A Thermodynamic Property Model for the R-134a/245fa Mixtures

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Presented at
\textsuperscript{15\textsuperscript{th}} International Refrigeration and Air Conditioning Conference at Purdue, July 14, 2014
R-134a (1,1,1,2-tetrafluoroethane)

- Broadly used refrigerant in many applications at medium- and high-temperatures
  - Stationary commercial refrigeration
  - Chiller equipment
  - Home applications
  - Mobile air conditioners
- The GWP (1450) is far from negligible.
R-245fa (1,1,1,3,3-pentafluoropropane)

- Mainly used for high-temperature applications
  - Centrifugal chillers
  - High-temperature heat pumps
  - Organic Rankin cycles for low-temperature heat sources
- The GWP (1030) is far from negligible.

RTVF type centrifugal chiller (Ebara Refrigeration Equipment & Systems)
1 Replacements for R-134a

- **R-1234yf**: lower performance, much higher production cost, slightly flammable
- **R-1234ze(E)**: lower performance, higher production cost
- **R-1243zf**: flammable

2 Replacements for R-245fa

- **R-1234ze(Z)**: higher production cost

Currently available replacements for R-134a and R-245fa are *still* in a development phase.

Therefore, in at least the next decade, the refrigeration industry will use R-134a, R-245fa, and their mixtures.
Air-sourced heat pump for supplying hot water (Kobe Steel, Ltd.). This heat pump uses the R-134a/245fa mixtures as working fluid.
Objectives

The following two projects are now ongoing:

1. **Development of a new equation of state for the thermodynamic properties of pure R-245fa** (Akasaka, Zhou, and Lemmon, to be submitted to JPCRD in 2014)

2. **Development of mixture models for the R-134a/245fa mixtures** (this work) and **R-134a/1234ze(Z) mixtures** (future work)

Currently, a reliable property model is not available for the R-134a/245fa mixtures.
Outline of the model development

1. The Kunz-Wagner (KW) mixture model was adopted with several departure functions (12-type KW models). This approach is often used in the recent developments of accurate mixture models.

2. The most reliable pure-fluid equations of state were incorporated for calculations of the Helmholtz energies of pure fluids.

3. Adjustable parameters of the KW models were optimized by a nonlinear fitting to the following experimental data:
   - Vapor-liquid equilibirum (Bobbo et al., 2001)
   - $pvTx$ property (Higashi and Akasaka, 2014)
   - Critical temperature, critical density, and saturated liquid and vapor densities (this work)

4. The best KW model was accordingly selected that minimizes the final sum of square of relative deviations between experimental and calculated values.
Multi-fluid model for binary mixtures

\[
\frac{a(T, v, x)}{RT} = \alpha = \alpha^{\text{idmix}} + \alpha^E
\]

- \(a\): Molar Helmholtz energy
- \(R\): Universal gas constant \((= 8.3144621 \text{ J mol}^{-1} \text{ K}^{-1})\)
- \(T\): Temperature
- \(v\): Molar volume
- \(x\): Mole fraction of the first component
- \(\alpha\): Reduced Helmholtz energy
- \(\alpha^{\text{idmix}}\): Ideal mixture contribution
- \(\alpha^E\): Contribution from mixing (Excess Helmholtz energy)
Ideal mixture contribution

\[ \alpha^{\text{idmix}}(T, v, x) = x [\alpha_1^o(T, v) + \alpha_1^r(\tau, \delta) + \ln x] \]
\[ + (1 - x) [\alpha_2^o(T, v) + \alpha_2^r(\tau, \delta) + \ln (1 - x)] \]

\[ \tau = \frac{T_{\text{red}}(x)}{T}, \quad \delta = \frac{v_{\text{red}}(x)}{v} \]

\( \alpha_i^o \) (\( i = 1, 2 \)) : Ideal gas part of component \( i \)
\( \alpha_i^r \) (\( i = 1, 2 \)) : Residual gas part of component \( i \)
\( \tau \) : Reduced mixture temperature
\( \delta \) : Reduced mixture volume
\( T_{\text{red}} \) : Reducing function for temperature
\( v_{\text{red}} \) : Reducing function for volume

Table 1: Pure-fluid equations of state for the calculations of \( \alpha_i^o \) and \( \alpha_i^r 

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>( i = 1 ) (R-134a)</td>
<td>Tillner-Roth and Baehr (1994)</td>
</tr>
<tr>
<td>( i = 2 ) (R-245fa)</td>
<td>Akasaka, Zhou, and Lemmon (2014)</td>
</tr>
</tbody>
</table>
Reducing functions

