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Final Report

PAVEMENT DRAINAGE AND PAVEMENT
SHOULDER JOINT EVALUATION

Numerical Modeling of Infiltration and
Drainage in Pavements

R. David Espinoza
Philippe L. Bourdeau
Thomas D. White

PURDUE UNIVERSITY

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**Numerical Analysis of Infiltration and
Drainage in Pavement Systems**

by

R.D. Espinoza

P.L. Bourdeau, and T.D. White

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The contents of this report reflects the views of the author who is responsible for the facts and accuracy of the data presented herein. The contents do not necessarily reflect the official views of policies of the federal highway administration. This report does not constitute a standard specification of regulation.

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<p>16. Abstract</p> <p>The continuing loss of pavement performance observed in the U.S. transportation infrastructure system raises the issue of the subdrainage design. Experience with the Indiana highways indicates that the lack of adequate sub-drainage is probably a major cause of pavement distress. In order to provide highway engineers with a methodology to analyze the water migration and drainage into pavement systems a new numerical model has been developed. The numerical scheme is a finite difference formulation of the equations of water flow in unsaturated porous media. The resulting system of equations used is nonlinear and state-of-the-art numerical techniques have been used in order to obtain solutions with accelerated convergency and improved stability of the algorithms employed.</p> <p>Theoretical background of unsaturated flow and its numerical analysis is included in the present report. A computer program, PURDRAIN, has been developed to implement the model in a software environment. The validity of proposed numerical model and the developed computer program have been tested using available experimental data. Examples of applications for analyzing moisture infiltration with different pavement geometries are presented.</p>			
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Implementation Report

In this study, it is shown that the simplifying assumptions made in the current practice by using saturated flow models in the assessment of the drainability characteristics of pavement bases lead to underestimating the drainage times and overestimating the amount of water drained in the designed structures. It is shown using numerical examples that the rate of drainage in a pavement system is not only dependent upon the soil saturated hydraulic conductivity but also on the soil water retention characteristics. Thus, in selecting draining materials, the water retention characteristics have to be taken into account. Based on this consideration only, soils exhibiting thick capillary fringes should not be recommended as subdrainage materials. Similarly, infiltration patterns are not only dependent on the amount of rainfall but also on the soil hydraulic parameters and their initial and boundary conditions. Depending upon the geometric and hydraulic characteristics of the saturated region, flow in the unsaturated region may be important. The significance of the flow in the unsaturated region can be assessed by comparing the capillary fringe to the thickness of the base course. As thicknesses of pavement bases usually are of the same order than the capillary fringes, accurate predictions of water flow require to take into consideration the unsaturated region. The quantities of flow computed in neglecting the unsaturated region may significantly differ from the actual ones. It is also shown using numerical examples that depending upon the unsaturated hydraulic characteristics, quasi-saturation may be expected to last even after several hours of drainage below pavement slabs.

Thus, the use of simplified models that only consider the saturated parameters to evaluate the pavement base drainability may lead to unconservative designs. These models usually predicts larger amounts of water drained in shorter times than it is actually the case in real structures.

Based on this study, the following recommendations are made:

1. Laboratory tests should be performed on the materials used in pavement systems to determine their saturated as well as unsaturated hydraulic

properties. These tests should provide the water retention curves near saturation. Suctions values ranging from 0 to 2 atms are recommended to be used. Values above 2 atms could also be obtained to completely define the water retention curves, but the values within the recommended range are particularly important.

2. Using a simplified geometry with the developed numerical model, drainability criteria for pavement bases can be established. The capillary fringe and the curvature can be used as indexes to represent the unsaturated characteristics of the soil. The use of a simplified geometry would allow to represent the results graphically in dimensionless form which would be convenient for practical design.
3. During the process of particular designs, the developed model and program PURDRAIN should be used to perform parametric and optimization studies.
4. Monitoring of pavement sections built with various materials characteristics should be performed over longer periods of time in order to better calibrate the developed model.

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CHAPTER 1 - INTRODUCTION

1.1 Background

This report constitutes the analytical and numerical counterpart of the experimental study presented in a separate report titled "Pavement Drainage and Pavement-Shoulder Joint Evaluation and Rehabilitation" by Zubair Ahmed, T. D. White and P. L. Bourdeau (Joint Highway Research Project, March 1993). Both the experimental and analytical studies were part of a concerted effort to help Indiana Highway Engineers to improve subdrainage of transportation infrastructures.

When excess moisture is retained in a pavement structure and its foundation early distress and loss of performance may result. In the case of rigid pavements, water collected in the subgrade or in the subbase may lead to pumping causing internal erosion. The resulting voids undermine the pavement slab and leave it unsupported. The pumping mechanism is related to the build-up of excess pore pressure under frequent, heavy traffic. Although pumping is not a normal occurrence in flexible pavements, excess water under and within the pavement structure is also a major factor of performance loss. Typical consequences are: reduction of strength of the subgrade and the granular base or subbase layers and stripping and reduction in modulus of the asphalt bound layers.

It has been recognized for many centuries that poor drainage leads to degradation of pavement performance, and the good drainability criteria have been applied with great success in the construction of major roads (Cedergren, 1988). The AASHTO Guide for the Design of Pavement Structures (1987) includes drainage as an essential criterion for pavement design. However, constructing with free draining materials is difficult because of their mechanical instability. A fraction of material with a large percentage of fines (i.e. passing a #200 sieve) has to be aggregated in order to increase the stability and workability of the base. The stability is then enhanced but, concurrently, the permeability is significantly reduced.

With the progress of materials testing techniques, the problem was stated from a different standpoint (Cedergren, 1988). Since adding fines decreases the drainability, the pavement structure could become completely saturated after rainfall infiltration and remain in that state for a long period of time. Consequently, laboratory tests for these extreme conditions should be performed to determine the soil strength parameters. The predicted bearing capacity of the base and subbase courses could then be compared to the expected equivalent static loads induced by the traffic. In this way, the effect of excess moisture could be accounted for either by an increase of design thickness of the base and subbase courses or by utilizing materials that are relatively insensitive to excess moisture.

In the United States it was observed that large number of roads designed using strength criteria rather than drainability criteria have deteriorated. In many instances, failures occurred although strength requirements had been met. Most of these premature pavement distress problems have been attributed to excess of water in the pavement system and questions have been raised regarding the suitability of the gradation of the materials commonly used in pavement designs (Forsyth, 1987; Cedergren, 1988). The theory behind the use of undrained material is that a strong base will provide uniform support for the pavement slab, but this theory remains true until the water penetrates the infrastructure. In 1820, John McAdam stated : "... if water pass through a road and fill the native soil, the road whatever may be its thickness loses its support and goes to pieces." (Pavement Newsletter, 1989). Trapped water causes severe damages in pavements such as cracking, rutting, pavement distortion, pumping, disintegration of cemented treated bases.

Failure is due to the dynamic action of the trapped water. When a pavement system becomes saturated with water and remains in this state, each time a vehicle passes an increase of pore pressure takes place. This sudden increase of the pore pressure in confined areas produces large pore pressure differences that are released at the adjacent joints and/or cracks. The water that seeps through the joints carries soil particles with it. If this process continues, a cavity under the pavement is eventually formed and any subsequent loading causes the destruction of the pavement. Additional cracks develop that serve as new sources of water infiltration into the pavement system, hence the destruction process is accelerated. This phenomenon is not taken into consideration in a conventional static analyses, therefore these

methods do not include rationale to increase the thickness of the base in order to address the problem created by the excess water.

A number of field studies have been conducted combined with laboratory and theoretical analyses to investigate the efficiency of subdrainage in preventing pavements distress. In most cases, the longitudinal pavement shoulder joint was found to be a source of infiltration (Barksdale and Hicks, 1977). The presence of excess moisture below the pavement edge and its movement toward the outside wheel path was clearly related to the pavement distress and the rate at which it develops. The moisture accelerated distress (MAD) identification system (Carpenter et. al., 1981) incorporates pavement's site consideration, material properties, visual condition and nondestructive testing results to assess the potential for subdrainage problems.

Road tests performed over a period of 35 years reported by Forsyth et al. (1987) have shown that much greater damage is caused during the wet periods than in the dry ones. The same study shows also that regardless of the pavement thickness or strength the damage under flooded conditions is much higher than when no water is trapped in the pavement structure. Field tests comparing the performance of pavements built using structural or drainage criteria show that larger permanent deflections develop with the less permeable materials (Hoffman, 1982; Hinshaw, 1988). Furthermore, some studies have shown that the elastic stiffness of some road bases varies slightly with moderate levels of moisture content but that saturation of poorly drained base layers has drastic effects on their elastic behavior (Thom & Brown, 1987). Similar findings regarding the resilient modulus of the materials were reported by Hicks and Monismith (1971).

Excess water in a pavement system may come from a number of sources. Some of the most common being surface water entering the pavement-shoulder joint, surface water entering longitudinal as well as transverse construction joints and pavement cracks. Presence of a near-surface ground water table can allow both capillary water or water in vapor phase to migrate toward the surface. Capillary water will rise to a certain height within the pavement foundation and structure, thus contributing to increasing the moisture content. Water vapor can migrate under the effect of temperature gradients. Hillside roadways may be subjected to groundwater flow and seepage forces. These sources of excess water can be at the origin of distress and reduced performance of pavements.

When frost penetration is possible, ice crystals or lenses may develop beneath the pavement. During the subsequent periods of thaw the melting of ice lenses results in significant excess of pore water. Because the thaw progresses from the pavement surface downwards, the melted excess water remains temporarily undrained. If during this period the pavement is subjected to heavy traffic loss of bearing capacity is likely to result.

Recommended guidelines for pavement subdrainage systems address the potential sources of water and the volume of water that each of these sources may generate (Carpenter et. al., 1981). Following the assessment of excess moisture, the pavement subdrainage system should be designed using materials with sufficient permeability. Guidelines are also provided for designing the collection and removal system. Cedergren et al. (1972) established the scope and provided the basis for modern subdrainage design for highway pavements. More recently Moulton (1980) presented a detailed and rational evaluation of material characteristic requirements and highway subdrainage component designs. The NCHRP Synthesis of Highway Practice 96 (Ridgeway, 1982) provides a comprehensive discussion of subsurface drainage design as well as installation of subdrainage as part of pavement rehabilitation projects.

Although the need for good internal drainage in pavement systems is now gaining recognition in the highway engineering community, a number of pavements currently designed do not actually have adequate drainage. This deficiency is often the consequence of a designing belief that resistance to free water infiltration is provided by the pavement surfacing and that sufficient sealing of the pavement can be achieved (Forsyth et al, 1987). Nevertheless good internal drainage is increasingly recommended in the recent literature (Hoffman, 1982; Highlands & Hoffman, 1988; Baldwin and Long, 1988; Mathis, 1989). Efforts to reduce, if not eliminate, the detrimental effects of entrapped water have been undertaken in several states such as California, Iowa, Kentucky, Michigan, Minnesota, New Jersey, Pennsylvania, North Carolina, West Virginia and Wisconsin. In these states, use of free draining material is being recommended for the construction of major roads. A review of these guidelines for design, use, construction, cost and performance of permeable bases was presented by Mathis (1989). No instability of the base materials under construction equipments was reported by any of the above mentioned States and only minor equipment modifications were required. However, the

construction of pavements with permeable bases requires more care than using conventional well-graded material (Mathis, 1989).

An economic study performed by Cedergren (1988) shows that an average of 15 billion dollars a year could be saved by good pavement drainage and subdrainage. This amount represents two-thirds of the total money spent by the FHWA to keep roads in serviceable conditions. Other studies indicate savings varying from 20 to 50% of the total money spent (Forsyth et al. 1987, Mathis, 1989).

The increasing use of geosynthetics in highway construction tends to alleviate the problems of cost and workability related to the placement of free draining material. The use of filter fabrics can greatly ease the cumbersome task of placing a granular filter layer to prevent subdrainage or permeable bases clogging.

It is anticipated that pavement performance can be greatly enhanced if free draining materials are used for base and subbase construction. A minimum of 50% extension of the pavement service life can be gained if infiltrated water can be promptly drained from beneath the pavement structure (Forsyth, 1987). However, several questions arise if free draining materials are used. Among these are: what would be the appropriate gradation of the material to be used, what combination of thickness and drain locations would be best from the hydraulic standpoint, what would be the drainability criteria to be adopted? Once a drainability criteria is defined, the aggregate material and its thickness can be determined based on an analytical model that simulates the hydraulic process of water movement into the pavement system.

The effect of excess moisture and the length of time it is retained in the pavement system are considered in the new 1986 AASHTO Guide for Pavement Design which contains specific structural requirements for moisture-induced weakened condition. The magnitude of these structural factors is directly related to the length of the time that the excess moisture remains in the pavement. They apply not only to designs of new pavements but also to the evaluation of existing pavements. Although new pavements may be designed in view of minimizing the use of material sensitive to excess moisture, existing pavements may have poor drainage conditions and unless complete reconstruction is undertaken these materials cannot be replaced.

A drainability criterion based on field performance needs to be developed and incorporated into guidelines for pavement design. The problem can be stated as to specifying

a given reduction of the degree of saturation to be achieved in a certain amount of time. For instance, in airfield design an adequate drainage is said to be achieved if 50% reduction of infiltrated water (drained by gravity) occurs within a 10-day period. If this criterion was applied to roadways, base course aggregates with fines materials in excess of 5% may not meet the requirement (Moyahanni and Stenberg, 1974). Such a criterion may not be applicable to highways and a proper monitoring of different pavements built with different base course aggregates should be performed. For pavement bases, Haynes and Yoder (1963) reported that bases composed of a mixture of gravel and crushed stone experience large deformations under cyclic loading conditions when the degree of saturation exceeds 85%. In summary, the final criterion for well drained pavements should be a good balance between structural stability and drainability. The requirements for a good pavement drainability can be summarized as follows: (1) the structural section of the pavement are not to become filled with excess water; and (2) the structural sections of the pavement system are not to carry heavy wheel loads for significant amount of time when there is excess water in the base. To satisfy these requirements the pavement system outflow capability must be at least equal to the inflows from various sources. The time required to drain at the end of inflow period must be short for the excess water not to remain in the structure long enough to freeze.

Despite the research results available so far, questions still remain unanswered regarding the suitability of current design procedures, material specifications, effect on pavement performance and the long term performance of the subdrainage systems. For this reason, the Strategic Highway Research Program (SHRP) has identified subdrainage as a high priority research item. One of the benefits expected from providing subdrainage for a pavement is to extend its performance by at least 50%.

An extensive literature review was performed in order to summarize the work done to date in the transportation field regarding analytical and numerical modeling of water movement in pavement systems. It was found that most of the current drainage criteria have been developed on the basis of experimental field results and theoretical analyses of infiltration under saturated conditions (Casagrande & Shannon, 1951; Cedergren, 1956 and 1972; Liu et al., 1983; Carpenter, 1990; Pufahl et al., 1990). In these previous works, material permeability and gravity action were identified as the controlling factors for the pavement drainability.

Full saturation of a pavement system can occur only under very specific circumstances. When raining starts following a dry period, the base and subbase are, in most cases, in unsaturated state. The pavement system may still be unsaturated at the end of the rainfall. The amount of water that can infiltrate within the soil is not only function of the permeability and the gravity forces, but also of the material matric suction (Philip, 1969). Furthermore, it is known that the permeability of a porous medium varies as a function of its degree of saturation. This variation can be dramatic. For instance, with a degree of saturation of 85% a soil may have a permeability only half its saturated value. These considerations show that models that assume fully saturated condition of the media under study are not valid for pavement infiltration.

For problems where the unsaturated zone is to be considered in the analysis, there is no simplified approach equivalent what can be done if only the saturated zone needs to be considered (e.g. Shannon and Casagrande, 1951). For instance, flow nets are very practical tools used in routine engineering design to analyze the variation of piezometric heads in saturated zones (Cedergren, 1956) but this technique is of little help for unsaturated flow analysis. In addition to the consideration of the unsaturated zone, the problem under consideration is also a transient flow problem in contrast with steady state flow problems for which classical solutions are available. Thus, the variable time is also introduced in the analysis. Although the account for temporal effects in the analysis of flow through unsaturated porous media adds more complexity to the problem, it is a necessary condition for realistically model of the process of moisture infiltration in the pavement system. Indeed, better description of the infiltration problem is at the cost of additional complexity in the analysis.

1.2 Purpose of the analysis

There is need for better understanding and predictive modeling of the infiltration and drainage mechanisms in order to evaluate the potential benefit of the use of subdrainage layers. Once such modeling capability is achieved, better understanding of the infiltration problem can be gained. Calibration of the models can be performed using field testing, and guidelines can be developed for choosing the base and subbase materials that would perform the best as well as for selecting the optimum location of the drains.

The present work aims to provide a rational tool to help in the analysis of the hydraulic behavior of a pavement system under conditions that best describe the field situation such as geometric characteristics, soil layering, different initial and boundary conditions. To this end, a numerical model and the associated computer program, PURDRAIN, were developed in this study. The program PURDRAIN solves the transient unsaturated flow equations using state-of-the-art numerical techniques. A detailed description of the mathematical model and the numerical algorithm used are presented later in this report.

The program PURDRAIN has been implemented in a "user friendly" PC-based environment. The program itself has been developed using a modular structure in order to allow future improvements and modifications. A detailed description of the available options of the program is presented in Appendix A which constitutes a User's guide. Once the input data have been defined (soil stratification, geometry, material parameters, rainfall event) a numerical simulation for a given period of time can be performed using PURDRAIN. If the output results (outflow capacity and time required to decrease the degree of saturation to an acceptable level) meet the performance specifications, then the design can be considered satisfactory. If the output results do not meet the specified requirements a modification of some of the design parameters (geometry, stratification, hydraulic properties) can be done and the analysis be repeated.

1.3 Scope of the study

1.3.1 One dimensional analysis

The numerical model developed in the present study enables one to handle one-dimensional analysis. A one-dimensional analysis is quite helpful when a parametric sensitivity study is needed. The data needed to define the geometry for the parametric study are simply the thickness of the layers inside the domain of interest. This simplified approach allows a better understanding of the infiltration when two- or three-dimensional attributes are not critical factors. For instance, unsaturated flow in a two layer system with varying permeability values and depths can be studied by this means. In doing so, an engineer can gain better understanding of the influence of a layer which is several times more/less permeable than the one overlying it.

1.3.2 Two dimensional analysis

The transient nature of the flow of water and the non-linearity of the differential equations that describes the flow process makes the problem very difficult to analyze in its full complexity. Research on this area was mostly done in the mid sixties, when high speed computers with large memory storage were not available. Thus, in order to make the computation technically possible, then analyses were usually restricted to one-dimensional modeling. With the advent of more powerful computers with larger memory storage and faster computation capabilities, more complex boundary and geometric conditions could be taken into account, and two and three dimensional analyses started to be considered more seriously (e.g., Freeze, 1971; Neuman, 1973; Wallace, 1977; Lam et al., 1987).

Following these earlier works, the numerical model developed in the present study is also able to handle two-dimensional analyses. Most current design procedures of pavement drainage and subdrainage are based on cross-section analyses. Thus, the developed two-dimensional model allows comparisons with conventional drainage designs. In addition, as one of the objectives of present work is to provide a rational basis for the development of design guidelines of pavement subdrainage in Indiana, the two dimensional analysis will provide a modeling framework needed to achieve this objective in the future.

1.3.3 Three dimensional analysis

A three-dimensional analysis is a better representation of actual conditions for cases where the material parameters, boundary and initial conditions can be assessed accurately. However, as pavement cross-sections are usually the same along the longitudinal direction of a particular road section, the use of a two-dimensional model is representative for most practical cases. Thus, a fully tridimensional model was not developed as part of the present study.

1.4 Limitations of the study

Due to large variability and uncertainty of the soil and other material properties involved in the problem, predictions based on numerical models cannot be made with certainty. Additional limitations are those related to the numerical methods used to solve the governing differential equations of unsaturated flow. Another limitation of the present model is that the

hysteretical behavior of the degree of saturation and the permeability in unsaturated flow is not taken into account (see section 3). Thus, the developed model is essentially an analytical tool to study how various design parameters can influence the moisture migration and redistribution underneath the pavement. Comparisons between different combinations of materials and cross-sections can be made to help design engineers making decisions on material and subdrainage pattern alternatives.

1.5 Application of infiltration process in pavement analysis - Review of previous works

1.5.1 Steady-state versus transient analysis

Although it is recognized that the use of free draining base and subbase materials helps in the durability of pavements, little has been done in order to model this process on a rational basis. Cedergren (1956) used flow nets techniques to obtain solutions for limiting cases of steady saturated flow. Casagrande and Shannon (1951) were the first to attempt an understanding of the process involved in the flow of water through the pavement base course taking its time-dependency into consideration. They assumed a saturated condition along with transient flow conditions with a linear free surface for the model (Fig. 1.1) which allowed them to develop solutions in terms of dimensionless parameters.

Figure 1.1 - Base drainage model (after Casagrande & Shannon, 1951)

Table 1.1 - Drainage Time (T) based on Casagrande & Shannon Model

	Horizontal Bases	Sloping Bases
$0 < U < 50\%$	$T = 2 U^2$	$T = 2US - S^2 \ln[1+2U/S]$
$50\% < U < 100\%$	$T = U/(2-2U)$	$T = S - S^2 \ln[1+1/S] + S \ln[(S - US + 1/2)/((1-U)(S+1))]$

where:

- U = Drained/Total area,
- S = $H/L \tan \alpha$,
- T = $tkH/n_e L^2$,
- H = Thickness of the base course,
- L = One half of the width of the pavement,
- α = slope angle,
- t = time,
- k = permeability of the base course,
- n_e = porosity of the base course,

In this model the center line of the base course and the bottom of the base course are considered impervious. The derived equations were presented in a dimensionless form (Table 3.1).

Based on Casagrande and Shannon's work, Liu et al. (1983) developed a model with a parabolic free surface (Fig. 1.2), permeable subgrade and other variations that made the model more appropriate to field conditions. In order to compare with the closed form solutions derived by Casagrande and Shannon, the base was assumed to be underlaid by an impervious subgrade. The equations corresponding to the situations described above are shown in Table 1.2.

Table 1.2 - Drainage Time (T) based on Liu et al. model

	Horizontal Bases	Sloping Bases
$0 < U < 33\%$	$T = 3 U^2$	$T = 3US/2 - 3/8 S^2 \ln[1+4U/S]$
$33\% < U < 100\%$	$T = (8/(9-9U))-1$	$T = S/2 - 3/8 S^2 \ln[1+4/3S] + S \ln[(9S - 9US + 8)/(3(1-U)(3S+4))]$

In order to better fit the field data, vertical drainage was allowed (Liu et al., 1983). An unknown location of the wetting front into the pervious subgrade and a one-dimensional infiltration into the subgrade were assumed. Due to the complexity of the modified differential equations, the solution was obtained using a numerical integration scheme. A computer program implementation of this model was written to perform this task and used to evaluate the moisture effects beneath pavements (Pufahl et al., 1990).

Figure 1.2 - Base drainage model (after Liu et al., 1983)

Although the simplifying assumptions used in order to model the drainage process might not fit the actual conditions found in the field, it is still a good attempt towards the understanding of the hydraulic behavior of pavements. The most restrictive simplifying assumption that is made is regarding the use of the saturated conditions. The equations derived are based on

saturation of the base and, as mentioned previously, this may not always be the case. In addition, assuming that the entrapped water in the pavement base can drain out freely (Fig. 1.2) considering only the gravitational forces may lead to a poor pavement design. The models described above assumed that the entrapped water can almost be 100% drained out of the system. As the water tries to go out, negative pore pressures (suction) are developed preventing, in some cases, drainage from occurring. For extreme cases, the capillary forces may be sufficiently large to prevent any drainage even for completely saturated soils. Despite the shortcomings of such models, they are routinely used in the design of drainage of pavement systems.

Furthermore, the soil underlying the pavement does not need to be fully saturated in order to damage the existing pavement. Hence a model that takes into account the unsaturated zone needs to be used in order to assess the degree of saturation that a pavement system may reach under certain design rainfall conditions. Other features that are considered to be of relevance in the hydraulic behavior of pavements need to be considered in the mathematical formulation. Some of these features are the soil stratification, anisotropy in the permeability parameters and location of drains. Thus an evaluation of the variation with time of the degree of saturation beneath the pavement can be done for different conditions. Such a procedure would allow the engineer to investigate whether the hydraulic behavior of a given pavement system is satisfactory and then decide on any appropriate modification if necessary in order to enhance the drainability of the system.

However, in order to use the procedure described herein, a tolerable range of degrees of saturation of the considered material layers, based on field experience, should be provided. Since rational analyses of pavement drainability based on unsaturated flow have been contemplated only recently, records of degrees of saturation, for which the pavement materials have proven to behave according to design requirements, are extremely sparse. Although this is a serious constraint at the present stage, once the suggested procedure is implemented, field measurements can be taken so as to relate them to the durability of the pavements in order to obtain suitable values of the maximum degrees of saturation allowable underneath pavements and the time required for drainage. In the mean time, the drainability criterion outlined by Carpenter et al. (1981) can be used. Accordingly, a base course design is considered acceptable if the time

to reach 85% saturation is five hours or less, it is considered marginal for a drainage time of five to ten hours and unacceptable for drainage time greater than ten hours.

1.5.2 Consideration of the unsaturated pavement conditions

The concept of infiltration under unsaturated conditions in pavements design was, apparently, first introduced by Wallace (1975, 1976, 1977). The solution of the model presented was based on finite difference approximation. A simple cross-section was used in the analysis; the pavement slab was assumed to have no thickness (Fig. 1.3). The main source of water was assumed to come from the surface of an unsealed road shoulder (Fig. 1.3). A simplified one-dimensional infiltration model was also introduced in order to facilitate the interpretation of more complex flow conditions. Different cases were analyzed and discussed therein.

Figure 1.3 - Idealized pavement cross-section (After, Wallace 1977)

Conceptually, the present work follows the same line as the work done by Wallace. A number of constraints of Wallace's work have been removed in the development of the present model. Some of these improvements are regarding the pavement geometry characterization and the material models used. The theory applied is described by Philip (1969). In this work, a

careful discussion is given on the infiltration process of water into porous media and the manner it is subsequently redistributed when an impervious boundary is reached.

In order to accommodate more realistic pavement geometries such as shown in Fig. 1.4 and implement the developed model, a new program, PURDRAIN, has been developed. The program is written in PASCAL language and it will provide a useful tool in the study of the behavior of water flowing through an unsaturated porous media. The program PURDRAIN, is user-friendly.

Figure 1.4 - Idealized Pavement cross section used in PURDRAIN

1.6 Report organization

The present report has been organized as follows: Chapter 2 is dedicated to the presentation of the theory of flow in unsaturated porous media and its limitations; Chapter 3 reviews the hydraulic parameters involved in the analysis; Chapter 4 discusses the numerical modeling; the model validation and applications are presented in Chapter 5. Details of the computer program and its User's guide are provided in Appendix A.

CHAPTER 2 - THEORY

2.1 Review of Previous studies on infiltration

2.1.1 One-dimensional analysis

Most of the studies performed on infiltration in unsaturated porous media have been performed by soil physicists for agricultural, hydrologic or environmental science purposes. The problems in these studies usually involve moisture movement over widely or deeply extended areas of ground. For these situations, one dimensional modeling can provide fair representations of water infiltration in the unsaturated zones. This explain the popularity of these one-dimensional solutions.

Unsaturated flow theory was initially applied to one-dimensional, horizontal and vertical, homogeneous cases, focusing on the amount of water infiltrated into the ground. Green and Ampt (1911) seemed to have made the first attempt to solve analytically the problem of water infiltrating an unsaturated media. They proposed an approximate solution based on one-dimensional vertical flow in which the infiltrating wetting front is defined by a water pressure on the wetting front (ψ_f), and in the soil behind the wetting front the water capacity ($\Delta\theta$) is uniform, coefficient of permeability (k) is a constant and the hydraulic head (h_o) is a constant throughout the infiltration process (Fig. 2.1a). It was assumed that on the moving water front a discontinuity of the degree of saturation occurs, that is behind the wetting front the soil is fully saturated whereas ahead of it, the degree of saturation is equal to the initial value (Fig. 2.1b). Applying Darcy's law and a continuity equation between the surface and the wetting front (Z_f), Green and Ampt (1911) derived the following expression for the amount of water infiltrated (I) as a function of time (t):

$$I = Z_f \Delta\theta = kt + \Delta\theta(\psi_f + h_o) \text{Ln} \left[1 - \frac{Z_f}{(\psi_f + h_o)} \right]$$

Figure 2.1 Piston-like model for one-dimensional infiltration analysis

Neglecting the effect of the gravitational forces, Philip (1969) derived a quasi-analytical solution for the Fokker-Plank equation expression for the infiltration (I) as a function of time as:

$$I = St^{1/2} + K$$

where S is called the sorptivity. Other simplified one-dimensional infiltration models for different initial and boundary conditions have been derived in the literature. A summary of the most commonly used infiltration models are presented by Delleur (1986).

