January 2015

DESIGN AND CONTROL OF A HUMMINGBIRD-SIZE FLAPPING WING MICRO AERIAL VEHICLE

Jian Zhang
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By Jian Zhang

Entitled
DESIGN AND CONTROL OF A HUMMINGBIRD-SIZE FLAPPING WING MICRO AERIAL VEHICLE

For the degree of Doctor of Philosophy

Is approved by the final examining committee:

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Head of the Departmental Graduate Program Date
DESIGN AND CONTROL OF A HUMMINGBIRD-SIZE FLAPPING WING

MICRO AERIAL VEHICLE

A Dissertation
Submitted to the Faculty
of
Purdue University
by
Jian Zhang

In Partial Fulfillment of the
Requirements for the Degree
of
Doctor of Philosophy

December 2015
Purdue University
West Lafayette, Indiana
To my beloved family, my girlfriend and friends.
ACKNOWLEDGMENTS

I would like to thank my advisor, Dr. Xinyan Deng for giving me the support, guidance as well as freedom to explore this exciting research field of flapping flight and bio-inspired micro aerial vehicles; it has been a wonderful journey. It is my great honor to have Dr. George T. Chiu, Dr. Dengfeng Sun, and Dr. Jianghai Hu as my exam committee members. I would also like to thank Dr. Bin Yao and Dr. Peter H. Meckl for their advice as my academic advisory committee members. Special thanks for Dr. Bin Yao for his support for my first year of study with funding from wind turbine diagnostics project. Special thanks to Dr. David Cappelleri for serving on my prelim exam committee. I would like to thank my fellow lab members and collaborators, Dr. Bo Cheng (now at Penn State University), Fan Fei, and Zhan Tu, for their helpful discussions and fruitful collaborations on this project. I would also like to thank my fellow lab members, Jesse Roll, Yun Liu, Ying Chen, Giovanni Barbera, Eric Anderson, and Bixing Yan, at Bio-Robotics Lab, for their help and collaborations along the way.
PREFACE

Animal flight has been a great source of inspiration for man-made flying vehicles and has led to all the great technical achievements in human aviation history. Now a new breed of air vehicle, the Flapping Wing Miro Aerial Vehicle (FWMAV), inspired by insects and hummingbirds, holds the promises of hovering and maneuverability that was perfected by their biological counterparts. The aim of this work is to design such man-made system and close the performance gap between engineering system and animal flight in nature.
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ABSTRACT

Zhang, Jian. PhD, Purdue University, December 2015. Design and Control of a Hummingbird-Size Flapping Wing Micro Aerial Vehicle. Major Professor: Xinyan Deng, School of Mechanical Engineering.

Flying animals with flapping wings may best exemplify the astonishing ability of natural selection on design optimization. They evince extraordinary prowess to control their flight, while demonstrating rich repertoire of agile maneuvers. They remain surprisingly stable during hover and can make sharp turns in a split second. Characterized by high-frequency flapping wing motion, unsteady aerodynamics, and the ability to hover and perform fast maneuvers, insect-like flapping flight presents an extraordinary aerial locomotion strategy perfected at small size scales. Flapping Wing Micro Aerial Vehicles (FWMAVs) hold great promise in bridging the performance gap between engineered flying vehicles and their natural counterparts. They are perfect candidates for potential applications such as fast response robots in search and rescue, environmental friendly agents in precision agriculture, surveillance and intelligence gathering MAVs, and miniature nodes in sensor networks.

Designing and developing such systems is a challenging problem under stringent constraints in size, weight and power (SWaP). In addition, the lagging in battery technology, requirement on miniature sensors and actuators for navigation, limited on-board computational power, and system integration all pose challenges in design. Under the SWaP constraints, balance and trade-off must be made among mechanical complexity, controllability, power, and weight. Otherwise, even producing enough lift to sustain the weight can be a challenge. In this thesis, we present a systematic approach of vehicle design and optimization, resonance design, and flight control.

Achieving resonance in flapping wings has been recognized as one of the most important principles to enhance power efficiency, lift generation, and flight control.
performance of high-frequency FWMAVs. Most of the work on the development of such vehicles have attempted to achieve wing flapping resonance. However, theoretical understanding of its effects on the response and energetics of flapping motion has lagged behind, leading to sub-optimal design decisions and misinterpretations of experimental results. In this thesis, we systematically model the dynamics of flapping wing as a forced nonlinear resonant system, using both nonlinear perturbation method and linear approximation approach. We derived analytic solution for steady-state flapping amplitude, energetics, and characteristic frequencies including natural frequency, damped natural frequency, and peak frequency. Validated with both simulation and experiments, our results showed that both aerodynamic lift and power efficiency are maximized by driving the wing at natural frequency, instead of other frequencies. Interestingly, the flapping velocity is maximized at natural frequency as well, which can lead to an easy experimental approach to identify natural frequency and follow the resonance design principle. The result can serve as a systematic design principle and guidance in the interpretations of empirical results.

For the vehicle design and prototype of FWMAV, we presented a complete, multidisciplinary formulation for system design optimization and integration for a Hummingbird size FWMAV. The formulation covers actuation, wing, battery, electronics, dynamics, flight stability and control. System parameters considered include parameters of wings, motors, gears, springs, batteries, control authorities, and locations of poles and zeros of the system dynamics. The formulation was validated by experimental data for both rigid and flexible wings, covering low to high wing loading. Based on the direct motor drive mechanism of this work, the optimization yields a prototype with on-board sensors, electronics, and computation. It has a wingbeat frequency of 30Hz to 40Hz, with 12 grams of total weight and 20 grams of maximum lift. Liftoff was demonstrated with extra payloads. Initial results of on-board state estimation and flight control were performed. Flapping wing platforms with different requirements and scales can now be systematically designed and optimized with parameter modifications of the proposed formulation.
Not only do we have to design and develop the system under the SWaP constraints, we also need to control the system under those tight constraints as well. The superior maneuverability of insect flight is enabled by rapid and significant changes in aerodynamic forces, a result of subtle and precise changes of wing kinematics. The high sensitivity of aerodynamic force to wing kinematic change demands precise and instantaneous feedback control of the wing motion trajectory, especially in the presence of various parameter uncertainties and environmental disturbances. Current work on flapping wing robots was limited to open-loop averaged wing kinematics control. Here we present instantaneous closed-loop wing trajectory tracking of a DC motor direct driven wing-thorax system under resonant flapping.

Finally, we present an analysis on fundamental limitations of flapping flight control and discovered, for the first time, the non-minimum phase nature of flapping flight when certain controls are used. We then presented full nonlinear attitude and position controller with exponentially stable and globally exponential attractive properties. The dynamics and flight control results were then illustrated by experimental results.
1. INTRODUCTION

1.1 Bioinspiration

At large scale, the ingenious human engineering has created aerial vehicles that outperform the natural flying animals in terms of speed, range, and payloads, manifesting the extraordinary capabilities with inventions such as commercial airlines, fighter jets, and helicopters. Yet at small scale, no man-made machines come even close on matching the compact design and exceptional performance of insects and hummingbirds [25, 35, 61, 5, 30], shown in Fig. 1.1.

Figure 1.1. Flapping wing animals (A) Hummingbird (image: Bret Tobalske), (B) Fruit fly (Michael Dickinson), (C) Hawkmoth (Wikipedia), (D) Honeybee (Laidlaw Facility).
Flapping wings generate lift and control torques entirely from the reciprocating flapping wing motion with multiple degrees of freedom. The principle motions are wing flapping(stroke), wing rotation(pitching), and wing deviation, as illustrated in Fig. 4.2. Flapping(stroke) angle is the main back-and-forth motion defined in the stroke plane. Rotation(pitch) denotes the angle rotated by wing relative to its rotation axis, creating angle of attack during flapping motion. The deviation is the angle with which the wing rotational axis deviates from the stroke plane.

![Figure 1.2](image)

**Figure 1.2.** (a) Definition of wing kinematic parameters. (b) The body xyz coordinate system is fixed to the vehicle airframe. The wing flaps about the $\phi$ axis, which remains parallel with the body $z$ axis. The wing rotation $\psi$ axis is parallel with respect to the wing leading edge. Wing can also deviates from stroke plane with angle $\theta$, but deviation is not considered here.

Flapping wing aerodynamics scales favorably to smaller sizes [30]. On the lower end of the spectrum in nature, the smallest hovering insects such as Dicopomorpha echmepterygis (fairyflies) have body lengths as short as 0.11 mm, while on the other end of the spectrum, the largest hovering animals are the Patagona gigas (giant hummingbird) and the Glossophagine phyllostomid (flower bat), weighing up to 22 grams and 32 grams, respectively [56, 28]. The aerodynamic force generated by flapping wing generally scales to $m^{0.67}$, where $m$ is body mass for isometrically similar systems. The flapping wings will eventually not be able to generate sufficient lift as the body weight increases [27].
For such small scale systems, forces generated by flapping wings are dominated by low Reynolds number aerodynamics. Reynolds number, Re, is the ratio of inertial to viscous fluid forces, and as size decreases, viscous effects tend to dominate over inertial effects. With advances in the research on flapping wing aerodynamics over the past couple of decades [31], [30], [101], [67], [25], [85], forces generated at small size and low Reynolds number of typical insects/hummingbird was revealed, i.e., the unique unsteady mechanisms facilitating force enhancements including rotational circulation, clap-and-fling, delayed stall (the leading edge vortex), Kramer effect, and wake-capture. These fluid dynamic phenomena underlying flapping flight are fundamentally different from those of non-flapping surface generated traditional aerodynamics that large man-made aircraft are based on. The unsteady aerodynamics mechanism greatly boosts the forces generated on the wing, thus enabling insects to hover or maneuver. Further studies [55, 115] demonstrated the favorable scaling of flapping wings at the small size of insects/hummingbird, compared with traditional rotary wings.

Over the last few decades, biologists have studied the exceptional flight stability and maneuverability of flapping-wing insects and hummingbirds [25, 35, 61, 5, 30]. One such example is the saccade maneuver with which the fruitfly can achieve 90 degree direction change within 50ms [35]. The source of this maneuverability lies in insect/hummingbird’s abilities to make precise and subtle changes of three degree-of-freedom (DOF) wing kinematics that result in rapid and significant changes in aerodynamic forces [5, 46]. The design of Flapping-wing Micro Aerial Vehicles (FW-MAV) with similar capabilities would be perfectly suited for applications where it is necessary to maneuver through very tight spaces and navigate cluttered environments. They are perfect for potential applications, fast response robots in search and rescue, environmental friendly agents in precision agriculture, surveillance and intelligence gathering MAVs, and miniature nodes in sensor networks.

The earliest bio-inspired flapping wing systems are bird-inspired ornithopters. With improved lift coefficients over fixed wings [80], they are successful in applications
ranging from surveillance to toys. However, such ornithopter systems are typically not capable of hovering and lose stability when forward flight velocity decreases to zero.

For applications that require hovering, traditional systems such as helicopters, quadrotors, and VTOL with directed jet propulsion are typically the top choices. For those systems, rotating wings (blades) generate lift, driven by either engines or electric motors. Even at small size, such systems have achieved commercial success. The the Prox Dynamics Pico-flyer weights 3.3 grams and is the smallest untethered helicopter to date [73]. The larger Prox Dynamics PD-100 Black Hornet incorporates an onboard GPS and weights 16 grams [29].

In general, high-speed rotary motion is difficult to realize as system size decrease further down. Electromagnetic forces scale down at a rate of $L^4$, where $L$ is the wire length in a magnetic field. As motors scales down, the torque drops significantly. Adding gear transmission can recover torque generation but at the cost of further increasing motor speed and frictional loss. The smallest rotary wing platform is the magnetically actuated device in [70, 69]. The system weighs only 3.5 mg and operates within a external magnetic coil system. At 3 grams and a width of 4 cm, the larger Mesicopter is composed of polymeric materials with four rotors and has demonstrated liftoff with a tether [54].

Compared to a fully actuated flapping wing, rotatory wing systems also suffer from reduced maneuverability limited by the producible body forces and moments. Swash plates are typically used to generate collective and cyclic control inputs for stabilizing the attitude of the helicopter. Cyclic change of blade angle of attack and the induced body angle change enable control of translation. Multi-rotor system such as quadrotor can be configured to have additional freedom on torque generation. This, however, increase the system complexity. In contrast, a fully controlled flapping wing has the capability of vectoring three degree-of-freedom forces and changing the three degree-of-freedom torques with subtle changes of wing kinematics. With typical two flapping wings driven by two single degree actuators, three degree of freedom torques
(roll, pitch, and yaw) and at least two degree of freedom forces \((x,z)\) can be generated. For multi-rotor vehicle, however, even with three to four rotor configuration, only two degree of freedom torques (roll and pitch) and one degree of freedom force \((z)\) can be directly produced; the under-actuation has to be overcome by adding additional vector thrust rotor or similar techniques to directly generate forces in desired directions.

At low Reynolds numbers, rotary wing systems can also enhance lift generation with delayed stall[59]. However, rotational circulation, wake capture and clap-and-fling seem to be unique to flapping wing motion. Each can significantly enhance lift production from 10 to over 50% [25, 30]. Recent works [55],[115] has demonstrated the favorable scaling of flapping wings at the small size of insects/hummingbird, compared with traditional rotary wings. The threshold Reynolds numbers has shown to be 100[115]. As Reynolds number decreased, viscous drag dominated in the rotary wing, significantly increasing power expenditure. In [55], a side-by-side aerodynamic performance comparison of hummingbird wings and an advanced micro helicopter rotor shows that they are remarkably similar when both are driven as a rotary wings. Even though no direct comparison of flapping and rotary wings are made, in consideration of the additional lift enhancements flapping motion achieves, one can anticipate the possible better aerodynamic performance of flapping wing at the sizes spanned by the hummingbird wings.

1.1.1 Flapping Wing Platforms

Designing and developing such systems is a challenging problem under stringent size, weight and power (SWaP) constraints, in addition to the difficulties in the lagging battery technology, miniature sensors for navigation, limited computation, and system integration all pose design challenges. Under the SWaP constraints, balance and trade-offs must be made among mechanical complexity, controllability, power, and weight. Otherwise, producing enough lift for sustaining weight could be a challenge. Adding actuators generally increase the controllable degrees of freedom, however, they
contribute significantly to weight and complexity. Moreover, the lack of accurate modeling, precise and repeatable fabrication techniques, and available off-the-shelf components all adds to the difficulties in design and control of FWMAV.

Flapping wings generate lift and control torques entirely from the reciprocating flapping wing motion. The designed system must be able to meet the costly power demand to support its own weight, and overcome both drag and inertia cost of driving the wings at high frequency oscillation [102]. Characteristics such as flapping resonance and passive wing rotation have been observed in natural systems and serve to reduce the actuation complexity and the power requirements for driving flapping wing motion [42, 6]. Utilizing passive wing rotation has become a recent trend in the effort to reduce system mass. In fact, recent studies have shown that at least a portion of insect wing rotation is passively achieved through inertial and aerodynamic forces [6]. Passive wing rotation, from either wing deformation [100],[18], [43] or through the use of a hinge [108], [51], [78] has been applied in a few man-made vehicles.

Despite the difficulties, substantial work from a few groups have been devoted to design and prototype FWMAVs over the past years. Examples of the platforms are compiled in Fig. 1.3. Most of the previous works focused on the mechanism design, few demonstrated vehicle design and integration [50, 18]. The efforts to develop FWMAV to date can be divided into two main categories: motor driven linkage [4, 50, 19, 57, 75, 51, 68] and piezoelectric cantilever [108, 2] mechanisms.

At sub-gram scale, PZT bending cantilever operates with high power density at high frequencies with high voltage input, as shown in [49] and [108] and Fig. 1.4. With low transmission losses and resonance, the power to generate flapping motion [22, 108, 2] were greatly reduced. The reciprocating bending motion, though small, is well suited to flapping systems after displacement amplification through a four-bar mechanism with flexural joints. The original Micromechanical Flying Insect (MFI) of UC Berkeley was the first such FWMAV system, designed to independently control each wings leading and trailing edges with a total of four PZT bending actuators [109, 106], as shown in Fig. 1.3(a). It flaps at 275 Hz with wingspan of 25 mm.
However, due to the complexity and weight of the four actuators, it was not able to produce sufficient lift. With simplified design and choice of passive wing rotation and resonant flapping, lift off [108] was successfully demonstrated for Harvard’s Robobee. As shown in Fig. 1.3(b), Harvard Robobee consists of a pair of wings, one PZT actuator, and airframe. It flaps at 110 Hz and weights 60 mg with wingspan of 3 cm. The actuator drives the leading edge of both wings while the trailing edge is allowed to passively rotate about a flexure hinge. In the later work [33], additional small bending and twisting PZT actuators are added to create wing flapping asymmetry, allowing
the generation of controlling torques. Later version with two actuators driving each wing individually have demonstrated controlled hovering [62][76]. Gradually, more sensors were added onboard individually [38, 47, 37]. However, due to its payload limitation and sub-gram size, the system integration of onboard electronics and power remains extremely challenging.

At grams to tens-of-gram scale, the DC motor driven actuators are operating at high efficiency and generating large output angles for higher payload with low drive voltage. Traditionally, linkage mechanisms were designed to transfer rotational motion from the motor to reciprocal motion of the wings [50, 19, 57, 68, 75, 51]. Delfly I, II, and Micro, from Technical University of Delft, are among the smallest flapping flight systems capable of autonomy. As shown in Fig. 1.3(f), with working principle of the ornithopters, the Delfy micro drives four wings with one dc motor. It has a wingspan of 10 cm and weights 3 grams, including battery, camera, and wireless transmitter [18]. The four wings adopt the clap-and-fling aerodynamic mechanism to increase lift generation. Air circulation about wing is enhanced by fluid flow at the opening gap, when the wings are flapped together and pulled apart. The flexibility

![Figure 1.4. Power density of various actuators from [49].](image-url)
of the wing creates the passive camber deformation induced by the wing loading. Nanohummingbird [50] of AeroVironment weights 19 g with wingspan of 16.5 cm. The platform is currently the only motordriven system capable of stable hovering with both lift and control torques generated by only two flapping wings [50]. The two wings are driven with a single DC motor through novel strings four bar with fixed flapping amplitude.

To improve controllability, passive stability and additional servo mechanisms were often incorporated for generating necessary control torques, despite further increased mechanical complexity. Cornell 3D printed FWMAV was designed to be passively stable about hover as shown in Fig. 1.3(e). With four wings or eight wings driven by a single dc motor, the passive stability is realized with symmetric placement of wings and added dampers at the top and bottom of the platform[82, 100]. Delfly circumvented some of the difficulties with clap-and-fling flapping motion, four bar mechanism, and passively stable low-speed flight. While not capable of true hovering, the systems are capable of very slow forward flight. To allow control, magnetic actuators are used to change the rudder and elevator at the tail. They also successfully demonstrated onboard power and remote control [18]. Nanohummingbird [50] by adopting additional servo mechanism for angle of attack control to produce roll, pitch, and yaw torques for body stabilization. Despite the mechanism complexity, they achieved enough lift to hover and high level of system integration with onboard power, altitude stabilization, and remote control. Outdoor flight in light wind conditions as well as maneuvers such as the 360 lateral flip were demonstrated.

However, linkage mechanisms, such as crank-rocker four-bar linkage, generally are subjected to limitations such as low efficiency, fixed trajectory, increased mechanical complexity, and reduced controllability. In the Nanohummingbird [50], the linkage was replaced by strings with negligible mass, therefore, reduced the inertial loss on transmission and the parasite structural vibrations due to asymmetric acceleration. Several works added elastic elements to improve the efficiency of four-bar linkage mechanism that resulted in limited success[97, 64, 53, 4, 65], largely due to
the fact that the highly nonlinear and asymmetric linkage transmissions hinder the full recovery of potential energy from sinusoidal wing motion. For example, in [4], an elastic component was introduced to achieve resonance of a motor driven slider-crank mechanism. In the Nano hummingbird [50], the linkage was replaced by strings with negligible mass, therefore, reduced the inertial loss on transmission and the parasite structural vibrations. In the ideal scenario, with elastic components and system resonance, the kinetic and potential energy of the mechanical components in the system are conserved, and therefore, all the power is spent on the non-conservative energy cost, such as friction, damping of the system and the aerodynamic damping acting on the wing.

Recently, directly driving each flapping wing using a geared DC motor without linkages is gaining popularity with works such as [8, 112, 113, 3, 48]. In this case, the motor undergoes reciprocating rotation with linear gear transmission, capable of almost full recovery of potential energy with resonance, and generating control torques similar to Harvard’s insect-size robot without any additional servo mechanisms.

In [112], a 2.5g brushless dc motor is coupled with gear ratio of 10 and torsional springs to drive flapping wings at resonance of 25Hz. In addition, onboard feedback control is achieved for precise kinematics control using PID, LQR and CPG. This prototype leads to the current proposed work in this thesis, shown in Fig. 1.3(g). The design optimization and prototype will be detail in the coming chapters. Whereas in [3], the wing is direct driven by a brushed dc motor without even gear transmission to resonate at around 42Hz. The setup was only for proof of concept, so the wing is not rotating and the angle of attack is fixed at 90 degree. Later in [48] as shown in Fig. 1.3(i), two wings are each directly driven by a geared pager motor by utilizing an elastic element for energy recovery, resulting in a maximum lift-to-weight ratio of 1.4 at 10 Hz for the 2.7 g system. With a series of varied prototypes, system performance is examined with change in wing offset from center of rotation and elastic element stiffness. A prototype that achieved liftoff was demonstrated in [48].
Others modified or customized electromagnetic actuators. Vanneste et al. developed a 22 mg flapping wing system with a resonant thorax, passive wing rotation using a compliant link [7], and polymer body. The system flapped at 30Hz with a lift-to-weight ratio of 0.75, with two wings driven by a single actuator. In contrast, a similar electromagnetic actuator was proposed and demonstrated in [83] to achieve lift-to-weight ratio of over one for a single actuator. Besides differences in scale and size, one of the key advantage of this actuator, compared to one in [7], is that the actuator drives only a single wing, thus double, independently driven wings is possible to give the platform freedom for control. The proposed single actuator is shown in Fig. 1.3(k). They also anticipated the possible scaled-down version to have better scaling of lift-to-weight-ratio.

1.1.2 Design Optimization for Flapping Wing Platforms

Flying animals with flapping wings may best exemplify the astonishing ability of natural selection on design optimization under various constrains, especially for flight. Flapping Wing Micro Air Vehicles (FWMAVs) holds great promise in bridging the performance gap between engineering system and their natural counterparts.

A systematic design under SWaP constraints for FWMAVs consists of mechanism design, modeling and characterization of vehicle components, formulation of optimization, and system integration.

Even though the work in [50] briefly introduced the vehicle design and integration for Nanohummingbird with stable hovering demonstrated using onboard sensor, electronics, computation, power, and remote control, the complicated mechanisms and lack of details on system characterization and design formulation provided limited information for the later development of similar platforms. Thus most other works of DC motor driven type were still focusing on solving lift and control torque generation problem with various mechanism designs.
For parametric study, in [48], the authors presented motor-driven platform design with a parametric study on wing offset from center of rotation and elastic element stiffness, and a impedance matching method was used to numerically study the design choices on those parameters. In [12], the authors discussed 16 different wings with various geometries and their effects on the lift and system resonance for their proposed electromagnetic actuator. In [23], wing characterization was though a quasi-steady model that computes the hinge stiffness that leads to optimal flapping kinematics, given a flapping frequency and a driving voltage. Then parameter study on frequencies and driving voltages is conducted to validate the method. Work such as [97, 64, 4, 65] investigated nonlinear modeling of actuation, with attention to design and optimization of elastic elements which allow a flapping system to be driven at resonance, thus reducing or eliminating the inertial cost associated with accelerating and decelerating the wing. Few works have done with optimization for flapping wing actuation. In [53], for the linkage motor driven vehicle, using a numerical nonlinear model, effects of system parameters were examined including wing shape, actuator and transmission elements, such as linkage system or gearbox. In [2], an optimization was performed to find the optimal wing for the PZT driven platform, with parameters such as wing length, maximum chord length and spanwise location of the maximum chord, and passive wing rotation joint stiffness.

So far most of the platforms relied on empirical testing, limited modeling and parametric studies. These approaches may be effective in designing specific platforms, it is often difficult to generalize the resulting design and possibly leads to sub-optimal design decisions. No work, thus far, has been attempting to formulate a general and comprehensive design optimization problem for the vehicle, which could be very beneficial for the future development, commercialization and wider adoption of FWMAV platforms.
1.1.3 Flapping Resonance

The ability of flapping wing to generate elaborate wing motion therefore produce rapidly changing instantaneous aerodynamic forces and torques with minimum actuation makes it an attractive alternative to conventional Micro Air Vehicles (MAVs) with rotary wing based solutions by providing great inspiration for the development of flapping wing MAVs. Actuating the reciprocal flapping wing motion involves rapid acceleration and deceleration during stroke reversals, and therefore, can be quite power intensive. Power is consumed in overcoming not only the aerodynamic drag, but also the substantial wing inertia due to constantly oscillating wings and accompanying mechanisms that move at high frequency [102]. Therefore, mechanical resonance has been recognized as one of the key principles for reducing wing actuation complexity and power consumption. In the ideal scenario, with elastic components and system operating at resonance, the kinetic and potential energy of the moving components would be conserved, and therefore, power is spent only on the non-conservative energy cost, such as friction and the aerodynamic damping from the wing.

Biological evidence such as that in [42, 27] speculated the flapping wing as an oscillator. Based solely on large amount of observation of different species, [42] speculated that the muscular energy should be at the minimum when the muscular driving force is in resonance with the system. A more recent study of [27] showed that the wingbeat frequency of individual insects are fairly constant with the variation within 5% in typical hovering flight. They hypothesized that at this relative constant frequency which may be the resonant frequency of oscillation, the energetic expenditure is minimized. Furthermore, by experimental perturbation of wing inertia, they showed predicted changes in resonant frequency. The direct implementation of resonant oscillation on flapping wing MAV, however, is complicated due to the lack of a comprehensive understanding of the wing-thorax system including the unsteady time-varying aerodynamics, complex fluid-structure interactions, and nonlinear linkage transmission, etc.
The widely adopted method to identify and validate the resonance of the system is by conducting a frequency sweep, during which the wing is driven by sinusoidal voltage inputs with a wide range of frequencies of interest[108, 48]. The steady state amplitudes of the responses in terms of flapping angles would then be plotted with respect to input frequencies. Other interested output such as the steady state velocities of the responses or stroke-averaged lifts can also be plotted similarly. A typical second order resonant system can be characterized by natural frequency, damped natural frequency, and peak frequency. Natural frequency is also called primary resonant frequency and is a function of system inertia and stiffness. For linear under-damped second order system, peak frequency is also called peak resonance frequency or resonant frequency, corresponding to the maximum amplitude (flapping angle for example) from frequency response. The term resonant frequency has often been loosely used to indicate the peak frequency. Moreover, it is also defined to be the natural frequency in the context of nonlinear vibration. To avoid the misinterpretation of resonance frequency, here we use natural frequency and peak frequency instead. Only when the system damping is sufficiently small, these three frequencies converge close to one other in their values. As shown in many previous work and this study, flapping wing typically is under-damped although it has sufficiently large damping from aerodynamics. So we’ll show the penalties of using the wrong frequency can not be ignored.

Due to its simplicity, linear model with passive rotation and quasi-steady aerodynamic model was adopted in previous work to model flapping wing. For PZT bending cantilever beam actuation mechanism [32], the nonlinear aerodynamic damping was linearized about an operating point at a fixed angle of attack with maximum drag, giving a ”worst case” estimate for aerodynamic damping. In [48], the author assumed zero rotational angles for the nonlinear aerodynamic damping. Those simplifications could lead to large discrepancies on the evaluation of aerodynamic damping effects. In this work, we’ll show the limitations of linear model and complement it with non-
linear modeling and analysis in order to gain a full picture of the system response at different frequencies around resonance.

Flapping wing resonance tuning, however, still suffer from sub-optimal design decisions and misinterpretations of experimental results, owing to the lack of theoretical analysis of resonance’s effect on the response and energetics of flapping wing. Currently, most work in this area still relies heavily on empirical testings. In order to achieve adequate passive rotation angle (therefore desired angle of attack) with sufficiently large flapping amplitude, one of the common design goal was to maximize the stroke amplitude, instead of maximizing the lift and efficiency[92, 108, 83, 112]. While work such as [63, 8] where the wing was driven at damped natural frequency with a goal to achieve optimal efficiency, other work measured peak frequency from the frequency response experiments to validate the calculation of the natural frequency [92, 107, 3]. In [48], the peak frequency was used as the indication of maximum lift and optimal efficiency, while in the frequency response experiment it was observed that the peak frequency was actually lower than the one under which maximum lift was produced (off-resonance maximum lift). Similar results were also found in works of [92, 107]. In this thesis, we will show that maximum lift is produced at natural frequency, which is normally higher than the peak frequency, thus explaining the observed discrepancies.

1.1.4 Kinematic Control and Force Generation

The remarkably maneuverable and stable flight of insects and hummingbirds [25, 5] are due to their abilities to make precise and subtle changes of wing kinematics that result in rapid and significant changes in instantaneous aerodynamic forces [5, 46]. The high sensitivity of aerodynamic forces to wing kinematic change demands precise and instantaneous control of the flapping wing kinematics. Nevertheless nature’s flyers master wing kinematics control remarkably well in spite of various changing parameters [105] and unexpected disturbances [11]. Wing parameters vary by mor-
phology, wear and tear, wind conditions, and variations of air density due to changing elevation. In addition, external disturbances such as wind gusts, rain drops, obstacles will also affect aerodynamic forces. Insects and hummingbirds are able to cope with these disturbances and recover their flight stability with ease [11]. Furthermore, it has been shown that the basic wing trajectory of some species of insects is driven by CPG in an adaptive feedforward manner [105] and then modified by sensory feedback control.

For wing kinematics control, the linkage transmission systems are often subjected to control limitations such as fixed wing trajectories and asymmetry in the kinematics. Therefore, kinematics control is limited to varying wing speed profile (e.g., split-cycle) and additional mechanisms are required to further modulate the wing trajectory and angle of attack [50], which could drastically increase the design complexity and becomes infeasible at high frequency. In fact, prior work on wing kinematics control was based on linkage transmissions system and open loop control. For example, [77] presented the modeling and open loop control of wing kinematics of PZT based FWMAV. Due to the size limitation of a feedback sensor at sub-gram scale, Perez-Arancibia and his coworkers [77] used the open-loop feedforward method which achieved control of aerodynamic forces and torques. Also in [17], a combined repetitive and minimum-variance adaptive control strategy was used to generate desired flapping trajectories.

As for the direct drive actuation, the inherent limitation of fixed and asymmetric wing trajectories on linkage transmission is avoided, and various kinematic control approaches can be applied, which will be demonstrated in this thesis.

1.1.5 Whole Body Dynamics and Control of Flapping Flight

Due to the lack of comprehensive understanding of the system dynamics, control performance limitations, complex time-variant aerodynamics, manufacturing imperfections, and additional platform limitations, control of FWMAV is a great challenge. One approach, implemented in Cornells platform, is to build passive stability into
the system to make hovering stable[82],[100]. Practically, however, one cannot make
the system too stable as this tends to eliminate all system maneuverability. In gen-
eral, flapping wings act to damp the systems motion, providing a stabilizing effect
about certain body axes. Both Hedrick et al. and Cheng et al. have investigated body
damping from wing and body motion, determining a linear dependence on translation
and angular velocity and an increase in damping from increased wing beat frequency
and amplitude [46],[10].

In actively controlled systems, averaging theory is widely used in control devel-
opment, allowing approximation of the time-variant system with its time-invariant
average [87]. In flapping systems, as long as flapping frequencies are sufficiently high
and the wing generated forces sufficiently filtered by the body dynamics, a controller
can be designed about the simplified system where wing forces are averaged over each
wing beat. Using a linear approximation, the MFI design team was able to demon-
strate controlled hover in simulation using linear-quadratic regulator (LQR) after
parameterizing their fully controlled wing trajectory [21]. Khan et al. demonstrated
longitudinal control in simulation of a flapping flight system using a differential flat-
ness based nonlinear controller, with the assumption that mean lift force could be
commanded [52]. A nonlinear robust controller was developed by Serrani for a robot
in the longitudinal plane, allowing change in wing beat frequency and variable stroke
plane angle [89].

The majority of previous work on flapping-wing system controllers has been per-
formed in simulation due to the lack of platforms that are capable of both liftoff and
controlling forces and torques. However, recently successful free flight control has
been demonstrated in experiment with a variation of the Harvard Robobee.

With the capability of generating three degree-of-freedom control torques and lift,
various controllers has been demonstrated for controlling Harvard Robobee. In [62],
tethered stable hovering of an insect scale robot was demonstrated for the first time,
with the dynamics of the robot mostly ignored. In order to cope with model uncertain-
ties from imperfect fabrication, adaptive control was developed in [14], with reduction
in position errors and demonstrations of vertical takeoff and landing flights. A model
free linear controller without any knowledge of the robot dynamics was shown in [76],
which were tuned sequentially for upright stable flight, straight vertical flight, and
stable hovering with altitude and position control. All aforementioned controllers
adopted a cascade controller structure with assumption of time scale separation be-
tween attitude loop and position loop. In [16], a single-loop adaptive flight controller
was designed in order to alleviate the cascade assumption. In [15], landing on a ver-
tical wall with magnetic foot was demonstrated with iterative tracking control of a
landing trajectory planned on the lateral 2D plane.

