2002

Polytropic Exponents for Common Refrigerants

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ABSTRACT

Analysis of compression mechanisms requires a prediction of the pressure during the compression and expansion processes. A common model is the polytropic process, \( PV^n = \text{constant} \). This paper presents a method for determining the best polytropic exponent to use and suggests values for some common refrigerants used in the air conditioning and refrigeration industry.

INTRODUCTION

When a gas undergoes a reversible process, the process frequently takes place in such a manner that a plot of \( \log P \) vs. \( \log V \) is a straight line\(^1\). For such a process \( PV^n = \text{constant} \). This is called a polytropic process. It can be shown that for an ideal gas with constant specific heats undergoing a reversible adiabatic (i.e. isentropic) process, \( n \) is equal to the ratio of specific heats, \( C_p/C_v \). Though technically a formidable list of requirements, this turns out to be an excellent model for the compression and expansion of a refrigerant in the cylinder of a compressor. Though rigorous models based on accurate thermodynamic equations of state are certainly available, the simple polytropic process is adequate for determining forces and moments on compressor parts. Any inaccuracy introduced by using this model will be small considering the uncertainty in predicting the actual operating conditions that the compressor will eventually experience in the field.

Specific heats \( C_p \) and \( C_v \) of real refrigerants vary with temperature and pressure. Some refrigerants have specific heats that vary more than others. This is not only a violation of the constant specific heat requirement it makes it ambiguous as to which temperature and pressure to use. This suggests the need for some form of regression or averaging method. The recommendation below is based on the idea that one of the most important predictions from a compressor mechanism simulation is the input power requirement.

It is proposed that the polytropic exponent \( n \) be chosen such that the average shaft input power of the simplified \( PV^n \) model match one using an accurate equation of state for a representative compression cycle. Specifically, the cycle chosen is the ideal isentropic compressor operating at the ARI rating conditions for which this refrigerant is most commonly used.
MATHEMATICAL MODELS

The Ideal PV diagram for \( PV^n = \text{constant} \)

Work done at the moving boundary for the compression process, going from state 1 (start of compression) to state 2 (start of discharge) is given by

\[
_jW_2 = \int_1^2 PdV
\]

The following relation can be written for a polytropic process

\[
P = P_1V_1^nV^{-n}
\]

Substituting (2) into (1) and integrating from state 1 to state 2

\[
_jW_2 = PV_1^n\int_1^2 V^{-n}dV = \frac{PV_1^n(V_2^{1-n} - V_1^{1-n})}{1-n}
\]

or

\[
_jW_2 = \frac{P_2V_2 - P_1V_1}{1-n}
\]

For the purposes of this analysis \( V_1 \) is taken as 1 in\(^3\). Work for the re-expansion process, going from state 3 (end of the discharge process) to state 4 (beginning of the suction process) is given by

\[
_3W_4 = \frac{PV_4 - P_3V_3}{1-n}
\]

For the purposes of this analysis, the clearance volume, \( V_3 \) is taken as 1 percent of the maximum, \( V_1 \). The ideal discharge process is a constant pressure process with \( P_2 = P_3 = P_{\text{discharge}} \). Work for the ideal discharge process is

\[
_2W_3 = P_2(V_3 - V_2)
\]

The ideal suction process is a constant pressure process with \( P_1 = P_4 = P_{\text{suction}} \). Work for the ideal suction process is

\[
_4W_1 = P_1(V_1 - V_4)
\]
The work for the complete cycle is the sum

\[ W = W_1 W_2 + W_3 W_4 + W_1 \]  

(8)

The ideal PV diagram using REFPROP

The ideal PV diagram is also integrated using the more accurate thermodynamic relations provided by REFPROP\(^2\). This integration is done numerically using the trapezoidal rule\(^3\). For the compression process

\[ w_2 = \sum_{i=0}^{N} \frac{1}{2} \left( P_i + P_{i-1} \right) (V_i - V_{i-1}) \]  

(9)

Here the volume is subdivided into \( N \) intervals between \( V_1 \) and \( V^* \) and \( P \) is calculated using REFPROP for an isentropic process. After some experimentation it was found that the integrated work per cycle results for \( N=800 \) were indistinguishable to 6 decimal places from results with \( N=400 \). Pressure as a function of volume was found by starting with density and specific entropy at suction line conditions and maximum volume and following a constant specific entropy process through the compression, discharge, re-expansion, and suction parts of the cycle. Note that \( V^* \) may be different from the \( V_2 \) used in the polytropic cycle.

REFPROP does not provide thermodynamic properties as a function of density and specific entropy. However REFPROP does provide specific entropy, \( s \), as a function of temperature, \( T \), and density, \( \rho \). Therefore an algorithm based on a secant method\(^3\) was used to numerically solve for \( T \) given \( \rho \) and \( s \). Progressively better estimates for \( T_j \) are found by applying the iterative correction \( dT \)

\[ T_{j+1} = T_j + dT \]  

(10)

where

\[ dT = (s - s_j) \left( \frac{(T_j - T_{j-1})}{(s_j - s_{j-1})} \right) \]  

(11)

and

\[ s_j = s(T_j, \rho) \]  

(12)
is provided by REFPROP. This iteration was applied until \( dT \) became less than \( 10^{-9} \). At this point \( P \) is found from REFPROP from \( T \) and \( v \).

The exact integration of the polytropic ideal PV diagram is compared with the numerically integrated ideal PV diagram using REFPROP thermodynamic relations. The goal is to find a polytropic exponent, \( n \), that makes the two equal. The same secant method employed above to solve for \( T \) given \( s \) and \( \rho \) is employed to find a value of \( n \) that makes this true. A starting guess of \( n=1.1 \) was used and the final value was easily found within 4 to 6 iterations.

