Modelling and Simulation of a R744 based Air Conditioning Unit

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Performance of transcritical R744 systems is highly sensitive to the high side pressure. An appropriate strategy is required to control the high side pressure.

In the present study, a R744 based air conditioning cycle with two expansion valves is considered for proper control of high side pressure and quality of refrigerant at evaporator exit.

A system simulation model is developed for this cycle considering detailed model for each component.

Using the developed model, the effects of variation of various important environmental parameters on system performance is analysed.
INTRODUCTION

- Increasing Installation of Heating, Ventilation and Air Conditioning equipments globally

- Use of synthetic refrigerants is harmful to environment

- Use natural refrigerants like air, water, CO\textsubscript{2}, ammonia etc.
Why CO₂?

CO₂ is a promising refrigerant -

**Advantages:**

- **Zero ODP and unity GWP.**
- **A non-flammable, non-toxic and naturally available refrigerant.**
- **Compatible with normal lubricants and common machine construction materials.**
- **Weight and space requirement is low due to high volumetric refrigerant capacity.**
- **No recycling required.**
Transcritical CO$_2$ system

- Low critical temperature
- High critical pressure
- High operating pressure
- Heat rejection at supercritical pressure

Fig. 1: p-h plot of transcritical CO$_2$ cycle
Control of high side pressure

- Performance of transcritical system is strongly dependent on **high side pressure** and **gas cooler exit temperature**

Fig. 2: p-h plot of transcritical CO₂ cycle for gas cooler exit temperature of 35°C

Fig. 3: p-h plot of transcritical CO₂ cycle for gas cooler exit temperature of 25°C
Objectives of present work

- A comprehensive mathematical model is developed for R744 based air conditioning cycle with two expansion valves.

- Design dimensions for different components are predicted using the developed model.

- Optimum operating parameters are identified from the results of numerical simulations.

- The effects of various important environmental parameters on system performance are analyzed.
System description

Fig. 4: Schematic of the R744 based air conditioning unit

Fig. 5: p-h plot of transcritical R744 based cycle
Design procedures adopted

- Elemental log mean temperature difference approach for designing gas cooler
- Elemental log mean enthalpy difference approach for designing evaporator
- Semi empirical model for compressor
Gas cooler

- Counter cross flow arrangement
- Discretized approach to design the gas cooler
- Guess Pressure and temperature at exit
- Node by node marching from exit to inlet
- Iterative procedure to match pressure and temperature obtained at the end of the march
Gas cooler (Mathematical model)

Procedure adopted for designing gas cooler:

- Calculation of surface geometrical characteristics for each node
- Calculation of air side heat transfer coefficient and pressure drop
- Calculation of overall fin efficiency
- Calculation of refrigerant side heat transfer coefficient and pressure drop
- Reduced governing equations for each discretized node
Surface geometrical characteristics [Shah et al. (2003)]:

Unfinned base surface area,

\[ A_b = n_t N_{row} \left( \pi d_o L - \left( \sqrt{f_p^2 + (\pi d_o)^2} \right) \times f_i \left( \frac{L}{f_p} \right) \right) \]

Fin surface area,

\[ A_f = n_t N_{row} \left( \frac{L}{f_p} \right) \times \left( \frac{1}{2} \pi (d_f^2 - d_o^2) + \pi d_f d_t \right) \]

Total surface area,

\[ A_o = A_b + A_f \]

Minimum free flow area,

\[ A_{min} = \left[ (n_t - 1) c' + (P_t - d_o) - (d_f - d_o) f_i \left( \frac{1}{f_p} \right) \right] \times L \]

\[ c' = \begin{cases} 2a' & \text{if } 2a' < 2b' \\ 2b' & \text{if } 2b' < 2a' \end{cases} \]

\[ 2a' = (P_t - d_o) - (d_f - d_o) f_i \left( \frac{1}{f_p} \right) \]