Harvard Robobee was also fitted with various individual sensors for onboard feed-
back. A wind-sensing antenna and a light-sensing ocelli was studied in [39]. Pitch
angle control on a wire was also demonstrated using the ocelli as feedback. Later,
using that ocelli-inspired visual sensor to stabilize the RoboBee was demonstrated
[38]. MEMS sensors were also added to RoboBee for angle feedback. In [47], pitch
and yaw control of the RoboBee using an onboard magnetometer was presented with
robot constrained to rotate only about its each axis. While in [37], the integration
of a MEMS gyroscope onto the RoboBee to provide attitude feedback in flight is
demonstrated. This enables 25 s hovering flights in which the motion capture system
provides only position feedback.

1.2 Challenges and Open Issues

During the past few years, we have witnessed the rising of the drone industry. A
true disruptive wave of innovation are forging and mostly facilitated by the mobile
phone industry, which brought down the cost of computation, sensors and electronics
dramatically. Compared to the mature technology of quadcoptors, the FWMAV is
still in its infant age. There remain a number of open issues that must be addressed
in order for flapping vehicles to achieve the acrobatic agility of a hummingbird in
autonomy due to the lack of general design and fabrication tools, new sensors and actuators, effective control methods, battery technologies, and navigation methods.

Designing and prototyping a fully autonomous flapping wing micro aerial vehicle (FWMAV) at the insect/hummingbird scale poses great challenges including battery technology, sensor design, computation, and system integration. It is a system design problem for under stringent size, weight and power (SWaP) constraints, referring to design, integration, and development restrictions inherent in many military and aerospace embedded applications. In addition, the lack of accurate modeling, precise and repeatable fabrication techniques, and available off-the-shelf components makes designing FWMAV even more difficult.

First of all, few work has been attempting to formulate a general and comprehensive design optimization problem, which could be very beneficial for the future development, commercialization and wider adoption of FWMAV platform. The advances in modeling provide design guidelines for whole system optimization. The model should be general and concise enough to be used with optimization routine efficiently. It has to be informative enough to capture all the important characteristics of the system. It should be conclusive, including not only actuator, wing, but also battery, and other electronics and sensors. The goal is to get a easy to use, verified design toolbox, that give the designer the ability to quickly design a FWMAV for a particular application. In the mean time, it should also be precise, general, versatile enough to be used in multiple optimization formulations and can be solved easily for the requirement of a specific application. With this tool, the difficulties and frustrations of trial-and-error approach of designing FWMAV should be eliminated and the FWMAV will gain popularity in wider applications for different communities with open-source access. Thus FWMAV can start to rival quadrotor in various application domains.

Second, precise manufacturing becomes critical in order for mechanisms to function as intended in platforms under one gram. The Smart Composite Microstructures (SCM) manufacturing technique allows millimeter scale structures and actuators to
be constructed with low error tolerance in sequential layering and laser cutting of materials [103], [95]. SCM works well for two dimensional, folded flexural mechanisms, but when considering components of more arbitrary three dimensional shape and material, challenges remain. Micro-forming, microassembly, and micro-molding are currently being investigated for continued reduction in size scale and increase in accuracy [81].

Third, miniaturized bio-inspired sensors to be used on-board are still under development. There have been a number researchers working on the development of small and biologically inspired sensors for miniature systems [71], [72], [79], [109]. The fly haltere and the moth antenna function as gyroscope sensors [24], [86], and efforts have been made to create devices that perform similar functions. Recent artificial halteres have been shown to well capture angular rates while maintaining a small device profile [109], [94]. Optical flow, or the direction of apparent motion of objects or edges in a field of view, has also been shown to be useful for both stable flight and navigation in insects [98], and has been developed and used for robot control [96], [84]. Optical flow is well suited for simple, light weight, low optical sensors, easing their incorporation into small platforms. Currently, both ocelli inspired optical sensors and magnetic field sensors have also been incorporated into the Harvard Robobee and used for system control [38], [47].

Fourth, wing morphological optimization with systematic tools is lacking. Aerodynamic studies continue to be of interest in order to better understand and usefully apply the mechanisms of lift production on flapping wings. While many wing optimizations have been done, they are primarily platform specific and traditionally encompass the experimentally testing a variety of manufactured wings. Improvements in manufacturing, as well as better understanding the impact of shape, flexibility, and surface texture on lift production are important for improving overall platform lift generation [107], [20], [114], [90], [91].

Fifth, in robotic systems under one gram, there is currently no existing feasible on-board power source. Batteries continue to decrease in size, but due to scaling limi-
tations, they cannot supply sufficient power for more than seconds of flight. Miniature fuel cells, supercapacitors, and radioactive thin films have been proposed as alternatives, but are yet unrealized [93]. In sub-ten gram systems performance is better, yet the Delfly Micro at 3 grams is still only capable of flights up to 3 minutes. New actuators and light weight power electronics to decrease power consumption and weight, and increase power output are also open topics of current research [9], [110], [40]. As weight and size constrained miniature systems, either battery technology improves or the system must be further optimized for efficiency. In terms of vehicle design, an optimization formulation based on flight time should be explored. For better management and utilization of the onboard power, a power management system (BMS) with a computation-efficient algorithms should be developed.

Finally, due to the computation and payload limitations, autonomous navigation of such small aerial mobile system is still an open question. The first step towards the navigation solution is to localize the vehicle in 3D space. Without using an external motion capture system such as a Vicon system, a typical solution for larger mobile robot is to add more sensors to do localization or Simultaneous localization and mapping (SLAM), which is not currently feasible for such small system. Equipped only with inertial measurement unit, radio communication module (bluetooth LE), and altimeter, the system will be localized relative to the base station/mobile mothership using above sensors and the novel localization methods, while the base station with multiple radio modules estimates the 3D location of the vehicle. In this thesis we also proposed a sensor fusion method and hardware setup to provide the vehicle with 20cm accuracy 3D location information to aim the navigation.

1.3 Thesis Outline

This thesis is divided into six chapters. In Chapter 2, the flapping wing aerodynamics model, the flapping wing actuation model, whole body dynamics of flapping flight, and the design and optimization of the FWMAV are detailed. In Chapter 3,
the study of flapping resonance is discussed. In Chapter 4, flapping wing kinematics and force control are discussed. In Chapter 5, the fundamental limitation of flapping flight is introduced. And finally, in Chapter 6, the thesis is summarized and future prospects are discussed.

1.4 Thesis Contributions

The major contributions of this work are in the development of a novel hummingbird-size FWMAV platform, specifically:

1. Flapping resonance has been recognized as a key principle for reducing the cost of driving high-frequency flapping wing motion. Numerous works have been done to design flapping wing system based on resonance. However, theoretical understanding of resonance’s effects on response and energetics of flapping motion has lagged behind, leading to sub-optimal design decisions and misinterpretations of experiment results. In this work, we analyzed dynamics of flapping wing as a nonlinear forced second order resonant system, with both nonlinear perturbation method and linear approximation approach. We derived analytic formulas for steady-state flapping amplitude, energetics, and characteristic frequencies, including natural frequency, damped natural frequency, and peak frequency. The analysis revealed that both lift and efficiency are maximized by driving the wing at natural frequency. Interestingly, the flapping velocity is maximized at natural frequency as well, which can serve as an convenient experiment approach to identify natural frequency and validate the resonance design. The modeling and analysis were validated with both simulations and experiments on ten different wings mounted on a direct-motor-drive flapping wing test setup. It is a systematic resonance design tool to aid the initial design decisions, and guide interpretations of empirical results.

2. We presented a complete, multidisciplinary formulation for system design optimization and integration of FWMAV. The formulation covers actuation, wing, battery, electronics, dynamics, stability and control. System parameters considered
include parameters of wing, motor, gear, spring, battery, control authorities, and locations of poles and zeros of the dynamics. The formulation was validated by experimental data. Based on the direct motor drive mechanism of our previous work, an optimization yields a prototype with on-board sensors, electronics, and computation, flapping at 30Hz to 40Hz, with 12 grams of weight and 20 grams of maximum lift. Initial results of onboard state estimation and control were discussed. Flapping wing platforms with different requirements and scales can now be easily designed and optimized with parameter modifications of the proposed formulation.

3. The high sensitivity of aerodynamic forces to wing kinematic changes demands precise and instantaneous feedback control of the flapping wing trajectories, especially in the presence of various types of uncertainties. In this work, we present a dynamic model of a pair of direct-motor-driven flapping wings while taking into consideration the parameter uncertainties and disturbances. We then present an adaptive robust controller to achieve robust performance of instantaneous wing trajectory tracking at over 30Hz. The proposed control algorithm was experimentally validated on the FWMAV which showed excellent tracking of various wing trajectories with different amplitude, bias, frequency, and split-cycles. Experimental results on various model wings demonstrated that the proposed controller can adapt to unknown parameters and show no performance degradation across wings of different geometries. The results of the proposed controller were also compared with those of open-loop and classical PID controllers.

4. For dynamics and control of flapping wing flight, we present an initial analysis on fundamental limitations of flapping flight control, discovering, for the first time, the non-minimum phase nature of flapping flight when certain controls are used. We also presented the first full nonlinear attitude and position controller with exponentially stable and globally exponential attractive properties.
2. DESIGN OPTIMIZATION AND SYSTEM INTEGRATION OF FLAPPING WING MICRO AIR VEHICLES

2.1 Introduction

Flying animals with flapping wings may best exemplify the astonishing ability of natural selection on design optimization. They evince extraordinary prowess to control their flight: while demonstrating rich repertoire of agile maneuvers, they remain surprisingly stable during hover. Flapping Wing Micro Air Vehicle (FWMAV) holds great promise in bridging the performance gap between engineering system and their natural counterparts. Designing and constructing such systems is a challenging task under stringent size, weight and power (SWaP) constraints.

![Figure 2.1. Photograph of one of the prototyped FWMAVs with flexible bistable wings and onboard electronics.](image)

In this chapter, we presented a complete, multidisciplinary modeling, characterization, and formulation for system design optimization and integration for direct-motor-driven FWMAVs. The formulation covers actuation, wing, battery, electronics, dynamics, flight stability and control. System parameters considered include parameters of wings, motors, gears, springs, batteries, control authorities, and location of poles.
and zeros of the system dynamics. A well-recognized actuation configuration is the adoption of the following methods: (1) resonance, (2) two wings independently driven by two actuator, (3) passive wing rotation. For such configuration, here we propose a design optimization modeling and formulation. To avoid the drawbacks of linkage mechanisms while at the same time to achieve a resonant system, as first presented in [112], we directly drive the flapping wing using DC motors coupled with torsion springs. Using a transmission gear, the motor is designed to operating at an efficient speed, but generates an overall reciprocal motion to the wing. The formulation was validated by experimental data for both rigid and flexible wings, covering a range of wing aerodynamic loading. Based on the direct motor drive mechanism of our work, the optimization yields 3 prototypes. One prototype is with rigid wing and generating 12 grams total lift. The other two prototypes are with flexible bi-stable wings and are with onborad sensors, electronics, and computation. The vehicles flaps at 30Hz to 40Hz wingbeat frequency with 12.5 grams system weight and 20 grams maximum lift. The two flexible wing prototypes are with different constrains on achievable control bandwidth and electronics. Liftoff was demonstrated with extra payloads for flexible wing prototypes. Initial results of onboard state estimation and flight control were demonstrated. Flapping wing platforms with different requirements and scales can now be designed and optimized with parametric modifications of proposed formulation. Another contribution of this work is that the robustness issue of the flapping flight both for insect and FWMAV is discussed. Important aspects of the intrinsic limitation of flapping flight is studied here using the available tools of robust control analysis. The non-minimum phase nature of flapping flight is then identified. A simple rule for evaluation of flapping flight is then established for studying the bandwidth limitation of insect and for guidance of designing more maneuverable FWMAV.
2.2 System Description

As illustrated in Fig. 2.1 and Fig. 4.1, we have developed two types of vehicles, one with flexible wings and the other with rigid wings and wing rotation stoppers. Version 1 is shown in Fig. 4.1(b). Version 2 is shown in Fig. 4.1(a). Version 3 is shown in Fig. 2.1. For both types of prototypes, two flapping wings are directly driven by two 2.5 gram, 6mm brushless DC motors coupled with torsion springs for kinetic energy restoring. Using a gear transmission, the motor was designed to generate an overall reciprocal motion of the wing. A portion of the gear on the load shaft was removed to reduce the weight and moment of inertia. The assembled flexible wing prototype with weight of 10.5 grams and wing span of 15cm and two compact onboard electronic boards is shown in Fig. 2.1. The assembled flexible wing prototype with weight of 11.5 grams and wing span of 15cm and a single electronic board is shown in Fig. 4.1 (a). The assembled rigid wing prototype with weight of 7.5 grams, rigid
wings and wing span of 15cm is shown in Fig. 4.1 (b). For the flexible wing, the wing membrane becomes cambered after assembly, thus forming the desired angle of attack. Whereas the rigid wing prototype, the wing was allowed to passively rotate up to a 45 degrees angle limited by a stopper fixed at the proximal end of the wing leading-edge spar.

2.3 System Modeling

![Diagram](image)

Figure 2.3. Schematic view of coordinate systems and kinematics. (a) Top view of the stroke plane coordinate frame (red) \((x_s, y_s, z_s)\). (b) Side view of body coordinate frame (blue) \((x_b, y_b, z_b)\). (c) Blade element (BE) cut view shows the (geometric) angle of attack \(a\). (d) The system diagram of FWMAV showing all components. (e) Parameters of wing shape. (f) Diagram of typical flapping wing motion projected onto a 2-D plane. (g) The wing flaps about the \(\phi\) axis, which remains parallel with \(z_s\) axis. The wing rotation \(\psi\) axis is parallel with respect to the wing leading edge.

The FWMAV vehicle system in consideration consists of flapping wing actuation subsystem, electronics, batteries, and airframe as shown in Fig. 4.2(d). The vehicle’s flight dynamics is first introduced.
2.3.1 Flight Dynamics

The FWMAV is modeled as a rigid body in three-dimensional space. The inertia effect of the wings is relatively small compared to the body. The oscillations due to high frequency flapping wings are thus commonly neglected. As shown in Fig. 4.2 (a), the stroke plane frame \((x_s, y_s, z_s)\) is fixed relative to the body with the origin located at the wing base while the origin of the body frame is located at the center of mass. The left and right wing frames with bases, \(O_l\) and \(O_r\) respectively, are at \(d_0\) distance to the wing common base \(O_s\). Wing kinematics are specified by the stroke angle \(\phi\). The positive direction of \(\phi\) is defined to be downstroke direction for both left and right wing. Wing coordinate frames (brown) \((x_{wl}, y_{wl}, z_s)\) and \((x_{wr}, y_{wr}, z_s)\) share the same \(z\) direction with stroke plane frame and are attached to the blade element (BE) on the wing at distant \(r\) from the wing base. As shown in Fig. 4.2 (c), angle of attack \(a\) is defined as the angle between the wing chord and the tangential of the wings trajectory (relative to the stroke plane), and instantaneous lift \(dF_L\) and drag forces \(dF_D\) on the BE are defined accordingly. The forces \((F_x, F_y, F_z)\) and torques \((T_x, T_y, T_z)\) that are produced by the wing pair are defined with respect to the stroke frame. Body coordinate frame (blue) \((x_b, y_b, z_b)\) has the same orientation as the stroke plane frame but with the origin that is located at the center of mass. The center of mass of the body is located at \(d_s\) below the wing common base \(O_s\) as shown in Fig.
4.2 (b). The inertial frame (world frame) is frame A, which is fixed with the ground. At time zero, the body frame B is coincident with frame A, denoted by \( \{e_1; e_2; e_3\} \). The flapping stroke plane is shown in Fig. 4.2 frame C attached. The origin of body frame B is denoted by \( \{b_1; b_2; b_3\} \). \( p = [x, y, z]^T \) is the Cartesian position of the vehicle expressed in the inertial frame. The orientation of the body with respect to the inertial frame is specified by Euler angles: the roll \( \alpha \), pitch \( \beta \), and yaw \( \gamma \). Thus the rotation matrix is

\[
R = \begin{bmatrix}
C\gamma C\beta & C\gamma S\beta S\alpha - S\gamma C\alpha & C\gamma S\beta C\alpha + S\gamma S\alpha \\
S\gamma C\beta & S\gamma S\beta S\alpha + C\gamma C\alpha & S\gamma S\beta C\alpha - C\gamma S\alpha \\
-S\beta & C\beta S\alpha & C\beta C\alpha
\end{bmatrix},
\]

(2.1)

where the short hand notation for \( \sin \) and \( \cos \) are used.

Stroke-averaged models were found to be sufficiently accurate to capture the dynamics of insects and robots of similar scales. The rigid body dynamics represented in the body frame is the Newton-Euler equation, i.e. a rigid body subject to an external wrench \( F \) applied at the center of mass and specified with respect to the body coordinate frame, written as

\[
\begin{bmatrix}
mI_{3\times3} & 0 \\
0 & \mathcal{I}
\end{bmatrix}
\begin{bmatrix}
v^b \\
\omega^b
\end{bmatrix}
+ \begin{bmatrix}
\omega^b \times m v^b \\
\omega^b \times \mathcal{I} \omega^b
\end{bmatrix} = F^b,
\]

(2.2)

where \( I_{3\times3} \) is 3 by 3 identity matrix and \( \mathcal{I} \) is the inertia matrix with respect to the body frame. Due to symmetry, we assume that the cross terms in the moment of inertia matrix \( \mathcal{I} \) are negligible and therein \( \mathcal{I} = \text{diag}[I_{xx}, I_{yy}, I_{zz}] \).

There exists the following transformation between the body velocity \( v^b = [u, v, w]^T \) in body frame B and the body velocity \( \dot{p} = [\dot{x}, \dot{y}, \dot{z}]^T \) in the inertial frame,

\[
\begin{bmatrix}
v^b \\
\omega^b
\end{bmatrix} = \begin{bmatrix}
R^T \dot{p} \\
(R^T \dot{R})^\gamma
\end{bmatrix},
\]

(2.3)
Similarly, the angular velocity in body frame $\omega^b = [p, q, r]$ is related to derivatives of Euler angles as

$$\omega^b = (R^T \dot{R})^\vee = \begin{bmatrix} 1 & 0 & -S\beta \\ 0 & C\alpha & C\beta S\alpha \\ 0 & -S\alpha & C\beta C\alpha \end{bmatrix} \begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \end{bmatrix} = W\dot{\Theta}. \quad (2.4)$$

 Whereas the rigid body dynamics represented in the inertial frame is written as

$$\mathcal{I}W\ddot{\theta} + \mathcal{I}\dot{W}\dot{\theta} + W\dot{\theta} \times \mathcal{I}W\dot{\theta} = \tau^b,$$

$$\ddot{\mathbf{p}} = \frac{1}{m} R^b. \quad (2.5)$$

With above definition, the full flight dynamics in the body frame is given by

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} + \begin{bmatrix} wq - vr \\ ur - wp \\ vp - uq \end{bmatrix} = \frac{1}{m} f^b + g \begin{bmatrix} S\beta \\ -C\beta S\alpha \\ -C\beta C\alpha \end{bmatrix},$$

$$\begin{bmatrix} I_{xx} \dot{p} \\ I_{yy} \dot{q} \\ I_{zz} \dot{r} \end{bmatrix} + \begin{bmatrix} (I_{zz} - I_{yy})qr \\ (I_{xx} - I_{zz})pr \\ (I_{yy} - I_{xx})qp \end{bmatrix} = \tau^b,$$

$$\begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \end{bmatrix} = \begin{bmatrix} 1 & \frac{S\beta}{C\beta} S\alpha & \frac{S\beta}{C\beta} C\alpha \\ 0 & C\alpha & -S\alpha \\ 0 & \frac{S\alpha}{C\beta} & \frac{C\alpha}{C\beta} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}. \quad (2.6)$$

For longitudinal motion in body frame, the flight dynamic model can be simplified to

$$\dot{u} + wq = \frac{1}{m} f^{bx} + gS\beta,$$

$$\dot{w} - uq = \frac{1}{m} f^{bz} - gC\beta,$$

$$I_{yy} \dot{q} = \tau^{by},$$

$$\dot{\beta} = q. \quad (2.7)$$
Similarly, for lateral motion in body frame, the flight dynamic model can be written as

\[
\begin{align*}
\dot{v} - wp &= \frac{1}{m} f^{by} - gS\alpha, \\
\dot{w} + vp &= \frac{1}{m} f^{bz} - gC\alpha, \\
I_{xx}\dot{\alpha} &= \tau^{bx}, \\
\dot{\alpha} &= p.
\end{align*}
\] (2.8)

Whereas for longitudinal motion in world frame, the flight dynamic model can be transformed as

\[
\begin{align*}
m\ddot{x} &= C\beta f^{bx} + S\beta f^{bz}, \\
m\ddot{z} &= C\beta f^{bz} - S\beta f^{bx} - mg, \\
I_{yy}\ddot{\beta} &= \tau^{by}.
\end{align*}
\] (2.9)

Similarly, for lateral motion flight dynamics in world frame, we have

\[
\begin{align*}
m\ddot{y} &= C\alpha f^{by} - S\alpha f^{bz}, \\
m\ddot{z} &= S\alpha f^{by} + C\alpha f^{bx} - mg, \\
I_{xx}\ddot{\alpha} &= \tau^{bx}.
\end{align*}
\] (2.10)

Transforms a wrench applied at the origin of the B frame into an equivalent wrench applied at the origin of the C frame, B and C are on the rigid body:

\[
\begin{align*}
F_c &= A_d^{T}g_{bc} F_b, \\
f_c &= \begin{bmatrix} R_{bc}^T & 0 \end{bmatrix} \begin{bmatrix} f_b \\ \tau_b \end{bmatrix}, \\
\tau_c &= \begin{bmatrix} -R_{bc}^T \hat{p}_{bc} \\ R_{bc}^T \end{bmatrix} \begin{bmatrix} f_b \\ \tau_b \end{bmatrix}.
\end{align*}
\] (2.11)

2.3.2 Model of Flapping Wing

The geometric quantities of the wing are defined with parameterization methods detailed in [30]. We have the moment of inertia of the wing with respect to the wing root as

\[
J_w = \int_0^{R_w} \rho_w c(r) r^2 \, dr = \rho_w R_w^3 \bar{c} \bar{r}_2^2,
\] (2.12)
where, as illustrated in Fig. 4.2(e), wing root is at \(O'\), i.e. the intersection of leading edge (LE) and trailing edge (TE), \(R_w\) is the wing length, \(\rho_w\) is the wing planar density, \(c(r)\) is the chord length at \(r\) distance from the wing root \(O'\), \(\bar{c} = \frac{1}{R_w} \int_0^{R_w} c(r) dr\) is the mean chord length, and \(\hat{r}_2^2\) and \(\hat{r}_3^3\) are the second and third moment of wing area, respectively, given by \(\hat{r}_2^2 = \int_0^{R_w} c(r) r^2 dr / \bar{c} R_w\) and \(\hat{r}_3^3 = \int_0^{R_w} c(r) r^3 dr / \bar{c} R_w^2\). \(O\) is the intersection of stroke axis \(z\) and LE, \(d_w\) is the wing offset of \(O'\) from \(O\). In consideration of the non-negligible wing offset \(d_w\), the wing geometric quantities are transformed according to

\[
R'_w = R_w + d_w, \quad \bar{c}' = \frac{R_w \bar{c}}{R'_w}, \quad \hat{r}_1' = \frac{R_w \hat{r}_1^1 + d_w}{R'_w},
\]

\[\hat{r}_2'^2 = \frac{R_w^2 \hat{r}_2^2 + 2d_w R_w \hat{r}_1^1 + d_w^2}{R'_w^2},\]

\[\hat{r}_3'^3 = \frac{R_w^3 \hat{r}_3^3 + 3d_w R_w^2 \hat{r}_2^2 + 3d_w^2 R_w \hat{r}_1^1 + d_w^3}{R'_w^3},\]

where \(\hat{r}_1^1\) are the first moment of wing area, variables without prime follow the original definitions, and ones with prime are corrected variables with wing offset considered. Once transformed, the primes are dropped for all following modeling steps, in favor of cleaner notations.

A general flapping wing is capable of three degrees of freedom (DoF) motion: wing stroke (flapping), wing rotation (pitching), and wing deviation. Stroke (\(\phi\)) is the back-and-forth wing motion defined in the stroke plane. Rotation (\(\psi\)) is the angle rotated by wing relative to its rotation axis, generally close and near the leading edge. The deviation is the angle with which the wing rotation axis deviates from the stroke plane, but due to added weight and negligible effect on lift production, the deviation angle is commonly not considered. Typical two DoF wing flapping motions are illustrated in Fig. 4.2(f) with the view of flapping wing motion projected onto a 2-D plane. Dash lines indicate the instantaneous position of the wing chord at temporally equidistant points during each half-stroke. Small circles mark the leading edge. Wing moves left to right during downstroke, right to left during upstroke.
With quasi-steady aerodynamic model [25], the instant lift force and aerodynamic damping torque on the wing at time \( t \) with wing flapping \( \phi(t) \), rotation \( \psi(t) \), and angle of attack \( a(t) \) [22] are given by

\[
F_L(t) = \frac{1}{2} \rho_a \bar{C}_L R_w^3 \bar{c} \dot{r}^2 \dot{\phi}^2, \\
\tau_d(t) = B_{s2} \dot{\phi}^2 \text{sign}(\dot{\phi}),
\]

where \( \rho_a \) is the air density, \( B_{s2} = \frac{1}{2} \rho_a \bar{C}_D R_w^4 \bar{c} \dot{r}^3(S) \) is the aerodynamic damping coefficient, \( \bar{C}_L, \bar{C}_D, \bar{C}_N \) and \( \bar{C}_T \) are the mean lift, drag, normal, and tangential coefficients averaged over one wing stroke, respectively, given by [25]

\[
\bar{C}_D = 1.92 - 1.55 \cos(2.04a - 9.82) \\
\bar{C}_L = 0.225 - 1.58 \sin(2.13a - 7.20) \\
\bar{C}_N = 3.4 \sin(a) \\
\bar{C}_T = 0.4 \cos^2(2a),
\]

where the angle of attack is \( a \). The quasi-steady model is adopted here due to the lack of simple closed form model for unsteady aerodynamics[25].

The center of pressure, i.e. the location where the aerodynamic forces acting on the wing is defined as

\[
r_{cp} = \frac{R_w \bar{r}^3(S)}{\bar{r}^2(S)} = \frac{\int_0^{R_w} c(r) r^3 dr}{\int_0^{R_w} c(r) r^2 dr}.
\]

2.3.3 Model of Wing Actuation

Over the years, studies in fly mechanics of flapping wing flyers ranging from hawk-moths [44], hummingbirds [99], flies [36], and dragonflies [6] have provided great insight into the flapping wing lift production and control. Flapping-wing lift generation comes entirely from the reciprocating wing motion; the system must be able to meet the costly power demand to support its own weight, overcome both induced drag and profile drag, and accelerate the wings which have non-negligible inertia [102] due to high frequency oscillation. Characteristics such as flapping resonance and passive wing rotation have each been observed in natural systems and serve to reduce the
actuation complexity and the power requirements for driving flapping wing motion [42, 6].

While the flapping wing system considered here has two degrees of freedom: wing stroke (flapping) and wing rotation, only flapping motion is actuated, while wing rotates passively due to aerodynamic and inertial forces. Passive wing rotation has been observed in biological systems to reduce the actuation complexity and the power requirements for driving flapping wing motion [42, 6], which has also been applied in a number of platforms as well[108, 48, 112]. The passive wing rotation is realized either though wing deformation [100, 18, 43], or flexture hinge [23, 48], or mechanical stopper that limits the free wing rotation up to a desired maximum angle of attack[112].

Simplifications have been made to reduce the complexity of the dynamics, typically with assumptions of constant angle of attack[32, 48]. As shown in Fig. 4.2(f) and (g) and also in study [25], the passive wing rotation consists mostly two parts: translation at optimal angle of attack for majority of the stroke motion and rotation to the other extreme during stroke reversals. In this work, for the rigid wing case, as the passive wing rotation is limited by stopper to optimal 45 degrees angle of attack during most of the stroke motion, a constant angle of attack of 45 degrees is assumed. This assumption implies that ideally the wing can rotate instantaneously from 45 degrees to -45 degrees, and vice versa, at the two extremes of stroke reversals. Similar assumption can be argued for the flexible bistable wing case. [32] also assumed fixed angle of attack of 45 degrees, while [48] adopted the angle of attack of 90 degrees.

The wing stroke dynamics is more dominant than the wing rotation dynamics. In [32, 104], it was shown that the majority of kinetic energy due to wing movement is stored in the flapping mode, about 50 times larger than that in the rotation mode. As a result, we assume that the behavior of the wing is modeled by a beam damped by quasi-steady aerodynamics, rotating about the stroke axis and resonating with torsion spring, as shown in Fig. 4.2(d), where the wing stroke (φ) is driven by the actuator under resonance with torsion spring stiffness (K_s) and passive wing rotation angle (ψ).
The wing stroke ($\phi$) is driven by DC motor under resonance with torsion spring ($K_s$), and passive wing rotation ($\psi$) is limited by the stopper to optimal angle of attack of 45 degrees. The system is normally excited with sinusoidal input $u = V_I \cos(\Omega t + \beta)$, where $V_I$ is the magnitude, $\Omega$ is the angular frequency, and $\beta$ is the phase.

As the inductance of the motor is negligible, the equation of motion for DC motor is

$$J_m \ddot{\phi}_m + B_m \dot{\phi}_m = K_a I_a - T_m,$$

(2.17)

where $J_m$ is the moment of inertia of the motor rotating elements, $\phi_m$ is the motor angle, $B_m$ is the damping coefficient of the motor rotating elements, $K_a$ is the torque constant, $I_a$ is armature current, and $T_m$ is the motor load torque. $I_a = \frac{u - K_s \dot{\phi}_m}{R_a}$ with $u$ being the input voltage to the motor and $R_a$ being the resistance of the motor.

Defining $B_{m1} = B_m + \frac{K_s^2}{R_a}$, we have

$$J_m \ddot{\phi}_m + B_{m1} \dot{\phi}_m = \frac{K_a}{R_a} u - T_m.$$

(2.18)

With gear transmission, we have $\phi_m = N_g \phi_l$, $\eta_g N_g T_m = T_l$ with $N_g$ being the gear ratio, $\phi_l$ being the load angle, $T_l$ being the load torque, and $\eta_g$ being the gear efficiency. Then the motor dynamics becomes

$$J_e \ddot{\phi}_l + B_e \dot{\phi}_l = K_u u - T_l,$$

(2.19)

where the effect moment of inertia $J_e = \eta_g N_g^2 J_m$, effective damping $B_e = B_{s1} = \eta_g N_g \left( B_m + \frac{K_s^2}{R_a} \right)$, and input gain $K_u = \eta_g N_g \frac{K_a}{R_a}$.

Directly driving the wing using geared motor with coupled parallel torsion spring as shown in Fig. 4.2(d), we have $T_l = J_w \ddot{\phi}_w + B_{s2} \dot{\phi}_w |\dot{\phi}_w| + K_s \dot{\phi}_w$ and $\phi_l = \phi_w = \phi$, thus

$$J_s \ddot{\phi} + B_{s1} \dot{\phi} + B_{s2} |\dot{\phi}| \dot{\phi} + K_s \phi = K_u u,$$

(2.20)

where the total moment of inertia is $J_s = N_g^2 J_m + J_w + J_g$. 
2.3.4 Steady-State Flapping Wing Response

Dynamics of the flapping wing actuation system in Equation (3.10) is nonlinear, and its response can not, in general, be expressed as simple analytic formulas. It is necessary to numerically simulate the dynamics to get the response. In order to provide computationally efficient solution of system response as functions of system parameters for design optimization, we use a nonlinear perturbation technique, i.e. method of multiple scales\textsuperscript{[74, 13]}, to analyze the nonlinear dynamics and obtain analytical predictions of forced response of Equation (3.10). The closed-form solutions capture the nonlinear vibration response as functions of system parameters, revealing insights about how system parameters affects the steady state response and energetics of flapping wing.

Excited with sinusoidal input $u = V_I \cos(\Omega t)$ with amplitude $V_I$ and frequency $\Omega$, the equation of motion in Equation (3.10) can be rewritten in the following dimensionless form

$$\ddot{\phi} + \omega_n^2 \phi = -2\epsilon \mu \dot{\phi} - \epsilon |\dot{\phi}| \dot{\phi} + E(t),$$  \hspace{1cm} (2.21)

which represents a forced oscillator with a quadratic damping and a spring. The natural frequency or primary resonance is $\omega_n = \sqrt{\frac{K_s}{J_s}}$. $\epsilon = B_s^2 J_s$ is a small dimensionless perturbation parameter\textsuperscript{[74]}. $\mu = B_s^2 J_s^2$ is the normalized linear damping coefficient. $E(t) = \frac{K_s u}{J_s} = k \cos(\omega_n t + \epsilon \sigma t)$ is the input excitation, where $\Omega = \omega_n + \epsilon \sigma$ and $k = \frac{K_s V_I}{J_s^2}$. $\sigma = O(1)$ is the detuning parameter [74], which quantifies the nearness of $\Omega$ to $\omega_n$.

The approximate solution of Equation (3.10) near $\omega_n$ can be obtained using method of multiple scales\textsuperscript{[74]}. Specifically, we first define new time scales $T_i = \epsilon^i t$, $i = 0, 1, 2...$ and the solution has the form

$$\phi(t, \epsilon) = \phi_0(T_0, T_1) + \epsilon \phi_1(T_0, T_1) + ...$$ \hspace{1cm} (2.22)

Then the first order approximation of the solution is:

$$\phi_0 = A \cos(\omega_n T_0 + B(T_1)) + O(\epsilon),$$  \hspace{1cm} (2.23)
which means the response of the system is approximately sinusoidal with approximation error quantified by parameter $\epsilon$. In this work, with the parameters of DC motor driven flapping wing system, $\epsilon = O(0.1)$.