This class of model can be used for cases where the rainfall rates are much higher than the infiltration capacity of the soil. In such cases it is very likely that full saturation above the wetting front will be achieved and variation from the fully saturated region to the in-situ degree of saturation occurs across a very narrow zone. If infiltration rate is less than the infiltration capacity of the soil, the hypotheses used on the derivation of the above expressions are more difficult to accept because full saturation of the region above the wetting front is not achieved and a thicker wetting front zone has to be expected.

For such cases a more general expression of the unsaturated flow is needed. The unsaturated flow into homogeneous isotropic non-selling soils is described by the well known Richards parabolic differential equation (e.g. Mein and Larson, 1973). Quasi-analytical solutions for nonlinear diffusion equations have been developed to a large extent for the Focker-Planck differential equation. In order to solve Richard's equation applying these quasi-analytical solutions, a number of hypotheses have to be assumed. However, because of the restrictive hypothesis involved in these quasi-analytical solutions, much emphasis have been put into the numerical solutions. The partial differential equation that describes the water flow into unsaturated is highly non-linear (section 2.2); therefore, a number of difficulties are usually encountered when solving the system of governing equations, especially when there is interaction between unsaturated and saturated zones of flow. Different one-dimensional numerical models have been proposed in the past. Rubin (1966) presented a finite difference model for homogeneous soils using the Kirkchoff transformation. Wishler and Klute (1969) presented a finite difference model to analyze a stratified column of soil using non-dimensional parameters. Main and Larson (1973) using a one-dimensional finite difference model compared test results obtained in the laboratory with numerical results and several closed form solutions of the infiltration rate. Morel-Seytoux (1973) proposed a one dimensional model taking explicitly the air phase into consideration, and a numerical procedure to solve the derived equation was presented by Morel-Seytoux and Vauclin (1983). Christe and Gaillard (1988) presented a one-dimensional finite element model and successfully compared its prediction to test results obtained in the laboratory.

2.1.2 Two and three-dimensional analysis

Close form solutions for two and three-dimensional analysis have been obtained for very special cases such as axisymmetrical flow into a homogeneous soil (Philip, 1969) which are not of much practical interest to civil engineers. A number of numerical models using finite difference and finite element formulations to describe the water flow have been implemented recently. Pioneer work towards two-dimensional unsaturated flow modeling was performed by Verma and Brutsaert (1971) and Neuman (1973), for

earth dam applications, using finite difference and finite element techniques respectively. The first three-dimensional unsaturated model using a finite element scheme was apparently proposed by Freeze (1971). Wallace (1977), Dempsey (1977a) have later presented two-dimensional finite difference models of water flow in unsaturated media for pavement applications. Winter (1983) presented a finite element model to study the influence of lakes in aquifers recharge. Lam et al. (1987) also presented a finite element model that is more amenable for geotechnical applications. Dysli and Rybisar (1987), Papagianakis and Fredlund (1984) have proposed computational techniques to determine the phreatic level (line of zero pressure) using commercially available finite element programs of unsaturated flow. Due to memory constraints, implementations of three-dimensional numerical models are being restricted to mainframe or supercomputers; thus, the application of such models to practical civil engineering problems is seldom.

2.2 Derivation of the general equation of flow in unsaturated porous media

Cases of practical interest may involve stratified soils along with more complex boundary conditions as it is the case of pavement structures. For such cases, it is not possible to use closed form solutions and a more general treatment of the unsaturated flow process including the extension of the Darcy's law to unsaturated flow is needed. A comprehensive review of this theory was presented by Philip (1969). Morel-Seytoux (1973) presented a comprehensive discussion of the theory of unsaturated flow taking the air phase explicitly into account. A synthesis of this theoretical background is presented herein.

The motion of a fluid can be described using two different methods. The first, in which the history of the individual particles is described, is called the Lagrangian method. Here, the coordinate system is not fixed in the space. The second, in which a fixed point in the space is chosen (i.e. control box), is called the Eulerian method. Here, the particles are described based on a fixed system of coordinates. The description of the system of equations of fluid motion using either methods depend upon the particular application, the Eulerian method being the most often used in fluids mechanics.

2.2.1 Conservation of mass

The following formulation follows the classical theory of mechanics of continuum media (Eringen, 1980; Bear, 1972) where the balance equation takes the form of a partial differential relationship describing the change of a considered quantity (J — mass per unit area per unit time) in a unit volume of porous media (control box) per unit time (Fig. 2.2). The equation that defines the conservation of mass flux entering, leaving and being stored in a control box can be expressed as follows:

$$\text{Flux in}_x = \left[J_x - \frac{\partial J_x}{\partial x} \frac{dx}{2} \right] dy dz \quad (2.1)$$

$$\text{Flux out}_x = \left[J_x + \frac{\partial J_x}{\partial x} \frac{dx}{2} \right] dy dz \quad (2.2)$$

thus, the total flow in the x direction is given by

$$q_x = \text{Flux in}_x - \text{Flux out}_x = -\frac{\partial J_x}{\partial x} dx dy dz \quad (2.3)$$

Figure 2.2 Elemental control volume of porous media (After Bear and Verruijt, 1987)

Using similar relationships for the y and z directions, the total change (q) in a control volume can be expressed as:

$$q = q_x + q_y + q_z = - \left[\frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z} \right] dx dy dz \quad (2.4)$$

which simplifies to

$$q = (-\nabla \cdot J) dV \quad (2.5)$$

where ∇ is the Laplacian operator

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right), \quad J = (J_x, J_y, J_z), \quad dV = dx dy dz$$

Additionally, the total flow (q) is equal to the change of mass (M) in the control box during the considered interval of time, thus:

$$q = \frac{\partial M}{\partial t} \quad (2.6)$$

thus, making Eq. (2.5) equal to Eq. (2.6) yields:

$$\frac{\partial M}{\partial t} = (-\nabla \cdot J) dV \quad (2.7)$$

The mass (M) of porous material in the control box can be expressed as

$$M = \left(\sum n_i \rho_i \right) dV \quad (2.8)$$

where $n_i = V_i/V$ is the volume fraction, i is the subscript representing the solid, water or air phase, V represents the total volume ($V_s + V_a + V_w$), $V_s = (1-n)V$ is the solid phase volume, $V_a = n(1-S)V$ is the air phase volume, and $V_w = nSV$ is the water phase volume. Equation (2.7) is the general equation of conservation of a given extensive quantity (i.e. mass) that travels at a certain velocity. In the analysis of unsaturated flow, three phases are present namely solid, water and air. Thus, the total mass is

$$M = [\rho_s(1-n) + \rho_w nS + \rho_a (1-S)n] dV \quad (2.9)$$

and the corresponding mass flux is

$$J = (1-n)\rho_s u_s + nS\rho_w u_w + (1-S)n\rho_a u_a \quad (2.10)$$

where n is the porosity of the soil, S represents the soil degree of saturation, the subscript s, a, w represents the solid, air and water phase respectively, ρ_i is the density of phase i ($i = a, w, s$), and u_i represents the actual microscopic velocity of phase i ($i = a, w, s$). Replacing Eq. (2.9) and Eq. (2.10) in Eq. (2.7) gives:

$$\frac{\partial}{\partial t}[\rho_s(1-n) + \rho_w nS + \rho_a(1-S)n] = -\nabla \cdot ((1-n)u_s \rho_s + q_w \rho_w + q_a \rho_a) \quad (2.11)$$

$$q_a = (1-S)n u_a \quad (2.12)$$

$$q_w = S n u_w \quad (2.13)$$

where q_i is the specific discharge (Darcian velocity) of the phase i ($i = a, w$). Eq. (2.11) is the explicit equation of conservation taking into account all three phases. In order to reduce Eq. (2.11) to a more tractable form, a number of hypothesis are introduced.

Hypothesis I (Solid phase incompressible)

The effective stresses induced by groundwater movement are usually small as compared to the solid compressibility. Hence, the solid phase can be assumed incompressible (i.e. $d\rho_s/dt = 0$) without introducing appreciable errors:

$$\begin{aligned} & [-\rho_s + \rho_w S + \rho_a(1-S)] \frac{\partial n}{\partial t} + \\ & n \frac{\partial}{\partial t} [\rho_w S + \rho_a(1-S)] = -\nabla \cdot ((1-n)u_s \rho_s + q_w \rho_w + q_a \rho_a) \end{aligned} \quad (2.14)$$

Hypothesis II (non-deformable soil matrix)

Assuming that the soil matrix does not deform (i.e. the porosity does not change as a function of time and the solid velocity u_s is nil), yields:

$$n \frac{\partial}{\partial t} [\rho_w S + \rho_a(1-S)] = -\nabla \cdot (q_w \rho_w + q_a \rho_a) \quad (2.15)$$

Hypothesis III (water incompressibility)

Dividing the right and left hand side of Eq. (2.15) by ρ_w and assuming the water density to be incompressible, gives:

$$n \frac{\partial}{\partial t} [S + \frac{\rho_a}{\rho_w} (1-S)] = -\nabla \cdot (q_w + q_a \frac{\rho_a}{\rho_w}) \quad (2.16)$$

Hypothesis IV

Since $\rho_w \gg \rho_a$ and assuming that the darcian velocities q_a and q_w are of the same order of magnitude, the above equation can be simplified to

$$\frac{\partial \theta_w}{\partial t} = -\nabla \cdot (q_w) \quad (2.17)$$

where $\theta_w = nS$ is the volumetric water content. Replacing Eq. (2.17) in Eq. (2.16) yields:

$$\frac{\partial (\rho_a \theta_a)}{\partial t} = -\nabla \cdot (\rho_a q_a) \quad (2.18)$$

where $\theta_a = n(1-S)$ is the volumetric air content. The derivation presented herein differs from the classical ones in which the continuity equation is applied independently to each phase (e.g. Bear, 1972; Bear and Verruijt, 1987; de Marsily, 1986; Şeker, 1983; Brutsaert and El-Kadi, 1984). More recent publications (e.g. Allen, 1986; de Boer et al., 1991; Gray and Hassanizadeh, 1991a and 1991b) analyzes the multiphase flow as a continuum mixture which is a set of overlapping continua called constituents. Eqs. (2.17) and (2.18) have been obtained from the general continuity equation (Eq. 2.11). Notice that Hypothesis I, II and III are not sufficient to obtain separate equations describing the fluid motion for air and water (Eqs. 2.17 and 2.18). The analyzed fluids must have density values of the different orders of magnitude (Hypothesis IV) as it is the case of air and water.

2.2.2 Conservation of linear momentum

From Newton's law, the fluid stresses, body forces, externally applied stresses and inertial terms of a moving phase must be in equilibrium. This is known as the conservation of linear momentum. The momentum balance for each phase relates the inertia ($n_i \rho_i du_i/dt$), the fluid stresses (σ_i), the body force (b_i), the rate of momentum exchange from other phases (m_i), r_i is the rate of mass transfer into phase i from other phases. Thus

$$n_i \rho_i \left(\frac{du_i}{dt} \right) - \nabla \sigma_i - n_i \rho_i b_i = m_i - u_i r_i \quad (2.19)$$

du_i/dt is the material derivative of the quantity u_i . Neglecting the momentum transfer within the fluid via shear stresses as compared with the momentum exchange to solid matrix, and assuming that each fluid is newtonian, i.e. the stresses are proportional to the rates of strains, $\sigma_i = p_i \mathbf{1}$ where p_i is the mechanical pressure in phase i and $\mathbf{1}$ is the unit isotropic tensor. If gravity force is the only body force acting on the phase i , then $n_i b_i = g \mathbf{R}z$ (g stands for the gravity acceleration and z is the vertical coordinates positive upwards). The momentum exchange losses at the solid-fluid interface can be written as a drag force (Stoke's law)

$$m_i = n(u_s - u_i) \Gamma_i^{-1} \quad (2.20)$$

in which $\Gamma_i (= k k_{ri}/\mu_i)$ is a tensor that measures the ease of a phase i to travel through the porous medium, the dimensionless parameter k_{ri} is the relative permeability with values ranging from 0 to 1 (Fig. 2.3), μ_i dynamic viscosity of fluid phase i [ML^2T^{-1}], and k a second order rank tensor of intrinsic conductivity of an non-homogeneous medium that depends only on the soil matrix [L^2].

Assuming further that no interphase mass transfer occurs (i.e. $r_i = 0$), the inertial effects are negligible compared with rock fluid interaction (that is $n_i \rho_i du_i/dt = 0$), the solid velocity negligible small ($u_s = 0$) and replacing Eq. (2.20) in Eq. (2.19) yields

$$u_i = -\frac{k k_{ri}}{\mu_i n_i} \nabla(p_i + \rho_i g z) \quad (2.21)$$

Figure 2.3 Typical relative hydraulic conductivity (K_r) vs. Saturation

Eq. (2.21) can be written in terms of the specific discharge (q_i) and the piezometric head $\phi (= z + p/\rho_i g)$,

$$q_i = -\frac{\gamma_i k k_{ri}}{\mu_i} \nabla \phi \quad (2.22)$$

where γ_i is the phase specific weight and the intrinsic permeability tensor is represented by

$$k = \begin{bmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{yx} & k_{yy} & k_{yz} \\ k_{zx} & k_{zy} & k_{zz} \end{bmatrix} \quad (2.23)$$

Equation (2.23) is known as Darcy's law for each constituent of the mixture. It has been derived herein from the general equation of linear momentum of the mixture. The applicability of Darcy's law to unsaturated flow was confirmed experimentally by Childs and Collis-George and others (as discussed by Philip, 1969).

2.2.3 Two-phase flow governing equations (Unsaturated zone)

A combination of Eq. (2.17) and Eq. (2.22), assuming that the principal directions of the hydraulic conductivity are used as coordinate system (i.e. $k_{xx}=k_{yy}=k_{zz}=k_{xy}=k_{yx}=k_{yz}=k_{zy}=0$), yields the general second-order partial differential equation that describes the water flow through unsaturated or saturated anisotropic media, sometimes referred to as the Richards' equation (Mein and Larson, 1973):

$$\frac{\partial \theta_w}{\partial t} = \frac{\partial}{\partial x} \left[\frac{\gamma_w k_x k_{rw}(\psi)}{\mu_w} \frac{\partial \phi}{\partial x} \right] + \frac{\partial}{\partial y} \left[\frac{\gamma_w k_y k_{rw}(\psi)}{\mu_w} \frac{\partial \phi}{\partial y} \right] + \frac{\partial}{\partial z} \left[\frac{\gamma_w k_z k_{rw}(\psi)}{\mu_w} \frac{\partial \phi}{\partial z} \right] \quad (2.24)$$

where x, y, z are the three space coordinates, and ψ is the suction ($z-\phi$). An expression similar to that of water flow can be obtained for an incompressible air phase, thus

$$\frac{\partial \theta_a}{\partial t} = \frac{\partial}{\partial x} \left[\frac{\gamma_a k_x k_{ra}(\psi)}{\mu_a} \frac{\partial \phi}{\partial x} \right] + \frac{\partial}{\partial y} \left[\frac{\gamma_a k_y k_{ra}(\psi)}{\mu_a} \frac{\partial \phi}{\partial y} \right] + \frac{\partial}{\partial z} \left[\frac{\gamma_a k_z k_{ra}(\psi)}{\mu_a} \frac{\partial \phi}{\partial z} \right] \quad (2.25)$$

Equations (2.24) and (Eq.2.25) describe the flow of water and air within an incompressible porous medium. Since they are coupled through the suction, ψ , and the piezometric head, ϕ , both equations must be solved simultaneously. The non-linearity of the system of differential equation poses serious difficulty for the analytical and numerical solutions. Morel-Seytoux (1973, 1982) presents a comprehensive discussion of two-phase unsaturated flow analysis into porous medium. A solution for one-dimensional case assuming both the water and air phase to be incompressible is presented in Appendix A.

In the present analysis, the motion of air is considered to have negligible effect on the motion of water, hence the water movement can be accurately predicted by Eq. (2.24) only (e.g, Philip, 1969; Neuman, 1973; Rubin, 1969; Wallace, 1975; Christe and Gaillard, 1989). Eq. (2.24) can be rearranged such that the main variable on the left hand side is written in terms of the piezometric head which is continuous throughout the

space domain, even for heterogeneous soils, unlike the volumetric moisture content which can be a discontinuous function. This can be achieved by using the definition of specific water capacity (C_w) as defined by Bear (1972),

$$C_w = -\frac{\partial \theta_w}{\partial \Psi} \quad (2.26)$$

thus, Eq. (2.24) can be written as

$$C_w \frac{\partial \phi}{\partial t} = \frac{\partial}{\partial x} \left[K_x(\psi) \frac{\partial \phi}{\partial x} \right] + \frac{\partial}{\partial y} \left[K_y(\psi) \frac{\partial \phi}{\partial y} \right] + \frac{\partial}{\partial z} \left[K_z(\psi) \frac{\partial \phi}{\partial z} \right] \quad (2.27)$$

where $K = \gamma_w k k_{rw} / \mu_w$. Eq. (2.27) is a second order differential equation with non-linear coefficients C_w and K . The cartesian two-dimensional form of Eq. (2.27) is,

$$C_w \frac{\partial \phi}{\partial t} = \frac{\partial}{\partial x} \left[K_x(\psi) \frac{\partial \phi}{\partial x} \right] + \frac{\partial}{\partial z} \left[K_z(\psi) \frac{\partial \phi}{\partial z} \right] \quad (2.28)$$

2.2.4 Laplace equation (Saturated zone)

Eq. (2.24) is the general expression for modeling three-dimensional unsaturated or saturated flow that describes the variation of volumetric moisture content as a function of time within the domain of interest. If saturation occurs, no variation of volumetric moisture content (θ) with respect to time occurs and the relative permeability matrix reaches its maximum value $k_{rw} = 1$, thus $K = K_s$ (the saturated permeability values) is constant, yielding,

$$K_{s_x} \left[\frac{\partial^2 \phi}{\partial x^2} \right] + K_{s_y} \left[\frac{\partial^2 \phi}{\partial y^2} \right] + K_{s_z} \left[\frac{\partial^2 \phi}{\partial z^2} \right] = 0 \quad (2.29)$$

Eq. (2.29) is the classical equation of flow under saturated conditions for anisotropic materials.

2.2.5 Infiltration and absorption

It is also important to define the terms infiltration and absorption which are commonly found in the literature (e.g. Philip, 1969). Infiltration is the process of water

moving through the soil as a result of interaction between capillarity and gravity forces. The absorption process occurs when the gravity forces are neglected either because they are small in comparison to capillarity forces (e.g. into fine texture soils at earlier stage of infiltration) or because no significant gradient is experienced in the applied gravity forces (i.e. infiltration into a horizontal system). Absorption can be understood as a special case of the more general infiltration process.

2.2.6 Other forms of the unsaturated flow equation

The derivation of Eq. (2.27) was derived from the principles of continuum mechanics. Thus taking into consideration the appropriate parameters, material constitutive equations and nature of flux, the same governing equation can be applied to a number of transport problems such as heat transfer, migration of pollutants, and so forth.

Depending upon the particular problem in hydraulics, Eq. (2.24) is found in the literature expressed in terms of volumetric moisture content (e.g. Philip, 1969; Dempsey and Elzeftawy, 1977; Warrick et al., 1991), pressure heads (e.g. Neuman, 1973; Freeze, 1971; Paniconi et al., 1991; Celia et al., 1987) or the piezometric heads (e.g. Wallace, 1977; Fredlund, 1989; Dysli and Rybisar, 1987) as a main variable. Because pioneer works were performed by soil physicists who focused mainly on the unsaturated zone response, the initial description of unsaturated flow was made in terms of volumetric moisture content. In some cases such formulations are equivalent to diffusion equations. In some instances, the techniques available for deriving closed form solutions for certain particular diffusion problems could be used for solving unsaturated flow problems (Philip, 1969). Recalling that the matric suction is related to the volumetric moisture content, Eq. (2.24) can be rewritten:

$$\frac{\partial \theta}{\partial t} = \frac{\partial K}{\partial \theta} \frac{\partial \theta}{\partial z} + \nabla(D(\theta)\nabla\theta) \quad (2.30)$$

where

$$D(\theta) = K(\theta) \frac{d\psi}{d\theta} \quad (2.31)$$

In Eq. (2.30), θ is the volumetric moisture content, and $D(\theta)$ the coefficient of water diffusion. If it is assumed that the flow is mainly produced by absorption forces, then the gravitational component can be neglected and Eq. (2.30) reduces to:

$$\frac{\partial \theta}{\partial t} = \nabla(D(\theta)\nabla\theta) \quad (2.32)$$

The versatility of Eq. (2.32) to describe different transport processes has contributed to its widespread use instead of the more general Eq. (2.24). Nevertheless, neither Eq. (2.30) nor Eq. (2.32), as presented, are suitable for a general numerical treatment. In moisture flow problems, saturation is likely to take place at some time during the analyzed process. At this stage, no further changes in moisture content would occur, thus the diffusion term in Eq. (2.30) will become infinite. Such a situation is evidenced in Fig. 2.4 by the vertical slope of the curve in the capillary fringe zone.

Figure 2.4 Typical soil retention curves

This mathematical anomaly will restrict the use of these equations to problems where saturation is not likely to occur. In practice, to avoid numerical difficulties, these equations are restricted to saturation values smaller than 85%. On the contrary, for modeling flow into very dry soils, the volumetric moisture content based equation seems to be the most appropriate one.

Unsaturated flow problems have also interested hydrologist who wrote the governing differential equation in terms of pressure head:

$$C_w \frac{\partial \psi}{\partial t} = \nabla(K \nabla(z-\psi)) = \frac{\partial K}{\partial z} + \nabla(K \nabla \psi) \quad (2.33)$$

Although Eqs. (2.24), (2.27), (2.30), and (2.33), are derived from the same principles they are not completely equivalent and they cannot be freely interchanged (Philip, 1969). For instance, Eq. (2.24) may present some restrictions when a heterogeneous soil is to be analyzed; since the left hand side of Eq. (2.24) is expressed in terms of the volumetric moisture content which can be discontinuous in the space domain. For instance, in a two-layer soil the volumetric moisture content at the boundary will not necessarily take the same value on both sides of the interface (Fig. 2.5). Discontinuity of the volumetric moisture content function is difficult to handle in numerical solutions. Thus, the choice of a particular expression will depend on the specific application.

Figure 2.5 Saturation discontinuity in a two-layer system

2.3 Effects of air phase on the unsaturated flow

In the analysis of unsaturated flow driven by gravity forces only, the air-phase is implicitly assumed to be continuous, at atmospheric pressure, and free to escape from or enter the soil. This simplifying assumption allows to solve the sole differential

equation of water movement (Eq. 2.4) and to disregard the air phase motion equation (Eq. 2.5). Laboratory and field tests show that in a number of instances no significant errors are introduced in the evaluation of the moisture content obtained in this way (e.g. Christe and Gaillard, 1989; Stauffer and Dracos, 1984; Watson, 1967; Mein and Larson, 1973; Thomas and Rees, 1990).

There are however cases of gravity driven unsaturated flow where the influence of the air-phase motion on the moisture movement cannot be neglected in the analysis (Vachaud et. al., 1973; Morel Seytoux, 1973). For such cases, a two-phase flow formulation allows a better formulation of the problem (Morel-Seytoux and Khanji, 1974; Morel-Seytoux and Vauclin, 1983). Unfortunately, a description of the unsaturated flow taking the air-phase into consideration requires an additional non-linear equation which is coupled with that of the water flow. Solutions of coupled partial differential equations are quite difficult to obtain. The difficulty is increased by their non-linearity. Brustkern and Morel-Seytoux (1970), Morel-Seytoux and Vauclin (1983a) have presented a numerical algorithm to solve a one-dimensional two-phase flow for particular cases. A detailed formulation for the air-water flow considering both fluids to be incompressible is presented in Appendix A.

When analyzing the deformation of an unsaturated porous medium, the air-phase response cannot be disregarded in the analysis since it is the main factor controlling the soil matrix compressibility. The solid and water-phase are for most engineering purposes considered to be incompressible. Thus, the air-phase needs to be taken into account to model realistically a three-phase medium (solid-water-air) (e.g. Seker, 1983; Fredlund and Morgenstern, 1976; Fredlund and Hasan, 1979; Schrefler and Simoni, 1988). Fortunately, for most practical cases, the soil matrix compressibility can be safely disregarded in the analysis of moisture movement (Brutsaert and El-Kadi, 1984).

2.4 Initial and boundary conditions

The governing equations listed in the previous sections are second order partial differential equations which express mass and momentum balances of an hypothetical flow passing through a domain of interest. If no additional information is provided, Eq.

(2.27) will have infinite number of possible solutions. Initial and boundary conditions constitute the additional information which makes the problem determinate. Such problems are known as initial and boundary value problems.

The initial conditions refer to values that the state variables take at the beginning of the analysis ($t=0$). This information is based on field measurements or on theoretical considerations. The boundary conditions describe the interaction between the domain being analyzed and the environment. These conditions may change with time. Different initial and boundary conditions will result in different solutions. Initial or boundary conditions can take the form of either numerical values assigned to the state variables or relationships between these variables. At the boundaries, such equations have often to be imposed so as to best simulate the actual conditions. Hence, it is important to examine the different types of boundary conditions that can be used in flow analysis and their applicability to the different cases.

The wide variety of practical situations that might occur at a boundary together with the influence that the boundary conditions may have on the solution makes it a critical factor of any analysis. From the physical point of view, the boundary conditions can be classified into four categories: (a) first kind or Dirichlet conditions, (b) second kind or Neumann conditions, (c) third kind or Cauchy conditions and (d) variable boundary conditions.

2.4.1 General boundary conditions

The general boundary condition states that the total flux of any extensive quantity, relative to the possibly moving boundary; remains unchanged as the boundary is crossed (Bear and Verruijt, 1990). This boundary conditions is often referred as the jump condition at a moving discontinuity surface (Eringen, 1980). The position of the boundary itself is part of the solution of the problem and the equation required to determine it is given by the jump conditions. It can be shown (Bear & Verruijt, 1987) that the boundary conditions listed below are particular cases of this more general one.

A typical example of a moving boundary condition is the top of the phreatic surface in a transient analysis when only saturated conditions are considered in the

analysis. The phreatic surface represents as the upper boundary of the saturated region within which Laplace's equation (Eq. 2.29) is applicable. Since this upper boundary is part of the solution of the problem a closed form solution cannot be obtained except in very particular cases. As the governing equation of flow (Eq. 2.27) is applicable for saturated and unsaturated porous media (using the appropriate parameters), Eq. (2.27) can be applied throughout the space domain; hence, the phreatic level is not a moving boundary and it can be determined by interpolating the values of zero pressure (Dysli and Rybisar, 1987). This technique avoids the use of special conditions for the moving boundary.

2.4.2 Boundary condition of the 1st kind (Dirichlet condition)

For the Dirichlet conditions the head is specified at the boundary:

$$\phi_B = f_1(P_B, t) \quad (2.33)$$

where f_1 is a known function valid at the boundary; P_B represents the coordinates of the boundary, and the subscript B indicates that the expression is evaluated at the boundary. A typical example of this type of boundary is the case of a soil with low permeability where ponding can occur immediately after a rainfall, thus $f_1(P_B, t) = H(P, t)$ where H is the value of the piezometric head. Segments A'A, AB, CG and GG' of Fig. 2.6 are examples for this type of boundary condition.

2.4.3 Boundary condition of the 2nd kind (Neumann condition)

For the Neumann conditions the pressure gradient along the boundary is prescribed. Thus, the boundary conditions are imposed on the head gradient which controls the flux.

$$\nabla \phi_B = f_2(P_B, t) \quad (2.34)$$

An example is that of an impervious material boundary where the normal flux is zero, $f_2(P_B, t) = 0$. Segment HI of Fig. 2.6 is an example of this type of boundary condition.

