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RANDOM VIBRATION FATIGUE TESTS TO PROVE INTEGRITY OF CANTILEVERED ATTACHMENTS ON COMPRESSOR SHELLS

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ABSTRACT

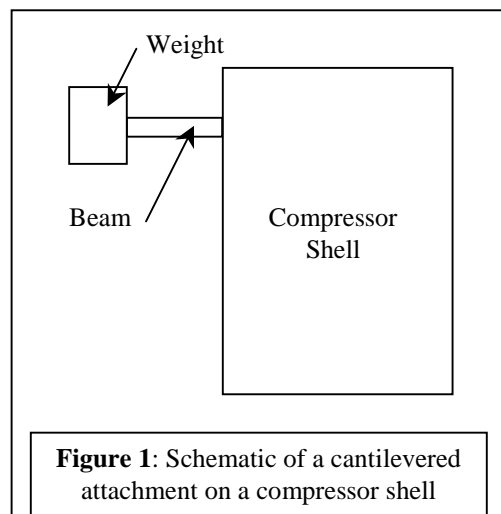
Many Air Conditioning and Refrigeration hermetic compressor shells are designed with cantilevered attachments such as control valves. These attachments are subject to several sources of loading such as shell pressure, external tube load or vibration. One of the most common failure modes yet the least predictable is fatigue due to vibration at the joint with the shell. The purpose of the paper is to provide a method for developing an accelerated random vibration test, which can be used to efficiently evaluate the joint reliability for these cantilevered compressor shell attachments.

By assuming a simple cantilever beam model, the cumulative fatigue damage can be estimated at the joint from the relative displacement at the beam-ends. The cumulative damage from a bench test is compared to the measured vibration of operating compressors subject to a representative sample of the compressor application. Finally a random vibration fatigue test is developed at higher-than-service loads to accelerate the test and reduce test cost. This fatigue test includes the specification of test load spectra, test duration and pass criteria that represents an acceptable probability of failure in the market place.

INTRODUCTION

A cantilevered compressor shell attachment consists of three major components: the compressor shell, a connection (the beam) and a weight at the end of the beam. In general, the weight is much heavier than the beam. A tube and valve assembly welded to the compressor shell is typical of such an application and is herein used as an example as schematically provided in Figure 1. In this example the vibration of the weight is assumed to be from the shell either from compressor vibration such as during flooded start or from shaker operation where the shell is attached to the shaker table during a test. This vibration is transmitted to the weight via the beam. The common failure mode is the fatigue failure at the joint of beam to shell where the maximum bending stress occurs. This procedure could be applied equally where the vibration source was external from the compressor shell, such as from the tubing, as long as the stress at the shell can be characterized in terms of the loading or movement on the shell appendage.

The compressor vibration at the shell contains a wide range of frequencies and these frequencies are present at all times in a broad distribution of intensities. Random vibration tests can effectively simulate such compressor vibrations in a controlled test environment. The objective of this analysis is to design an accelerated random vibration test to evaluate the reliability of the joint.



The frequency band for the vibratory response at the weight is much narrower than at the shell, depending on the modes of vibration of the cantilever assembly. As the weight is much heavier than the beam, the fatigue stresses at the joint can be represented by the displacements of the weight relative to the shell. Assuming the first order cantilever mode to be the predominant destructive behavior is a reasonable and effective approach. Thus a simple proportional relation between fatigue stress and the relative displacement provides a way to assess the cumulative fatigue damage from relative displacement that can be determined by double integration of the vibratory acceleration. This approach has been proven reasonably accurate by strain gage data.

Experience has taught us that the bending stress is the predominant fatigue stress involved in the observed failures. Thus the beam torsion is neglected. The analysis is focused on the vibrations in two perpendicular directions: the vertical direction (or shell axial direction for a vertical scroll compressor) and the horizontal or shell tangential direction. The vibration in each direction is analyzed and tested separately.