**RESULTS**

The above procedures were applied to a number of common refrigerants. For each refrigerant, suction pressure, discharge pressure, and return gas temperature are taken from the ARI standard rating condition for the application in which it is primarily used\(^4\). Table 1 summarizes the resulting polytropic exponent, \( n \), for each refrigerant. Though each is done for the specific condition listed, it is proposed that these values are applicable for other points, such as endurance test conditions. Also presented are the isentropic discharge temperature and work per cycle for 1 in\(^3\) compressor displacement. Both of these values were found in the course of doing this analysis using REFPROP thermodynamic relations.

**REFERENCES**


Table 1: Resulting Polytropic Exponents

<table>
<thead>
<tr>
<th>Refrigerant</th>
<th>Suction Pressure (psi)</th>
<th>Discharge Pressure (psi)</th>
<th>Isentropic Disch Temp (°F)</th>
<th>Work/cycle for 1 cu.in disp in-lbf (J)</th>
<th>Polytropic Exponent</th>
</tr>
</thead>
</table>
| Low Temperature Applications: $T_{evap} = -10^\circ F (-23.3^\circ C)$  
$T_{cond} = 120^\circ F (48.9^\circ C)$, $T_{return\ gas} = 40^\circ F (4.4^\circ C)$  
R12 | 19.16 (132.10) | 171.97 (1185.70) | 194.08 (90.04) | 43.16 (4.88) | 1.091104 |
| R134A | 16.63 (114.67) | 185.86 (1281.48) | 186.39 (85.77) | 39.80 (4.50) | 1.070567 |
| ISOBUTANE | 9.04 (62.34) | 96.57 (665.83) | 160.34 (71.30) | 21.07 (2.38) | 1.062463 |
| Medium Temperature Applications: $T_{evap} = 20^\circ F (-6.67^\circ C)$  
$T_{cond} = 120^\circ F (48.9^\circ C)$, $T_{return\ gas} = 40^\circ F (4.4^\circ C)$  
R123 | 3.46 (23.86) | 29.78 (205.30) | 140.01 (60.01) | 7.49 (0.85) | 1.071638 |
| R401A | 35.13 (242.22) | 209.52 (1444.59) | 177.00 (80.56) | 64.73 (7.31) | 1.088923 |
| R401B | 37.79 (260.58) | 220.59 (1520.90) | 179.95 (82.19) | 69.13 (7.81) | 1.094488 |
| R402A | 74.65 (514.67) | 347.42 (2395.40) | 156.32 (69.07) | 113.30 (12.80) | 1.029437 |
| R402B | 68.34 (471.21) | 324.94 (2240.36) | 166.46 (74.70) | 107.91 (12.19) | 1.063055 |
| R404A | 70.31 (484.80) | 326.47 (2250.95) | 149.14 (65.08) | 104.45 (11.80) | 1.004830 |
| R407A | 57.08 (393.55) | 327.73 (2259.62) | 172.64 (78.13) | 100.17 (11.32) | 1.056713 |
| R407B | 65.52 (451.72) | 344.39 (2374.48) | 157.33 (69.63) | 106.10 (11.99) | 1.021157 |
| R407D | 44.20 (304.76) | 268.93 (1854.24) | 171.37 (77.43) | 80.42 (9.09) | 1.062161 |
| R408A | 65.38 (450.78) | 303.70 (2093.95) | 164.63 (73.68) | 101.69 (11.49) | 1.062702 |
| R409A | 33.17 (228.70) | 213.61 (1472.81) | 183.60 (84.22) | 63.90 (7.22) | 1.092439 |
| R409B | 35.92 (247.67) | 223.24 (1539.20) | 184.10 (84.50) | 68.05 (7.69) | 1.094980 |
| R411B | 56.80 (391.62) | 275.38 (1898.70) | 185.19 (85.11) | 94.62 (10.69) | 1.119151 |
| R502 | 66.45 (450.78) | 299.29 (2063.56) | 153.57 (67.54) | 99.24 (11.21) | 1.035137 |
| R507A | 73.30 (505.40) | 333.64 (2300.35) | 147.81 (64.34) | 107.13 (12.10) | 0.999919 |
| BUTANE | 11.57 (79.76) | 69.98 (482.49) | 130.87 (54.93) | 20.88 (2.36) | 1.056592 |
| PROPANE | 55.81 (384.81) | 242.48 (1671.83) | 148.65 (64.81) | 82.20 (9.29) | 1.047834 |
| Air Conditioning: $T_{evap} = 45^\circ F (7.22^\circ C)$  
$T_{cond} = 130^\circ F (54.4^\circ C)$, $T_{return\ gas} = 65^\circ F (18.3^\circ C)$  
R22 | 90.76 (625.78) | 311.58 (2148.28) | 185.20 (85.11) | 116.37 (13.15) | 1.105588 |
| R407C | 85.43 (589.04) | 354.57 (2444.66) | 181.30 (82.94) | 121.93 (13.78) | 1.047152 |
| R407E | 81.68 (563.19) | 343.64 (2369.29) | 183.80 (84.34) | 118.36 (13.37) | 1.055449 |
| R410A | 144.46 (996.03) | 491.20 (3386.73) | 185.03 (85.02) | 180.53 (20.40) | 1.073859 |
| R410B | 143.48 (989.25) | 487.93 (3364.18) | 182.03 (83.35) | 178.10 (20.12) | 1.061954 |
| R417A | 75.46 (520.28) | 295.62 (2038.21) | 157.92 (69.96) | 99.59 (11.25) | 0.994024 |
| R744 | 608.46 (4195.21) | 1650.00 (11376.35) | 208.64 (98.14) | 672.37 (75.97) | 1.289373 |