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ABSTRACT

Frequently, a design or R&D engineer is asked a question: "Will that compressor start if we change, if we use, etc." Usually the answer may be given after long testing. This paper offers a method of fast assessment of compressor startability by means of computer simulation. The core of the paper is a simple fractional-polynomial model of an electrical induction motor that is suitable for modeling compressor start up and run. The torque-slip or torque-rpm dynamometer test data are fitted to the model that is based on the equivalent circuit diagram of an induction motor. Only two torque-slip or torque-rpm pairs that are measured at only one voltage and one frequency suffice to find all constants of the model. The identified model generates full range continuous, from zero rpm to synchronous rpm, torque-slip or torque-rpm function for any other voltage and frequency. The model constants have physical meaning of resistance and reactance of the circuit diagram. The model was also successfully used to identify friction loses of the compressor.

EQUATIONS OF MOTION OF A COMPRESSOR

Most frequently, a hermetic compressor is flexibly supported on suspension springs inside the shell, and the shell is flexibly supported by rubber grommets. This mechanical system of three rigid bodies has fourteen degrees of freedom. In this analysis, we restrict our analysis to the rotational motion of the rotor only as it is depicted in Figure A1 in the appendix.

When such a hermetic compressor that is driven by an electric induction motor is switched on, the torque that develops between the stator and the rotor accelerates the rotor in accordance with the equation (1).

$$J_1(\varphi)\cdot\dot{\omega} = T(\varphi,\omega); \qquad \dot{\omega} = \frac{d\omega}{dt}; \qquad \omega = \dot{\varphi} = \frac{d\varphi}{dt} \tag{1}$$

Where is

$J_1(\varphi)$	instantaneous moment of inertia of the rotor about the axis of rotation [kg.m^2]		
T	is torque acting upon the shaft [N.m]	φ	is angle of rotation as shown in Figure A1 [rad]
t	is time [s].		

Instantaneous moment of inertia of the rotor is

$$J_1(\varphi) = J_R + J_C\left(\frac{\dot{\psi}}{\dot{\varphi}}\right)^2 + m_C\left(\frac{\dot{x}_C^2+\dot{y}_C^2}{\dot{\varphi}}\right) + m_P\left(\frac{\dot{x}_B}{\dot{\varphi}}\right)^2 \tag{2}$$

Where

m_C	mass of connecting rod [kg]	m_P	mass of piston [kg]
J_R	is moment of inertia of the rotor proper [kg.m^2]		
J_C	moment of inertia of connecting rod with respect to its center of gravity [k.m^2]		

Coordinates of the position of the piston x_B, position of the CG of connecting rod x_c and y_c, and the corresponding velocities can be found from the geometry of the slider crank mechanism that is shown in Figure A1.

The torque acting between the rotor and the stator consists of several terms

$$T(\varphi) = T_m - T_C - T_P - T_B \tag{3}$$

Where

T_m	is torque of electric motor [N.m]	T_C	is compression torque [N.m]
T_P	is torque due to piston friction [N.m]	T_B	is torque of bearing friction [N.m]

For the purpose of the simulation of compressor start up, the compressor is assumed to have ideal valves. This means that the maximum pressure in the cylinder during the discharge period is constant and equal to the condensing pressure. Similarly, the minimum pressure in the cylinder during the suction period is also constant and equal to the evaporating pressure. To take into account function of valves, one can increase condensing pressure and decrease evaporating pressure by several percentage points.

The torque T_B due to bearing friction is

$$T_B = \sum_i \frac{\pi \cdot \mu \cdot d_i^3 \cdot L_{Bi}}{2 \cdot c_i} \cdot \dot{\varphi} \tag{4}$$

Where

d_i	is shaft diameter [m]	L_{Bi}	is length of bearing [m]
μ	is dynamic viscosity of oil [Pa.s]	c	is radial bearing clearance [m]

The torque due to the viscous friction of piston movement is

$$T_P = F_P \cdot R \cdot \sin\varphi \cdot \left(1 + \frac{R}{L} \cdot \frac{\cos\varphi}{\cos\psi}\right); \qquad F_P = \frac{\pi \cdot \mu \cdot d_P \cdot L_P}{d_C - d_P} \cdot \dot{X}_B \tag{5}$$

