})} = \begin{cases} 
\frac{1}{\rho} & \text{if } i \neq j \text{ and } j \in \Gamma(i), \text{ where } \rho \geq d_{\text{max}} \\
1 - d_{\text{max}}/\rho & \text{if } i = j \\
0 & \text{otherwise.} 
\end{cases} \]

Here, \( \rho \) is a static system parameter with the constraint that it should be greater than \( d_{\text{max}} \). This parameter is static because we can sufficiently overestimate \( d_{\text{max}} \) knowing system properties (e.g., popular P2P clients have a maximum connection limit [17]). Note that this phase results in a high self-transition probability for low degree node. Also note that the resulting transition probability matrix is symmetric.

After the initialization is complete, each node attempts to distribute its self-transition probability randomly and symmetrically to its neighbors. The term ‘weight of a node’ refers to the self-transition probability of the node at any given time during the execution of the algorithm. At a node \( i \), the algorithm terminates when either the weight of the node becomes zero or the weight of all nodes \( j \in \Gamma(i) \) becomes zero. Intuitively, a low self-transition probability implies that the walk mixes faster and converges to a stationary uniform distribution with a fewer number of steps. The pseudo code for the complete RWD algorithm is shown in Figure 1.

![Figure 1. The Random Weight Distribution algorithm.](image)

**Remark 4.1** Each step in the RWD algorithm maintains symmetry in the global transition probability matrix \( P_{\text{unif}} \). Therefore, the transition probability matrix remains symmetric when the algorithm terminates. Thus, a random walk based on \( P_{\text{unif}} \) will have stationary distribution \( \pi_{\text{unif}} \).

The overhead of messages due to our algorithm are minimal as explored in our earlier work [2].

### 5 Experimental Evaluation

We present here detailed simulation results for various performance aspects of our system. First, we evaluate the efficiency of our system for different job loads. We also show that job allocation using random walks with transition matrix generated using the RWD algorithm yields good load balance due to uniform sampling. In compar-
son job allocation using simple random walk based sampling yields a highly skewed load balance. A comparison of the overhead associated is low, compared to replication-at-initiation using a high replication level. The main contribution of this experimental study is that it serves as a proof-of-concept of the feasibility of a randomization-based unstructured P2P computing environment. It also identifies key factors in the design of such architectures.

5.1 Experimental Setup

Our simulation testbed implements the RWD algorithm for uniform sampling of nodes, the two job distribution protocols discussed in Section 3.2, and the rendezvous service for job aggregation and progress monitoring. We use a power-law random topology for the network. In a power-law random graph, if the nodes are sorted in descending order of degree the density distribution of the node degrees follows a power-law distribution of the form $D = a\cdot n^{-\beta}$, where $D$ is the degree distribution of the nodes in the network.

For each node using a power-law distribution and then connecting them randomly. Motivated by real-world systems [7], these restrictions are often specified in evaluation studies of P2P networks [9]. The underlying topology is constructed by first selecting the degree of each node using a power-law distribution and then connecting them randomly. Motivated by real-world systems [7], we fix the network size to $N = 10000$ nodes. This network size corresponds to an optimal number of tasks in the system.

### Table 1. Number of jobs and batch size parameters used for evaluating the efficiency of our system.

<table>
<thead>
<tr>
<th>Number of jobs, $N$</th>
<th>Cluster size, $K$</th>
<th>Number of batches</th>
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<tbody>
<tr>
<td>$N = 10000$</td>
<td>$K = 100$</td>
<td>$b = 100$</td>
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Table 1. Numbers of jobs and batch size parameters used for evaluating the efficiency of our system.

2. As predicted the system has the highest efficiency when $K = \log^2 N$ and the number of batches is $N = \log^2 N$. The efficiency achieved in this case is 45%, which is excellent for such loosely coupled dynamic environments. We expect that in the real world there would usually be enough job submission requests to meet the optimum value of number of tasks in the system.