2.4.4 Boundary condition of the 3rd kind (Cauchy condition)

For the Cauchy conditions the gradient of head, normal to the boundary, and the head at the boundary are related in a prescribed manner.

$$\nabla\phi_B = f_3(P_B, \phi_B, t) \quad (2.35)$$

Examples of this type of boundary condition are found in aquifers which are underlaid by a semipermeable thin layer. Thus the unknown flux normal to the boundary can be expressed in terms of the unknown piezometric head at the boundary by $f_3(P_B, \phi_B, t) = (K/D)(\phi_B - \phi_o)$, where K is the saturated permeability of the thin layer of thickness D , ϕ_B and ϕ_o are the piezometric heads at the top and bottom of the semipervious layer, respectively. Segment DD' of Fig. 2.6 is an example of this type of boundary condition.

2.4.5 Variable boundary conditions

On a variable boundary condition either Cauchy or Dirichlet conditions may prevail depending on the characteristics of the boundary itself. The variable boundary condition is described by Eq. (2.33) or by Eq. (2.35) and it cannot be predicted a priori which one of the two equations will need to be used. The change from one to the other will be dictated by the solution of the equation itself. An example of this type of boundary condition is that of a rainfall on top of an unsaturated medium. If saturation of the boundary has not been reached at a give time, then a Cauchy condition should be used:

$$(K(\phi_B)\nabla\phi_B) \cdot \nu = N \cdot \nu \quad (2.36)$$

where N is a vector representing the rate of precipitation as function of time, ν is a unit vector normal to the surface of infiltration (Fig. 2.6).

If the boundary becomes saturated at a given time, Eq. (2.36) cannot be used any longer and a Dirichlet boundary condition will have to be written instead. Segments BE, EF, and FC of Fig. 2.6 are examples of variable boundary condition.

Two possible causes of deviation from Darcy's law are the inertia effects and the non-Newtonian behavior of the water flow. When the ratio of inertial to viscous forces acting on the fluid become important, the Navier-Stokes equation (equation of momentum) is not linear, and a 'nonlinear laminar' regime is imposed. This ratio is known as the Reynolds number (Re) which was defined for flow through pipes as

$$Re = \frac{qd}{\nu} \quad (2.37)$$

in which q is the specific discharge [LT^{-1}], d is the pipe diameter, and ν is the kinematic viscosity [L^2T^{-1}]. In a porous medium, by analogy, a representative pore size length is used. The inertial effect can be disregarded when the Reynolds number does not exceed some value between 1 to 10. For such cases the flow velocity can be written proportional to the potential gradient. In most practical cases of steady flow, the Reynolds number occurs in this range except in the vicinity of large pumping wells or in a very porous media such as gravel. A careful examination is required in order to evaluate the influence of inertia on transient flows. Nevertheless, inertia forces are unlikely to be of much significance in either saturated or unsaturated transient flow (Philip, 1969).

The Navier-Stokes equation validity also depends on the Newtonian behavior of the fluid (rate of stress proportional to rate of strain). Water is a newtonian fluid. However, its behavior may be altered if a large fraction of the soil-water is subject to large forces. This can occur in soils with a high colloid content and large specific surface.

Soils with high colloid content also present problems of swelling and shrinkage involving particle motion. Thus, a description of the flow in terms of relative displacements to the soil particles should be considered. The use of Eq. (2.27) implicitly assumes that the change in the mass is only due to the change in water content. In other words no compression or expansion of the soil due to the water flow is allowed. This

aspect is fundamental in the application of the derived governing equations. The analysis is also restricted to an isothermic process.

It is also known that there is no unique relationship between the volumetric moisture content and the matric suction (Philip, 1969; Bear, 1972; Mualen, 1974). The process of wetting-up and drying is not a reversible process (Fig. 2.7). The evolution depends on the initial values and on whether the volumetric moisture content at a given point is increased or reduced. In other words, the behavior is path-dependant. This hysteretical behavior has a marked influence in the determination of the parameter C_w . It is noted that the hysteretical nature of this curve does not invalidate the use of equation (2.27) as long as the appropriate branch of the patterns is used.

Figure 2.7 Hysteresis of the retention curve (after Bear and Verruijt, 1987)

It is noted that there is ambiguity in the definition of the volumetric water content. Richards (1974) defines the volumetric water content as the ratio between the volume of water and the volume of solids contained in a unit volume. This definition is somewhat similar to that of the void ratio. Bear and Verruijt (1988) define the volumetric content as the ratio between the volume of water and the total volume. This definition is somewhat similar to that of the porosity. Although the numerical values of both definitions are not the same, and since it is the value of the degree of saturation that is important when analyzing flow through unsaturated media, if the soil is assumed to be incompressible both definitions become equivalent and either may be used to determine the degree of saturation and the net change per unit time of the volumetric water content ($\partial\theta/\partial t$). Nevertheless, caution is advised when comparisons are done with previously published data especially when an explicit definition for volumetric water content is not given.

2.6 Paradoxes of the current unsaturated flow theory

Current knowledge of unsaturated flow is based on the extension of equations and ideas from simpler systems such as saturated porous media. Multiphase flow in porous media differs from a single-phase flow mainly because of the presence of interfaces which are dependent on the amount of each component (Gray and Hassanizadeh, 1991b). Neglecting the influence of the interfaces in the formulation of the mathematical model of multiphase flow may be oversimplifying.

A typical paradox resulting from the current theory of single-phase flow is the range of validity of the capillary pressure (p_c). The capillary pressure is defined as the difference between the air and water pressures, $p_c = p_a - p_w$. In order to have positive capillary pressure in natural soils, the pressure equilibrium of the water phase (p_w) must be less than that of the air-phase (p_a). If the air-phase is assumed to be at 1 atm. pressure, the capillary pressure should be in the range from 0 to 1 atm. However, it is very common to find values of capillarity, reported in the literature, much higher than 1 atm. (e.g Fredlund, 1991) indicating that pressures smaller than the absolute zero must exist. De Marsily (1986, pp. 33) agrees with the idea of negative absolute pressures.

Another common paradox is regarding the applicability of the hydrostatic pressure gradient at equilibrium. At equilibrium, the water darcian velocity is zero, thus from Eq. (2.22)

$$\nabla p_w = \rho_w g \quad (2.38)$$

thus assuming constant ρ_w , $p_w = p_a$ at $z=0$, and integrating in the vertical direction yields

$$p_w - p_a = -\rho_w g z \quad (2.39)$$

As suction is also defined as $(p_a - p_w)/\rho_w g$, Eq. (2.38) implies that $\psi = z$; indicating that, at equilibrium, in the unsaturated zone, suction is a linear function of elevation and as long as the water is continuous, the pressure difference between two points separated by a vertical distance Δz is given by $\rho_w g \Delta z$. Gray and Hassanizadeh (1991b) state that this model does not hold true for unsaturated media because of the attraction of the water to the soil grains, thus the pressure gradient should be less than hydrostatic. Gray and Hassanizadeh (1991a) have proposed a revised model for unsaturated porous media where the equations for mass, momentum and energy are examined in light of the second law of thermodynamics. They have introduced additional terms that take into account the phases interface presence. The paradoxes presented above have been explained based upon the new expressions derived. Although the model of Gray and Hassanizadeh is theoretically sound, its mathematical complexity along with the number of material parameters required prevent its practical application.

CHAPTER 3 - HYDRAULIC CHARACTERISTICS OF UNSATURATED SOILS

3.1 Introduction

Continuum mechanics states that the basic equations derived in the previous section are valid for all type of media irrespectively of their internal constitution. Different soils with the same mass and geometric characteristics respond differently to the same boundary conditions and applied energy. The reason for this individual attribute is the internal constitution of the soil.

In the foregoing development, Eq. (2.27) is the main governing differential equation which is applied throughout the domain analysis. In the derivation of Eq. (2.27) it was implicitly assumed that analytical or experimental relationships that describe the variation of the permeability and matric suction as functions of the degree of saturation of the soil mass were available. In the present section, a description of the two main equations of state, that are $k_w = f(S_e)$ and $\psi = f(S_e)$, used in the solution of the governing differential equation are presented.

3.2 Capillary suction

A discontinuity in pressure exists across the boundary of two immiscible fluids in contact. This discontinuity is due to the interfacial tension that tends to keep in equilibrium the two fluid phases. The difference in pressure, $p_{air} - p_{water}$, is called capillary pressure, p_c , where p_{air} is the non-wetting fluid pressure and p_{water} is the wetting one. A mathematical expression for p_c can be obtained from force balance across the interface (Fig. 3.1):

$$p_c = \sigma_{aw} \left(\frac{1}{r'} + \frac{1}{r''} \right) \quad (3.1)$$

where σ_{aw} denotes the constant air-water interfacial tension (which depends on the fluids in contact), r' and r'' are the principal radii of curvature of the interface. For an interface boundary of single curvature the above expression reduces to:

$$p_c = \frac{\sigma_{aw}}{r'} \quad (3.2)$$

Figure 3.1 Capillary forces at the air-water interface

3.2.1 Theoretical evaluation of capillarity

The capillary tube model (Fig. 3.2) is commonly used (Taylor, 1948) to evaluate the capillary rise. The following expression can be derived from force equilibrium in the vertical direction

$$P_c = \psi \gamma_w = \frac{2T \cos \alpha}{r} \quad (3.3)$$

where ψ is the capillary rise (suction), α is the contact angle between the solid phase and the boundary interface measured through the denser fluid; T is the air-water interfacial

force per unit length and P_c is the capillary pressure. For a given fluid, $2T\cos\alpha/\gamma_w$ is a constant (approximately 0.15 cm^2 for water); thus, the maximum capillary rise will depend only on the value of the tube radius (r),

$$\psi = \frac{0.15 \text{ cm}^2}{r} \quad (3.4)$$

Figure 3.2 A simple capillary tube model

3.2.2 Soil suction

A concept similar to the capillary tube has been used to describe the water rise above the phreatic level. The negative value of the capillarity is often called by the soil physicist 'suction'. As shown in the previous section for a tube with a constant radius, the value of the capillary rise is known and unique regardless the tube is filled from the bottom (imbibition) or from the top (drainage). For a tube with a variable cross section, the actual capillary rise cannot be predicted simply using an equation such as Eq. 3.4. As the capillary rise on a tube of variable cross section depends on whether the tube is filled from the top (infiltration) or from the bottom (imbibition), it is necessary to know the wetting history. Fig. 3.3 shows the actual capillary rise for various possible cases.

Figure 3.3 Influence of the tube cross-section variation (after Taylor, 1948)

A porous medium can be regarded as a series of interconnected tubes so that a uniform location of the capillary fringe cannot be estimated by means of a simple expression such as Eq. (3.4). To overcome this difficulty, a number of empirical models based on laboratory tests have been proposed in the literature (e.g. Bear and Verruijt, 1987; El-Kadi, 1985b, Bumb et al., 1992). Theoretical relationships that take into account the randomly distributed porosity have also been developed (e.g. Brutsaert, 1966). Fig. (3.4) show typical water retention curves (S_e - ψ) measured for different soils. It is noted that the matric suction seem to increase exponentially with decreasing values of the moisture content.

Figure 3.4 Experimental retention curves for different soils (after Bear, 1972)

The choice of a given analytical relationship to model the water retention curves will depend on its ability to fit the laboratory data and, sometimes, on its ability to yield a closed form solution for the Richard's equation. Extensive laboratory tests on different soils to compare the different candidate functions were performed by several researchers (La Liberte et. al., 1966, El-Kadi, 1985a, Bumb et. al., 1992; Van Genuchten, 1980) and the results are discussed in section 3.2.3.6.

3.2.3 Soil-water characteristic curves

The water retention curves (ψ - θ) are also described in the literature as the variation of the suction with varying effective degree of saturation (S_e). The effective degree of saturation (S_e) is related to the degree of saturation (S) and the volumetric water content (θ) by

$$S_e = \frac{(S - S_r)}{(1 - S_r)} = \frac{(\theta - \theta_r)}{(\theta_s - \theta_r)} = \frac{(\theta - \theta_r)}{\Delta\theta} \quad (3.5)$$

where θ_s is the volumetric moisture content at saturation, θ_r is the residual volumetric moisture content (which is the amount of water retained in the pore space even at high suction values), S_r is the residual saturation corresponding to the value of θ_r , S is saturation corresponding to the value of θ , and $\Delta\theta$ is the effective porosity ($=\theta_s-\theta_r$). The value of residual water content θ_r is easily obtained in the laboratory. From the practical standpoint, it seems sufficient to define S_r (or θ_r) as the value of the degree of saturation (or water content) obtained for a large value of the matric suction (Van Genuchten, 1980), e.g. at the permanent wilting point ($\psi = 15,000$ cm).

Typical curves for different soils are found in the literature (e.g. Brooks and Corey, 1964; Vachaud, et al., 1973; Wallace and Leonardi, 1976a; Dempsey, 1979; Van Genuchten, 1980; Znidarcic et al., 1991; Milly, 1987, Bumb et al., 1992). As both the values of S_e and its derivative are needed to solve the unsaturated flow equations, a mathematical representation of these curves are almost a necessity. Piecewise

representations of the water retention curve requires of large number of data at the location of major curvature (Freeze, 1971).

Significant computation effort can be saved and numerical oscillations avoided when solving the Richard's equation (Eq. 2.24), if continuous functional forms of the ψ - S_e curve are used (Freeze, 1971; Felton and Nieber, 1991) instead of a piecewise representation of this curve. A summary of such models are presented below.

3.2.3.1 Brooks and Corey model

Brooks and Corey (1964) proposed the following relationship between the degree of saturation and the matric suction based on substantial experimental data:

$$S_e = \left(\frac{\psi}{PB}\right)^{-\frac{1}{\nu}} \quad \text{for } \psi \geq PB \quad (3.6)$$

$$S_e = 1 \quad \text{for } \psi < PB$$

where S_e ¹ is the effective degree of saturation, PB the bubbling pressure of the soil and ν the pore size distribution index parameter. The parameter PB (bubbling pressure) is the height of the capillary fringe and the pore size distribution index parameter is a measure of the soil grain uniformity.

Using the definition of the water capacity, C_w (Eq. 3.26), and Eqs. (3.5) and (3.6), an expression for this parameter can be written:

$$C_w = \frac{\Delta\theta S_e}{\nu\psi} \quad (3.7)$$

A set of representative values of the parameters ν and η , for different soils, is presented by Wallace and Leonardi (1976a). Fig. 3.5 shows the influence of these parameters on the S_e - ψ response. An extensive discussion of their determination in the laboratory is presented by La Liberte et al. (1966).

¹ The effective degree of saturation should not be confused with the actual value of the saturation that it is evaluated considering oven dried soils.

Figure 3.5 Typical values for the Brooks and Corey model for water retention

3.2.3.2 Van Genuchten model

Van Genuchten (1980) has proposed the following empirical equation to relate the matric suction and the moisture content:

$$S_e = \frac{1}{(1 + (\alpha\psi)^\beta)^\gamma} \quad \left(\gamma = 1 - \frac{1}{\beta}\right), \quad (3.8)$$

$$S_e = 1 \quad \text{for } \psi < 0$$

where β and γ are dimensionless coefficients and α is a coefficient which has the dimension of the inverse of the piezometric head. A procedure to evaluate these parameters from experimental soil-water retention curves is given by Van Genuchten (1980). Eq. (3.8) reduces to Eq. (3.5) for large values of the matric suction (ψ), i.e. small values of the moisture content. For such limiting cases, the parameters used in the Brooks and Corey model can be related to those used in Van Genuchten's model (Van Genuchten, 1980). For instance, the parameter α is inversely proportional to the

bubbling pressure (PB). However, large deviations between the two models occur when the moisture content approaches saturation. Similarly, using Eqs. (2.26), (3.6), and (3.8), an expression of the water capacity can be obtained based on the Van Genuchten model as follows:

$$C_w = \alpha \Delta \theta (\beta - 1) (\alpha \psi)^{\beta-1} S_e^{1+1/\gamma} \quad (3.9)$$

Fig. (3.6) shows the variation of the matric suction vs. effective degree of saturation for a range of β values that may be found in practice.

Figure 3.6 Typical values for the Van Genuchten model for water retention

3.2.3.3 Brutsaert model

Using the pore size distribution and statistical arguments, another relationship between the degree of saturation and the matric suction was proposed by

Brutsaert (1966). It is a inverse power relationship with two empirical coefficients, α and β :

$$S_e = \frac{\alpha}{(\alpha + \psi^\beta)} \quad \text{for } \psi > 0 \quad (3.10)$$

$$S_e = 1 \quad \text{for } \psi \leq 0$$

The dimension of α depends on the suction units whereas β is dimensionless. A log-log transformation can be applied to the experimental soil-water retention data to evaluate these coefficients by fitting. The corresponding water capacity equation for the Brutsaert's model is given by:

$$C_w = \frac{\Delta\theta \alpha \beta \psi^{\beta-1}}{(\alpha + \psi^\beta)^2} = \frac{\Delta\theta \beta}{\alpha} S_e^2 \psi^{\beta-1} \quad \text{for } \psi > 0 \quad (3.11)$$

It is noted that the symbols α and β are used herein for the empirical coefficients although they do not have the meaning than for the Van Genuchten model.

3.2.3.4 Vauclin model

Another empirical equation relating the degree of saturation and the matric suction, similar to that proposed by Brutsaert, was later introduced by Vauclin et al. (1979a):

$$S_e = \frac{\alpha}{(\alpha + (\ln/\psi)^\beta)} \quad \text{for } \psi > 1\text{cm} \quad (3.12)$$

$$S_e = 1 \quad \text{for } \psi \leq 1\text{cm}$$

and the corresponding water capacity equation for this model is given by:

$$C_w = \frac{\Delta\theta \alpha \beta (\ln\psi)^{\beta-1}}{\psi (\alpha + (\ln\psi)^\beta)^2} = \frac{\Delta\theta \beta}{\alpha \psi} S_e^2 (\ln\psi)^{\beta-1} \quad \text{for } \psi \geq 1 \quad (3.13)$$

where α and β are fitting coefficients.

3.2.3.5 Other hydraulic models

In addition to the models described above, there are several other models have been proposed in the literature to represent the S_e - ψ curves. For instance, because of the ability of exponential functions to yield analytical solutions of the Richards equation, Bumb et al. (1992) proposed the following expression:

$$S_e = \exp \left[\frac{-(\psi - PB)}{\beta} \right] \quad \text{for } \psi \geq PB \quad (3.14)$$

where β has the dimension of suction. In general β tends to become smaller when the pore size distribution becomes more uniform (Bumb et. al, 1992). The PB coefficient represent the bubbling pressure.

A total of 113 data set representing 12 different soils were used by Bumb et al. (1992) and the proposed equation (Eq. 3.14) was found as good as the Brooks and Corey model (Eq. 3.6) to fit experimental data. As Eq. (3.14) is only valid for pressures greater than the bubbling pressure, a more general form of this equation was proposed (Bumb et al., 1992)

$$S_e = \frac{1}{\left[1 + \exp[(\psi - PB)/\beta] \right]} \quad (3.15)$$

Generalization of existing models have also been proposed in order to better fit laboratory data. For instance, Cooley (1983) used in his study a generalization of Brutsaert's model:

$$S_e = \frac{\alpha}{\left[\alpha + (\psi - PB)^\beta \right]} \quad \psi \geq PB \quad (3.16)$$

In general, power law functions such as the Brooks and Corey model (Eq. 3.6) can fit reasonably well to most of the experimental data, except, in a range close to saturation where large deviations may occur (Felton and Nieber, 1991). This problem has been overcome by combining the Brooks and Corey model with a parabolic function that represents the water retention curve in the range $0 \leq \psi \leq \psi_i$, where ψ_i is the

capillary pressure head at which the inflection in the retention curve occurs. At the inflection point, continuity of the curve and its first derivative must be provided by the parabola (Clapp and Hornberger, 1978).

3.2.3.6 Summary of the water retention models

El-Kadi (1985a) performed extensive tests in the laboratory to examine the capability of various forms of the water retention function to represent the measured data. A total of 448 sample data were used in the study representing the three main soil types (i.e. sand, silt and clay). The models used were those given by Brooks and Corey (Eq. 3.5), Van Genuchten (Eq. 3.8), Brutsaert (Eq. 3.10), and Vauclin (Eq. 3.12). Comparisons were made in terms of mean values and standard deviations of the error obtained using least-square regression techniques between measured and predicted values of suction. El-Kadi concluded that the four equations provide comparable quality of fit. However, the Brutsaert model was found the most sensitive to the values of suction close to saturation. In other words, the coefficients of Brutsaert model was largely dependant on the values obtained close to saturation. Conversely, Brooks and Corey model was the most insensitive to both frequency and measurement close to saturation. The Van Genuchten and Vauclin models better describe laboratory curves at small suction.

Bumb et al. (1992) also performed comparisons of various types of water retention relationships. Taking 10 samples for each of 12 soil texture of the USDA soil classification except for the silt for which only 3 samples were available, a total of 113 samples were tested. The results show that, on the average, the Brooks and Corey model fit the laboratory data better.

Although the choice of the analytical relationship depends on its ability to represent the experimental data, some relationships may be preferred because they make analytical solutions of the Richard's equation possible (Bumb et al., 1992). Although the computational effort is becoming less of a concern as computer technology evolves, this might still be a valid criteria for electing a particular model. Felton and Nieber (1991) have shown that different water retention curve models required significantly different computation effort to obtain equivalent solutions.

3.2.4 Determination of soil suction in the laboratory

Experimental water retention curves can be obtained by two procedures in the laboratory: a) by taking a initially saturated soil sample and increasingly drying the soil sample (desorption) and b) by gradually wetting an initially dry sample (imbibition). A number of tests are currently available for the determination of the soil suction; the most popular one consists of applying increasing air pressures to drive the water out of the soil sample and measuring the water content at equilibrium (Şeker, 1983). This test provides only the suction value within the sample; hence, the water content of the soil sample has to be measured indirectly. Measurement devices such as porous blocks, neutron probes, gamma rays are commonly used to measure electrical resistivity, dielectric constant, or thermal conductivity (Loiseau et al., 1990).

Indirect techniques can also be used to measure the matric suction. Balbour et al. (1992) describes the use of the thermal conductivity matric suction sensors for evaluating the soil matric suction. This device consists of a porous ceramic block containing a temperature sensing device and a miniature heater. The rate of heat is directly proportional to the water content of the block. Another technique that is also commonly used that described by Richards (1974). A soil sample is placed on top of the ceramic plate which is saturated, and the upper part of the sample is included in a vessel where the air pressured is controlled. Both the air and water pressure can be measured, and hence the suction. Variation in the moisture content is obtained by oven-drying the sample.

A technique called the filter paper method is also used for the determination of the soil suction. The technique consists of introducing a filter paper in the soil sample and, at equilibrium, the water content measured of the filter paper is measured and compared to a water retention curve of the filter paper (Fry, 1990). A more sophisticated technique is the psychrometer procedure which consists in measuring the moisture content around the capillary meniscus and in computing the suction using Kelvin's law. Richards (1974) presents a complete description of this laboratory test. Recently, Prapaharan et al. (1985) have shown that the mercury intrusion method provides a quick and accurate way to determine the water retention curves for clayey

soils. Tensiometer devices are also used to measure the water pressure especially on sandy soils. Other procedures such as the osmotic method, and the hygrometric method are described by Fry (1990).

3.2.5 Effects of hysteresis

It is well known that the relationship between the degree of saturation and the suction is not unique and depends on whether wetting or drainage is taking place. For a given degree of saturation, the value of suction during drainage is greater than during wetting (Fig. 3.7). Several models have been proposed to explain the hysteresis phenomenon. Bear and Verruijt (1990) attributes this effect to two phenomena: (a) the ink-bottle effect and (b) the raindrop effect. The first results from the capillary tube cross section variation resulting in a variation of the meniscus radius. The later is related to the variation of the contact angle, the angle on a wetting interface differs from a draining one (Fig. 3.8). It has been suggested that entrapped air also causes hysteresis (Mualen and Morel-Seytoux, 1980).

Figure 3.7 Hysteretical effects - Main drainage and wetting curves

The hysteretical attribute of the water retention curves indicates that the suction is not a unique function of the moisture content. Based on the thermodynamic principles and momentum balance equations Gray and Hassanizadeh (1991b) proposed a general expression for the capillarity. Thus, suction was defined as the net energy per unit volume of pore space that would be released per unit change in saturation. As a result, a unique suction curve is obtained as a function of water and air density, temperature, degree of saturation, and area of air-water interface per unit of porous medium. However, disregarding the interface area influence, the matric suction (p_c) is usually determined in function of the degree of saturation only; values of p_c vs S_e showing effects of hysteresis are obtained on this basis. From the microscopic viewpoint, hysteresis can be attributed to the different contact angle values that the air-water interface with solid may take on. This effect should be reflected at the macroscale in a parameter that measures the area of air-water interface per unit of porous medium (Gray and Hassanizadeh, 1991b).

Figure 3.8 Variation of the air-water interface due to raindrop effect (After Bear, 1972)

3.2.6 Mualem hysteretical model

Although the formulation for the capillary pressure presented by Gray and Hassanizadeh (1991b) is a way to avoid theoretically the hysteresis by properly accounting for the air-water interface, the parameters required to apply it are not easy

to obtain using current laboratory tests. Hence, the effect of hysteresis is accounted for in numerical analysis by using curves obtained during cycles of wetting and drying (e.g. Whisler and Klute, 1969; Giesel et al, 1973; Hoa et. al., 1977). Obtaining a curve containing several scanning curves is nevertheless prohibitively expensive and cannot be performed for every situation. Therefore, considerable effort made by researchers to develop semi-empirical relationships that take into account the phenomenon of hysteresis using the minimum data possible.

Mualen (1973) presented a model based on the independent domain theory to describe the scanning curves obtained from the main wetting and drying curves. The domain theory conceptualizes the porous medium as a system of pore domains with characteristic wetting and drying pore radii such that the pore volume can be expressed as:

$$\theta = \int \int f(\psi_w, \psi_d) d\psi_w d\psi_d \quad (3.17)$$

where ψ_w and ψ_d are independent variables representing the suction on the main wetting and drying curves, ψ_w being always greater than or equal to ψ_d , and f is a bivariate probability distribution function. The distribution function $f(\psi_w, \psi_d)$ is represented as the product of two independent marginal functions and the procedure to determine these functions is presented by Mualen (1973).

The resulting expression for the moisture content has the advantages of (a) representing the hysteretical behavior of the ψ - S_e from the main drying and wetting curves; (b) representing analytically the scanning curves, and (c) the scanning curves fall within the main loop. Mualen (1974) improved his 1973 model by including a reversible contribution of pore volume to the hysteresis mechanism (Fig. 3.9).

As the previous model was not accurate for soils with narrow pore size distributions, Mualen and Dagan (1975) introduced a dependent domain model which takes into account the air pore blockage against air entry during the drying process. This model requires in addition to the main wetting and drying curves, one wetting and drying

scanning curve. Mualen's 1984 model allows to overcome this difficulty, and provides results comparable to those of the 1975 model.

Figure 3.9 Mualen hysteretical model for the secondary wetting and drying curves

The dependent domain types of models are the most accurate in predicting the scanning curves for different media, although, since they require the history of wetting and drying, these models may not be the easiest to use for practical numerical applications (Kool and Parker, 1987; Izadi and Klute, 1990). Recently, it has been proposed a new treatment of the hysteretical effects in which the secondary wetting and drying curves are scaled from the main wetting and drying curves (Scott et al., 1983; Curtis and Watson, 1984). Although these models do not represent the hysteretical behavior of the scanning curves (Fig. 3.10) as accurately as the Mualen models (1973, 1984), the overall accuracy of the infiltration profiles using the simplified models were similar to that of the more complex Mualen's models.

Using the drying and wetting curves (Fig. 3.7) and the equations (e.g. Brooks and Corey model, Eq. 3.6) with their corresponding parameters, the secondary wetting curves are defined by the parameters of the main wetting curve but with the value of the residual

moisture content θ_r replaced by a value θ_r^* . Similarly, the scanning drying curves are defined by the parameters of the main drying curve but with the value of θ_r replaced by θ_r^* . The modified values of residual and saturated volumetric moisture content can be estimated by scaling the wetting (or drying) curve shifted to the reversal point (θ, ψ) and by using the definition of the effective degree of saturation (Eq. 3.5). This transformation leads to

$$\theta_r^* = \frac{(\theta - \theta_r S_e^w)}{(1 - S_e^w)} \quad (3.18)$$

$$\theta_s^* = \frac{(\theta - \theta_r(1 - S_e^d))}{S_e^d} \quad (3.19)$$

where S_e^d and S_e^w are the effective degree of saturation on the wetting and drainage curves respectively, evaluated at the reversal point (θ, ψ) . Consequently, the model is such that all secondary wetting scanning curves pass through the point of maximum volumetric moisture content (θ_s) and all secondary scanning drying curves pass through the point of minimum volumetric moisture content (θ_r). As a result, a closed hysteretical loop is not obtained (Fig. 3.10).

