SIMULATED GPS OBSERVATION OF TRAVELING IONOSPHERIC DISTURBANCE FROM GROUND BASED RECEIVERS

SEE-CHEN GARY LEE

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By SEE-CHEN GARY LEE

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SIMULATED GPS OBSERVATION OF TRAVELING IONOSPHERIC DISTURBANCE FROM GROUND BASED RECEIVERS

For the degree of Doctor of Philosophy

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SIMULATED GPS OBSERVATION OF TRAVELING IONOSPHERIC
DISTURBANCE FROM GROUND BASED RECEIVERS

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Submitted to the Faculty
of
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of
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ABSTRACT

Lee, See-Chen G. PhD, Purdue University, December 2015. Simulated GPS Observation of Traveling Ionospheric Disturbance from Ground based Receivers. Major Professor: James L. Garrison.

Traveling Ionospheric Disturbances (TID) can be induced by acoustic waves in the neutral atmosphere, allowing such process to be observed as changes in the trans-ionospheric GNSS signal. Coherence between measurements from different stations within a large, dense GNSS network has been used to identify and characterize various TIDs. In order to evaluate the sensitivity, accuracy and possible biases of this technique, a simulator has been developed. The magneto-hydrodynamic (MHD) equations were simplified through assumptions about the time scales of various processes, and the interactions between ions and electrons in the ionosphere. A ray-trace method was used propagate the acoustic wave from the surface and the problem was approximated as axially symmetric about the point source location, allowing a two-dimensional cylindrical coordinate system to be used. These model simplifications were necessary to produce a numerically efficient algorithm capable of simulating hundreds of GNSS ray paths, in order to represent the response over a large network. Ground motion observed during the 2011 Tohoku earthquake and tsunami was used to define the input signal. Synthetic waveforms produced from the model were found to agree well with the GNSS observations made in Japan during that event. Arrival time errors showed geographical correlation with the distance from the epicenter, indicating the limitations of the point-source approximation.
1. INTRODUCTION

The Earth’s atmosphere is approximately horizontally stratified (i.e. its properties depend on altitude), therefore the atmospheric structure can usually be represented by the vertical temperature and density profile. The temperature reaches its minima at approximately 80-90 km, which is called mesopause. For altitudes higher than this boundary the temperature increase rapidly due to high energy absorption from solar photons. An ionized plasma layer is created by this absorption of photons and collision of charged particles from the solar wind with the neutral atmospheric particles, this layer is known as the ionosphere. Since the net rate of ionolization depends of temperature and density which are both horizontally stratified (i.e. only depends on local altitude), the ionospheric structure can also be represented by the vertical plasma density profile as shown in Figure 1.1.

The ionosphere plays an important part in the concept of space weather. A variety of physical phenomena are associated with space weather, including geomagnetic storms. These phenomena cause variations of particle distribution in the ionosphere causing ionospheric disturbances and scintillations.

1.1 Traveling Ionospheric disturbance

Traveling Ionospheric disturbance could be caused by various means, such as solar flares [1], geomagnetic storms [2], and atmospheric waves. Three types of atmospheric waves are known to induce TIDs: gravity waves, acoustic waves and shocks. Short period (usually less than about 10 min) disturbances, propagating near the speed of sound have been associated with acoustic waves in the atmosphere. Several known sources are responsible for generating strong enough acoustic waves that could induce TIDs: earthquakes [3] [4], large chemical explosions [5] [6], superbolide meteor [7],
and nuclear weapon tests [8]. Longer period (usually more than about 10 min) disturbances, propagating slower than the speed of sound has also been associated with large explosions events, geomagnetic storm or gravity waves [9] [10] [11]. Shocks induced disturbances have been reported from rocket launches [12] [13] [14], they attenuate very fast and thus are rarely observed.

One single event could produce multiple TID phenomenon. The 2011 Tohoku earthquake serves an example in such observation: this earthquake triggered direct acoustic waves from the epicenter and the Rayleigh surface wave. The earthquake also triggered tsunami that induce gravity wave propagates to ionospheric height [15]. Those atmospheric waves induced different types of TIDs and can be observed and distinguished with wavelet filters [16]. Since natural hazards and weapon testing can
be associated with TIDs, a real time remote sensing system for detecting TIDs could served as a useful tool to monitor those hazards in the future [17].

1.2 GPS measurement of TEC

Dual-frequency GPS receivers are commonly used to measure the integrated electron content (IEC) through a linear combination of the pseudorange and carrier phase from the L1 and L2 frequencies (1575.42 MHz and 1227.6 MHz, respectively). In 1993, Mannucci et al. [18] introduced a method to monitor the IEC using those GPS measurements:

\[
IEC(t) = \frac{1}{40.308} \frac{f_1^2 f_2^2}{f_1^2 - f_2^2} (\rho_1 - \rho_2 + nL)
\] (1.1)

The integrated electron content (IEC), a popular measurement of the ionosphere, represents the integration of electron density along the path of line of sight (LOS) between the satellite and the receiver. \( f_1 \) and \( f_2 \) are the two carrier frequencies (Hz) of GPS, \( \rho_1 \) and \( \rho_2 \) are the GPS observables (see Appendix B for detail) (m), \( nL \) is unknown bias which is usually constant.

In 2003, [19] applied an array processing technique, using a 3 – 10 min band-pass filter and measurements from the Southern California Integrated GPS Network (SCIGN) to search for disturbances following the 16 October 1999 Hector Mines earthquake. The amplitudes of these disturbances are typically small compared with the diurnal variation in the ionosphere or the change of elevation angle of the LOS. Use of a band-pass or wavelet filter, combined with some test for signal coherence, is thus necessary in order to detect these disturbances in the presence of the much larger long-period variations. Variations observed in the IEC could be the result of changes in the concentration of electron anywhere along the LOS. Most researchers usually have approximated the total change in the IEC as a change within a two-dimensional thin layer, located at a fixed altitude of 250 to 400 km (approximately the altitude of the F2 layer) [10] [19]. The measurements of IEC variation were usually assumed
to take place at the pierce point where the LOS intersects this thin ionospheric layer, also known as the ionospheric pierce point (IPP).

1.3 TID velocity estimation

Several different methods of TID detection and its velocity estimation are known \cite{20} \cite{21} \cite{22} \cite{23}. They all assume the TIDs propagate along a horizontal plane at ionospheric height, thus it is actually an estimation of horizontal velocity. The cross-correlation method \cite{20} will be briefly discussed in this section.

By cross-correlating two pairs of IEC time series, viewing from the same satellite but different receivers, we can find the IEC with similar waveforms and the time-delay measurement between them.

A forward model, relating the TID propagation velocity $\vec{V}_p$ to the time-delay measurement, $\Delta t_i$, for the $i^{th}$ station pair, was based upon a simplified geometry. In this model, the disturbance is assumed to be a plane wave, propagating at the ionospheric height, with a constant velocity, over a locally-flat-Earth. It is also assumed that only one disturbance is present within the time range and the area of study. This method does not required quasi-monochromatic assumptions of the TID, but was assumed to be non-dispersive. The displacement of the TID wave during the time $\Delta t_i$ is the projection of the displacement vector between the IPPs $\Delta \vec{x}_i$ corresponding to the two stations. The measurement, $\Delta t_i$, produced from the maximum cross-correlation, can thus be related to the unknown propagation velocity, $\vec{V}_p$, by:

$$\Delta t_i = \frac{\Delta \vec{x}_i \cdot \vec{V}_p}{\|\vec{V}_p\|^2}$$

$\vec{V}_p$ can be solved by linear regression with a system of known $\Delta t_i$ and $\Delta \vec{x}_i$ and change of variables. There is, however, one unsolved problem for velocity estimation. This speed estimation depends on the assumption of ionospheric height almost linearly. Furthermore, this ionospheric height assumption may be invalid in some cases, especially for TID induced by acoustic waves. A more complete, three-dimensional
model of the ionospheric wave propagation would thus be useful for checking the validity of this ionospheric height approximation and increasing the sensitivity and accuracy of GNSS-based methods.

### 1.4 Observation bias

Several possible TID detection biases have being studied when using a ground based receivers:

The Doppler-like effect due to the relative motion between the velocity of the satellite and the TID propagation velocity. That is, the TID frequency will increase if the satellite velocity is in the opposite direction of the TID velocity. It has been shown to substantially change the detectability of disturbances, through shifting the dominant frequencies outside of the filter band [20]. This effect could contribute to a directional bias since the frequency shift depends upon the component of propagation velocity in the direction of the SIP velocity. The resulting frequency change, however, will only shift the wave to a higher or lower frequency, so that it should be detected through a change in the bandwidth of the filter. This effect could only produce a directional bias if the most likely disturbances were within a narrow band of frequencies and thus only the frequency shift resulting from motion aligned with the SIP would put them within the bandwidth of the filter.

[24] identified directional biases inherent in any measurement of perturbations in the ionosphere which are based upon IEC. Equation (9) in that reference [24] describes the response of the IEC to a propagating wave disturbance in electron density, described by a general model that could represent either gravity or acoustic waves.

As a simple summary of [24] there are three types of bias:

a. Electrons can only be perturbed significantly in direction parallel to magnetic field. b. Geometric bias due to the orientation of LOS (electrons perturbed along LOS produce no change in IEC). c. Phase cancellation effect (depletion of electrons
at one point along the line of sight is replaced by accumulation of electrons at another point, tending to reduce the net effect of the perturbation on the IEC.