GENERAL PROCEDURE

The vibratory acceleration is the only measurement required by the analysis. A computer program is then used to process the time-domain acceleration signals. This program includes the following:

- Calculate the power spectral density (PSD) from the acceleration measurements.
- Filter the displacement to the frequency range of interest.
- Double integrate the acceleration measurements to obtain the displacement.
- Calculate the cumulative fatigue damage from the relative displacement of the beam-ends using the rain flow cycle counting method and a linear damage rule.

Finally, the acceleration factor for the bench test is then calculated as the ratio of cumulative fatigue damages from the bench test versus the cumulative fatigue damage from the compressor operation. The determination of acceleration factor is the key to the design of an accelerated bench test that represents the anticipated deployment scenario for the product.

Relative End Displacement Can Represent Fatigue Stress

The beam vibrates in several modes, which makes the relationship between the beam end displacements and the joint stress complex. But as previously noted, the higher modes of the beam can be neglected. Similarly the weight has several degrees of freedom. But forces and bending moments can represent the interaction between the weight and the beam without knowing the details of this motion. Consequently the dynamic problem can be reduced to a simple one-dimensional cantilever with end loads. From classic beam theory, the relative displacement in this case is linearly and directly proportional to the joint stress even though this proportion may vary due to the source of the loading. If both loads, concentrated force and bending moment, are simultaneously involved, the relationship between resultant stress and the relative displacement is not always a constant proportion. However, the linear trend of larger relative displacement corresponding to higher stress is generally valid.

Relative Displacement Is Derived from Acceleration Measurements

Consider a vibrating shell on a shaker with random vibration

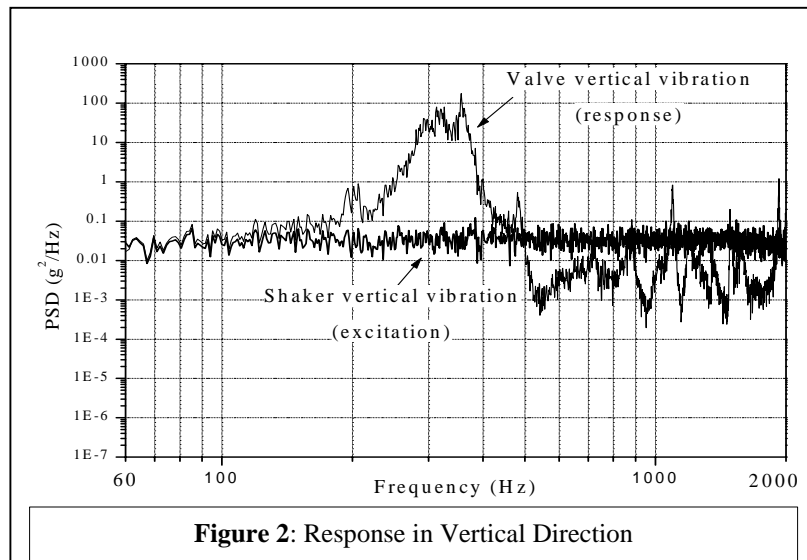


Figure 2: Response in Vertical Direction

input. The response of the cantilevered attachment is apparent in certain frequency regions as noted in Figure 2. In this particular example, the weight is a control valve body.

This 140-Hz to 500-Hz response is mainly dependent on the attachment modal frequencies and damping. The frequency higher than 500 Hz is of little interest since they represent small displacement for this mechanical structure and thus small stresses at the attachment location.

