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P. Cyklis
Cracow University of Technology

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A CFD BASED IDENTIFICATION METHOD OF THE TRANSMITTANCES FOR THE PULSATING GAS INSTALLATION ELEMENT PART I – THEORY

Piotr Cyklis, PhD, Cracow University of Technology, al. Jana Pawla II 37
31-864 Krakow, POLAND; Tel.: 048-607573473; Fax: 048-126485771
E-Mail: pcyklis@mech.pk.edu.pl

ABSTRACT

Most commercial programs for calculating pressure pulsation propagation in volumetric compressors manifolds are based on the Helmholtz method. In this method the assumption is made that any installation element may be simplified and treated as a straight pipe with given diameter and length. In my paper a new method based on Computational Fluid Dynamics (CFD) simulation is presented. The main idea is to use CFD simulation instead of experimental measurements. The impulse or step function flow excitation is introduced as a source. The results of simulation are averaged in the inlet and outlet cross sections, so time only dependent functions at the inlet and outlet of the simulated element are determined. The transmittances of special form are introduced. In this part of the paper the theoretical basis of the worked out method is shown.

NOMENCLATURE

b : flow damping coefficient
c : sound velocity,
$\dot{m}$: mass flow rate,
L : length,
p : pressure,
S : cross section area,
w : velocity,
τ : time,
ω : frequency.

Complex values
$\boldsymbol{j} = \sqrt{-1}$: imaginary unit,
P: complex pressure (after FFT),
M: complex mass flow rate (after FFT),
T: transmittance,
$\mathbf{A} = \{a_{ij}\}$: four pole matrix,
$\mathbf{Z} = \{z_{ij}\}$: impedance matrix,

INTRODUCTION

The periodicity of compressor operation is a source of pressure pulsations in the volumetric compressor manifolds. This problem concerns mainly piston type compressors but to a lesser degree also screw and other types of volumetric compressors.

An analysis of pressure pulsations in volumetric compressor manifolds is important due to several reasons:

- they have a direct influence on the amount of energy needed for compression of a medium due to such phenomena as dynamic supercharging or dynamic attenuation of the suction and discharge processes;

- they excite mechanical vibrations inside the compressed gas installation leading sometimes to damages;
- they are a source of aerodynamic and mechanical noise;
- they affect performance of operating valves in valve compressors which results in dynamic leaks and premature valve failure;
- they enhance heat convection processes in the heat exchangers.

Theoretical methods for analysis of the interaction between a manifold component and the propagation of a pressure pulsation wave can be divided into two groups:

a) Helmholtz analysis and a solution of the telegraph equation in the complex space;

b) One-dimensional (differential, characteristics) or multi-dimensional simulation methods based on CFD simulation (Computational Fluid Dynamics – Finite Element Method, Boundary Element Method, Finite Difference Method).

The advantage of CFD methods is that they enable the simulation of any geometry and that they take into account a non-linearity of the propagation of finite amplitude disturbances. These methods have been successfully applied in solving automotive exhaust silencers. However, compressor manifolds are much more complex. In natural gas pumping stations where pressure pulsations are really a serious problem, the manifolds can be composed of many different elements and their total length can be of the order of several hundred meters. Reading in the entire geometry of such an installation into the CFD program would be very arduous in itself, even if in the future the now unrealistically long computation times become shorter. In addition, any modification of the installation geometry, for example due to the change of a collector or an oil separator, would require a new simulation of the entire manifold. Therefore CFD method alone seems to be difficult for application in the case of vast manifolds.

On the other hand, the Helmholtz method – applied in all commercial programs offered by the firms professionally dealing with damping of pressure pulsations - contains numerous simplifying assumptions. Each element of the manifold is replaced by a straight section of a pipeline having a known length and diameter or by a lumped volume. In the case of oil separators, pressure pulsation mufflers of special design and even for collectors such simplifications are inadmissible because of geometrical reasons. Attempts of the theoretical analysis of other shapes were undertaken in [1][2][3], but only simple geometries (sphere, cylinder) were considered. Additional difficulty is related with a medium that not always can be treated as an ideal gas.