Figure 3.10 Scott Hysteretical model for the secondary wetting and drying curves

3.3 Soil hydraulic conductivity

The property of the soil which measures the facility with which water is able to travel through the pores is called permeability. This is one of the most important parameters in civil engineering. The permeability was defined by Henry Darcy in 1856, in Dijon, Burgundy, when he established that the flux of water through a sample of sandy soil (Fig. 3.11) can be calculated by means of the following empirical relationship:

$$Q = KA \frac{\Delta\phi}{L} = KAi \quad (3.21)$$

where L and A are the thickness and cross section area of the soil sample respectively, K a constant that depends on the porous medium, temperature and viscosity of the fluid, $\Delta\phi$ is the difference in hydraulic head between the top and bottom of the soil sample, and i is the hydraulic gradient.

Figure 3.11 Schematic representation of Darcy's experiments (de Marsily, 1986)

Darcy's experiment was in one direction. Because soil deposits are not in general isotropic media, it is reasonable to expect that their permeability is not the same in every

direction. Thus, it is also reasonable to consider the permeability has a tensorial property. The second order permeability (K) tensor is defined as:

$$K = \frac{1}{\mu} \begin{bmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{yx} & k_{yy} & k_{yz} \\ k_{zx} & k_{zy} & k_{zz} \end{bmatrix} \quad (3.22)$$

A number of authors have tried to formally demonstrate the tensorial nature of the permeability but with little success so far (de Marsily, 1986).

3.3.1 Theoretical derivation of soil permeability

Using an analogy with flow through circular tubes, attempts have been made to relate the permeability to the porosity of the pores. In most cases, the flow through a soil formation can be assumed to be laminar.

3.3.1.1 Saturated soils

Based on Poiseuille's law for a flow cylindrical capillary tubes, the discharge can be computed as:

$$Q = \frac{\gamma_w R^2}{8\mu} i A_v \quad (3.23)$$

where A_v is the area of the cylindrical tube (or the cross section area of the voids through a soil sample), R the radius of the tube, and μ the fluid viscosity. It can also be shown that a flow in a tube of irregular shape can be formulated in a similar manner as Eq. (3.23) using the hydraulic radius (R_H) instead of R , the effective hydraulic gradient i_e instead of i , and the coefficient C_s instead of $1/8$, yielding

$$Q = \frac{C_s \gamma_w R_H^2}{\mu} i_e A_v \quad (3.24)$$

For a soil sample the area A_v can be written as:

$$A_v = nA = \frac{e}{(1+e)}A \quad (3.25)$$

The hydraulic radius can be expressed as

$$R_H = \frac{A_v}{P_s} = \frac{A_v L}{P_s L} = \frac{V_v}{A_s} = e \frac{V_s}{A_s} \quad (3.26)$$

where A_v and V_v are the area and volume of the voids; L is the length of the sample; P_s , A_s , V_s are the perimeter, area and volume of the solids respectively and e is the void ratio. Assuming the soil particles to be spherical, the hydraulic radius becomes:

$$R_H = \frac{D_s}{6} \quad (3.27)$$

Substituting Eqs. (3.26) and (3.27) in (3.24) and expressing the effective hydraulic gradient as $i_e = i/T$, where T is the tortuosity coefficient, gives:

$$Q = D_s^2 \frac{C}{T} \frac{\gamma_w}{\mu} \frac{e^3}{(1+e)} iA \quad (3.28)$$

comparing Eq. (3.28) with Eq. (3.23), it follows

$$K = D_s^2 \frac{C}{T} \frac{\gamma_w}{\mu} \frac{e^3}{(1+e)} \quad (3.29)$$

Equation (3.29), known as the Kozeny-Karma equation, shows that the soil permeability depends on the soil structure soil (D_s , e , C , T) and fluid characteristics (μ , γ_w). This equation only provides an insight of the factors affecting the permeability. It can be seen that keeping all other parameters constant, the permeability depends on the second power of a representative grain diameter. Laboratory tests have shown that the above expression is only applicable to sandy soils. The thin layer of water attached to clay particles is the probable cause of its inability to describe the permeability of fined grained soils (Taylor, 1948). Following a similar idea, Hazen (de Marsily, 1986) expressed the permeability for sands in terms of the effective sand diameter (D_{10}) as

$$k = 100 D_{10}^2 \quad (3.30)$$

where cm and seconds are used as units for all terms. Eq. (3.30) was experimentally verified for sands with effective sizes between 0.1 and 3 mm.

3.3.1.2 Unsaturated soils

It is known that the total permeability is reduced due to the presence of air in the porous medium. In unsaturated soils, it is common to use the concept of relative permeability which is the ratio between the unsaturated permeability and the saturated permeability

$$k_r(S_e) = \frac{K(S_e)}{K} \quad (3.31)$$

where k_r is the relative permeability ranging from 0 to 1 and K is the saturated permeability. Different models have been proposed to represent the relative permeability as a function of the effective degree of saturation. Most of these models assume the porous medium is equivalent to a set of tubes connected randomly in parallel-series, each tube having a different permeability. By this mean, the random variations of the pores sizes normal to and along the direction of flow can be accounted for (Brutsaert, 1968).

Using the series-parallel model Burdine (1953) related the permeability to the water retention curves. He assumed that the discharge of two pores connected in series was controlled by the narrower pore, and that during desaturation the pores are emptied one after the other starting with the wider ones. Using the definition of the relative permeability (Eq. (3.31)), he obtained

$$k_r(S_e) = \frac{S_e^2 \int_0^{S_e} \frac{dS_e}{\psi^2(S_e)}}{\int_0^{1.0} \frac{dS_e}{\psi^2(S_e)}} \quad (3.32)$$

where $S_e(r) = dS_e/dr$ is the effective pore size density function. Based on the same principle of the series-parallel model, Brutsaert (1968) derived a double integral expression for the relative permeability:

$$k_r(S_e) = \int_0^r S_e(x) \int_0^x S_e(y) y^2 dy dx + \int_0^r S_e(x) x^2 \int_x^r S_e(y) dy dx \quad (3.33)$$

where r is the pore size that can be defined as the hydraulic radius (Eq. 3.26). More recently, Mualen (1976) derived an analytical expression for the permeability which essentially is similar to that proposed by Burdine (1953) but is based on a different assumption regarding the hydraulic conductivity of pores connected in series in order to take into account the larger diameter pores. According to this model the hydraulic conductivity is not controlled by the radius of the narrower element.

$$K_r(S_e) = S_e^n \left[\frac{\int_0^{S_e} \frac{dS_e}{\psi(S_e)}}{\int_0^{1.0} \frac{dS_e}{\psi(S_e)}} \right]^2 \quad (3.34)$$

Mualen (1976) also showed that for a value of the coefficient n equal to 0.5, Eq. (3.34) predicts reasonably well most hydraulic conductivity curves obtained in the laboratory.

3.3.2 Determination of soil permeability in the laboratory

A number of different testing procedures to determine the hydraulic conductivity of soils have been to date. The choice of an appropriate testing procedure depends in particular upon the magnitude of the soil hydraulic conductivity to be determined.

3.3.2.1 Saturated permeability

The falling and constant head permeameter tests, where the difference in hydraulic head are generated only by gravity, are commonly used to determine the permeability of sandy soils. For lower hydraulic conductivities, larger pressure differences need to be applied using pumps. Either flexible and rigid wall can be used in permeameters. More recently, Olsen (1988) has described a testing apparatus (flow pump) for a rapid evaluation of low hydraulic conductivities at substantially smaller gradients than in conventional permeameters. The consolidation test can also be used to determine the hydraulic conductivity of soils of low permeability (Richards, 1974).

3.3.2.2 Unsaturated permeability

A number of different techniques are available for the determination of the unsaturated permeability. Most of these procedures consist of determining the hydraulic permeability indirectly by using the water retention curve (Şeker, 1983) or by applying a controlled flux at the upper end of the soil sample and measuring locally the head gradients (de Marsily, 1986). Among the most common techniques are:

- (a) Transient flow in soil columns: the solution of Eq. (2.30a) for a known flow in a column of unsaturated soil is used to determine the permeability function. The suction gradients are measured using tensiometers (e.g. Perroux et al., 1981).
- (b) The pressure plate outflow technique: this technique is quite similar to the consolidation technique. The outflow from a pressure plate is measured when a suction gradient is created by increasing the air pressure. The advantages and disadvantages of this technique are discussed by Richards (1974).
- (c) The osmotic permeameter: an odometer cell is adapted by setting two osmotic independent circuits such that the two phases (i.e. air and water) can be measured (Fry, 1990).
- (d) Permeameter apparatus: consists of applying different air and water pressures through a semi-impervious membrane and then measuring the permeability at equilibrium (El Tani, 1991).

3.3.3 Hydraulic conductivity curves

Semi-empirical expressions for the hydraulic conductivity can be obtained by using the water retention expressed in the equations of section 3.3.2.2.

3.3.3.1 Brutsaert model

Using various probability distributions for the pore size (e.g. the incomplete gamma function, the beta function) and the equation derived based on the series-parallel model (Eq. 3.33), Brutsaert (1968) derived several expressions for the permeability as a function of the degree of saturation. He showed that Eq. (3.33) used to represent the relative permeability as a function of S_e practically was insensitive to the choice of the

probability functions. The complex analytical expressions derived by Brutsaert (1968) can be cumbersome to use in practical applications. Fortunately, for all cases the analyzed therein, the permeability functions derived using Eq. (3.33) can be approximated by the Averjanov-Irmay power function:

$$K = K_s S_e^\eta \quad (3.35)$$

where η is a parameter that depends only on the pore size distribution and can be obtained by fitting to test data, and K_s is the saturated permeability.

3.3.3.2 Brooks and Corey model

Using Eq. (3.6) along with the relative permeability equation proposed by Burdine (Eq. 3.32), Brooks and Corey (1964) arrived at an empirical equation similar to Eq. (3.35). They related the hydraulic conductivity parameter (η) to the pore size distribution parameter (ν) by the expression $\eta = 3 + 2\nu$. In practice, the determination in the laboratory of the hydraulic parameter (η) is more reliable than that of ν (La Liberte et al., 1966).

3.3.3.3 Van Genuchten models

Van Genuchten-Mualen model: Based on Mualen's model (Eq. 3.34) and Eq. (3.8) for the water retention curve, a relationship between the permeability and the volumetric moisture content was also derived by Van Genuchten (1980):

$$K(\psi) = K_s S_e^{1/2} (1 - (1 - S_e^{1/\gamma})^\gamma)^2 \quad (\gamma = 1 - 1/\beta) \quad (3.36)$$

Van Genuchten-Burdine model: Using Burdine's theory (Eq. 3.32) and Eq. (3.8) for the water retention curve, a similar expression for the relative permeability was also derived by Genuchten (1980):

$$K(\psi) = K_s S_e^2 (1 - (1 - S_e^{1/\gamma})^\gamma) \quad (\gamma = 1 - 2/\beta) \quad (3.37)$$

Based upon an extensive comparative study, Mualen (1976) showed that the Burdine-based expressions were in lesser agreement with experimental data than the Mualen-based expressions. Therefore, Eq. (3.36) was recommended to describe the variation of permeability with the effective degree of saturation.

3.3.3.4 Gardner model

In contrast with the models described above, Gardner (1956) represented the relative permeability directly as a smooth function of suction instead of degree of saturation. An empirical relationship was proposed,

$$K(\psi) = \frac{K_s A}{A + \psi^\beta} \quad \text{for } \psi \geq 0 \quad (3.38)$$

where β is a positive dimensionless coefficient and A is a coefficient with its dimension depending on the units of suction, permeability and the value of β . This expression can provide good approximations of the actual relationship between the permeability and suction (La Liberte et al., 1966). This model is often used when hydraulic conductivity curves are available in the laboratory.

A quasi-linear exponential model that is simple to use for deriving close form solutions (Warrick, 1991) and is also a reasonable model for fitting experimental data was also proposed by Gardner (1958)

$$K(\psi) = K_s \exp(\alpha \psi) \quad (3.39)$$

where α is a curve fitting coefficient that depends on the sample pore size distribution and has the dimension $[\text{length}]^{-1}$. Typical values of α range from 0.05 to 0.002 cm^{-1} for most practical cases. The parameter α is a measure of the relative importance of gravity and capillarity in the water movement, smaller values of α corresponds to fine-textured soils and, conversely, larger values of α are for coarse-textured soils.

Owing to its simplicity, the above expression (Eq. 3.39) has been used extensively in theoretical and numerical modelling (e.g. Philip, 1969, Raats, 1976; Warrick, 1991). More recently, stochastic modeling using Eq. (3.39) has also been attempted (White and Sully, 1992).

3.3.3.5 El-Kadi model

Another relationship between permeability and suction has been proposed by El-Kadi (1985b).

$$K(\psi) = K_s \psi^\alpha \quad (3.40)$$

where α is a dimensionless negative parameter related to the soil type. This expression can provide a fairly good approximation of the hydraulic conductivity curve whenever insufficient experimental data is available (El-Kadi, 1985b).

3.3.3.6 Summary of the permeability relationships

A study performed by El-Kadi (1985b) shows that, overall, the Brooks and corey formulation provides the best modeling for sands and silts whereas the Van Genuchten formulation is more satisfying for clay soils. Useful predictions of the permeability as function of the degree of saturation can be obtained using the Averjanov-Irmay function (Eq. 3.35). If laboratory test results are available, the choice of one of the models above will depend only upon its capability to fit the data. If only the experimental water retention curve is available, the permeability function will have to be chosen consistently to the model chosen to represent the water retention curve.

3.3.4 Hysteretical effects

Similarly to the soil suction, the permeability curve also depends of the direction of flow. Fortunately, the relative permeability changes only marginally with the degree of effective saturation (Mualen, 1976; Philip, 1969). Using a model made of a cubic network of cylindrical capillary tubes, Wise (1992) has shown that variations of the permeability due to variations of the contact angle of the air-water interface is bounded within one order of magnitude. Thus, for most cases, the hysteretical behavior of the permeability curves can be neglected. If necessary, the hysteretical effects could be taken into account by expressing the permeability as a function of suction.

CHAPTER 4 - NUMERICAL MODELING

4.1 Introduction

As mentioned earlier, a closed form solution of Richard's equation, taking into account various boundaries conditions and complex geometries, is at present impossible to obtain. Hence, a numerical solution has to be developed. Different numerical techniques are available to date, e.g. finite difference method (e.g. Carnahan et al., 1969), finite element method (e.g. Reddy, 1984), the integrated finite difference method (Narasimhan and Witherspoon, 1977), the boundary element method (Huyakorn and Pinder, 1983). The selection of the numerical technique to be used depends on the class of application to be addressed.

Once the governing differential equation is available, a set of constitutive parameters chosen, and the initial and boundary conditions identified, a numerical scheme has to be formulated in order to solve the problem. The solution of the partial differential equation (PDE) can then be obtained at a finite number of points called nodes.

Due to the highly nonlinear constitutive relationships of the Richard's equation, obtaining reliable numerical solutions is difficult in presence of analyzing complex boundary conditions and material heterogeneity. Discretization forms of nonlinear differential equations generally lead to nonlinear algebraic equations, which in turn, requires iterative solutions.

The number of numerical schemes proposed in the literature for unsaturated flow problems reveals the importance as well as the difficulty of solving topic for soil physicist and agricultural engineers, and more recently for civil and environmental engineers. Pioneering work towards the numerical solution of Richards equation using

finite difference approximation was apparently made in the mid 1960's, mostly for one dimensional vertical infiltration (e.g. Watson, 1967; Rubin, 1969; Whisler and Klute, 1969).

These works suggested the methodology had for predicting unsaturated flow response encouraged the development more sophisticated numerical schemes. Consequently, various finite differences models based on pressure heads (e.g. Freeze, 1971; Haverkamp and Vauclin, 1981; Vauclin et al., 1979a and 1979b), piezometric heads (e.g Wallace, 1977), and volumetric moisture content formulations (e.g. Dempsey and Elzeftawy, 1977) were proposed. Narasimhan and Witherspoon (1976, 1977) presented a model based on the integrated finite difference scheme. Neuman (1973) proposed a finite element method to analyze water flow through dams taking into consideration the unsaturated zone. Other finite element models formulated in terms of pressures heads (e.g. Cooley, 1983; Celia et al., 1988; Van Genuchten, 1982; Winter, 1987) and piezometric heads (Christe and Gaillard, 1989; Dysli and Rybisar, 1987; Lam et al., 1987; Yeh, 1987a and 1987b) have been described in the literature. A moving finite element mesh formulation has also been proposed (Thomson et al., 1984) to model the water movement in a system of a Lagrangian coordinates in which the stream lines are used as main variable. This procedure presents major technical difficulties but can model effectively the propagation of abrupt wetting fronts.

Although the Richard's equation (Eq. 2.24), or any of its modified forms, has been the basis for the development of most numerical models, they differ by the numerical technique used, the number of space dimension considered, the treatment of the initial and boundary conditions, and the algorithm utilized for solving the derived system of equations. The finite element and finite difference techniques are the most popular for the solution of unsaturated problems. Because of its apparent simplicity, a finite difference approach is often more appealing. However, in cases involving irregular geometries, a finite element approach may be the most efficient—or even the only—way. In the present chapter, a description of both techniques will be presented as well as the theory involved in the solution of linear and non-linear equations.

4.2 Solution of linear and non-linear systems of equations

Many boundary value problems can be expressed in matrix form via finite difference or finite element schemes (Dalquist and Bjorck, 1974). The discretization of a particular PDE leads to a system of equations that can be linear or non-linear. The solution of differential equations using numerical techniques requires the use of the theory of linear algebra and its application to solving systems of linear equations. A popular procedure, suitable for computer programming, consists of writing the coefficients of the system of equations in a matrix format. In addition to its numerical reliability and matrix characteristics, the choice of a particular procedure to solve the system of equations depends on computational factors such as the speed of the hardware process and the memory storage capacity of the computer.

4.2.1 Linear system of equations

A system of equations is said to be linear if solution can be achieved after a fixed number of operations, and non-linear otherwise. The system of equations obtained from the spatial discretization of Laplace's equation (Eq. 2.29) is an example of a linear system of equations:

$$[A]\{\phi\} = \{f\} \quad (4.1)$$

where $[A]$ is known as the 'stiffness' matrix, $\{\phi\}$ is the vector representing the unknown variable, and $\{f\}$ is a vector with elements representing the boundary conditions or equal to zero.

Differential equations involving time, known as initial value problems, with material parameters that are constants throughout the analysis (e.g. Poisson's equation) can also be written in matrix form and solved using the techniques of linear algebra. In this case the main variable (i.e. the piezometric head in this case) also varies with time. The system is written:

$$[C] \frac{\{\phi^{t+\Delta t}\} - \{\phi^t\}}{\Delta t} = [A]\{\phi\} - \{f\} \quad (4.2)$$

where the vectors $\{\phi^i\}$ and $\{\phi^{i+\Delta t}\}$ represent the piezometric heads at times t and $t+\Delta t$ respectively, Δt is the time increment, and $[C]$ is a diagonal matrix usually referred to as the mass matrix.

Once the geometric discretization is performed and the corresponding matrix-form expression of the PDE is obtained (e.g. Eq. 4.2), one has to decide on what values of the piezometric head should be written on the right hand side of the equation. This has been solved by using a linear combination of $\{\phi\}$ the values obtained at the present and subsequent time steps, as follows:

$$\{\phi\} = \mu \{\phi^{i+\Delta t}\} + (1-\mu) \{\phi^i\} \quad (4.3)$$

where μ is a dimensionless weighing factor that ranges from 0 to 1. This approximation leads to the following matrix equation for the Poisson's equation with material parameters that are only function of the geometry (i.e. $[A]$ and $[C]$ are independent of time):

$$([C] - \mu \Delta t [A]) \{\phi^{i+\Delta t}\} = ([C] + (1-\mu) \Delta t [A]) \{\phi^i\} + \Delta t \{f\} \quad (4.4)$$

The right hand side in Eq. (4.4) is known at a given time step (t) and the corresponding unknowns, $\{\phi^{i+\Delta t}\}$, at the subsequent time step ($t+\Delta t$) can be obtained by solving the system of equations. The three values of μ usually employed in the solution of initial-value problems (i.e. transient problems) are:

(a) fully explicit ($\mu=0$);

$$[C] \{\phi^{i+\Delta t}\} = ([C] + \Delta t [A]) \{\phi^i\} + \Delta t \{f\} \quad (4.5)$$

(b) fully implicit ($\mu=1$);

$$([C] - \Delta t [A]) \{\phi^{i+\Delta t}\} = ([C]) \{\phi^i\} + \Delta t \{f\} \quad (4.6)$$

(c) Crank-Nicolson scheme ($\mu=0.5$);

$$([C] - \frac{\Delta t}{2} [A]) \{\phi^{i+\Delta t}\} = ([C] + \frac{\Delta t}{2} [A]) \{\phi^i\} + \Delta t \{f\} \quad (4.7)$$

Using $\mu=0$, at first glance, seems to be attractive because it reduces the number of matrix operations to a minimum since $[C]$ is a diagonal matrix. However, it can be shown that the errors made using the explicit formulation remain bounded within a finite range only under certain particular conditions (Huyakorn and Pinder, 1983). The values of the time step required to make the system stable are quite small and, therefore, the number of times that Eq. (4.5) has to be solved may be quite large. Values of μ other than zero will lead to a sparse matrix on the left hand side of the system. Thus, the solution of Eq. (4.4) at every time step will take more computational effort than for the case of μ equal to zero. However, for values of μ greater than 0.5, the system of equations represented by Eq. (4.3) is unconditionally stable, i.e. no matter how large the time steps are taken, no stability problem will occur (Huyakorn and Pinder, 1983). This interesting characteristic allows to increase the time step, reducing, as a consequence, the number of times that Eq. (4.4) needs to be solved. Hence, the implicit (Eq. 4.6) and Crank-Nicolson (Eq. 4.7) approach are preferred. However, oscillations around the exact solution may occur as the time step becomes larger (Zienckiewicz, 1978).

4.2.2 Non-linear system of equations

In Eq. (4.4), the material parameters represented by the matrices $[A]$ and $[C]$ are constants. A PDE with parameters that are dependent on the solution of the problem itself leads to a nonlinear system of equations (e.g. Richards' equation) taking the form:

$$([C(\phi)] - \mu[A(\phi)])\{\phi^{t+\Delta t}\} = ([C(\phi)] + (1-\mu)[A(\phi)])\{\phi^t\} + \{f\} \quad (4.8)$$

In Eq. 4.8 the matrices $[A(\phi)]$ and $[C(\phi)]$ are no longer constant but functions of ϕ which varies with time. As the values of $\{\phi\}$ are not known at the subsequent time step, the matrices $[A]$ and $[C]$ are also unknown; hence, an iterative procedure is necessary to solve the problem. The system of equations is solved by starting with initial values of the matrices of coefficients which are then updated every time the equation is solved. This iterative process is repeated until the solution converges to a fixed value within a

tolerable margin of error. Then, the time is incremented and the procedure outlined above is repeated.

As in the case of linear system of equations, the explicit, implicit and Crank-Nicolson schemes can be used to solve Eq. (4.8). It is noted from Eq. (4.8) that the iterative procedure described above can be avoided if time steps small enough are used so that the explicit scheme ($\mu = 0$) is appropriate. However, to avoid numerical instabilities, the appropriate time step might be so small that the number of steps required makes the procedure unpracticable from the computational standpoint.

4.2.3 Direct methods

Linear and nonlinear systems of equations can be solved using direct, iterative or a combination of both methods. Direct methods, for linear systems of equations, involve a finite number of operations after which, disregarding the round-off errors, an exact solution is achieved. The most popular direct method for solving a general linear system of equations is the Gauss elimination algorithm which is commonly used for general matrices, i.e. matrices without any special characteristic (Dalquist and Bjork, 1974).

For sparse matrices (i.e., containing a large amount of zeros), the Gaussian elimination by the wave front technique is an elegant and efficient way of using the sparseness characteristic of the matrix. Another direct method to solve linear systems of equations often used is the LDU decomposition (e.g. Dalquist and Bjork, 1974). This method consist of writing any matrix as the product of three matrices ($LxDxU$) in which L is a lower-diagonal matrix, U is an upper-diagonal matrix and D is a diagonal matrix. An special case of this method is the Cholesky decomposition (e.g. Dalquist and Bjork, 1974) which is usually applied to symmetric matrices.

The advantages of direct method are: (1) the efficiency in the computation for matrices with small bandwidth and (2) "exact" solution are obtained for linear equations and (3) only limited convergency problems are encountered with nonlinear equations. The major disadvantage of direct methods is their important requirements on of memory storage and computation time with large matrices.

In summary, two types of systems of equations (linear and non-linear) can be solved using direct methods. When solving a system of linear equations using direct methods, an exact solution of the system of equations obtained after a fixed number of operations. For nonlinear system of equations, the system is solved by recursive schemes in which direct matrix solution techniques are used at each iteration and the result is considered satisfactory when a specified error criteria has been met. Thus, when solving a system of nonlinear equations, only approximate solutions (in addition to the rounding off errors) are obtained.

4.2.4 Iterative methods

So far, the term iteration has only been used in the context of solving a system of nonlinear equations. However, iterative methods may also be used to solve a system of linear equations. Solving a linear system of equations using iterative methods takes, in general, more computation time than the direct ones. When the number of equations is large, direct procedures such as Gauss elimination would require using a large amount of memory storage. In such cases indirect procedures may be preferred. As mentioned earlier, partial differential equations can be written in matrix form. The system of equations obtained using a finite difference or finite element approach usually leads to a large sparse linear system where the use of indirect procedures is particularly attractive. The larger is the sparse matrix the greater become the advantages of indirect procedures. It can be shown (Rice, 1981) that for matrices developed from partial differential equations, the number of operations is approximately n^2 with a direct method (n is the order of the matrix) whereas the number of operations for iterative methods is $6ni$ (i is the number of iterations). Thus the number of iterations (i) has to be less than $n/6$ for an iterative method to be considered advantageous over a Gauss elimination in order to solve a system of linear equations. Nevertheless, the main consideration might be the memory space required which is pn (p is the non-zero elements per row) for the iterative procedures instead of n^2 for a Gauss elimination procedure.

The basic idea behind iterative procedures for solving system of linear equations is to decompose the original matrix $[A]$ as follows:

$$[A] = [M] - [N] \quad (4.9)$$

this allows a linear equation of the form of Eq. (4.1) to be written in a recursive form as:

$$[M]\{\phi^{m+1}\} = [N]\{\phi^m\} + \{f\} \quad (4.10)$$

where the superscript m is used to denote the iteration number. Convergency of the recursive solution of equations (Eq. 4.10) is not guaranteed for any decomposition of the matrix $[A]$. The matrices $[M]$ and $[N]$ must have some special characteristics to lead to convergency (Rice, 1986). In order to ensure convergency the maximum eigenvalue of the matrix product $M^{-1}N$ has to be smaller than unity, thus

$$\lambda_{\max} = \text{Max}(\det(M^{-1}N - \lambda I) = 0) \leq 1 \quad (4.11)$$

where I is the identity matrix, and λ is the associated eigenvalue. The choices of the decomposition matrices influences the rate of convergency (if convergency occurs). The smaller the value of $\lambda_{\max}$ is, the faster the rate of convergency will be. An interesting study of algorithm instabilities for the Richard's equation based on determination of the eigenvalues has been presented by Diment and Watson (1983). Unfortunately, determination of eigenvalues is not always straight forward and involves itself time consuming computations. Thus, this criterion is not routinely used for the assessment of convergency of algorithms.