1.5 objective

As stated from previous sections, there are remaining problems from TID detection and velocity estimation. The main objective of this research is therefore as followed:

1. Simulate an TID observation induced by acoustic wave and reduce the computational cost. In 2009, [25] successfully demonstrated the IEC simulation induced by acoustic wave with a GPS network near the eruption of Soufrie Hills Volcano. In 2013, [26] include an estimation of energy lost for a event during the 2011 Van earthquake in Turkey. In this research we will presents a method, similar to [25] and [26] and but with different assumptions and initial conditions, of acoustic TID simulation. A spherical Earth model is used to construct the ray trace map and atmospheric properties. We introduce different assumptions to simplify the MHD equations, thus making the system linear. And the initial waveform is based on ground motion measurement instead a simple N-wave.

2. Apply a velocity estimation method to the TID model. Base on our model we are capable to estimate the locations where the LOS and ray paths intersect, thus obtain a better approximation of ionospheric height. Using this model we could also the bias of current estimation methods thus reduce errors.

1.5.1 An overview of the model

Although there are many causes of TIDs, this simple model focuses only on propagation of acoustic waves from a point source. The overall flowchart for the model is shown in Figure 1.2. The propagation path of acoustic wave can be computed with ray-trace method described in Chapter 2, thus, we can calculate propagation time and propagation velocity of an acoustic wave as a function of position. Then we can
estimate the time and location at the intersection between the acoustic wave and LOS. The ray-trace map and atmospheric profile together also provide us the vibration amplitude of air particles as a function of position. The waveform of acoustic wave can be represented by a neutral atmospheric particle velocity profile, which can be modeled as a N-wave or the waveform from ground motion data. We could also use a recorded particle velocity as the initial waveform (if such data are available). Once the acoustic wave propagates to the ionosphere, the neutral atmospheric particles and the ions and electrons collide and interact with each other. This behavior can be described by Magnetohydrodynamics (MHD) equations, as described in Chapter 3. With continuity equation the electron/ion density variation can be estimated from its velocity. In the end we integrate the electron density along the LOS to obtain the observed TEC series.

Fig. 1.2. Flow chart of the model. The input (initial condition) of the system is the neutral particle velocity profile. The output of the system is IEC.
2. RAY TRACE

In fluid-dynamics, ray tracing is a method of computing the path of wave propagation through the fluid which may have varying wave velocity and reflecting surfaces. Since the medium is non-uniform, the wavefronts may bend and change directions. Using this we can estimate the wave velocity and the time-relationship between the propagating wave and the LOS.

Fig. 2.1. 3D illustration of LOS and ray trace for a ground receiver. $x_3$ is the axis normal to the surface of Earth at the source. $x_1$ is the radial axis for the cylindrical coordinate. All atmospheric properties and wave properties can be expressed as function of $x_1$ and $x_3$. 
2.1 Assumption of Atmosphere Model

We now make the three following assumptions in order to simplify the model:

1. The acoustic wave source is treated as a point source located at the surface of the Earth, the reflection from the ground is ignored.
2. The atmosphere is locally horizontally stratified, i.e. the atmospheric properties only depend on the local altitude.
3. There is no wind.

This way we can assume the ray path has axial symmetry about the source and only need to compute the two dimensional ray trace instead of three dimensional, as illustrated in Figure 2.1. In other words, we can generate the wave properties (time, velocity, magnitude... etc.) profiles as functions of two dimensional position, and simply project the LOS onto the 2D plane to extract the corresponding data, as described in the Appendix C. There are some disadvantage about those assumptions. The shattering ground motions of underground explosion and earthquake are clearly not point sources at the surface of the Earth [27]. The atmospheric properties changes not only across horizontal range but also time, especially if the wave propagates to more than 1000 km away from the source. And winds are always present, however, it may be reasonable to neglect them for acoustic waves near their source.

2.2 Acoustic Ray Tracing for Flat Earth Model

The acoustic wave propagation is treated as a point source located at the surface of the Earth, the reflection from the ground is ignored. Thus the pressure perturbation propagates semi-spherically from the source if the atmosphere is an uniform medium. Here we use the MSIS-E-90 atmospheric model, and assume that the sound speed only depends on the altitude.
The speed of sound, $C_s$, in idea gas is approximately:

$$C_s = \sqrt{\frac{\gamma T R_m}{M_m}}$$

where $\gamma$ is adiabatic index, $T$ is absolute temperature, $R_m$ is molar gas constant, $M_m$ is molar mass. Temperature is obtained from MSIS-E-90 atmospheric model. $\gamma$ and $M_m$ are nearly constant in lower altitude but not for high altitude. These values can be approximated by [28]:

$$\gamma = 1.4 + 0.135(1 + \tanh (z - 300)/100)$$

$$M_m = 28.9 - 6.45(1 + \tanh (z - 300)/100)$$

where $z$ is local altitude (km).

In general, for any arbitrary sound speed profile the path must satisfy Fermats principle, i.e. the propagation path is such that the travel time between two points is minimized. This argument has to satisfy the Euler-Lagrange equation, in this case, the linear ray-path equation:

$$\frac{d}{ds} \left( \frac{1}{C_s} \frac{dx_1}{ds} \right) = \frac{\partial}{\partial x_1} \left( \frac{1}{C_s} \right)$$

where sound speed $C_s$ only depends on altitude $C_s(x_3)$, and $s$ is the arc length along the ray path which described by horizontal range $x_1 = r$ and altitude $x_3 = z$ in 2D plane. The $C_s(x_3)$ assumption is not exactly true in spherical Earth model because the altitude in flat-Earth and altitude in spherical Earth are close but not the same.

Define $y_1 = r = x_1, y_2 = z = x_3, y_3 = dr/ds, y_4 = dz/ds$

When $i = 1$,

$$\frac{d}{ds} \left( \frac{1}{C_s} \frac{dx_1}{ds} \right) = \frac{\partial}{\partial x_1} \left( \frac{1}{C_s} \right) = 0$$

$$\Rightarrow \frac{d}{ds} \left( \frac{1}{C_s} \right) = \frac{d}{ds} \left( \frac{1}{C_s} \right) \frac{dx_1}{ds} + \frac{1}{C_s} \frac{d^2 x_1}{ds^2} = 0$$
\[ \Rightarrow \frac{d^2 x_1}{ds^2} = -C_s \frac{d}{ds} \left( \frac{1}{C_s} \right) \frac{dx_1}{ds} \]

where \( \frac{d}{ds} \left( \frac{1}{C_s} \right) = -\frac{1}{C_s^2} \frac{dC_s}{dx_3} \frac{dx_3}{ds} \)

thus

\[ \frac{d^2 x_1}{ds^2} = \frac{1}{C_s} \frac{dC_s}{dx_3} \frac{dx_3}{ds} \frac{dx_1}{ds} \]

when \( i = 3 \),

\[
\frac{d}{ds} \left( \frac{1}{C_s} \frac{dx_3}{ds} \right) = \frac{\partial}{\partial x_3} \left( \frac{1}{C_s} \right) = -\frac{1}{C_s^2} \frac{dC_s}{dx_3} \frac{dx_3}{ds}
\]

Therefore the overall differential equation in 1st order form is:

\[
\begin{align*}
\frac{dy_1}{ds} &= \frac{dx_1}{ds} = \frac{dr}{ds} = y_3 \\
\frac{dy_2}{ds} &= \frac{dx_3}{ds} = \frac{dz}{ds} = y_4 \\
\frac{dy_3}{ds} &= \frac{d^2 x_1}{ds^2} = \frac{1}{C_s} \frac{dC_s}{dx_3} \frac{dx_3}{ds} y_4 y_3 \\
\frac{dy_4}{ds} &= \frac{d^2 x_3}{ds^2} = -\frac{1}{C_s} \frac{dC_s}{dx_3} \frac{dx_3}{ds} (1 - y_4^2)
\end{align*}
\]

The sound speed term \( \frac{1}{C_s} \frac{dC_s}{dx_3} \) is interpolation of its altitude profile, therefore the altitude need to add an extra term while doing interpolation in Spherical Earth model.

### 2.3 Acoustic Ray Tracing for Spherical Earth Model

The two dimensional (radial range and altitude measures from the source) ray trace profile is assumed in flat-Earth-model. However the horizontal range can be up to 1000 km long - roughly 77 km of altitude error on spherical Earth, which is quite significant (max electron density is about at 300-400 km of altitude, so 77 km of error cannot be neglected). Note that 1000 km of horizontal range is most likely the worst scenario, most satellites detected the disturbance at 300-500 km from the source, the altitude error is about 7-20 km which is not that bad. Figure 2.2 shows the altitude error from flat-Earth-model near the equator.
The altitude difference between flat and spheric Earth models is:

\[ error_{flat} = \sqrt{r^2 + (E_{lr} + z)^2} - E_{lr} - z \]

where \( r \) is the range on flat-Earth, \( z \) is the altitude on flat-Earth, \( E_{lr} \) is the locally Earth radius depends on latitude.

### 2.3.1 Polar Coordinate

Re-write the linear ray equation in polar coordinate (where the origin is the center of the Earth):

\[
\frac{d}{ds}\left(\frac{1}{C_s} \frac{dx_1}{ds}\right) = \frac{\partial}{\partial x_1} \left(\frac{1}{C_s}\right) \implies \nabla n = \frac{d}{ds} \left(n \frac{d\vec{x}}{ds}\right)
\]
Fig. 2.3. Define $x_1$ and $x_3$ coordinate in spherical Earth.

$$\nabla n = \frac{\partial n}{\partial R} \hat{e}_R + \frac{1}{R} \frac{\partial n}{\partial \theta} \hat{e}_\theta = \frac{d}{ds}(n(\frac{dR}{ds} \hat{e}_R + R\frac{d\theta}{ds} \hat{e}_\theta))$$

where $n = 1/C_s$, $R = \sqrt{(E_{lr} + z)^2 + r^2}$ and $\theta = \arctan\left(\frac{r}{E_{lr} + z}\right)$

Note that the this polar coordinate is a little bit different from common definition
where $\theta$ is the opposite direction of usual polar angle.

