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The Astrophysical Journal, 668: L1–L4, 2007 October 10

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Received 2007 July 16; accepted 2007 August 23; published 2007 September 24

ABSTRACT

We discuss dissipative processes in strongly gyrotropic, nearly collisionless plasma in clusters of galaxies (the ICM). First, we point out that Braginskii's theory, which assumes that collisions are more frequent that the system's dynamical timescale, is inapplicable to fast, subviscous ICM motion. Most importantly, the electron contribution to collisional magnetoviscosity dominates over that of ions for short-scale Alfvénic motions with wavelength satisfying $l \leq (\lambda/\sqrt{\beta})(m_e/m_p)^{1/4} \sim 1$ kpc (where λ is the particle's mean free path, β is the plasma pressure parameter, and m_e and m_p are electron and proton masses). Thus, if a turbulent cascade develops in the ICM and propagates down to scales $\lesssim$1 kpc, it is damped collisionally not on ions, but on electrons. Second, in high-β plasma of the ICM, small variations of the magnetic field strength, of relative value $\sim 1/\beta$, lead to the development of anisotropic pressure instabilities (firehose, mirror, and cyclotron). Unstable wave modes may provide additional resonant scattering of particles, effectively keeping the plasma in a state of marginal stability. We show that in this case the dissipation rate of a laminar, subsonic, incompressible flows scales as the inverse of the plasma β parameter. We discuss application to the problem of ICM heating.

Subject headings: galaxies: clusters: general

1. INTRODUCTION

One of the key problems in the physics of the intercluster medium (ICM) is the absence of strong cooling flows at the centers of galaxy clusters (see, e.g., Peterson & Fabian 2006 for a review). It has been proposed that the heating of the ICM by active galactic nuclei (AGNs) may be sufficient to offset the cooling (e.g., Begelman 2004). While the total energy budget of AGNs is, in principal, sufficient to offset the radiative cooling, details of how this is achieved are far from clear.

The observational confirmation of the AGN heating model comes from the ubiquitous presence of AGN-blown bubbles, identified by decreased X-ray emission in *Chandra* and *XMM* maps (McNamara et al. 2000). These bubbles expand and rise in the cluster potentials, transferring part of their energy to the internal energy of the ICM. It has been suggested that this process can be very efficient, so that a large fraction of the power released by AGNs ends up as internal energy of the ICM.

The high efficiency of energy dissipation is far from obvious. What is required is a distributed increase of the entropy of the gas, not just of the internal or bulk energy (entropy floor problem; Lloyd-Davies et al. 2000). The main problem is that these AGN-blown bubbles expand, typically, *subsonically*, as is indicated by the general absence of shock signatures ahead of the bubbles. In laminar flows at small Reynolds numbers, $\mathrm{Re} \leq \mathrm{Re}_{\rm crit} \sim 10$–100, the dissipation efficiency is $\propto 1/\mathrm{Re}$. For $\mathrm{Re} \sim 100$, such a low efficiency puts unreasonable demands on AGN luminosity.

2. COLLISIONAL DISSIPATION IN GRYOTROPIC PLASMA: GYRORELAXATIONAL HEATING

Ion Larmor radii in the ICM, $r_{\rm L} \sim 10^8$ cm, is some 15 orders of magnitude smaller than the system size, $L \sim$ hundreds of kpc, and Coulomb mean free path, $\lambda \sim 10$–30 kpc, for a typical density $n \sim 10^{-3}$ cm^{-3}, magnetic fields $\sim$1–10 μG, and temperatures in the keV range (e.g., Carilli & Taylor 2002). Thus the ICM is weakly collisional, $r_{\rm L} \ll \lambda$. In addition, it is weakly magnetized, in the sense that magnetic field energy is smaller than plasma pressure, $\beta = 8\pi P/B^2 \geq 1$. Below we refer to this regime as "a strongly gyrotropic plasma."