\[ b' = \left( \left( \frac{P_t}{2} \right)^2 + P_t^2 \right)^{1/2} - d_o - (d_f - d_o) f_i \left( \frac{1}{f_p} \right) \]
Air side heat transfer coefficient and air side pressure drop [Pongsoi et al. (2013)]:

\[
\alpha_a = \frac{j \rho_a V_{max} C_{p,a}}{Pr^{2/3}} \\
\Delta P = f \times \left( \rho_a V_{max}^2 \right) \times \left( \frac{A_o}{A_{min}} \right)
\]

\[
j = 0.215 \text{Re}_{do}^{-0.4059} \quad f = 0.4852 \text{Re}_{do}^{-0.2156} \left( \frac{f_p}{d_o} \right)^{0.4771}
\]

Fin efficiency [Pongsoi et al. (2013)]:

\[
\eta_f = \frac{2\psi}{\phi(1+\psi)} \frac{I_1(\phi R_o)K_1(\phi R_i) - I_1(\phi R_i)K_1(\phi R_o)}{I_0(\phi R_i)K_1(\phi R_o) + I_1(\phi R_o)K_0(\phi R_i)}
\]

\[
\phi = \left( r_o - r_i \right)^{3/2} \left( \frac{2\alpha_o}{k_f A_p} \right)^{1/2}
\]

\[
A_p = f_t \left( r_o - r_i \right) \quad \psi = \frac{r_i}{r_o} \quad R_o = \frac{1}{1-\psi} \quad R_i = \frac{\psi}{1-\psi}
\]

Overall fin efficiency:

\[
\eta_o = 1 - \left( \frac{a_f}{a_o} \right) \left( 1 - \eta_f \right)
\]
Refrigerant side heat transfer coefficient and pressure drop are predicted using correlation proposed by Pitla et al. (2002):

\[ Nu = \left( \frac{Nu_{\text{wall}} + Nu_{\text{bulk}}}{2} \right) \frac{k_{\text{wall}}}{k_{\text{bulk}}} \]

\( Nu_{\text{wall}} \) and \( Nu_{\text{bulk}} \) are calculated using Gnielinski correlation within the range \( 2300 < \text{Re} < 10^6 \) and \( 0.6 < \text{Pr} < 10^5 \):

\[ Nu = \frac{(f/8)(\text{Re} - 1000)\text{Pr}}{12.7 \sqrt{f/8(\text{Pr}^{2/3} - 1)} + 1.07} \]

For \( \text{Re} > 10^6 \), \( Nu_{\text{wall}} \) and \( Nu_{\text{bulk}} \) are calculated using Petukhov-Popov-Kirilov correlation:

\[ Nu = \frac{(f/8)\text{Re}\text{Pr}}{12.7 \sqrt{f/8(\text{Pr}^{2/3} - 1)} + 1.07} \]

Friction factor \( 'f' \) is given by: \( f = (0.79 \ln(\text{Re}) - 1.64)^{-2} \)
Governing equations for each node [Yin et al. (2001)]:

Refrigerant side:

\[ Q_{node} = m_{ref} \left( h_{i+1, j, k} - h_{i, j, k} \right) \]

\[ Q_{node} = \alpha_{ref} \rho_{ref} LMTD_{ref} \]

\[ LMTD_{ref} = \frac{(T_{ref(i+1, j, k)} - T_{s(i, j, k)}) - (T_{ref(i, j, k)} - T_{s(i, j, k)})}{\ln \left( \frac{T_{ref(i+1, j, k)} - T_{s(i, j, k)}}{T_{ref(i, j, k)} - T_{s(i, j, k)}} \right)} \]

\[ P_{ref(i+1, j, k)} - P_{ref(i, j, k)} = \Delta P_{f} = f \frac{G^2 \times dL}{2 \times \rho \times d_i} \]