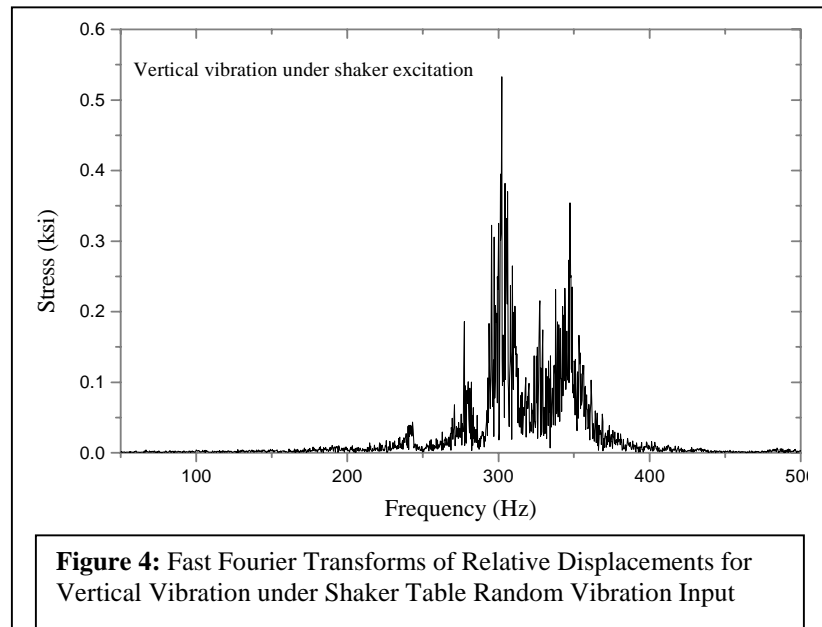
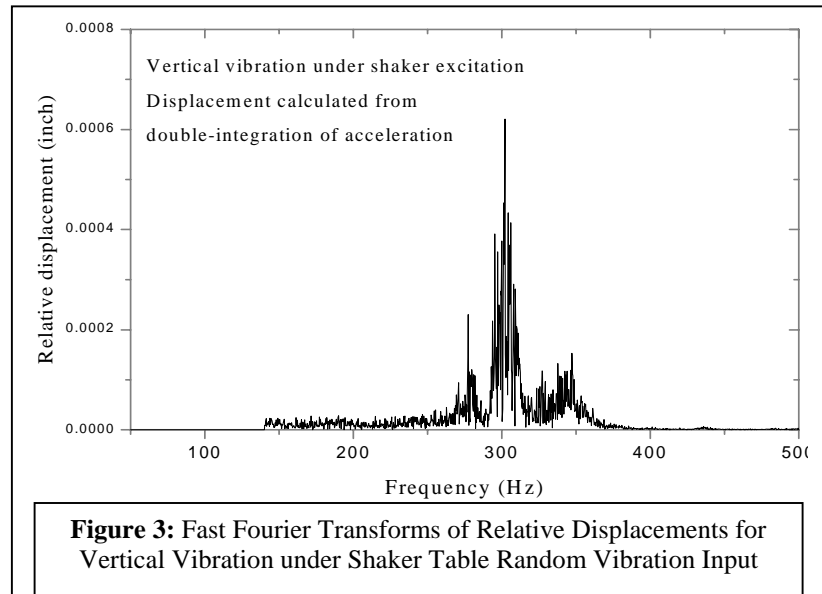
Relative displacement measurements of the shell versus the attached valve require a high-speed data logger and two accelerometers. Figure 3 provides a FFT of the relative displacement that was derived from the double integration of the accelerations measured at the beam-ends. The bending stress was also measured

by strain gages near the joint but away from the stress concentration area. This is a typical practice in tubing strain analysis since it allows the isolation of the bending loads without the complexity of the high and complex stress gradients in the attachment region. Several time domain traces were transformed to the frequency domain and then averaged in the frequency domain to provide the mean system response for this graphical comparison of displacement and stress.

A Fast Fourier transform of the stress at the joint that was measured simultaneously with the relative displacement of the beam-ends, is plotted in Figures 4. Outside the frequency region where the response is noted in Figure 2, the relative displacement and the bending stresses are trivial. Note also the similarity of the displacement and the measured stress spectra provided in Figures 3 and Figure 4.

Since significant relative displacements are only present in certain frequency region, the relative displacement between the valve body and the attachment at the shell in this frequency region only needs to be considered.