Dissipation in a strongly gyrotropic plasma proceeds in a qualitatively different way from the isotropic case, as is exemplified by so-called gyrorelaxational heating. If in an initially pressure-isotropic plasma the absolute value of magnetic field oscillates with a frequency ω and relative amplitude $\delta = \delta B/B_0$, then the dissipation rate α (so that energy of a particle $\mathcal{E}$ changes according to $d\mathcal{E}/dt = \alpha\mathcal{E}$) in a cycle is (e.g., Borovsky 1986)

$$\alpha \approx \frac{\omega^2 \nu_c}{(9/4)\nu_c^2 + \omega^2}\frac{\delta^2}{6}, \tag{1}$$

where ν_c is collision frequency. Dissipation of energy occurs due to both electron and ion collisions, so that $\alpha = \alpha_e + \alpha_i$, calculated with corresponding collision frequencies ν_e and ν_i. In the high collision frequency regime, $\nu_c \gg \omega$, equation (1) approximates Braginskii's result, $\alpha \propto 1/\nu_c$ (Braginskii 1965). Since ions have smaller collision frequency, dissipation in this limit is dominated by ions. On the other hand, *for rare collisions*, $\nu_c \ll \omega$, *the dissipation rate is proportional to collision frequency*, $\alpha \propto \nu_c$, *and is thus dominated by electrons* for $t_e\omega > (3/2)(m_e/m_i)^{1/4} \approx 0.2$.

Consider subviscous turbulent motion of the ICM occurring on scale l smaller than the mean free path λ and mediated by Alfvén waves, so that a typical wave frequency is $\omega = V_{\rm A}/l \sim c_s/(\sqrt{\beta}l)$. Then for waves satisfying $t_e\omega > (3/2)(m_e/m_i)^{1/4}$, or for

$$l \leq \frac{\lambda}{\sqrt{\beta}}\left(\frac{m_e}{m_p}\right)^{1/4} \sim 1 \text{ kpc}, \tag{2}$$

electron viscosity dominates over ion viscosity. For numerical estimates we assumed $T = 10^8$ K, $n = 10^{-3}$ cm^{-3}, $B = 5$ μG, so that $\beta \sim 10$ and the mean free path $\lambda = 23$ kpc.

Thus, if a turbulent cascade develops in the ICM and propagates down to scales $\lesssim$1 kpc, it is damped collisionally not on ions, but on electrons. Thus, Braginskii's (1965) theory, which assumes frequent collisions, $t_{\rm coll}\omega \gg 1$, *is inapplicable to fast, subviscous ICM motion.*

3. HEATING IN A BOUND ANISOTROPY MODEL

Besides binary collisions, plasma can be heated through the development of electromagnetic turbulence which resonantly scatters particles and, thus, provides an additional dissipation mechanism. In this section we describe this mechanism of dissipation through the development of anisotropic plasma instabilities.

3.1. *Viscosity due to Binary Collisions in a Gyrotropic Plasma*

When the Coulomb collision frequency ν_c is much smaller than the cyclotron frequency, $\omega_B/\nu_c \gg 1$, plasma viscosity is strongly anisotropic, determined by seven coefficients (Braginskii 1965). In the limit $\omega_B \to \infty$ and slow changes of the magnetic field, $\omega \ll \nu_c$, the only remaining coefficient is η_0, which is responsible for the viscosity along the field lines. In this case the viscous stress tensor becomes (Landau & Lifshitz 1982)

$$\sigma_{ij} = \eta_0(3b_ib_j - \delta_{ij})\left(b_lb_k\partial_l v_k - \frac{1}{3}\,\mathrm{div}\,\boldsymbol{v}\right), \tag{3}$$

where b_i is a unit vector along the local magnetic field and $\eta_0 = p/\nu_c$, where $p = (P_\parallel + 2P_\perp)/3$ is the total pressure. Below we concentrate on the incompressible limit, div $\boldsymbol{v} = 0$, which eliminates reversible compressional heating. For incompressible plasma without conductivity, using equation (3), the volumetric dissipation and entropy generation rates due to viscosity are (Landau & Lifshitz 1975)

$$\rho\frac{dE}{dt} = \rho T\frac{dS}{dt} = \sigma_{ij}\partial_i v_j = 3\eta_0\left(\boldsymbol{b}\cdot(\boldsymbol{b}\boldsymbol{\nabla})\boldsymbol{v}\right)^2. \tag{4}$$

Using the induction equation, $d\boldsymbol{B}/dt = (\boldsymbol{B}\boldsymbol{\nabla})\boldsymbol{v}$, the entropy generation rate can be related to the rate of change of the magnetic field (Schekochihin & Cowley 2006):

$$\rho\frac{dE}{dt} = 3\eta_0\left(\frac{1}{B}\frac{dB}{dt}\right)^2. \tag{5}$$

The dissipated power of a gyrotropic fluid is solely due to the changing magnetic field, which is very different from the isotropic case. This result can also be verified if we note that in a gyrotropic plasma the entropy is $S \propto \frac{1}{2}\ln P_\perp P_\parallel^2$ (assuming constant density). The entropy production is then

$$\frac{dS}{dt} = \frac{1}{3}\frac{(P_\perp - P_\parallel)^2}{P_\perp P_\parallel}\,\nu. \tag{6}$$