The amplitude $A$ and phase $B$ are solved by two ordinary differential equations

$$
\dot{A} = -\mu A - \frac{4}{3\pi} A|A|\omega_n + \frac{k}{2\omega_n} \sin(\gamma),
$$

$$
A\dot{\gamma} = A\sigma - \frac{k}{2\omega_n} \cos(\gamma),
$$

(2.24)

where $\gamma = \sigma T_1 - B$.

The steady-state solution is solved by setting time derivatives to zeros, i.e. $\dot{A} = 0$ and $\dot{\gamma} = 0$, we have

$$
A^2 \left[ \left( \mu + \frac{4}{3\pi} A|A|\omega_n \right)^2 + \sigma^2 \right] = \frac{k^2}{4\omega_n^2},
$$

(2.25)

which is commonly called frequency-response equation. The steady state amplitude of the response $A$ can be solved given the driving frequency $\Omega$ (or $\sigma$).

When the system is driven at natural frequency, i.e. $\sigma = 0$ and $\Omega = \omega_n$, we have the amplitude prediction $A$ is

$$
A = -\mu + \sqrt{\mu^2 + \frac{8k}{3\pi\omega_n}},
$$

(2.26)

which can be transformed into

$$
A\omega_n = \frac{1}{2B_2} \left( \sqrt{B_{s1}^2 + 4B_2 V_I K_u} - B_{s1} \right), \text{ or }
$$

$$
A\omega_n = \frac{2V_I K_u}{B_{s1} + \sqrt{B_{s1}^2 + 4B_2 V_I K_u}},
$$

(2.27)

with the original system parameters and lumped parameter $B_2 = B_{s2} \frac{8}{3\pi}$.

With the first order approximation of the solution in Equation (3.14), the stroke-averaged powers drained by aerodynamic damping is

$$
\overline{P_d} = \frac{1}{T} \int_T B_{s2} \dot{\phi} |\dot{\phi}| d\tau = \frac{4}{3\pi} B_{s2} (A\omega_n)^3 + O(\epsilon^3),
$$

(2.28)

the total power is

$$
\overline{P_{total}} = \frac{1}{T} \int_T uI_a d\tau
$$

$$
= \frac{1}{2} \left( \frac{V_I}{R_a} - N_g \frac{K_u}{R_a} \omega_n A \right) V_I + O(\epsilon),
$$

(2.29)
and mean efficiency is

\[
E_{ff} = \frac{\bar{P}_d}{\bar{P}_{total}} = \frac{B_s 2^\frac{8}{3n} (A\omega_n)^3 + O(\epsilon^3)}{\left(\frac{V_I}{R_a} - N_g \frac{K_a}{R_a} A\omega_n\right) V_I + O(\epsilon)}.
\]  (2.30)

Substituting \(A\omega_n\) from Equation (3.27) gives the closed-form formulas for energetics of flapping wing.

With analytical solution for kinematics in Equation (3.27), the lift from a single wing is

\[
\bar{F}_L = \frac{1}{4} \rho_a \bar{C}_L R_w^2 \bar{c} r_2^2 (S)(A\omega_n)^2
= \frac{1}{16 B_s^2} \rho_a \bar{C}_L R_w^3 \bar{c} r_2^2 (S) \left(\sqrt{B_s^2 + 4 B_2 V_I K_u - B_s^1}\right)^2.
\]  (2.31)

2.3.5 Model of Control Authorities

Previous studies on insect flight [5] show that the subtle changes of kinematics can lead to large variations of the resulting aerodynamic forces and torques. In this section, to facilitate later design optimization, we derive the force and torque generation as functions of wing kinematics and input voltages.

To quantify the ability of the proposed mechanism to generate forces and torques, we define the kinematic-force gains as the ratio between resulting forces/torques and kinematics change from trim condition (stable hovering).

The input voltages for the two motors take the following sinusoidal forms,

\[
u_i = \begin{cases} 
 V_I \cos \left(\frac{2\pi f t}{2\sigma_i^v} + b_i\right) + V_0, & if \ 0 \leq t < \frac{\sigma_i^v}{f} \\
 V_I \cos \left(\frac{2\pi f t - 2\pi}{2(1-\sigma_i^v)} + b_i\right) + V_0, & if \ \frac{\sigma_i^v}{f} \leq t < \frac{1}{f}
\end{cases}
\]  (2.32)

where \(i\) represents the right \((i = r)\) and left motor \((i = l)\), \(V_I\) is the input voltage amplitude, \(b_i\) is the phase angle, \(V_0\) is the bias voltage, and \(\sigma_i^v\) is the split cycle parameter for the voltage wave forms.

It is easy to verify from simulation and experiment that the wing kinematic response reach steady-state in 2 to 3 wing strokes. Therefore we assume the actuator and wing dynamics reach steady state when we evaluate the stroke-averaged forces.
and torques, which typically span at least 3 to 5 wing strokes for the average flight dynamics. The stroke-averaged forces and torques under consideration are $F_z$, $F_x$, roll torque $T_x$, pitch torque $T_y$ and yaw torque $T_z$ defined similar to [10] as shown in Fig. 4.2. With a fixed angle of attack $a$, the flapping kinematics of each wing are uniquely defined through its stroke angle, which is assumed to be generated by

$$
\phi_i = \begin{cases} 
A_i \cos \left( \frac{2\pi ft}{2\sigma_i} + \psi_i \right) + \phi_{0i}, & \text{if } 0 \leq t < \frac{\sigma_i}{f} \\
A_i \cos \left( \frac{2\pi ft - 2\pi}{2(1-\sigma_i)} + \psi_i \right) + \phi_{0i}, & \text{if } \frac{\sigma_i}{f} \leq t < \frac{1}{f} 
\end{cases}
$$

(2.33)

where $i$ represents the right ($i = r$) and left wing ($i = l$), $A_i$ is the flapping amplitude, $\psi_i$ is the phase angle, $\phi_{0i}$ is the bias angle, and $\sigma_i$ is the resulting split cycle parameter for the wing kinematics.
The stable hovering condition of $A_i = A_0$, $\psi_i = 0$, $\phi_0 = 0$ and $\sigma_i = 0.5$ is assumed as the nominal kinematics. Under this condition, $F_x = F_y = 0$, $T_x = T_y = T_z = 0$, and the lift generated by the wing pair is balanced by the body weight $mg$ of the MAV/insect, i.e., $mg = \frac{1}{2} \rho_a C_L R_w \bar{r}_2^2(S) \omega_w^2 A_0^2$, where $C_L$ is the mean lift coefficient averaged over one wing stroke [25], and $\omega_w = 2\pi f$ is the wing angular velocity.

The control inputs were obtained when kinematic parameters deviated from their nominal values. Based on the assumption of near-hovering condition and the method in [26, 10], for small deviations from the nominal kinematics parameters in amplitude $\delta A$, bias $\delta \phi_0$, and split cycle $\delta \sigma$, it can be shown that

1) Lift force $F_z$ due to symmetric amplitude changes of the left and right wing, i.e., $A_l = A_r = A_c = A_0 + \delta A_0$, as shown in Fig. 2.5(a), is

$$F_z = \frac{1}{2} \rho_a C_L R_w \bar{r}_2^2(S) \omega_w^2 A_c^2$$

$$= F_0 \left( \frac{\delta A_0}{A_0} + 1 \right)^2 = F_0 v_1^2$$

$$v_1 = \frac{\delta A_0}{A_0} + 1 \approx \frac{1}{A_0 \omega_n} \frac{K_u}{\sqrt{B_{s1}^2 + 4B_2 V_0 K_u}} \partial V_{10} + 1,$$

where $F_0 = \frac{1}{2} \rho_a C_L R_w \bar{r}_2^2(S) \omega_w^2 A_0^2$ is nominal lift force, and $F_0 = mg$ at nominal hovering condition.

2) The roll torque $T_x$ due to asymmetric amplitude changes of the left and right wing, i.e., $A_l = A_c + \delta A_d$ and $A_r = A_c - \delta A_d$, as shown in Fig. 2.5(b), is

$$T_x = \frac{1}{2} \rho_a C_L R_w \bar{r}_2^2(S) \omega_w^2 A_c^2 r_{cp} \left( \frac{2 \delta A_d}{A_c} \right)$$

$$= r_{cp} F_z \left( \frac{2 \delta A_d}{A_c} \right) = r_{cp} F_0 A_c \left( \frac{2 \delta A_d}{A_0} \right) = r_{cp} F_0 v_1 v_2$$

$$v_2 = \left( \frac{2 \delta A_d}{A_0} \right) \approx \frac{2}{A_0 \omega_n} \frac{K_u}{\sqrt{B_{s1}^2 + 4B_2 V_0 K_u}} \partial V_{10}. $$

where $r_{cp} = \frac{\bar{r}_2(S)}{\bar{r}_2^2(S)} R_w$ is the center of pressure on the wing. $\delta A_d$ denotes the differential changes of amplitude.
3) The pitch torque $T_y$ due to symmetric bias changes of the left and right wing, i.e., $\phi_{0l} = \delta\phi_0$ and $\phi_{0r} = \delta\phi_0$, as shown in Fig. 2.5(c), is

$$T_y = -r_{cp} F_z \sin(\delta\phi_0) = r_{cp} F_0 v_1^2 v_3, \quad (2.36)$$

$$v_3 = -\sin(\delta\phi_0) \approx -\delta\phi_0 = -\frac{K_u}{K_s} \delta V_i$$

4) The yaw torque $T_z$ cannot be realized by the amplitude and bias change, so the split cycle method introduced in [26] is adopted here to generate yaw torque. Specifically, when the left and right wing are anti-symmetric for split cycle, i.e., $\sigma_l = \sigma = 0.5 - \delta\sigma$ and $\sigma_r = 1 - \sigma = 0.5 + \delta\sigma$, as shown in Fig. 2.5(d), it can be shown that

$$T_z = \frac{1}{8} \rho_a C_D R_w \bar{c}^3 \bar{r}^3 (S) \omega_w^2 A_c^2 \left( \frac{1 - 2\sigma}{\sigma(1 - \sigma)} \right)$$

$$= r_{cp} F_z \frac{C_D}{C_L} v_4 = r_{cp} F_0 \frac{C_D}{C_L} v_1^2 v_4 \quad (2.37)$$

$$v_4 = \frac{C_D}{C_L} \left( \frac{1 - 2\sigma}{4\sigma(1 - \sigma)} \right) \approx 2\delta\sigma,$$

where $\sigma = 0.5 - \delta\sigma$, for small $\delta\sigma$, $\frac{1 - 2\sigma}{\sigma(1 - \sigma)} \approx \frac{2\delta\sigma}{0.25} = 8\delta\sigma$.

The effects of split cycle can never reach steady state, as the excitation changes for every half of the stroke. The response for split cycle is the transient solution of the flapping wing, excited by the input wave forms. The dynamics of wing has strong attenuation effects on the input voltage split cycle. From experiments and simulations, it can be shown that the resulting split cycle parameter for the wing kinematics $\sigma_i = k_{sc} \sigma_i^v$, where scaling $k_{sc} \approx 0.1$.

From 1)-5) and , we can derive all force and torque model as functions of changes of input voltage, summarized in the Table 2.2.
Table 2.1. Wing Kinematics and Forces.

<table>
<thead>
<tr>
<th>Forces</th>
<th>Kinematics</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_z$</td>
<td>Amplitude</td>
<td>$F_0v_1^2 = u_1$</td>
</tr>
<tr>
<td>$T_x$</td>
<td>Amplitude</td>
<td>$r_{cp}F_0v_1v_2 = u_2$</td>
</tr>
<tr>
<td>$T_y$</td>
<td>Bias</td>
<td>$r_{cp}F_0v_1^2v_3 = u_3$</td>
</tr>
<tr>
<td>$T_z$</td>
<td>Split Cycle</td>
<td>$r_{cp}F_0v_1^2v_4 = u_4$</td>
</tr>
</tbody>
</table>

Table 2.2. Wing Kinematics and Input.

<table>
<thead>
<tr>
<th>Forces</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_1$</td>
<td>$\frac{\delta A_0}{A_0} + 1 = \frac{K_u}{A_0\omega_n\sqrt{B_{s1}^2 + 4B_2V_{10}K_u}}\partial V_{10} + 1$</td>
</tr>
<tr>
<td>$v_2$</td>
<td>$\frac{2\delta A_0}{A_0} = \frac{2}{A_0\omega_n\sqrt{B_{s1}^2 + 4B_2V_{10}K_u}}\partial V_{1d}$</td>
</tr>
<tr>
<td>$v_3$</td>
<td>$-sin(\delta \phi_0) \approx -\delta \phi_0 = -\frac{K_u}{K_z}\delta V_i$</td>
</tr>
<tr>
<td>$v_4$</td>
<td>$\left(\frac{1-2\sigma}{4\sigma(1-\sigma)}\right) \approx 2\delta \sigma$</td>
</tr>
</tbody>
</table>

With above wrench model, we have the nominal wrench for the flight dynamics,

\[
\begin{align*}
f_n^b &= f_n^s = \begin{bmatrix} 0 \\ 0 \\ F_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ u_1 \end{bmatrix} \\
\tau_n^b &= \tau_n^s = \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} = \begin{bmatrix} u_2 \\ u_3 \\ u_4 \end{bmatrix}
\end{align*}
\]  

(2.38)

where $u_1$, $u_2$, $u_3$, $u_4$ are defined for the concise notations. The overall vehicle system has those four inputs.
### 2.3.6 Flapping Wing Passive Damping

Even though the flapping wing aerodynamics are highly nonlinear and oscillating in nature, it is well known that the body motion will induce additional forces and torques on the flapping wing [10]. Those effects can be approximated by damping wrenches termed Flight Counter Forces (FCF) and Flight Counter Torques (FCT)[10], we have the additional damping forces and torques, so the overall wrenches from the flapping wings are as follows,

\[
\begin{align*}
\mathbf{f}_b &= \mathbf{f}_n + \begin{bmatrix}
-c_x(u + d_s q) \\
-c_y(v - d_s p) \\
-c_z w 
\end{bmatrix} \\
\tau_b &= \tau_n + \begin{bmatrix}
-c_{yt}(v - d_s p) \\
-c_{xt}(u + d_s q) \\
0
\end{bmatrix} + \begin{bmatrix}
-c_{rol} p \\
-c_{pit} q \\
-c_{yaw} r
\end{bmatrix} 
\end{align*}
\tag{2.39}
\]

Table 2.3. FCF and FCT.

<table>
<thead>
<tr>
<th>Forces</th>
<th>DoF</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_x$</td>
<td>x</td>
<td>$2\rho_a R^2_w \tilde{c} A_0 \omega_w \tilde{r}_1(S) C_D(a_0)\cos^2(\phi)\left</td>
</tr>
<tr>
<td>$c_y$</td>
<td>y</td>
<td>$2\rho_a R^2_w \tilde{c} A_0 \omega_w \tilde{r}_1(S) C_D(a_0)\sin^2(\phi)\left</td>
</tr>
<tr>
<td>$c_z$</td>
<td>z</td>
<td>$\rho_a R^2_w \tilde{c} A_0 \omega_w \tilde{r}_1(S) \frac{dC_N(a)}{da} \bigg</td>
</tr>
<tr>
<td>$c_{rol}$</td>
<td>Roll</td>
<td>$\rho_a R^4_w \tilde{c} A_0 \omega_w \tilde{r}_3(S) \frac{dC_N(a)}{da} \bigg</td>
</tr>
<tr>
<td>$c_{pit}$</td>
<td>Pitch</td>
<td>$\rho_a R^4_w \tilde{c} A_0 \omega_w \tilde{r}_3(S) \frac{dC_N(a)}{da} \bigg</td>
</tr>
<tr>
<td>$c_{yaw}$</td>
<td>Yaw</td>
<td>$2\rho_a R^4_w \tilde{c} A_0 \omega_w \tilde{r}_3(S) C_D(a_0)\left</td>
</tr>
</tbody>
</table>

With assumption of near-hovering condition, as derived in [10], for a pair of wings, we have the damping coefficients summarized in Table 5.1. As in [10], $\hat{t} = \omega_w t$ is the nondimensional time, $a$ is the effective angle of attack, $\frac{d\phi}{dt}$ is the nondimensional flapping velocity of the wing, $\phi$ and $n$ are, respectively, the wing-flapping amplitude
and frequency, and \( \hat{r}^1(S) \) and \( \hat{r}^2(S) \) are, respectively, the nondimensional first and second moments of the wing area. In addition, during the forward and backward translation, we have damping torque coefficient due to the offset \( d_s \) between the stroke plane and the COM, i.e. \( c_{xt} = d_s c_x \), and similarly, during lateral translation, we have \( c_{yt} = -d_s c_y \).

### 2.3.7 Stability and Maneuverability

To facilitate the discussion of flight stability and maneuverability, we first obtain the linearized dynamics around the hovering condition. For stability, we look at the poles of the longitudinal and lateral dynamics. As the RHP-zeroes and RHP-poles pose fundamental limitations on the achievable bandwidth of the controlled flight, we thus quantify maneuverability of flapping flight with zeroes and poles of the linearized dynamics.

**Linearization of Flight Dynamics:** The wrenches of the pair of flapping wings are

\[
\begin{align*}
\mathbf{f}^b &= \begin{bmatrix} -c_x(u + d_s q) \\ -c_y(v - d_s p) \\ u_1 - c_z w \end{bmatrix}, \\
\mathbf{\tau}^b &= \begin{bmatrix} u_2 - c_{yt}(v - d_s p) - c_{rod} p \\ u_3 - c_{xt}(u + d_s q) - c_{pit} q \\ u_4 - c_{yaw} r \end{bmatrix}.
\end{align*}
\]

For longitudinal motion in world frame, we have

\[
\begin{align*}
mx\ddot{x} &= C\beta f^{bx} + S\beta f^{bz}, \\
m\ddot{z} &= C\beta f^{bz} - S\beta f^{bx} - mg, \\
I_{yy}\ddot{\beta} &= \tau^{by}.
\end{align*}
\]
For lateral motion in world frame, we have
\[ m \ddot{y} = C \alpha f^y - S \alpha f^z, \]
\[ m \ddot{z} = S \alpha f^y + C \alpha f^z - mg, \] (2.42)
\[ I_{xx} \ddot{\alpha} = \tau^x. \]

Assuming small angle \( \beta \), the longitudinal dynamic model in world frame can be linearized as
\[ \ddot{x} = -\frac{c_x}{m} \dot{x} - \frac{c_x d_s}{m} \dot{\beta} + g \beta, \]
\[ \ddot{z} = -\frac{c_z}{m} \dot{z} + \frac{1}{m} u_1 - g, \]
\[ \ddot{\beta} = -\frac{c_{pit} + c_{xt} d_s}{I_{yy}} \dot{\beta} - \frac{c_{xt}}{I_{yy}} \dot{x} + \frac{1}{I_{yy}} u_3, \] (2.43)
which can be put into state space form, \( \dot{X} = AX + BU + \Delta \), with \( U = [u_1 - mg, u_3]^T \), \( X = [x, \dot{x}, z, \dot{z}, \beta, \dot{\beta}]^T \), \( \Delta = [0, 0, 0, -g, 0, 0] \)

\[
A = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
0 & -\frac{c_x}{m} & 0 & 0 & g & -\frac{c_x d_s}{m} \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & -\frac{c_z}{m} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & -\frac{c_{xt}}{I_{yy}} & 0 & 0 & 0 & -(c_{pit} + c_{xt} d_s)/I_{yy}
\end{bmatrix},
\]

(2.44)

\[
B = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
1/m \\
0 \\
0 \\
0 \\
0 \\
0 \\
1/I_{yy}
\end{bmatrix},
\]

(2.45)

and
\[
C = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}.
\]

(2.46)
Similarly, assuming small angle \( \alpha \), the lateral dynamic model in world frame can be linearized as
\[
\begin{align*}
\ddot{y} &= -\frac{c_y}{m} \dot{y} + \frac{c_y d_s}{m} \dot{\alpha} + g \alpha, \\
\ddot{z} &= -\frac{c_z}{m} \dot{z} + \frac{1}{m} u_1 - g, \\
\ddot{\alpha} &= -\frac{c_{rot} + c_{yt} d_s}{I_{xx}} \dot{\alpha} - \frac{c_{yt}}{I_{xx}} \dot{y} + \frac{1}{I_{xx}} u_2,
\end{align*}
\]
which can be put into state space form, \( \dot{X}_2 = A_2 X_2 + B_2 U_2 + \Delta_2 \), with \( U_2 = [u_1 - mg, u_2]^T \), \( X_2 = [y, \dot{y}, z, \dot{z}, \alpha, \dot{\alpha}]^T \), \( \Delta_2 = [0, 0, 0, -g, 0, 0] \)

\[
A_2 = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
0 & -\frac{c_y}{m} & 0 & 0 & g & -\frac{c_y d_s}{m} \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & -\frac{c_z}{m} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & -\frac{c_{yt}}{I_{xx}} & 0 & 0 & 0 & \left(-\frac{c_{rot} + c_{yt} d_s}{I_{xx}}\right)
\end{bmatrix}, \quad (2.48)
\]

\[
B_2 = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
1/m & 0 \\
0 & 0 \\
0 & 1/I_{xx}
\end{bmatrix}, \quad (2.49)
\]

and
\[
C_2 = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0
\end{bmatrix} \quad (2.50)
\]

**Poles and Zeros:** Since the longitudinal dynamics is highly similar to the lateral dynamics. The detailed derivation is only shown for the longitudinal dynamics. Since \((A,B)\) is state controllable and \((A,C)\) is state observable, this state-space realization
is minimal. To obtain the poles and zeros of the linearized longitudinal dynamics, we first obtain the transfer functions matrix as

\[ G(s) = C(sI - A)^{-1}B + D \]

and we have

\[
\text{det}(G) = \frac{(c_x d_s s - g)}{s^2(s + \frac{c_x}{m})(s^3 + (\frac{c_{pit}}{I_{yy}} + \frac{c_x}{m} + \frac{c_{xt} d_s}{I_{yy}})s^2 + \frac{c_{pit} c_x}{m I_{yy}} s + \frac{c_{xt} g}{I_{yy}})}.
\]

which shows that the system for individual transfer function has two unstable poles and one unstable zero. The RHP-zero is

\[
z_u = \frac{mg}{c_x d_s} = \frac{mg}{2d_s \rho_a R_0^2 c A_0 \omega_u r_1^1(S)C_D(a_0)\cos(\phi)\frac{d\phi}{dt}},
\]

and the RHP-poles \( p_u \) are solutions of

\[
s^3 + (\frac{c_{pit}}{I_{yy}} + \frac{c_x}{m} + \frac{c_{xt} d_s}{I_{yy}})s^2 + \frac{c_{pit} c_x}{m I_{yy}} s + \frac{c_{xt} g}{I_{yy}} = 0.
\]

\( G(s) \) has normal rank of 2 and zero polynomial of \( z(s) = \frac{c_x d_s}{m} s - g \). So the RHP-zero is also a transmission zero and the system is non-minimum phase.

Similarly, for lateral dynamics, \( G_2(s) = C_2(sI - A_2)^{-1}B_2 + D_2 \) and

\[
\text{det}(G_2) = \frac{(c_y d_s s - g)}{s^2(s + \frac{c_y}{m})(s^3 + (\frac{c_y}{m} - \frac{c_y d_s}{I_{xx}})s^2 + \frac{c_y c_y}{m I_{xx}} s + \frac{c_y g}{I_{xx}})}.
\]

which shows that the system for individual transfer function has two unstable poles and one unstable zero. The RHP-zero is

\[
z_{u2} = \frac{mg}{c_y d_s} = \frac{mg}{2d_s \rho_a R_0^2 c A_0 \omega_u r_1^1(S)C_D(a_0)\sin(\phi)\frac{d\phi}{dt}},
\]

and the RHP-poles \( p_{u2} \) are solutions of

\[
s^3 + (\frac{c_{rol}}{I_{xx}} + \frac{c_y}{m} - \frac{c_y d_s}{I_{xx}})s^2 + \frac{c_{rol} c_y}{m I_{xx}} s + \frac{c_y g}{I_{xx}} = 0.
\]

\( G_2(s) \) has normal rank of 2 and zero polynomial of \( z_2(s) = \frac{c_y d_s}{m} s - g \). So the RHP-zero is also a transmission zero and the system is non-minimum phase.
### Table 2.4. Morphology of Insects.

<table>
<thead>
<tr>
<th>Species</th>
<th>Fruit Fly</th>
<th>Hawkmoth</th>
<th>Stalk-eyed fly</th>
<th>Bumblebee</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass (g)</td>
<td>9.6e-4</td>
<td>1.62</td>
<td>7e-3</td>
<td>0.175</td>
</tr>
<tr>
<td>$I_{xx}(Nms^2)$</td>
<td>3.06e-13</td>
<td>2.55e-7</td>
<td>2.67e-11</td>
<td>3.70e-9</td>
</tr>
<tr>
<td>$I_{yy}(Nms^2)$</td>
<td>5.06e-13</td>
<td>2.83e-7</td>
<td>2.95e-11</td>
<td>2.13e-9</td>
</tr>
<tr>
<td>$I_{zz}(Nms^2)$</td>
<td>3.06e-13</td>
<td>2.43e-7</td>
<td>8.48e-12</td>
<td>2.08e-9</td>
</tr>
<tr>
<td>$I_{xz}(Nms^2)$</td>
<td>-1.91e-13</td>
<td>-3.34e-8</td>
<td>-7.65e-12</td>
<td>-1.73e-9</td>
</tr>
<tr>
<td>$\chi_o(\circ)$</td>
<td>45</td>
<td>50</td>
<td>70</td>
<td>57.5</td>
</tr>
<tr>
<td>$\dot{d}_s$</td>
<td>0.20</td>
<td>0.27</td>
<td>0.25</td>
<td>0.21</td>
</tr>
<tr>
<td>$L(mm)$</td>
<td>2.5</td>
<td>39.5</td>
<td>6.23</td>
<td>18.6</td>
</tr>
<tr>
<td>$R_w(mm)$</td>
<td>2.39</td>
<td>48.8</td>
<td>4.46</td>
<td>13.2</td>
</tr>
<tr>
<td>$\bar{c}(mm)$</td>
<td>0.8</td>
<td>18.6</td>
<td>0.94</td>
<td>4.01</td>
</tr>
<tr>
<td>$n(Hz)$</td>
<td>218</td>
<td>26</td>
<td>170</td>
<td>155</td>
</tr>
<tr>
<td>$\Phi(\circ)$</td>
<td>140</td>
<td>98</td>
<td>140</td>
<td>116</td>
</tr>
<tr>
<td>$\hat{r}_1(S)$</td>
<td>0.49</td>
<td>0.435</td>
<td>0.568</td>
<td>0.49</td>
</tr>
<tr>
<td>$\hat{r}_2(S)$</td>
<td>0.545</td>
<td>0.505</td>
<td>0.614</td>
<td>0.55</td>
</tr>
<tr>
<td>$\hat{r}_3(S)$</td>
<td>0.59</td>
<td>0.56</td>
<td>0.64</td>
<td>0.60</td>
</tr>
<tr>
<td>$\eta_{aero}$</td>
<td>0.45</td>
<td>0.27</td>
<td>0.31</td>
<td>0.13</td>
</tr>
</tbody>
</table>

### Table 2.5. Poles and Zeros of Insects (unit: rad/s).

<table>
<thead>
<tr>
<th>Species</th>
<th>Fruit Fly</th>
<th>Hawkmoth</th>
<th>Stalk-eyed fly</th>
<th>Bumblebee</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_u$</td>
<td>$14.5 \pm 33.6i$</td>
<td>$4.0 \pm 10.8i$</td>
<td>$9.2 \pm 19.6i$</td>
<td>$7.7 \pm 19.2i$</td>
</tr>
<tr>
<td>$z_u$</td>
<td>3041.7</td>
<td>268.1</td>
<td>1925.0</td>
<td>689.5</td>
</tr>
<tr>
<td>$p_{u2}$</td>
<td>31.3</td>
<td>6.0</td>
<td>17.7</td>
<td>11.4</td>
</tr>
<tr>
<td>$z_{u2}$</td>
<td>5226.2</td>
<td>1007.3</td>
<td>3307.5</td>
<td>1794.3</td>
</tr>
</tbody>
</table>
From Table 2.4, we can calculate the unstable poles and zeros for 4 example animals.

**Non-minimum Phase characteristics**: From control theory, the sensitivity peak can be very large if the plant have both RHP-zero and RHP-pole. The RHP poles and RHP zero put strict constrains on the achievable bandwidth of the system, so overall the bandwidth $\omega_B^*$ of the body control has to be

$$2|p_u| < \omega_B^* < \frac{z_u}{2}, \quad (2.57)$$

which provides a fundamental limitation on the maneuvering performance of the flapping flight on the longitudinal dynamics. Similar results can be obtained for lateral dynamics. From Table 2.8, we can see that the poles and zeros put a tighter constrains on the bandwidth of longitudinal dynamics, compared to that of lateral dynamics.

From the observation, for fast maneuvering, the body of the insect indeed shows the characteristics of the non-minimum phase system, which consists of overshoot and undershoot. Those movements show as dancing movement of the insect/hummingbird. To further answer the question: Why flapping wing animal dance when maneuvering? This questions is important because it is directly related with insects ability to fast maneuvering for survival, feeding, escape, catching a prey, etc. To see the effect on different size of insects, the scaling of the RHP-zero is calculated as

$$z_u = \frac{mg}{c_2d_s} = \frac{R_w A_0 n C_L \dot{r}_2^2(S)}{8d_s \dot{r}_1(S) C_D \cos^2(\phi) |\frac{d\phi}{dt}|} \approx \frac{R_w A_0 n}{d_s} \quad (2.58)$$

Similarly, for lateral RHP-zero,

$$z_{u2} = \frac{mg}{c_2d_s} \approx \frac{R_w A_0 n}{d_s} \quad (2.59)$$

Basically, the lower the frequency, the more dance. This phenomenon is unique to flapping wing flyer due to the unbalanced air flow on the flapping wing from the body movement.

This also have implications for flapping wing MAV design, if $d_s = 0$, this zero will disappear, along with all the limitations associated with it. This is a very simple yet
effective guideline for design better, more maneuverable FWMAV. It is not achievable for some constrains on the weight distribution. The second implication is to have a control authority that directly generating forces on x and y directions, to make this zero not a transmission zero of the system, so the effect of this zero will not show on the final response.

This is a novel finding in flapping flight field of research. Most of previous works focused on flight stabilization, few has attempted to quantify the intrinsic and fundamental limitations of flapping flight.

2.3.8 Weight Scaling

![Graphs showing weight scaling](image)

Figure 2.6. (a) The plot of motor power to weight using Faulhaber and Maxon Data. (b) The predicted spring stiffness (red line) vs. the measured values (blue dots) for 16 springs. (c) The scaling of Li Battery.

The proposed FWMAV is comprised of wings, motor, gear, spring, electronics, battery, and frames. The modeling and especially the scaling of all the major components are crucial for both quickly sizing the overall system and optimization formulation. In this section, the scaling of different components are presented. The electronics including sensors are considered a fixed weight.

**Scaling of Wing:** The planar density of the wing with carbon fiber bar and vine, and membrane is experimentally determined to have a average value, e.g. of
\( \rho_w = 0.09018(kg/m^2) \) for Mylar membrane. Thus for a wing with wing length \( R_w \) and mean chord length \( \bar{c} \), the mass of the wing \( w_w = \rho_w R_w \bar{c} \).

**Scaling of Motor:** Motor’s weight is difficult to model from the geometric property. Guided by the simple model, the empirical scaling model is obtained using data sheet information from Faulhaber motor, Maxon motor, among others.

From the data sheet, we obtain the polynomial curve fitting, \( w_w = c_1 p^2 + c_2 p + c_3 \), where \( p \) is the motor rated power. The coefficients (with 95% confidence bounds) are determined to be \( c_1 = 0.06719(0.05589, 0.07849) \), \( c_2 = 0.508(0.1966, 0.8194) \), \( c_3 = 1.368(−0.1333, 2.87) \), with R-square of 0.9967, thus

\[
w_m = 0.06719p^2 + 0.508p + 1.368 \tag{2.60}
\]

where \( p \) is the motor rated power in unit of \( W \) and \( w_m \) is the weight of the motor in unit of gram.

Motor’s power limitation can be determined from the thermal behavior, which is generally complicated. The max continuous current without overheating is \( I_c \), and the heat generated \( I_c R^2 \) has to be lower than the maximum dissipation for maximum allowable temperature. The calculated current for a motor at the thermal limit is

\[
I_c = \sqrt{\frac{T_{125} - T_{22} - \omega0.45R_{th2}(C_0 + B_m\omega)}{R(1 + \alpha_{22}(T_{125} - T_{22}))(R_{th1} + 0.45R_{th2})}} \tag{2.61}
\]

where \( \alpha_{22} \) is the temperature coefficient for terminal resistance, which is directly affected by the coil temperature, and \( \alpha_{22} = 0.004K^{-1} \).

The typical maximum continuous current at steady state resulting from the rated continuous duty torque. This value includes the effects of a loss of \( K_a \) (torque constant) as it relates to the temperature coefficient of the winding as well as the thermal characteristics of the given magnet material. This value can be safely exceeded if the motor is operated intermittently, during start / stop, in the ramp up phases of the operating cycle and/or if more cooling is applied.