The acceleration data is filtered using a FFT digital band filter with low and high cut-off frequencies of 140 and 500 Hz respectively. The low-pass filter can reduce the high frequency noise that can occur in this calculation. The high-pass filtering eliminates drift from the DC component that can bias the signal. Then the relative displacement of the beam-ends is obtained from the difference in the double integration in the time domain of the filtered accelerations at the beam-ends.



As noted previously, the vertical and horizontal directions are developed and tested separately. Since bending alone is considered, the mutual interaction in the neutral plane is negligible. The horizontal, or shell tangential direction, results, as noted in Figure 5 show a similar resemblance.

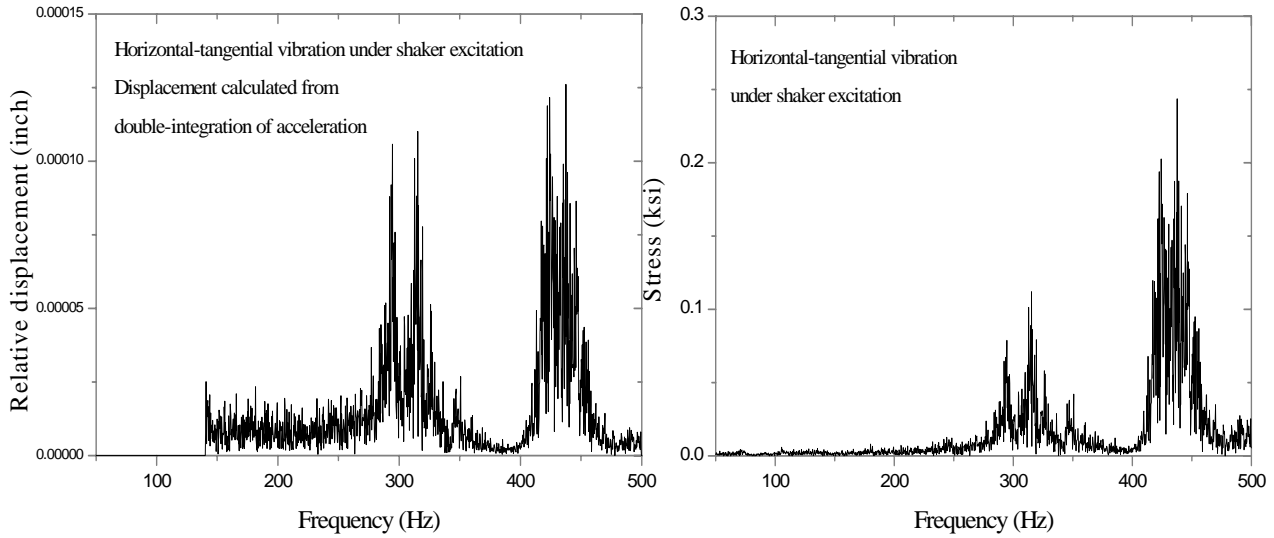


Figure 5: FFT of Relative Displacement and Stress for the Horizontal or Shell Tangential Direction

Cumulative Fatigue Damage from Relative Displacements

The power law can be used to express the relationship between an alternating fatigue stress (σ) and the median fatigue life (N - cycles) at which a part would fail.

$$\sigma = a \cdot N^b \quad (1)$$

The parameter “a” is a constant of proportionality and “b” is the power exponent that describes the material response. This is a common statement used to describe the linear elastic behavior of a material. Palmgren-Miner rule provides a concept that the damage (D) produced by one cycle of stress is equal to the inverse of the median fatigue life at that same stress. This concept is restated to express fatigue damage as a function of the fatigue stress on the component.

$$D = 1/N = a^{1/b} * \sigma^{-1/b} \quad (2)$$

The fatigue stress can be linearly represented by the relative displacement (d) of the beam-ends.