Air side:

\[ Q_{node} = m_{a} C_{pa} \left( T_{a,ex(i, j, k)} - T_{a,in(i, j, k)} \right) \]

\[ Q_{node} = \alpha_{a} \rho_{a} LMTD_{a} \]

\[ LMTD_{a} = \frac{(T_{s(i, j, k)} - T_{a,in(i, j, k)}) - (T_{s(i, j, k)} - T_{a,ex(i, j, k)})}{\ln \left( \frac{T_{s(i, j, k)} - T_{a,in(i, j, k)}}{T_{s(i, j, k)} - T_{a,ex(i, j, k)}} \right)} \]
## Gas cooler (Dimensions predicted)

<table>
<thead>
<tr>
<th>Tube</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>Copper</td>
</tr>
<tr>
<td>Inner diameter</td>
<td>5.5 mm</td>
</tr>
<tr>
<td>Outer diameter</td>
<td>9.5 mm</td>
</tr>
<tr>
<td>Longitudinal tube pitch ($P_l$)</td>
<td>30 mm</td>
</tr>
<tr>
<td>Transverse tube pitch ($P_t$)</td>
<td>27 mm</td>
</tr>
<tr>
<td>Finning length of tube</td>
<td>480 mm</td>
</tr>
<tr>
<td>Number of tubes in each row</td>
<td>18</td>
</tr>
<tr>
<td>Number of tube rows</td>
<td>3</td>
</tr>
<tr>
<td><strong>Fin</strong></td>
<td></td>
</tr>
<tr>
<td>Material</td>
<td>Copper</td>
</tr>
<tr>
<td>Outer diameter</td>
<td>26 mm</td>
</tr>
<tr>
<td>Fin thickness</td>
<td>0.19 mm</td>
</tr>
<tr>
<td>Fin pitch</td>
<td>3 mm</td>
</tr>
</tbody>
</table>

### Surface area

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>Total fin surface area</td>
<td>8.0837 m$^2$</td>
</tr>
<tr>
<td>Unfinned base surface area</td>
<td>0.7243 m$^2$</td>
</tr>
<tr>
<td>Total surface area, A</td>
<td>8.8080 m$^2$</td>
</tr>
</tbody>
</table>
Evaporator

Like gas cooler similar discretized approach is adopted for designing the evaporator

Evaporator coil is divided into two different zones based on the quality of refrigerant:

• Two phase zone
• Single phase vapour zone

Each zone is then further subdivided based on the occurrence of condensation of moisture on coil surface:

• Wet surface (for water film temperature less than DPT of air)
• Dry surface (for water film temperature greater than DPT of air)
Evaporator (Mathematical model)

Procedure adopted for designing the evaporator

- Calculation of surface geometrical characteristics for each node
- Calculation of air side heat transfer coefficient and pressure drop
- Prediction of zone based on quality of refrigerant
- Prediction of Wet/dry region based on condensation of moisture on surface
For refrigerant in **two phase region**, heat transfer coefficient is estimated using the correlation proposed by *Yoon et al. (2004)*.

Critical quality at which liquid film breaks down is calculated from:

\[
x_{cr,t} = 38.27 \text{Re}_i^{2.12} (1000 \text{Bo})^{1.64} B_d^{-4.7}
\]

For \(x < x_{cr,t}\)

\[
\alpha_i = \left[ (S \alpha_{NB})^2 + (E \alpha_i)^2 \right]^{1/2}
\]

\[
\alpha_{NB} = 55 P^{0.12} \left( - \log P^* \right)^{-0.55} M^{-0.5} q^{0.67}
\]

\[
S = \frac{1}{1 + 1.62 \times 10^{-6} E^{0.69} \text{Re}_i^{1.11}}
\]

\[
E = \left[ 1 + 9.36 \times 10^3 x \text{Pr}_i \left( \frac{\rho_l}{\rho_g} - 1 \right) \right]^{0.11}
\]

\[
\alpha_i = 0.023 \frac{k_i}{d_i} \text{Re}^{0.8}_i \text{Pr}_i^{0.4}
\]