The compression torque is

$$T_C = F_C \cdot R \cdot \sin\varphi \left(1 + \frac{R}{L} \cdot \frac{\cos\varphi}{\cos\psi}\right); \qquad F_C = \frac{\pi \cdot d_P^2}{4} \cdot \left(p_C - p_S\right) \tag{6}$$

$$p_C = p_S \cdot \left(\frac{2 \cdot R + L + S_0}{R \cdot (1 - \cos\varphi) + L \cdot (1 - \cos\psi) + x_0}\right)^{k_S}; \qquad \pi \le \varphi \le 2\pi$$

$$p_C = p_D \cdot \left(\frac{S_0}{R \cdot (1 - \cos\varphi) + L \cdot (1 - \cos\psi) + x_0}\right)^{k_D}; \qquad 0 \le \varphi \le \pi$$

Where is:

p_C	instantaneous pressure in the cylinder [Pa]	p_S	suction pressure [Pa]
p_D	discharge pressure [Pa]	S_0	head clearance [m]
k_S	isentropic coefficient of suction gas [ ]	k_D	isentropic coefficient of discharge gas [ ]

TORQUE OF AN INDUCTION MOTOR

The torque-speed characteristic (or torque-slip curve) is the most important characteristic of an electric motor. It tells the magnitude of starting torque (locked rotor torque), breakdown torque (pullout, pull-down, or breakdown torque) and the magnitude of the torque at any other speed (slip) between zero and the synchronous speed. In order to develop en equation suitable for the start up simulation we start with a circuit model of an electrical induction motor.

CIRCUIT MODEL OF AN IDUCTION MOTOR

Figure 1 shows the Steinmetz circuit model of one phase of a polyphase induction machine. The expression for the torque as derived in [2] is

$$T_m = \frac{i \cdot V_{Th}^2 \cdot \frac{r_2}{s}}{\omega_s \cdot \left[\left(R_1 + \frac{r}{s}\right)^2 + \left(X_1 + x_2\right)^2\right]}; \qquad s = \frac{\omega_s - \omega}{\omega_s} \tag{7}$$

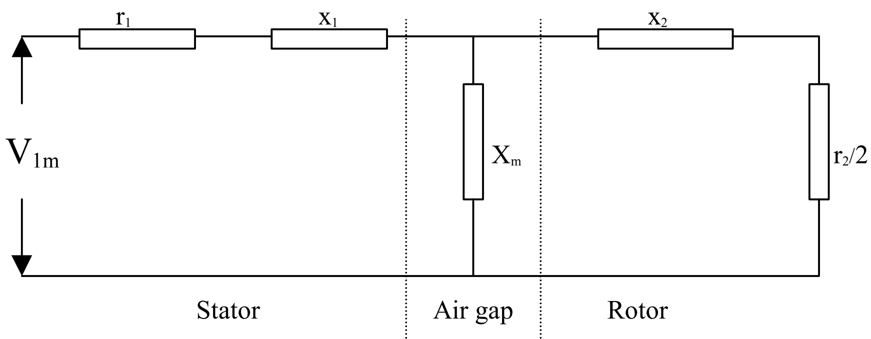

Figure1: Steinmetz model of one phase of an induction motor

In the equation (7) and in the Figure 1 there is:

$$V_{Th} \cong V_{1m} \cdot \frac{j \cdot X_m}{r_1 + j \cdot (x_1 + X_m)}; \qquad R_1 \cong r_1 \cdot \left(\frac{X_m}{x_1 + X_m} \right)^2; \qquad X_1 \cong x_1;$$

T_m	is torque [N.m]	i	number of electrical phases [1,2,3]
r_2	rotor resistance [Ω]	x_2	rotor reactance [Ω]
s	slip	ω_s	synchronous angular velocity [s^{-1}]
ω	actual angular velocity [s^{-1}]	V_{Th}	voltage of equivalent Thevenin generator [V]
V_{1m}	voltage applied to one phase [V]	j	imaginary unit
X_m	reactance of air gap [Ω]	r_1	resistance of stator winding [Ω]
x_1	reactance of stator winding [Ω]		