**Scaling of Gear:** To model the gear, we define the number of teeth for the pinion gear as \( N_{gp} \), number of teeth for the spur gear as \( N_{gs} \). The gear ratio is \( N_g = N_{gs}/N_{gp} \).
and modulus of gear is \( m_g = 0.12 - 0.2 \). The distance between the two axis becomes \( a_{ps} = m_g(N_{gs} + N_{gp})/2 \). The radius of pinion gear is \( d_{gp} = m_gN_{gp} \). The radius of spur gear is \( d_{gs} = m_gN_{gs} \). The thickness of the gear is determined by the required rigidness of the material, for nylon gear, \( t_{gp} = 3(mm) \) and \( t_{gs} = 0.8(mm) \). With the density of the gear material \( \rho_g \), the weight of the spur gear is

\[
 w_{gs} = \rho_g t_{gs} \pi \frac{d_{gs}^2}{4} = \rho_g t_{gs} \pi \frac{(m_gN_{gs})^2}{4} \]

As the spur gear is often cut with holes to reduce the weight and when it is not rotating fully, part of the gear can be cut to reduce the weight. Thus a scaling ratio \( k_c \) can be used for the weight of the spur gear. Here \( k_c = 0.6 \times 0.5 \). The weight of the pinion gear is

\[
 w_{gp} = \rho_g t_{gp} \pi \frac{d_{gp}^2}{4} = \rho_g t_{gp} \pi \frac{(m_gN_{gp})^2}{4} \]

For example, for a nylon spur gear, \( \rho_g = 1150 kg/m^3 \), \( t_{gs} = 0.8mm \), \( m_g = 0.2 \), \( N_{gs} = 90 \), the full gear weight is 0.117\( g \), assuming 50\% percent of the gear weight is saved by cutting holes \( k_c = 0.5 \), compared with the measured value 0.12\( g \).

**Scaling of Spring**: The linearity of the torsional spring is validated experimentally by measuring the spring torque at different angular positions. The corresponding experimental setup and results are shown in [112]. The spring torque is shown to be proportional to the angular position from \(-75^\circ\) to \(75^\circ\) with good linearity, and the measured spring coefficient matches well with the theoretical calculation.

The characterization of elastic elements is essential for design and validation of flapping resonance. For torsion springs (from McMaster-Carr) used in this study, the theoretical model of the spring stiffness is given by

\[
 K_s = \frac{Ed^4}{64Dn},
\]

with wire diameters \( d \), outer diameter \( D \), Youngs modulus \( E \), and number of winding \( n \).

The linearity of the torsional spring is validated experimentally by measuring the spring torque at different angular positions. The corresponding experimental setup
is shown in Fig. 3.5(a). As a example, for one torsional spring with wire diameters \( d = 0.3\text{mm} \), outer diameter \( D = 2.67\text{mm} \), Youngs modulus \( E = 193\text{Gpa} \), and number of winding \( n = 4.25 \), the experiment result is shown in Fig. 3.5(b). The spring torque is shown to be proportional to the angular position from \(-75^\circ\) to \(75^\circ\) with good linearity, and the measured spring stiffness \( K_s = 0.0019 \) matches well with the theoretical calculation \( K_s = 0.002 \).

Total 16 torsion springs with different geometries and spring stiffness from McMaster-Carr were tested. The measured and predicted spring stiffness were plotted together in Fig. 2.6(b) to show consistent matches of spring model of Equation (3.44) and experiment testing.

![Figure 2.7](image)

**Figure 2.7.** (a) Experiment setup for testing springs. (b) Linearity test of spring.

Theoretical weight of the spring can be calculated as

\[
w_s = \rho_s n \pi \frac{d^2}{4} \pi D,
\]

where \( \rho_s = 8050\text{kg/m}^3 \).

The ratio between stiffness and weight gives the unit weight stiffness of the spring, given by

\[
\frac{K_s}{w_s} = \frac{E}{16\rho_s \pi^2} \left( \frac{d}{Dn} \right)^2.
\]

In consideration of the weight, springs that have smaller \( D \), \( n \), and larger \( d \) is preferred, as this give the spring the smallest weight for generating certain stiffness. This makes sense as \( d \) dominates the stiffness of the spring, but larger \( D \) and \( n \) have more impact on increasing the weight.
Scaling of Battery: Compiled from datasheets of Fullriver Battery, the following model is fitted

\[ w_b = 0.02284 \times C_{bat} + 0.3304, \]  

(2.67)

where \( C_{bat} \) is the battery capacity in unit of \( mAh \) and \( w_b \) is in unit of gram. Goodness of fit is \( R - square = 0.9718 \).

2.4 Model Validation Experiment

In this section, the proposed modeling and analytical solution are validated with simulations and experiments.

2.4.1 Materials and Methods

The simulations and experiments were conducted on two FWMAV prototypes with two independent, motor-driven flapping wing subsystems, as shown in Fig. 4.1.
Table 2.6. Wings Parameters.

<table>
<thead>
<tr>
<th>Wing</th>
<th>m</th>
<th>R_w</th>
<th>d_w</th>
<th>(\bar{c})</th>
<th>(\hat{r}_2)</th>
<th>(\hat{r}_3)</th>
<th>J_w</th>
<th>(f_n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>42</td>
<td>47.0</td>
<td>12</td>
<td>8.2</td>
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<td>2</td>
<td>40</td>
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<td>36.41</td>
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<td>3</td>
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<tr>
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<td>9</td>
<td>88</td>
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<td>0.63</td>
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<tr>
<td>10</td>
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<td>11</td>
<td>86</td>
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<td>13</td>
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<td>90.0</td>
<td>0</td>
<td>27.7</td>
<td>0.53</td>
<td>0.59</td>
<td>319582</td>
<td>21.3</td>
</tr>
</tbody>
</table>

For ease of instrumentation, the flapping wing subsystem was separately mounted onto the single wing testing setup. The block diagrams of the single wing testing setup are shown in Fig 2.8. FAULHABER Brushless DC-Servomotors 0620B has following parameters: nominal voltage 6V, resistance \(R_a = 8.8\)ohm, torque constant \(K_a = 1.09e-3Nm/A\), moment of inertia \(J_m = 9.5e - 10kgm^2\), and friction damping coefficient \(B_m = 9.74e-9Nms/rad\). The gear ratio is 10, with moment of inertia \(J_g = 5e - 9kgm^2\) and efficiency \(\eta_g = 0.8\). Parameter estimations and system identification were specified in [112].

Simulations and experiments were tested for a total of 14 wings. The tested wings have different geometries as shown in Fig. 3.4 with parameters detailed in Table 2.6).
In Table 2.6), parameters are corrected with wing offsets, and units of parameters are \( m(\text{mg}) \), \( R_w(\text{mm}) \), \( d_w(\text{mm}) \), \( c(\text{mm}) \), \( J_w(\text{mg.mm}^2) \), and \( f_n(\text{Hz}) \), respectively. \( \hat{r}_2 \) and \( \hat{r}_3 \) are unitless. For the flexible wing was fabricated from CTF3 (Cuben Fiber), a high-performance light weight non-woven fabric. The wing membrane becomes cambered after assembly, thus forming the desired angle of attack. Whereas the rigid wing prototype, the wing was constructed from carbon fiber-reinforced polymer and Mylar membrane. The wing was allowed to passively rotate up to a 45 degrees angle limited by a stopper fixed at the proximal end of the wing leading-edge spar.

All experiments were recorded by real-time dSPACE DS1103 PPC DAQ Board with sampling frequency \( f_s = 5kHz \). The brushless DC motor three-phase commutation was implemented on a 72 MHz cortex M3 board (NXP Semiconductors, San Jose, CA, USA) at rate of 50kHz. The three phase drive electronics was custom-made for the motor. The wing stroke angle was measured with motor magnetic encoder at the bottom of the motor (FAULHABER Brushless DC-Servomotors 0620B) with 256 counts/rev and calculated according to the gear transmission with gear ratio of 10:1, which gives total \( 2\pi/2056 \) rad resolution on angle measurement. The wing stroke angular velocity is calculated with simple first order Euler method. The encoder reading was recorded with the encoder interface on the dSPACE DAQ system along with the force measurements. All the measurement data are synchronized to be recorded real-time at 5000Hz. Force measurement was performed using a six component force/torque transducer (Nano17, ATI Ind. Automation). Due to limited resolution of Nano17 (0.3g resolution on the force and 1/64Nmm resolution on the torque measurement), a rigid 150mm beam setup was used to amplify the lift measurement as shown in Fig. 4.1. The improved resolution was about 0.0106g. The force sensor and beam setup was calibrated with precision weights of 0.1g, 0.5g, 5g and 20g and verified the resolution of at least 0.03g. When calculating the time-averaged force, sufficient numbers of wing -beat cycles at steady state were averaged to guarantee the reliability of the results. The raw data was filtered with cut-off frequency of 150Hz. The power source used during experiments is a DC power supply HY-5003.
from MASTECH, with current and voltage measurements accurate up to \(0.01A\) and \(0.1V\), respectively.

All numerical simulations were implemented with MATLAB ode45 solver, dynamic models in Equation (3.10) and parameters specified above.

![Image of 14 wings tested with 10 rigid carbon-fiber-Mylar wings and 4 bistable flexible cambered wings.](image)

Figure 2.9. 14 wings tested with 10 rigid carbon-fiber-Mylar wings and 4 bistable flexible cambered wings.

### 2.4.2 Experimental Results

To validate the modeling and analytical solutions, a total of 14 wings of different geometries and sizes are tested as shown in Fig. 3.4 with parameters detailed in Table 2.6.

To obtain and validate the natural frequency for each wing, first, a the frequency sweep test was conducted for each wing. For each frequency sweep test, the wing was driven at a sinusoidal input voltage wave form \(u = V_I \cos(\omega t + \beta)\) with suitable amplitude \(V_I\), while the frequency of the wave form varied from 20Hz to 45Hz. Each test run over 40 sec with frequency step size of 1Hz. Between frequency steps, the input voltage and frequency was maintained for 5 sec, which is much longer than the wingbeat period, so that the wing had sufficient time to reach steady state response.
The forces were measured with the beam setup shown in Fig. 2.8 and the wing flapping angle was recorded with the motor encoder. For processing of data, during each step, the later 2-second force measurements were averaged to get the average lift. Amplitudes of angle and the angular velocity of the stroke motion are extracted from the maximum value of the steady-state responses. The results for wing #1 to #14 are compiled to show the frequency response of the stroke amplitude, stroke angular velocity amplitude, and the mean lift force as functions of frequencies. For each wing, the natural frequency corresponding to the peak value of the stroke angular velocity and the maximum lift value. When driving the system at natural frequency \( \omega_n \) (not peak frequency \( \omega_p \)), we observed that the maximum lift, maximum efficiency, and maximum velocity (but not amplitude) were obtained for the flapping wing system. The spring stiffness for prototype with rigid wing is \( K_s = 0.00588 Nm/rad \). The spring stiffness for prototype with flexible wing is \( K_s = 0.01338 Nm/rad \).

For wing 11, the measured \( \omega_n \) is 37Hz, compared to the calculated value of 37.6Hz. For wing 12, the measured \( \omega_n \) is 34Hz, compared to the calculated value of 33.9Hz. For wing 13Hz, the measured \( \omega_n \) is 27Hz, compared to the calculated value of 28.6Hz. For wing 14, the measured \( \omega_n \) is 22Hz, compared to the calculated value of 23.9Hz.

Next, at the natural frequency of each wing, the input voltage amplitude sweep test was conducted. For each wing, the amplitude \( V_I \) of the sinusoidal input voltage wave form \( u = V_I \cos(\omega t + \beta) \) was varied, and the upper bound on the voltage amplitude depended on the wing size and limitation on the maximum allowable stroke amplitude. The input voltage drives the wing at corresponding natural frequency \( \omega_n \) determined from the driving frequency sweep test. Each test run over 40 sec with voltage step size of 0.5V. Between steps, the input voltage amplitude was maintained for 5 sec, which is much longer than the wingbeat period, so that the wing had sufficient time to reach steady state response. The forces were measured with the beam setup shown in Fig. 2.8 and the wing flapping angle was recorded with the motor encoder and calculated according to the gear transmission. For processing of data, during each step, the later 2-second force measurements were averaged to get the
average lift. Amplitudes of angle and the angular velocity of the stroke motion are extracted from the maximum value of the steady-state responses. The results for wing #6,#7,#11,#12,#13,#14 are shown in Fig. 2.10, Fig. 2.11, Fig. 2.12, Fig. 2.13, Fig. 2.14, and Fig. 2.15, respectively. As a comparison, the analytical solution in Equation (3.27) was used to estimate the stroke amplitude and lift, which were plotted together with experiment results in red line. Similar to all the other wings, it is clear that the model and analytical solution predicts the steady state response and mean lift reasonably well over all tested wings and voltages. The discrepancies between the model and experiments are within reasonable levels as the quasi-steady model is inherently an approximation with possible 30% error comparing to experiments as examined in [25, 85]. But the results here are considered sufficient accurate and informative for design optimization.

Overall, the results demonstrated the well matched predictions of the modeling, analysis, and closed-form solutions, compared to the experimental testing. Another important implication of the testing is that the rigid wings are best for low loading. The results from wing 9, and 10 suddenly deteriorated as the loading become larger. The flexing of the rigid wing under high loading is the cause of this reduced performance. However, the results of the flexible wings are shown to be better for larger loading, from wing 11 to wing 14. The flexible wings under low loading were shown to be not as good as the rigid wings due to required loading for bi-stable wing to fully pop out and form ideal angle of attack. As a result, the flexible and rigid wings form a spectrum of wings that covers from low loading of as low as 1 grams to high loading of as high as 15 grams. Our proposed model can be used accurately across at least the tested range.

2.5 Design Optimizaiton

In this section, we present the formulation and solution of the design optimization based on the modeling from previous sections.
2.5.1 Performance Matrices

Following 3 types of performance matrices are introduced for evaluating the FW-MAV.
Figure 2.12. Responses and lift for wing 11 with different input voltage.

Figure 2.13. Responses and lift for wing 12 with different input voltage.

1. **Aerodynamic Efficiency** $\eta_{aero}$: defined to be the ratio of lift and aerodynamic power (output power viewing from the actuation’s perspective). This is for evaluation of wing geometry and kinematics of flapping wing, not relevant to actuation yet. It
only provides partial evaluation of the total performance. With the assumption of sinusoidal flapping motion, $\eta_{\text{aero}}$ is summarized as

$$\eta_{\text{aero}} = \frac{F_L}{P_d} = \frac{3\pi}{8} \left( \frac{C_L}{C_D} \right) \left( \frac{r_2^2(S)}{r_3^2(S)R_w} \right) \left( \frac{1}{A\omega_0} \right)$$

(2.68)
or

$$\eta_{aero} = \frac{3\pi}{8} \left( \frac{C_L}{C_D} \right) \left( \frac{\dot{r}_2(S)}{\dot{r}_3(S)} \right) \left( \frac{1}{A\omega_0 R_w} \right)$$

(2.69) Lift-to-drag ratio Wing geometry Wing tip velocity

The aerodynamic efficiency is first tested on the kinematics data on 4 types of insects 2.4, and the results suggest that, as expected from previous observation [36], fruit fly has the highest aerodynamic efficiency among the the 4 types of insects. And the fact that Bumblebee has lower aerodynamic efficiency also supporter the speculations in [1] that Bumblebee sacrifices efficiency for higher lift in order to carry more load.

The lift-to-drag ratio has value of 1 at 45 degree and 1.94 at 18.5 degree. As lift-to-drag heavily affects Equation (2.68), a optimization can be made for certain vehicle with fixed weight, fixed wing geometry. But the kinematic parameter of $A$, $\omega_0$, and AOA $a$ need to be determined to minimize power requirement or maximize $\eta_{aero}$ for a fixed lift generation. It can be shown that all that is required is to maximizing $\frac{C_L^{1.5}}{C_D}$, which give the AOA of 24 degree, and then determine the $A$, $\omega_0$ from the lift. This can be used to also explain and check if certain animal actually choose those kinetics parameters based on efficiency or simply using the maximum lift which happens at AOA of 45 degree. For MAV design, however, a more complete optimization problem can be formulated to include other parameters, which will be the topic of next section.

Note this is similar to the power loading parameter in helicopter analysis, which is one measure of rotor efficiency because a vehicle of a given weight should be designed to hover with the minimum power requirements; that is, PL should be made as large as possible. However, PL is a dimensional quantity and so a standard non-dimensional measure of hovering efficiency called the figure of merit has be adopted. FM is defined to be ideal power over actual power required to hover.

2. Motor efficiency: Partial only for motor, the efficiency for the motor when driving the nonlinear aerodynamic load.

$$\eta_{ma} = \frac{P_{out}}{P_{total}} = \frac{B_{g2} \frac{8}{\pi^2} (A\omega_0)^3}{\left( \frac{V_I}{K_e} - N g \frac{K_e}{T_e} A\omega_0 \right) V_I}$$

(2.70)
3. **Flapping wing efficiency**: This is the overall, combined efficiency for actuator driving a wing with certain geometry with certain kinematics.

\[
\eta_l = \frac{F_L}{P_{total}} = \frac{\frac{1}{2} \rho a C_L \bar{\delta}_{2} R^3 (S) (A \omega_0)^2}{\left( \frac{V_I}{R_a} - N_g \frac{K_a}{R_a} A \omega_0 \right) V_I}
\]  \(2.71\)

### 2.5.2 Initial Design Decisions

<table>
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<th>(U_n)</th>
<th>(R_a)</th>
<th>(K_a)</th>
<th>(T_s)</th>
<th>(V_0)</th>
<th>(B_m)</th>
<th>(I_{max})</th>
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<td>8.8</td>
<td>1.09e-3</td>
<td>7.32e-4</td>
<td>5.089e3</td>
<td>9.74e-9</td>
<td>0.367</td>
</tr>
</tbody>
</table>

To gain insight into the problem, a quick initial design is shown in the following section. To numerically determine all the components of the system, a simple procedure can be used for a quick design of the system. This initial design provides valuable information and intuition for the later optimization formulation, especially for sizing the boundaries and constrains for design variables.

Here, a bio-inspired approach is adopted for initial design. The maximum size of the vehicle is often known beforehand as the specific application dictates. Here the maximum wing-to-wing dimension is around 15cm, which is a typical size for MAV. Take inspiration from nature, the hummingbird is a good template, for the size, 10gram to 15gram is a typical weight of the hummingbird. Considering the desired the payload for electronics and batteries and according to the scaling studies in previous section, we pick 12gram as the target weight.

First, a wing is selected similar to hummingbird and its parameters are shown in Table 2.6 for wing 4. To determine the flapping frequency and amplitude, a simulation is setup to calculate the possible mean lift, required motor power and torque for different frequencies and peak-to-peak stroke amplitudes. When resonance is achieved,
the torque terms $J_s\ddot{\phi} + B_s\dot{\phi} + K_s\phi$ reduces to only $B_s\dot{\phi}$, with $J_s\ddot{\phi}$ and $K_s\phi$ canceling out. The output torque becomes $B_s\dot{\phi}\dot{\phi}$ and the output power is thus $B_s\dot{\phi}\dot{\phi}$, which can be obtained numerically based on wing parameters, amplitude and frequency. To lift target weight around 12g, from Fig. 2.16A, considering the need for biasing, we choose 160\textit{degree} stroke amplitude and 46Hz, which give 7.1g lift for one wing and 14.2g for a pair of wings, required mean power of 0.95W and maximum torque of 5.5mNm. Note here that the number don’t have to be exact, a slight larger wing of $R_w = 6.5cm$ and $\bar{c} = 1.5cm$ with 150\textit{degree} stroke amplitude and 40Hz, give a lift of 7.2g for single wing, required mean power of 0.82W and maximum torque of 5.9mNm, which are also good choice for the targeted vehicle. Clearly, even for generating same lift, the wing geometry and kinematics incur differences in power and torque requirement.

The resulting average output power contour as a function of amplitude and frequency are shown in Fig. 2.16B. Based on weight and power considerations, according to the motor weight scaling in Fig. 2.6 and Equation (2.60), a 2.5\textit{gram} 6\textit{mm} brushless DC motor (FAULHABER, Clearwater, Florida USA) with nominal output power of 1.47W is chosen, which in theory covers large region of the possible frequencies and amplitudes with margins for disturbance, control and future optimization.

Second, proper gear transmission ensures that the motor provides sufficient driving torque, thereby increase the efficiency. The simulation of flapping wing model also gives the maximum output torque (including transmission gear) as a function of frequency and amplitude shown in Fig. 2.16C. According to the torque limit (stall torque for nominal voltage of 6V) of 0.73mNm from the motor specification, the gear ratio of 10 is chosen, which gives the maximum output torque at 7.3mNm. Similar to those shown in Fig. 2.16B, a large region of possible kinematics is covered by this value (Fig. 2.16C). Note that the stall torque here is limited by nominal voltage. If a higher nominal voltage is chosen, the stall torque will be higher accordingly. The maximum continuous current is more meaningful and inherent metric for current limitation to prevent the motor from overheating. So the stall torque is good metric
if the driving voltage will not exceed the nominal voltage, but one still has to check for average current for overheating. A proper chosen gear will ensure that the motor will not overheat.

With the above design consideration, in the following section a optimization formulation is proposed to further refine the design.
2.5.3 Optimization Problem Formulation and Solutions

In general, following parameters are to be determined: wing kinematics \( A, \omega_0 \), wing geometry \( R_w, \bar{c}, \hat{r}_2, \hat{r}_3 \), gear ratio \( N_g \), motor \( K_a, R_a \), and input voltage magnitude \( V_I \). The spring constant is determined subsequently to guarantee resonance. Here we formulate a constrained optimization problem with design freedoms on partial or all above parameters.

General Optimization formulation: the problem of finding a vector \( x \) that is a local minimum to a scalar function \( f(x) \) subject to constraints on the allowable \( x \):

\[
\begin{align*}
\text{minimize} & \quad f(x) \\
\text{subject to} & \quad c(x) \leq 0, \\
& \quad ceq(x) = 0, \\
& \quad Acx \leq b, \\
& \quad Aceqx = b, \\
& \quad xl \leq x \leq xu,
\end{align*}
\]

(2.72)

where nonlinear inequality constrain is \( c(x) \leq 0 \), nonlinear equality constrain is \( ceq(x) = 0 \), linear equality constrain is \( Aceqx = b \), linear inequality constrain is \( Acx \leq b \), and bounds on variables \( xl \leq x \leq xu \).

Here as a example, following the initial design in last section, the motor parameters \( K_a \) and \( R_a \) are fix due to the availability of commercial motor. Due to the prior knowledge on the wing performances [30], wing parameters \( \hat{r}_2 \) and \( \hat{r}_3 \) are pre-selected as well. So the following optimization is formulated accordingly.

Optimization formulation 1: maximizing lift to weight ratio \( \gamma_{l2w} \), i.e. \( f(x) = -\gamma_{l2w} \), where

\[
\gamma_{l2w} = \frac{\bar{F}_L}{w},
\]

\[
\bar{F}_L = \frac{1}{4} \rho_a \bar{C}_L R_w^3 \bar{c} \hat{r}_2^2 (S)(A \omega_n)^2,
\]

\[
A \omega_n = -\frac{B_{21}}{2B_{2,2}} + \sqrt{\left(\frac{B_{21}}{2B_{2,2}}\right)^2 + \frac{8}{3\pi} \frac{V_L}{B_{2,2}} K_u},
\]

(2.73)
where the total weight is \( w = w_e + w_w + w_g + w_m \) (Note here the battery weight \( w_b \) is not yet included), \( x = [R_w, \bar{c}, \omega_n, A_m, V_I, N_g]^T \), there is no linear equality constrain \( Aeqx = b \) and no linear inequality constrain \( Ax \leq b \). The nonlinear inequality constrain \( c(x) \leq 0 \) is define as

\[
\begin{align*}
  c(1) &= I_{crms} - I_{max} \leq 0 \\
  c(2) &= A - A_m \leq 0 \\
  c(3) &= (A\omega_n N_g) - v_{max} \leq 0
\end{align*}
\]

where \( A_m \) is the maximum amplitude and \( I_{crms} \) is the rms of the estimated maximum current defined as

\[
I_{crms} = \frac{\sqrt{2}}{2} \frac{(V_I - N_g K_a A \omega_n)}{R_a},
\]

\( I_{max} \) is the maximum allowing continuous current for the motor, and \( v_{max} \) is the maximum angular speed limit for the motor.

The bounds on the variables are selected according to the intuition and insight provided by the initial design, thus

\[
\begin{align*}
  lb &= [0.030, 0.001, 2\pi, 60/180 \times \pi, 0, 1] \\
  ub &= [0.100, 0.015, 2\pi \times 40, 160/180 \times \pi, 60, 10]
\end{align*}
\]

For solution method for this problem, we use a Hybrid Method, i.e. Generic Algorithm and then sequential quadratic programming method \texttt{fmincon} with Interior point algorithms. We obtain the following optimal results for formulation 1, \( x^* = [0.0610, 0.0150, \omega_n, \phi_m, 8.6772, N_g]^T \), with maximum lift of 6.7869\textit{gram} and maximum lift to weight ratio 1.6967.

Optimization formulation 2: to balance the lift generation and the control force generation, the design goal is

\[
f(x) = -\gamma_{f2w} + c_{u1} v_1 + c_{u2} v_2 + c_{u3} v_3 + c_{u4} v_4,
\]

where \( c_{ui}, i = 1, 2, 3, 4 \) are weight constants for each control inputs. The solution of this problem generate the prototype 2.
Optimization formulation 3: to balance the lift generation and the control force
generation, the design goal is

\[ f(x) = -\gamma l_{2w} + c_z |z_u| - c_p |p_u|, \]  

(2.78)

where battery weight \( w_b \) relates to \( C_{bat} \) by Equation (2.67) and \( \bar{P}_{total} \) is given by
Equation (3.42). The solution of this problem generate the prototype 3.

In formulation 1,2,and 3, batteries are not considered and the hope is that by
optimizing lift, the vehicle will be able to carry more payload, thus large battery
can be onboard. However, in light of the trade-off between the lift maximization
and efficiency reduction, the added energy cost may outweigh the benefit of larger
battery. So here, in the next formulation, the battery is added to the system, thus
system weight is \( w = w_e + w_w + w_g + w_m + w_b \).

Optimization formulation 4: flight time can be guaranteed if a nonlinear inequality
constrain is added as \( t_{min} - t_{opt} \leq 0 \), where \( t_{min} \) is the minimum flight time and \( t_{opt} \)
is the optimal flight time defined as

\[ t_{opt} = C_{bat}/\bar{P}_{total}; \]  

(2.79)

where battery weight \( w_b \) relates to \( C_{bat} \) by Equation (2.67) and \( \bar{P}_{total} \) is given by
Equation (3.42). Similarly, the flight time can be one of the design goals instead of
constrains.

In summary, formulation 1 is optimized for lift-to-weight-ratio; formulation 2 is
optimized for lift-to-weight-ratio and control authorities; formulation 3 is optimized
for lift-to-weight-ratio and control bandwidth; formulation 4 is optimized with con-
strained flight time. With this framework, stringent size, weight and power (SWaP)
constraints can be traded off with proper design of optimization goal and constrains.
Other formulations can be mixed and matched based on specific requirements of a
particular application.
2.6 System Prototype

There are total 3 prototypes with different goals optimized from previous section. The solution of formulation 1 is used to design the prototype with rigid wings. The solutions of formulation 2 and 3 are used to for the two versions of prototypes with flexible wings.

![Prototype images](image)

Figure 2.17. The prototype 2 with flexible wings. (a) Perspective view of the CAD drawing of the vehicle. (b) The picture of the vehicle. (c) The exploded view of the components. (d) The prototyped 2.3 electronic board. (e) The weight distributions of different components of the vehicle.

2.6.1 Mechanical Design and Prototype

The assembled flexible wing prototype with weight of 11.5 grams and wing span of 15cm and onboard electronics is shown in Fig. 2.17. The assembled rigid wing prototype with weight of 7.5 grams, rigid wings and wing span of 15cm is shown in Fig. 2.18.

Prototypes with flexible wings and with rigid wings share some common features. The flapping wings are directly driven by two 2.5 gram, 6mm brushless DC motors
(FAULHABER, Clearwater, Florida USA) coupled with torsional springs for kinetic energy restoring. Linear torsion springs mounted on the load shaft creates restoring torque when the wing is displaced from its mid-stroke position. Using a gear transmission, the motor was designed to generate an overall reciprocal motion of the wing. The frame structure, and spring holders were prototyped by 3D printing using a multipurpose transparent resin. A portion of the gear on the load shaft was removed to reduce the weight and moment of inertia. Two miniature ball bearings were used to support the load shaft. A pair of torsion springs was mounted on the bottom of the shaft and oriented in such a way that rotation to one direction compresses one spring and extends the other.

The rigid wing was constructed from carbon fiber-reinforced polymer and Mylar membrane. The rigid wing consists of wing frame and wing membrane attached with polyurethane adhesive. The rigid wing frame is extremely strong and light carbon fiber-reinforced polymer formed using unidirectional carbon fiber segments impregnated with an epoxy binder. The rigid wing shape for the veins and spars of the wing frame was constrained by a silicone modeling process, common for constructions of fiber-reinforced polymers. The wing membrane is $3\mu m$ thin BoPET(Mylar) sheet trimmed to the wing shape. Carbon fiber rod was fitted to the wing leading edge with cyanoacrylate adhesive to provide stopper attachment and shaft for wing rotation.
The rigid wing was allowed to passively rotate, up to a 45-degree angle limited by a stopper fixed at the proximal end of the wing leading-edge spar. The rigid wing stopper was prototyped by 3D printing using a multipurpose transparent resin.

The flexible wing was fabricated from CTF3 (Cuben Fiber), a laminated high-performance light weight non-woven fabric constructed from Ultra High Molecular Weight Polyethylene (UHMWPE) fiber monofilaments and polyester, PVF etc. films. The flexible wing has two sleeves, one on the leading edge and one on the root edge (close to the body), that accommodate the leading edge and root edge carbon-fiber bars. Since the angle between the sleeves is greater than the angle between the bars the wing becomes cambered and twisted after the assembly. The camber is bistable - it pops passively from one side to another depending on the direction of motion. The shape of the flexible wings is maintained by printing the desired shape on a sheet of paper that is attached under the membrane and used as a template for cutting. The sleeves at the leading edge and at the root edge are reinforced with superglue to increase their durability. They allow an easy assembly and disassembly as well as free rotation around the 0.8 mm leading edge and 0.8 mm root edge carbon fiber rods.

A high speed camera (Photron FASTCAM, resolution 2048 x 2048 pixels) was used to observe the wing behavior throughout the stroke and to track wing kinematics at 2000fps and 4000fps, as shown in Fig. 2.20.

2.6.2 Electronic Design and Prototype

As shown in Fig. 2.17, the prototyped electronic board (total weight 2.3 gram) is equipped with two 55mm DMOS fully integrated 3-phase motor driver with 2.8A overcurrent protection (L6230Q), a 77mm 48pin STM32F303CCT6 microcontroller with CortexM4 32-bit CPU, FPU, DSP instruction, memory protection unit and up to 13 timers, a MPU9150 Inertial Measurement Unit (IMU) with 3-axis gyroscope, 3-axis accelerometer, a 3-axis digital magnetometer. On version 3 of the prototype, the electronic board is further miniaturized as shown in Fig. 2.1.
The brushless DC motor three-phase commutation was implemented onboard of the microcontroller at rate of 50kHz. The wing stroke angle was measured with motor magnetic encoder at the bottom of the motor (FAULHABER Brushless DC-Servomotors 0620B) with 256 counts/rev and calculated according to the gear transmission with gear ratio of 10:1, which gives total $2\pi/2056$ rad resolution on angle measurement. The IMU data fusion is implemented onboard of the microcontroller at the rate of 1KHz. The motor control loop is implemented onboard of the microcontroller at the rate of 2KHz. The body controller is implemented onboard of the microcontroller at the rate of 500Hz.

2.6.3 Liftoff Results

Due to the inherent instability of the flapping flight during hovering, the take-off experiments were conducted with assistant setups. For flexible wing prototype, a 1.2 meter balanced long beam setup was used. The robot was release from the balance position in order to keep the lift vertically aligned with the direction of gravity. Taking off along a balanced beam for the prototype with battery and onboard electronics were demonstrated in Fig. 2.19 (a) (b). As shown in Fig. 2.19 (c), a 2.3 gram of additional weight (lead ball) was hanged on the robot, circled with red mark, and the lift off was still achieved. The high speed video sequences of the flapping wing at 34Hz with 170 degrees of peak-to-peak amplitude is shown in Fig. 2.20.

For the rigid wing prototype, the lift-off was demonstrated with the help of guide wire that axes the prototype’s yaw rotation with free movement along the vertical axis as well as free yaw rotation, as shown in Fig. 2.19 (d). The prototype was flapping at 30Hz with 150 degrees peak-to-peak amplitude generating 12 grams of lift.

The RHP-poles and RHP-zeros for the longitudinal and lateral dynamics are calculated for the prototype 2 and 3 as shown in Table 2.8. The prototype 3 achieved better poles and zeros locations and looser constrains on the control bandwidth than that of prototype 2, as intended by the optimization formulation 2 and 3.
Figure 2.19. Taking off along a balanced beam for the prototype with battery and onboard electronics: (a) side view, (b) front view, and (c) with 2.3gram of additional weight, circled with red mark. (d) Taking off along guid wire of current design flapping at 30Hz with 150 degrees peak-to-peak amplitude generating 12 grams of lift (0.5 sec between each sequence).