Using chain rule we get

$$\frac{dn}{ds} = \frac{\partial n}{\partial R} \frac{dR}{ds} + \frac{\partial n}{\partial \theta} \frac{d\theta}{ds} = \frac{\partial n}{\partial R} \frac{dR}{ds}$$

because

$$\frac{\partial n}{\partial \theta} = 0$$

For $\hat{e}_R$:

$$\frac{\partial n}{\partial R} = \frac{d}{ds}(n \frac{dR}{ds}) = \frac{dn}{ds} \frac{dR}{ds} + n \frac{d^2 R}{ds^2} = \frac{\partial n}{\partial R} \frac{dR}{ds} + n \frac{d^2 R}{ds^2}$$

$$\Rightarrow \frac{\partial n}{\partial R} - \frac{\partial n}{\partial R} \left(\frac{dR}{ds}\right)^2 = n \frac{d^2 R}{ds^2}$$

For $\hat{e}_\theta$:

$$\frac{1}{R} \frac{\partial n}{\partial \theta} = R \frac{d}{ds}(n \frac{d\theta}{ds}) + n \frac{\partial \theta}{\partial s} \frac{\partial R}{\partial s} = R \frac{dn}{ds} \frac{d\theta}{ds} + R n \frac{d^2 \theta}{ds^2} + n \frac{\partial \theta}{\partial s} \frac{\partial R}{\partial s}$$
\[ \implies R \frac{\partial n}{\partial R} \frac{dR}{ds} ds + Rn \frac{d^2 \theta}{ds^2} + n \frac{\partial \theta}{\partial R} \frac{dR}{ds} \frac{ds}{ds} = 0 \]

\[ \implies \frac{d^2 \theta}{ds^2} = -\frac{1}{n} \frac{\partial n}{\partial R} \frac{dR}{ds} \frac{ds}{ds} - \frac{1}{R} \frac{\partial \theta}{\partial s} \frac{dR}{ds} \frac{ds}{ds} \]

Define \( y_1 = R, y_2 = \theta, y_3 = dR/ds, y_4 = d\theta/ds \) Then the overall differential equations in 1st order form become:

\[
\begin{align*}
\frac{dy_1}{ds} &= \frac{dR}{ds} = y_3 \\
\frac{dy_2}{ds} &= \frac{d\theta}{ds} = y_4 \\
\frac{dy_3}{ds} &= \frac{d^2 R}{ds^2} = \frac{1}{n} \frac{\partial n}{\partial R} \frac{dR}{ds} \frac{ds}{ds} (1 - y_5^2) \\
\frac{dy_4}{ds} &= \frac{d^2 \theta}{ds^2} = -\left( \frac{1}{n} \frac{\partial n}{\partial R} + \frac{1}{y_1} \right) y_3 y_4
\end{align*}
\]

We also need to consider the initial conditions, which are the location of the epicenter and the launch angle of the ray, \( \alpha \). To make this problem simple the epicenter is set at \( R(0) = E_{lr}, \theta(0) = 0 \). In Cartesian coordinate the launch angle is \( dx_1/ds(0) = \sin \alpha \) and \( dx_3/ds(0) = \cos \alpha \), so that the ray is vertical if \( \alpha = 0 \) and horizontal if \( \alpha = \pi/2 \).

The I.C. becomes complicate in polar coordinate:

\[ R = \sqrt{(E_{lr} + x_3)^2 + x_1^2} \]

\[ \frac{dR}{ds} = \frac{\partial R}{\partial x_1} \frac{dx_1}{ds} + \frac{\partial R}{\partial x_3} \frac{dx_3}{ds} \]

\[ \implies \frac{\partial R}{\partial x_1} = \frac{x_1}{\sqrt{(E_{lr} + x_3)^2 + x_1^2}} \]

and

\[ \frac{\partial R}{\partial x_3} = \frac{E_{lr} + x_3}{\sqrt{(E_{lr} + x_3)^2 + x_1^2}} \]

at \( s = 0, x_1 = x_3 = 0 \), this becomes \( \frac{dR}{ds}(0) = \cos \alpha \)

\[ \theta = \arctan\left( \frac{x_1}{E_{lr} + x_3} \right) = \arctan(u(x_1, x_3)) \]
where \( u(0,0) = 0 \)

chain rule:

\[
\frac{d\theta}{ds} = \frac{\partial \theta}{\partial x_1} \frac{dx_1}{ds} + \frac{\partial \theta}{\partial x_3} \frac{dx_3}{ds}
\]

\[\Rightarrow \frac{\partial \theta}{\partial x_1} = \frac{1}{E_{br} + x_3} \frac{1}{1 + u^2}\]

and

\[
\frac{\partial \theta}{\partial x_3} = -x_1 \frac{1}{(E_{br} + x_3)^2} \frac{1}{1 + u^2}
\]

at \( s = 0 \), \( x_1 = x_3 = 0 \), this becomes \( \frac{d\theta}{ds}(0) = \frac{1}{E_{br}} \sin \alpha \)

At the end, we transfer variables \((R, \theta)\) to \((x_1, x_3)\) in order to compare the results:

\(x_1 = R \sin \theta\)

\(x_3 = R \cos \theta - E_{br}\)

Unfortunately, this polar form does not generate reasonably result (except for the trivial solution), one of the possible reasons is that the numerical error is too sensitive.

We are dealing with large number like the Earth’s radius and small number as \(\theta\) which has the initial condition \( \frac{d\theta}{ds}(0) = \frac{1}{E_{br}} \sin \alpha \). Therefore the Cartesian form of Spherical model is introduced in the next section to reduce the numerical error.

### 2.3.2 Cartesian Coordinate

Consider the linear ray equation:

\[
\nabla n = \frac{d}{ds} \left( n \left( \frac{d\vec{x}}{ds} \right) \right)
\]

\[\Rightarrow \frac{\partial n}{\partial x_i} = \frac{dn}{ds} \frac{dx_i}{ds} + n \frac{d^2 x_i}{ds^2}\]

where \( n = 1/C_s \) is the inverse of sound speed

Applying chain rule for \( \frac{\partial n}{\partial x_1} \neq 0 \):

\[
\frac{dn}{ds} = \frac{\partial n}{\partial x_1} \frac{dx_1}{ds} + \frac{\partial n}{\partial x_3} \frac{dx_3}{ds}
\]

\[
\frac{\partial n}{\partial x_1} = \frac{\partial n}{\partial R} \frac{\partial R}{\partial x_1} + \frac{\partial n}{\partial \theta} \frac{\partial \theta}{\partial x_1} = \frac{\partial n}{\partial R} \frac{\partial R}{\partial x_1}
\]
\[
\frac{\partial n}{\partial x_3} = \frac{\partial n}{\partial R} \frac{\partial R}{\partial x_3} + \frac{\partial n}{\partial \theta} \frac{\partial \theta}{\partial x_3} = \frac{\partial n}{\partial R} \frac{\partial R}{\partial x_3}
\]

From previous section we know that:

\[
\frac{\partial R}{\partial x_1} = \frac{x_1}{\sqrt{(E_{lr} + x_3)^2 + x_1^2}} = \sin(\theta)
\]

\[
\frac{\partial R}{\partial x_3} = \frac{E_{lr} + x_3}{\sqrt{(E_{lr} + x_3)^2 + x_1^2}} = \cos(\theta)
\]

This implies:

\[
\frac{dn}{ds} = \frac{\partial n}{\partial R} \sin \theta \frac{dx_1}{ds} + \frac{\partial n}{\partial R} \cos \theta \frac{dx_3}{ds}
\]

Therefore:

\[
\frac{\partial n}{\partial R} \sin \theta = \left( \frac{\partial n}{\partial R} \sin \theta \frac{dx_1}{ds} + \frac{\partial n}{\partial R} \cos \theta \frac{dx_3}{ds} \right) \frac{dx_1}{ds} + n \frac{d^2 x_1}{ds^2}
\]

\[
\frac{\partial n}{\partial R} \cos \theta = \left( \frac{\partial n}{\partial R} \sin \theta \frac{dx_1}{ds} + \frac{\partial n}{\partial R} \cos \theta \frac{dx_3}{ds} \right) \frac{dx_3}{ds} + n \frac{d^2 x_3}{ds^2}
\]

Let \( y_1 = x_1, y_2 = x_3, y_3 = \frac{dx_1}{ds}, y_4 = \frac{dx_3}{ds}, y_5 = t \)

The 1st order ODE becomes:

\[
\frac{dy_1}{ds} = \frac{dx_1}{ds} = y_3
\]

\[
\frac{dy_2}{ds} = \frac{dx_3}{ds} = y_4
\]

\[
\frac{dy_3}{ds} = \frac{d^2 x_1}{ds^2} = \frac{1}{n} \frac{\partial n}{\partial R} (\sin \theta - \sin \theta y_3^2 - \cos \theta y_4 y_3)
\]

\[
\frac{dy_4}{ds} = \frac{d^2 x_3}{ds^2} = \frac{1}{n} \frac{\partial n}{\partial R} (\cos \theta - \sin \theta y_3 y_4 - \cos \theta y_4^2)
\]

\[
\frac{dy_5}{ds} = \frac{dt}{ds} = n
\]

and \( \theta = \arctan(\frac{x_1}{E_{lr} + x_3}) \)

This set of equations is computed by Matlab ODE45 solver.