The equation (7) can be rearranged into a more convenient form

$$T_m = \frac{V_{1m}^2}{\omega_s} \cdot \frac{a_1 \cdot s}{\left(b_2 \cdot s^2 + b_1 \cdot s + 1\right)}; \qquad a_1 = \frac{i}{r_2}; \; b_2 = \frac{R_1^2 + (X_1 + x_1)^2}{r_2^2}; \qquad b_1 = \frac{2 \cdot R_1}{r_2} \tag{8}$$

It is possible to find constants a_1, b_1 and b_2 either by using motor design data, or by electrical measurements of the resistance and reactance (generally impedance) of the motor [1]. Nevertheless, in most cases these data ere not available or a detailed electrical measurement of all electrical parameters is not feasible. On the other hand, the electric motor manufacturers frequently supply the torque-slip or torque-rpm data. Thus, the easiest way of finding model constants a_1, b_1 and b_2 is to fit these data into equation (8).

TORQUE-SPEED MODEL OF AN INDUCTION MOTOR

The use of a commercial software such as EXCEL spreadsheet to fit the full-range torque-slip data into a polynomial equation of the type

$$T_m(s) = a_n \cdot s^n + a_{n-1} \cdot s^{n-1} + \cdot \cdot + a_1 \cdot s + a_0$$

is almost impossible. On the other hand, to fit torque-slip data into equation (8) is easy and accurate.

In order to fit torque-slip data into an equation similar to equation (8), we assume the terminal voltage of all stator windings is the same, and the frequency of electric current is constant, which is the usual case. These two assumptions enables us to write equation (8) in a shorter form

$$T_m = V^2 \cdot \frac{A_1 \cdot s}{B_2 \cdot s^2 + B_1 \cdot s + 1} \tag{9}$$

Where

V	is applied terminal voltage [V]	A_1	is constant [$N.m.V^{-2}$]
B_1,B_2	are dimensionless constants		

Only three torque-speed pairs are necessary to find all three unknown constants A_1, B_1 and B_2. One pair is always known. It is zero torque at synchronous speed where slip is equal to zero. Second pair is locked-rotor torque at which the slip is equal to one. The third torque-speed pair is the breakdown torque and breakdown slip at which the torque-speed curve has its local maximum.

Constant B_2 can be conveniently found by taking a derivative of equation (9)

$$\frac{dT_m}{ds} = 1 - B_2 \cdot s_M^2 = 0 \quad \Rightarrow B_2 = \frac{1}{s_M^2} \tag{10}$$

Thus, the constant B_2 is always equal to the squared reciprocal of breakdown slip.

Substituting constant B_2 into equation (9) and rearranging, the two other conditions yield

$$\begin{vmatrix} V^2 & -T_L \\ V^2 \cdot s_M & -T_M \cdot s_M \end{vmatrix} \cdot \begin{vmatrix} A_1 \\ B_1 \end{vmatrix} = \begin{vmatrix} T_L \cdot \left(1 + \frac{1}{s_M^2}\right) \\ 2 \cdot T_M \end{vmatrix} \tag{12}$$

Where is:

V	terminal voltage [V]	T_L	locked-rotor torque [N.m]
T_M	maximum (breakdown) torque [N.m]	s_M	slip at maximum (breakdown) torque

Solving equation (12) for unknown constants A_1 and B_1, we get

$$A_1 = \frac{T_L \cdot T_M \cdot \left(1 + 2 \cdot s_M - s_M^2\right)}{V^2 \cdot s_M \cdot \left(T_L - T_M\right)}; \qquad B_1 = \frac{2 \cdot T_M \cdot s_M - T_L \cdot \left(s_M^2 + 1\right)}{s_M^2 \cdot \left(T_L - T_M\right)} \tag{13}$$

Equation (9) is the torque speed model of an induction motor class A, B or C of NEMA classification [6]. These motors always have distinct local maximum (breakdown torque) on the torque-speed curve. This model can not be used to model NEMA class D induction motors since they do not have any local maximum on the torque-speed curve. Instead, these motors have maximum torque equal to the locked-rotor torque. NEMA class D motors are rarely used to drive compressors because of their poor efficiency.