$$D = c^{1/b} * d^{-1/b} \quad (3)$$

This damage can be accumulated as a function of stress as measured with strain gages or relative displacement of the beam-ends as note in equation 4. The damaged can be accumulated for every event in the study as noted in the sited equation or can be grouped with like events. In either case the sampling is assumed to repeat and be a direct portion of the number of repeats that make up the life of the environmental exposure.

$$\sum_{i=1}^n D_i = \frac{1}{a^{1/b}} \cdot \sum_{i=1}^n \sigma_i^{-1/b} \quad \text{or} \quad = \frac{1}{c^{1/b}} \cdot \sum_{i=1}^n d_i^{-1/b} \quad \text{as appropriate.} \quad (4)$$

As previously noted the parameter “*b*” is the slope of *S-N* curve, and “*a*” and “*c*” are proportional constants. Since only ratios of cumulative fatigue damages are required in determining the acceleration factor, the unit of fatigue damages is immaterial. The proportional constants “*a*” and “*c*” are not of interest. A ratio of damage under different conditions are used to compare a test event and actual condition, thus these proportional constants cancel out of the equations. Therefore for convenience in the calculations the constants can be set to one.

To calculate the cumulative fatigue damage of a sample subjected to a variable load history, it is necessary to reduce the complex load history into a number of events. The rain flow method of cycle counting is used to obtain “*σ*” and “*d*” in Equations (2) and (3) for each cycle. The mean stress effect on fatigue life is not considered since the vibration is generally reversible, that is *R* = -1. For the purpose of comparative tabulations, the cumulative fatigue damage is given per unit time.

Design of an Accelerated Random Vibration Testing

In most engineering tests, it is unrealistic to run the samples under normal operating conditions within a reasonable time frame. For this reason, bench tests are usually accelerated. That is, a higher-than-service load is applied in the bench tests. The reliability of test samples can thus be evaluated within a period of time, which is much shorter than the demanded service life. The acceleration factor is calculated as the ratio of cumulative fatigue damage on the tube joint during the bench test to the cumulative damage of the joint during its service life.

$$\text{Acceleration Factor} = \frac{\text{Shaker test cumulative damage per unit time}}{\text{Service cumulative damage per unit time}} \quad (5)$$

To determine the acceleration factor, the reliability objective of test samples should be predetermined, which includes the demanded lives at different operating conditions and probabilities of failure. This service history can be assessed, for example, from measured data on field installations. This is a difficult and a costly procedure. Copeland Reliability Department defines the criteria against which all products are audit-tested. These standards are built on decades of experience with the equipment in the markets the company serves. These are based on typically 0.1% probability of failure for a 15-year service life, or on a specific number of stop-start cycles, or a specific number of heavy-duty cycle requirements. This provides a vehicle, which specifically defines the limiting duty on service life; plus it also provides access to test machinery and test conditions with availability to measure what is required. Through measurement of the beam-end relative displacement under all the Reliability conditions on the test stands the worst case-operating scenario was chosen based on the cumulative damage calculation noted previously. When calculating the actual target service life, the life for 0.1% probability of failure was extrapolated to a life for a 50% probability of failure to simplify test observations. For copper or low carbon steel components, a normal distribution is assumed with a 12% or 8% standard deviation respectively. For welds the distribution is usually larger and should be based on actual test data since the value can be joint-design dependent.

Similarly, the cumulative fatigue damages from shaker vibration at various “*g*” levels are also calculated and stated as the cumulative damage per unit time. A uniform PSD profile with a frequency range sufficiently wide to cover the attachment response can be used to simulate the compressor vibration. Beyond the range of interest attenuation is provided. On the low frequency side, this attenuation is usually determined by equipment capability. On the upper frequency side this attenuation is usually determined by avoidance of fixture or other higher end resonance that is noise to this specific test. This high-end resonance can adversely affect the test controls. The control feedback should hold the input to + or -10% of the required PSD in the specified response interval. Note, that by design, the cantilevered attachment response in the compressor operating speed range is non-resonant.