Figure 2.4 shows an example of spherical Earth acoustic ray-trace using the EOM.

The ray trace map sometimes generates refraction caustics points which could become a problem, an example is shown in Figure 2.5. We can usually ignore caustics because they appear mostly below the ionosphere, or they never reach the LOS. But we still need to be careful to check each simulation cases to make sure the refraction
caustics point does not distort the result. We now have the arrival time, velocity and ray Jacobian of the acoustic wave as function of $x_1$ and $x_3$. The next step is to find the waveform of the neutral particle velocity and the magnitude change along the ray path.
Fig. 2.5. An example of local refraction caustics points appear on the ray-trace map. The caustics points are located where the rays cross with each other.
3. WAVE PROPAGATION AND THE DISTURBED ELECTRON DENSITY

In this chapter we study the wave propagation along each each ray traces, and the resultant electron density variation along the LOS. For acoustic wave, we assume an initial neutral particle velocity profile (usually a N-wave-shape) near the source, and estimate the neutral particle velocity profile near the intersection of the ray and the LOS. We then use magnetohydrodynamics (MHD) equation to compute the electron velocity near the intersection. This MHD equation can be further simplified by adding several more optional assumptions to reduce numerical error and computational cost, as described in each sections of this chapter.

3.1 Initial condition from ground motion

The system input for the entire simulation model is the initial neutral particle velocity profile near the source. Since there is no direct measurement of such profile, we have to model it with some unknown parameters. These unknown parameters can be estimated (regression or iteration) by comparing the synthetic IEC results and the IEC from GPS observations. It is a common strategy to model the neutral particle velocity of a large amplitude acoustic wave as a N-wave [25], which is the first derivative of a Gaussian function:

\[
|\vec{U}(t)| = A_0 \sqrt{2} \frac{t - t_0}{\pi^{1/4} \sigma^{3/2}} e^{-\frac{(t-t_0)^2}{\sigma^2}}
\]

where \( \sigma \) describes the width (standard deviation) of the N-wave, and \( A_0 \) is the initial magnitude. Both \( \sigma \) and \( A_0 \) are unknown, they can be iterated by comparing the simulated and observed IEC at the final stage of the model. With the atmospheric
dispersion effect, the waveform magnitude of the neutral particle velocity will keep behave as a N-wave except that its width $\sigma$ will grow with time [25] [30]:

$$\sigma = \sqrt{a_d + b_d t^2}$$

Parameter $a_d$ is the initial width of the source function at $t = 0$, while $b_d$ parametrizes the broadening of the pulse as the acoustic wave propagates. If we want to include dispersion in the model, both parameters $a_d$ and $b_d$ depend on initial conditions (spatial and time) and local atmospheric properties, which are usually unknown. Thus there are 3 unknown parameters in the final stage of iteration, and it is very computational expensive. Note that for close distance propagation the dispersion effect may be small (i.e. $b_d = 0$). In this case we only need to estimate $a_d$ and $A_0$, or we could use a more complex waveform to replace the N-wave approximation.

The alternative method to simulate the initial waveform profile is to study the ground motion. The 2011 earthquake off the Pacific coast of Tohoku provides a rare opportunity to estimate the waveform. This earthquake was a magnitude 9.0 (Mw) undersea earthquake off the coast of Japan that occurred at 05:46 UTC on Friday, 11 March 2011, with the epicenter approximately at 38.322 and 142.369 degree of latitude and longitude [27]. Since there are thousands of GPS ground receivers in Japan and the epicenter is not far from them, we have an opportunity to recover the initial ground motion near the epicenter. Figure 3.1 shows the locations of GPS receivers (blue and green) in Japan and the epicenter (red). Figure 3.2 shows the vertical ground motion time series estimated at each individual GPS receiver with 30-sec time interval, they are sorted by the distances between the receivers and epicenter. We normalized and cross-correlated all waveforms to find the ground motions those are consistence to each other (the green receiver shown in Figure 3.1). Then we stacked those waveforms to generate our initial waveform profile at the epicenter, as shown in Figure 3.3. The stacking method was detailed in Garrison et al. [20].
Fig. 3.1. GPS sites locations at Japan, the green dots are the sites close to the epicenter (red star) with available data. Some sites near the epicenter are not used because of damaged data due to the earthquake.

However, there are several issues that might not be consistence to our assumptions, as shown from the ground motion data. The seismic wave in this event propagated in non-uniform medium (from undersea to lands and mountains), and the epicenter is not a point but only an approximation of the fault region:

1. From Figure 3.2 we can see that the waves do not necessary arrive earlier to the receiver closer to the epicenter (most waves do). This indicates that this Rayleigh surface wave did not propagate radially from the source. Note: a detailed sea-floor displacement study can be found from Simons et al. [27], which indicates the earthquake is not simply a point source.

2. For a surface Rayleigh wave the amplitude should decay as $1/\sqrt{r}$, where $r$ is the radius from the source. However in this case the magnitude of ground motions
Fig. 3.2. Vertical ground motion time series estimated at each individual GPS receiver with 30-sec time interval, they are sorted by the distance between the receiver and epicenter in y-axis. The magnitudes are normalized by each individual’s square root of signal energy (i.e. root mean square) during the first 400 sec.

Data shows very weak correlation to the factor of $1/\sqrt{r}$. This also indicates that we cannot track back the initial magnitude near the source. Thus the initial neutral particle velocity profile is the average of the versicle ground velocity scaled with an unknown parameter $A_0$. This parameter can be iterated by comparing the simulated and observed IEC data. If we carefully simplify our model to linear system, $A_0$ can be estimated by linear regression, thus there is no need of iteration and save a lot of computational time. Note that in practice the initial pulse of neutral velocity profile is assumed to be somewhere very close (1 km) to the epicenter, but not exactly on top of it. Because in the model the magnitude of the initial pulse will be approach to infinity at $r = 0$. 
Fig. 3.3. The stacked waveforms of ground motion after correlation. Green lines are the individual normalized ground motion time series with corrected time shift. Red line is the average of the greens.

3.2 Amplitude change along the propagation

The amplitude change of this neutral particle velocity profile along the ray path, can be computed from the transport equation, assuming energy is conserved [31]:

$$\frac{A(s_2)}{A(s_1)} = \sqrt{\frac{\rho(s_1)C_s(s_1)J(s_1)}{\rho(s_2)C_s(s_2)J(s_2)}}$$

This equation describes the amplitude ratio between any two points $s_1$ and $s_2$ along the ray path, where $\rho$ is density, $C_s$ is sound speed, $J$ is the ray Jacobian. The ratio of ray Jacobian is approximated by ratio of cross-sectional area of a ray tube at $s_1$ and $s_2$. Note that the amplitude ratio only depends on $x_1$ and $x_3$ for a fixed atmospheric properties profile. As described from previous section, the initial pulse
of neutral velocity profile is assumed to be someplace very close \((s_0 = 1 \text{ km})\) to the epicenter. Although the amplitude of neutral velocity profile is decreased by spherical spreading, the very thin atmosphere density in high altitude amplifies the amplitude. Again, conservation of energy is assumed thus the viscosity and ray reflections are ignored in the transport equation. This assumption becomes unrealistic after the wave travels more than 90 km [32], thus we also have to include an estimation of energy lost in the next section.

3.3 Atmospheric absorption of acoustic wave

The absorption coefficient of sound (i.e. intensity attenuation due to energy lost) \(\alpha\) in still air can be estimated from [33] and [34].

The amplitude ratio between two points \(s_1\) and \(s_2\) along a ray path is:

\[
\frac{A(s_2)}{A(s_1)} = 10^{-\alpha s/20}
\]

where \(s\) is the path length between \(s_1\) and \(s_2\).

Figure 3.4 shows a general relationship between the absorption coefficient and altitude for different frequencies. This equation will need modification for high altitude atmosphere but still able to provide a rough estimation of energy lost. The modification for high altitude atmosphere is provided by [34], although this modification only valid up to 160 km of altitude it can still provide a rough estimation of \(\alpha\) for higher altitude.

Figure 3.5 shows the amplitude change of acoustic wave with transport equation and energy lost. This amplitude attenuated to insignificant amount for wave propagate higher than 350 km altitude, thus the high altitude error of absorption of sound can generally be neglected. In our study we only deal with the acoustic wave with low frequency since the sampling time interval is 30-sec. The frequency we used to generate energy absorption is about 1/120 Hz, higher frequency components of the acoustic wave will attenuate faster and become insignificant before reaching the ionosphere. This energy lost estimation agrees with the results generated by Mai [32],
Fig. 3.4. Sound absorption coefficient vs. altitude for different frequency

who computed the energy lost based on model of viscosity and heat transfer. Now we have the magnitude change of neutral particle velocity profile as function of $x_1$ and $x_3$.