Since linear systems are usually solved using backward or forward substitution, a natural choice for decomposing the matrix $[A]$ is:

$$[A] = [D] - [L] - [U] \quad (4.12)$$

where $[D]$, $[-U]$, $[-L]$ are, respectively, the diagonal, upper-diagonal and lower-diagonal submatrices of $[A]$. Thus, a linear equation such as Eq. (4.1) can be written in a

recursive manner using the following decomposition $[M] = [D]$ and $[N] = [L] + [U]$, which is known as the Jacobi decomposition

$$[D]\{\phi^{m+1}\} = ([L] + [U])\{\phi^m\} + \{f\} \quad (4.13)$$

Another choice for decomposing the matrix $[A]$ is $[M] = [D] - [L]$ and $[N] = [U]$, known as the Gauss-Seidel decomposition, yielding

$$([D] - [L])\{\phi^{m+1}\} = [U]\{\phi^m\} + \{f\} \quad (4.14)$$

Eqs. (4.13) and (4.14) are related to the well known Jacobi and Gauss-Seidel iterative methods respectively. In the Gauss-Seidel method, the newly-computed components of the $\{\phi\}$ vector are used in the right hand side as soon they are obtained whereas in the Jacobi method the new computed values are not used until all components have been found. Consequently, the Gauss-Seidel iteration scheme converges faster than the Jacobi method and therefore is often preferred.

In some cases the rate of convergency can be improved by the use of weighing factors that are applied to the decomposed matrices $[M]$ and $[N]$:

$$\frac{1}{w}([D] - r[L])\{\phi^{m+1}\} = \frac{1}{w}((1-w)[D] + (w-r)[L] + w[U])\{\phi^m\} + \{f\} \quad (4.15)$$

where w, r are weighing factors within a range of 0 to 2. Different combinations of these factors reduces Eq. (4.15) to well known iterative methods for solving linear equations, these are listed in Table 4.1. The rate of convergency depends upon these weighing factors. Optimal values for these factors can be chosen exactly only in very simple cases. For general matrices, these optimal values are usually chosen after trial and error. This choice is made even more complicated when one has to analyze non-linear systems of equations where the matrix coefficients are not constant but functions of the unknown variables.

A nonlinear system that has been linearized by writing it in matrix form such as Eq. 4.8 can be solved using the iterative scheme shown in Eq. (4.10). In this case, the recursive scheme will handle both, the nonlinearity of the matrix $[A]$ and its decomposition in terms of $[M]$ and $[N]$. In summary, iterative methods can be used to solve both linear and non-linear systems of equations. In both cases only approximate solutions are obtained. For a general linear system of equations where the matrix $[A]$ is full the solution using direct method is faster than using any iterative method. There is a trade-off when using indirect method: memory space saving versus rapidity of computation.

Table 4.1 - Weighing Factors for iterative procedures

Method	w	r
Jacobi	1	0
Gauss-Seidel	1	1
Successive Overrelaxation (SOR)	$a^\dagger$	a
Extrapolated Jacobi	0-2	0
Extrapolated Gauss-Seidel	0-2	1
Extrapolated SOR	0-2	0-2

† a can take any value from 0 to 2

4.3 Finite difference method

The method consist of discretizing the space and time domain by a finite grid (Fig. 4.1). Approximations of the partial derivatives by algebraic expressions at a selected number of points called nodes are made using Taylor's series expansion. Thus, a system of finite difference equations that hold at discrete points (equivalent to the PDE) is obtained. As a result, a system of algebraic equations which can be written in matrix form is obtained in which the values at the selected points are the unknowns.

Figure 4.1 Geometric discretization - Typical finite difference mesh

4.3.1 Consistency, stability and convergency

Although a problem may be correctly formulated in the mathematical sense, the computer implementation of the numerical solution problem may not always lead to the expected results. In any computer programming language, real numbers are represented by a set of finite digits with a limited precision, thus round-off errors and machine precision may sometimes play a role in the computed result. In addition, as mentioned earlier, not all systems of equations written in matrix form can be solved by iterative methods. When using an iterative method for solving a system of equations, the matrix obtained by discretization of a partial differential equation must have certain properties to ensure convergency. Therefore, it is important to define concepts of consistency, convergency and stability in relationship to machine precision. Consistency and stability are necessary conditions to ensure convergency of a solution algorithm.

Consistency means that the discretized expressions should reduce to the exact continuum expression as all finite intervals tend to zero. Convergency implies that the exact continuum solution of the partial differential equation written in discrete form should be achieved as all finite intervals (in time and distance) approach zero. This property is more difficult to verify strictly speaking, this can only be done if closed form solutions are available. The stability property, a necessary condition for convergency,

ensures that errors remain bounded and the solution itself should remain bounded as the time increment tends to zero.

A numerical process which is consistent, convergent, and stable does not necessarily provide a difference between the exact solution and the finite difference solution which is small. In fact, the difference between the exact and numerical solutions could be quite large. This latter characteristic is related to the accuracy of the algorithm. For instance, using the Taylor series expansion, the second derivative of a function $F(x)$, for a uniform grid (h), is given by

$$F''(x) = \frac{F(x+h) - 2F(x) + F(x-h)}{h^2} + \frac{h^2}{12} F''''(x) + \dots \quad (4.16)$$

Numerical approximation of the second derivative limited to the first term on the right hand side involving an error which is proportional to the second power of the mesh size and the fourth-order derivative of the function $F(x)$. The accuracy can be of course improved by choosing smaller grid sizes. It is important to remark that for a non uniform grid, the third derivative term would appear in equation (4.16) and hence the truncation error would be proportional to the mesh size itself, thus a gain in geometric flexibility is counter-balanced by a lost in accuracy.

4.3.2 Different forms of the Richards equation

Different forms of the Richards equation are presented in section 2.2.5. Discontinuities in volumetric water content may be expected at the soil interface in heterogenous porous media, thus the Richards equation is easier to handle in terms of piezometric or pressure heads which are continuous in the space domain. Because piezometric heads more relevant to civil engineering applications, in what follows, the formulation used herein will be based on this variable given by (Eq. 2.27). In two dimension, it reduces to:

$$C_w \frac{\partial \phi}{\partial t} = \frac{\partial}{\partial x} \left[K_x(\psi) \frac{\partial \phi}{\partial x} \right] + \frac{\partial}{\partial z} \left[k_z(\psi) \frac{\partial \phi}{\partial z} \right] \quad (4.17a)$$

The above equation can be decomposed using the chain rule, yielding:

$$C_w \frac{\partial \phi}{\partial t} = \frac{\partial K_x}{\partial x} \frac{\partial \phi}{\partial x} + \frac{\partial K_z}{\partial z} \frac{\partial \phi}{\partial z} + K_x \frac{\partial^2 \phi}{\partial x^2} + K_z \frac{\partial^2 \phi}{\partial z^2} \quad (4.17b)$$

It is noted that Eq. (4.17b) implies that the permeability function is continuous and differentiable in the (x,z) domain. Hence, the above expression cannot be used at the interface of stratified soils where continuity conditions has to be specified. Another form of Richard's equation can be obtained by introducing the Kirchoff integral transformation (Rubin, 1969), giving

$$F(U) \frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial z^2} - G(U) \frac{\partial U}{\partial z} \quad (4.18)$$

where

$$U(\psi) = \int_0^\psi k(\psi) d\psi, \quad F(U) = \frac{C_w(\psi)}{K(\psi)}, \quad G(U) = \frac{1}{K} \frac{dK}{d\psi}$$

Because the variations of U are much slower than those of ψ , the numerical scheme for the differential equation obtained using the Kirchoff transformation is more advantageous than any of the other forms (Vauclin et al., 1979b). However, the application of Eq. (4.18) is limited to homogeneous soils and, therefore, it will not be further discussed.

Theoretically Eqs. (4.17a) and (4.17b) are equivalent and should provide similar results. However, the numerical solutions is very sensitive to the choice of the initial form of the Richard's equation (Haverkamp and Vauclin, 1981).

4.3.3 Richard's equation numerical model

The form of the partial differential equation has been chosen (Eq. 4.17a or Eq. 4.17b), the spatial and time derivatives are replaced by their the approximate numerical values.

4.3.3.1 Decomposed formulation

To provide flexibility to the numerical characterization of the derivatives a non uniform mesh is assumed. The discretization of Eq. (4.17) gives:

$$C_w \frac{\phi^{i+\Delta t} - \phi^i}{\Delta t} = \frac{\Delta K x_i}{2\Delta \bar{x}_i} \frac{\Delta \phi_i}{2\Delta \bar{x}_i} + \frac{\Delta K z_k}{2\Delta \bar{z}_k} \frac{\Delta \phi_k}{2\Delta \bar{z}_k} + 2k_{i,k} \left[\frac{\bar{\phi}_i - \phi_i}{(\Pi x)_i^2} \right] + 2k_{i,k} \left[\frac{\bar{\phi}_k - \phi_k}{(\Pi z)_k^2} \right] \quad (4.19)$$

where the subscripts i and k represents the node location (Fig. 4.2) and the following short-hand notation has been used:

$$\Delta \phi_i = (\phi_{i+1,k} - \phi_{i-1,k}), \quad \Delta \phi_k = (\phi_{i,k+1} - \phi_{i,k-1}) \quad (4.20)$$

$$\Delta \bar{x}_i = \frac{1}{2}(\Delta x_{i+1} + \Delta x_i), \quad \Delta \bar{z}_k = \frac{1}{2}(\Delta z_{k+1} + \Delta z_k) \quad (4.21)$$

$$(\Pi x_i)^2 = (\Delta x_{i+1} \Delta x_i), \quad (\Pi z_k)^2 = (\Delta z_{k+1} \Delta z_k) \quad (4.22)$$

$$\Delta x_i = (x_{i,k} - x_{i-1,k}), \quad \Delta z_k = (z_{i,k} - z_{i,k-1}) \quad (4.23)$$

$$\Delta K x_i = (K x_{i+1} - K x_{i-1}), \quad \Delta K z_k = (K z_{k+1} - K z_{k-1}) \quad (4.24)$$

$$\bar{\phi}_i = \frac{1}{2}(\phi_{i+1,k} r_i^0 + \phi_{i-1,k} r_i^1), \quad \bar{\phi}_k = \frac{1}{2}(\phi_{i,k+1} r_k^0 + \phi_{i,k-1} r_k^1) \quad (4.25)$$

$$r_i^p = \frac{\Delta x_{i+p}}{\Delta x_i}, \quad r_k^p = \frac{\Delta z_{k+p}}{\Delta z_k}, \quad p=0,1 \quad (4.26)$$

Figure 4.2 Nodes numbering scheme and notation used

Eq. (4.19) can be reduced to

$$C_w \phi_{i,j}^{t+\Delta t} = C_w \phi_{i,j}^t + A_x \Delta \phi_i + A_z \Delta \phi_k + B_x (\bar{\phi}_i - \phi_{i,j}) + B_k (\bar{\phi}_k - \phi_{i,j}) \quad (4.27)$$

where:

$$A_x = \frac{\Delta t \Delta K x_i}{4 \Delta \bar{x}^2}, \quad B_x = \frac{2 K x_i \Delta t}{(\Pi x)^2}, \quad A_z = \frac{\Delta t \Delta K z_k}{4 \Delta \bar{z}^2}, \quad B_z = \frac{2 K z_k \Delta t}{(\Pi z)^2} \quad (4.28)$$

The values of the piezometric head at each node at time $t + \Delta t$ are unknown and they are part of the solution. In order to solve Eq. (4.27), the piezometric heads have to be approximated using the linear relationship given by Eq. (4.3). The following equivalent expressions for Eqs. (4.20) and (4.25) are obtained:

$$\Delta \phi = \mu \Delta \phi^{t+\Delta t} + (1-\mu) \Delta \phi^t \quad \bar{\phi} = \mu \bar{\phi}^{t+\Delta t} + (1-\mu) \bar{\phi}^t \quad (4.29)$$

Substituting Eq. (4.29) in Eq. (4.27) and grouping conveniently, leads to

$$(\mu(B_x+B_z) + C_w)\phi_{i,k}^{t+\Delta t} - \mu[A_x\Delta\phi_i^{t+\Delta t} - A_z\Delta\phi_k^{t+\Delta t} - B_x\bar{\phi}_i^{t+\Delta t} + B_z\bar{\phi}_k^{t+\Delta t}] = \quad (4.30)$$

$$(C_w - (1-\mu)(B_x+B_z))\phi_{i,k}^t + (1-\mu)[A_x\Delta\phi_i^t + A_z\Delta\phi_k^t + B_x\bar{\phi}_i^t + B_z\bar{\phi}_k^t]$$

The right-hand side of Eq. (4.30) is known at a given time step whereas the unknown piezometric heads have been grouped on the left-hand side. The matrix equation should be similar to Eq. (4.8). The diagonal mass matrix coefficient $[C_{pq}]$ is equal to $(C_w + \mu(B_x + B_z))$. The matrix $[A_{pq}]$ has four elements which are linear combinations of the coefficients μA_x , μB_x , μA_z , and μB_z grouped around $\phi_{i,k-1}$, $\phi_{i,k+1}$, $\phi_{i-1,k}$, $\phi_{i+1,k}$. In order to solve the linearized finite difference equation given by Eq. (4.30) by iterative methods, Eq. (4.27) is written as follows:

$$\phi_{i,k}^{t+\Delta t} = \frac{[(C_w - (1-\mu)(B_x+B_z))\phi_{i,k}^t + A_x\Delta\phi_i^t + A_z\Delta\phi_k^t + B_x\bar{\phi}_i^t + B_z\bar{\phi}_k^t]}{(C_w + \mu(B_x+B_z))} \quad (4.31)$$

In Eq. (4.31) the values ϕ_k depends on the of ϕ at the surrounding nodes. Eq. (4.31) is equivalent to the Gauss-Seidel formulation.

4.3.3.2 Local formulation

An equivalent finite difference formulation can be obtained by discretizing the original Richard's equation (Eq. 4.17a), as

$$C_w \frac{\phi_{i,k}^{t+\Delta t} - \phi_{i,k}^t}{\Delta t} = \frac{\left[Kx_{i+1/2} \frac{\phi_{i+1} - \phi_i}{\Delta x_{i+1}} + Kx_{i-1/2} \frac{\phi_i - \phi_{i-1}}{\Delta x_i} \right]}{\Delta x_i} + \frac{\left[Kz_{k+1/2} \frac{\phi_{k+1} - \phi_k}{\Delta z_{k+1}} + Kz_{k-1/2} \frac{\phi_k - \phi_{k-1}}{\Delta z_k} \right]}{\Delta z_k} \quad (4.32)$$

where $Kx_{i+1/2}$, $Kx_{i-1/2}$, $Kz_{k+1/2}$, $Kz_{k-1/2}$ are the permeability values at internodal calculation points located at half-way between two neighboring points. Eq. (4.32) can be simplified as

$$C_w \frac{\phi^{t+\Delta t} - \phi^t}{\Delta t} = A\phi_{i,k} + B \quad (4.33)$$

where

$$A = \left[\frac{Kx_{i+1/2}}{\Delta x \Delta x_{i+1}} + \frac{Kx_{i-1/2}}{\Delta x \Delta x_i} + \frac{Kz_{k+1/2}}{\Delta z \Delta z_{k+1}} + \frac{Kz_{k-1/2}}{\Delta z \Delta z_k} \right] \quad (4.34)$$

$$B = \left[\frac{Kx_{i+1/2} \phi_{i+1}}{\Delta x \Delta x_{i+1}} + \frac{Kx_{i-1/2} \phi_{i-1}}{\Delta x \Delta x_i} + \frac{Kz_{k+1/2} \phi_{k+1}}{\Delta z \Delta z_{k+1}} + \frac{Kz_{k-1/2} \phi_{k-1}}{\Delta z \Delta z_k} \right] \quad (4.35)$$

In writing the piezometric head ϕ as a linear combination of the piezometric heads at the present and previous time (Eq. 4.3), the following iterative expression is obtained

$$\phi^{t+\Delta t} = \frac{[(C_w - (1-\mu)A\Delta t)\phi^t + B\Delta t]}{C_w + \mu A\Delta t} \quad (4.36)$$

It is noted that Eq. (4.36) does not require the 1st derivative of the permeability function, hence, this expression can also be used for non-homogeneous soils. The main drawback of Eq. (4.36) is that it requires the evaluation of the values of permeability ($Kx_{i+1/2}$, $Kx_{i-1/2}$, $Kz_{k+1/2}$, $Kz_{k-1/2}$) at the internodal points. At first glance one would be tempted to use the arithmetic mean of the permeability evaluated at the neighboring nodes. However, such a approximation would result in a bias towards the wetter node, and lead to overestimating the rate of the infiltration process. (Schnabel and Richie, 1984). As large variations of the permeability may be expected at the wetting front,

especially during infiltration through very dry soils, large discrepancies with the exact solution may be obtained depending upon the interpolation used for the permeability.

Studies on the influence of different assumptions made to evaluate the internodal conductivities have been presented by several authors (e.g. Haverkamp and Vauclin, 1979; Schnabel and Richie, 1984; Warrick; 1991). A comprehensive study was performed by Haverkamp and Vauclin (1979) who tested 9 different weighing average criteria and reached to the conclusion that the geometric average is preferable. Similar findings were made by Schnabel and Richie (1984) for the cases they analyzed. Hornung and Messing (1982) also showed that errors were considerably reduced when a geometric mean of the permeability was used. Warrick (1991) presented an interesting analysis of the numerical solution of Richard's equation using arithmetic, geometric, and upstream averaging criteria for computing the permeability at the internodal points; Warrick concluded that regardless of how the permeability is computed, the finite different approximations can lead to erroneous velocities, especially for flows along large pressure gradients. It is pointed out that in all these studies, the Richard's equation was written in terms of pressure head instead of piezometric head.

4.3.4 Laplace's equation numerical model

When the explicit or Crank-Nicolson scheme is used, the derived discretized expressions for Richards equation become undefined at saturation. Thus, a different numerical expression for the saturated zone based on Laplace's equation has to be used. The resulting discretized form of Laplace's equation expressed in an iterative manner is given as,

$$\phi_{i,k} = \frac{(C_x \bar{\phi}_l + C_z \bar{\phi}_k)}{C_x + C_z} \quad (4.37)$$

where:

$$C_x = \frac{Kx_i}{\Pi x_i^2}, \quad C_z = \frac{Kz_k}{\Pi z_k^2} \quad (4.38)$$

Ks_x and Ks_z are the saturated permeability in the horizontal direction and vertical directions respectively.

4.3.5 Boundary conditions

Special equations need to be written at the boundaries (Fig. 4.3), especially where infiltration due to rainfall needs to be specified. Assuming the rainfall to be vertical, the differential equation (Eq. 2.36) can be written:

$$-K_x \sin \alpha \frac{\partial \phi}{\partial x} + K_z \cos \alpha \frac{\partial \phi}{\partial z} = N(t) \cos \alpha \quad (4.39)$$

where $N(t)$ is the infiltration rate as function of time and α is the slope angle of the surface defined positive counter-clockwise. A discretized form of Eq. (4.39) can be written:

$$-K_x \tan \alpha \left[\frac{\phi_{i+1,j} - \phi_{i,j}}{\Delta x_i} \right] + K_z \left[\frac{\phi_{i,j} - \phi_{i,j-1}}{\Delta z_k} \right] = N(t) \quad (4.40)$$

Grouping conveniently yields:

$$\phi_{i,j} = \frac{(D_z \phi_{i,j-1} + D_x \tan \alpha \phi_{i+1,j} + N(t))}{(D_z + D_x \tan \alpha)} \quad (4.41)$$

where:

$$D_x = \frac{Kx_i}{\Delta x_i}, \quad D_z = \frac{Kz_i}{\Delta z_i} \quad (4.42)$$

Eq. (4.41) is to be applied at the boundary to express the piezometric head. If the calculated value is found to correspond to a 100% saturation, ponding of the rainfall water will occur. When this situation takes place, Eq. (4.39) is no longer valid and the value of the piezometric head is simply evaluated by:

$$\phi_{i,j} = \phi_{ponding} \quad (4.43)$$

Figure 4.3 Relationship between boundary and interior nodes

An impervious boundary is represented as $\nabla\phi = 0$ in the direction perpendicular to the boundary. If the boundaries at the right, left and bottom of the analyzed region (Fig. 4.3), are assumed to be impervious, this can be expressed numerically as:

$$\phi_{0,k} = \phi_{1,k} \quad \phi_{n+1,k} = \phi_{n,k} \quad \phi_{i,0} = \phi_{i,1} \quad (4.44)$$

for the left, right and bottom boundaries respectively (Fig. 4.3). To describe a boundary staying in atmospheric pressure conditions (Fig. 4.4), a variable boundary condition is required. For negative pressure heads computed at this boundary, no flow is physically possible, then the boundary acts as if it were impervious. Flow will not occur unless the pressure is at least atmospheric, if the computed pressure heads at the boundary are

greater than zero, then they must be reset to zero, indicating that flow is taking place.

Figure 4.4 Soil-Atmosphere boundary type (e.g. Segment EF)

4.4 Convergency criteria

In solving non-linear partial differential equations using any numerical scheme, the convergence criterion is as important as the numerical model itself. A well posed problem may lead to wrong results if the criteria for convergency is not properly set.

4.4.1 Head convergency criterion

Since solutions of water flow problems are often sought in terms of pressure heads, a number of authors prescribe convergency criteria in terms of pressure heads values (e.g. Neuman, 1973; Van Genuchten, 1982; Vauclin et al., 1979b). Whenever an exact solution is available, the convergency criterion can be established as follows (Haverkamp and Vauclin, 1981):

$$\max \left| \frac{\psi_a - \psi_n^m}{\psi_n^m} \right| \leq \varepsilon \quad (4.45)$$

where ψ_n^m and ψ_a are the pressure heads obtained numerically and analytically respectively, and ε is the relative error. If a more general assessment of the numerical

precision is desired, the magnitude of the errors at locations other than the maximum error point need to be taken into account. The criterion is then

$$\frac{\sum |\psi_a - \psi_n^m|}{\sum |\psi_n^m|} \leq \varepsilon \quad (4.46)$$

Convergency criteria similar to those depicted by Eqs. (4.45), (4.46) can also be established for the piezometric heads $\{\phi\}$ and applied to assess the convergency of numerical schemes expressed in terms of this variable. In most practical cases, closed form solutions are not available, thus the usual criterion consists of comparing the pressure heads computed at two consecutive iterations. Van Genuchten (1980) expressed this criterion mathematically:

$$|\psi_j^{m+1} - \psi_j^m| \leq \varepsilon_1 + \varepsilon_2 \psi_j^{m+1} \quad (4.47a)$$

$$|\phi_j^{m+1} - \phi_j^m| \leq \varepsilon_1 + \varepsilon_2 \phi_j^{m+1} \quad (4.47b)$$

where j represents the node number, m is the iteration number, ε_1 and ε_2 are the selected absolute and relative error criteria which can be applied to each node or to specific ones. Absolute error criteria are often used as convergency criteria (e.g. Giesel et al., 1973; Freeze; 1971; Vauclin et al., 1979a and 1979b; Hornung and Messing, 1983; Paniconi et al. 1991; Christe and Gaillard, 1989; Thomas and Rees, 1990; Lam et al., 1987; Papagianakis and Fredlund, 1984).

4.4.2 Effective degree of saturation criterion

Similarly to Eqs. (4.47), a convergency criterion based on volumetric water content can be written

$$|S_e^{m+1} - S_e^m| \leq \varepsilon_1 + \varepsilon_2 S_e^{m+1} \quad (4.48)$$

This convergency criterion should be used only when the Richards equation is described in terms of volumetric water content. For numerical models based on pressure or piezometric head formulation, the major drawback of this criterion is that near saturation where little or no changes in the degree of saturation occurs, the piezometric heads may be unbalanced in the saturated region.

4.4.3 Total water balance criterion

A more comprehensive convergency criterion that takes the global mass balance into account as well as the change in piezometric head is expressed as (Celia et al., 1987; Allen, 1986)

$$R^m = C_w(\psi^m) \frac{\phi^{t+\Delta t, m} - \phi^{t, m}}{\Delta t} - \frac{\partial}{\partial x} \left[K_x(\psi^m) \frac{\partial \phi^m}{\partial x} \right] + \frac{\partial}{\partial z} \left[k_z(\psi^m) \frac{\partial \phi^m}{\partial z} \right] \quad (4.49)$$

where R^m , called the residual, is a measure of the amount by which the approximation at the m th iteration fails to satisfy the discretized numerical equation (Eq 4.31 or Eq. 4.36). Making R^m arbitrarily small, the resulting increment in piezometric head will be also small. The integral of the residuals (R^m) over the spatial domain D of the problem provides a global measure of the mass balance. Hence, mass balance can be conserved within the limits imposed by the convergency criterion (Celia et al. 1987). The inconvenience of this method is that an additional matrix computation has to be performed.

4.5 Comments on numerical techniques

A summary of the two most popular procedures used for the numerical solutions of the equation flow into unsaturated porous medium has been presented together with the procedures available for the solution of system of linear equations. The main characteristic of the finite difference formulation is that its formulation is based on local

water balance (e.g. Eq. 4.17) whereas the finite element formulation is based on the total water balance (Eq. 4.50). In other words, the finite difference method approximates the function derivatives whereas the finite element model approximates the function itself.

Both formulations can be expressed in matrix form and be solved using the Gaussian elimination technique in an iterative manner. As the solution of systems of equations using direct technique, even if the matrix sparseness is taken into account, may require large amount of memory storage, iterative procedures such as Jacobi or Gauss-Seidel are often preferred. The Gaussian elimination technique may lead to faster solution than its counterparts the Jacobi or Gauss-Seidel techniques, especially when saturated zones are present in the domain of interest. The finite difference formulation allows a more natural development of iterative techniques for solving linear equations. The implementation of such iterative techniques in a finite element formulation requires a judicious memory mapping of the matrix coefficients.

The mass matrix $[C]$ obtained from the finite difference formulation is always diagonal whereas the one obtained using the finite element formulation, the consistent mass matrix, contains off-diagonal terms that make the system of equations less stable. This problem has been overcome by lumping the mass matrix $[C]$, this procedure results in a diagonal matrix.

As the matrix assemblage is based on continuity requirements, theoretically the total water balance is ensured in the finite element procedure. However, due to solution techniques used (i.e. conformal mapping, gaussian integration points, shape functions and so forth), violations of the continuity requirements may happen especially for those cases where a sharp wetting front is developed.

The main drawback of a finite difference computation is its limitations to handle complicated geometries and boundary conditions. Different matrix equations can be obtained depending upon the time weighing factor used to evaluate the piezometric (or pressure) heads. The three main weighing techniques are the explicit, the Crank-Nicolson and the implicit schemes. The explicit scheme is often avoided because of its instability with large time steps. The Crank-Nicolson leads to a numerical scheme that is correct up to the second order in time and therefore preferred to the implicit scheme

that is only first order correct in time. However, the Crank-Nicolson scheme may give oscillatory solutions for large time steps. These oscillations are damped when the implicit scheme is used and the non-linear coefficients are still evaluated at the mid-time step.

The parabolic Richards equation becomes elliptic when saturation occurs. For saturated regions of the analyzed domain, the Crank-Nicolson scheme combined with direct matrix solution algorithms cannot be used. For such situations, the only appropriate numerical analog to the Laplace's equation is obtained using the implicit scheme. Different time stepping schemes can be used depending upon the node conditions at a given time interval for (1) nodes remaining saturated, (2) nodes remaining unsaturated, and (3) nodes that undergo a change from saturated to unsaturated during the time interval.

4.6 Proposed Numerical Model

4.6.1 The Decomposed-Local Mixed (DLM) formulation

To develop the numerical model of unsaturated flow in porous media presented herein, the finite difference method was chosen as the analytical framework. Several options are available for developing a finite difference model using the piezometric heads as field variable. These methodologies have been described in section 4.3.3. The two principal numerical models are obtained by the discretization of either the local balance equation (Eq. 4.17a) or of its decomposed form (Eq. 4.17b) resulting in either Eq. (4.36) or Eq. (4.31) respectively.

Experience has shown that numerical models based on the local formulation (Eq. 4.36) are simpler to implement in a computer environment. Hence, although their numerical precision is sensitive to the permeability weighing method used, most previously published finite difference models were based on the local formulation. Even using the geometric mean to compute the permeability at the internodal points the local formulation may give unrealistic results when a sharp wetting front develops (Hornung and Messing, 1983; Warrick, 1991). This happens for instance with initially very dry

soils. These previously reported observations were confirmed by preliminary results obtained during the present study.