Figure 2 is an example of torque-speed characteristic of an induction motor. The input data were: Test voltage 115V, test frequency 60 Hz, breakdown torque 1.25Nm, breakdown slip 0.2, and locked-rotor torque 0.5Nm. Torque curves at 97.75 and 126.5 volts (-15%, +10%) are also shown. The three torque-rpm pairs used to simulate continuous torque-speed are depicted by diamonds on the 115V test torque-rpm curve.

The same characteristics (for nominal test voltage and frequency) as those in Figure 2 are valid for any induction motor which speed is controlled by variable frequency, i.e. by an inverter, provided the inverter automatically adjusts voltage with frequency so that

$$\frac{V_1}{f_1} = \frac{V_2}{f_2} = \text{const.} \tag{15}$$

In such a case, the air-gap flux per pole is kept constant, and the torque-slip curve does not change. If the voltage is not adjusted with the frequency, then the constant A_1 in equation (9) should be replaced with constant A_2 that is equal to

$$A_2 = A_1 \cdot \frac{\omega_{s1}}{\omega_{s2}} \tag{16}$$

Where

ω_{s1} is synchronous angular frequency of the test current [s^{-1}]

ω_{s2} is synchronous angular frequency of the actual operating current [s^{-1}]

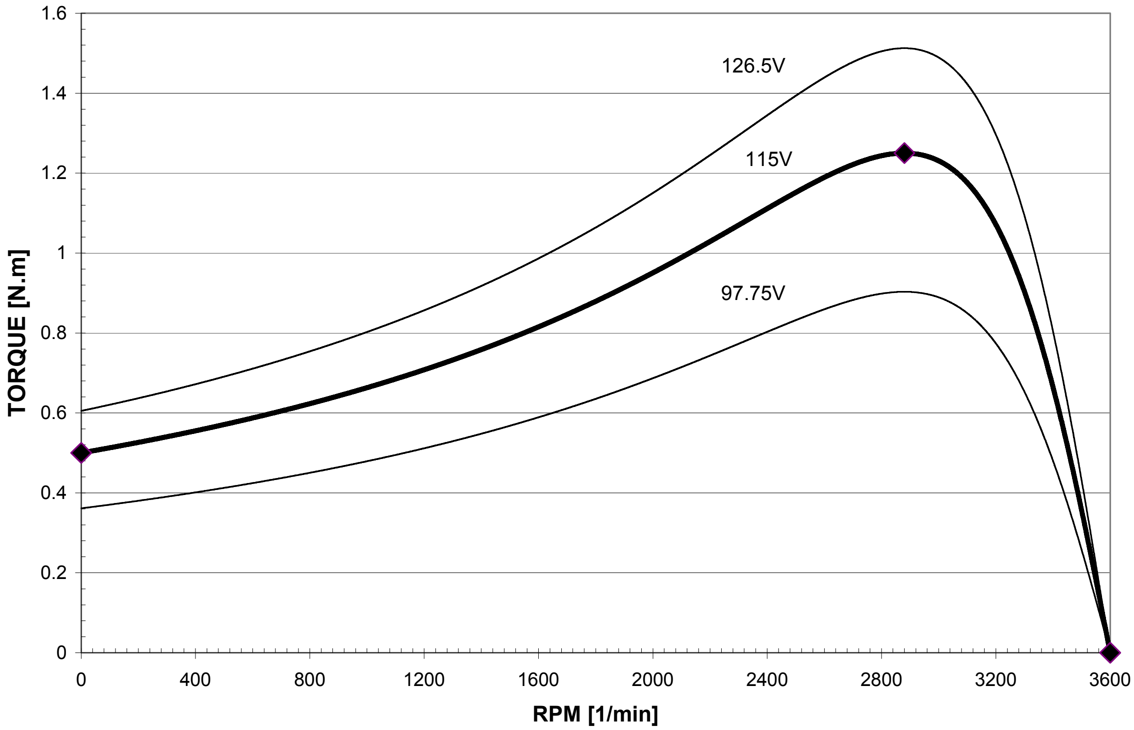

Figure 2: Simulated torque-RPM characteristics of an induction motor

When the actual synchronous frequency is lower than the test synchronous frequency, and the voltage is not adjusted in accordance with equation (15), the motor can overheat, and the stator winding can burnout. Nevertheless, in this case, the torque-speed curve will go up above the test torque-speed curve, and motor will become stronger.