### 3.4 Coupling between neutral and charged particles for acoustic wave

This step we are going to estimate the velocity profile of the charged particles when they collide with the neutral particles, described by magnetohydrodynamics (MHD) equations. Note that we only need to process the data point along the LOS instead of the entire region on $x_1 - x_3$ plane. The complete MHD equations usually include the continuity equation, the ideal gas law, heat transfer and the momentum equations. These equations describe the relationship between the charged particles and neutral
Fig. 3.5. An example for acoustic amplitude change due to atmospheric properties change, spreading and energy lost (assume wave central frequency at 7.5e-3 Hz). The amplitude attenuated almost completely for more than 450 km above the source. The small high peak at lower altitude is a zone of refraction caustics points. Note that this is the maximum possible amplitude change, if the neutral particles approach speed of sound, the energy lost will be greater than what estimated here.

particles in the ionosphere. However, in our study we neglect the heat transfer effect in our model to simplify the problem. The continuity equation is simply:

\[
\frac{d\rho_j}{dt} + \nabla \cdot (\rho_j \vec{V}_j) = \Delta M_j
\]

where \(\rho\) is particle density, \(\vec{V}\) is particle velocity, \(\Delta M\) is net rate of ion/electron production. For most cases of acoustic-wave-generated TID, the ion/electron production rate is relatively small [32].
The momentum equation for magnetohydrodynamic (MHD) between particle $j$ and $k$ is [35] (ignore viscosity):

$$
\rho_j \frac{D\vec{V}_j}{Dt} = -\nabla p_j + \rho_j \vec{g} + N_j e (\vec{V}_j \times \vec{B} + \vec{E}) - \rho_j \nu_{jk}(\vec{V}_j - \vec{V}_k)
$$

where $\vec{V}_j$ is the particle velocity as an output function of time, (m/s)
$
\vec{U}$ is the neutral air velocity as an input function of time, (m/s)
$
\vec{B}$ is local magnetic field (T) (depends on altitude),
$
\nu_{jk}$ is the local particle collision rate (1/s) (depends on altitude),
$
\rho_j$ is local particle mass density (kg/m$^3$) (depends on altitude),
$
N_j$ is local particle number density (1/m$^3$) (depends on altitude),
$
\vec{g}$ is local gravitational acceleration (m/s$^2$) (depends on altitude),
$
\nabla p_j$ is the local partial pressure gradient (assume only have z component)
$
e$ is the electron charge (C)

According to Hooke [36] the electric field term, $E$, can be neglected when the frequency is small ($\ll 1$ Hz), and our sampling rate is limited at 30-sec. Consider a particular ion (subscript $i$) and neutral particle (subscript $n$):

$$
\rho_i \frac{D\vec{V}_i}{Dt} = -\nabla p + \rho_i \vec{g} + N_i e (\vec{V}_i \times \vec{B}) - \rho_i \nu_m (\vec{V}_i - \vec{U})
$$

Rearrange this MHD equation we got:

$$
\frac{DV_{ix}}{Dt} = \frac{N_i}{\rho} (V_{i,y}B_z - V_{i,z}B_y) - \nu_m (V_{i,x} - U_x)
$$
$$
\frac{DV_{iy}}{Dt} = \frac{N_i}{\rho} (V_{i,z}B_x - V_{i,x}B_z) - \nu_m (V_{i,y} - U_y)
$$
$$
\frac{DV_{iz}}{Dt} = \frac{N_i}{\rho} (V_{i,x}B_y - V_{i,y}B_x) - \nu_m (V_{i,z} - U_z) - \frac{1}{\rho} \frac{\partial p}{\partial z} - g
$$

In matrix form:
\[
\begin{bmatrix}
\frac{DV_x}{Dt} \\
\frac{DV_y}{Dt} \\
\frac{DV_z}{Dt}
\end{bmatrix}
= \begin{bmatrix}
-\nu_m & \frac{N_i}{\rho} B_z & -\frac{N_i}{\rho} B_y \\
-\frac{N_i}{\rho} B_z & -\nu_m & \frac{N_i}{\rho} B_x \\
\frac{N_i}{\rho} B_y & -\frac{N_i}{\rho} B_x & -\nu_m
\end{bmatrix}
\begin{bmatrix}
V_x \\
V_y \\
V_z
\end{bmatrix}
+ \begin{bmatrix}
U_x \\
U_y \\
U_z
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
0 \\
K_{MHD}
\end{bmatrix}
\]

where \( K_{MHD} = -\left( \frac{1}{\rho} \frac{\partial p}{\partial z} + g \right) \)

In standard states-space form:

\[
\frac{D\vec{V}}{Dt} = [A_{MHD}]\vec{V} + \nu_m \vec{U} + [0 \ 0 \ K_{MHD}]^T
\]

This is very close to linear system except for the \( K_{MHD} \) term. Note that the 3-by-3 matrix, \( A_{MHD} \), is not symmetric.

In principle, the motions of changed particles spiral along the magnetic field lines. This frequency of the gyro-motion perpendicular to the magnetic field line is usually in the order of \( 1e6 \) Hz. Therefore the minimum time step for solving this system has to be less then \( 1e-6 \) sec. However the full duration of the propagation N-wave (within \( 4\sigma \)) is usually in the order of \( 1e3 \) sec. The number of data points (in the order of \( 1e9 \)) will be too large to be solved in a reasonable amount of time. Considering that the gyro-frequency is much higher than the sampling frequency of the GPS receivers (in the order of \( 1 \) Hz), we can assume that the gyro-motion will appear as a random noise in GPS receiver, with some small bias. Thus we can ignore the gyro-motion and only solve for the component parallel to the magnetic field lines.

The equation can be rewritten as:

\[
\frac{D\vec{V}_i}{Dt} = \frac{-1}{\rho_i} \nabla p + \vec{g} + \Omega_i(\vec{V}_i \times \vec{B}) - \nu_m(\vec{V}_i - \vec{U})
\]

where \( \Omega_i \) is the gyrofrequency, usually in the order of \( ~1e6 \) Hz.

Now taking dot product with the magnetic field, \( \vec{B} \) :

\[
\frac{D\vec{V}_i}{Dt} = \frac{-1}{\rho_i} \nabla p + \vec{g} + \Omega_i(\vec{V}_i \times \vec{B}) - \nu_m(\vec{V}_i - \vec{U})
\]

\[
\Rightarrow \frac{D\vec{V}_{i,B}}{Dt} = (\vec{g} - \frac{1}{\rho_i} \nabla p) \cdot \vec{B} - \nu_m(\vec{V}_{i,B} - \vec{U}_B)
\]
Let \( f_i(t) = (\vec{g} - \frac{1}{\rho_i} \nabla p) \cdot \hat{B} + \nu_{in} U_B \), then:

\[
\frac{DV_{i,B}}{Dt} + \nu_{in} V_{i,B} = f_i(t)
\]

Note that a useful approximation for ion-neutral collision frequency is [35]:

\[
\nu_{in} = (2.6 e^{-9}) \left( \frac{N_n + N_i}{\sqrt{A}} \right)
\]

where \( N_n \) and \( N_i \) are number of neutral particles and ions per volume, \( A \) denotes the mean neutral molecular mass in atomic mass unit.

### 3.4.1 Solve the MHD assuming \( \dot{V}_i \) is small

If we assume \( \dot{V}_i \) is small then the momentum equation can be reduced to (\( \hat{B} \) component only):

\[
\left( \frac{\partial p_i}{\partial z} \right) - \rho_i g = \rho_i \nu_{in} (V_i - U) \quad (3.1)
\]

This equation is used by [37] to simulate acoustic wave model.

If we further assume system is initially at hydrostatic equilibrium, that is, \( \left( \frac{\partial p_i}{\partial z} \right)_{eq} = \rho_i g \), then the equation is reduced to:

\[
V_i = U \quad (3.2)
\]

This means ions and neutral particles are at the same speed in \( \hat{B} \) direction. This also implies the electron speed \( V_e \) is also the same as \( U \) in \( \hat{B} \) direction, due to conservation of charges. Several researchers have used this simple equation to simulate their model [24] [2] [21], because this simplified set of MHD equations only model electrons and ignore the various ions. Note: there are four types of ions are considered in the calculation: O+, H+, O2+ and NO+.
The IEC perturbation is simply the integration of electron density perturbation \( \delta N_e \) along the LOS:

\[
\Delta TEC = \int_{\text{LOS}} \delta N_e \, ds \tag{3.3}
\]

The electron density perturbation \( \delta N_e \) can be computed from the sum of each individual type of ion density perturbations \( \delta N_i \) (conservation of charges):

\[
\delta N_e = \sum \delta N_i \tag{3.4}
\]

The density perturbation for a specific ion, \( \delta N_i \), can be found from conservation of mass, once we have the ion velocity \( \vec{V}_i \):

\[
\frac{\partial N_i}{\partial t} + \nabla \cdot (N_i \vec{V}_i) = 0 \tag{3.5}
\]

Assuming the perturbation of background ion density \( N_i \) is small, and \( N_i \) only depends on altitude \( z \), the continuity equation becomes [25]:

\[
\delta N_i(t) = - \frac{\partial N_i}{\partial z} \int_{t_0}^{t} V_{i,z}(t) \, dt - N_i \int_{t_0}^{t} \nabla \cdot \vec{V}_i(t) \, dt \tag{3.6}
\]

If equation 3.2 is used, the change of ion density is:

\[
\delta N_i(t) = - \frac{\partial N_i}{\partial z} \int_{t_0}^{t} U_{i}(t) \, dt - N_i \int_{t_0}^{t} \nabla \cdot \vec{U}(t) \, dt \tag{3.7}
\]

If equation 3.1 is used, the change of ion density has one extra term:

\[
\delta N_i(t) = - \frac{\partial N_i}{\partial z} \int_{t_0}^{t} U_{i}(t) \, dt - N_i \int_{t_0}^{t} \nabla \cdot \vec{U}(t) \, dt - [t - t_0] \frac{\partial \Omega_i}{\partial z} \tag{3.8}
\]

This extra term describes the change of ion density if no acoustic wave present. Note that \( \Omega_i \) in the extra term only depends on altitude:

\[
\Omega_i = \frac{N_i}{\nu_{in}} \left( -\frac{1}{\rho_i} \frac{dp_i}{dz} - |g| \right) (\dot{z} \cdot \vec{B}) \tag{3.9}
\]
This means the ion density difference between using equation 3.1 and 3.2 does not depend on the neutral particle velocity $\vec{U}$.