As yet, experimental methods have been developed consisting in measuring pressure pulsations at several, specially determined locations of a manifold [4][5][6][7]. However, a theoretical method enabling the determination of parameters of a given element of the manifold already at design phase is still lacking.

The aim of the present paper is to show a new theoretical method, developed by the author, for the identification of any manifold element. This method allows determining the appropriate complex transmittances using CFD (Computer Fluid Dynamics) simulation.

2. THEORETICAL BASIS OF THE METHOD

The classic Helmholtz model is based on a solution, for a straight section of a pipeline, of the wave equation [7]. As a result, a four-pole matrix of the form (2.1) is obtained. This four-pole matrix $\{a_{ij}\}$ has a general form defined by (2.2). Concurrently with a four-pole matrix, a complex impedance matrix **Z** having the elements $\{z_{ij}\}$ is defined by the relation (2.3).

$$\begin{bmatrix} P_1 \\ M_1 \end{bmatrix} = \begin{bmatrix} ch\gamma L & Z_f sh\gamma L \\ \frac{1}{Z_f} sh\gamma L & ch\gamma L \end{bmatrix} \cdot \begin{bmatrix} P_2 \\ M_2 \end{bmatrix} \tag{2.1}$$

$$\begin{bmatrix} P_1 \\ M_1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \cdot \begin{bmatrix} P_2 \\ M_2 \end{bmatrix} \tag{2.2}$$

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \cdot \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} \tag{2.3}$$

Where:

$$\left. \begin{aligned} \gamma &= \sqrt{(b + j\omega)\left(\frac{j\omega}{c^2}\right)} \\ Z_f &= \frac{1}{S}\sqrt{\frac{(b + j\omega)}{\frac{j\omega}{c^2}}} \end{aligned} \right\} \tag{2.4}$$

In order to generalize the model for an arbitrary geometry it has been assumed that the forms of matrices $\{a_{ij}\}$, $\{z_{ij}\}$ will not be based on equation (2.1), but that they will have a completely free form. This assumption means that for a unique determination of the interaction of an arbitrary component of a manifold it is sufficient to know four elements of the matrix, which must be identified for this component. The aim of the present work was to develop a theoretical method of the identification of these matrix elements.

The concept of the method developed here is the following: for a considered element of a manifold a full multi-dimensional CFD simulation is carried out, solving the Navier-Stokes set of equations numerically together with the necessary closing models, i.e. gas equation of state, turbulence model, boundary conditions. The results obtained are averaged at the inlet and outlet of the analyzed element and next a complex transformation of the results is carried out, so that elements consistent with the generalized form of matrices $\{a_{ij}\}$, $\{z_{ij}\}$ are obtained. In this way the advantages of both methods can be combined: the Helmholtz model possibility of analysis of any vast installations and the possibility of studying geometry of any complex geometry element, without a priori simplifications.

For further considerations it is necessary to define complex transmittances. Below, complex transmittances are defined: a flow and flow-pressure one for excitation directed along the flow direction in the manifold and oppositely to it.

- flow transmittance $$T_M(i\omega) = -\frac{M_{i+1}}{M_i} \qquad (2.5)$$

- flow-pressure transmittance $$T_{MP}(i\omega) = \frac{P_{i+1}}{M_i} \qquad (2.6)$$

For a symmetrical muffler (or other compressor manifold element) two transmittances would be sufficient (along the flow direction), for unsymmetrical four transmittances are necessary. These transmittances can be derived using CFD simulation, as all of them are determined having the flow excitation boundary condition: $\dot{m}_i$, $\dot{m}_{i+1}$ or w_i, w_{i+1} . The second boundary condition is a condition of a free outflow or a total closing. Next, complex impedances $\mathbf{Z}_0$ and $\mathbf{Z}_k$ closing the studied muffler from both sides (Fig.2.1) have been defined.

$$Z_{0,k} = \frac{P_i}{M_i} \qquad (2.7)$$

where 0 – means inlet, and k – outlet of the element along the flow direction.