For binary collisions using $P_\perp - P_\parallel = 3\eta_0 d_t \ln B$ (eq. [12]), this gives

$$\frac{dS}{dt} = 3(d_t\ln B)^2\eta_0^2\frac{\nu}{P_\perp P_\parallel} \approx 3\,\frac{(d_t\ln B)^2}{\nu}, \tag{7}$$

consistent with equation (5).

The differences between the dissipation rates calculated using isotropic and anisotropic viscosities can be dramatic. For example, for spherical expansion of a bubble into incompressible fluid, in the absence of magnetic field, the dissipated power is zero (flow field is irrotational). Introduction of a weak (in the sense that $\beta \gg 1$) magnetic field changes this picture completely. In a kinematic approximation (neglecting its dynamical effects, so that field lines are just advected with the flow, satisfying the frozen-in condition), expansion of a bubble into a constant magnetic field creates magnetic fields

$$B_\theta = \frac{\sin\theta}{(1-\xi^{-3})^{1/3}}B_0, \quad B_r = -\cos\theta(1-\xi^{-3})^{1/3}B_0, \tag{8}$$

where $\xi = r/R(t) > 1$ and B_θ and B_r are components of the magnetic field in a spherical system of coordinates aligned with the initial direction of the field. Although the tangential component of the magnetic field diverges on the contact $\xi = 1$ (the magnetic draping effect), the increase in B-field energy over the initial homogeneous field is finite, $\frac{1}{9}B_0^2R^3$, and the total heating rate is $d_tE = 3\eta_0 R^3\int d^3\xi\,(d_t\ln B)^2 = 9.54\eta_0 R^3 \times (d\ln R/dt)^2$.

This example illustrates an important point: even a weak magnetic field may considerably affect plasma dissipative properties. The inverse situation, in which a dissipative flow with isotropic viscosity becomes nondissipative in the strongly gyrotropic limit, is also possible. The example is a longitudinal shear, when the magnetic field is directed along velocity. In the absence of cross-field viscosity there is no dissipation.

3.2. *Anisotropic Pressure Instabilities*

In collisionless plasma, particles in magnetic fields tend to conserve their adiabatic invariants (Chew et al. 1956). In the case of rare collisions the equations describing the evolution of pressures becomes (e.g., Hollweg 1985)

$$\begin{aligned}\frac{d\ln P_\perp/B}{dt} &= \frac{\nu}{3}\frac{P_\parallel - P_\perp}{P_\perp},\\ \frac{d\ln P_\parallel B^2}{dt} &= -\frac{2\nu}{3}\frac{P_\parallel - P_\perp}{P_\parallel},\end{aligned} \tag{9}$$

where $P_\perp$ and $P_\parallel$ are pressure across and along the magnetic field.

In a $\beta \gg 1$ plasma, the development of pressure anisotropy may lead to firehose, mirror, and ion cyclotron instabilities when the following conditions are satisfied:

$$\begin{cases}\beta_\parallel - \beta_\perp > 2, & \text{firehose,}\\ \beta_\perp/\beta_\parallel > 1 + 1/\beta_\perp, & \text{mirror,}\\ \beta_\perp/\beta_\parallel > 1 + k/\beta_\parallel^m, & \text{cyclotron,}\end{cases} \tag{10}$$

where $\beta_\parallel = 8\pi P_\parallel/B^2$, $\beta_\perp = 8\pi P_\perp/B^2$, $0.35 \leq k \leq 0.65$, and $0.4 \leq m \leq 0.42$ (Gary et al. 1994). The cyclotron instability has growth rate larger than the mirror instability for $\beta \leq 6$ and $p_\perp > p_\parallel$. If initially the plasma pressure is isotropic, the firehose and mirror instabilities occur when $\delta B/B = -2/(3\beta_0)$ (firehose) and $\delta B/B = +1/(3\beta_0)$ (mirror), and according to a similar expression for the ion cyclotron instability (Gary et al. 1994); for clarity we do not consider the latter here.