The IEC difference $\Delta IEC_D$ is integration of this extra term along the LOS for all ions:

$$\Delta IEC_D(t) = -[t - t_0] \sum_i \int_{LOS} \frac{\partial \Omega_i}{\partial z} ds$$

(3.10)

$\Delta IEC_D$ describes the change of IEC without the acoustic wave presents. If the LOS moves slowly and has very high elevation angle, the integral part in equation 3.10 will be nearly constant. Therefore $\Delta IEC_D(t)$ is simply a linear time function. In this case there will be no difference between equation 3.1 and 3.2 after detrending the data. If the LOS has low elevation angle, equation 3.1 and 3.2 could make some big difference.

Natural neighbor method is used for data interpolation, and Savitzky-Golay filter [38] is applied before integration to reduce numerical error.

### 3.5.1 Compute the divergence of ion velocity

We define the spatial grid points along the LOS, then estimate the corresponding neutral velocity time series $U(t)$. This estimation is done by interpolating values of propagation direction, propagation time and amplitude ratio from the known scattered data points of ray traces in the $x_1 - x_3$ plane, as described in previous sections.

Delaunay triangulation method is used to interpolate those scattered data. Once we know $U(t)$ we can compute $V_i(t)$ for every grid points on the LOS. Then divergence of a specific grid point can be estimated from the neighborhood grid points.

The divergence of ion velocity $\nabla \cdot \vec{V}_i$ at each grid point is critical in the numerical integration in the continuity equation. A rough estimation of $\nabla \cdot \vec{V}_i$ at a grid point can be done by generating $\vec{V}_i$ at three orthogonal direction ($x_1,x_2,x_3$) then computing the difference:
\[ \nabla \cdot \vec{V}_i = \frac{\Delta V_{i,1}}{\Delta x_1} + \frac{\Delta V_{i,2}}{\Delta x_2} + \frac{\Delta V_{i,3}}{\Delta x_3} \]

In order to reduce the numerical error we use \( \vec{V}_i \) values at six directions \( (x_1, x_2, x_3 \) and their opposite directions) instead of just three.

Since \( \nabla \cdot \vec{V}_i \) is a scalar, we can compute its value in any coordinate system as long as the system is orthogonal. In other word, we can define a new coordinate system using the direction of LOS as one of the orthogonal axises, transform \( \vec{V}_i \) into the new coordinate LOS frame, then compute the divergence. The advantage is that the \( \vec{V}_i \) values at other grid points could serve as neighborhood points, only 4 additional neighborhood points are needed for each grid point instead of 6 (as shown in Figure 3.6), and thus reduce the computational time.

Fig. 3.6. Evently spaced grid points (light grey) and its neighborhood points (dark grey) long the LOS, each grid point also serve as a neighborhood point for other grid points. The neighborhood points are in the direction orthogonal to the LOS, and are used to compute the divergence for the corresponding grid point. Compute the electron/ion velocity and the divergence at each grid point will give us the \( \delta N \) distribution along the LOS.
3.6 Estimate the input amplitude

The entire simulation process is a linear system, i.e, the output $IEC_{sim}(t)$ is linearly dependent of the input (initial particle velocity profile $U_0(t)$ near the epicenter), regardless of using equation 3.1 or 3.2.

$$A_0 U_0(t) \leftrightarrow A_0 IEC_{sim}(t) \quad (3.11)$$

Unfortunately this system is time variant due to integration along time-varying LOS, thus cannot modeled as an LTI system. With the linear property we can easily estimate the amplitude parameter without iteration. The estimation of this amplitude factor $A_i$ for the $i$th LOS is done by minimizing the error between the simulated and observed IEC time series in a least squares sense:

$$Error_i = \sum_{n=1}^{m} [IEC_{obs}(t_n) - A_i IEC_{sim}(t_n)]^2 \quad (3.12)$$

However there is an extra parameter we need to consider: the arrival time difference between the simulated and observed IEC. In other word, we need to align them first before we can compare the difference:

$$Error_i = \sum_{n=1}^{m} [IEC_{obs}(t_n) - A_i IEC_{sim}(t_n - t_{e,i})]^2 \quad (3.13)$$

This error term $t_{e,i}$ mainly arise from the breakdown in our assumption of axial-symmetry, which becomes more significant when the epicenter is not a point source. $t_{e,i}$ can be estimated without iteration by using cross-correlation methods described by [20].

Then the estimated $\hat{A}_i$ is computed from simple linear regression:

$$\hat{A}_i = \frac{IEC_{obs}(t_n) IEC_{sim}(t_n - t_{e,i})}{IEC_{sim}^2(t_n - t_{e,i})} \quad (3.14)$$

where the over bar represents mean value over the time series. The estimated amplitude factor $\hat{A}_0$ for every LOS is simply the average of $\hat{A}_i$, if all LOS weighted equally.
4. ESTIMATE THE CIRCULAR PROPAGATION SPEED

An estimation method of TID propagation speed is present in this chapter. Instead of a simple plane wave, we assume the TID propagates at ionospheric height as a circular wave from the center. If we have a circular TID detection, without knowing the source and velocity, the only information we have are the IEC time series and the LOS geometry.

If the observed TID location is represented by IPP location, then the IPP is the location where LOS and acoustic wavefront intersect. Location of intersection, \( \vec{r}_{int}(t) \), can be easily found from the ray-trace map from Figure 2.4. Sometimes there are two intersection points for a specific pair of wavefront and LOS, we pick the point correspond to stronger electron density changed.

Note the idea of ionospheric height is not very good for acoustic waves because the height of \( \vec{r}_{int}(t) \) can ranges from 200 km to 500 km, but it is a necessity. Because when a TID is detected, the only information available are usually only the IEC time series and the LOS geometry, not the actual location of \( \vec{r}_{int}(t) \). For this reason current methods of TID velocity estimation always assume a ionospheric height [20] [21] [22] [23].

The least square cost function of circular wave model propagating at ionospheric height is therefore:

\[
SS(t_0, x_0, y_0, v_p) = \sum_k \| (t_k - t_0)v_p - \sqrt{(x_k - x_0)^2 + (y_k - y_0)^2} \|^2 \quad (4.1)
\]

There are four unknowns we are trying to estimate: \((x_0, y_0)\) is the location of the source, \(t_0\) is the starting time of propagation from the source, \(v_p\) is the constant propagation speed. The observed variables are: \((x_k, y_k)\) are locations of IPP with strong TID disturbance, \(t_k\) is the time of strong TID detection. Note that in this
estimation method we assume the wave is two-dimensional thus \( t_0 \) is actually the average traveling time from the surface of Earth to ionospheric height. This non-linear least square is computational expensive but we could break it into two different parts: finding the center \((x_0, y_0)\) and computing the speed \( v_p \).

If the cross-correlation of any pair of IEC time series has zero time lag, this indicates the corresponding IPPs lay on the same wavefront. The wavefront at any given time of a circular wave are all concentric with each other, thus the perpendicular bisectors of two IPPs at any single wavefront will always go through the center, forming a classic linear least square problem, as illustrated with Figure 4.1. Note that this method is unstable \[39\] thus we only use the 80 receivers that are closest to the epicenter to reduce the error. An example of this method for PRN 15 is shown in Figure 4.2.

Once the center location is obtained the cost function from equation 4.1 reduced to:

\[
SS(t_0, v_p) = \sum_k \|(t_k - t_0)v_p - w_k\|^2 \tag{4.2}
\]

where \( w_k = \sqrt{(x_k - x_0)^2 + (y_k - y_0)^2} \).

Express it in matrix form:

\[
\begin{bmatrix}
  t_1 \\
  t_2 \\
  \vdots \\
  t_n
\end{bmatrix} =
\begin{bmatrix}
  w_1 & 1 \\
  w_2 & 1 \\
  \vdots & \vdots \\
  w_n & 1
\end{bmatrix}
\begin{bmatrix}
  \frac{1}{v_p} \\
  t_0
\end{bmatrix}
\]

then this is also another linear regression problem, which requires no iteration.
Fig. 4.1. Method of estimating the source location. Assume 2D propagation and constant speed from a point source. Then all wavefronts (blue) at different time are concentric. All perpendicular bisectors (red dash line) between IPPs (dark points) belong to the same wavefront always intersect at the center.
Fig. 4.2. An example of estimating the epicenter using the method described in Figure 4.1, for PRN 15 with 355 km of ionospheric height. Red dots are the IPP locations corresponding to strong TID variations; green dash lines are the connection between the red dots with zero time lag; and the blue dash lines are the perpendicular bisectors of the greens. Red star represents the estimated center and the ellipse represents 2 STD of error.
5. RESULTS AND DISCUSSION

This chapter presents the results of the simulated IEC from our model, compared to the observed IEC data from the receiver array, for PRN 15 and PRN 26. The estimation method of propagation speed and source of circular wave is tested with the synthetic IEC time series for both PRN 15 and PRN 26.

5.1 Simulation results

5.1.1 Results from Japan 2011 Tohoku Earthquake, PRN 15

The IEC time series have been observed from PRN 15 during the 2011 Tohoku earthquake with 80 receivers that are close to the epicenter showing TID detections. Since the receivers and epicenter are not far away from each other, we could assume the dispersion is small and use the normalized vertical ground motion as the initial condition of the acoustic wave. The initial condition is shown in Fig. 3.3 with an unknown amplitude factor $A_0$, which is an universal scalar for the simulations of all receiver/satellite pairs.