Table 2.8. Poles and Zeros of Prototypes (unit: rad/s).

<table>
<thead>
<tr>
<th>Species</th>
<th>Prototype 2</th>
<th>Prototype 3</th>
</tr>
</thead>
<tbody>
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<td>$p_u$</td>
<td>$3.1 \pm 8.2i$</td>
<td>$2.5 \pm 6.6$</td>
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<tr>
<td>$z_u$</td>
<td>220.7</td>
<td>403.4</td>
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<td>$p_{u2}$</td>
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<tr>
<td>$z_{u2}$</td>
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<td>751.1</td>
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</table>
2.6.4 Development of Onboard Sensor Fusion

The MPU9150 Inertial Measurement Unit (IMU) onboard the FWMAV consists of 3-axis gyroscope, 3-axis accelerometer and a 3-axis digital magnetometer. The widespread use of Kalman-based solutions are a testament to their accuracy and effectiveness, however, they incur high computation load at high sampling rate. For onboard implementation of IMU sensor fusion to get orientation estimation of the robot, a computationally efficient algorithm [66] is implemented onboard.

The algorithm was implemented onboard the FWMAV, and the pitch angle estimation is shown in Fig. 2.21(a), while the wings were not flapping. Fig. 2.21(a) demonstrated the fast convergence of the estimation with low steady state errors. However, as shown in Fig. 2.21(b), when the wings were flapping, due to high frequency noise induced by the flapping wing motion, the results deteriorated. Future work will be needed for improve the sensor fusion.

Figure 2.20. The high speed video sequences of the flexible flapping wing at 34Hz with 170 degrees of peak-to-peak amplitude.
2.7 Chapter Summary

In this chapter, we presented a complete, multidisciplinary formulation for system design optimization and system integration for a hummingbird-size Flapping Wing Micro Air Vehicles. The formulation covers actuation, wing, battery, electronics, dynamics, flight stability and control. System parameters considered include parameters of wings, motors, gears, springs, batteries, control authorities, and locations of poles and zeros of system dynamics. The formulation was validated by experimental data for both rigid and flexible wings, covering wing loading from low to high. Based on the direct motor drive mechanism of our previous work, the optimization
yields a prototype with on-board sensors, electronics, and computation. It flaps at 30Hz to 40Hz with 12 grams of system weight and 20 grams of maximum lift. Liftoff was demonstrated with extra payloads. Initial results of on-board state estimation and flight control were demonstrated. Flapping wing platforms with different requirements and scales can now be designed and optimized with light modifications of proposed formulation.
3. FLAPPING RESONANCE DESIGN PRINCIPLE FOR FLAPPING WING MICRO AIR VEHICLES

3.1 Introduction

Achieving resonance in flapping wings has been recognized as one of the most important principles to enhance power efficiency, lift generation, and flight control performance of high-frequency flapping wing micro air vehicles (MAVs).

Figure 3.1. Flapping wing robots: (1) Berkely MFI [22], (2) Harvard Robobee [108], (3) CMU [2], (4) Nano Humminbird [50], (5) Cornell [82], (6) Delfly [18], (7) Purdue [112, 113], (8) Beelte [75], (9) CMU [48], (10) NTU [57], (11) Purdue [83].
Despite the lack of theoretical guidance and the difficulty in resonance tuning, the past several years have seen numerous advances in designing and prototyping high-frequency flapping wing actuation and mechanisms across different scales. At sub-gram scale, in order to reduce the required power for flapping motion, the Piezo electric (PZT) bending cantilever has been developed to resonate with actuator’s material property[108, 2, 34]. With this design, the Harvard’s insect-size robot has demonstrated lifting off [108] and controlled flight [62]. Meanwhile, at grams to tens-of-gram scale, motor driven thorax-wing systems were successful developed, operating at low input voltage with higher payload. This form of actuation can be further divided into linkage-mechanisms and direct-drive types. Linkage mechanisms transfer rotational motion from the motor to reciprocal motion of the wings through four-bar or equivalent four-bar mechanisms. Several works added elastic elements to improve the four-bar linkage mechanism that resulted in limited success[97, 64, 53, 4, 65], largely due to the fact that the highly nonlinear and asymmetric linkage transmissions hinder the full recovery of potential energy from sinusoidal wing motion. Recently, direct-drive type is gaining popularity with works such as[112, 3, 48], where each flapping wing is driven directly using a geared DC motor without linkages, while the motor undergoes reciprocating rotation. Prototypes that achieved liftoff was demonstrated in [48].

However, the theoretical understanding of its effects on the response and energetics of flapping motion has lagged behind, leading to sub-optimal design decisions and misinterpretations of experimental results. In addition, for most of the previous works, the flapping resonance was designed as an independent variable, thereby requiring a large number of tests to characterize and verify choice of system parameters and yielding little insight for the overall systematic design.

In this chapter, we systematically model the dynamics of flapping wing as a forced nonlinear resonant system, using both nonlinear perturbation method and linear approximation approach. We derived analytic solution for steady-state flapping amplitude, energetics, and characteristic frequencies including natural frequency, damped
natural frequency, and peak frequency. Our results showed that both aerodynamic
lift and power efficiency are maximized by driving the wing at natural frequency,
instead of other frequencies. Interestingly, the flapping velocity is maximized at nat-
ural frequency as well, which can lead to an easy experimental approach to identify
natural frequency and follow the resonance design principle. Our models and analysis
were validated with both simulation and experiments on ten different wings mounted
a direct-motor-drive flapping wing MAV. The result can serve as a systematic design
principle and guidance in the interpretations of empirical results.

The rest of the chapter is organized as follows. Section 3.2 presents the modeling
of flapping wing. Section 3.3 gives the theoretical analysis of flapping resonance.
Section 3.4 details the experiment results. The chapter is concluded with discussions
and future works.

3.2 Dynamic Model of Flapping Wing Actuation

3.2.1 Flapping Wing System for Case Study

The flapping wing MAV used for experiment validation and case studies in this
work is introduced here. As illustrated in Fig. 4.1(7), two flapping wings are directly
driven by two 2.5 gram, 6mm brushless DC motors coupled with torsional springs
for kinetic energy restoration. Using a gear transmission, the motor was designed
to generate reciprocal motion of the wing. The wing was constructed from carbon
fiber-reinforced polymer and Mylar membrane. The frame structure, wing stoppers,
and spring holders were prototyped by 3D printing using a multipurpose transparent
resin. A portion of the gear on the load shaft was removed to reduce the weight and
moment of inertia. Two miniature ball bearings support each load shaft. A torsion
spring was mounted on the bottom of each shaft and oriented in such a way that
rotation to one direction compresses one spring and extends the other. The wing was
allowed to passively rotate up to a 45 degrees angle limited by a stopper fixed at the
proximal end of the wing leading-edge spar. The assembled FWMAV with weight of
7.5 grams and wing span of 15cm is capable of taking off. As a result, the experiment studies in this work were conducted at scale, and the results can be directly applied with insect/hummingbird-size flapping wing robots.

### 3.2.2 Flapping Wing Aerodynamic Model

![Diagram](image)

Figure 3.2. (a) Parameters of wing shape: $R_w$ is the wing length, $O$ is wing root, i.e. the intersection of leading edge (LE) and trailing edge (TE), $c(r)$ is the chord length at distance $r$ from $O$, $O'$ is the intersection of stroke axis $z$ and LE, $d_w$ is the wing offset of $O$ from $O'$, stroke angle is $\phi$, and rotation angle is $\psi$. (b) The body $xyz$ coordinate system is fixed to the vehicle airframe. The wing flaps about the $\phi$ axis, which remains parallel with the body $z$ axis. The wing rotation $\psi$ axis is parallel with respect to the wing leading edge. (c) Diagram of typical flapping wing motion projected onto a 2-D plane. (d) The diagram of generalized flapping wing actuation system.
The geometric quantities of the wing are defined with parameterization methods detailed in [30]. We have the moment of inertia of the wing with respect to the wing root as

\[ J_w = \int_0^{R_w} \rho_w c(r)r^2 dr = \rho_w R_w^3 \bar{c} \hat{r}_2^2, \]  

(3.1)

where, as illustrated in Fig. 4.2(a), wing root is at \( O \), \( R_w \) is the wing length, \( \rho_w \) is the wing planar density, \( c(r) \) is the chord length at \( r \) distance from the wing root \( O \), \( \bar{c} = \frac{1}{R_w} \int_0^{R_w} c(r) dr \) is the mean chord length, and \( \hat{r}_2^2 \) and \( \hat{r}_3^3 \) are the second and third moment of wing area, respectively, given by

\[ \hat{r}_2^2 = \frac{\int_0^{R_w} c(r)r^2 dr}{R_w^3 \bar{c}}, \quad \hat{r}_3^3 = \frac{\int_0^{R_w} c(r)r^3 dr}{R_w^4 \bar{c}}. \]

Typical two DoF wing flapping motions are illustrated in Fig 4.2(c) with the cut view of flapping wing motion projected onto a 2-D plane. Dash lines indicate the instantaneous position of the wing chord at temporally equidistant points during each half-stroke. Small circles mark the leading edge. Wing moves left to right during downstroke, right to left during upstroke.

With quasi-steady aerodynamic model [25], the instant lift force and aerodynamic damping torque on the wing at time \( t \) with wing flapping \( \phi(t) \), rotation \( \psi(t) \), and angle of attack \( \alpha(t) \) [22] are given by

\[ F_L(t) = \frac{1}{2} \rho_a \bar{C}_L R_w^3 \bar{c} \hat{r}_2^2 \dot{\phi}^2, \]

\[ \tau_d(t) = B_{s2} \dot{\phi}^2 \text{sign}(\dot{\phi}), \]  

(3.2)

where \( \rho_a \) is the air density, \( B_{s2} = \frac{1}{2} \rho_a \bar{C}_D R_w^4 \bar{c} \hat{r}_3^3(S) \) is the aerodynamic damping coefficient, \( \bar{C}_L \) and \( \bar{C}_D \) are the mean lift and drag coefficients averaged over one wing stroke, respectively, given by [25]

\[ \bar{C}_D = 1.92 - 1.55 \cos(2.04\alpha - 9.82) \]

\[ \bar{C}_L = 0.225 - 1.58 \sin(2.13\alpha - 7.20), \]  

(3.3)

where the angle of attack is \( \alpha \). The quasi-steady model is adopted here due to the lack of simple closed form model for unsteady aerodynamics[25].
The center of pressure [22], i.e. the location where the aerodynamic forces acting on the wing is defined as

\[ r_{cp} = \frac{R_w \hat{r}_3^3(S)}{\hat{r}_2^2(S)} = \frac{\int_0^{R_w} c(r)r^3 dr}{\int_0^{R_w} c(r)r^2 dr}. \] (3.4)

3.2.3 General Flapping Wing Actuation Model

While the flapping wing system considered here has two degrees of freedom: wing stroke (flapping) and wing rotation, only flapping motion is actuated, while wing rotates passively due to aerodynamic and inertial forces. Passive wing rotation has been observed in most insect flight to reduce the actuation complexity and the power requirements [42, 6], which has also been applied in a number of robotic platforms as well[108, 48, 112]. The passive wing rotation is realized either though flexural hinge [23, 48] or mechanical stopper that limits the free wing rotation up to a desired maximum angle of attack[112].

As shown in Fig. 4.2(b) and (c) and also in study [25], the passive wing rotation consists mostly two parts: translation at optimal angle of attack for majority of the stroke motion and rotation to the other extreme during stroke reversals. The angle of attack will not be constant at least during stroke reversals. [48] has studied the model of such two degrees of freedom system.

In order to reveal the underlying flapping resonance, simplifications have been made to reduce the complexity of the dynamics, typically with assumptions of constant angle of attack[32, 48]. In this work, as the passive wing rotation is limited by stopper to optimal 45 degrees angle of attack during most of the stroke motion, a constant angle of attack of 45 degree is assumed. This assumption implies that ideally the wing can rotate instantaneously from 45 degrees to -45 degrees, and vice versa, at the two extremes of stroke reversals to always deliver the constant angle of attack of 45 degrees. [32] also assumed fixed angle of attack of 45 degrees, while [48] adopted the angle of attack of 90 degrees.
The wing stroke dynamics is more dominate than the wing rotation dynamics. In [32], it was shown that the majority of kinetic energy due to wing movement is stored in the flapping mode, about 50 times larger than that in the rotation mode. As a result, we assume that the behavior of the wing is modeled by a beam damped by quasi-steady aerodynamics, rotating about the stroke axis and resonating with torsional spring, as shown in Fig. 4.2(d), where the wing stroke angle ($\phi$) is driven by the actuator under resonance with torsional spring stiffness ($K_s$) and passive wing rotation angle ($\psi$). The behavior of the wing is modeled by

$$J_s\ddot{\phi} + B_{s1}\dot{\phi} + T_a + T_r = T_s,$$  \hspace{1cm} (3.5)

where $\phi$ is the flapping/stroke angle in rad, $J_s$ is the total moment of inertia of rotational elements, $B_{s1}$ is the lumped linear damping coefficients, $T_a$ is the aerodynamic damping torque on the wing as it flaps, $T_r$ is the elastic restoring torque, and $T_s$ the input torque applied by the actuator, respectively. From the aerodynamic model of flapping wing, $T_a = B_{s2} |\dot{\phi}| \dot{\phi}$, where $B_{s2}$ is the aerodynamic damping coefficients. The aerodynamic damping term $B_{s2} |\dot{\phi}| \dot{\phi}$ is estimated based on a quasi-steady aerodynamic model using blade element theory (BET) [25]. If the elastic elements is or can be approximated as linear, $T_r = K_s \dot{\phi}$, with stiffness $K_s$. Without loss of generality, input torque can be rewritten as product of linear input gain $K_u$ and a physical quantity $u$, $T_s = K_u u$. Equation (3.5) becomes

$$J_s\ddot{\phi} + B_{s1}\dot{\phi} + B_{s2} |\dot{\phi}| \dot{\phi} + K_s \dot{\phi} = K_u u.$$ \hspace{1cm} (3.6)

The system is normally excited with sinusoidal input $u = V_{in} \cos(\Omega t + \beta)$, where $V_{in}$ is the magnitude, $\Omega$ is the angular frequency, and $\beta$ is the phase.

This general wing actuation model can capture the nonlinear dynamics of motor direct-driven flapping wing [48, 112, 8], Piezo-driven flapping wing [32], and even insect wings [42].
3.2.4 Flapping Wing Actuation Model with DC Motor

The flapping wing actuation system with DC motor is illustrated in Fig. 3.3. The wing stroke ($\phi$) is driven by DC motor under resonance with torsion spring ($K_s$), and passive wing rotation ($\psi$) is limited by the stopper to optimal angle of attack of 45 degrees.

As the inductance of the motor is negligible, the equation of motion for DC motor is

$$J_m \ddot{\phi}_m + B_m \dot{\phi}_m = K_a I_a - T_m,$$

(3.7)

where $J_m$ is the moment of inertia of the motor rotating elements, $\phi_m$ is the motor angle, $B_m$ is the damping coefficient of the motor rotating elements, $K_a$ is the torque constant, $I_a$ is armature current, and $T_m$ is the motor load torque. $I_a = \frac{u - K_a \phi_m}{R_a}$ with $u$ being the input voltage to the motor and $R_a$ being the resistance of the motor.

Defining $B_{m1} = B_m + \frac{K_a^2}{R_a}$, we have

$$J_m \ddot{\phi}_m + B_{m1} \dot{\phi}_m = \frac{K_a}{R_a} u - T_m.$$  

(3.8)
With gear transmission, we have \( \phi_m = N_g \phi_l \), \( \eta_g N_g T_m = T_l \) with \( N_g \) being the gear ratio, \( \phi_l \) being the load angle, \( T_l \) being the load torque, and \( \eta_g \) being the gear efficiency. Then the motor dynamics becomes

\[
J_e \ddot{\phi}_l + B_e \dot{\phi}_l = K_u u - T_l,
\]

(3.9)

where the effect moment of inertia \( J_e = \eta_g N_g^2 J_m \), effective damping \( B_e = B_{s1} = \eta_g N_g^2 (B_m + \frac{K_a^2}{K_a}) \), and input gain \( K_u = \eta_g N_g \frac{K_a}{R_a} \).

Directly driving the wing using geared motor with coupled parallel torsion spring as shown in Fig. 3.3, we have \( T_l = J_w \ddot{\phi}_w + B_{s2} \dot{\phi}_w |\dot{\phi}_w| + K_s \phi_w \) and \( \phi_l = \phi_w = \phi \), thus

\[
J_s \ddot{\phi} + B_{s1} \dot{\phi} + B_{s2} |\dot{\phi}| \dot{\phi} + K_s \phi = K_u u,
\]

(3.10)

where the total moment of inertia is \( J_s = N_g^2 J_m + J_w + J_g \). Compared with the general flapping wing actuation system model of Equation (4.1), in the case of DC motor, input torque \( T_s = K_u u \) is different from the motor torque \( T_m \).

### 3.2.5 Flapping Wing Actuation Model with Current-Controlled DC Motor

As a special case, for DC motor with fast current feedback control loop, the input to the system is the regulated output current \( I_a \) from the current control loop. The differences of the corresponding system parameters for Equation (3.10) are \( B_{s1} = \eta_g N_g B_m \), \( K_u = \eta_g N_g K_a \), and \( u = I_a \).

### 3.2.6 Flapping Wing Actuation Model with Piezoelectric Cantilever Beam

In [32], the actuation based on piezoelectric cantilever beam delivers an oscillatory mechanical input to a four-bar transmission, converting linear motion of the tip of the actuator to flapping motion of the wings. Each wing is attached to the transmission through a flexure hinge that acts as a torsion spring, allowing the wing to rotate passively due to aerodynamic and inertial forces as it flaps. Under the assumption
of small displacements, the four-bar transmission can be approximated as a linear transmission, which effectively function similarly to the gear transmission of the DC motor case. With the model of piezoelectric cantilever beam as a spring mass subsystem, the overall wing actuation model with piezoelectric cantilever beam can still be accurately captured with the general model in Equation (4.1) as shown in [32].

3.2.7 Flapping Wing Actuation Model of Flapping Wing Animals

Flapping wing insects have two types of muscles actuating their wings: synchronous and asynchronous muscles. The synchronous muscles are mostly responsible for modulating the wing rotation and fine-tuning the flapping motion, while asynchronous muscles provide the majority of power for flapping motion, indirectly moving the wings by exciting a resonant mechanical load in the body structure. The asynchronous muscles are analogous to the DC motor and torsion spring in previous section. Biological study in [42] modeled the muscle elasticity and its force-extension relation as elastic polymers such as rubber, and the restoring force $f_r$ can be described as

$$f_r = E_0 A_0 (\beta - \beta^{-2}),$$

(3.11)

where $E_0$ is the elastic modulus of the muscle, $A_0$ is the cross section of the unstrained muscle, and $\beta$ is the extension ratio $b/b_0$, where $b$ and $b_0$ are the length of the strained and unstrained muscle relative to the equilibrium positions, respectively. For small extension ratios, the expression $\beta - \beta^{-2}$ reduces to $3\Delta b/b_0$, and the muscle can be approximated as a linear spring similar to the torsion spring in the DC motor case. Without a good understanding of the unsteady aerodynamics of flapping wing at the time, [42] modeled the flapping wing aerodynamics directly as an unknown linear damping term. Incorporating the aerodynamic modeling from previous section, the general model of Equation (4.1) applies to biological flapping wing system as well.
3.3 Effects of Resonance on Flapping Wing Response

Dynamics of the flapping wing actuation system in Equation (4.1) is nonlinear, and its response can not, in general, be expressed as simple analytic formulas. It is necessary to numerically simulate the dynamics to get the response.

To facilitate the analysis, especially of the influences of resonance and system parameters on the system response, linearization methods were regularly utilized[32, 8, 104]. In [32], the nonlinear aerodynamic damping was linearized about an operating point of maximum drag and fixed 45 degrees angle of attack, while in [8, 104], a linear damping coefficient is defined to approximate the nonlinear aerodynamic damping effect to provide the same average power over a wingbeat period.

In order to provide a theoretical basis for linear approximation and quantify the associated errors, we use a nonlinear perturbation method, i.e. method of multiple scales[74, 13], to analyze the nonlinear dynamics and obtain analytical predictions of forced responses of the dynamics of Equation (4.1). The closed-form solutions capture the nonlinear vibration response as functions of system parameters, revealing insights about how system parameters and resonance affects the steady state response and energetics of flapping wing.

3.3.1 Nonlinear Analysis of Flapping Wing Response

Excited with sinusoidal input $u = V_{in} \cos(\Omega t)$ with amplitude $V_{in}$ and frequency $\Omega$, the equation of motion in Equation (4.1) can be rewritten in the following dimensionless form

$$\ddot{\phi} + \omega_n^2 \phi = -2\epsilon \mu \dot{\phi} - \epsilon |\dot{\phi}| \dot{\phi} + E(t), \quad (3.12)$$

which represents a forced oscillator with a quadratic damping and a spring. The natural frequency or primary resonance is $\omega_n = \sqrt{\frac{K_s}{J_s}}$. $\epsilon = \frac{B_s^2}{J_s}$ is a small dimensionless perturbation parameter[74]. $\mu = \frac{B_s \omega}{2J_s \epsilon}$ is the normalized linear damping coefficient. $E(t) = \frac{K_s}{J_s} u = \epsilon k \cos(\omega_n t + \epsilon \sigma t)$ is the input excitation, where $\Omega = \omega_n + \epsilon \sigma$ and
\[ k = \frac{K_u V_{in}}{J_\sigma}; \]
\[ \sigma = O(1) \] is the detuning parameter \cite{74}, which quantifies the nearness of \( \Omega \) to \( \omega_n \).

The approximate solution of Equation (4.1) near \( \omega_n \) can be obtained using method of multiple scales\cite{74}. Specifically, we first define new time scales \( T_i = \epsilon^i t, i = 0, 1, 2... \) and the solution has the form

\[ \phi(t, \epsilon) = \phi_0(T_0, T_1) + \epsilon \phi_1(T_0, T_1) + ... \] (3.13)

Then the first order approximation of the solution is:

\[ \phi_0 = A \cos(\omega_n T_0 + B(T_1)) + O(\epsilon), \] (3.14)

which means the response of the system is approximately sinusoidal with approximation error quantified by parameter \( \epsilon \). In this work, with the parameters of DC motor driven flapping wing system, \( \epsilon = O(0.1) \).

The amplitude \( A \) and phase \( B \) are solved by two ordinary differential equations

\[ \dot{A} = -\mu A - \frac{4}{3\pi} A |A| \omega_n + \frac{k}{2\omega_n} \sin(\gamma), \]
\[ A \dot{\gamma} = A \sigma - \frac{k}{2\omega_n} \cos(\gamma), \] (3.15)

where \( \gamma = \sigma T_1 - B \).

The steady-state solution is solved by setting time derivatives to zeros, i.e. \( \dot{A} = 0 \) and \( \dot{\gamma} = 0 \), we have

\[ A^2 \left[ (\mu + \frac{4}{3\pi} A |A| \omega_n)^2 + \sigma^2 \right] = \frac{k^2}{4\omega_n^2}, \] (3.16)

which is commonly called frequency-response equation. The steady state amplitude of the response \( A \) can be solved given the driving frequency \( \Omega \) (or \( \sigma \)).

When the system is driven at natural frequency, i.e. \( \sigma = 0 \) and \( \Omega = \omega_n \), we have the amplitude prediction \( A = A_n \) is

\[ A_n = \frac{-\mu + \sqrt{\mu^2 + \frac{8k}{3\pi}}}{\frac{8}{3\pi} \omega_n}, \] (3.17)

which can be transformed into

\[ A_n \omega_n = \frac{1}{2B_2} \left( \sqrt{B_2^2 + 4B_2 V_{in} K_u} - B_{s1} \right), \] (3.18)
with the original system parameters and lumped parameter $B_2 = B s^2 \frac{8}{3\pi}$.

With the first order approximation of the solution in Equation (3.14), the stroke-averaged powers drained by aerodynamic damping is

$$\overline{P_d} = \frac{1}{T} \int_T B_s^2 \dot{\phi} |\dot{\phi}| dt = \frac{4}{3\pi} B s^2 (A\Omega)^3 + O(\epsilon^3),$$

(3.19)

the total power is

$$\overline{P_{\text{total}}} = \frac{1}{T} \int_T u I_a dt = \frac{1}{2} \left( \frac{V_{\text{in}}}{R_a} - N_g \frac{K_a}{R_a} \Omega A \right) V_{\text{in}} + O(\epsilon),$$

(3.20)

and mean efficiency is

$$\overline{E_{\text{ff}}} = \frac{\overline{P_d}}{\overline{P_{\text{total}}}} = \frac{B s^2 \frac{8}{3\pi} (A\Omega)^3 + O(\epsilon^3)}{\left( \frac{V_{\text{in}}}{R_a} - N_g \frac{K_a}{R_a} \Omega A \right) V_{\text{in}} + O(\epsilon)}.$$  

(3.21)

Substituting $A_n \omega_n$ from Equation (3.18) gives the closed-form formulas for energetics of flapping wing.

### 3.3.2 Linear Approximation of Flapping Wing Response

The nonlinear analysis above well captures the system response excited at/near natural frequency. We have $\epsilon = \frac{B_2 s^2}{J_s}$ to quantify approximation errors when sinusoidal output is assumed as well.

A typical 2nd-order resonant system can, in addition, be characterized by damped natural frequency and peak frequency. To facilitate the definition of these frequencies and to characterize the associated system response, here we adapt the linear approximation methods from [8, 104, 32], despite the differences on actuation and dynamics. Analysis of the system response especially at damped natural frequency and peak frequency was performed. Moreover, we quantify the errors and identify the limitations of the linear approximation based on results of previous nonlinear analysis.

When the system is excited with sinusoidal input $u = V_{\text{in}} \cos(\Omega t)$ with amplitude $V_{\text{in}}$ and frequency $\Omega$, as shown in [8, 104], a linear damping $B_1$ can be defined to
capture the nonlinear aerodynamic damping effects to provide the same average power over a wingbeat period, i.e. \( \frac{1}{T} \int_T B_1 \dot{\phi} \dot{\phi} dt = \frac{1}{T} \int_T B_{s2} \dot{\phi} |\dot{\phi}| dt \). With the assumption of sinusoidal response, it can be shown that \( B_1 = B_{s2} \frac{8}{3\pi} \Omega A = B_2 \Omega A \).

To quantify approximation errors, in this work, the criteria for evaluating the linear approximation accuracy is defined as the time-averaged squared error over one wingbeat for the nonlinear damping torque. With the assumption of sinusoidal output, we have

\[
Err = \frac{1}{T} \int_T (B_1 \dot{\phi} - B_{s2} |\dot{\phi}|)^2 dt
\]

\[
= \frac{A^2 \Omega^2}{2} (B_1^2 + \frac{3}{4} B_{s2}^2 A^2 \Omega^2 - \frac{16}{3\pi} B_1 B_{s2} A \Omega),
\]

which is minimized to

\[
Err_{min} = (3/4 - (8/3/\pi)^2) B_{s2}^2 \Omega^2 A^2 \approx 0.03(B_{s2} \Omega A)^2,
\]

with \( B_1 = B_{s2} \frac{8}{3\pi} \Omega A = B_2 \Omega A \). This showed the equivalent average power approach of \([8, 104]\) minimized the wingbeat-averaged squared approximation error as well.

Essentially, the nonlinear aerodynamic damping is linearized here, not about an operating point, but with steady state of the response, as \( B_1 \) is a function of the frequency \( \Omega \) and steady state amplitude of the response \( A \). In comparison, \([32]\) had the nonlinear aerodynamic damping linearized about an operating point of maximum drag (in the middle of the half stroke with maximum flapping velocity) and fixed 45 degrees angle of attack, giving a ”worst case” estimate for aerodynamic damping. The approach in \([32]\) could lead to conservative estimates of the flapping damping effects: the effective linear damping coefficient in \([32]\) is \(3\pi/4 \approx 2.356\) times of current value \( B_1 \).

With the linear damping \( B_1 \), the equation of motion Equation (4.1) can be approximated by the following linear dynamics,

\[
J_s \ddot{\phi} + B_t \dot{\phi} + K_s \phi = K_u u,
\]
with the transfer function as

\[ G(s) = \frac{\Phi(s)}{U(s)} = \frac{K_u}{J_s s^2 + B_l s + K_s} \]

\[ = \frac{K_0}{s^2 + 2\xi\omega_n s + \omega_n^2}, \]  

where \( K_0 = \frac{K_u}{J_s} \), natural frequency \( \omega_n = \sqrt{(K_s/J_s)} \), \( B_l = B_{s1} + B_2\Omega A \), and damping ratio \( \xi = \frac{B_2}{2\sqrt{J_s K_s}} \).

From linear system theory, we have the following definition for frequencies related to resonance:

- **Natural frequency**: \( \omega_n = \sqrt{(K_s/J_s)} \).
- **Damped natural frequency**: \( \omega_d = \omega_n \sqrt{1 - \xi^2} \).
- **Peak frequency**: \( \omega_p \), for moderately under-damped (\( \xi < 1/\sqrt{2} \)) 2nd-order harmonic oscillator having its maximum gain \( |G(\Omega_j)|_{max} \) when driven by a sinusoidal input. All the wings tested in this study didn’t exceed this critical value for damping ratio.

The response is typically what to be determined and unknown in advance. This linear model will not be useful for predicting the steady state response or even running simulation, as its own parameters depend on the response. At particular characteristic frequencies, however, this difficulty can be alleviated. Next we exam the system responses driven at different frequencies. Here we use \( A_i \) to represent the amplitude of system response at frequency \( \omega_i \), with \( i = n, d, p \) for natural frequency, damped natural frequency and peak frequency, respectively.

**Natural Frequency**: When the system is driven at natural frequency, i.e. \( \Omega = \omega_n \), we can substitute the closed-form solution of \( A_n \omega_n \) of Equation (3.18) into \( B_1 \) for the linear model Equation (3.24) and get

\[ J_s \ddot{\phi} + \frac{1}{2} \left( B_{s1} + \sqrt{B_{s1}^2 + 4B_2 V_{in} K_u} \right) \dot{\phi} + K_s \phi = K_u u, \]  

(3.26)
where at natural frequency, \( \angle G(\omega_n j) = -\frac{\pi}{2} \), \( A_n = V_{in}|G(\omega_n j)| = \frac{V_{in}K_u}{\frac{1}{2}(B_{s1} + \sqrt{B_{s1}^2 + 4B_2V_{in}K_u})\omega_n} \),
which is rewritten as
\[
A_n \omega_n = \frac{2V_{in}K_u}{B_{s1} + \sqrt{B_{s1}^2 + 4B_2V_{in}K_u}}. \tag{3.27}
\]
Equation (3.27) can be shown to be the same as Equation (3.18), which connects the nonlinear analysis with the linear approximation. It shows that the linear approximation method is accurate at least when the system is driven at natural frequency. The approximation error can also be characterized with the same parameters \( \epsilon = \frac{B_{s2}}{J_s} \) and the big O notation used in the nonlinear analysis.

**Damped Natural Frequency and Peak Frequency:** When the system is driven at other frequency \( \Omega \), i.e. \( u = V_{in}\cos(\Omega t) \), we have
\[
\phi = |G(\Omega j)|V_{in}\cos(\Omega t + \angle G(\Omega j)), \tag{3.28}
\]
where the response amplitude is
\[
A = |G(\Omega j)|V_{in} = \frac{K_uV_{in}}{\sqrt{(K_s - J_s\Omega^2)^2 + B_1^2\Omega^2}}. \tag{3.29}
\]
Equation (3.29) can be rewritten as
\[
B_2^2\Omega^4A^4 + 2B_{s1}B_2\Omega^3A^3 + (K_s^2 + J_s\Omega^2)A^2
+ (B_{s1}^2 - 2J_sK_s)\Omega^2A^2 - (K_uV_{in})^2 = 0, \tag{3.30}
\]
which is a nonlinear function of amplitude and frequency. Numerical solution can be obtained for amplitude \( A \), given driving frequency \( \Omega \), even though simple closed-form solution may not exist.

For the special case when \( B_{s1} \approx 0 \), e.g. current-controlled DC motor with low frictional damping on the shaft and flapping wing animals, however, Equation (3.30) becomes
\[
c_{01}A^4 + c_{02}A^2 + c_{03} = 0, \tag{3.31}
\]
where \( c_{01} = B_2^2\Omega^4 \), \( c_{02} = (K_s - J_s\Omega^2)^2 \), and \( c_{03} = -(K_uV_{in})^2 \), respectively. Given frequency \( \Omega \), the closed-form solution for Equation (3.31) is
\[
A = \sqrt{-\frac{c_{02} + \sqrt{c_{02}^2 - 4c_{01}c_{03}}}{2c_{01}}}. \tag{3.32}
\]
For the system response driven at damped natural frequency, $\Omega = \omega_d = \omega_n \sqrt{1 - \xi^2}$, numerical solution for amplitude $A_d$ can be obtained from Equation (3.30).

When the system is driven at peak frequency, i.e. $\Omega = \omega_p$, from traditional linear theory, peak frequency is typically defined as $\omega_p = \omega_n \sqrt{1 - 2\xi^2}$, with maximum gain $|G(\Omega j)|_{max} = \frac{K_s}{2\xi \sqrt{1 - \xi^2}}$. Here we show that this is actually not the case, as damping term $B_1$, $B_l$, and $\xi$ are function of resulting amplitude $A$ and driving frequency $\Omega$. We need to derive the maximum amplitude and peak frequency accordingly.