Abrupt changes in the degree of saturation occurring in the space during an infiltration process make the reliable computation of the prediction of internodal permeability coefficients extremely difficult. In such cases, the numerical evaluation of the permeability gradients used in the decomposed formulation (Eq. 4.17b) can be performed more accurately. Hence, a numerical formulation based on the decomposed formulation (Eq. 4.17b) can be more advantageous. However, the Decomposed formulation has two major drawbacks: (1) it requires the permeability function to be first order continuous inside the analyzed domain; thus, at the interface between two soils with different permeability coefficients, Eq. (4.17b) cannot be utilized; and (2) significant errors at the saturated-unsaturated interface may be introduced by the central difference scheme commonly used to compute the total head gradients.

Regardless of the numerical scheme being used, errors in the numerical simulation of the infiltration are usually concentrated at the beginning of the process that is before the upper surface boundary has reached a steady state. Once the upper boundary has reached a steady state infiltration, lesser discrepancies result from using the different schemes (Haverkamp and Vauclin, 1981).

The major numerical difficulties experienced during modelling of sharp wetting fronts using a ψ or ϕ -based Richards equation can be lessened by using a θ -based equation, Eq. 2.30b, (Narasimhan and Witherspoon, 1977). The primary disadvantage of using Eq. 2.30b is its inability to handle materially heterogeneous media such as stratified soil structures. A review of different numerical models presented in the literature also indicates difficulties in modeling the saturated-unsaturated interface (e.g. Watson, 1967; Neuman, 1973; Cooley, 1983; Vauclin et al., 1979a) which is usually treated as an especial boundary condition.

Haverkamp and Vauclin (1981) have performed a comparative study of three different ψ -based numerical formulations used in unsaturated flow analysis: (a) the local, (b) the decomposed, and (c) the Kirchoff formulation. Based on the examples analyzed, they concluded that the Kirchoff formulation provides better predictions followed by the

local formulation. However, the Kirchoff formulation is restricted to the analysis of flow in a homogeneous media and is therefore unapplicable in the present study. The reliability of the local formulation was found to depend largely on the averaging procedure used for computing the internodal permeability. It should be noted that Haverkamp and Vauclin (1981) assumed for their comparative study C_w and $K(\psi)$ to be constant during the time interval and equal to their values computed at the preceding time step.

In the present study, numerical simulations performed to test different algorithms for one-dimensional infiltration in homogeneous soils here have shown that for a ϕ -based equation, the decomposed formulation (Eq. 4.17b), or its equivalent discretized form (Eq. 4.31) is more reliable in presence of sharp wetting fronts than the local formulation (Eq. 4.17a), or its equivalent numerical form (Eq. 4.36). For these homogeneous soils when the initial conditions are such that no wetting front develops, both formulations gave comparable results. On this later point, it seems that the conclusion presented by Vauclin et al. (1979a) on the superiority of the local formulation cannot be confirmed. However, it should be noted that the basic differential equation used in their study was expressed in terms of pressure heads instead of the one expressed in terms of piezometric head used herein. In addition, during the present study, the values of C_w and $K(\psi)$ were constantly updated at each iteration instead of being kept constant.

Based on this preliminary study, it appears that the choice of a sole formulation to develop a numerical model does not provide optimal solutions in the case of infiltration through stratified media with a range of possible boundary and initial conditions. In previously, published models it is nevertheless customary that once the mathematical representation of the Richards equation has been chosen, the corresponding numerical model is developed based on this single formulation for the entire analyzed domain regardless of the flow conditions. A fundamental advantage of model proposed herein over previously published solutions is that it is based on an association two different expressions of the Richards equation (Eqs. 4.17a and 4.17b) such that the optimal applicability and reliability of the solution can be obtained.

Because of its capability to handle sharp wetting front situations, the decomposed formulation was chosen as the main numerical scheme for locations inside homogeneous layers. To overcome the difficulty created by the inadequacy of the decomposed formulation at the interface of two different soils or when ponding develops, the continuity of the quantity of flow (i.e. $K(\psi)\nabla\phi$) has to be verified in the solution at these particular locations, hence a numerical expression based on the local formulation is used at this interfaces and ponding surfaces. This association of both solution schemes in a single numerical model will be referred to as the Decomposed-Local Mixed formulation (DLM).

The implementation into a computer environment of both expressions (Eq. 4.31 and Eq. 4.36) requires constant monitoring of the situation at each node. A node is considered of the homogeneous kind if it is surrounded by nodes with similar soil properties, in this case Eq. (4.31) is applied otherwise the node will be considered non-homogeneous and Eq. (4.36) be utilized. The node will be also considered non-homogeneous, only for computational purposes, at a surface where ponding is occurring.

Nodes located at soil layer interfaces are clearly identified whereas the automatic monitoring of ponding surfaces requires a more elaborate programming. Ponding usually occurs when the infiltrating water reaches a less pervious layer causing automatically an increase in the moisture content at the interface which propagates upwards in the case of one-dimensional infiltration and also to laterally in a two-dimensional process. Ponding usually goes along with changes in concavity of the piezometric heads (see section 5.1.6). This effect can be detected by monitoring in the computer program the sign of the derivative $\partial^2\phi/\partial z^2$.

Within this framework, special cases can be treated by introducing simplifications to the general formulation two examples are outlined below (a) saturated conditions since the developed expressions presented in section 4.3 are based on Eq. (2.27), and Laplace's equation (Eq. 2.29) is an special case of Eq. (2.27), saturated conditions can be analyzed by replacing in Eq (4.31) or (4.36) C_w by zero and the permeability functions by the values of saturated permeability . It should be noted, however, that when the soil becomes saturated, the piezometric head is no longer a continuous function

of time and depends on changes of the boundary conditions. Sudden changes at the boundaries will induce immediate responses within the saturated zone (Neuman, 1973). Thus, if variations at the boundary are discontinuous in time, Eq. (4.31) or (4.36) may no longer be applied. (b) Steady state conditions taking into consideration the saturated and unsaturated zones can also be simulated replacing the parameter C_w by 0 in Eqs. (4.31) or (4.36), but still leaving the permeability as functions of the degree of saturation.

4.6.2 Main characteristics of the PURDRAIN finite different model

A computer program was developed to implement the numerical model in a microcomputer environment. The finite difference analysis is based on the DLM formulation described in the previous section. The other main characteristics of the model are as follows:

- Four models to describe the water retention curves (Brooks and Corey, Van Genuchten, Brutsaert, and Vauclin's models) are available.
- Three models to describe the water retention curves (Brooks and Corey, Van Genuchten-Mualem, and Gardner's models) are available.
- The implicit, explicit and Crank-Nicolson time weighing schemes can be used in the numerical model.
- Impervious, open to atmospheric or constant piezometric heads can be assumed as boundary conditions.
- Initial conditions can be specified on the values of piezometric heads or effective degree of saturation.
- Two convergency criteria are available, one in terms of piezometric heads and the other in terms of volumetric water content.

A more detailed description of the available options of the PURDRAIN computer program can be found in Appendix A.

CHAPTER 5 - NUMERICAL MODEL VALIDATION AND APPLICATIONS

5.1 One dimensional model validation

In this chapter, validations of the proposed model are performed by comparing its computed response to previously published available in the technical literature. The developed finite difference model is validated by performing several numerical simulations of one-dimensional infiltration of several examples cases described in previously published works. The available data relating to these examples utilized herein were obtained by computer and/or laboratory simulations. The selected cases allow to test four of the most common water retention models (i.e. Brooks and Corey, Van Genuchten, Brutsaert, and Vauclin models) as well as three of the most popular hydraulic conductivity models (i.e. Brooks and Corey, Van Genuchten-Mualen and Gardner models) together with different stratigraphy and boundary conditions.

Because of the widespread application of one-dimensional analyses in agricultural engineering and soil physics, a number of numerical simulations and experimental studies of one-dimensional vertical infiltrations is readily available from published works (e.g. Rubin, 1969; Watson, 1967; Dempsey and Elzaeftawy, 1977; Van Genuchten, 1982; Christe and Gaillard, 1989; Poulovassilis et al., 1991). Unfortunately, complete information on the hydraulic and numerical parameters used is not always well documented in the publications.

5.1.1 Watson (1967)

One-dimensional drainage in a 57 cm high column of sand was studied experimentally by Watson (1967). Botany sand was carefully placed under water in an acrylic column (15cm x 10 cm). Uniformity of the sand placement was verified by measuring the bulk density using a gamma-ray absorption apparatus. Prior to starting

the drainage test, steady state flow was established by supplying excess water on top of the column. Water was allow to drain through the bottom of the soil column which was left open to atmospheric conditions. The variations of moisture content and matric suction with time at different depths were measured.

The hydraulic characteristic curves of the botany sand, experimentally determined by Watson (1966), are shown on Fig. (5.1). The Brooks and Corey's model (Eqs. 3.6 and Eq. 3.35) was used to represent these curves using the parameters indicated in Fig. 5.1. It is also observed from Fig. 5.1 that the maximum volumetric moisture content is 0.35 and the residual volumetric water content (θ_r) is 0.05. Figs. 5.2a and 5.2b show the values of moisture content and piezometric heads measured during the drainage test. This experiment was simulated using the developed finite difference program PURDRAIN. These simulations compare favorably to the measured values (Figs. 5.3a and 5.3b).

Figure 5.1 Water retention and hydraulic characteristic of Botany sand (After Watson, 1967)

Figure 5.2 Measured (a) water content profiles and (b) total head distribution (After Watson, 1967)

Figure 5.3 Computed (a) water content profiles and (b) total head distribution

It is noted that according to the computed results full hydraulic equilibrium is reached after 1 hour of drainage. Because of the high permeability of the sand ($K_s = 0.0186$ cm/s), the water located at an elevation greater than the bubbling pressure is quickly drained but then, despite the bottom boundary open to atmosphere, full drainage of the column sand does not occur and a zone of fully saturated sand remains below the 40 cm level (Figs. 5.2 and 5.3). Both measured and computed thickness of the capillary fringe are practically equal to the bubbling pressure ($PB=39.7$ cm), this result is consistent with the theory. An extreme case would be that of the bubbling pressure of the sand greater than that of the thickness of the sample, then no drainage would occur under the action of gravity.

5.1.2 Poulouvassilis et al. (1991).

This case is a one-dimensional vertical infiltration in a column of study on Yolo-Light clay performed by Poulouvassilis et al. (1992) who studied the physics the infiltration under a constant flux larger than the soil saturated permeability. The theoretical study was complemented with numerical simulations for several soils hydraulic characteristics with satisfying results. The water retention curve characteristics of this soil was represented by Vauclin's model (Eq. 3.12) and the hydraulic conductivity was represented by Gardner's model (Eq. 3.39); the corresponding parameters are shown on Table 5.1. The initial moisture content (θ_i) is equal to 0.2376. A constant flux equal to $1.637K_s$ is applied on the top of the soil column. The resulting infiltration profiles for a continuous 127.5 hours long are shown on Fig. 5.4.

Table 5.1 - Soil hydraulics parameters for Yolo-Light Clay

	Vauclin Model				Gardner Model		
Formation	θ_s	θ_r	α	$\beta[-]$	$K_s(\text{cm/s})$	A	B
Yolo Clay	0.495	0.125	737	3.98	1.2E-6	124.6	1.77

Numerical simulation of their experiment was performed using the program PURDRAIN and the results at similar times are shown in Fig. 5.4. Good correspondence between the numerical simulation and the theoretical results is obtained.

Figure 5.4 Infiltration profiles at different times as computed by Poulouvasilis et al. (dotted lines) and as obtained by the PURDRAIN finite difference model (solid lines)

5.1.3 Christe and Gaillard (1989).

Christe & Gaillard (1989) have simulated experimentally and numerically the water infiltration and redistribution in a column of two-layer stratified soil. The numerical simulation was performed using a finite element program. The water retention curve and the hydraulic conductivity were represented using the Van Genuchten model (Eq. 3.8) and Van Genuchten-Mualen model (Eq. 3.36) respectively. The parameters for the two soils are shown on Table 5.2. The boundary conditions were an applied flux at the top of the soil column and atmospheric pressure at the bottom.

Table 5.2 - Soil hydraulics parameters for Botany Sand

Depth(cm)	θ_s	θ_r	$\alpha(\text{cm}^{-1})$	$\beta[-]$	$K_s(\text{cm/s})$
0-125	0.306	0.0	0.05465	2.30	1.2E-2
125-300	0.250	0.0	0.09366	1.52	3.9E-2

The results obtained for several phases of infiltration and subsequent redistribution are discussed below.

5.1.3.1 Short term infiltration process

The first experiment corresponds to one dimensional infiltration in a two-layer soil. A constant flux $q = 1.43\text{E-}3 \text{ cm/s}$ at the top of the column was applied for a period of 2h46 allowing the wetting front to reach the interface between the two soils. The bottom boundary was assumed to drain freely. Fig 5.5a shows several water content profiles measured at different times in the laboratory and computed using a finite element model (Christe and Gaillard, 1989). It is noted that as the applied flux is less than the maximum water capacity of the soil, full saturation of the soil ($\theta_s = 0.306$) is not reached, the steady state at the top is reached at a moisture content approximately $\theta = 0.22$ and the wetting front moves downwards without fully saturating the column of soil. There is good agreement among the results obtained using the finite element program

used by Christe and Gaillard (Fig. 5.5a), the values measured, and the results obtained using the PURDRAIN finite difference program (Fig. 5.6a). Logically, a discontinuity in the moisture content profile is observed at elevation 175 cm where the interface between the two soils is located.

5.1.3.2 Redistribution process

Following the infiltration process described above, the constant flux applied at the top boundary was stopped and the redistribution of the water content was monitored. Fig. 5.5b shows the measured and computed values by the finite element model. Similar case was analyzed using the finite difference program and the results are shown on Fig. 5.6b. The patterns of the numerically simulated redistribution process agrees well with the results obtained by Christe and Gaillard. It should be noted however that although the underlying soil is more permeable, the finite element model developed by Christe and Gaillard (1989) predicts ponding (i.e. an upward increase of the water content) at the interface of the two soils which is not reflected by the measured values. The reasons of this discrepancy are discussed by Christe and Gaillard (1987). The results of the PURDRAIN finite difference model are in better agreement with the measured values.

5.1.3.3 Long term infiltration process

The third experiment corresponds to one dimensional infiltration in a two-layer soil with constant flux $q = 1.15E-3$ cm/s applied at the top of the column for a period of 8h30. The wetting front reached the bottom of the soil column after almost 7h. Fig 5.5c shows several water content profiles measured for different times measured in the laboratory and computed using a finite element model (Christe and Gaillard, 1989) and Fig. 5.6c shows the results obtained using the PURDRAIN finite difference program. Good agreement is observed among the results obtained using the finite element method by Christe and Gaillard, the values measured, and the results obtained using the PURDRAIN finite difference program.

Figure 5.5 Water content profiles obtained by Christe and Gaillard during (a) short infiltration period (2h46), (b) redistribution process, and (c) long infiltration period (8h30). The Horizontal segments indicate the measurements uncertainty

Figure 5.6 Water content profiles obtained using PURDRAIN during (a) short infiltration period (2h46), (b) redistribution process, and (c) long infiltration period (8h30)

5.2 Two-dimensional model validation

In contrast with one dimensional infiltration, only a very number of well documented two-dimensional infiltration cases analysis is available in the literature.

5.2.1 Cooley (1983)

Cooley (1983) presented a numerical model based on the finite element method using linear triangular elements. The numerical model was validated using previous one and two-dimensional theoretical and experimental results. Results of two examples are presented and compared herein with the results obtained using the program PURDRAIN. Consistent units were used by Cooley (1983); thus no units appear in the following discussion.

5.2.1.1 Determination of the water table-Steady state flow

A classical application in unsaturated flow is the determination of the phreatic surface for steady state conditions. The usual approach consists of assuming in Richards equation the coefficient of water retention (C_w) is equal to zero and considering the permeability as a function of suction. The resulting expression is a Laplace's elliptic equation with non-linear parameters. Its solution has to be found iteratively; however, depending upon the soil characteristics, the geometry and boundary conditions, stability or/and convergency problems are often encounter. This difficulty can be overcome by performing instead a transient analysis until the steady state solution is reached.

Such an analysis was performed by Cooley (1983) for the geometry shown in Fig. 5.7a using the finite element method. The boundary conditions were a constant head equal to 10 on the left hand side boundary, impervious conditions for the upper and bottom boundaries, and a constant head equal to 2 for z smaller than 2 and atmospheric pressure for z greater than 2 along the right hand side boundary. The water retention curve was represented using the Brutsaert model (Eq. 3.10) and the hydraulic conductivity represented by a power function (Eq. 3.35). Table 5.3 shows the hydraulic parameters used in the analysis. Fig. 5.7 shows the piezometric head contours obtained using the finite element method by Cooley (1983) and using the finite difference model

developed herein. Comparison of these two sets of results show very good agreement between the finite element and finite difference models.

Table 5.3 - Soil hydraulics parameters (Cooley, 1983)

Model	Brutsaert Model				Irmay-Averjnov	
Parameter	θ_s	θ_r	α	$B[-]$	$K_{xx}=K_{yy}$	η
Data	0.25	0.05	10	4	0.01	4

Figure 5.7 Steady state flow through a square embankment. Distribution of piezometric heads obtained by Finite element model by Cooley (solid lines) and (b) by PURDRAIN finite difference model (Dashed lines).

Traditional saturated flow models consider that moisture movement occurs only below the phreatic level which is assumed to be a streamline (i.e. no flow across this surface could occur). As it was shown by Freeze (1971), flow across the phreatic line can occur and the use of saturated-unsaturated theory allows to compute the piezometric head distribution without the above restrictive assumption regarding the phreatic level. Depending upon the aquifer and the soil water retention characteristics, the quantity of flow occurring above the phreatic level might be significant. Brutsaert and El-Kadi (1984) found that if D is a characteristic vertical dimension of the flow region, for instance its thickness, then the criterion $\psi_t/D \ll 1$ (where $\psi_t = \alpha^\beta$, α and β are the water retention parameters of Brutsaert's model) indicates that the unsaturated flow above the water table does not significantly affect the overall flow in the aquifer. In the above ratio ψ_t/D , ψ_t represents the contribution of the capillary fringe to the aquifer behavior. When ψ_t/D is very small, the zone with saturation values close to unity is very thin and most of the flow takes place in the saturated zone. Similar criteria can be applied to other hydraulic models; for instance, the value ψ_t can be computed equal to PB for the Brooks and Corey model and equal to α for the Van Genuchten's model. For the problem analyzed the values of D and ψ_t equal 10 and 1.77 respectively, yielding $\psi_t/D = 0.17$; thus, according to the above criteria, the flow above the water table cannot be neglected.

5.2.1.2 2-D Drainage process

An analysis was also presented Cooley (1983) for a two-dimensional drainage from a homogeneous isotropic soil with the same geometry as the preceding analysis but different boundary conditions (Fig. 5.8). As in the previous example, the water retention curve and the hydraulic conductivity were represented by the Brutsaert model and the power function respectively. Table 5.4 shows the hydraulic parameters used in the analysis. By letting q_a be the applied flux, H_s the height of the seepage face, the initial and boundary conditions are as follows:

$q_a=0$ $x=0$ $0 < z < 10$ $t > 0$ (no flow through the left hand side boundary)

$q_a=0$ $z=0$ and $z=10$ $0 < x < 10$ $t > 0$ (no flow through the upper and bottom boundaries)

$\phi = z$ $x = 10$ $0 < z < H_s$ $t > 0$ (atmospheric pressure below the seepage face)

$q_n = 0 \quad x = 10 \quad H_s < z < 10 \quad t > 0$ (impervious condition above the seepage face)

$\phi = 10 \quad 0 < x < 10 \quad 0 < z < 10 \quad t = 0$ (Hydrostatic initial conditions)

Table 5.4 - Soil hydraulics parameters for 2D Drainage (Cooley, 1983)

Model	Brutsaert Model				Irmay-Averjnov	
Parameter	θ_s	θ_r	α	$B[-]$	$K_{xx}=K_{yy}$	η
Data	0.25	0.05	10	4	0.01	1

Fig. 5.8 shows the phreatic lines obtained by Cooley (1983) at different times using his finite element model. The results obtained using a finite difference program developed herein are superimposed in Fig. 5.8 to Cooley's curves. The computed phreatic level at similar times obtained using the PURDRAIN finite difference model differ from the ones obtained by Cooley (1983). This discrepancy might be due to the convergency criteria used in the simulations. Cooley (1983) used the absolute convergence criterion in his analysis (Eq. 4.75b) and the combination of relative and total water balance criteria was used in PURDRAIN. A more detailed discussion on the influence of the convergence criteria is presented in Chapter 6.

Figure 5.8 Piezometric line variation during transient drainage of a homogeneous square block as obtained by Cooley (solid lines) and by the PURDRAIN model (Dashed line).

5.2.2 Vauclin et al. (1979b).

A transient two dimensional flow experiment was performed in the laboratory with a layer of soil 300 cm long, 200 cm high and 5cm thick. The soil was prepared as uniformly as possible between two walls supported by a frame and laid on a impervious horizontal boundary. The right end boundary was connected to a constant head reservoir (Fig. 5.9). Water was supplied at a constant rate $q_0 = 14.8$ cm/hr from the top. The water pressure heads were measured using 20 tensiometers mounted on one of the walls. The water content at the top corresponding to the applied flux was equal to 0.26. A complete description of the laboratory test and its results is presented by Vauclin et al. (1979b).

Figure 5.9 (a) Geometry and Flow patterns (b) Experimental setup (After Vauclin et al., 1979).

Piezometric Head Variation at $t = 3$ hours

Figure 5.10 (a) Computed piezometric heads at $t = 3$ hours and (b) Variation with time of the water table as measured by Vauclin et al. (1979) and as computed using the PURDRAIN finite difference model.

Suction and hydraulic conductivity curves were determined in the laboratory as part of the experimental program and fitted by regression analysis to the Brutsaert's and Gardner's models respectively. Table 5.5 shows the values obtained. The experiment was simulated using the PURDRAIN finite difference model, the results show excellent agreement between the measured and calculated positions of the water table (Figs. 5.10a and 5.10b). The water table started to move 1.75h after the beginning of the infiltration. Rapid variations of the water table after 1.75h were observed (Fig. 5.10) and the rate of variation was gradually reduced until the phreatic level reached its steady state position 8h after the beginning of the test.

Table 5.5 - Soil hydraulics parameters (Vauclin et al, 1979b)

	Brutsaert Model				Gardner Model		
Formation	θ_s	θ_r	α	$\beta[-]$	$K_s(\text{cm/h})$	A	B
Sand	0.30	0.0	40,000	2.9	35	2.99E6	5.0

It is interesting that the criterion proposed by Brutsaert and El-Kadi (1984), for $D = 65 \text{ cm}$ and $\psi_t = 38.62$, corresponds to ψ_t/D equal to 0.598. This indicated for this particular case, the importance of the flow occurring in the unsaturated region.

5.3 Application examples

5.3.1 Drainage of a base course with simplified geometry

In order to assess the influence of the unsaturated hydraulic parameters on the drainability characteristics of pavement bases, an analysis of the drainage process with a simplified rectangular cross-section was performed. The geometry, initial and boundary conditions are similar to those used in the development of Casagrande and Shanon's model as well as of Liu et al's model (Figures 1.1 and 1.2 respectively). Accordingly, full saturation of the base is assumed as initial condition; impervious conditions are assumed for the upper, bottom and left side boundaries, and the right side boundary is set to atmospheric pressure and open for drainage. The dimensions of the

example problem are those corresponding to typical sizes of pavement bases (Fig. 5.11). The parameters describing the water retention and hydraulic conductivity curves were obtained from laboratory tests performed on typical soils used in Indiana pavement bases and then fitted to the Van Genuchten model (Ahmed et al., 1993). Table 5.6 shows the saturated-unsaturated hydraulic characteristics for this example.

Table 5.6 - Soil hydraulics parameters (Van Genuchten model)

Parameter	θ_s	θ_r	$\alpha(\text{cm}^{-1})$	$\beta[-]$	$\gamma[-]$	$K_s(\text{cm/s})$
Data	0.11	0.02	0.0052	1.74	0.42	0.1

Figure 5.11 shows the variation of the piezometric heads as well as the phreatic level at different times as drainage takes place. It is observed that the piezometric heads contours for the different times shown in Fig. 5.11 are almost vertical. Consideration of the unsaturated region allows to describe flow occurring above the phreatic level. Similarly, the variations of the effective degree of saturation at different times are shown in Fig. 5.12.

As the assessment of the drainability characteristics are often performed based on saturated flow models such as that of Casagrande and Shanon, it is useful to compare the results obtained using both saturated and unsaturated models. In order to compute the time required to drain a given percentage of the saturated base area, using either the Casagrande and Shanon or the Liu et al. model, the saturated permeability and effective porosity (n_e) are required together with the geometric characteristics of the base. Based on Casagrande and Shanon's model (Table 1.1), the time required to drain 50% of the total area (i.e. $U = 0.5$) is equal to 24 minutes. Similarly, based on Liu et al.'s model (Table 1.2) the time corresponding to $U = 50\%$ is 37 minutes. Applying similar criteria of percentage of drained area to the computed results, a value of drained area (U) approximately equal to 50% is shown in Fig. 5.12d which is obtained 3 hours after the drainage process was initiated. Despite the drained area is approximately 50% of the

total base area, due to the unsaturated hydraulic characteristics of the soil base, the effective degree of saturation above the phreatic level is still quite high.

Fig. 5.11 Variation of the piezometric heads at different times (0.75h, 1.5h, 2.25h, 3h)

Fig. 5.12 Variation of the degree of saturation at different times (0.75h, 1.5h, 2.25h, 3h)

The saturated flow models assume that the amount of water available for drainage above water table is equal to the specific yield, n_e . As shown in the above example, this assumption might be unrealistic since it can be shown, based on the unsaturated flow model, that the variation of the degree of saturation from fully saturated to its residual value does not occur abruptly (Fig. 5.12). It can also be seen from Fig. 5.11 that as drainage takes place the water table does not pass through the right bottom corner (where it is open to atmospheric conditions) as it is assumed in saturated flow models (Figs. 1.1. and 1.2). This later assumption yields higher hydraulic gradients and therefore unconservative predictions of the drainability characteristics of the base. Finally, saturated flow models do not allow for flow across the phreatic level, this assumption implies that a larger amount of water is available for drainage.

In summary, the assumptions made in the development of the saturated models (i.e Casagrande and Shanon, 1951; and Liu et al., 1981) commonly used in the assessment of the drainability characteristics of pavement bases yield shorter drainage times and larger amount of water drained, therefore these models are unconservative from the design and drainage performance standpoints. Thus caution should be exercised when using such models.

5.3.2 Infiltration in a simplified geometry pavement structure

It is also interesting to model the infiltration process that pavement structures may undergo during rainfall events. This example represents the case of a two dimensional infiltration in a simplified geometry pavement structure made of a wearing course overlaying an homogeneous base material. The Brooks and Corey model is used in this example to represent the unsaturated hydraulic characteristics. Geometric and hydraulics characteristics of the system are shown in Fig. 5.13. The base is modeled as an anisotropic material, with horizontal conductivity ten times larger than the vertical one. The structure rests on an impermeable subgrade and the lateral boundaries are also impermeable. Symmetry with respect to the longitudinal axis of the road is assumed. The wearing course is modeled as an impermeable layer, 2.5 cm thick, leaving the shoulder as the only source of water. As an initial condition, the effective degree of

saturation is specified to be 50%. A rainfall of constant intensity is applied during six hours, giving a total of 6 cm of cumulative precipitation.

Figure 5.13 Simplified homogeneous pavement geometry

Figures 5.14 and 5.15 show the evolution with time of the piezometric heads and effective degree of saturation respectively for the six hours infiltration process. Due to gravity effects, the wetting front advances faster in the vertical direction than in the horizontal one. In the horizontal direction, once the front has reached the impervious bottom, moisture migrates beneath the pavement slab, showing the influence of low permeable subgrades located below pavement bases.

Fig. 5.14 Variation of the piezometric heads at different times (1.5h, 3h, 4.5h and 6h)

Fig. 5.15 Variation of the degree of saturation at different times (1.5h, 3h, 4.5h and 6h)

5.3.3 Infiltration in a two-layer pavement structure

This example corresponds to a case of two dimensional infiltration into a non-homogeneous isotropic structure which simulates the case of a subdrainage layer with high permeability. The Brooks and Corey model was used to represent the unsaturated soil characteristics. The geometric characteristics and hydraulic parameters of this example are shown on Fig. 5.16. As in the previous case, the pavement slab, the bottom and both lateral boundaries are assumed to be perfectly impervious. Symmetry was assumed for the cross-section. The lower layer (subbase) was assumed to be approximately 100 times as permeable as the upper layer (base) course. Both base and subbase materials are assumed to be at 50% saturation at the beginning of the analysis. The shoulder is considered to be the source of infiltration with a rate of 0.012 cm/sec.