5.2 Efficiency and Computational Throughput

We study the efficiency of our system in comparison to an ideal $N$ processor parallel ensemble, which computes $n$ tasks in $\mu N$ time. As stated in the discussion in Section 3, if $N \log N$ batches are submitted then, w.h.p, each processor has a task to perform. Similarly, if possible we would like to have batch sizes approaching $K = \log^N N$ tasks (which translates to an optimal $n = N \log^2 N$ in the cases where $n/N < K$ we use $K = \frac{\log^2 n}{N} = 1$). To evaluate the efficiency of our system for different job sizes and batch sizes we use the parameters described in Table 1. Note that although we would like to have $N \log N$ batches each time, due to substantial network overhead we prioritize larger batch size over the number of batches that can be submitted. As a result when there are only 10,000 batches, $\approx 30\%$ of the machines in the network had no tasks to process. This is consistent with the predictions from Section 3.2.1.

The resulting efficiency of the system is plotted in Figure 3. As predicted the system has the highest efficiency when $K = \log^2 N$ and the number of batches is $N = \log^2 N$. The efficiency achieved in this case is 45%, which is excellent for such loosely coupled dynamic environments. We expect that in the real world there would usually be enough job submission requests to meet the optimum value of number of tasks in the system.

5.2.1 Effect of Sampling Techniques

Uniform node sampling is the underlying substrate for all our randomized algorithms. Our RWD algorithm computes transition probabilities in such a way that a random walk yields a uniform sample. In comparison a simple random walk (SRW) is biased towards high degree nodes. We compare the load balance achieved using these two strategies and evaluate the impact of load imbalance due to the SRW algorithm.

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The main contribution of this experimental study is that it serves as a proof-of-concept of the feasibility of a randomization-based unstructured P2P computing environment. It also identifies key factors in the design of such architectures.
Figure 2. Efficiency of our system when compared with an ideal parallel ensemble.

To compare the load balance of the two schemes we use the optimal parameters \( n = N \log^2 N \) and \( K = \log^2 N \). The resulting percentage of the total number of tasks assigned to each machine are plotted in Figure 3. The x-axis of the plot represents nodes sorted in ascending order by degree. On the other hand the number of tasks assigned to a machine by sampling using SRW is biased to the degree of the node. Thus, some nodes end up receiving almost 5 times higher load than other nodes.

A load imbalance implies that the job would take longer time to finish. We compare the system efficiency when batches are allocated with sampling using the two techniques. The number of jobs used are \( n = N \log^3 N, N \log^2 N \), and the cluster sizes used in both cases is \( \log^2 N \). The plot in Figure 4 shows this comparison. The system using RWD performs much better irrespective of the loads assigned to it and the performance advantage increases as the load increases.

5.3 Performance Under Node Failures

An important aspect of our architecture is its performance under node failures. The key parameters of interest are: percentage of tasks that are successfully retrieved by the owner of the job, increase in completion time due to replication, and performance under varying failure rates.

For these experiments we use \( K = \log^2 N, n = N \log^3 N \) and \( N = 10,000 \) nodes.

5.3.1 Node Failure Model

Node failure is modeled using a parameter \( \alpha \), which represents the fraction of nodes that fail in the time it would have taken for the job to complete in an environment without failure. This definition is useful for modeling node lifetime in comparison to the lifetime of the job. Node lifetimes are modeled as zipf random variables which are correlated to the degree of the node. The parameter \( \alpha \) is used to normalize these lifetimes in relation to the job makespan time. The number of nodes in the network is kept roughly constant by matching the arrival and failure rates.

5.3.2 Evaluation

We compare the performance of the two schemes described in the paper, namely replication-at-initiation and multi-step replication. For the replication-at-initiation method we use a replication factor of two (i.e., two copies of the replicated processes are submitted to the system). The effect of higher replication levels is discussed later. We let the owner query the system (through the rendezvous service) for its running tasks, often enough so that it knows almost instantaneously if no more of its tasks are running. This is unrealistic in a real-world environment, but our results here are meant to provide a bound on how well the system can perform. For the multi-step replication method the system assumes an ef-
Figure 4. Performance advantage of uniform sampling using random walk with RWD versus simple random walk.

Figure 5. Percentage of unique task results successfully retrieved by the owner, for the replication-at-initiation scheme.