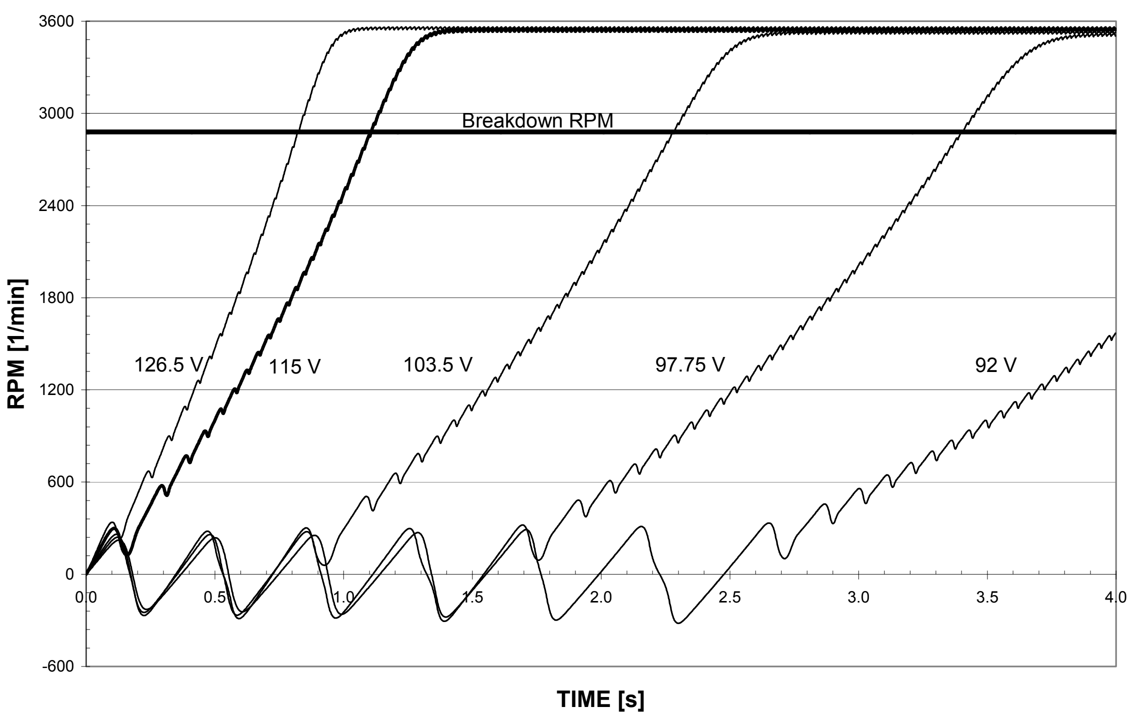

Figure 3: Simulated start of a single-cylinder refrigeration compressor

When the actual synchronous frequency is higher than the test synchronous frequency, and the voltage is not adjusted in accordance with the equation (15), nothing can happen to the stator winding, only torque-speed curve will go lower bellow the test torque-speed curve, and the motor will become weaker.