When the transmittances $\boldsymbol{T}$ defined above are determined for specific known values of $\boldsymbol{Z}_0$ and $\boldsymbol{Z}_k$, a relation between matrix $\boldsymbol{Z}$ and transmittances $\boldsymbol{T}$ is unique and can be derived, which means that transmittances $\boldsymbol{T}\big|_{Z_0 Z_k}$ can be used for calculating matrices $\boldsymbol{Z}$ and $\boldsymbol{A}$. The simplest case is when $\boldsymbol{Z}_k = 0$ or $\boldsymbol{Z}_k = \infty$ or, for an opposite flow, $\boldsymbol{Z}_0 = 0$ or $\boldsymbol{Z}_0 = \infty$.

The four cases planned for the CFD simulation of the unsymmetrical compressor manifold element are shown on the figure 2.1. Each simulation allows determining one transmittance according to the drawing, named respectively $\boldsymbol{T}_a$, $\boldsymbol{T}_b$, $\boldsymbol{T}_c$, $\boldsymbol{T}_d$.

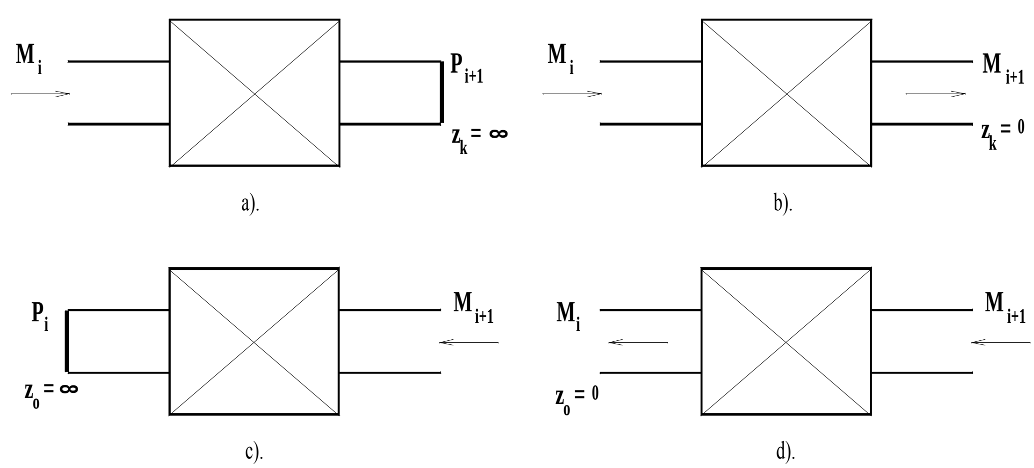

Figure 2.1. Four cases used for CFD simulation shown with boundary conditions and quantities used for transmittance determination.

Writing down impedance relations (2.3) for the cases a, b, c, d as defined in Fig.2.1 and using (2.5), (2.6) one obtains:

$$\left.\begin{aligned} \boldsymbol{P}_i &= z_{ii} \cdot \boldsymbol{M}_i \\ \boldsymbol{P}_{i+1} &= z_{i+1,i} \cdot \boldsymbol{M}_i \end{aligned}\right\} \Rightarrow z_{i+1,i} = \boldsymbol{T}_a \tag{2.8a}$$

$$\left.\begin{aligned} \boldsymbol{P}_i &= z_{ii} \cdot \boldsymbol{M}_i + z_{i+1,i} \cdot \boldsymbol{M}_{i+1} \\ 0 &= z_{i+1,i} \cdot \boldsymbol{M}_i + z_{i+1,i+1} \cdot \boldsymbol{M}_{i+1} \end{aligned}\right\} \Rightarrow z_{i+1,i+1} = \frac{\boldsymbol{T}_a}{\boldsymbol{T}_b} \tag{2.8b}$$