For \(x \geq x_{cr,t}\)

\[
\alpha_i = \frac{\theta_{dry} \alpha_g + (2\pi - \theta_{dry}) \alpha_{wet}}{2\pi}
\]

\[
\alpha_g = 0.023 \frac{k_g}{d_i} \text{Re}_g^{0.8} \text{Pr}_g^{0.4}
\]

\[
\alpha_{wet} = E \alpha_i
\]

\[
E = 1 + 3000 \text{Bo}^{0.86} + 1.12 \left( \frac{x}{1-x} \right)^{0.75} \left( \frac{\rho_l}{\rho_g} \right)^{0.41}
\]

\[
\frac{\theta_{dry}}{2\pi} = 36.23 \text{Re}_i^{3.47} \text{Bo}^{4.84} B_d^{-0.27} \left( \frac{1}{X_{\mu}} \right)^{2.6}
\]
For **wet zone**, where condensation occurs on external surface, heat as well as mass transfer takes place. Method proposed by Threlkeld (1998) is adopted here to incorporate the mass transfer effects.

**Governing equations for each node of evaporator:**

\[
m_a h_{a,in} = m_a h_{a,ex} + m_a (w_{in} - w_{ex})h_f + dQ \\
\Rightarrow dQ = -m_a (h_{a,ex} - h_{a,in}) + (m_a dw)h_f \\
dQ = \alpha_D A (w - w_{sat})h_{fg} + \alpha_{c,o} A (T - T_{sat}) \\
- m_a (w_{ex} - w_{in}) = \alpha_D A (w - w_{sat})
\]

In reduced form, the above equations are expressed as:

\[
dQ = \frac{\alpha_{c,o} A}{C_{p,a}} (h - h_{sat}) \\
\frac{dh}{dw} = Le \left( \frac{h - h_{sat}}{w - w_{sat}} \right) + \left( h_g - 2501 \times Le \right)
\]
Heat transfer rate through fin surface:

\[
dQ_f = \frac{\alpha_o a_f}{C_{p,a}} (h - h_w) = \frac{k_w a_f}{y_w} (T_w - T_f) = \frac{k_w a_f}{y_w b_f} (h_w - h_f)
\]

\[
= \left( \frac{1}{\frac{C_{p,a}}{\alpha_o a_f} + \frac{y_w b_f}{k_w a_f}} \right) \times (h - h_f) = \left( \frac{\alpha_o f a_f}{b_f} \right) \times (h - h_f)
\]

Heat transfer rate through unfinned base surface:

\[
dQ_s = \left( \frac{\alpha_o s a_s}{b_s} \right) \times (h - h_s) \quad \text{where,} \quad \alpha_o s = \frac{1}{C_{p,a} + \frac{y_w}{b_s \alpha_o / k_w}}
\]
Total heat transfer rate for each node:

\[ dQ_{node} = dQ_s + dQ_f = \left( \frac{\alpha_{o,s} a_s}{b_s} \right) (h - h_s) + \left( \frac{\alpha_{o,f} a_f}{b_f} \right) (h - h_f) \]

\[ = \left( \frac{\alpha_{o,s} a_s}{b_s} \right) (h - h_s) + \left( \frac{\alpha_{o,f} a_f \eta_f}{b_f} \right) (h - h_s) = \left( \frac{\alpha_{o,w}}{b_w} \right) (a_s + a_f \eta_f) (h - h_s) \]

\[ = \left( \frac{\alpha_{o,w} \eta_o a_{node}}{b_w} \right) (h - h_s) \]

where, \( b_s = b_f = b_w \) and \( \alpha_{o,s} = \alpha_{o,f} = \alpha_{o,w} \)

Refrigerant side heat transfer rate:

\[ dQ_{node} = \alpha_i a_{ref} (T_s - T_{ref}) = \frac{\alpha_i a_{ref}}{b_{ref}} \left( \frac{h_s - h_{ref}}{T_s - T_{ref}} \right) \]

where, \( b_{ref} = \left( \frac{h_s - h_{ref}}{T_s - T_{ref}} \right) \)
Now equating air side heat transfer rate and refrigerant side heat transfer rate we get:

\[ dQ_{\text{node}} = \left( \frac{\alpha_{o,w} \eta_o a_{\text{node}}}{b_w} \right) \times (h - h_s) = \frac{\alpha_i a_{\text{ref}}}{b_{\text{ref}}} \times (h_s - h_{\text{ref}}) \]

where,

\[ U_o = \left[ \frac{1}{\frac{b_w}{\alpha_{o,w} \eta_o a_{\text{node}}} + \frac{b_{\text{ref}}}{\alpha_i a_{\text{ref}}}} \right] \times (h - h_{\text{ref}}) = U_o a_{\text{node}} \times (h - h_{\text{ref}}) \]