The acceleration factor and the test duration are determined together. The bench test duration is a function of the total service life and the acceleration factor.

$$\text{Bench test duration} = \frac{\text{Total demanded service life}}{\text{Acceleration Factor}} \quad (6)$$

Both the acceleration factor and the bench test duration in Equations (5) and (6) are functions of “g” intensity of shaker random vibration. Herein “g” intensity or “g” level refers to the magnitude of the integral of the PSD input signal expressed in g-RMS (universal gravity constant multiples, root mean square). Raising the *g* intensity of the shaker random vibration can increase the acceleration factor and shorten the test duration. In choosing a good compromise between acceleration and bench test time, both physics and economics are considered. Care must be taken not to change the mode of failure of the parts or to drive linear elastic fatigue behavior into low cycle plastic fatigue behavior. Examination of failure modes and material surfaces provide the best evidence that the test integrity has been maintained. Bench test time is typically chosen to favor an eight-hour work shift in the laboratory if possible. Consideration is given to allowance for test set up and test observations upon test completion. Three, four or six-hour tests are typical.

There are two test scenarios. If there are sufficient test samples available test to failure can be used along with a Weibull or other appropriate distribution to define the life characteristic. This statistical distribution can be used to determine the reliability for the required service life, or the life at a given reliability. The alternate test method, which is more common for random vibration testing, is where a specific number of parts are tested for a specific time and the pass or fail is based on the observed number of failures or lack of failures. This is used when it is not feasible to shut down the shaker when cracks are initiated. Inspection is carried out after each test to determine whether or not the sample has failed. A non-parametric, median-rank statistical method, the C-rank theorem, can be used to determine the reliability when *k* failures occur out of *n* samples tested to the test duration at a given confidence (2). An alternate treatment is possible if the failure characteristic is statistically known; that is, to use WeiBayes. Since the test condition was shifted to a 50% probability of failure and the test criteria was derived from reliability test limits which already have test margins, the success or pass condition is three or less failures in six test samples or four or less failures in eight test samples. The number of samples is governed by the confidence limit desired for the test.

VERIFICATION USING STRAIN GAGE DATA

Part of the verification of the procedure has already been presented in Figure 3 and 4. Two strain gages are installed on the tube to measure nominal axial bending stresses near the joint, one for the vertical direction and one for the horizontal or shell tangential direction. Simultaneously the differential displacement was measured between the tube ends with accelerometers. As noted previously the Fast Fourier Transform results show that both stress and relative displacement have very similar frequency contents. The relative amplitudes are different for the higher modes. An alternate procedure would be to account for these differences by using a different relationship between the stress and displacement for these higher modes. Then sum the damage components for each distinct frequency. This added complexity gains little in terms of the resulting go-no-go test that is established at the end of the calculation. The simple assumption provides an effective solution as noted below.

To further verify the approach, stresses and accelerations at various “g” levels on the shaker table were measured. Cumulative fatigue damages were then estimated using rain flow cycle counting method and Equations (4). The estimated