Figure 5.16 Simplified two-layer pavement geometry

Fig. 5.17 and 5.18 shows the evolution of the piezometric heads and effective degree of saturation at different times (i.e. 1.5 hr, 3hr, 4.5hr, 6 hr). At the early stages of infiltration (e.g. 1.5 hr., Fig. 5.17a), the wetting front is not affected by the underlying layer and is similar to that for the previous case (Fig. 5.14a).

Fig. 5.17 Variation of the piezometric heads at different times (1.5h, 3h, 4.5h and 6h)

Fig. 5.18 Variation of the degree of saturation at different times (1.5h, 3h, 4.5h and 6h)

Once the wetting front reaches the lower layer, a sudden decrease in the degree of saturation is observed at the boundary. This is due to the high permeability of the underlying soil. The water seeps downward much faster than the upper layer can supply. An increase in the degree of saturation from bottom to top is then experienced (Fig. 5.18b, c, and d). It is also noted that the horizontal displacement of the water front is much faster in the lower layer. The effect of this layer on the patterns of infiltration is evidenced by the comparison between Figures 5.15 (a to d) and Figures 5.18 (a to d).

This example shows the benefit of having a more permeable layer beneath the base for cases where the shoulder is the main source of water. Ponding in the soil base is avoided preventing the infiltrating water from migrating below the pavement slab as it was the case in the example analyzed previously.

5.3.4 Infiltration process in a typical pavement structure (1st case)

Analysis of simplified pavement structures is advantageous in understanding the influence of the different parameters involved in the process of infiltration and drainage. However, as modeling of moisture migration in actual pavement structures is one of the goals of the present project, the computer program, PURDRAIN, was implemented such that pavement sections can be represented as closely as possible.

This example represents the geometry of a typical pavement structure (Figures 5.19) that can be found in Indiana highways. The geometric characteristics and hydraulic parameters are shown in Fig. 5.19. As in the previous case, the bottom and both lateral boundaries are assumed to be perfectly impervious, and symmetry was assumed for the cross-section. The assumed initial effective degree of saturation is also shown in Fig. 19. Unlike the previous case, the shoulder is not the only source, water coming from the simulated rainfall is allowed to infiltrate through the pavement slab. A macroscopic pavement permeability equal to 0.01cm/sec is assumed in this example. Approximately one hour of infiltration was analyzed. Figures 5.20 and 5.21 show the evolution with time of the piezometric head and the effective degree of saturation of the pavement structure respectively. The non-homogeneous conditions of the analyzed problem is reflected in the variation of the piezometric heads (Fig. 5.20). Different patterns of

infiltration are observed in Fig. 20. Similarly, the presence of the drainage trench is also reflected in Figs. 5.20 and 5.21. The soil located above the drainage trench experience lower effective degree of saturation during the infiltration period.

	Soil 1	Soil 2	Soil 3	Soil 4
Kh (cm/s)	2.3×10^{-3}	0.04	1.3×10^{-6}	4.0×10^{-3}
Kv (cm/s)	2.3×10^{-3}	0.04	1.3×10^{-6}	4.0×10^{-3}
θ_0	0.04	0.1	0.196	0.04
$\Delta\theta$	0.01	0.19	0.171	0.01
α (cm-l)	0.006	0.006	0.001	0.006
β	3.5	3.5	2.0	2.5
γ	0.72	0.28	0.5	0.6

Fig. 5.19 Multilayer pavement geometry and hydraulic characteristics (case 1)

Fig. 5.20 Variation of the piezometric heads at different times

Fig. 5.21 Variation of the degree of saturation at different times

5.3.5 Infiltration process in a typical pavement structure (2nd case)

This example also represents the geometry of a pavement system (Figures 5.22) that can be found in Indiana highways. The bottom and lateral boundaries are assumed to be perfectly impervious. Symmetry was assumed for the cross-section. The geometric characteristics and hydraulic parameters are shown on Fig. 5.22. In order to examine the two dimensional drainage process of a stratified soil system, the worst scenario, full saturation of the pavement system, was assumed as initial conditions.

	Soil 1	Soil 2	Soil 3	Soil 4	Soil 5
Kh (cm/s)	2.0×10^{-4}	8×10^{-7}	0.07	3.6×10^{-5}	0.022
Kv (cm/s)	2.0×10^{-4}	8×10^{-7}	0.07	3.6×10^{-5}	0.022
θ_0	0.02	0.095	0.033	0.019	0.042
$\Delta\theta$	0.009	0.198	0.103	0.089	0.01
α (cm-l)	0.0028	0.008	0.012	0.0052	0.0028
β	1.68	1.45	1.58	1.74	1.68
γ	0.4	0.31	0.37	0.42	0.4

Fig. 5.22 Multilayer pavement geometry and hydraulic characteristics (case 2)

Fig. 5.23 Piezometric heads and degree of saturation after 45 minutes of drainage

Figure 5.23 show the piezometric head and effective degree of saturation of the pavement system recorded 45 min after the drainage process was initiated. Notice that the phreatic level is lowered at the drain location (Fig. 5.23). However, due to the unsaturated characteristics of the soils and despite the high coefficient of permeability of the edge drain the effective degree of saturation within the base is still quite large. No significant volume of water located above the drain pipe is removed (Fig. 5.23b). This example also demonstrate the importance of the unsaturated soil characteristics in the process of drainage.

5.4 Discussion

One-dimensional and two-dimensional examples of infiltration previously published in the literature have been re-analyzed using the developed model. In cases where a direct comparison with experimental data was feasible, the developed finite difference model has been found to give correct results. In cases where comparisons with previously proposed numerical models was feasible, The PURDRAIN finite difference model was found to give consistent results (Espinoza, 1993).

Several types of applications the proposed model have been outlined. The phreatic level can easily be determined by using the unsaturated-saturated flow model. Application of the unsaturated-saturated flow model shows that the phreatic line is not a streamline and that flow across this line can occur. The importance of the flow in the unsaturated region can be assessed by comparing the capillary fringe to the thickness of the aquifer.

It has also been shown using numerical examples that the rate of drainage is not only depending upon the soil hydraulic conductivity but also on the soil water retention characteristics. Thus, in choosing draining materials, its water retention characteristics have to be taken into account.

CHAPTER 6 - SUMMARY AND CONCLUSIONS

6.1 Summary

A two dimensional finite difference model for the analysis of moisture migration in pavement systems that can handle various boundary, initial and stratigraphic conditions has been presented. The analytical formulation is based on the theory of continuum for multi-phase flow. Appropriate simplifications have been introduced in order to model the pore water phase flow. The system of equations is solved by using the finite difference method. Explicit, implicit and the Crank-Nicolson algorithms have been implemented. The resulting matrix equation has been solved using indirect procedures using the Gauss-Seidel iterative techniques. Two different mathematical expressions of the Richard's equation were combined in the Decomposed-Local mixed formulation to yield more modeling flexibility.

A computer program, PURDRAIN, has been developed to implement the proposed model in a software environment. A total of four water retention models and three hydraulic conductivity models are available in the program. The numerical model has been validated using different one-dimensional and two-dimensional data obtained from previously published results. The proposed numerical model can be used to analyze simplified pavement sections as well as more realistic ones. Simplified pavement sections allows an easy interpretation of the results and the performance of parameter sensitivity studies.

6.2 Conclusions

Using the proposed methodology, the phreatic surface can be determined easily by applying unsaturated-saturated flow theory for several cases. The unsaturated-saturated flow model shows that the phreatic line is not a streamline and flow across this

line can occur. Depending upon the aquifer geometric and hydraulic characteristics, flow in the unsaturated region may be important. The importance of the flow in the unsaturated region can be assessed by comparing the capillary fringe to the thickness of the aquifer. As the thickness of pavement bases is usually of the same order as the capillary fringe height, accurate predictions of water flow require to take into consideration the unsaturated region. The quantity of flow computed in neglecting the unsaturated region may significantly differ from the actual ones.

It is shown using numerical examples that the rate of drainage is not only dependent upon the soil hydraulic conductivity but also on the soil water retention characteristics. Thus, in selecting draining materials, the water retention characteristics has also to be taken into account. Based on this considerations only, soils with large capillary fringes are not recommended as draining materials. Similarly, infiltration patterns are not only dependent on the amount of rainfall but also on the soil hydraulic parameters and initial and boundary conditions.

It is also shown using numerical examples that depending on the unsaturated hydraulic characteristics, large degrees of saturation may be expected below the pavement slab even after several hours of drainage.

The use of simplified models that only considers the saturated parameters (e.g. Casagrande and Shanon's) to evaluate the pavement base drainability may lead to unconservative results. These models usually predict larger amount of water drained in shorter times than is actually the case.

Modeling coupled saturated-unsaturated flow problems is in general more difficult than modeling separately saturated or unsaturated conditions. This is due to the degeneration of the governing Richards equation when crossing from an unsaturated to a saturated zone. The differential equation changes from a parabolic type to an elliptic one, this results in serious numerical difficulties. These difficulties are increased when sudden changes in flow occur.

Except for the Brooks and Corey model, most discussed material models can be used to represent adequately the soil hydraulic characteristics and used in the numerical solution of the Richards equation. However, solving the highly non-linear Richards

differential equation may, in some cases, pose extreme numerical difficulties solution leading unrealistic results. The proposed numerical technique tends to alleviate these numerical difficulties.

6.3 Recommendations for future research

Although it is difficult to measure soil water retention hysteresis in the laboratory, a numerical model that incorporates this effect would be desirable. To achieve this, the Scott's model may be preferred to the more exact but more troublesome Mualem's model.

Extensive laboratory tests on the different soils and materials utilized in typical pavement systems would be necessary. As the unsaturated-saturated analysis is critical for the drainage process, these tests should concentrate on defining the water retention curves near saturation; suctions values ranging from 0 to 2 atms are recommended. Values above 2 atms could also be obtained to completely define the water retention curves, but the fitting for the different models should concentrate on the recommended range.

Using a simplified geometry, currently used drainability criteria for pavement bases that can be improved drastically. The capillary fringe and the curvature can be used to represent the unsaturated characteristics of the soil. The use of a simplified geometry would allow to represent the results graphically in dimensionless form which would be convenient for practical design.

As hydraulic soil properties are usually spatially variable and difficult to measure in practice, uncertainty is one of their major attributes. Such uncertainty should be accounted for in view of predictively modeling unsaturated flow by incorporating probabilistic arguments into the numerical model presented in this report.

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APPENDIX A - PROGRAM PURDRAIN USERS GUIDE

A.1 INTRODUCTION

As the main objective of this research is the study of the hydraulic behavior of pavement bases and sub-bases —taking into account the unsaturated zone— the input data necessary to define the geometry has been customized to specifically address this issue. The restrictions imposed on the geometric characteristics of the region under study does not prevent the study of other problems of practical interest which can be accommodated within the framework of this work. In addition, the governing equations employed in the development of the present computer program are quite general. Thus, further modifications of the program can be easily done to encompass other problems of practical interest.

This User's Guide provides a comprehensive description of the different options available in the program. These options can be classified into five main groups: (a) preparing the geometry of the problem (e.g., coordinates of the surface, layer thickness, geometry of the drain, location of the pavement); (b) file handling (input and output files); (c) graphic representation of the input and output data; (d) dealing with the material parameters used for the hydraulic description of the soil (permeability, porosity, etc); and (e) exclusively necessary for the numerical algorithm used in the program (e.g., grid sizes, time step, tolerances, etc.). All this options have been grouped conveniently in different menus in order to facilitate the editing process as well as to enhance the understanding of the meaning and function of the different parameters.

The options encompassed in groups (a),(b) and (c) can be explained within the framework of this appendix. An understanding of the theory of flow through unsaturated porous media is required to fully understand the meaning of the options classified in group (d). For that matter, the reader should refer to Sections 2 and 3 of this document. Similarly, the options classified in group (e) require a basic understanding of numerical analysis and the different algorithms available for solving partial differential equations. Details of the numerical analysis are presented in Section 4 and 6. For a complete discussion, the above mentioned sections should be referred to as well as the references indicated therein .

The program PURDRAIN provides a user friendly environment and the various options are presented in a sequence of windows which have been designed following the standards found in most commercial programs. The menu structure of the main program along with its corresponding submenus are discussed in detail in the subsequent sections.

A.2 STARTING PURDRAIN

The program can be run on a DOS compatible PC with at least 512K of RAM. A PC equipped with the Intel 80486 processor and 4Mb and sufficient hard disk is preferable. The only file required to run the program is the PURDRAIN.EXE file. The program is started by typing "PURDRAIN". Once the program is loaded, the cursor appears on the main menu screen (Fig. A.2.1). One can choose any of the items just by pressing the key corresponding to the capital letter (highlighted) of the items shown in the menu (Creating/editing, Execute, Files, Graphics, Quit). The Up/Down/Left/Right arrow keys can also be used to move the highlighted bar to the desired item which is then selected by pressing the RETURN key. Every option of the main menu encompasses a number of other options that can be combined to suit the users needs.

Large amounts of data input is quite common in computer programs that deal with numerical algorithms for solving partial differential equations. This problem is partially overcome by using input data files which are pre-processed in a given format so they can be read by the program. Although the use of such files facilitates the input data processing, there are still some difficulties involved in their preparation. These difficulties are minimized in PURDRAIN by a flexible input data processor which enables the user to navigate through the different options available with ease. The implementation of user-friendly menus in the program expedites the editing process. The amount of information required to define the geometry of the problem has been reduced to a minimum. The mesh is automatically generated based on this information.

A.3 EDITING THE INPUT DATA

This section provides a description of the editing facilities available in this program. A number of routines have been developed to make data editing a fairly simple task. Each element of the data to be input to the program is presented in a field with a given width which has

Figure A.2.1 - Main Menu Layout

already been customized according to the expected needs of the user.

Three data types are implemented in the program and these are: Real, Integer, and String (Fig. A.3.1). The data type can be easily recognized by simple visual inspection of the field shown on the screen. A Real type usually has a decimal point with a specified number of decimal places. A real type can be input in exponential form which is usually converted to a pre-specified format. An Integer does not have any decimal point. When a data type is numeric (Real or Integer) the cursor is always shown at the beginning of the field. An string type can be recognized by the position of the cursor which is always placed after the last character.

In order to avoid trivial mistakes regarding the data type, an error trap routine has been implemented. Thus, if the required data is supposed to be an integer and accidentally a letter or decimal point is typed a beep sound will warn the user and the character just typed will not be accepted.

Point	Y-value
1).-	0.00
2).-	8.38
3).-	8.58
4).-	8.88
5).-	8.96
6).-	1.88

Figure A.3.1 - Typical one-column data

The available field that has been assigned to each variable is shown on the screen with a different color from that of the screen background. When a group of fields are shown on the screen the active field can be recognized by the location of the cursor as well as by the color of the field. The Left/Right arrow keys can be used to move the cursor to any position within a field, without overwriting the existing data. The default mode during the editing process is overwrite. The INSERT key can be used to switch from insert to overwrite mode. A thicker cursor indicates that the editing is being done in the insert mode. The DELETE and BACKSPACE keys can be used to erase part of the field. The edited field is not updated until the Return or the Up/Down arrow key is pressed. If the key ESC (Escape) is used instead, the changes performed will not be stored. The HOME key can be used to take the cursor to the beginning of the field and the END key takes the cursor to the end of the field. All these options make it easy to edit a given variable.

The format for input data shown on the screen is usually presented in one column (Fig. A.3.1), and in two columns (Fig. A.3.2) when the input data required are coordinates. When the data is presented in a single column, the user can go from one field to the next just by pressing the ENTER key or by pressing the Down arrow key. To go to a previous field, the Up arrow or TAB key can be used. When the data is presented in more than one column the Up/Down arrow keys can be used in a similar way as before. To move to the right or left fields the Right/Left keys cannot be used; instead the RETURN key will send the cursor to the field located to the right whereas the TAB key will move it to the field located to the left. In a group

of fields, the Page Up key will take the cursor to the first field of the group whereas the Page Down key will take the cursor to the last field of the group.

Coordinates X-Z of the Pavement Surface:		
Number of Points		
Point	X(cm)	Z(cm)
1).-	8.500	20.00
2).-	8.500	20.00
3).-	8.500	8.00
4).-	8.500	8.00
5).-	8.500	8.00

Figure A.3.2 - Typical 2-columns input data

A.4 MAIN MENU

The main menu is the menu that it is visible on the screen after loading the PURDRAIN program (Fig. A.2.1). The different options available have been grouped according to their functions. Fig. A.4.1 shows the structure of the menu system implemented in the program. There are five options shown in the main menu and a summary is presented next:

- 1) **Creating/editing Data** - The geometric characteristics of the pavement as well as the hydraulic parameters of the different soil types constituting the pavement system are specified. The data describing the rainfall event is also input in this menu. Once the pavement system has been completely defined a mesh is automatically generated.
- 2) **Execute** - Encompasses a number of options that are relevant for the execution of the program. Before the analysis is started, the initial and boundary conditions must be specified as well as the parameters that are necessary for the numerical algorithm. A Plot on the screen of the input geometry can also be performed within this menu.
- 3) **Files** - The main purpose of this option is the management of the input and output files. The data generated for a given problem is stored into specified files and can be later

retrieved for further analysis. To further facilitate the file operation a number of features usually found in the disk operating system (DOS) have been incorporated in this option.

- 4) **Graphics** - This option provides a powerful tool for the analysis of water infiltration in unsaturated porous media. Different plotting options are provided, thus the values of the piezometric heads, volumetric moisture content, degree of saturation, or matric suction can be plotted in different formats. The implementation of this tool within the framework of the program allows a quick evaluation of the physical admissibility of the results. Further data processing using more sophisticated commercial graphic packages can be performed separately.
- 5) **Quit** - This option sends the prompt back to the DOS commands.

These options can be easily accessed by typing the highlighted letter or by going to the desired option using the arrows keys.

A.5 CREATING/EDITING DATA MENU

This is the first option available in the main menu (Fig. A.5.1). It handles data regarding general aspects of the computation (**General**), the geometry of the pavement system (**Geometry**), the rainfall characteristics (**Rainfall**), and the hydraulic characteristics of the soil (**Material**).

Figure A.5.1 - Creating/editing Menu

Figure A.4.1 - Layout of the PURDRAIN menu system

A.5.1 The General Menu

This is the first option in the main menu and it contains the general settings for the program. It is subdivided into two different menus: Global and Time (Fig. A.5.1.1).

Figure A.5.1.1 - Creating/editing Menu

A.5.1.1 The Option Global

When Global is chosen a set of parameters with default values are shown on the screen (Fig. A.5.1.2):

- a) Job Remarks - It allows the user to input a short description (up to 70 characters) of the work to be performed.
- b) Root File - It provides the root name for the files that are to be generated during the execution of the program. Appropriate extensions will be added to each file generated. Although file names in DOS operating systems can have up to eight characters, the maximum length allowed herein is seven characters. This restriction is needed for the generation of plotting files. A more detailed explanation is given in section 7.0.
- c) Number of soils in the system - Indicates the number of soils that is going to be used to describe the pavement system (up to ten different types of soil can be specified).
- d) Period of Analysis (hr) - Specifies the total time (in hours) to be analyzed. This is not the time that the computer analysis will take, but the time period for which flow modeling is done.

- e) Water retention curve models [1,4] - The hydraulic behavior of an unsaturated soil can be described by different expressions that describe the degree of saturation of the soil as a function of the matric suction. Four of such models have been implemented in the program: Brooks & Corey (option 1), Van Genuchten (option 2), Brutsaert (option 3), and Vauclin (4). Depending on the model chosen, the corresponding parameters are required. A detailed discussion of the required parameters and their determination in the laboratory can be found in Section 3.2 of this document. The default value is [1], i.e. the Brooks & Corey model.
- f) Hydraulic models [1,3] - The variation of the hydraulic conductivity of an unsaturated soil can be described by different models. The permeability of the soil can be described as a function of degree of saturation or the matric suction. Three of such models have been implemented in the program: Brooks & Corey (option 1), Van Genuchten-Mualen (option 2), and Gardner (option 3). Depending on the model chosen, the corresponding parameters are required. A detailed discussion of the required parameters and their determination in the laboratory can be found in section 3.2. The default value is [1], i.e. the Brooks & Corey model.

08-Jul-93		PURDRAIN - 2D PAVEMENT INFILTRATION ANALYSIS		4:22 am	
Job remarks:					
ID Analysis for homogeneous soil - Brooks and Corey					
Name of the root file	DATA1				
Number of soils in the system	4				
Max simulation time (hr)	4.00				
Water retention curve model [1..4]	1				
Hydraulic model [1..3]	1				
Maximum number of records [1..10]	10				
Print on Screen 0,head,suction, or Sr [1..4] ...	0				
Initial conditions: constant (1)/from file (2)...	1				
Initial conditions Type: Sr(1)/Head (2).....	1				
Boundary conditions: Constant (0)/from file(1)...	0				

Figure A.5.1.2 - General Data

- g) Number of Records [1..10].- This parameter indicates how many times the computed values of piezometric head and degree of saturation is going to be stored in a given file. The default value is 8.
- h) Print (w/c(1), head(2), suction(3), Sr(4) - Partial results at each time step are shown on the screen during the infiltration analysis. This output can be in terms of moisture content (1), piezometric head (2), matric suction (3), or degree of saturation (4). An extra option (option 5), not shown on the menu legend, available when one dimensional analysis is performed, can be used to show the variation of the degree of saturation as well as the piezometric head at the same time. By printing the partial results on the screen the program allows the user to observe the development in time of the variable of interest and to assess whether the partial results are satisfactory (i.e. stable). If unsatisfactory results are displayed, the program can be halted and the proper adjustment of the numerical parameters performed. When the number of vertical and horizontal nodes are more than 6 and 15 respectively, the partial results at each node cannot be shown on the screen at once. In this case, only the results of selected horizontal and vertical nodes are shown on the screen. The program chooses automatically the nodes that will be shown on the screen.
- i) Initial conditions: constant (1)/ from file (2) - Two types of initial conditions can be input in the program: (1) constant within each layer or (2) variable along the whole region in that case, the initial condition is read from a file (see Section 6.2).
- j) Initial conditions type: Sr(1)/ Head (2) - The initial conditions can be specified in terms of (1) effective degree of saturation or (2) piezometric heads (see Section 6.2).
- k) Boundary Conditions: constant (0)/from file (1)- The bottom, left and right boundary conditions can be specified to be constant (0), i.e. the same boundary condition applies to a given boundary or variable (1) in which case the data has to be input from a file havin the extension '.BC' (see Section 6.2).

A.5.1.2 The Recording Option

Once all of the above information has been provided, the cursor goes back to the previous menu where the option Recording is shown. This option allows the user to input the

time at which the piezometric head and degree of saturation is going to be recorded. By default the specific times at which the computed results are to be recorded in files is computed by the program by dividing the total time to be analyzed by the number of records wanted. Consequently, the recorded values of piezometric heads (and degree of saturation) are uniformly distributed in time. Some flexibility is added to this option by allowing input of a non-uniform recording time distribution. Thus, the times at which the piezometric head and the degree of saturation are to be stored can be specified by the user.

A.5.2 The Geometry Menu

This is the second option found inside the Editing/creating menu. It allows the input of the geometric characteristics of the pavement system. It is divided for convenience into six sub menus: Surface, Layers, Pavement, Drains, Sources, and Mesh (Fig. A.5.2.1).

Figure A.5.2.1 - Geometry Menu

A.5.2.1 The Surface Option

The option Surface allows the user to input the coordinates defining the surface of the pavement system. Fig. A.5.2.2 shows an example of a profile defining the surface (points 1-6). The program requests the number of points defining the surface and, subsequently, displays two columns of fields which corresponds to the horizontal and vertical coordinates as in Fig. A.5.2.3. After the last coordinate has been filled, the command goes back to the previous menu. As road slope in the transversal direction is of the order of 3% to 5% which does not affect the infiltration patterns into the pavement system, these are input as horizontal lines (e.g.

section 3-4, Fig. A.5.2.2). For one dimensional analysis only two points defining the surface are required.

Figure A.5.2.2 - Location of the critical points

14-Mar-92 2-D PAVEMENT INFILTRATION ANALYSIS 8:08 pm		
Coordinates X-Z of the Pavement Surface:		
Number of Points		
Point	X(cm)	Z(cm)
1).-	0.00	20.00
2).-	10.00	20.00
3).-	0.00	0.00
4).-	0.00	0.00
5).-	0.00	0.00

Figure - A.5.2.3 - Typical ground surface input data

5.2.2 The Layers Option

The option Layers allows the user to input information regarding the geometric arrangement of the soil layers composing the pavement system which are characterized in

horizontal layers. According to the information required for pavement geometries, two types of horizontal layers have been implemented: the shoulder layers (Fig. A.5.2.4) and the layers beneath the shoulder-pavement system (Fig. A.5.2.5).

Figure A.5.2.4 - Pavement shoulder

Figure A.5.2.5 - Layers geometry

08-Jul-93		PURDRAIN - 2D PAVEMENT INFILTRATION ANALYSIS		5:00 am	
Drain number1					
Coordinates (x,z) of :					
the upper left corner.....		275.00	45.00		
the bottom right corner...		270.00	5.00		
Soil Type inside Trench.....					
Number of Grids (1,3,5....)					
Pipe Diameter (cm)			15.00		

Figure A.5.2.6 - Typical input data for each layer

For each layer the following information (Fig. A.5.2.6) is required:

- Layer thickness (cm) - It furnishes the thickness of the layer in centimeters. In case of shoulder layers, the total thickness must be equal to the pavement slab thickness.
- Minimum number of vertical grids - This option provides information regarding the numerical discretization of each layer. A coarse grid will expedite the calculation but it may give inaccurate results due to truncation errors. The appropriate grid size will depend on several factors which include the soil hydraulic and rainfall characteristics, and the pavement geometry. The computer memory is also a constraint to be taken into account.
- Left boundary of the layer (cm) - This option is available for layers beneath the pavement-shoulder system. These layers are usually composed of a different material at the layer mid-section (Fig. A.5.2.5). If a different material is to be specified, the left boundary of the soil placed in the mid-section of the layer has to be input (Fig. A.5.2.5).
- Right boundary of the layer (cm) - It indicates the right abscissa of the soil at the mid-section (Fig. A.5.2.5). This option is only available for layers beneath the pavement-shoulder system.

- e) Soil type at the mid-section - It allows the user to choose the soil type that composes the central region of the horizontal layer. This option is not available for shoulder layers.
- f) Soil type on the sides - It allows the user to choose the material type that composes both side regions of the horizontal layers which are assumed to be composed of the same material.

A.5.2.3 The Pavement Option

The option Pavement allows the user to input the geometric and hydraulic characteristics of the pavement slab which are defined by four parameters (Fig. A.5.2.7):

08-Jul-93		PURDRAIN - 2D PAVEMENT INFILTRATION ANALYSIS		4:54 am	
Macroscopic Pavement Permeability (cm/sec)...	0.0100000				
Pavement Thickness (cm).....	27.88				
Abscissa of the left Side of the Pavement.....	6.88				
Abscissa of the right Side of the Pavement ...	118.88				

Fig. A.5.2.7 - Typical Pavement Data

- a) Macroscopic pavement permeability (cm/sec) - This value mainly refers to the permeability obtained due to the presence of micro-cracks in the pavement slab. For simplicity, it is assumed that all micro-cracks are uniformly distributed along the pavement slab. This parameter should not be mistaken as the permeability of the material composing the pavement which is, for most practical purposes, impervious as compared to the surrounding soil.
- b) Pavement thickness (cm) - It specifies the slab thickness and should be the same as the sum of the thickness of the layers composing the shoulder otherwise an error will occur.

- c) Abscissa of the left side of the pavement.- It requests information regarding the horizontal coordinates of the left side of the pavement slab (Fig. A.5.2.8).
- d) Abscissa of the right side of the pavement.- It requests information regarding the horizontal coordinates of the right side of the pavement slab (Fig. A.5.2.8).