As $B_l$ is also a function of $A$ and $\Omega$, we have $\frac{\partial B_l}{\partial \Omega} = B_2 A^2 + B_2 \omega_p \frac{\partial A}{\partial \Omega}$. The maximum amplitude is obtained by taking the partial derivative of Equation (3.29) with respect to $\Omega$ and setting $\frac{\partial A}{\partial \Omega} = 0$, and we have

$$c_{11} \omega_p^2 + c_{12} \omega_p + c_{13} = 0,$$  
(3.33)

where $c_{11} = 2(B_2^2 A_p^2 + J_s^2)$, $c_{12} = 3B_{s1}B_2 A_p$, and $c_{13} = (B_{s1}^2 - 2J_s K_s)$, respectively. If $A_p$ is known, we can solve the peak frequency as

$$\omega_p = \frac{-c_{12} + \sqrt{c_{12}^2 - 4c_{11}c_{13}}}{2c_{11}},$$  
(3.34)

With Equation (3.30) and Equation (3.33), we cancel out the $A^3$ term and have

$$c_{21} A_p^4 - c_{22} A_p^2 + c_{23} = 0,$$  
(3.35)

where $c_{21} = B_2^2 \omega_p^4$, $c_{22} = (3K_s^2 + B_{s1}^2 \omega_p - J_{s1}^2 A_p^4 - 2J_s K_s \omega_p^2) A_p^2$, and $c_{23} = 3(K_s V_{in})^2$, respectively. If $\omega_p$ is known, we can solve the maximum amplitude as

$$A_p = \sqrt{\frac{c_{22} - \sqrt{c_{22}^2 - 4c_{21}c_{23}}}{2c_{21}}}.$$  
(3.36)

However, $A_p$ and $\omega_p$ need to be solved together. The system of nonlinear equations consisting of Equation (3.33) and Equation (3.35) with two variables $[\omega_p, A_p]$ can be solved numerically, for example, with MATLAB function fsolve. To speed up the convergence, the initial solution can be set to $[\omega_n, A_n]$ according to closed-form solution in Equation (3.18) and $\omega_n = \sqrt{(K_s/J_s)}$.

**Summary of Analytical Results:** The analytic formulas for predicting characteristic frequencies and system responses are summarized here:
(a) At natural frequency, $\omega_n = \sqrt{K_s/J_s}$, closed-form solution for amplitude $A_n$ is

$$A_n\omega_n = \frac{1}{2B_2} \left( \sqrt{B_{s1}^2 + 4B_2V_inK_u - B_{s1}} \right), \text{ or}$$

$$A_n\omega_n = \frac{2V_inK_u}{B_{s1} + \sqrt{B_{s1}^2 + 4B_2V_inK_u}},$$

and the linear model is

$$J_s\ddot{\phi} + \frac{1}{2} \left( B_{s1} + \sqrt{B_{s1}^2 + 4B_2V_inK_u} \right) \dot{\phi} + K_s\phi = K_uu,$$  

(3.38)

(b) At damped natural frequency, $\omega_d = \omega_n\sqrt{1 - \xi^2}$, numerical solution for amplitude $A_d$ can be obtained from

$$B_2^2\omega_d^4A_d^4 + 2B_{s1}B_2\omega_d^3A_d^3 + (K_s^2 + J_s^2\omega_d^2)A_d^2$$

$$+ (B_{s1}^2 - 2J_sK_s)\omega_d^2A_d^2 - (K_uV_in)^2 = 0;$$

(3.39)

(c) At peak frequency, $\omega_p$ and $A_p$ can be solved numerically from the system of nonlinear equations:

$$c_{11}\omega_p^2 + c_{12}\omega_p + c_{13} = 0,$$

$$c_{21}A_p^4 - c_{22}A_p^2 + c_{23} = 0,$$

where $c_{11} = 2(B_2^2A_p^2 + J_s^2)$, $c_{12} = 3B_{s1}B_2A_p$, $c_{13} = (B_{s1}^2 - 2J_sK_s)$, $c_{21} = B_2^2\omega_p^4$, $c_{22} = (3K_s^2 + B_{s1}^2\omega_p^2 - J_s^2\omega_p^4 - 2J_sK_s\omega_p^2)A_p^2$, and $c_{23} = 3(K_uV_in)^2$, respectively.

(d) Flapping wing energetics: the stroke-averaged power requirement of flapping wing drained by aerodynamic damping is

$$P_d = \frac{4}{3\pi}B_{s2}(A\Omega)^3 + O(\epsilon^3),$$

(3.41)

the total power is

$$P_{total} = \frac{1}{2} \left( \frac{V_{in}}{R_a} - N_g\frac{K_a}{R_a}\Omega A \right) V_{in} + O(\epsilon),$$

(3.42)

and mean efficiency is

$$E_{eff} = \frac{P_d}{P_{total}} = \frac{B_{s2}^{8/3\pi}(A\Omega)^3 + O(\epsilon^3)}{\left( \frac{V_{in}}{R_a} - N_g\frac{K_a}{R_a}A\Omega \right) V_{in} + O(\epsilon)},$$

(3.43)
Theoretical Analysis: The term resonant frequency has been loosely used and referring to different quantities by various research communities, it is often referring to peak frequency, corresponding to the largest flapping angle from experiment frequency response. However, resonant frequency is also defined to be the natural frequency in the context of nonlinear vibration. To avoid this confusion, we use natural frequency and peak frequency instead. Only when the damping is sufficiently small (e.g. < 0.1), the three frequencies come close to each other. Flapping wing normally has sufficient large damping due to aerodynamic damping, but still mostly under-damped.

The widely adopted method to identify and validate the resonance of the system is by conducting a frequency sweep, during which the wing is driven by a sinusoidal voltage input with different frequencies from a range of interest[108, 48]. The steady state amplitudes of the responses in terms of flapping angle will be compile together and plotted with respect to input frequencies. The same can be done in terms of velocity of response and mean lift with respect to frequencies. In the current research among FWMAV community, the relationships were not clear among natural frequency, damped natural frequency, peak frequency, the maximum flapping amplitudes, maximum lift, and maximum efficiency. Here, the relationship will be first shown with theoretical analysis and then verified with experiment results.

From the modeling and analysis above, we have following observations:

(a) The peak value of flapping angle in frequency response does not correspond to natural frequency of the system and \( \omega_p < \omega_n \), unless \( \xi \) is really small, which is generally not true for high power and high damping flapping motion. The peak value of the flapping angular velocity, however, corresponds to natural frequency \( \omega_n \). The transfer function from input to velocity indicates that \( G_v(s) = \frac{V(s)}{U(s)} = \frac{K_u\omega}{J_3s^2+Bls+K_s} \) with \( |G_v(\omega_j)| = \frac{K_u\omega}{\sqrt{(K_s-J_3\omega^2)^2+B_3^2\omega^2}} \), which reach maximum when \( \omega = \omega_n \).

(b) Maximum lift occurs when driving the system at natural frequency, not the peak frequency. As \( F_L = \frac{1}{4}\rho_aC_LR_w^3\bar{c}\bar{r}^2(S)(A\Omega)^2 \), where \( A\Omega \) is the magnitude of
the flapping angular velocity, maximum lift happens at the peak value of the flapping angular velocity, which occurs at $\omega_n$ as discussed in (1).

(c) Maximum efficiency is achieved when the system is driven at natural frequency, not the peak frequency. As the efficiency is defined to be $E_{ff} = \frac{P_{out}}{P_{total}} \approx \frac{B_s^2 \pi (\Omega)^3}{(N_g V_a - N_g \frac{k_g A\Omega}{\eta}) V_{in}}$, the maximum efficiency happens for maximum velocity $\Omega$, thus at resonance $\omega_n$.

As a result, when driving the system at natural frequency $\omega_n$ (not peak frequency $\omega_p$), we expect that the maximum lift, maximum efficiency, and maximum velocity (not amplitude) should be achieved for the flapping wing system. The correct method to identify and validate the resonance of the system is by conducting a frequency sweep, after which, the frequency corresponding to the maximum value of the steady state magnitude of the flapping velocity is the natural frequency of the system. Driving the system at this natural frequency, the maximum lift and maximum efficient will be achieved.

3.4 Experiment Results

The simulations and experiments aim to justify following simplifications:

- **Assumption 1**: With reasonable accuracy, the quasi-steady aerodynamic model can capture the effects of the aerodynamic force on the wing exerted by unsteady aerodynamics.

- **Assumption 2**: With reasonable accuracy, the two degree-of-freedom dynamics of flapping wing with passive wing rotation can be simplified to the one degree-of-freedom dynamic model.

- **Assumption 3**: With reasonable accuracy, the response of the system driven by sinusoidal input can be well approximated as sinusoidal.

Moreover, the goal is to validate the modeling and analysis from previous sections.
3.4.1 Materials and Methods

Figure 3.4. 10 wings tested with the experiment setup.

The simulations and experiments were conducted, as a case study, on a Flapping Wing Micro Aerial Vehicle with two independent, motor-driven flapping wing subsystems, as shown in Fig. 4.1(7). For ease of instrumentation, the flapping wing subsystem was separately mounted onto the single wing testing setup. The block diagrams of the single wing testing setup are shown in Fig 2.8. FAULHABER Brushless DC-Servomotors 0620B has following parameters: nominal voltage $6V$, resistance $R_a = 8.8\,\text{ohm}$, torque constant $K_a = 1.09e-3\,\text{Nm/A}$, moment of inertia $J_m = 9.5e-10\,\text{kgm}^2$, and friction damping coefficient $B_m = 9.74e-9\,\text{Nms/rad}$. The gear ratio is 10, with moment of inertia $J_g = 5e-9\,\text{kgm}^2$ and efficiency $\eta_g = 0.8$. Parameter estimations and system identification were specified in [112]. Simulations and experiments were tested for total ten wings. The tested wings have different geometries as shown in Fig. 3.4. The wing consists of wing frame and wing membrane attached with polyurethane adhesive. The wing frame is extremely strong and light.
carbon fiber-reinforced polymer formed using unidirectional carbon fiber segments impregnated with an epoxy binder. The wing shape for the veins and spars of the wing frame was constrained by a silicone modeling process, common for constructions of fiber-reinforced polymers. The wing membrane is 3µm thin BoPET(Mylar) sheet trimmed to the wing shape. Carbon fiber rod was fitted to the wing leading edge with cyanoacrylate adhesive to provide stopper attachment and shaft for wing rotation. Parameters of the ten different wings are detailed in Table 3.1. In Table 3.1), parameters are corrected with wing offsets, and units of parameters are $m (mg)$, $R_w (mm)$, $d_w (mm)$, $\bar{c} (mm)$, $J_w (mg-mm^2)$, and $f_n (Hz)$, respectively. $\hat{r}_2$ and $\hat{r}_3$ are unitless.

All experiments were recorded by real-time dSPACE DS1103 PPC DAQ Board with sampling frequency $f_s = 5kHz$. The brushless DC motor three-phase commutation was implemented on a 72 MHz cortex M3 board (NXP Semiconductors, San Jose, CA, USA) at rate of 50kHz. The three phase drive electronics was custom-made.

Table 3.1. Wings Parameters.

<table>
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<tr>
<th>Wing</th>
<th>$m$</th>
<th>$R_w$</th>
<th>$d_w$</th>
<th>$\bar{c}$</th>
<th>$\hat{r}_2$</th>
<th>$\hat{r}_3$</th>
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<td>0.67</td>
<td>171147</td>
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</tr>
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</table>
for the motor. The wing stroke angle was measured with motor magnetic encoder at the bottom of the motor (FAULHABER Brushless DC-Servomotors 0620B) with 256 counts/rev and calculated according to the gear transmission with gear ratio of 10:1, which gives total $2\pi/2056$ rad resolution on angle measurement. The wing stroke angular velocity is calculated with simple first order Euler method. The encoder reading was recorded with the encoder interface on the dSPACE DAQ system along with the force measurements. All the measurement data are synchronized to be recorded real-time at 5000Hz. Force measurement was performed using a six component force/torque transducer (Nano17, ATI Ind. Automation). Due to limited resolution of Nano17 (0.3g resolution on the force and 1/64Nmm resolution on the torque measurement), a rigid 150mm beam setup was used to amplify the lift measurement as shown in Fig. 2.8. The improved resolution was about 0.0106g. The force sensor and beam setup was calibrated with precision weights of 0.1g, 0.5g, 5g and 20g and verified the resolution of at least 0.03g. When calculating the time-averaged force, sufficient numbers of wing-beat cycles at steady state were averaged to guarantee the reliability of the results. The raw data was filtered with cut-off frequency of 150Hz. The power source used during experiments is a DC power supply HY-5003 from MASTECH, with current and voltage measurements accurate up to 0.01A and 0.1V, respectively.

All numerical simulations were implemented with MATLAB ode45 solver, dynamic models of Equation (3.10) or Equation (3.38), and parameters specified above.

3.4.2 Spring Modeling and Testing

The characterization of elastic elements is essential for design and validation of flapping resonance. For torsion springs (from McMaster-Carr) used in this study, the theoretical model of the spring stiffness is given by

$$K_s = \frac{Ed^4}{64Dn}, \quad (3.44)$$
with wire diameters \( d \), outer diameter \( D \), Young's modulus \( E \), and number of winding \( n \).

The linearity of the torsional spring is validated experimentally by measuring the spring torque at different angular positions. The corresponding experimental setup is shown in Fig. 3.5(a). As an example, for one torsional spring with wire diameters \( d = 0.3 \text{mm} \), outer diameter \( D = 2.67 \text{mm} \), Young's modulus \( E = 193 \text{GPa} \), and number of winding \( n = 4.25 \), the experiment result is shown in Fig. 3.5(b). The spring torque is shown to be proportional to the angular position from \(-75^\circ\) to \(75^\circ\) with good linearity, and the measured spring stiffness \( K_s = 0.0019 \) matches well with the theoretical calculation \( K_s = 0.002 \).

Total 16 torsion springs with different geometries and spring stiffness from McMaster-Carr were tested. The measured and predicted spring stiffness were plotted together in Fig. 3.5(c) to show consistent matches of spring model of Equation (3.44) and experiment testing.

3.4.3 Model Validation

First, we demonstrate that the nonlinear model of Equation (3.10) and linear model of Equation (3.38) capture the real system with reasonable accuracy. The
Figure 3.6. (a) Nonlinear model simulation, linear model simulation, and experiment data. (b) Line 1: nonlinear damping torque, Line 2: linear damping torque, Line 3: nonlinear conservative torque, Line 4: linear conservative torque. (c) Linear model with Equation (3.38) only approximates nonlinear model well at natural frequency. (d) Predicted and simulated amplitudes at natural frequency and peak frequency. Single-sided amplitude spectrum of input voltage \( u(t) \) and stroke angle \( \phi(t) \) scaled to unity for better comparison: (e) is from experiment data at natural frequency; (f) is from simulation data at other frequency.

modeling errors between the real system and the nonlinear model of Equation (3.10) are mainly due to the one DoF simplification and quasi-steady aerodynamic model. The linear approximation introduces additional errors as specified by Equation (3.22). The numerical simulation of the nonlinear model of Equation (3.10) and linear approximation of Equation (3.38), and the experiment response are compared in Fig. 3.6(a). When simulating the linear model of Equation (3.24), natural frequency and amplitude solution of Equation (3.37) were used for parameter \( B_1 \), as the amplitude
was unknown in advance. Fig. 3.6(a) demonstrates the accuracy of the nonlinear and linear modeling, despite the simplifications. Fig. 3.6(b) illustrates the working mechanism of the linear approximation of damping torques. The linear model approximates the nonlinear damping torque with a linear damping torque. The approximation error is compensated by the difference in the conservative torques from the not-perfect cancellation of the inertial torque and the spring torque. Fig. 3.6(c) shows the limitations of the linear approximation model with fixed damping coefficient $B_l$. As the operating frequency deviated from the natural frequency of the system, the modeling errors grew larger. Understanding the assumptions, errors sources, and limitations are important for proper use of the proposed models.

Second, we show that system responses at natural frequency and peak frequency can be predicted with precision by the closed-form solutions in Equation (3.37) and analytic formulas in Equation (3.40), respectively. For sinusoidal inputs at corresponding system natural frequencies, the steady-state amplitudes of the responses from numerical simulations of the nonlinear model in Equation (3.10) were extracted for all ten wings. The amplitude predictions from Equation (3.37) were compared with simulated ones in the upper plots of Fig. 3.6(d). Similarly, for peak frequencies, results from simulations and solutions of Equation (3.40) were compared in the lower plots of Fig. 3.6(d). Fig. 3.6(d) illustrated consistent matches between simulations and amplitude predictions for all ten wings. Similar results can be obtained for the formula of damped natural frequency in Equation (3.39), which are not detailed here.

Last, to verify the assumption of the sinusoidal response, the Fast Fourier Transforms of the input voltages and output flapping motions is shown in Fig. 3.6(e) for experiment and Fig. 3.6(f) for simulation. The input and output were scaled to unity and plotted together for better comparison. For experiment, a sinusoidal input with amplitude $6\text{V}$ and at natural frequency was applied to the DC motor and the flapping wing motion was recorded with motor encoder. For simulation, a sinusoidal input at frequency different from natural frequency, around 0.6 times of natural frequency, was run with nonlinear model in Equation (3.10). It is shown that the output responses
of the flapping motions to the sinusoidal input voltages were mostly sinusoidal, for both experiment and simulation, at and off natural frequency.

3.4.4 Frequency Sweep Test

Figure 3.7. Plots of frequency response for the frequency sweep experiments (blue) and simulations (red) (#4, #6, and #8 shown here as examples).

To verify the proposed method, frequency responses for total ten wings were obtained. For frequency sweep test for each wing, the wing was driven at a sinusoidal input voltage $u = V_{in} \cos(\Omega t + \beta)$ with sufficient large amplitude $V_{in}$, while the frequency $\Omega$ varied from 20Hz to 45Hz. Each test run over 40 sec with frequency step size of 1Hz. Between frequency steps, the input voltage frequency was maintained for 5 sec, which is much longer than the wingbeat period, so that the wing response had sufficient time to reach steady state. The forces were measured with the beam setup shown in Fig. 2.8 and the wing flapping angle was recorded with the motor encoder.

For data processing, during each step, the later 2 seconds of force measurements were averaged to get the average lift. Amplitudes of angle and the angular velocity of the stroke motion were extracted from the maximum values of the steady-state responses. The results for wing #1 to #10 are plotted to show the frequency response
of the stroke amplitude, stroke angular velocity amplitude, and the mean lift force as functions of frequencies, as shown in Fig. 3.7, respectively.

As a comparison, numerical simulations with the nonlinear flapping wing dynamics of Equation (3.24) were run. Parameters for each simulation was based on the corresponding experiment. The frequency responses from experiments and simulations were plotted together with experiment results in red and simulation in blue, as shown in Fig. 3.7, respectively. Among all the wings, it is clear that the model of Equation (3.24) predicted the steady state response and mean lift reasonably well over all tested wings and frequencies, as blue and green data points were relatively close. The discrepancies between the model and experiments are within reasonable levels, despite the quasi-steady model is inherently an approximation to the unsteady aerodynamics with possible 30% error comparing to experiments as examined in [25].

3.4.5 Examination of Resonance

From frequency responses in Fig. 3.7, following key quantities were extracted: the peak frequency \( \omega_p \) where stroke amplitude is maximum in frequency responses, denoted as peak freq. in the figure; the frequency that corresponds to maximum angular velocity in frequency responses, denoted as max.-velocity freq.; the frequency that gives the maximum lift in frequency responses, denoted as max. lift freq.; the lift at peak freq.; the lift at max. velocity; the lift peak, denoted as max. lift. Along with the calculated natural frequency \( \omega_n \), above quantities for all 10 wings were compiled to Fig. 3.8(c)(d). Corresponding quantities extracted from simulations of the nonlinear model were compiled to Fig. 3.8(a)(b). We have following observations.

First, Fig. 3.8(a)(b)(c)(d) confirmed that maximum lift and maximum angular velocity occur when driving the system at natural frequency, not the peak frequency. In Fig. 3.8(e)(f), for better comparisons between simulations and experiments, natural frequencies, maximum-velocity frequencies, maximum-lift frequencies, lifts at
maximum-velocity frequencies, and maximum lifts were plotted together to show consistent matches.

Second, we validated the formula in Equation (3.40) for predicting peak frequency \( \omega_p \). In addition, we illustrated that the formula \( \omega'_p = \sqrt{1 - 2\xi^2 \omega_n} \) from traditional linear system theory is not valid for this system, compared to the proposed one in Equation (3.40). We denoted the peak frequencies from formula \( \omega'_p = \sqrt{1 - 2\xi^2 \omega_n} \) as linear peak freq. and ones from Equation (3.40) as corrected peak freq.. The measured and predicted ones were compared side by side in Fig. 3.8(g) for experiments and Fig. 3.8(h) for simulations. Predictions from formula in Equation (3.40) matched with better precision than formula \( \omega'_p = \sqrt{1 - 2\xi^2 \omega_n} \) for both experiments and simulations for all wings. This is due to the fact that linear model in Equation (3.38) is only accurate around natural frequency \( \omega_n \), as the damping ratio \( \xi \) depends on linear damping term, which in turn is a function of \( A\Omega \). As \( \omega_p \) is smaller than \( \omega_n \), the linear model is not accurate anymore, discrepancies between the measured ones from experiment and predicted ones based on the wrong formula were clearly shown in Fig 3.8(g) and (h).

Last, to show that maximum efficiency is achieved when the system is driven at natural frequency, not the peak frequency, experiments showed the total power of the flapping wing system as a function of deviation from natural frequency, in 3.9 for wing #6.

3.4.6 Discussion

Fig. 3.8 and Fig. 3.7 showed that the wing #5 had largest discrepancies. We speculate that the discrepancies were due to the relatively long chord length of wing #5, which induced large chordwise flexing of the wing.

From Fig. 3.8(a)(b)(c)(d), the general trend of the separations between natural frequency and peak frequency grew as the wing size increased and frequency
decreased. This means that the above discussion will be more important for low frequency flapping wings.

We here clarify the design goal for flapping resonance. Previous works such as [63, 8] were driving the wing at damped natural frequency in the hope of getting optimal efficiency. We showed from theory and experiments that the best choice should be natural frequency for better efficiency. It’s well known that sufficient large flapping amplitude ensures good passive rotation and thus good lift generation. Several previous works [92, 108, 83, 112] adopted maximizing the stroke amplitude as their design goal, even though the real design target should be to maximize the lift generation and optimize efficiency. Instead of driving the system at natural frequency, previous works commonly resorted to damped natural frequency or peak frequency to generate larger amplitudes. In this work, we showed that this is a sub-optimal design decision. From the closed-form solution in Equation (3.37), we argue that this seeming trade-off can be resolved simply by increasing the input amplitude $V_{in}$ or choosing actuator parameters to optimize $K_n$ or $B_s$1. From the analysis and experiments, we argue that the design goal for flapping resonance should be to maximize lift and efficiency by driving the wing at natural frequency, where the flapping velocity is also maximized. Increasing input voltage can ensure that the flapping amplitude is sufficiently large for good passive rotation.

We here re-exam some of the possible misinterpretations of previous experiment results. Previous works such as [92, 107, 3] confused natural frequency with peak frequency, using peak frequency from experiment frequency response to validate the value calculated from the equation of natural frequency. Here we propose the correct experimental approach to validate natural frequency is, from frequency response, to find the frequency that maximizes flapping velocity, instead of flapping angle. In [48], the peak frequency was used as the indication of maximum lift and best efficiency, but later in the experiment frequency response, it was observed that the peak frequency actually appeared lower than the one under which maximum lift is produced (off-resonance maximum lift). In this study, we show that maximum lift is produced at
natural frequency, which is typically higher than the peak frequency, thus explaining the observed discrepancy. Similar cases can also be seen in [92, 107].

Last but not least, we propose the correct experimental approach to identify and validate the resonance of the system (natural frequency) is by conducting a frequency sweep, from the resulting frequency response, the frequency corresponding to the maximum value of the steady state flapping velocity is the natural frequency of the system. Driving the system at this natural frequency, the maximum lift and maximum efficient will be achieved.

3.5 Chapter Summary

Mechanical resonance has been recognized as a key principle both by biologists and engineers for reducing the cost of driving high-frequency flapping wing motions. Elastic elements are typically incorporated for restoring the high inertial energy of oscillating wing. In this chapter, we analyzed dynamics of flapping wing as a nonlinear forced 2nd order resonant system, with both nonlinear perturbation method and linear approximation approach. We derived analytic formulas for steady-state flapping amplitude, energetics, and characteristic frequencies, including natural frequency, damped natural frequency, and peak frequency. The analysis revealed that both lift and efficiency are maximized by driving the wing at natural frequency. Interestingly, the flapping velocity is maximized at natural frequency as well, which can serve as an convenient experiment approach to identify natural frequency and validate the resonance design. The modeling and analysis were validated with both simulations and experiments on ten different wings and a direct-motor-drive Flapping Wing Micro Air Vehicle. Finally, we explained some lingering questions on experiment results of previous works using proposed method. Current study can clarify the confusions about resonance for designing and prototyping of Flapping Wing Micro Air Vehicle, as well as help biologists study resonance of insects’ flapping wing system.
Figure 3.8. Comparisons of peak frequencies, natural frequencies, maximum-velocity frequencies, and maximum-lift frequencies from simulations (a) and experiments (c) for 10 wings. Comparisons of lifts at peak frequencies, lift at maximum-velocity frequencies, and maximum lift from simulations (b) and experiments (d) for 10 wings. For better comparisons between simulations and experiments, natural frequencies, maximum-velocity frequencies, and maximum-lift frequencies are plotted together in (e) to show their match, where Line 11 is natural frequencies from experiments, Line 12 is maximum-velocity frequencies from experiments, Line 13 is maximum-lift frequencies from experiments, Line 14 is natural frequencies from simulations, Line 15 is maximum-velocity frequencies from simulations, and Line 16 is maximum-lift frequencies from simulations; lifts at maximum-velocity frequencies and maximum lifts from simulations and experiments are plotted together in (f) to show their match, where Line 21 is lifts at maximum-velocity frequencies from experiments, Line 22 is maximum lift from experiments, Line 23 is lifts at maximum-velocity frequencies from simulations, and Line 24 is maximum lift from simulations. (g) A comparison of measured, estimated and corrected peak frequencies from experiments for 10 wings. (h) A comparison of measured, estimated and corrected peak frequencies from simulations for 10 wings.
Figure 3.9. Experiment shows the total power of the flapping wing system as a function of deviation from natural frequency.
4. FLAPPING WING KINEMATICS AND FORCE CONTROL

4.1 Introduction

The superior maneuverability of insect flight is enabled by rapid and significant changes in aerodynamic forces, a result of subtle and precise changes of wing kinematics. The high sensitivity of aerodynamic force to wing kinematic change demands precise and instantaneous feedback control of the wing motion trajectory, especially in the presence of various parameter uncertainties and environmental disturbances.

In terms of actuators of FWMAVs, the effort to date can be divided into two main categories: motor driven [48, 4, 50, 19, 57] and piezoelectric cantilever [108, 2] mechanisms. The latter has been proven to be effective as a flapping actuator at sub-gram scale [108] because of its high power density at high frequencies (using high voltage) and low transmission losses, leading to the lift off [108] and controlled flight [62] of Harvard’s insect-size robot.

On the other hand, motor driven actuators are successful at larger scales, operating at high efficiency and generating large output angles with low drive voltage [50]. Linkage mechanisms are commonly used to transform rotational motion from the motor to reciprocal motion of the wings, which ensures the motor to operate at its efficient speed. However, they are also subject to limitations such as fixed output kinematics without additional mechanisms, asymmetry in the kinematics without additional variable speed control, parasite structural vibration due to asymmetric acceleration and the linkage system operating at high frequency, and no elastic component in the system to preserve wing kinetic energy for efficiency [4, 50]. In the ideal scenario, with elastic components and system resonance, the kinetic and potential energy of the mechanical components in the system can be conserved, and power is spent only on the non-conservative energy cost such as friction, damping of the system and the
aerodynamic damping acting on the wing. Several modifications to the linkage system have been proposed and tested in previous studies [50, 4] that result in efficiency improvements. For example, in [4], an elastic component was introduced to achieve resonance of a motor driven slider-crank mechanism. In the Nano hummingbird [50], the linkage was replaced by strings with negligible mass therefore reduced the inertial loss on transmission and the parasite structural vibrations.

Recently, motor driven wings with direct-drive transmission are gaining popularity with works such as [112, 3, 48], where each flapping wing is driven directly using a geared DC motor without linkages, and the motor undergoes reciprocating rotation. As for the direct drive actuation, the inherent limitation of fixed and asymmetric wing trajectories on linkage transmission is avoided, and various kinematic control approaches can be applied. For example, driven by DC motors coupled with torsion springs with a transmission gear [112], the motor can operate at an efficient speed while generate reciprocal wing motion with varying wingbeat frequency tunable by changing the spring stiffness.

Current work on flapping wing robots was limited to open-loop averaged wing kinematics control.

In this chapter, we present instantaneous closed-loop wing trajectory tracking of a DC motor direct driven wing-thorax system under resonant flapping. A dynamic model with parameter uncertainties and disturbances was developed and validated through system identification. For wing trajectory generation, we designed a Hopf oscillator based central pattern generator with smooth convergence. Using the linearized model while treating the nonlinearity as disturbance, we designed a proportional-integral-derivative (PID) controller and a linear quadratic regulator (LQR) for instantaneous wing trajectory tracking at 24Hz; Using the original nonlinear model, we designed a nonlinear controller to achieve robust performance at over 30Hz. The control algorithms were implemented and compared experimentally on a 7.5 gram Flapping Wing Micro Air Vehicle (MAV). The experiments showed that the PID and nonlinear controls resulted in precise trajectory tracking; while LQR
controller tracked with less precision but with smaller input effort. In addition, the nonlinear control algorithm achieved better tracking of wing trajectories with varying amplitude, bias, frequency, and split-cycles while adapting to the variations on wing morphological parameters such as wing geometry and stiffness. Furthermore, the lift force measurements of the nonlinear control results were compared with those of open-loop average wing kinematics control commonly adopted in current designs.

The rest of the chapter is organized as follows. Section 4.2 presents the model of the system and a numerical analysis for selecting main system components such as motor sizing, gear ratio, and spring stiffness. In Section 4.3, the design and prototype were verified by system identification experiments. In Section 4.4, we formulate the sensitivity functions of aerodynamic forces to wing kinematic changes. Section 4.5 presents the control algorithms and Section 4.6 presents the control experiment results. Section 4.7 discusses conclusion.

4.2 System Modeling, Design, and Fabrication

Figure 4.1. (a) Assembled FWMAV. (b) Solidworks model of the FWMAV and the wing stopper.
4.2.1 Wing Thorax System

The flapping wing MAV with the proposed mechanism, capable of lifting off and closed-loop wing kinematics control, is shown in Fig. 4.1. The flapping wings are directly driven by two 2.5 gram, 6mm brushless DC motors (FAULHABER, Clearwater, Florida USA) coupled with torsional springs for kinetic energy restoration. Using a gear transmission, the motor was designed to generate an overall reciprocal motion of the wing. The frame structure, wing stopper, and spring holders were prototyped by 3D printing. A portion of the gear on the load shaft was removed to reduce the weight and moment of inertia. Two miniature ball bearings were used to support the load shaft. A pair of torsional springs was mounted on the bottom of the shaft and oriented in such a way that rotation to one direction compresses one spring and extends the other. The wing was allowed to passively rotate, up to a 45 degrees angle limited by a stopper fixed at the proximal end of the wing leading-edge spar.

4.2.2 Wing Thorax System Dynamic Model

The block diagram of flapping wing actuation system is illustrated in Fig. 4.3. The wing has two degrees of freedom motion: wing stroke ($\phi$) driven by dc motor under resonance with torsion spring ($K_s$) and passive wing rotation ($\psi$) limited by stopper to optimal 45degree angle of attack. Typical two DoF wing flapping motions are illustrated in Fig 4.2(c). Characteristics such as flapping resonance and passive wing rotation have each been observed in nature and serve to reduce the actuation complexity and the power requirements [42, 6], and has been applied to a number of platforms [108, 51, 78]. In [32], it is shown that the majority of kinetic energy due to wing movement is stored in the flapping mode, about 50 times larger than rotational mode. Thus, similar to [104], we make the simplifying assumption that the behavior of the wing is modeled by a beam damped by quasi-steady aerodynamics, rotating about the stroke axis and resonating with torsion spring.
Figure 4.2. (a) Parameters of wing shape: \( R_w \) is the wing length, \( O \) wing root, i.e. the intersection of leading edge (LE) and trailing edge (TE), \( c(r) \) is the chord length at distance \( r \) from \( O \), \( O' \) is the intersection of stroke axis \( z \) and LE, \( d_w \) is the wing offset of \( O \) from \( O' \), stroke angle is \( \phi \), and rotation angle is \( \psi \). (b) The body \( xyz \) coordinate system is fixed to the vehicle airframe. The wing flaps about the \( \phi \) axis, which remains parallel with the body \( z \) axis. The wing rotation \( \psi \) axis is parallel with respect to the wing leading edge. Wing can also deviates from stroke plane with angle \( \theta \), but deviation is not considered here. (c) Diagram of typical flapping wing motion. Dash lines indicate the instantaneous position of the wing chord at temporally equidistant points during each half-stroke. Small circles mark the leading edge. Wing moves left to right during downstroke, right to left during upstroke.