Figure 5.1 show some of the filtered simulation IEC results and the filtered IEC measurements from satellite PRN15. The arrival time and magnitude of results are of the same order of magnitude. However there are some duration of oscillatory signal recorded after the initial pulses, this is likely because the source of acoustic wave does not only come from the epicenter, but also from a Rayleigh wave that travels on the surface of Earth (land and ocean) [40]. Some other possible sources for those oscillatory are aftershocks of the earthquake and tsunami [27]. The left part of Figure 5.2 shows the color map of time error ($|\tau_c|$) with the corresponding IPP location at time of detection for PRN15. The right part of Figure 5.2 shows the color map of
RMS of residual with the corresponding IPP location at time of detection. This $RMS(\text{Error}_i)$ is defined as:

$$RMS(\text{Error})_i = \sqrt{\frac{1}{m} \sum_{n=1}^{m} [IEC_{\text{obs}}(t_n) - \hat{A}_0 IEC_{\text{sim}}(t_n - t_{e,n})]^2}$$

We can see that both time error and residual is small at the region closest to the epicenter. There are 5 stations with zero time errors ($t_{e,i} = 0$) at that region, the comparison of simulated and observed IEC of those receivers are shown in Figure 5.3.

![Fig. 5.1. Multiple results of LOSs between satellite PRN15 and several different receivers during 2011 Tohoku earthquake, using universal $\hat{A}_0$. Red lines are simulated results and black lines are IEC from GPS observations, using the same bandpass filter.](image-url)
Fig. 5.2. The left figure shows the time error $\|t_e\|$ map at the corresponding IPP location (maximum time error is 180 sec). The earthquake slip is also shown here (extract from CIT Tectonics Observatory, maximum slip is 30 m). The right figure shows the RMS of residual $RMS(\text{Error})_i$ at the corresponding IPP location (maximum RMS is 0.375 TEC).

The error increase further distance from the source, especially at the northern and southern sides. This is probably because the epicenter is not a point, instead, the fault region is an irregular shape nearly parallel to the Japan coast line as shown in Figure 5.2 (the slip data is provided from Tectonics Observatory at California Institute of Technology). Therefore the acoustic waves from the north and south region arrived earlier then the estimation from a point source, and thus the estimated magnitude is also greater in those region. Another possible explanation will be the effect of acoustic wave generated from the surface Rayleigh wave, which travel horizontally with greater speed. In very close range the acoustic wave directly from the epicenter
will reach ionosphere faster than those generated from the Rayleigh wave, both took
time to propagate vertically. In farther range the acoustic wave directly from the
epicenter start to fall behind those generated from the Rayleigh wave and create the
arrival time error ($t_{e,i}$).

![Graph showing simulated and observed filtered IEC at five stations closest to the epicenter.]

**Fig. 5.3.** The simulated (red) and observed (green) filtered IEC at the five station closest to the epicenter.

### 5.1.2 Results from Japan 2011 Tohoku Earthquake, PRN 26

The IPP locations of PRN 15 and PRN 26 are shown in Figure 5.4. Unlike the
IPPs of PRN 15, which locate at the west side of epicenter with high elevation angles,
the IPPs of PRN 26 are at east side of epicenter with low elevation angles (about 40
deg). Some of the LOS actually go through right on top of the epicenter.
Figure 5.4. IPP location (blue line) of PRN 15 at the west and PRN 26 at the east for 2000 sec after the event. Red star represents the epicenter. Ionospheric height assume at 355 km.

Figure 5.5 shows some simulation result for PRN 26 using individual magnitude scale, i.e. \( \hat{A}_i \) from equation 3.14 were used instead of using an universal \( \hat{A}_0 \). It shows the synthetic PRN 26 waveform and arrival time generally agree with the observation. This time error map in Figure 5.6 shows the time error increase further away from the source, agrees with the time error map shown in Figure 5.2 from PRN 15. However a huge magnitude disagreement between the synthetic and observation is shown in Figure 5.7, using the same universal \( \hat{A}_0 \) obtained from PRN 15. The magnitudes of observed IEC variation have very large variance, some IEC time series are in the order of 0.2 TECU and some time series are in the order of 1.0 TECU, as shown in Figure 5.5. The synthetic IEC magnitude of the northern group from PRN 26 agrees with the
Fig. 5.5. Observation (blue) and simulation (red) results for PRN 26 using individual magnitude scale ($\hat{A}_i$), the left figure shows the southern group with large observed IEC variation, the right figure shows the northern group with small observed IEC variation. The intermediate group is not shown here. The arrival times and waveforms of observations and simulations generally are consistent. However the magnitudes show very large differences between observations and simulations, those errors are not shown here because the magnitude of each IEC time series are calibrated (by $\hat{A}_i$) in order to compare the waveforms.

magnitude from PRN 15, but the southern group doesn‘t. One possible explanation for the magnitude error is that the absorption coefficient of sound is underestimated, since the method in [34] is only valid up to 160 km of altitude. Another possible explanation would be the effect due to the extra term, $[t - t_0] \frac{\partial \Omega_i}{\partial z}$, in equation 3.8 becomes significant in low elevation angle.
5.2 Error of velocity estimation

The estimated epicenter location and circular propagation speed from synthetic IEC time series (PRN 15 and 26) are listed on Table 5.1, using the estimation method described in section 4 and different ionospheric height assumptions. The estimated speed decreases with increasing ionospheric height, and agree with the conclusion from [20]. The order of magnitude also agrees with the speed of sound profile. The estimated epicenter location from PRN 15, however, shifts to the north with increasing ionospheric height assumptions. This is an observation bias due to the LOS geometry,
as illustrates by Figure 5.8. Figure 5.8 shows that, increasing the ionospheric height also shift all IPP location to the satellite direction (for PRN 15 it is north), thus shifting all the red dots in Figure 4.2 to the north and its estimated center. Therefore the directional bias does not effect the estimated result perpendicular to the LOS direction, i.e. east-west direction for PRN 15. As for PRN 26, the directional bias also exist to the LOS direction. Figure 5.9 and Figure 5.10 show the error ellipses of finding the source are much larger due to the low elevation angle (about 40 deg) for PRN 26, especially in the North-south direction. Since the directional bias from one satellite always parallel to the LOS, two satellites could be used to greatly reduce this bias, as shown by Figure 5.11.

Fig. 5.8. An illustration of directional bias due to the change of ionospheric height. The IPP locations will shift to the satellite direction if the ionospheric height increased.

This estimated speed can be verified by using a traditional method of plotting the IEC in time and traveling distance, as shown in Figure 5.12. This method assumes the source is already known, plots the IPP distance from the source vs. time, and represents IEC variation with color at the corresponding point. Then the coherent color slope represent the propagation speed, which is between 550 m/s to 800 m/s for PRN 15. The propagation speed estimated by circular wave method does agree with the traditional method for PRN 15. However the speed from PRN 26 is obviously
Fig. 5.9. Estimating the epicenter using the method described in Figure 4.1, for PRN 26 with 100 km of ionospheric height. Red dots are the IPP locations corresponding to strong TID variations; green dash lines are the connection between the red dots with zero time lag; and the blue dash lines are the perpendicular bisectors of the greens. Red star represents the estimated center and the ellipse represents 2 STD of error. Note that the error ellipse has high uncertainty in the north-south direction.

overestimated, since they are greater than the speed of sound at that altitude. This could be due to the large error ellipse of center estimation, and thus greatly degraded the speed estimation.

5.3 Conclusion

There are a few conclusions which can be drawn from these results. First, the arrival times and waveforms of initial pulse from our simulation agree with the obser-
Table 5.1.
Table of estimated source locations and propagation speeds with different ionospheric height setting and different PRN

<table>
<thead>
<tr>
<th>PRN</th>
<th>15</th>
<th>15</th>
<th>15</th>
<th>26</th>
<th>26</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ionospheric height (km)</td>
<td>100</td>
<td>216</td>
<td>355</td>
<td>100</td>
<td>300</td>
</tr>
<tr>
<td>Estimated source location w.r.t. true source (km)</td>
<td>-20.9 East, 35.5 North</td>
<td>-20.8 East, 9.2 North</td>
<td>-20.7 East, 62.5 North</td>
<td>-11.6 East, -9.0 North</td>
<td>198.6 East, 72.9 North</td>
</tr>
<tr>
<td>Estimated speed (m/s)</td>
<td>790.9 ± 60.9</td>
<td>775.3 ± 60.3</td>
<td>756.6 ± 59.3</td>
<td>957.0 ± 15.4</td>
<td>968.0 ± 18.7</td>
</tr>
</tbody>
</table>
Fig. 5.10. This is the same figure as Figure 5.9, except assumes 300 km of ionospheric height. Note that the error ellipse has very large uncertainty in the north-south direction.

vation from the Tohoku Earthquake in the correct frequency range. This confirmed that strong earthquake can indeed generate acoustic waves that induce TIDs that are observed from GPS measurements. The time and magnitude error increases further away from the epicenter, especially to the northern and southern region since the slip region is nearly parallel to the eastern coast line of japan.

Second, the dispersion effect is insignificant when the LOS and the sources are in close range, such as the results from PRN15 of Tohoku Earthquake. Even for farther range where dispersion might become significant, the results will be mixed with acoustic wave from sources other than the epicenter, such as the case of PRN18. Without dispersion, only one unknown parameter is required in the initial waveform,
\( A_0 U_o(t) \) the magnitude. If the dispersion effect is taken into account there will be an extra unknown parameter which would need to be estimated [25].