Figure A.5.2.8 - Geometry of the pavement slab

A.5.2.4 The Drains Option

The option Drains allows the user to indicate the number of drains to be placed in the pavement system. For each drain the following information is requested (Fig. A.5.2.9):

- a) Coordinates (x,z) of the upper left corner and the bottom right corner - These two points define the boundaries of a box within which the drain is located (Fig. A.5.2.10). The program automatically chooses the location of the drainage pipe which is at the center of the defined box and one half of the pipe diameter above the trench bottom.
- b) Soil type inside the trench - The soil surrounding the drain pipe is usually different from the one composing the base of the pavement. The program allows the user to specify the

14-Mar-92		2-D PAVEMENT INFILTRATION ANALYSIS		8:14 pm	
Drain number1					
Coordinates (x,z) of :					
the upper left corner.....	225.00	45.00			
the bottom right corner...	245.00	9.00			
Soil Type inside Trench.....	2				
Max size of the mesh in the trench (cm)....	15.00				
Pipe Diameter (cm)	15.00				

Figure A.5.2.9 - Typical input data for a drain

Figure A.5.2.10 - Typical cross section for a drain and sources

soil type used in the trench.

- c) Number of grids inside the trench - As in the case of the vertical layers, information regarding the discretization of the geometry is required.

- d) Diameter of the Pipe (cm) - The diameter of the drain pipe is required to be input by the user. This parameter is used to locate vertically the draining node in the numerical model.

A.5.2.5 The Sources Option

This option allows the user to simulate the presence of concentrated pavement crackings where water ponds and it can be modeled as sources of water. For each source the coordinates (x,z) of the upper left corner and the bottom right corner is requested. These two points define the boundaries of a box within which the source is located (Fig. A.5.2.10). The program automatically sets the piezometric heads of all nodes interior to this box equal to the maximum possible value of the piezometric head (i.e. the crack is full of water).

A.5.2.6 The Mesh Option

This option automatically generates the mesh based on the data provided to define the pavement geometry. Prior to the mesh generation, a minimum number of horizontal grids to be used in the horizontal direction is requested by the program. An average width of the mesh is then computed by dividing the total horizontal length into the requested number of grids. As the mesh is generated taking into account critical points of the geometry (corner points, change in soil characteristics, pavement boundaries, etc) the actual number of nodes in the horizontal and vertical directions may not be the same as the specified minimum values. The average width is used by the program to compute the number of grids between two adjacent critical points (Fig. A.5.2.2).

A number of different problems can be analyzed within this program by choosing an appropriate combination of the options described above. For example, a one-dimensional analysis can be simulated by choosing the number of horizontal grids to be equal to unity. If an unpaved road is to be analyzed, the pavement thickness must be specified to be zero as well as the remaining parameters dealing with the pavement.

A.5.3 The Rainfall Option

As rainfall events are commonly recorded in terms of precipitation vs. time, the required input is also in the same format. The time is registered in hours whereas the net precipitation in centimeters. This allows the user specification of variable rainfall rates (Fig. A.5.3.1). A positive slope will indicate a rainfall event whereas a flat horizontal slope will indicate that no rainfall has occurred in that period of time.

Figure A.5.3.1 - Typical rainfall data

A.5.4 The Material Option

This option is for input of all the parameters necessary to describe the hydraulic behavior of each type of soil. In unsaturated soils the hydraulic behavior is governed not only by the values of the saturated permeability but also by the values of the degree of saturation. The values of the permeability and degree of saturation are related to the values of matric suction through a mathematical function (Fig. A.5.4.1). There are different mathematical representations describing these variations and the most popular ones have been implemented in the program. A total of four different models describing the variation of the degree of saturation with suction and three describing the variation of hydraulic conductivity have been implemented in the present computer program.

Figure A.5.3.2 - Typical precipitation curve

The parameters have been divided into two groups (Fig. A.5.4.2). The first group consists of those parameters that are independent of the material model used. The first group of parameters are:

- a) Horizontal Saturated Permeability (cm/s) - This represents the maximum value of the horizontal permeability which occurs when the soil is fully saturated.
- b) Vertical Saturated Permeability (cm/s) - This is the maximum value of the vertical permeability which occurs when the soil is fully saturated.
- c) Irreducible Volumetric Content - This is a non-dimensional parameter which represents the value of volumetric moisture content that is always present in the soil. This is the value of the moisture content obtained in the laboratory for suction values of 15,000 cm (15 atm) (Van Genuchten, 1980).
- d) Volumetric water capacity - This quantity represents the capacity of the soil to store water in its voids. This value can be computed from the difference of the maximum moisture content value of fully saturated soils (i.e. the soil porosity) minus the value of the irreducible volumetric content.
- e) Maximum matric suction.- This value provides an upper limit for the suction values to avoid computer overflow. The default value is 15,000 cm.

Figure 5.4.1 - Typical variation of matrix suction and permeability vs. saturation

The second group of parameters are specific to the material model used to describe the variation of the matrix suction (ψ) —or permeability (k)— with the degree of saturation (S_e). The questions displayed on the screen will depend on the material model chosen for the analysis. Some researchers, e.g. Brooks & Corey (1964), have developed models describing both $\psi(S_e)$ and $k(S_e)$. For instance if Brook and Corey (1964) model is used to describe both the matrix

Fig. A.5.4.2 - Typical material parameters

suction and the permeability as function of the effective degree of saturation (Option 1 for both cases, see section A.5.1), the following information must be provided:

- Bubbling Pressure Head (cm) - This represents the maximum value of the matric suction (PB) having a value of the degree of saturation equal to unity (full saturation).
- Pore index parameter (ν) - This parameter is used to represent the behavior of the degree of saturation as a function of the matric suction.
- Unsaturated hydraulic parameter (n) - This parameter is used to describe the behavior of the soil permeability as a function of the matric suction.

A set of values of these parameters for different soil characteristics can be found elsewhere (Wallace & Leonardi, 1976). A comprehensive discussion of the laboratory procedure to obtain these parameters is presented by Laliberte et al. (1966).

Similarly, if Van Genuchten (1980) model is used to describe both the matric suction and permeability as function of the effective degree of saturation (Option 2 for both cases, see Section A.5.1), the following information must be provided:

- a) The α parameter - This value is used to represent the behavior of the degree of saturation with matric suction. The units of α has the inverse units of that of the piezometric head.
- b) The β parameter - This non-dimensional parameter is used to describe the behavior of the soil permeability as a function of the matric suction.

Other options Brutsaert (Option 3), and Vauclin (Option 4) are also available for the description of the suction as function of the degree of saturation. A description of the hydraulic conductivity of an unsaturated soil as function of the matric suction (Gardner, 1962) is also widely used. The Brutsaert and Vauclin models are usually combined with Gardner's model to provide a full description of the unsaturated soil hydraulic characteristics. A detailed discussion of the mathematical representation, the required parameters and their determination in the laboratory can be found in section 3.2.

A.6 THE EXECUTE MENU

Five options are available in this menu (Fig. A.6.1) and these are related to the numerical algorithm (Numerical), initial and boundary conditions (BC), screen preview of the geometry of the problem (Plot), and execution of the analysis (Steady and Transient).

Fig. A.6.1 - The Execute menu

A.6.1 The Numerical Parameters Option

As discussed in Section 2.0, the governing differential equation of water flow through unsaturated porous media is a highly non-linear differential equation and analytic solutions of the above equation for a variety of problems of interest cannot be obtained. Hence, the differential equation has to be solved numerically. The continuous domain in time and space is discretized reducing the problem to solving a non-linear system of equations. The different algorithms and methods for solving a system of equations have been obtained from the theory of numerical analysis for solving partial differential equations and are discussed in detail by Espinoza et al. (1992).

The parameters of this option have been further divided into three groups as shown in Fig. A.6.1.1. The first group contains those parameters that deal with the time domain discretization; the second group contains those parameters that are exclusively related to the numerical solution and the assigned default values could be used in most problems; and the third group refers to the criteria used for convergence. In order to ensure convergence of a given problem a suitable combination of these parameters has to be chosen. These parameters can be modified by the experienced user so as to expedite the rate at which the program converges to a stable solution.

Time Discretization Parameters

Selection of the time step (Δt) is a key factor in avoiding numerical instability in the solution of the system of equations. The time step is not constant throughout the analysis but is varied according to the partial results obtained at each iteration. The program automatically increases or decreases this time step according to a given criteria. The criteria used to adjust the time step is based upon the computed values of the piezometric heads. At each time step, the values of the piezometric head and the corresponding values of the degree of saturation are evaluated. The values of the piezometric heads are compared with those obtained in the previous time step. If the calculated variation is less than a specified minimum (provided by the user), the time step to be used in the next iteration is automatically increased. On the other hand, if the calculated variation is larger than a specified maximum, the time step to be used

in the next iteration is automatically decreased. The factor use to increase or decrease the time step is equal to 1.3.

08-Jul-93		PURDRAIN - 2D PAVEMENT INFILTRATION ANALYSIS		6:04 am	
PARAMETERS FOR TIME DISCRETIZATION					
Time Increment (sec)					0.00
Max Time Increment (sec)					0.00
Min Time Increment (sec)					0.00
Total time simulated (sec)					0.00
Maximum increment allowed at each iteration ..					0.00000000
Minimum increment allowed at each iteration ..					0.00000000
PARAMETERS FOR THE NUMERICAL ALGORITHM					
Weighting parameter u [0,1]					1.00
SOR parameter w [0,2]					1.00
AOR parameter r [w,2]					1.00
TOLERANCES FOR THE CONVERGENCE CRITERIA					
Tolerance for head and Volume convergence ...					0.00000000
Maximum Number of Iterations					6

Fig. A.6.1.1 - Numerical parameters

A varying time step is important when analyzing a heterogenous region composed of soils with different hydraulic characteristics. For instance, if the infiltrated water is passing through a low permeability zone, changes in the degree of saturation take place slowly and the time step can be increased without any loss of accuracy. On the other hand, when the infiltrating water front reaches a highly permeable region, changes in the degree of saturation take place rapidly and large time steps might not be adequate to describe the variation of the degree of saturation with time. This inadequacy can cause numerical problems and it would be prudent to reduce the time step.

The set of parameters that are related with the time domain discretization are the following (Fig. A.6.1.1):

- a) Time increment (sec) - This is the initial time step used by the program in the analysis.

- b) Max Time increment (sec) - It sets an upper bound for the time step. If the computed time step is larger than this value, it is reset equal to this value.
- c) Min Time Increment (sec) - It sets a lower bound for the time step. If the computed time step is lower than this value, it is reset equal to this value.
- e) Total Time analyzed - As the execution can be stopped at any time, this option indicates the total time for which the analysis has been done.
- f) Maximum increment per iteration - This provides a criterion for reducing the value of the time step. If the total change in the piezometric heads at a given time step is greater than this value, a reduction in the time step is automatically performed.
- g) Minimum Increment per iteration.- It provides a criterion for increasing the value of the time step. If the change in the piezometric heads a given time step is smaller than this specified value, the time step is automatically incremented for the next iteration.

Numerical Algorithm Parameters

The parameters of the second group are related to the different choices available for solving the partial differential equation. The differential equations in the time domain can be solved using the explicit, implicit or Crank-Nicholson scheme. A more complete discussion of the different procedures usually employed for time domain discretization can be found in texts dealing with numerical analysis (e.g., Dahlquist & Bjork, 1974; Carnahan et al., 1969).

Similarly, the resulting system of equations in the space domain can be solved using well known iterative procedures, namely, Jacoby, Gauss-Seidel, or SOR (Rice, 1981). All of these procedures have been implemented in the program and they can be easily chosen by selecting the appropriate parameters.

- a) Weighting Parameter, u [0,1] - This is used to indicate how the piezometric heads are going to be updated in the time domain. This parameter is a real number that varies from 0 to 1. The most common values are 0, 0.5 and 1 that indicate fully explicit, Crank-Nicholson and Fully Implicit analysis schemes respectively. Other values different from the ones presented can also be used. The default value of this parameter is 1.0 which leads to the implicit scheme.

- b) SOR parameters, w [0,2] - When the soil mass becomes saturated the system of equations becomes independent of time; thus, the weighting parameter (u) does not have any effect in the iterative process. Theoretically, the system of equation could be solved without iterations. Nonetheless, due to memory constraints, iterative procedures are used to solve the resulting system of equations. To this end, most common procedures for solving linear equations have been incorporated within this program and appropriate selection of the SOR parameter will lead to these procedures. This parameter varies from 0 to 2. The most common values are 0 for Jacobi's procedure, and 1 for Gauss-Seidel's procedure. The rate of convergence depends upon this value and there is an optimum value of this parameter (the one that gives the fastest rate of convergence) but its analytical evaluation is quite complex.
- c) AOR parameter, r [w ,2] - This is the very well known accelerated overrelaxation method (AOR). The valid range for this parameter is from w (the SOR parameter) to 2. This parameter is also used to accelerate the convergency of the linear system of equations. Although the full range is implemented in the program, it is recommended that set be equal to the SOR parameter equal to unity.

Convergency Criteria Parameters

The error criteria is also very important in the numerical solution of partial differential equations. Convergence to a solution is said to be achieved when a given parameter, namely, the piezometric head, matric suction or degree of saturation does not change in subsequent iterations.

When the soil is unsaturated the values of the matric suction, degree of saturation and piezometric head are related to each other through a mathematical function. Thus, theoretically, any of these parameters could be used to evaluate whether the error tolerance are met. However, due to the non-linearity of the governing equation, in order to ensure equilibrium at every time step, two error criteria have been implemented to monitor the convergency of both the piezometric heads and the volumetric water content (see section 6.5.2). Two parameters for convergence required by the program:

- j) Tolerance for head and volume convergence.- This parameter is usually set equal to 1%.

- k) Maximum number of iterations.- This parameter allows to set a maximum number of iterations per time step.

A.6.2 The BC Menu

This option refers to the initial and boundary conditions (Fig. A.6.1). To completely define the problem the initial and boundary conditions have to be specified before any computation is performed. The initial conditions are the values of the piezometric head that are assigned at the beginning of the analysis whereas the boundary conditions are the conditions that are specified for the piezometric head at the boundary. This menu is divided into three menus: Initial, Boundary, and View (Figure A.6.2.1).

A.6.2.1 The Initial Option

For the initial conditions, two options have been implemented in the program. The first option assumes that the initial conditions are the same for a given soil type. To give more flexibility to the program, an alternative option which allows the user to input a variable initial conditions is also available. In either case, the initial conditions can be specified in terms of degree of saturation or piezometric heads.

As the mesh is generated by the program, a file containing the initial conditions cannot be created prior to the mesh generation. As partial results at pre-selected times are stored in different files, a number of files containing the values of the degree of saturation and the piezometric heads are generated. Any of these generated files can be later used as initial conditions in a subsequent analysis.

A.6.2.2 The Boundary Option

The boundary conditions at the left, right or bottom side can be assumed to be impervious (1), open to the atmosphere (2), or constant a head (3). If the boundary conditions at any of the sides has more than one option (e.g. half of the boundary is assumed to be impervious and the rest open to atmosphere), the input data has to be given in a input file that has an extension '.BC'. The file can be prepared using the program PURDRAIN

Fig. A.6.2.1 - The initial conditions menu

A.6.2.3 The View Option

This option (Fig. A.6.2.1) allows the user to print the values of the specified initial conditions on the monitor screen.

A.6.3 The Plot Menu

Before any computation is performed, it is recommended that the user preview the geometry of the problem to be analyzed using the Plot command (Fig. A.6.1). This provides an insight regarding the geometric input data and whether they were correctly furnished. The graphic options available in this menu are essentially similar to those explained in section A.8 and the reader is referred to that section.

A.6.4 The Steady and Transient Options

After the boundary and initial conditions have been completely defined two options Steady (for steady state problems) and Transient (evaluates the variation of the piezometric heads with time) can be used to start the analysis (Fig. A.6.1). For transient problems, the partial results obtained at each time step are shown on the screen. The computation can be stopped at any time by pressing the SLASH ('/') key and re-started by choosing the corresponding option. This feature has been provided to halt the computation when any noticeable error of the partial results shown on the screen is observed.

A.7 The Menu Files

Due to the large amount of data generated by this program in a given analysis, partial results at each time step cannot be stored in memory. Thus, a number of input and output files are generated during the analysis. To facilitate the file operation a number of options have been incorporated in the present program. The identification of the files generated by the program is quite easy as they have the same name as the one furnished in the General menu but with different extensions.

So far, there are four main files extensions used by the program, these are: PRN, INP, PLT, BC. The file with extension '.INP' is the input data that has been generated by the user and stored for further analysis. Once the data is generated using the PURDRAIN computer program, it is recommended that it be stored in a given file before any computation is performed, so if anything goes wrong while computing the heads, the user can easily retrieve the input file.

The file with extension '.PRN' is a file generated by the program and it contains all the information regarding a given analysis. First, a description of the input data is presented in this file. This is followed by the values of piezometric head and degree of saturation at different times (as requested prior to the execution of the program, see section A.5.1).

The files with extension '.PLT' contain the information regarding the piezometric heads and degree of saturation for different times. The format used in this files is such that the information is easy to retrieve for plotting purposes. The first value in the file indicates the time at which the piezometric heads and degree of saturation were stored. Following that, the data is presented in four columns; the first two columns indicates the x and z coordinates, the third column indicates the value of the piezometric head and the fourth column the value of the degree of saturation.

The files with extension '.BC' contain the information regarding the boundary conditions at the left, bottom and right boundary for cases where complex boundary conditions are specified.

For instance, if the root file name is called DATA1, the output file will be called DATA1.PRN. The name of the file used for storing the input data is called DATA1.INP and the file name of the output data generated in a $(x,y,f(x,y))$ format for plotting purposes are called

DATA1A.PLT, DATA1B.PLT, DATA1C.PLT, and so forth, where the suffixes A,B,C, add to the root name DATA1 is to differentiate the output data obtained at different times. As a letter is added to the file name, the original root file name should not be larger than seven characters (8 is the maximum number of characters for a valid file name allowed by DOS operating system).

The default file extensions can be changed but it is recommended that the suggested extensions be maintained so as to standardize the files names, making it easier to search for a particular one. The following options are available in this menu (Fig. A.7.1):

- a) **Save** - Once the data required for a given problem has been furnished, it can be saved in a specified file (with extension [.INP]) so it can be used again without having to re-type all the information. It is recommended that the input data file be saved before any calculation is started. The program warns the user if a file with the same name is already present in the work directory. The file can be overwritten or the file name changed.
- b) **Read** - Loads any file previously saved with the option Save. This option facilitates input data processing. If an error occurs during this operation, a message will be displayed on the screen with the corresponding error code and error message, to assist in modifying the mistake.

Figure A.7.1 - The file menu

- c) **Directory** - Allows changing the current work directory to a specified one. It also shows the files available in the work directory. Valid prompts to be used in this option are those found in the DOS commands, e.g. `*.*`, `*.exe`, `DATA1.*` and so forth. For more details, refer to the `Dir` command in a DOS manual. If only the directory path is given, the path should end by a back slash character (i.e. `c:\dat1\`). The default directory is the current one from which the program is started.
- d) **Mkdir** - This option enables the user to create a directory that can be used later to store all the information of a given analysis, avoiding undesirable file overwriting which can result in the loss of valuable information.
- e) **Rmdir** - This option is equivalent to that found in the DOS commands and it is added to the program to add more flexibility to the available options.

A.8 The Graphics Menu

As in most numerical procedures, the analyzed geometric domain is discretized and represented by a finite number of points called nodes. The coordinates and the piezometric heads are obtained only at the nodes. As the number of nodes increases the interpretation of the values at each node becomes more cumbersome. The powerful graphic capabilities provided by PASCAL language allow for the implementation of a graphical tool within the framework of this program so as to facilitate the interpretation of the final results.

Four independent variables are present in the analysis: the horizontal and vertical coordinates of the node, the piezometric head and the time. Thus, one of the variables must be kept fixed and the remaining three can be represented as a contour plot. Since the node location are independent of time, there are only three different combinations that can be graphically represented (Fig. A.8.1): (a) fixing the horizontal coordinate, and representing the variation of the piezometric head with depths for different times (X-fixed); (b) fixing the vertical coordinate, and representing the variation of the piezometric head with the horizontal distance for different times (to be implemented in the future) and (c) fixing the time, and representing the variation of the piezometric head in the (X,Y) domain (T-fixed).

Figure A.8.1 - The graphics option menu

A.8.1.1 The Choose Option

As the matric suction, degree of saturation, and volumetric moisture content are related to the piezometric head through a given mathematical relationship, any of these variable can be chosen to be graphically represented instead of the piezometric head. This can be done through the option Choose (Fig. A.8.1.1).

Figure A.8.1.1 - Available options for plotting

A.8.1.2 The X-fixed Option

It allows the user to represent any of the variables chosen with the option Choose (Fig. A.8.1.1) as a function of depth and time. The program goes to the next menu where the options Global, Change and View are shown (Fig. A.8.1.2). The option global is described in section A.8.2. When analyzing 2D problems, profiles for different x-coordinates can be plotted. The

option Change allows to choose a node in the horizontal direction for which saturation (or other variable) vs. depth for different times will be plotted when using the option View.

Figure A.8.1.2 - The X-fixed option

A.8.1.3 The T-fixed Options

It allows the user to represent the variation in the X-Z coordinates of degree of saturation or the piezometric head (chosen with the option Choose). The program goes to the next menu where the options Global, Color-Range and View are shown (Fig. A.8.1.3). The user specifies the intervals within which a given color is to be used during the plotting (Fig. A.8.1.4).

Figure A.8.1.3 - T-fixed option

A description of the different options available for customizing the screen output is presented next. More elaborate figures can be obtained by using standard graphical packages such as SURFER, AUTOCAD, and SIGMAPLOT. Nonetheless, the graphical tools

incorporated within this program (PURDRAIN) allow a quick evaluation of the feasibility/accuracy of the results obtained.

Input the Color Range	
Point	Y-value
1).-	0.00
2).-	8.28
3).-	8.48
4).-	8.68
5).-	8.88
6).-	1.88

Figure A.8.1.4 - Typical color range

A.8.2 The Global Settings

The options available in this menu (Fig. A.8.2.1) are intended to give some flexibility to the format of the graphical output. At present the following options are available:

Figure A.8.2.1 - The global menu

8.2.1 The Layout Settings

The first option found in this menu (Layout) allows the user to resize and locate the plot on the monitor screen. The options available are (Fig. A.8.2.2):

- % of Horizontal Screen - sets the proportion of the screen that the figure will occupy in the horizontal direction.

08-Jul-93	PURDRAIN - 2D PAVEMENT INFILTRATION ANALYSIS	6:18 am
% of the Horizontal Screen [30-100%]....	40.00	
% of the Vertical Screen [30-100%]....	10.00	
X Location of the Plot	0	
Y Location of the Plot	0	

Figure A.8.2.2 - Typical screen layout settings

- b) % of Vertical Screen - sets the proportion of screen that the figure will occupy in the vertical direction.

This two options are useful to establish a suitable ratio between the Horizontal and Vertical LENGTHS in order to minimize the distortion which is introduced when different vertical and horizontal scales are used.

- c) X Location of the Plot Upper Left Corner - allows the user to indicate the location of the horizontal upper left corner of the figure. Suitable values vary according to the type of monitor available (i.e. an EGA monitor with 640x480 will allow any value within 0 to 639). If a value of zero is chosen the figure will be centered in the horizontal direction.
- d) Y Location of the Plot Upper Left Corner - allows the user to select the location of the vertical upper left corner of the figure. Suitable values vary according to the type of monitor available (i.e. an EGA monitor with 640x480 will allow any value within 0 to 479). If a value of zero is chosen the figure will be centered in the vertical direction.

A.8.2.2 The X-settings

It has several options to customize the horizontal axis output (Fig. A.8.2.3). They are the following:

- a) Legend - contains the legend for the X-axis. It is automatically customize by the program, but it can be easily modified by the user.

- b) Minimum Value - sets the minimum value of the X-axis. The minimum values are computed by the program according to the type of output. This values can be modified to satisfy the user's needs.
- c) Maximum Value - sets the maximum value of the X-axis. This value is automatically evaluated by the program but it can be modified.
- d) Automatic scale - This option allows to choose whether the maximum and minimum values as well as the number of division will be computed automatically.
- e) Number of Divisions - The number of ticks to be shown on the axis is set by this option.

A.8.2.3 The Y-Settings

The options available in this option are the same as for X-settings.

A.8.2.4 The Z-Settings

The options available in this option are the same as for X-settings.

The screenshot shows a terminal window for the program 'PURDRAIN - 2D-PAVEMENT INFILTRATION ANALYSIS'. The window title bar includes the date '08-Jul-93' and the time '6:21 am'. The main content area displays the 'Legend.....:' section with the following settings:

Min Value	2.58
Max Value	12.00
Automatic Scale (Y/N).....	N
Number of Divisions	10

Figure A.8.2.3 - Typical axis settings

8.3 The Option View

This options allows the user to plot on the screen monitor the specified graph. The parameters for plotting are those that the user has previously furnished to the program by means of the other above described options.

A.9 Example

An example is presented to illustrate the use of the program PURDRAIN as well as the interpretation of the input and output files created during the computation. This example represents a typical one dimensional case. Two dimensional examples with variable stratigraphic conditions are shown in chapter 5.

A.9.1 One dimensional analysis

The geometric characteristics as well as the hydraulic parameters of the one dimensional example are shown on Fig. A.9.1.1 This example corresponds to a typical one dimensional infiltration in a homogeneous soil. The input data corresponding to this problem has been prepared using the PURDRAIN program and has been saved in a file named DATA1.INP. To facilitate the identification of the files corresponding to this example, they have been stored in a subdirectory named A:\DAT1. The files required to run this example are assumed to be stored in the A drive. It must be kept in mind that if the program resides on the hard drive within a given sub-directory, the appropriate path must be given to start the program.

After loading the program into memory, choose first the option 'Files' followed by the option 'Directory' (to change the working directory to a:\DAT1\). All files contained in that directory will be shown on the screen. After that, the option 'File' must be selected again and then within it the option 'Read' (to read the input data file name DATA1). Supplying the extension is not necessary since the extension is the same as the default one ('.INP').

Once the input data file has been read, the user can go to the option Creating/editing to see the data assigned to the different variables. As this is one dimensional problem, the number of horizontal grid is equal to unity. The analysis is started by quitting from this menu and choosing the option Execute (in that option, after choosing the appropriate initial conditions ('Homogeneous' in this case), the assigned values at some of the nodes can be seen by choosing the option 'View'). The analysis is now started by choosing the Run option.

A file named DATA1.PRN containing a list of the input data used in the program as well as the values of the piezometric heads and degree of saturation is generated. In addition, files with extensions PLT (DATA1A.PLT, DATA1B.PLT, ..., DATA1I.PLT) are also created in the directory A:\DAT1.

Figure A.10.1.1 - Geometry and soil characteristics of Example 1

After the computation has been performed the results can be observed by going to the option Graphics. By choosing the option 3 (to see the variation of the degree of saturation) and then the option 'X-fixed', the variation of the degree of saturation with depth for different times can be seen. Next, by choosing the option 'T-fixed' and choosing the file DATA1H a plot is observed on the screen.

Once the analysis is completed the files generated with the extension '.PLT' can be used as input data files. To this end, the option 'Variable' initial condition can be chosen. It is important to point out that if the analysis is performed again the previous files are erased and replaced by the new results generated. Hence, if the previous results are not to be lost any of the following options should be used: (1) go to the 'Editing/creating' menu, from where the

option 'General' is chosen and the root file name is changed; or (2) the work directory can be changed to some other name.

A.9.2 Two dimensional example

The geometric characteristics and hydraulic parameters of this example are shown on Fig. 5.13. This example corresponds to a two dimensional infiltration into a homogeneous soil. The pavement was assumed to be perfectly impervious, thus the only source of water is at the shoulder. The input data corresponding for this problem was prepared using the program PURDRAIN and it is stored in a file named DATA2.INP. The files corresponding this example are stored in a subdirectory named A:\DAT2.

Similar to the previous example, the work directory should be changed to 'A:\DAT2\' and the input file DATA2.INP read. Next, the user must go to the different options available in the Creating/editing menu to see the values assigned to this problem and to become familiar with the options available in the program. The geometry of the region to be analyzed can be seen by choosing the option 'Plot' available in the Execute menu. From within the menu 'Initialize', 'Homogeneous' is to be chosen to set the initial conditions and the analysis performed by choosing the option Run. The files containing the results of this analysis can be found in the directory a:\DAT2.

Once the analysis are done finished, the option graphics can be selected to plot the results obtained. Choosing the option 3, and 'X-fixed' option the variation of the degree of saturation with time is observed. Next, selecting the option 'T-fixed' and the file name DATA2F.PLT, the variation of the degree of saturation in the space domain can be seen.

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