Assuming the inductance of the motor is negligible, the equation of motion for the system (including motor, transmission and wing) is given by [112]

\[
J_s \ddot{\phi} + B_{s1} \dot{\phi} + B_{s2} |\dot{\phi}| \dot{\phi} + K_s \phi + T_f \text{sign}(\dot{\phi}) + \Delta = K_u u \tag{4.1}
\]

where \( \phi \) is the flapping/stroke angle in rad, \( J_s \) is the total moment of inertia, \( B_{s1} \) and \( B_{s2} \) represent the lumped linear and aerodynamic damping coefficients respectively, \( K_s \) is the torsional spring coefficient, \( T_f \text{sign}(\dot{y}) \) is the nonlinear friction, \( K_u \) is the lumped control input gain, and \( \Delta \) is a lumped uncertain nonlinearities presenting unstructured nature of disturbances and modeling errors. The modeling of aerodynamics, especially due to the unsteady nature of the flapping wings, is subject to relatively large modeling errors when quasi-steady model is used[25].
In Equation (4.1), $J_s = N_g^2 J_m + J_w + J_g$, in which $N_g$ is the gear ratio and $J_m$, $J_w$ and $J_g$ are the moments of inertia of the motor’s rotational components, the wing, and the gears respectively. $J_w = \rho_w R_w^3 \bar{c} \bar{r}_2^2 (S)$ with $\rho_w$, $R_w$, $\bar{c}$ and $\bar{r}_2^2 (S)$ being the wing density, the wing length, the wing mean chord length and the 2nd dimensionless moments of wing area respectively [30]. $B_{s1} = N_g^2 (B_{m1} + \frac{K_a^2}{R_a})$, in which $B_{m1}$ is the linear damping of the motor’s rotational components, $K_a$ is the torque constant, and $R_a$ is the winding resistance. $B_{s2} = 0.5 \rho_{air} \bar{C}_D R_w^4 \bar{c} \bar{r}_3^2 (S)$, where $\rho_{air}$ is air density, $\bar{C}_D$ is the mean drag coefficient and $\bar{r}_3^2 (S)$ is the 3rd dimensionless moments of wing area [30]. The aerodynamic drag $B_{s2} |\dot{\phi}|$ is estimated based on a quasi-steady aerodynamic model using blade element theory (BET) [25], and $\bar{C}_D$ is the mean drag coefficient averaged over one wing stroke estimated by [25] $\bar{C}_D = 1.92 - 1.55 \cos (2.04 \alpha - 9.82)$, where the angle of attack $\alpha$ is assumed to be fixed at 45 degrees. The input gain is $K_u = N_g \frac{K_a}{R_a}$. Note that the quasi-steady model is used here due to the lack of simple closed form for unsteady aerodynamic models [25]. The actuation system diagram for one wing is shown in Fig. 4.3.

![Figure 4.3](image-url)  

Figure 4.3. The diagram of flapping wing actuation system consists of (1) motor with voltage input $u$, resistance $R_a$, inductance $L_a$, back EMF $e$, motor moment of inertia $J_m$, shaft damping $B_m$, and motor angle $\phi_m$, (2) gear with gear ratio $N_g$, gear moment of inertia $J_g$, (3) torsion spring with spring constant $K_s$, and (4) wing with stroke angle $\phi$, rotation angle $\psi$, and parameters defined in Fig. 4.2(1).
4.2.3 Selection of System Components

For a given wing, to numerically determine the optimal motor, gear transmission, and spring, the following procedures were used. To illustrate the procedure, the wing 1 with parameters shown in Table 4.3 is used as an example.

First, a simulation is setup to calculate the required motor power for different frequencies and peak-to-peak flapping amplitudes. As it will be shown later, when resonance is achieved, the torque terms $J_s \ddot{\phi} + B_s \dot{\phi} + K_s \phi$ reduces to only $B_s \dot{\phi}$, with $J_s \ddot{\phi}$ and $K_s \phi$ canceling out. The output torque becomes $B_s |\dot{\phi}| \dot{\phi}^2$ and the output power is thus $B_s |\dot{\phi}| \dot{\phi}^2$, which can be obtained numerically based on wing parameters, amplitude and frequency. The resulting average output power contour as a function of amplitude and frequency are shown in Fig. 4.4A. Based on weight and power considerations, a 2.5gram 6mm brushless DC motor (FAULHABER, Clearwater, Florida USA) with nominal output power of 1.47W is chosen, which in theory covers large region of the possible frequencies and amplitudes with margins for disturbance, control and future optimization.

Second, proper gear transmission ensures that the motor provides sufficient driving torque, thereby increase the efficiency. The simulation of Equation (4.1) also gives the maximum output torque (including transmission gear) as a function of frequency and amplitude shown in Fig. 4.4B. According to the torque limit of 0.73mNm from the motor specification, the gear ratio of 10 is chosen, which gives the maximum output torque at 7.3mNm. Similar to those shown in Fig. 4.4A, a large region of possible kinematics is covered by this value (Fig. 4.4B).

Parameters ($J_s^n$ and $B_s^{n1}$) were then calculated and shown in Table 4.1. The optimal operating frequency is determined based on the moment of inertial $J$ and spring stiffness $K_s$. We choose two springs with stiffness 0.004Nm/rad and 0.006Nm/rad, and obtain operating frequency of 24Hz and 30Hz, respectively.

Finally, with the selections of wing, motor and gears, a 3D CAD model was designed and shown in Fig 4.1.
Figure 4.4. Simulation results as function of frequencies and amplitudes for (A) Average output power (B) Maximum output torque (C) predicted mean lift (quasi-steady model[2]) (D) Optimal Ks value for minimum input.

4.3 System Identification

4.3.1 Frequency Response

Two sets of frequency response tests were performed to characterize the open-loop system responses for the system with stiffness of 0.004 Nm/rad. In the first test, the wing and stopper were removed from the mechanism, and then 8V peak-to-peak sinusoidal inputs with different frequencies were individually applied to the system.
In Fig. 4.5, the amplitude of the steady-state responses is plotted as a function of frequency, which shows peak amplitude corresponding to a frequency of 31.5 Hz.

In the second test, with the wing and stopper assembled, the same procedure in the first tests is performed but with 8V peak-to-peak sinusoidal input. Results are presented in Fig. 4.5 which shows peak amplitude corresponding to a frequency of 24 Hz.

### 4.3.2 Least Square Parameters Estimation

In this section, the parameters in Equation (4.1) were obtained using least square estimation for the system with stiffness of 0.004 Nm/rad. Based on the persistent excitation condition [60], a square wave with sufficient time length can be used to identify multiple parameters, while one sinusoidal input can only identify 2 parameters. Thus the parameter estimation will take two steps: First, we use square wave to identify 4 parameters of the system without wing. Then, with wing installed, the last unknown parameter $B_{s2}$ will be identified by controlling the system to track a sinusoidal trajectory. Therefore, we first consider a system without the wing, i.e. $B_{s2} = 0$. 

![Figure 4.5. Frequency Responses.](image)
Table 4.1. Open Loop Parameter Estimation (SI units).

<table>
<thead>
<tr>
<th>Wing</th>
<th>$J$</th>
<th>$B_{s1}$</th>
<th>$B_{s2}$</th>
<th>$K_s$</th>
<th>$d$</th>
<th>$T_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.55e-7</td>
<td>9.60e-6</td>
<td>3.47e-8</td>
<td>6.00e-3</td>
<td>1.22e-4</td>
<td>2.31e-5</td>
</tr>
<tr>
<td>2</td>
<td>1.60e-7</td>
<td>9.56e-6</td>
<td>7.48e-8</td>
<td>6.00e-3</td>
<td>1.20e-4</td>
<td>2.30e-5</td>
</tr>
<tr>
<td>$\hat{\theta}_0$</td>
<td>1.50e-7</td>
<td>1.00e-5</td>
<td>3.70e-8</td>
<td>5.70e-3</td>
<td>1.00e-4</td>
<td>2.00e-5</td>
</tr>
</tbody>
</table>

The system dynamics without wing can be normalized as the following linear regression form:

$$J_s\ddot{\phi} + B_{s1}\dot{\phi} + K_s\phi + d = u = \psi^T \beta,$$

written as a linear regression form, where $\psi^T = [\ddot{\phi} \dot{\phi} \phi 1]$ and $\beta = [J_s B_{s1} K_s d]^T$. $\psi^T$ is the basis function, which is obtained from experiments; and $\beta$ represents the parameter vector to be identified. The input $u$ to the system is a 2Hz square wave with 4V amplitude. This signal is filtered by a low pass filter (cutoff frequency at 500Hz) to reduce higher-order excitation, and therefore avoiding the un-modeled high order dynamics in the system to be excited. The estimation results are presented in Table 4.1, and are compared with theoretical values used in the numerical study. Based on these parameter estimates the peak frequency can be calculated by $\omega_p = \sqrt{1 - 2\eta^2 \omega_n}$ as 32Hz, which matches the one obtained from frequency response experiment for the system without the wing (Fig. 4.5). Again, the comparison justifies the results from the numerical simulation, and therefore the design decisions guided by the numerical simulation, in which the absolute value of the parameter $K_u$ is taken from the datasheet. In addition, to estimate the damping term due to wing drag, we assume all the parameters except $B_{s2}$ in Equation (4.1) are known (from the last step). With the wing attached, $B_{s2}$ is estimated using close loop least square estimation, basing on the input and output data of a PID closed loop control that tracks a 24Hz sinusoidal trajectory with amplitude of 120°. Note that the total moment of inertia $J_s$ equals...
to the sum of $J_s'$ and the wing moment of inertial. Similar least square estimation is conducted for only $B_{s2}$, which give the value $B_{s2} = 1.02E - 8$.

### 4.4 Control Problem Formulation

The system model and the robust control problem formulation are introduced in this section, together with a mathematical formulation of sensitivity analysis of force generation with respect to wing kinematic changes.

#### 4.4.1 Uncertainty Characterization

Uncertainties of the systems were first grouped into two types: parametric uncertainties and uncertain nonlinearities, according to their effects on the dynamics of Equation (4.1).

**Parametric uncertainties**: Parametric uncertainties capture the unknown changes of the system parameters, such as $J_s$, $B_{s1}$, $B_{s2}$, $K_s$ and $T_f$. Wing geometric dimensions, wing mass, motor and other changes of component parameters all belong to this category.

**Uncertain nonlinearities**: Disturbances are often unknown, unstructured and thus hard to be described with a fixed model structure. The modeling of aerodynamics, especially due to the unsteady nature of flapping wings, is subject to relatively large modeling errors when quasi-steady model is used[25]. Due to the unstructured nature of disturbances and modeling errors, their lumped effect, as denoted by $\Delta$ in Equation (4.1), belongs to this type of uncertainties.

#### 4.4.2 Open-Loop Wing Kinematics and Force Control

Fig. 4.6 shows the overall flight control system structure for flapping-wing MAVs when the open-loop wing kinematics and force control is adopted as a subsystem. The body controller will generate the control command in terms of driving voltage,
which is a prescribed sinusoidal wave with adjustable parameters, such as amplitude, phase and bias. The resulting dynamic equation is

\[
J_s \ddot{\phi} + B_s \dot{\phi} + B_s |\dot{\phi}| \dot{\phi} + K_s \phi + T_f \text{sign}(\dot{\phi}) + \Delta = K_u (U_a \sin(2\pi ft + \psi) + U_b),
\]

where \(U_b\) is the bias voltage and \(U_a\) is the voltage amplitude. The response of dynamic Equation (4.3) will be the wing kinematics \(\phi(t)\), which in turn determines the forces and torques according to flapping wing aerodynamics.

It is clear that all the disturbances and uncertainties have direct impacts on the resulting kinematics, thus the generated aerodynamic forces will be subjected to possible large errors from the commanded ones by the body controller, especially for operation in unstructured environments. For example, a simple unknown constant input disturbance will be equivalent to a change in bias voltage \(U_b\) that results in a constant unknown wing bias angle in output trajectory. This will directly affect the pitch torque generation, and cause unstable body motion due to high sensitivity.
of aerodynamic forces to wing kinematic changes. The sensitivities of the resulting forces with respect to changes in wing kinematics are described in subsection 4.4.3. The second issue with the open-loop method is that it sets a bandwidth limitation for the body controller due to the time it takes for the wing dynamic response to reach steady state. This issue will become more prominent for high bandwidth control of dynamic maneuvering. For example, for a 30Hz flapping wing, simulations show that it usually takes 3 cycles to reach steady-state, which means the update rate for the body controller should be less than 10Hz for a predictable behavior. This poses a severe controller design limitation for light-weight and fast maneuvering FWMAV.

The forces generated by the wing motion is determined by the relative instantaneous velocity of the aerodynamic surface to the incoming air flow [46, 10], which is subjected to the effects of the body velocity as shown in Fig. 4.6. In order to have a mathematically tractable problem, the theory used in [46, 10] to predict additional aerodynamic forces and torques caused by body velocity is based on the assumption that the interaction between the wing and body is only unidirectional, i.e., the wing motion is prescribed and unaffected by the body velocity. The possibility of having actual strong bidirectional interactions between the body velocity and the wing kinematics of the open-loop control, especially during high speed maneuvering, will make it questionable to apply the theory in [46, 10] to accurately model the aerodynamic forces. This will highly complicate the dynamic modeling and the control of the flapping wing MAVs.

4.4.3 Sensitivity of Force and Torque Generation to Wing Kinematics

Previous studies on insect flight show that the subtle changes of kinematics can lead to large variations of the resulting aerodynamic forces and torques[5]. For controller design and control performance evaluation, it is necessary to precisely quantify such high sensitivity of the force and torque generation to wing kinematics, which is carried out in this subsection as follows.
The stroke-averaged forces and torques under consideration are $F_z$, $F_x$, roll torque $T_x$, pitch torque $T_y$ and yaw torque $T_z$ defined similar to [10] as shown in Fig. 4.7. With a fixed angle of attack $\alpha$, the flapping kinematics of each wing are uniquely defined through its stroke angle, which is assumed to be generated by

$$
\phi_i = \begin{cases} 
A_i \cos \left( \frac{2\pi ft}{2\sigma_i} + \psi_i \right) + \phi_{0i}, & \text{if } 0 \leq t < \frac{\sigma_i}{f} \\
A_i \cos \left( \frac{2\pi ft - 2\pi}{2(1-\sigma_i)} + \psi_i \right) + \phi_{0i}, & \text{if } \frac{\sigma_i}{f} \leq t < \frac{1}{f}, 
\end{cases} 
$$

(4.4)

where $i$ represents the right ($i = r$) and left wing ($i = l$), $A_i$ is the flapping amplitude, $\psi_i$ is the phase angle, $\phi_{0i}$ is the bias angle, and $\sigma_i$ is the split cycle parameter.

The stable hovering condition of $A_i = A_0$, $\psi_i = 0$, $\phi_{0i} = 0$ and $\sigma_i = 0.5$ is assumed as the nominal kinematics. Under this condition, $F_x = F_y = 0$, $T_x = T_y = T_z = 0$, and the lift generated by the wing pair is balanced by the body weight $mg$ of the MAV/insect, i.e., $mg = \frac{1}{2}\rho_{\text{air}} CL R_3^3 \bar{c}\bar{r}_2^2(S)\omega_w^2 A_0^2$, where $CL$ is the mean lift coefficient averaged over one wing stroke [25], and $\omega_w = 2\pi f$ is the wing angular velocity.

Sensitivities are defined when kinematic parameters are deviated from their nominal values. Based on the assumption of near-hovering condition and the method in [26, 10], for small deviations from the nominal kinematics parameters in amplitude $\delta A$, bias $\delta \phi_0$, and split cycle $\delta \sigma$, it can be shown that

1) Lift force $F_z$ due to symmetric amplitude changes of the left and right wing, i.e., $A_l = A_0 + \delta A$ and $A_r = A_0 + \delta A$, as shown in Fig. 2.5(a), is

$$
\delta F_z = \frac{1}{2}\rho_{\text{air}} CL R_3^3 \bar{c}\bar{r}_2^2(S)\omega_w^2 A_0^2 \left( \frac{2\delta A}{A_0} \right). 
$$

(4.5)

2) The roll torque $T_x$ due to asymmetric amplitude changes of the left and right wing, i.e., $A_l = A_0 + \delta A$ and $A_r = A_0 - \delta A$, as shown in Fig. 2.5(b), is

$$
\delta T_x = \frac{1}{2}\rho_{\text{air}} CL R_3^3 \bar{c}\bar{r}_2^2(S)\omega_w^2 A_0^2 r_{\text{cp}} \left( \frac{2\delta A}{A_0} \right), 
$$

(4.6)

where $r_{\text{cp}} = \frac{\bar{c}_3(S)}{\bar{r}_2^2(S)} R_w$ is the center of pressure on the wing.

3) The pitch torque $T_y$ due to symmetric bias changes of the left and right wing, i.e., $\phi_{0l} = \delta \phi_0$ and $\phi_{0r} = \delta \phi_0$, as shown in Fig. 2.5(c), is

$$
\delta T_y = -r_{\text{cp}} F_z \sin(\delta \phi_0). 
$$

(4.7)
Figure 4.7. Schematic view of coordinate systems and kinematics. (a) Top View shows the stroke plane coordinate frame \((x_s, y_s, z_s)\) that originated from wing base. Wing kinematics are specified by the stroke angle \(\phi\). The positive direction of \(\phi\) is defined to be upstroke direction for both left and right wing. Wing coordinate frames (brown) \((x_{wl}, y_{wl}, z_s)\) and \((x_{wr}, y_{wr}, z_s)\) share the same \(z\) direction with stroke plane frame and are attached to the blade element (BE) on the wing at distant \(r\) from the wing base. (b) Body coordinate frame (blue) \((x_b, y_b, z_b)\) has the same orientation as the stroke plane frame but with the origin that is located at the center of mass. The offset between wing base and center of mass is \(l_s\). The forces \((F_x, F_y, F_z)\) and torques \((T_x, T_y, T_z)\) that are produced by the wing pair are defined with respect to the stroke frame. (c) Blade element (BE) cut view shows the (geometric) angle of attack \(\alpha\), which is defined as the angle between the wing chord and the tangential of the wings trajectory (relative to the stroke plane), and instantaneous lift \(dF_L\) and drag forces \(dF_D\) on the BE.

\[ \delta T_z = \frac{1}{8} \rho_{\text{air}} C_D R_w^4 \bar{c}_3^3(S) \omega_w^2 A_0^2 \left( \frac{1 - 2\sigma}{\sigma(1 - \sigma)} \right), \]

where \(\sigma = 0.5 - \delta\sigma\), for small \(\delta\sigma, \left( \frac{1 - 2\sigma}{\sigma(1 - \sigma)} \right) \approx \frac{2\delta\sigma}{0.25} = 8\delta\sigma.\]
5) Similarly, longitudinal horizontal force $F_x$ can be generated with symmetric split cycle on the left and right wings, i.e., $\sigma_l = \sigma_r = \sigma$ and $\sigma = 0.5 - \delta \sigma$, as shown in Fig. 2.5(e):

$$\delta F_x = \frac{1}{4} \rho_{air} C_D R_w^3 \bar{c}_r^2 \bar{S}(S) \omega_w^2 A_0^2 C_{scx} \left( \frac{1 - 2 \sigma}{\sigma (1 - \sigma)} \right), \quad (4.9)$$

where $4C_{scx} \frac{C_D}{C_L} \approx 1.5$ and $\frac{C_D}{C_L} \approx 1$.

6) Previous work [26] showed that the longitudinal horizontal force $F_y$ can be generated with split cycle method, using Bessel function to evaluate integrals involved. As there is no simple closed-form formula for the evaluation, we will not consider $F_y$ in this work.

<table>
<thead>
<tr>
<th>Forces</th>
<th>Kinematics</th>
<th>Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_z$</td>
<td>Amplitude</td>
<td>$S_{F_z</td>
</tr>
<tr>
<td>$F_x$</td>
<td>Split Cycle</td>
<td>$S_{F_x</td>
</tr>
<tr>
<td>$T_x$</td>
<td>Amplitude</td>
<td>$S_{T_x</td>
</tr>
<tr>
<td>$T_y$</td>
<td>Bias</td>
<td>$S_{T_y</td>
</tr>
<tr>
<td>$T_z$</td>
<td>Split Cycle</td>
<td>$S_{T_z</td>
</tr>
</tbody>
</table>

From 1)-5) and $mg = \frac{1}{2} \rho_{air} C_L R_w^3 \bar{c}_w^2 \bar{S}(S) \omega_w^2 A_0^2$, we can derive all the sensitivity functions summarized in the Table 4.2. To illustrate the large value of sensitivities, consider small change of kinematics, for example, $\frac{\delta A}{A_0} = 6deg/60deg = 0.1$ and $\delta \phi_0 = 5.7deg \approx 0.1$. With such small change of kinematics, we have $\delta F_z = 20\% mg$, $\delta T_x = 20\% mgr_{cp}$ and $\delta T_x = -10\% mgr_{cp}$, i.e. lift has a 20% of variation relative to the body weight, and roll torque and pitch torque all have very large variations. Similar results can be obtained for other parameters and their sensitivities.
4.5 Controller Design

4.5.1 Linear Control of Wing Kinematics and Force Generation

Based on the identified model parameters, closed-loop control is applied to achieve instantaneous wing trajectory tracking of the desired waveforms. Let $D = B_s|\dot{\phi}|\dot{\phi} + T_f\text{sign}(\dot{\phi}) + \Delta$, and the system dynamics in Equation (4.1) can be written as

$$J_s\ddot{\phi} + B_s\dot{\phi} + K_s\phi + D = K_uu. \quad (4.10)$$

Two types of linear controllers, i.e. PID and LQR, are designed for linear system of Equation (4.10) with disturbance D.

The PID (proportional-integral-derivative) controller with nominal model compensation was used,

$$u = J_s\ddot{\phi}_d + B_s\dot{\phi}_d - K_pe - K_i\int e\,dx - K_d\dot{e}. \quad (4.11)$$

where the design is based on pole placement design technique [41] to get control parameters $K_p$, $K_i$, and $K_d$ that achieves closed loop poles at $-200, -200 \pm 200i$.

LQR determines state feedback gain vector $K$ for input form of $u = -Kx$ + feedforward to minimize the cost function [41],

$$J(u) = \int_0^\infty (x^TQx + u^TRu)dt. \quad (4.12)$$

In order to reduce the control effort spent on tracking and therefore increase the achievable trajectory amplitude, a large $R$ is chosen to penalize the expenditure of the control effort. $R = 1000$ and $Q = [10; 01]$ are used to get feedback gain vector $K$.

4.5.2 Nonlinear Control of Wing Kinematics and Force Generation

The high sensitivity of aerodynamic force to kinematic changes imposes a stringent requirement on the wing kinematic control, in addition to all the uncertainties. In order to precisely control the force, a simple linear controller is not sufficient and thus a nonlinear control is chosen, which can provide following advantages: 1) Guaranteed
Transient and steady state performance, thus the transient can be as fast as physically permitted. The bandwidth of the controlled wing dynamics can be greatly increased with high (local) gain feedback. 2) Robust control attenuates the uncertain nonlinearities, including disturbances, modelling errors, wing-body velocity interactions, etc, which is more robust compared to other methods such as L1-adaptive control or an indirect method like model predictive adaptive control, etc. 3) Adaptation for parametric uncertainties.

By closing the loop with the wing kinematic feedback information, the structure of the resulting system is illustrated in Fig. 4.6(b), where due to the high closed-loop bandwidth and the strong attenuation of uncertainties on the wing kinematics using the proposed the proposed nonlinear controller in the inner-loop, the outer loop design can be greatly simplified by treating the inner loop as a high performance servo mechanism without uncertainties. This cascade structure is ubiquitous for robotic systems, such as industrial robotic manipulators. The resulting force/torque will be more precise and the effects of body velocity interaction can be well-predicted by FCF and FCT [10].

Following the design procedure in [111], we design the adaptive robust controller for the wing kinematics model of Equation (4.1) in the state space form,

\[ \dot{x}_1 = x_2 \]
\[ \theta_1 \dot{x}_2 = K_u u - \theta_2 x_2 - \theta_3 x_2^2 \text{sign}(x_2) - \theta_4 x_1 - \theta_6 \text{sign}(x_2) - \theta_5 + \tilde{d} \]

where \( x = [x_1, x_2]^T = [\phi, \dot{\phi}]^T \) represents the state vector of stroke angle and angular velocity, and \( \tilde{d} = \Delta - d \) is the uncertain nonlinearities with \( d \) as the slow changing components of the uncertainty that can be adapted. To linearly parameterize the state space equation in terms of a set of unknown parameters, define \( \theta = [\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6]^T \) as \( \theta_1 = J_s, \theta_2 = B_{s1}, \theta_3 = B_{s2}, \theta_4 = K_s, \theta_5 = d \) and \( \theta_6 = T_f \). Thus parametric uncertainties are equivalent to the variation of unknown \( \theta \).

As the system suffers from both parametric uncertainties \( \theta \in \mathbb{R}^p \) and uncertain nonlinearities \( \tilde{d} \), the following practical assumption can be made:
Assumption 4.5.1 Both parametric and nonlinear uncertainties are bounded, i.e.,

\[ \theta \in \Omega_\theta \triangleq \{ \theta : \theta_{\text{min}} \leq \theta \leq \theta_{\text{max}} \} \]  

(4.14)

\[ \tilde{d} \in \Omega_d \triangleq \{ \tilde{d} : ||\tilde{d}|| \leq \delta_d \} \]  

(4.15)

where \( \theta_{\text{min}} = [\theta_{1\text{min}}, ..., \theta_{p\text{min}}]^T \), \( \theta_{\text{max}} = [\theta_{1\text{max}}, ..., \theta_{p\text{max}}]^T \) are known constants, the operation \( \leq \) for two vectors is performed component-wisely, and \( \delta_d \) is a known bounding function.

Parametric uncertainties are treated by adaptation and projection. Let \( \hat{\theta} \) denote the estimate of \( \theta \) and \( \tilde{\theta} \) is the estimation error (i.e., \( \tilde{\theta} = \theta - \hat{\theta} \)). In view of Equation (4.14), the following adaptation law with discontinuous projection modification in [111] can be used

\[ \dot{\hat{\theta}} = \text{Proj}_{\hat{\theta}}(\Gamma \tau) \]  

(4.16)

where \( \Gamma > 0 \) is a diagonal matrix, \( \tau \in \mathbb{R}^p \) is a vector of adaptation functions to be synthesized later. The projection mapping \( \text{Proj}_{\hat{\theta}}(\bullet) = [\text{Proj}_{\hat{\theta}_1}(\bullet_1), ..., \text{Proj}_{\hat{\theta}_p}(\bullet_p)]^T \) is taken as follows

\[ \text{Proj}_{\hat{\theta}_i}(\bullet_i) = \begin{cases} 0 & \text{if } \hat{\theta}_i = \theta_{\text{imax}} \text{ and } \bullet_i > 0 \\ 0 & \text{if } \hat{\theta}_i = \theta_{\text{imin}} \text{ and } \bullet_i < 0 \\ \bullet_i & \text{otherwise} \end{cases} \]  

(4.17)

It can be shown that for any \( \tau \), the adaption law (4.16)-(4.17) ensures (4.14) and

\[ \hat{\theta} \in \Omega_\theta \triangleq \{ \hat{\theta} : \theta_{\text{imin}} \leq \hat{\theta} \leq \theta_{\text{imax}} \} \]  

(4.18)

\[ \hat{\theta}^T(\Gamma^{-1}\text{Proj}_{\hat{\theta}}(\Gamma \tau) - \tau) \leq 0, \ \forall \tau. \]  

(4.19)

Define a sliding surface

\[ p = \dot{e} + k_1 e = x_2 - \dot{y}_n + k_1 e = x_2 - x_{2eq} \]  

(4.20)
where \( e = x_1 - y_d(t) \) is the output tracking error, \( y_d(t) \) is the desired trajectory and \( k_1 \) is any positive feedback gain. The goal of making \( e \) as small as possible is equivalent to reducing \( p \), since \( G_p(s) = \frac{e(s)}{p(s)} = \frac{1}{s + k_1} \) is a stable transfer function. Then

\[
J_s \dot{p} = K_u u - \theta_1 \dot{x}_2 e - \theta_2 x_2 - \theta_3 x_2^2 \text{sign}(x_2)
- \theta_4 x_1 - \theta_5 \text{sign}(x_2) - \theta_6 + \tilde{d}
= K_u u + \Phi^T \hat{\theta} + \tilde{d}
\] (4.21)

where

\[
\Phi = [- (\ddot{y}_d - k_1 \dot{e}), -x_2, -x_2^3 S_f(x_2), -x_1, -1, -\text{sign}(x_2)]^T
\] (4.22)

The following nonlinear control is proposed:

\[
K_u u = u_a + u_s, \quad u_a = -\Phi^T \hat{\theta}
\] (4.23)

where \( u_a \) is a feedforward model compensation term, and the robust control term \( u_s \) is

\[
u_s = u_{s1} + u_{s2}, \quad u_{s1} = -k_2 p
\] (4.24)

where \( u_{s1} \) is used to stabilize the nominal system and \( u_{s2} \) is a robust feedback term used to attenuate the effect of model uncertainties. Substituting Equation (4.24) into Equation (4.21),

\[
J_s \dot{p} + k_2 p = u_{s2} - \Phi^T \hat{\theta} + \tilde{d}
\] (4.25)

With the Assumption 4.5.1 and P1, \( u_{s2} \) can be synthesized to dominate the model uncertainties from both parametric uncertainties \( \hat{\theta} \) and uncertain nonlinearities \( \tilde{d} \), which satisfies the following two conditions:

- \( p(u_{s2} - \Phi^T \hat{\theta} + \tilde{d}) \leq \epsilon \)
- \( pu_{s2} \leq 0 \)

where \( \epsilon \) is a design parameter which should be sufficiently small. One example of \( u_{s2} \) that satisfies above conditions can be taken as follows

\[
u_{s2} = -\frac{1}{4\epsilon} h^2 p
\] (4.26)
where \( h(x,t) = |\Phi^T \theta_{\text{max}} - \theta_{\text{min}}| + \delta_d \). Other designs of \( u_{s2} \) can be found in [111].

Transient and steady state tracking performance are obtained with above proposed controller design:

**Theorem 4.5.2** If the adaptation function is chosen as

\[
\tau = \Phi p
\]  

(4.27)

Then the proposed control law guarantees the following as in [111]:

1. In general, all signals are bounded. Furthermore, the positive definite function \( V_s \) defined by \( V_s = \frac{1}{2} J_s p^2 \) is bounded by

\[
V_s \leq \exp(-\lambda t)V_s(0) + \frac{\epsilon}{\lambda}[1 - \exp(-\lambda t)]
\]  

(4.28)

with \( \lambda = 2k_2/\theta_{\text{max}} \).

2. If after a finite time, there exist parametric uncertainties only (i.e., \( \tilde{d} = 0 \)), in addition to results in 1), zero final tracking error is also achieved, i.e., \( e \to 0 \) and \( p \to 0 \), as \( t \to \infty \).

The proofs are as follows:

Proof of 1. Given the Lyapunov function candidate \( V_s \) and Assumption 4.5.1, we have

\[
\dot{V}_s = J_s \dot{p}^T = -k_2 p^2 + (u_{s2} - \phi^T \tilde{d} p)
\]  

\[
\leq -k_2 p^2 + \epsilon \leq -\frac{2k_2}{\theta_{\text{max}}} V_s + \epsilon
\]  

(4.29)

Proof of 2. If \( \tilde{d} = 0 \), \( \forall t \geq t_0 \), \( V_{as} = V_s + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} \)

\[
\dot{V}_{as} \leq -k_2 p^2 + \tilde{\theta}^T \Gamma^{-1} (\dot{\theta} - \Gamma \tau) \leq -k_2 p^2
\]  

(4.30)

We have \( p \in L_2 \), \( \dot{p} \) is bounded, thus \( p \) is uniformly continuous. From Barbalat’s lemma, \( p \to 0 \), as \( t \to \infty \).

The diagram of the proposed nonlinear controller is shown in Fig. 4.8.
4.6 Control Experiment Results

All control experiments were conducted on dSPACE DS1103 PPC Controller Board with sampling frequency \( f_s = 5kHz \). The brushless dc motor commutation was implemented on a 72 MHz cortex M3 board (NXP Semiconductors, San Jose, CA, USA) at rate of 50kHz. Motor commutation loop, control loop and data acquisition loop are all implemented on the same board with different rate and levels of priorities. Commutation loop runs at rate of 50000 samples/sec with highest priority; control loop with lower priority is executed at rate of 2000 samples/sec; data acquisition loop sends encoder counts, desired trajectory and control input counts via serial communication with lowest priority at rate of 400 samples/sec. The drive electronics was custom made for the motor. Two different wing models were used for control experiments, and their parameters are shown in Table 4.3. The angle feedback signals are obtained by magnetic encoder at the bottom of the motor (FAULHABER Brushless DC-Servomotors 0620B). The block diagram of the single wing testing platform are shown in Fig 2.8. The wing fabrication process was described previously in [112]. Least square off-line parameter estimations were performed to obtain the nominal system parameters shown in Table 4.4. Wing #1 was used as the nominal system for controller design, while the parameters of wing #2 are assumed unknown.
and will be used to demonstrate the effects of controller to cope with the parametric uncertainties.