Third, that assumption of electrons move along with neutral particles \( V_e = U \) at ionospheric height is sufficient for our simulation, at least within the close range. In fact, since the variation of collision frequency \( \nu_{in} \) is very large [35], numerical errors can become very large in some cases. For the sake of computational resources and numerical error, this \( v_e = U \) is a better assumption than the method used by [37], which computes the velocity for all different type of ions as described in equation 3.1. Note that these two methods are equivalent for high elevation angles of LOS.

Forth, the entire system is linear but time variant for a specific LOS and event, thus the unknown magnitude scalar of initial condition \( A_0 \) can be estimated by linear regression instead of iteration, and saving computational power.
Fig. 5.12. The traditional method of estimating propagation speed (if source is known) for PRN15. The color variation represents IEC variation. Y-axis represent the distance between IPP and the source. X-axis represent time. Thus the coherent color slope represent the propagation speed.

Fifth, the amplitude attenuation tells us the traveling distance of acoustic waves directly from the epicenter are limited, as the case with central frequency of 7.5e-3 Hz shown from Figure 3.5. Higher frequency waves attenuate faster and lower frequency waves can travel farther. However the acoustic waves not directly from the epicenter (such as generated from the surface Rayleigh wave) could travel farther as well. This amplitude attenuation could also be underestimated since there is a huge magnitude difference between the northern and southern group of IEC variation from PRN 26.

Last, an estimation method of circular propagation is tested with the synthetic IEC time series. The estimated speeds decreases with increasing ionospheric height assumption, agrees with the conclusion from [20]. The estimated center location shows a directional bias parallel to the LOS direction. This bias could be fixed by using observations from multiple satellites. The propagation speed estimation from PRN 15 agrees with the traditional method, as shown in Figure 5.12.
REFERENCES


APPENDICES
A. SMOOTHING THE AMPLITUDE RATIO

The ray trace profile contains three information: propagation time, propagation direction, and the amplitude ratio.

The amplitude change of this N-wave along the ray trace, can be compute from the transport equation, as described from previous section:

$$A(r) = \sqrt{\frac{\rho(r_0)C_s(r_0)J(r_0)}{\rho(r)C_s(r)J(r)}}$$

The Jacobian ratio $J(r_0)/J(r)$ describes the geometric spread of the energy. This can be estimated from the ratio of surface areas of the wavefront at two different location $r_0$ and $r$. Some numerical error exist when we estimated the surface areas from the finite difference between each ray trace. Those errors are generally small (less than 5%) and its variation are also small. However when we compute the electron density from the continuum equation, this error variation will be enlarged and become significant. Therefore we need a method to smooth the amplitude ratio profile before we procee our data. This amplitude ratio profile is a 3-D surface which depends on the location of $x_1$ and $x_3$. As shown in Figure 2.4, the blue lines represent the acoustic ray trace, the green lines shows the 3-min time grid of the wave, those green lines are also equivalent to wavefronts.

The error variation of amplitude ratio for a specific LOS depends on the 3D location of the LOS, in other words, the projection of a specific LOS in ECEF frame onto the $x_1$-$x_3$ ray trace map may is an arbitrary curve in Fig 2.4.

In order to see the error variation of amplitude ratio in a general sense, we show the error variation along the curves of wavefront, instead of some arbitrary curves. This is because the amplitude ratio should be a constant along the wavefront if the medium is isotropic.
Several different smoothing filters have been tested: moving average filter, local weighted linear regression with 1st and 2nd degree polynomial model, and Savitzky-Golay filter (local polynomial regression of degree k). Figure A.1 shows an amplitude ratio along an arbitrary wavefront.

Fig. A.1. Amplitude ratio along an arbitrary wavefront, smoothed by moving average filter and local linear regression

Figure A.3 compares the amplitude ratio along an arbitrary ray-trace (instead of a wavefront) before and after the using local linear regression smoothing. This plot confirms that smoothing amplitude ratio along the wavefronts does not significantly change the amplitude ratio along the ray-trace. Figure A.4 and Figure A.5 also compare the electron speed generated with raw and smoothed amplitude ratio profiles.
Fig. A.2. The results of all smoothed amplitude ratio along wavefronts in 30-sec interval, using local linear regression smoothing.
Fig. A.3. Amplitude ratio along an arbitrary ray-trace, before (blue) and after (red) using local linear regression smoothing.
Fig. A.4. Electron speed history along the LOS, generated by an unsmoothed amplitude ratio profile. Note the velocity scale is arbitrary for testing purpose, it does not reflect the actual physical value.
Fig. A.5. Electron speed history along the LOS, generated by a smoothed amplitude ratio profile. Note the velocity scale is arbitrary for testing purpose, it does not reflect the actual physical value.
B. GPS MEASUREMENT OF IEC

The IEC measurement is derived here [18]:

The propagation speed of electromagnetic wave in dispersive medium depends on the wave frequency. If there exist two radio signals with different frequencies share the same LOS, the difference of traveling time of these two signals can be used to estimate the refractive index of the medium.

The phase refractive index $n_p$ for a dispersive medium is:

$$n_p = \frac{c}{v_p} = \frac{ck}{\omega}$$

where $v_p$ is the phase velocity, $c$ is speed of light, $k$ is wave number and $\omega$ is frequency in radian per second.

The ionized gas is a dispersive medium for electromagnetic waves, where the dispersion relation is:

$$n^2 = 1 - \frac{\omega_p^2}{\omega^2}$$

where $\omega_p = \sqrt{\kappa_1 N_e}$ is the plasma frequency, and $N_e$ is electron density (number of electron per $m^3$). $\kappa_1$ is a constant.

Taking binomial approximation we get:

$$n_p \approx 1 - \frac{\omega_p^2}{2\omega^2} = 1 - \frac{40.3N_e}{f^2}$$

where $f$ is frequency in Hz.

Therefore the phase delay $\Delta \tau_p$ along the propagation path is:

$$\Delta \tau_p = \frac{1}{c} \int \{n_p(l) - 1\} dl = \frac{-40.3}{cf^2} \int N_e(l) dl$$

Now consider the refractive index of group velocity:

$$n_g = c \frac{dk}{d\omega} = \frac{d(ck)}{d\omega} = \frac{d(n_p \omega)}{d\omega} = n_p + \omega \frac{dn_p}{d\omega}$$
Therefore the group delay $\Delta \tau_g$ along the propagation path is:

$$\Delta \tau_g = \frac{1}{c} \int \{n_g(l) - 1\}dl = \frac{40.3}{cf^2} \int N_e(l)dl$$

Thus:

$$\Delta \tau_g = -\Delta \tau_p$$

Note: $\Delta \tau_g$ and $\Delta \tau_g$ have unit in sec.

Using linear combination of GPS duel-frequency system we get:

$$IEC(t) = \frac{1}{40.308} \frac{f_1^2 f_2^2}{f_1^2 - f_2^2} (\Delta \tau_1 - \Delta \tau_2 + nL)$$

Units: $\Delta \tau_1$ in meter, $f$ in Hz, $IEC$ in $e/m^2$

The GPS RENIX data file contains observables of $\Delta \tau_1$ and $\Delta \tau_2$.

Since our research only focus at the filtered IEC, not the the absolute values of raw IEC, we could use the carrier phase delay, instead of pseudorange, for $\Delta \tau_1$ and $\Delta \tau_2$ for more precise measurement.
C. PROJECTION OF PARAMETERS

Define Cartesian coordinate where the origin is located at the source/epicenter: \( \hat{c}_3 \): Vertical axis about the source. \( \hat{c}_1 \): Arbitrary axis that tangents to the surface of the Earth and perpendicular to \( \hat{c}_3 \). \( \hat{c}_2 \): Axis that satisfies \( \hat{c}_3 = \hat{c}_1 \times \hat{c}_2 \).

A LOS can be described at such Cartesian coordinate as:

\[
\vec{x}_{LOS} = \vec{x}_R + \hat{L}t = \begin{pmatrix} c_{1R} \\ c_{2R} \\ c_{3R} \end{pmatrix} + \begin{pmatrix} c_{1L} \\ c_{2L} \\ c_{3L} \end{pmatrix} t
\]

where \( \vec{x}_R \) is the position of the receiver, \( \hat{L} \) is the direction of the LOS, and \( t \) is a scaler variable.

Define a polar coordinate system as shown in Fig. C.1:
\( \hat{x}_3 \): Vertical axis about the source. \( \hat{x}_1 \): Radial axis that tangents to the surface of the Earth and perpendicular to \( \hat{x}_3 \). \( \hat{\theta} \): polar angle.

Project LOS onto the \( \hat{x}_1-\hat{x}_3 \) plane:

\[
x_1 = \sqrt{(c_{1R} + c_{1L}t)^2 + (c_{2R} + c_{2L}t)^2} \\
x_3 = c_{3L}t
\]

Therefore the linear LOS in Cartesian coordinate will appear as a non-linear curve on \( \hat{x}_1-\hat{x}_3 \) plane.
Fig. C.1. Projection between polar and Cartesian coordinate. The projection of LOS onto the $\hat{x}_1-\hat{x}_3$ plane (yellow panel) will be curved.
D. RESULTS FROM SHINAGAWA’S METHOD

The synthetic IEC results for PRN 15, generated using Shinagawa’s Method, as described by equation 3.1, is shown here. The results agree with observed IEC initially, then the numerical error start to increase to chaos. This method requires more computational time but produces more numerical errors thus is abandoned.
Fig. D.1. Example of synthetic and observed IEC, using equation 3.1
Fig. D.2. Example of synthetic and observed IEC, using equation 3.1
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