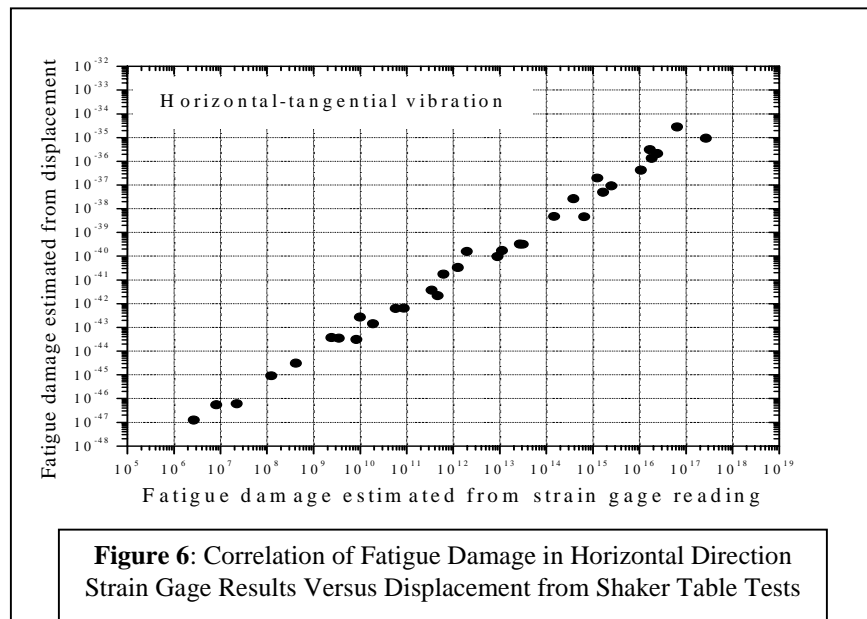
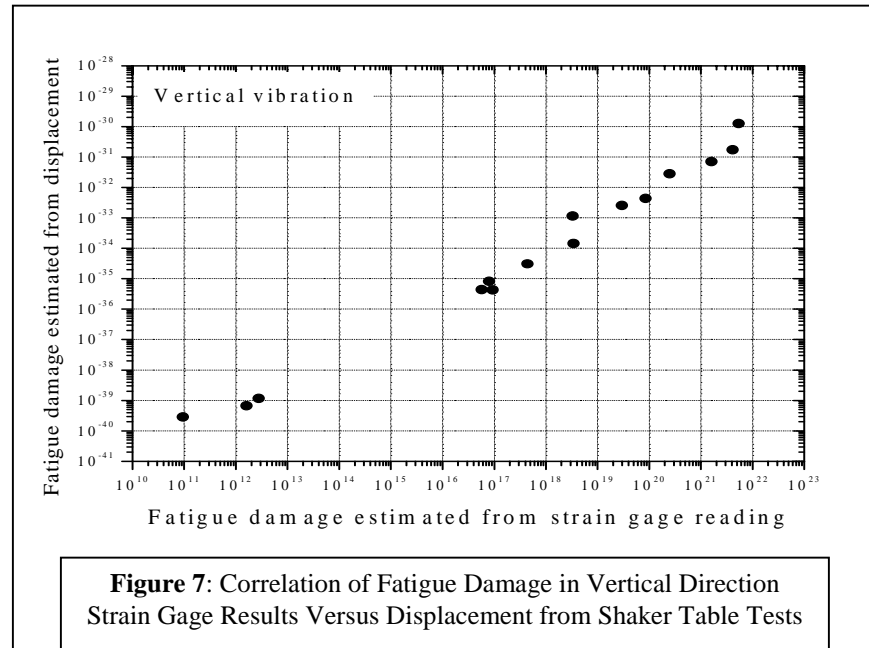


Figure 6: Correlation of Fatigue Damage in Horizontal Direction Strain Gage Results Versus Displacement from Shaker Table Tests

fatigue damages per unit time from the stresses and from the relative displacements are compared in Figures 6 for the horizontal or shell tangential direction and in Figure 7 for the vertical or shell axial direction. The data points are in good agreement with straight lines having a unit slope in the log-log plots. This provides additional proof that the presumption that the relative displacement can represent the fatigue stress in a cumulative damage calculation was well founded.



CONCLUSION

This paper has provided a method for developing an accelerated random vibration test, which can be used to efficiently evaluate the joint reliability for cantilevered compressor shell attachments. This procedure provides a random vibration fatigue test with higher-than-service loads to accelerate the test and reduce test cost. This fatigue test includes the specification of test load spectra, test duration and pass criteria that represents an acceptable predefined probability of failure. The paper provides a detailed methodology that can be used to develop such tests. The assumption of a simple behavior of the cantilever on which the development of the test is based is shown as being well founded.

This procedure has been applied successfully to develop test procedures at Copeland Corporations, Applied Mechanics Fatigue Test Laboratory.

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