Based on the identified model parameters of the nominal system (wing #1), the proposed controller parameters were chosen as $k_1 = 1000$, $k_2 = 200\hat{J}_s = 3.0920e-5$, and $\epsilon = 0.12$. Specifically, $k_1$ was chosen to be as large as possible but 5 times smaller than loop sampling frequency $f_s$ to avoid any digital effect. $k_2$ was chosen according to $u_{s1s}$ contribution to the nominal bandwidth of 200rad/s. $\epsilon$ was selected according to the overall desired local bandwidth and the balance between $k_2$ and $\epsilon$. If $k_2$ is too large, when the error is large, control saturation will occur. For parameter adaptation, the initial values were chosen not to be the same as the identified values, but the value shown in Table 4.3 as $\hat{\theta}_0$. Adaptation rates $\Gamma=\text{diag}(4.5307e-12, 2.9416e-08, 6.4701e-13, 0.0065, 5e-05, 2e-07)$ is selected according to general gradient type adaptation method. Bounds for the parameters and the uncertain are chosen to be rather conservative according to the prior knowledge of physical parameters.

### Table 4.3. Wings Parameters.

<table>
<thead>
<tr>
<th>Wing</th>
<th>$m$ (mg)</th>
<th>$R_w$ (mm)</th>
<th>$\bar{c}$ (mm)</th>
<th>$\hat{r}_2^2$</th>
<th>$\hat{r}_3^3$</th>
<th>$J_w$ (mg.mm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>62</td>
<td>62.8</td>
<td>11</td>
<td>0.29</td>
<td>0.20</td>
<td>69718</td>
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<tr>
<td>2</td>
<td>66</td>
<td>69.3</td>
<td>16</td>
<td>0.25</td>
<td>0.20</td>
<td>79241</td>
</tr>
</tbody>
</table>

### Table 4.4. System Parameters (SI units).

<table>
<thead>
<tr>
<th>Wing</th>
<th>$J_s$</th>
<th>$B_{s1}$</th>
<th>$B_{s2}$</th>
<th>$K_s$</th>
<th>$d$</th>
<th>$T_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.55e-7</td>
<td>9.60e-6</td>
<td>3.47e-8</td>
<td>6.00e-3</td>
<td>1.22e-4</td>
<td>2.31e-5</td>
</tr>
<tr>
<td>2</td>
<td>1.60e-7</td>
<td>9.56e-6</td>
<td>7.48e-8</td>
<td>6.00e-3</td>
<td>1.2e-4</td>
<td>2.30e-5</td>
</tr>
<tr>
<td>$\hat{\theta}_0$</td>
<td>1.50e-7</td>
<td>1.00e-5</td>
<td>3.70e-8</td>
<td>5.70e-3</td>
<td>1.00e-4</td>
<td>2.00e-5</td>
</tr>
</tbody>
</table>
4.6.1 Linear Kinematics Control Results

One PID control result is shown in Fig. 4.9, where the system was tracking a 24Hz sinusoidal trajectory with peak-to-peak amplitude of 90°. The controller is proven to be effective in that tracking error and phase lag are small and the control input effort is within the saturation limit of 9V. The encoder reading of the wing angular position is compared with the wing tip position measured from high speed camera images (Fastec Imaging Corp.) with a Nikon 35mm AF/1.8G (Nikon Incorp.) lens capturing 1280 × 512 images at 1000fps from top view of the flapper. The wing angular position from the images were calculated based on a vector from wing base to a point located at 15% (wing length) to the wing tip, for image digitization refer to [45]. Due to minor wing deflection along its leading edge, especially at wing stroke reversals, the encoder-based flapping amplitude is about 13% less than that based on images as shown in Fig. 4.11.

![Tracking Results PID](image)

Figure 4.9. PID Tracking 24Hz 90deg Sinusoidal Trajectory.

Using LQR controller, the experimental result is shown in Fig. 4.10. The output is a 24Hz sinusoidal with fixed $\pi$ phase lag, compared with the desired trajectory, but the achievable amplitude is much larger. It is important to note that, although LQR
does not achieve an accurate tracking of the phase, it is able to track a kinematics with larger amplitude and less power consumption, which better fulfill the design requirement for MAV.

Next, the LQR controller is used to track a sinusoidal signal with different amplitude and bias. Fig. 4.10 shows the LQR controller successfully tracks wing kinematics with amplitudes of 120° and 150°, respectively. Wing stroke amplitude difference between left and right wings is expected to generate roll torque acting on the body of
the MAV. Fig. 4.10 shows LQR successfully tracks sinusoidal wing motions with 5° and 10° bias. This asymmetry for both left and right wings will generate pitch torque on the MAV.

![Graph showing Angle(deg) vs Time(sec)](image)

Figure 4.11. Encoder and video for wing spar deflection.

Finally, a comparison between the encoder-based and image-based wing angular positions for the LQR case is performed which yields similar results as with the PID case.

Nonlinear oscillator based central pattern generator (CPG) is widely used in robot locomotion control due to its stable limit cycle and synchronization of coupled networks [88]. Here a simple one degree of freedom CPG, realized by a Hopf oscillator is used as an alternative to the sinusoidal trajectory. It will serve as an online trajectory generator that can be implemented efficiently on microcontroller using Euler method. The time it takes to run the CPG code is close to that needed to evaluate the math function \( \sin() \) on the microcontroller. Here the CPG code is running in the
same loop as the controller at rate of 2000 sample/sec. The Hopf oscillator has the following dynamics [88],

\[
\begin{align*}
\dot{x}_1 &= -\omega x_2 - \lambda \left( \frac{x_1^2 + x_2^2}{\rho^2} - 1 \right) x_1 \\
\dot{x}_2 &= \omega x_1 - \lambda \left( \frac{x_1^2 + x_2^2}{\rho^2} - 1 \right) x_2
\end{align*}
\] (4.31)
where $x_1$ and $x_2$ are states of the oscillator dynamics, $\omega = 2\pi f$ is the angular velocity of the trajectory, $\rho$ is the amplitude of the trajectory, and $\lambda$ is the rate of convergence for the limit cycle. The desired trajectory is thus $\phi_d = c \cdot x_1$, where $c$ is scaling constant.

The CPG tracking results for PID and LQR are illustrated in Fig. 4.12 and 4.13, respectively. As seen from Fig. 4.12, CPG will converge to sinusoidal motion and the controller successfully tracks this trajectory. Fig. 4.13 shows the limit cycle nature of the trajectory: starting at any initial condition, the trajectory converges smoothly back to its stable sinusoidal limit cycle. The rate of convergence can be tuned by changing parameter $\lambda$. This is desirable for our system because unmatched initial condition results in large initial tracking error that in turn leads to large control input and thus input saturation.

### 4.6.2 Nonlinear Kinematics Control Results

The nominal control performance comparison between PID and the proposed nonlinear controller are shown in Fig. 4.14. The control results show that the proposed nonlinear controller achieves a much smaller tracking error within 1deg, while the error for PID controller has a peak value of almost 5deg. The presence of the small higher-frequency correction signal is the evidence of the proposed nonlinear controller’s robust control $u_{s2}$ in action, which gives an effective local nonlinear high gain and avoids control saturation associated with linear high gain control. Due to adoption of smoothed version of the robust sliding mode control term $u_{s2}$, the proposed nonlinear controller exhibits little control input chattering.

In order to demonstrate the proposed nonlinear controller’s ability to effectively handle parametric uncertainties through online adaptation, with two controllers remaining unchanged, the nominal wing #1 was swapped with ‘unknown’ wing #2. The control results after the converged adaptation are shown in Fig. 4.15. Compared with the results of nominal wing (Fig. 4.14), it is clear that the proposed nonlinear
controller shows no performance degradation in that there is no increase of control input level and tracking error. However, the control performance of the non-adapting PID controller deteriorated.

With $A/A_0 = 1/60 = 0.017$, the lift generation sensitivity for the proposed nonlinear controller is $\delta\bar{F}_z = mg(2\delta A/A_0) = 3.3\%mg$, i.e. the maximum lift variation is only 3.3%. For PID, $\delta A/A_0 = 5.5/60 = 0.0917$ and the maximum lift variation is as large as 18.3%. Therefore, the proposed nonlinear controller is proven to be effective in handling parametric uncertainties and delivering guaranteed robust force generation. To further prove that the proposed nonlinear controller is suitable for aerodynamic force generation, the following trajectories were tracked:

- **T1**: amplitude variation, from 54.27deg to 65.73deg.
- **T2**: frequency variation, from 28Hz to 33Hz.

![Figure 4.14. Tracking performance before parameter change.](image-url)
Figure 4.15. Tracking performance after parameter change.

- T3: bias, from -0.1 rad to 0.1 rad.
- T4: split-cycle, from 0.45 to 0.55.

All trajectory parameters were updated to new values at increments of 1 sec. The tracking errors are shown in Fig. 4.16a-d. For all the trajectories, different parameters, and even during the transient portions between parameter updates, the errors are within the consistent range of ±1 deg. Next, the input disturbance rejection results of the proposed nonlinear controller are shown in Fig. 4.16e. A 1 V input disturbance was applied at 0.5 sec and removed at 1 sec. The resulting error shows a slight offset from 1 deg to 1.5 deg, but is still very small. As a comparison, the same input disturbance results in a larger angle offset around $K_u/K_s \approx 9$ deg in openloop experiments.
4.6.3 Lift Force Generation Results

Force measurement was performed using a six component force/torque transducer (Nano17, ATI Ind. Automation). Due to limited resolution of Nano17 (0.3g resolution on the force and 1/64Nmm resolution on the torque measurement), a rigid 150mm beam setup was used to amplify the lift measurement as shown in Fig. 4.1. The improved resolution was about 0.0106g. The force sensor and beam setup was calibrated with precision weights of 0.1g, 0.5g, 5g and 20g and verified the resolution of at least 0.03g. When calculating the time-averaged force, sufficient large number of wing-beat cycles at steady state were used to guarantee the reliability of the results.

To test the performance of the lift force generation from the proposed nonlinear controller controlled kinematics, the lift force at various amplitudes and frequencies were measured and the average lift forces obtained are shown in Fig. 4.17 and Fig. 4.18. For comparison, the lift force in openloop experiments were also measured and
Figure 4.17. Openloop and the proposed nonlinear controller lift generation for different amplitudes.

plotted. The amplitude of the open-loop response was matched with the proposed nonlinear controller controlled result for fair comparison. The results clearly demonstrate the excellent performance of the proposed nonlinear controller in aerodynamic force generation with better consistency, accuracy, smoothness and linearity across the tested amplitude and frequency range. In contrast, the open-loop generated force not only varied in an unpredicted way, but also did not generate as much lift as the proposed nonlinear controller controlled wing kinematics.

4.7 Chapter Summary

In this chapter, we present a flapping-wing actuator capable of instantaneous closed-loop wing trajectories tracking. The flapping wing is driven by a DC motor directly through gear transmission, and the system is designed to resonant with torsion springs. We present a dynamic model with parameter uncertainties and disturbances, validated with system identifications. Treating the nonlinearity as disturbances, we
Figure 4.18. Openloop and the proposed nonlinear controller lift generation for different frequencies.

designed a proportional-integral-derivative (PID) controller and a linear quadratic regulator (LQR), for instantaneous wing trajectory tracking at 24Hz; with the nonlinear model, we designed a nonlinear controller to achieve robust performance at over 30Hz with stiffer springs. The algorithms were compared experimentally on a 7.5 gram Flapping Wing MAV. The experiments showed PID and nonlinear controller precisely tracked the wing trajectories; while LQR tracked with less precision but smaller input effort. In addition, the nonlinear controller achieved better tracking of various wing trajectories with different amplitude, bias, frequency, and split-cycles and with adaptations to unknown wing morphological parameters. For wing trajectory generation, we designed a Hopf oscillator based central pattern generator with smooth convergence. The lift force measurements of nonlinear controller were then compared with those of open-loop methods. Future work will focus on the implementation of the the proposed wing controllers as a subsystem of the vehicle controller to achieve and stable hovering and maneuvering ability.
5. SYSTEM IDENTIFICATION AND ALTITUDE CONTROL OF FLAPPING WING INSECT MAV

5.1 Introduction

Due to the lack of comprehensive understanding of the system dynamics, control performance limitations, complex time-variant aerodynamics, manufacturing imperfections, and additional platform limitations, control of FWMAV is a great challenge.

In this chapter, we first formulate the full nonlinear dynamics of FWMAV with exact nonlinear force mapping, and then present full nonlinear attitude and position controller with exponentially stable and globally exponential attractive properties.

5.2 Development of Onboard Attitude Control Algorithms

Figure 5.1. Video sequences of the unstable fight without any feedback control (0.1sec in between sequences).
5.2.1 Flight Dynamics

Due to the inherent instability of the flapping flight, without feedback control, the FWMAV quickly went unstable, as illustrated by the video sequences of the unstable flight in Fig. 5.1.

For controller design, we consider the full dynamic model of the vehicle, given by

\[
\begin{align*}
\dot{p} &= v, \\
m\ddot{p} &= Rf^b + mg e_3, \\
\dot{R} &= R\dot{\omega}^b, \\
\mathcal{I}\ddot{\omega}^b + \omega^b \times \mathcal{I}\omega^b &= \tau^b.
\end{align*}
\] (5.1)

As shown from previous chapter, so the overall wrenches from the flapping wings are given by

\[
\begin{align*}
f^b &= f^b_n + f^b_d, \\
\tau^b &= \tau^b_n + \tau^b_d,
\end{align*}
\]

\[
f^b_d = \begin{bmatrix}
-c_x u - c_x d_s q \\
-c_y v + c_y d_s p \\
-c_z w
\end{bmatrix}
\] (5.2)

\[
\tau^b_d = \begin{bmatrix}
d_s c_y v - (d_s^2 c_y + c_{roll}) p \\
-d_s c_x u - (d_s^2 c_x + c_{pit}) q \\
-c_{yaw} r
\end{bmatrix}
\]

The nominal wrenches for the flight dynamics are given by

\[
\begin{align*}
f^b_n &= \begin{bmatrix} 0 \\ 0 \\ F_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \\
\tau^b_n &= \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} = \begin{bmatrix} u_2 \\ u_3 \\ u_4 \end{bmatrix}
\end{align*}
\] (5.3)
The control inputs to the system is given by

\[
\begin{bmatrix}
F_z \\
T_x \\
T_y \\
T_z
\end{bmatrix} =
\begin{bmatrix}
F_0v_1^2 \\
r_cp F_0 v_1 v_2 \\
r_cp F_0 v_2^2 v_3 \\
r_cp F_0 v_2^2 v_4
\end{bmatrix} =
\begin{bmatrix}
u_1 \\
u_2 \\
u_3 \\
u_4
\end{bmatrix}.
\tag{5.4}
\]

Table 5.1. FCF and FCT.

<table>
<thead>
<tr>
<th>Forces</th>
<th>DoF</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c_z)</td>
<td>(x)</td>
<td>(2\rho a R_w^2 \bar{c} A_0 \omega_w \hat{r}_1^1(S) C_D(a_0) \cos^2(\phi)</td>
</tr>
<tr>
<td>(c_y)</td>
<td>(y)</td>
<td>(2\rho a R_w^2 \bar{c} A_0 \omega_w \hat{r}_1^1(S) C_D(a_0) \sin^2(\phi)</td>
</tr>
<tr>
<td>(c_z)</td>
<td>(z)</td>
<td>(\rho_a R_w^2 \bar{c} A_0 \omega_w \hat{r}_3^1(S) \frac{dC_S(a)}{da} \big</td>
</tr>
<tr>
<td>(c_{roll})</td>
<td>Roll</td>
<td>(\rho_a R_w^4 \bar{c} A_0 \omega_w \hat{r}_3^3(S) \frac{dC_S(a)}{da} \big</td>
</tr>
<tr>
<td>(c_{pitch})</td>
<td>Pitch</td>
<td>(\rho_a R_w^4 \bar{c} A_0 \omega_w \hat{r}_3^3(S) \frac{dC_N(a)}{da} \big</td>
</tr>
<tr>
<td>(c_{yaw})</td>
<td>Yaw</td>
<td>(2\rho a R_w^4 \bar{c} A_0 \omega_w \hat{r}_3^3(S) C_D(a_0)</td>
</tr>
</tbody>
</table>

With assumption of near-hovering condition, as derived in [10], for a pair of wings, we have the damping coefficients summarized in Table 5.1. As in [10], \(\hat{t} = \omega_w t\) is the nondimensional time, \(a\) is the effective angle of attack, \(\frac{d\phi}{dt}\) is the nondimensional flapping velocity of the wing, \(\phi\) and \(n\) are, respectively, the wing-flapping amplitude and frequency, and \(\hat{r}_1^1(S)\) and \(\hat{r}_2^2(S)\) are, respectively, the nondimensional first and second moments of the wing area.

5.2.2 Nonlinear Controller Design

With control input mapping of Equation (5.4), control of dynamics Equation (5.1) is similar to that of quadrotors, except the passive flapping wing damping terms and the additional control input for yaw. Following the methods of [58], we design a
nonlinear controller for our FWMAV. The control inputs $u_1$ and $\tau^b_n = [u_2, u_3, u_4]$ are chosen as

$$\tau^b_n = -k_R e_R - k_{\omega^b} e_{\omega^b} + \omega^b \times I \omega^b$$

$$- I (\dot{\omega}^b R^T R_c \dot{\omega}^b_c - R^T R_c \omega^b_c) - \tau^b_{d},$$

$$u_1 = -(-k_p e_p - k_v e_v - mg e_3 + m \ddot{p} - f^b_{d3}),$$

where $p_d$ is the desired acceleration; $g$ is the gravity acceleration; $k_p$, $k_v$, $k_R$, and $k_{\omega^b}$ are positive definite control gains; $f^b_{d3}$ is the 3rd components of $f^b_d$; and the subscript $c$ denotes reference command. The quantities

$$e_R = \frac{1}{2} (R_c^T R - R^T R_c)^\top, e_{\omega^b} = \omega^b - R^T R_c \omega^b_c$$

represent the orientation and angular rate errors, respectively, whereas translation errors are represented by

$$e_p = p - p_d, e_v = v - v_d$$

If the initial attitude error is less than 90°, the zero equilibrium of the tracking errors is exponentially stable. Furthermore, if the initial attitude error is between 90° and 180°, then the zero equilibrium of the tracking errors is almost globally exponentially attractive. See [58] for algorithm proof and analysis.

### 5.2.3 Control Experiment Setup

As the sensor fusion has large errors due to flapping wing vibration and there is no position information available from onboard sensors, an external measurement setup was used, as shown in Fig. 5.3. 6 Vicon motion-capture-system cameras covered a working area of 50cm by 60cm volume. The Vicon motion-capture system (http://www.vicon.com) provides a 6 degree-of-freedom state estimate for the vehicle, which is considered ground truth and control feedback signals at 200 Hz. The microcontroller onboard the vehicle receives the feedback control signal at 200Hz with UART communication consists physically two thin wire. The standard deviation for single-marker position estimates is 50 nm, which is well beyond the accuracy required for flight.
Three single axis setups for system identification and control of the three Euler angles were shown in Fig. 5.2.

5.3 System Identification

The force mapping was first validated with experiments. The force measurements were performed while the input voltages were varied according to the input mapping. The results are shown in Fig. 5.4. The gains and slopes are shown to be consistent with predictions and the offset trims were extracted and applied for trimming the control inputs for control experiments.
As the dynamics is not stable, closed-loop system identification for each axis was conduced. Let the general angle to be $\psi$, thus for roll $\psi = \alpha$, pitch $\psi = \beta$, and yaw angle $\psi = \gamma$. For single axis dynamics,

$$I\ddot{\psi} + B\dot{\psi} = Ku,$$  \hspace{1cm} (5.8)

With input $u = -k_p e - k_d \dot{e}$ used to stabilize the corresponding dynamics and $e = \psi - \psi_d$, the dynamic model can be transformed as

$$I\ddot{\psi} + B\dot{\psi} = K_o(-k_p(\psi - \psi_d) - k_d(\dot{\psi} - \dot{\psi}_d)),$$  \hspace{1cm} (5.9)

with transfer functions as

$$\frac{\Psi(s)}{\Psi_d(s)} = \frac{K_oK_ds + K_oK_p}{Is^2 + (B + K_oK_d)s + (K_s + K_oK_p)}.$$  \hspace{1cm} (5.10)

Closed-loop system identification was performed for each axis, and the results for roll dynamics is shown in Fig. 5.5. The red line shows the reference step command. The blue line is the system response, and the blue line is the fitted model.
5.3.1 Control Results

The roll, pitch and yaw control results are shown in Fig. 5.6, Fig. 5.7 and Fig. 5.8, respectively. During each test, the angle was commanded to track a step reference with magnitude of 20 degrees. For roll control, the angle converged relatively fast and light oscillation was observed at steady state. The pitch control resulted in an undershoot before overshoot, which indicates the possible influence of the non-minimum phase nature of the longitudinal dynamics, since the single axis setup was not completely constrained. The yaw angle control result is similarly to that of roll angle which slightly large oscillation at steady state.
Figure 5.5. Closed-loop system identification of roll dynamics.

Figure 5.6. Single axis control- step response for roll angle.
Figure 5.7. Single axis control- step response for pitch angle.

Figure 5.8. (a) Single axis control- step response for yaw angle.
Next, the free flight attitude stabilization experiments were conducted. As shown in Fig. 5.1, without feedback control, the original open-loop flight dynamics are highly unstable. With the proposed attitude controller, the flight is stable for all three Euler angles during the whole flight duration, as shown in Fig. 5.9 for low speed video sequence and in Fig. 5.10 for high speed video sequence of the same flight.

Figure 5.9. Time sequences of the attitude controlled fight (0.3sec in between sequences) - experiment 2 from normal speed video.
Figure 5.10. Time sequences of the attitude controlled flight (0.3sec in between sequences) - experiment 2 from high speed video.

Total five attitude control tests are shown here, as shown in Fig. 5.12, Fig. 5.14, Fig. 5.16, Fig. 5.18 and Fig. 5.20. All angle response were within maximum 20 degrees bounds. All control inputs were not saturated, exception for the yaw degree of freedom in experiment 4. Experiments in Fig. 5.14 are corresponding to that in Fig. 5.9 and Fig. 5.10.
Figure 5.11. Attitude Stabilization control- experiment 1 response.

Figure 5.12. Attitude Stabilization control- experiment 1 control input.
Figure 5.13. Attitude Stabilization control- experiment 2 response.

Figure 5.14. Attitude Stabilization control- experiment 2 control input.
Figure 5.15. Attitude Stabilization control- experiment 3 response.

Figure 5.16. Attitude Stabilization control- experiment 3 control input.
Figure 5.17. Attitude Stabilization control- experiment 4 response.

Figure 5.18. Attitude Stabilization control- experiment 4 control input.
Figure 5.19. Attitude Stabilization control- experiment 5 response.

Figure 5.20. Attitude Stabilization control- experiment 5 control input.
5.3.2 Discussion

Figure 5.21. (a) The output stroke angle compared to the input voltage with split cycle. (b) FFT of input voltage with split cycle. (c) FFT of output stroke angle.

From the control results, we observed that the yaw torque generation was saturating and limiting the control performance. As shown in Fig. 5.21, even though the input voltage wave form is with large split cycle parameter, the output stroke angle is not showing the same effects of the split cycle. This is due to the strong attenuation of the flapping wing dynamics to the non-sinusoidal excitation with split cycle. The FFT of Fig. 5.21 showed the higher harmonic components of the input split cycle was attenuated to smaller value on the output side. This confirmed with the modeling of
the previous chapter, where a scaling parameter of around 0.1 has to be set for the output split cycle of the stroke kinematics.

To improve performance, consistent fabrication is the key, as the large trimming errors of the system depends heavily on the fabrication of the system. The current prototype, especially the wings, was partially made by hand. This induced large inconsistencies for the control force generation, which impacted the trimming tuning and control performance. For control of x,y,z of the vehicle, a bigger Vicon setup will be needed to provide large volume. To improve the yaw torque generation, two approaches can be further explored: change of stroke plane with a differential mechanism and one servo motor, or direct/indirect angle of attack (wing rotation) control. The latter can also improve roll, pitch, and yaw torque generation in the meantime. To improve the motor control, current feedback/regulation can be added. Wing loading can also be sensed from the current measurement, which can be used to infer the condition of the force generation, etc.
6. SUMMARY

Flying animals with flapping wings may best exemplify the astonishing ability of natural selection on design optimization. They evince extraordinary prowess to control their flight, while demonstrating rich repertoire of agile maneuvers. They remain surprisingly stable during hover and can make sharp turns in a split second. Characterized by high-frequency flapping wing motion, unsteady aerodynamics, and the ability to hover and perform fast maneuvers, insect-like flapping flight presents an extraordinary aerial locomotion strategy perfected at small size scales. Flapping Wing Micro Aerial Vehicles (FWMAVs) hold great promise in bridging the performance gap between engineered flying vehicles and their natural counterparts. They are perfect candidates for potential applications such as fast response robots in search and rescue, environmental friendly agents in precision agriculture, surveillance and intelligence gathering MAVs, and miniature nodes in sensor networks.

In this thesis, we set out to develop not only the state-of-art 12g Hummingbird size Flapping Wing Micro Aerial Vehicle, but also the general optimization solution that has the potential to completely solve the system design problem, even under stringent size, weight and power constraints. In the meantime, we presented a comprehensive study of the flapping resonance, identified both theoretically and experimentally the key principle for resonance design and experiment validation, explaining some lingering questions on experiment results of previous works. For the system software development, we studied the high-frequency wing kinematics control problem, discovered for the first time the non-minimum phase nature of flapping flight, and presented novel full nonlinear attitude and position controller with exponentially stable and globally exponential attractive properties. The multidisciplinary research generated fruitful novel results and concrete interesting scientific contributions, specifically:
1. Designing and developing such systems is a challenging task under stringent constraints in size, weight and power (SWaP). In addition, the lagging in battery technology, requirement on miniature sensors and actuators for navigation, limited on-board computational power, and system integration all pose challenges in design. Under the SWaP constraints, balance and trade-off must be made among mechanical complexity, controllability, power, and weight. Otherwise, even producing enough lift to sustain the weight can be a challenge. In this work, we chose to utilize passive wing rotation and mechanical resonance for the optimal trade-off. Achieving resonance in flapping wings has been recognized as one of the most important principles to enhance power efficiency, lift generation, and flight control performance of high-frequency FWMAVs. Most of the work on the development of such vehicles have attempted to achieve wing flapping resonance. However, theoretical understanding of its effects on the response and energetics of flapping motion has lagged behind, leading to sub-optimal design decisions and misinterpretations of experimental results. In this thesis, we systematically model the dynamics of flapping wing as a forced non-linear resonant system, using both nonlinear perturbation method and linear approximation approach. We derived analytic solution for steady-state flapping amplitude, energetics, and characteristic frequencies including natural frequency, damped natural frequency, and peak frequency. Our results showed that both aerodynamic lift and power efficiency are maximized by driving the wing at natural frequency, instead of other frequencies. Interestingly, the flapping velocity is maximized at natural frequency as well, which can lead to an easy experimental approach to identify natural frequency and follow the resonance design principle. Our models and analysis were validated with both simulation and experiments on ten different wings mounted a direct-motor-drive flapping wing MAV. The result can serve as a systematic design principle and guidance in the interpretations of empirical results.

2. For the vehicle design and prototype of FWMAV, we presented a complete, multidisciplinary formulation for system design optimization and integration for a Hummingbird-size FWMAV. The formulation covers actuation, wing, battery, elec-
tronics, dynamics, flight stability and control. System parameters considered include parameters of wings, motors, gears, springs, batteries, control authorities, and locations of poles and zeros of the dynamics. The formulation was validated by experimental data for both rigid and flexible wings, covering from low to high wing loading. Based on the direct motor drive mechanism of this work, the optimization yields a prototype with on-board sensors, electronics, and computation. It has a wingbeat frequency of 30Hz to 40Hz, with 12 grams of total weight and 20 grams of maximum lift. Liftoff was demonstrated with extra payloads. Initial results of on-board state estimation and flight control were performed. Flapping wing platforms with different requirements and scales can now be systematically designed and optimized with parameter modifications of the proposed formulation.

3. Not only do we have to design and develop the system under the SWaP constraints, we also need to control the system under those tight constraints as well. The superior maneuverability of insect flight is enabled by rapid and significant changes in aerodynamic forces, a result of subtle and precise changes of wing kinematics. The high sensitivity of aerodynamic force to wing kinematic change demands precise and instantaneous feedback control of the wing motion trajectory, especially in the presence of various parameter uncertainties and environmental disturbances. Current work on flapping wing robots was limited to open-loop averaged wing kinematics control. Here we present instantaneous closed-loop wing trajectory tracking of a DC motor direct driven wing-thorax system under resonant flapping. A dynamic model with parameter uncertainties and disturbances was developed and validated through system identification. For wing trajectory generation, we designed a Hopf oscillator based central pattern generator with smooth convergence. Using the linearized model while treating the nonlinearity as disturbance, we designed a proportional-integral-derivative (PID) controller and a linear quadratic regulator (LQR) for instantaneous wing trajectory tracking at 24Hz; Using the original nonlinear model, we designed a nonlinear controller to achieve robust performance at over 30Hz. The control algorithms were implemented and compared experimentally on a 7.5 gram Flapping
Wing Micro Air Vehicle (MAV). The experiments showed that the PID and nonlinear controls resulted in precise trajectory tracking; while LQR controller tracked with less precision but with smaller input effort. In addition, the nonlinear control algorithm achieved better tracking of wing trajectories with varying amplitude, bias, frequency, and split-cycles while adapting to the variations on wing morphological parameters such as wing geometry and stiffness. Furthermore, the lift force measurements of the nonlinear control results were compared with those of open-loop average wing kinematics control commonly adopted in current designs. Finally, we present an analysis on fundamental limitations of flapping flight control and discovered, for the first time, the non-minimum phase nature of flapping flight when certain controls are used. We then presented full nonlinear attitude and position controller with exponentially stable and globally exponential attractive properties. The dynamics and flight control results were then illustrated by experimental results.

To improve performance of the system, we’ve identified several important directions to pursue for future research.

1. Consistent fabrication has been a hallmark of the Harvard Robobee due to the small size of their robots and their state-of-art high precision micro-machining facilities. Here in this thesis, we found that the fine tuning of the trim condition of the flight and potential large error and discrepancies between trial flights are mainly caused by the quality and repeatability of the fabrication process. Currently our technique is a combined method of laser cutting and manual fabrication and assembly. The force and torque generated by the hand-made wings, were very sensitive to the minor changes of the wing, such as geometric asymmetry, the mount of carbon fiber and its placement, the quality of mechanical assembly, the amount of glue, etc. Low-quality fabrication caused large inconsistencies for the control force generation, which then impacted the flight trimming condition tuning and control performance. Developing an advanced manufacturing process, coupled with software and/or algorithm solution, can be complementary to the improved fabrication and consistent quality of
parts. For example, adaptation or estimation can be added to estimate the force and torque trims online to further improve the performance while the system is running.

2. The attitude control results revealed that one of the drawbacks of the passive wing rotation and flapping resonance is the attenuation of non-sinusoidal excitation, such as split cycle method used for yaw torque generation. Our study showed that the attenuation is as large as 90%. With higher energy cost, it was shown in the kinematics control section that this problem can be solved with high gain feedback tracking control of the wing stroke kinematics, where the error was reduced from 90% to within one degree all the time (2%). To improve the yaw torque generation, other approaches can be further explored, for example, changes of two stroke plane with a differential mechanism and one servo motor, or individual changes of two stroke plane with two servo motors, or direct/indirect angle of attack (wing rotation) control. The latter can also improve roll, pitch, and yaw torque generation in the same time.

3. To improve the motor control, current feedback/regulation can be incorporated for motor torque regulation. The modification to electronics is minor: two current feedback resistors are needed and the signal conditioning for the current feedback signal is to be added. Wing loading can also be sensed from the current measurements, which can be used to infer the condition of the force generation and wind gust, etc.

4. The body control results also showed robustness towards digital implementation. Even with 34Hz flapping wing frequency, the body control loops with high sampling rate of 100Hz and low sampling rate of 10Hz were tested with both digital control and analog emulation control implementations. No difference was observed for control performance, even the signal quality was not showing any substantial difference.

5. The state estimation from MEMS IMU sensors presented an interesting and unique research problem due to high frequency excitation from the flapping wings. One solution is to design a mechanical spring-damping vibration isolation or suppression mechanism between the IMU sensors and the flapping wing actuation subsystem. Further improvements can be obtained with signal processing methods, such as a
notch filter, to filter out the noise signal induced by the 34Hz flapping wings. The high impact of flapping wing vibration is more prominent for accelerometer than gyrometer. Sensor fusion without using accelerometer can partially alleviate the problem, but drifting on the integration of gyrometer will not be corrected. Further study can also adopt additional sensors to improve the sensor fusion performance, one candidate is the Vishay IR distance sensor (SparkFun Electronics), which can be very precise with resolution around 1mm but with limited range of 10cm to 30cm. The size of small IR distance sensor is around 3mm to 5mm, small enough for current prototypes to carry. Different complementary sensors can be fused with different combinations to provide better state estimations.

6 Due to computation and payload limitations, autonomous navigation of such small aerial mobile system is still an open question. The first step towards the navigation solution is to localize the vehicle in 3D space. Without using an external motion capture system, a typical solution for larger mobile robot is to add more sensors to do localization or Simultaneous localization and mapping (SLAM), which is not currently feasible for such small system. Equipped only with inertial measurement unit, radio communication module (bluetooth LE), and altimeter, the system will be localized relative to the base station/mobile mothership using above sensors and the novel method we proposed, while the base station with multiple radio modules estimates the 3D location of the vehicle. Future study can use a sensor fusion method and hardware setup to provide the vehicle with 20cm accuracy 3D location information to aim the navigation.
LIST OF REFERENCES
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