Kunz et al. (2007) (KW model)

\[ T_{\text{red}}(x) = x^2 T_{c,1} + (1 - x)^2 T_{c,2} + 2 \beta_T \gamma_T \left( \frac{x(1 - x)}{(\beta_T^2 - 1)x + 1} \right) T_{c,12} \]

\[ v_{\text{red}}(x) = x^2 v_{c,1} + (1 - x)^2 v_{c,2} + 2 \beta_v \gamma_v \left( \frac{x(1 - x)}{(\beta_v^2 - 1)x + 1} \right) v_{c,12} \]

\[ T_{c,12} = \sqrt{T_{c,1} T_{c,2}} \]

\[ v_{c,12} = \frac{1}{8} \left( v_{c,1}^{1/3} + v_{c,2}^{1/3} \right)^3 \]

\( \beta_T, \gamma_T, \beta_v, \) and \( \gamma_v \): Parameters determined by the fitting to experimental data
Contribution from mixing

\[ \alpha^E(\tau, \delta, x) = x(1-x)F_{12} \alpha_{12}^r(\tau, \delta) \]

\( \alpha_{12}^r \): Departure function  
\( F_{12} \): Scaling factor

- Several departure functions have so far been proposed for binary mixtures including hydrocarbons, noble gases, and refrigerants.
- The scaling factor \( F_{12} \) is used to adjust the magnitude of \( \alpha_{12}^r \) for a mixture of interest.
A multi-fluid model is combination of departure functions for temperature and volume and a departure function representing the contribution from mixing.
### Table 2: 12-type KW models

<table>
<thead>
<tr>
<th>Designation</th>
<th>Reducing functions</th>
<th>Departure function</th>
</tr>
</thead>
<tbody>
<tr>
<td>KWG</td>
<td>KW</td>
<td>generalized for binary pairs in methane, ethane, propane, n-butane, isobutane, ethylene, nitrogen, argon, oxygen, and carbon dioxide</td>
</tr>
<tr>
<td>KWR</td>
<td>KW</td>
<td>specified for R-32 + R-125 mixtures</td>
</tr>
<tr>
<td>KWS</td>
<td>KW</td>
<td>specified for R-32 + R-134a mixtures</td>
</tr>
<tr>
<td>KWT</td>
<td>KW</td>
<td>generalized for R-125 + R-134a, R-125 + R-143a, R-134a + R-143a, and R-134a + R-152a mixtures</td>
</tr>
<tr>
<td>KW0</td>
<td>KW</td>
<td>generalized for methane + n-butane, methane + isobutane, ethane + propane, ethane + n-butane, ethane + isobutane, propane + n-butane, propane + isobutane, and n-butane + isobutane mixtures</td>
</tr>
<tr>
<td>KW1</td>
<td>KW</td>
<td>specified for methane + ethane mixtures</td>
</tr>
<tr>
<td>KW2</td>
<td>KW</td>
<td>specified for methane + propane mixtures</td>
</tr>
<tr>
<td>KW3</td>
<td>KW</td>
<td>specified for methane + nitrogen mixtures</td>
</tr>
<tr>
<td>KW4</td>
<td>KW</td>
<td>specified for methane + carbon dioxide mixtures</td>
</tr>
<tr>
<td>KW5</td>
<td>KW</td>
<td>specified for nitrogen + carbon dioxide mixtures</td>
</tr>
<tr>
<td>KW6</td>
<td>KW</td>
<td>specified for nitrogen + ethane mixtures</td>
</tr>
<tr>
<td>KW7</td>
<td>KW</td>
<td>specified for methane + hydrogen mixtures</td>
</tr>
</tbody>
</table>
Nonlinear least square fitting of the five adjustable parameters

- Five adjustable parameters: $\beta_T$, $\gamma_T$, $\beta_v$, $\gamma_v$, and $F_{12}$
- Modified Levenberg-Marquardt algorithm
- Objective function

$$S = \frac{1}{n_{p_b}} \sum W_{p_b} X_{p_b}^2 + \frac{1}{n_{p_d}} \sum W_{p_d} X_{p_d}^2 + \frac{1}{n_{\rho}} \sum W_{\rho} X_{\rho}^2$$
  \[+ \frac{1}{n_{T_c}} \sum W_{T_c} X_{T_c}^2 + \frac{1}{n_{v_c}} \sum W_{v_c} X_{v_c}^2\]

$X$ : Relative deviation, e.g. $X_{p_b} = (p_{b,exp} - p_{b,cal})/p_{b,cal}$
$W$ : Weighting factor
$n$ : Number of data points
Optimum values of the adjustable parameters