SIMULATING COMPRESSOR START UP

To simulate a compressor start up one needs to integrate equation (1). This is a nonlinear differential equation of the first order in the angular velocity ω. As equations (3), (5) and (6) show, all the terms in equation (1) are nonlinear. Although some sophisticated methods of integration of nonlinear equations are available [2], that have better accuracy, the fast Euler integration method has been chosen here

$$\omega_{n+1} = \omega_{n-1} + 2 \cdot \frac{T_n}{J_1} \cdot \Delta t \qquad (17)$$

Where is:

ω_{n+1} future angular velocity of the rotor at time $t+\Delta t$, ($n+1^{st}$ time step) of integration [s^{-1}]

ω_n present angular velocity of the rotor at the time t

Δt time step of integration [2]

The initial conditions yield angular velocity at the time Δt

$$\omega_1 = \frac{T_L}{J_R} \cdot \Delta t \qquad (18)$$

This method is fast and when the time step is sufficiently small, it has good accuracy too. A program written in Visual Basic for Applications (VBA) as a macro in an EXCEL spreadsheet was used to simulate start up of a small single-cylinder refrigeration compressor. The result of simulation is shown in Figure 3. It shows the compressor will start easily at 126.5 and 115 volts, and definitely, it will not start at 92 volts. This was experimentally verified.

APPENDIX

Expressions used in equations (2) through (6) are presented here [5]. Figure A1 shows geometry of slider-crank mechanism

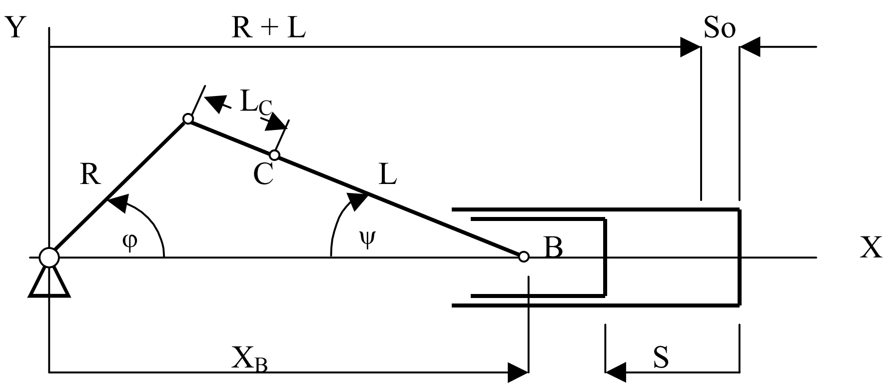

Figure A1: Geometry of slider-crank mechanism

Location of the piston

$$x_B = R \cdot \cos\varphi + L \cdot \cos\psi \qquad (A1)$$

$$y_B = 0$$

Location of the center of gravity C of connecting rod

$$x_C = R \cdot \cos\varphi + L_C \cdot \cos\psi \qquad (A2)$$

$$y_C = R \cdot \sin\varphi - L_C \cdot \cos\psi$$

Velocity of the piston

$$\dot{x}_B = -\left(\dot{\varphi} \cdot R \cdot \sin\varphi + \dot{\psi} \cdot L \cdot \sin\psi\right) \tag{A3}$$

Velocity of center of gravity C of connecting rod

$$\dot{x}_C = -\left(\dot{\varphi} \cdot R \sin\varphi + \dot{\psi} \cdot L_C \cdot \sin\psi\right) \tag{A4}$$

$$\dot{y}_C = \dot{\varphi} \cdot R \cdot \cos\varphi - \dot{\psi} \cdot L_C \cdot \cos\psi$$

Relation between φ and ψ

$$\sin\psi = \frac{R}{L} \cdot \sin\psi; \qquad \cos\psi = \sqrt{1 - \sin^2\psi}$$

Angular velocity of connecting rod

$$\dot{\psi} = \dot{\varphi} \cdot R \cdot \frac{\cos\varphi}{\cos\psi} \tag{A5}$$

Equivalent instantaneous moment of inertia of the rotor is found by summing kinetic energy of all moving parts

$$\frac{1}{2} \cdot J_1 \cdot \dot{\varphi}^2 = \frac{1}{2} \cdot \left(J_R \cdot \dot{\varphi}^2 + J_C \cdot \dot{\psi}^2 + m_P \cdot \dot{X}_B^2 + m_C \cdot \left(\dot{x}_C^2 + \dot{y}_C^2\right)\right) \tag{A6}$$

Equation (A6) multiplied by 2 and divided by $\dot{\varphi}^2$ yields equation (2).

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