The plot in Figure 5 shows the percentage of unique tasks successfully retrieved by the owner, for the replication-at-initiation scheme. As the fraction of failing nodes increase, the success ratio decreases rapidly. In comparison the multi-step protocol achieves a 100% completion rate. The plot on the top in Figure 6 compares the job completion time of the replication-at-initiation with two replicas and the multi-step algorithm. The job completion times are normalized with respect to the time it would take the job to finish in an environment with no failures. The plot shows that for $\alpha \leq 0.1$ the multi-step algorithm performs slightly better. However, as $\alpha$ increases the time taken by the multi-step algorithm increases. On the other hand the time for the replication-at-initiation scheme decreases gradually. This gradual decrease is because many tasks are lost with the failing nodes and thus the time for the network to have no tasks executing is reached earlier. Note also that the time taken by the multi-step technique levels off when the rate of failure is higher. This is because the multi-step technique has an increasing degree of replication at each step. Once the replication level becomes high it over compensates for the high failure rate.

Figure 6. (Top) Completion time for the two schemes. (Bottom) Number of replicas found by the master for the two schemes.

The plot on the bottom, in Figure 6 shows the average number of replicas found. The trends seen are interesting, because the average number of replicas found by the replication-at-initiation scheme decrease because of increased failures. On the other hand for the multi-step scheme, each iteration of replication submission increases the replication rate. With high failure rates the number of replicas submitted is higher and thus the average number of replicas received is higher. Quantitatively, the multi-step replication scheme has fewer redundant replicas and hence use resources more efficiently. However, as discussed earlier the replicas received can be useful for validating results. Nevertheless, the multi-step scheme can be easily modified to increase the redundancy in a controlled fashion, simply by submitting replicated tasks even after one result for that task has been successfully received.
Conclusion

To evaluate the benefit of using different levels of replication in the replication-at-initiation scheme, we repeat the experiment with $\alpha = 0.5$, while varying the replication level. The number of replicas submitted is increased from 1 (i.e., no redundancy) to 9 i.e., $10 \log N$. The plots in Figure 7 summarize the results. The plot on the top, shows that increasing the replication level results in the number of tasks completed to asymptotically approach 100%. However, a perfect result is not achieved even with very high replication levels. The plot in the middle shows that average number of results retrieved from replicas approaches 50% of the number of replicas submitted. This is consistent with the failure fraction $\alpha = 0.5$. When more replicas are submitted there is a higher chance of the replicas reaching a long lived node. Due to the zipf distribution such long lived nodes might have a much higher lifetime compared to the average. Finally, the plot at the bottom shows that the time for completion (i.e., no tasks of this job remain on the network) proportionally increases as the number of replicas increase. An important contrast can be drawn here to the performance of the multi-step technique. For $\alpha = 0.5$, the time taken by 7 level replication, which achieves 99.2% completion, is almost 2.4 times more than the multi-step technique, which always achieves 100% replication.

6 Conclusion

In this paper we presented a distributed architecture for sharing processor cycles in unstructured P2P networks. The use of unstructured P2P networks is motivated by the success of massive real-world networks for file sharing. We present randomized algorithms for allocating tasks in the network, which achieve a good load balance and low job makespan. We analytically show that random job allocation using uniform sampling achieves good load balance. We present two protocols that incorporate redundancy for resilience against frequent node departures and validation of the execution output. The parameters that affect job throughput are discussed in the context of our allocation scheme. Our architecture includes a rendezvous service that allows job progress monitoring, aggregation of tasks, node reputation management, and context-based communication between oblivious hosts. We show that the rendezvous service provides probabilistic guarantees for locating resources.

Our algorithms are built on the premise of uniform sampling in an unstructured network. We show that random walks are ideal for random sampling, however, the resulting samples are affected by the topology of the network. We present an algorithm that allows uniform sampling via random walks irrespective of the underlying network topology. This is done by building a transition matrix for the walk in a distributed fashion. The resulting random probability matrix also reduces the length of the random walk required to converge to uniform stationarity. The efficiency of the resulting cycle sharing system is evaluated using comprehensive simulation. The system is also evaluated with varying rates of node failures. The simulation results reflect the efficiency and robustness of our randomization based protocols.
References