Table 3: Optimum parameters and final sum of square

<table>
<thead>
<tr>
<th>Mixture model</th>
<th>$\beta_T$</th>
<th>$\gamma_T$</th>
<th>$\beta_v$</th>
<th>$\gamma_v$</th>
<th>$F_{12}$</th>
<th>$S_{\text{final}} \times 10^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>KWG</td>
<td>1.</td>
<td>1.00372</td>
<td>0.992392</td>
<td>0.998056</td>
<td>0.145638</td>
<td>0.1136</td>
</tr>
<tr>
<td>KWR</td>
<td>1.</td>
<td>1.01541</td>
<td>0.992526</td>
<td>1.00950</td>
<td>0.185932</td>
<td>0.4036</td>
</tr>
<tr>
<td>KWS</td>
<td>1.</td>
<td>1.01264</td>
<td>0.993027</td>
<td>1.01662</td>
<td>0.539071</td>
<td>0.5048</td>
</tr>
<tr>
<td>KWT</td>
<td>1.</td>
<td>1.00717</td>
<td>0.992132</td>
<td>1.00391</td>
<td>0.909890</td>
<td>0.2710</td>
</tr>
<tr>
<td>KW0</td>
<td>1.</td>
<td>1.00608</td>
<td>0.993058</td>
<td>1.00117</td>
<td>0.0529635</td>
<td>0.1222</td>
</tr>
<tr>
<td><strong>KW1</strong></td>
<td>1.</td>
<td><strong>1.00643</strong></td>
<td><strong>0.992025</strong></td>
<td>1.</td>
<td><strong>0.107754</strong></td>
<td><strong>0.1128</strong></td>
</tr>
<tr>
<td>KW2</td>
<td>1.</td>
<td>1.00592</td>
<td>0.993705</td>
<td>1.00213</td>
<td>0.0500026</td>
<td>0.1394</td>
</tr>
<tr>
<td>KW3</td>
<td>0.999303</td>
<td>1.00604</td>
<td>0.993438</td>
<td>0.999266</td>
<td>0.120055</td>
<td>0.2690</td>
</tr>
<tr>
<td>KW4</td>
<td>0.999366</td>
<td>1.00774</td>
<td>0.992810</td>
<td>0.995019</td>
<td>0.123985</td>
<td>0.2695</td>
</tr>
<tr>
<td>KW5</td>
<td>1.</td>
<td>1.00714</td>
<td>0.992583</td>
<td>0.997638</td>
<td>0.0561002</td>
<td>0.1470</td>
</tr>
<tr>
<td>KW6</td>
<td>1.</td>
<td>1.00682</td>
<td>0.992581</td>
<td>0.997262</td>
<td>0.057564</td>
<td>0.1379</td>
</tr>
<tr>
<td>KW7</td>
<td>1.</td>
<td>0.998221</td>
<td>0.994407</td>
<td>1.01430</td>
<td>0.116853</td>
<td>0.3022</td>
</tr>
</tbody>
</table>

- The KW1 model is the best in 12 models for the R-134a/245fa mixtures.
Introduction
Model development
Vapor-liquid equilibrium
Comparison to experimental data
Densities
Conclusions

Mole fraction of R-134a

$P$ (MPa)

Bobbo et al. (2001)
REFPROP 9.1 (Lemmon et al., 2013)
KW1 model (This work)

313.15 K
303.15 K
293.15 K

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1

0 0.2 0.4 0.6 0.8 1
Vapor-liquid equilibrium

Densities

Critical point

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R-134a/245fa = 10/90 mass%  30/70 mass%  50/50 mass%
### Vapor-liquid equilibrium

#### Densities

- **Saturated vapor and liquid densities** (This work)
- **REFPROP 9.1** (Lemmon et al., 2013)
- **KW1 model** (This work)

#### Comparison to experimental data

- **R-134a/245fa**
- **R-245fa**
- **R-134a**

#### Critical point

**30/70 mass%**

**50/50 mass%**

**R-134a**

**R-245fa**

= 10/90 mass%
Applying the multi-fluid model, this work successfully modeled the thermodynamic properties of the R-134a/245fa mixtures.

The estimated uncertainties in calculated properties from the model are

- ±0.20 % for the bubble and dew point pressures
- ±0.24 % for the liquid and vapor densities

The critical parameters of the mixtures are also properly represented with the model. The calculated critical temperatures correspond to experimental values within ±0.5 K.

The model allows reliable analysis of refrigeration systems or heat pumps using the mixtures.
The parameters determined in this work can be applied to REFPROP.
Thank you.
Any questions?

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