The instabilities' increment is maximal at the cyclotron frequency, which is very fast compared to any dynamical time. Further change of the magnetic field, beyond the limits given in equation (10), will be accompanied by the development of instabilities which will lead to an increased scattering rate, due to either quasi-linear diffusion or a fully developed turbulence. As a result, the system dissipates quickly any free energy in excess of the instability threshold and relaxes to a marginally stable state. We expect that the system remains at threshold of instability.

3.2.1. *Binary Collisions in the Subcritical Regime*

Binary collisions decrease the level of anisotropy and may stabilize plasma. Redefining pressures $P_\perp$ and $P_\parallel$ in terms of total pressure p (a trace of the pressure tensor) and pressure disbalance $(P_\perp - P_\parallel)/p = \Delta$, $P_\parallel = p - \frac{2}{3}\Delta p$, $P_\perp = p + \frac{1}{3}\Delta p$, we find

$$2p\Delta \frac{dB}{dt} = 3B\frac{dp}{dt},$$

$$\frac{d\Delta}{dt} + \nu\Delta - \frac{9 - 3\Delta - 2\Delta^2}{3}\frac{d\ln B}{dt} = 0. \tag{11}$$

In a $\beta \gg 1$ plasma at the moment of instability Δ is small, $|\Delta| \ll 1$. For slow changes $d/dt \ll \nu$ this gives

$$\nu\Delta = 3\frac{d\ln B}{dt}. \tag{12}$$

This implies that for the development of instabilities the dynamical time $t_{\rm dyn} \sim 1/d_t \ln B$ should be relatively short, $t_{\rm dyn}\nu_c \leq \beta$. This condition is satisfied by most scales of interest in ICM plasma.

3.3. *Dissipation at Marginal Stability*

As we argued in the previous section, a changing magnetic field will lead to the development of instabilities that will keep the plasma anisotropy at the critical values given by equation (10). Equations (9) and (10) may be regarded as defining effective scattering rates

$$\nu_{\rm eff,firehose} = \frac{3}{2}\beta d_t \ln B, \quad \nu_{\rm eff,mirror} = 3\beta d_t \ln B. \tag{13}$$

At a critically balanced case, the entropy generation rate (eq. [6])

$$d_t S \approx \frac{2}{\beta} d_t \ln B \times \begin{pmatrix} 1 \\ 1/2 \end{pmatrix} \tag{14}$$

for the firehose and mirror regimes.

We have arrived at an important result related to the efficiency of dissipation: *in a gyrotropic plasma, efficiency of dissipation is determined not by the Reynolds number but by the plasma β parameter*. The typical dissipation timescale is β times dynamical time, not Re times dynamical time.

The role of effective collisions in energy dissipation in a marginally stable regime is, in some sense, opposite to the role of binary collisions in a subcritical regime. The entropy production rate and corresponding volumetric dissipated power (eq. [6]) are proportional to pressure anisotropy and collision frequency, $\propto\nu(\Delta P)^2$, where ΔP is the difference in parallel and transverse pressures. If the pressure disbalance is due to binary collisions, then $\Delta P \propto 1/\nu$ so that the dissipation rate is $\propto 1/\nu$ (Braginskii 1965). Thus, before the instabilities are reached, increasing collision rate leads to decreasing dissipation. On the other hand, for the marginally stable case $\Delta P \sim$ constant, so that dissipated power is *proportional* to the effective collision rate (eq. [7]).

3.4. *Damping of Waves at Marginal Stability*

For Alfvén waves, perturbations of the magnetic field are orthogonal to the initial magnetic field, so that variations of the absolute value of the field are second order in amplitude. For large enough amplitude, satisfying the condition $(\delta B/B_0)^2 \equiv \delta^2 > 1/\beta$, this creates conditions favorable for the mirror instability. The entropy production rate over the period is

$$\frac{dS}{dt} = \frac{4}{\beta}\frac{\delta^2}{1+\delta^2}\omega\left[\frac{2\arccos(1/\beta\delta^2)}{\pi}\right], \tag{15}$$

where the term in square brackets takes into account phases when the amplitude of fluctuations satisfies the mirror instability criterion. With the Braginskii viscosity, the collisional damping of Alfvén waves is a nonlinear effect as well, but it has a much steeper dependence of wave amplitude and frequency. From equation (5) we find

$$\frac{dS}{dt} = \frac{3\delta^4\omega^2}{(1+\delta^2)^2\nu_c}. \tag{16}$$

For comparison, in isotropic MHD Alfvén waves are damped at a rate (Landau & Lifshitz 1982) $dS/dt = \delta^2\omega^2/\nu_c$.