$$\left.\begin{aligned} \boldsymbol{P}_i &= z_{i,i+1} \cdot \boldsymbol{M}_{i+1} \\ \boldsymbol{P}_{i+1} &= z_{i+1,i+1} \cdot \boldsymbol{M}_{i+1} \end{aligned}\right\} \Rightarrow z_{i,i+1} = \boldsymbol{T}_c \tag{2.8c}$$

$$\left.\begin{aligned} 0 &= z_{ii} \cdot \boldsymbol{M}_i + z_{i,i+1} \cdot \boldsymbol{M}_{i+1} \\ \boldsymbol{P}_{i+1} &= z_{i+1,i} \cdot \boldsymbol{M}_i + z_{i+1,i+1} \cdot \boldsymbol{M}_{i+1} \end{aligned}\right\} \Rightarrow z_{i,i} = \frac{\boldsymbol{T}_c}{\boldsymbol{T}_d} \tag{2.8d}$$

The derived relationships (2.8) allow a unique determination of complex impedance matrix $\boldsymbol{Z}$ that defines a linear lumped acoustic element.

The evaluation of transmittances $\boldsymbol{T}(\mathrm{i}\omega)$ for each of the four cases and for a dozen or so significant harmonics is very arduous and time consuming. Therefore it is better to use a system response for a unit step function or impulse function excitation. A unit step function is defined as follows:

$$1\dot{\boldsymbol{m}}(\tau) = \begin{cases} 0 & \text{for} \quad \tau<0 \\ 1 & \text{for} \quad \tau>0 \end{cases} \tag{2.9}$$

Using Laplace transform one can write:

$$1\boldsymbol{M}(\boldsymbol{s}) = \mathcal{L}\left\{1\dot{\boldsymbol{m}}(\tau)\right\} = \frac{1}{\boldsymbol{s}} \tag{2.10}$$

System responses for unit step excitation $1\boldsymbol{M}_i$ or $1\boldsymbol{M}_{i+1}$ depending on the case (a, b, c, d from Fig.2.1) are $1\boldsymbol{P}_{i+1}$, $1\boldsymbol{M}_{i+1}$, $1\boldsymbol{P}_i$, $1\boldsymbol{M}_i$, respectively:

$$\boldsymbol{Y}(\boldsymbol{s}) = \boldsymbol{T}(\boldsymbol{s}) \cdot \boldsymbol{X}(\boldsymbol{s}) \tag{2.11}$$

where $\boldsymbol{X}$(s) denotes excitation ($1\boldsymbol{M}_i$ or $1\boldsymbol{M}_{i+1}$ respectively) and $\boldsymbol{Y}$(s) is a system response ($1\boldsymbol{P}_{i+1}$, $1\boldsymbol{M}_{i+1}$, $1\boldsymbol{P}_i$, $1\boldsymbol{M}_i$ respectively).

For a known unit step response of a system $1\boldsymbol{Y}$(s) (where $\boldsymbol{Y}$ is $\boldsymbol{P}_i$, $\boldsymbol{P}_{i+1}$, $\boldsymbol{M}_i$, $\boldsymbol{M}_{i+1}$, respectively):

$$\overline{\boldsymbol{T}}(\boldsymbol{s}) = \boldsymbol{s} \cdot 1\boldsymbol{Y}(\boldsymbol{s}) \tag{2.12}$$

This means that based on a system response to a flow unit step excitation in each of the four cases a, b, c, d one can determine transmittance of the studied element.

Similar considerations apply to the case when an impulse δ-Dirac function is used as the flow excitation. Since the mathematical formulation of this approach is similar to that presented above it will not be given here.

3. CONCLUSION

The theory shown in this part of my paper is based on the assumption that "post factum" linearization of the CFD simulation results for the non-linear problem leads to better results than "a priori" linear and geometrically simplified models used so far. The use of this method for the muffler of special design with the comparison with classic method and experimental results are shown in the part II of this work.

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