In order to account for the variation in air enthalpy, logarithmic enthalpy difference is introduced in the above equation:

\[ dQ_{\text{node}} = U_o a_{\text{node}} \times \Delta h = U_o a_{\text{node}} \times \frac{h_{a,\text{in}} - h_{a,\text{ex}}}{\log \frac{h_{a,\text{in}} - h_{\text{ref}}}{h_{a,\text{ex}} - h_{\text{ref}}}} \]
### Evaporator (Predicted dimensions)

<table>
<thead>
<tr>
<th><strong>Tube</strong></th>
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</tr>
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<tbody>
<tr>
<td>Material</td>
<td>Copper</td>
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<tr>
<td>Inner diameter</td>
<td>5.5 mm</td>
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<tr>
<td>Outer diameter</td>
<td>9.5 mm</td>
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<tr>
<td>Longitudinal tube pitch ($P_l$)</td>
<td>30 mm</td>
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<tr>
<td>Transverse tube pitch ($P_t$)</td>
<td>30 mm</td>
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<tr>
<td>Finning length of tube</td>
<td>300 mm</td>
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<tr>
<td>Number of tubes in each row</td>
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</tr>
<tr>
<td>Number of tube rows</td>
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</tbody>
</table>

<table>
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<tbody>
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<td>Material</td>
<td>Copper</td>
</tr>
<tr>
<td>Outer diameter</td>
<td>26 mm</td>
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<tr>
<td>Fin thickness</td>
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<table>
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<th><strong>Surface area</strong></th>
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<tbody>
<tr>
<td>Total fin surface area</td>
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<tr>
<td>Total surface area, $A$</td>
<td>3.0691 m$^2$</td>
</tr>
</tbody>
</table>
Flowchart of the model

Start

Enter dimensions of Gas Cooler, Evaporator and Compressor

Enter discharge pressure and degree of superheat (t)

Guess Evaporator exit pressure

Compressor model: Calculate mass flow rate of refrigerant

Counter cross flow Gas Cooler model: Calculate exit condition

First stage of expansion

Second stage of expansion

If t'' = t

Calculate superheat at compressor inlet (t'')

Update evaporator exit pressure

Cross flow evaporator model: Calculate exit condition

Yes

No

End
Results & discussions

Figure 6: Variation of evaporator pressure with change in temperature at gas cooler inlet

Figure 7: Variation of evaporator capacity with change in temperature at gas cooler inlet
Contd.

Figure 8: Variation of compressor power with change in temperature at gas cooler inlet

Figure 9: Variation of COP with change in temperature at gas cooler inlet
Figure 10: Variation of evaporator pressure and cooling capacity with changes in DBT of air at evaporator inlet

Figure 11: Variation of compressor power and COP with changes in DBT of air at evaporator inlet
## Optimum operating parameters

<table>
<thead>
<tr>
<th>Ambient temperature (°C)</th>
<th>Performance of system with double stage expansion</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Optimum discharge pressure (bar)</td>
<td>Differential pressure drop (bar)</td>
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<td>95</td>
<td>20.97</td>
</tr>
<tr>
<td>34</td>
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<td>39</td>
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Conclusions

- Comprehensive mathematical model has been developed for R744 based air conditioning cycle with two expansion valves.

- Numerical simulations are carried out using the model.

- From the results obtained, the effects of variation of various important environmental parameters on system performance are analyzed.

- Optimum operating conditions are also identified for which the system attains maximum COP.
REFERENCES

REFERENCES


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THANK YOU