4. DISCUSSION

Our approach follows a long-established procedure of marginal stability (Kennel & Petscheck 1966; Manheimer & Boris 1977; Gary et al. 1994; Denton et al. 1994), when the instability threshold becomes the limiting value of anisotropy. In particular, Quest & Shapiro (1996) and Gary et al. (1998) applied a bounded anisotropy model to the measurements of parallel and perpendicular temperatures in the solar bow shock region near the Earth magnetosphere. It was found that an initial rapid growth of unstable waves indeed brings the system back to approximate marginal stability.

What is the relation of the marginal stability condition and the conventional quasi-linear and turbulence theories? According to Manheimer & Boris (1977), both predict some level of turbulent fluctuations. The marginal stability approach is applicable if the level of those fluctuations is smaller than the one calculated from nonlinear theory. This, typically, happens when the driver of the instability (in our case a large-scale motion of ICM plasma) is not strong. Assessing whether this is satisfied in the case of ICM plasma requires full-scale calculations of nonlinear turbulence levels, a prohibitively complicated task given the uncertainties in both plasma microphysics and the details of ICM plasma motions.

The most important effect that has not been taken into account in the present work is thermal conduction. The double-adiabatic equations are valid only when heat flux along magnetic field lines can be neglected. This is the main reason why the theory may fail (e.g., the notorious results of Kulsrud et al. 1965). Neglect of heat flux requires that phase velocity of the perturbations be much larger than the speed of heat carriers, electrons: $(\omega/k)^2 \sim V^2 \gg v_{T,e}^2$. This condition may be broken in the ICM, especially outside of cluster cores. On the other hand, an enhanced scattering rate suppresses conductivity (Levinson & Eichler 1992). The conduction coefficient is $\kappa \sim n_e v_{T,e}^2/\nu_{\rm eff}$ (assuming that the saturated conductivity regime [Cowie & McKee 1977] is not reached). The effective scattering frequency due to the development of electromagnetic instabilities (eq. [13]) may be higher than the binary collision rate, so that the conduction coefficient will be smaller, $\kappa \sim n_e v_{T,e}^2 L/(\beta V)$. Increased scattering will also inhibit the onset of the saturated regime.

There are a number of challenges that heating models should overcome. Primarily, the heating must be both widely distributed and gentle. It is hardly achievable with shocks, which provide very concentrated heating at the shock location, deposit most of the energy in the core, and generally contradict the observational absence of shock signatures. This, combined with low heat conductivity in the cores, leads to plasma overheating and the creation of inverted entropy gradients, contrary to observations (e.g., Voit & Bryan 2001).

The heating in the bound anisotropy model may be distributed. Consider a cluster with a typical density profile $\rho \propto 1/r$. Then if bremsstrahlung dominates over line emission, the cooling rate is $\propto r^{-2}$ (for nearly constant temperature in the cores). Since an energy flux from central source scales as $\propto r^{-2}$ as well, this implies that a heating rate should be independent of radius, and thus independent of the local plasma properties. Collisional dissipation clearly cannot produce this. On the other hand, if β is nearly constant, the heating rate will be nearly independent of radius. Thus, at least in principle, heating and cooling can be balanced in the bound anisotropy model.

One of the main drawbacks of many simulations of the ICM is that they use isotropic Spitzer viscosity. Examples in § 3 show that this can produce (at least locally) drastically incorrect results, which may either overestimate or underestimate the real collisional magnetoviscosity (we are not aware of any ICM-related simulations with anisotropic viscosity; see, however, Sharma et al. 2006). As for the value of the coefficient of viscosity, we argue that for binary collision it generally depends on electron and ion temperatures and dynamical timescales, while in case of marginal stability it is actually unrelated to the Spitzer value. Parameterization with respect to Spitzer may be useful, but we should not put too much physical emphasis on it.

REFERENCES

Begelman, M. C. 2004, in Coevolution of Black Holes and Galaxies, ed. L. C. Ho (Cambridge: Cambridge Univ. Press), 374

Borovsky, J. E. 1986, Phys. Fluids, 29, 3245

Braginskii, S. I. 1965, Rev. Plasma Phys., 1, 205

Carilli, C. L., & Taylor, G. B. 2002, ARA&A, 40, 319

Chew, G. F., Goldberger, M. L., & Low, F. E. 1956, Proc. R. Soc. London A, 236, 112

Cowie, L. L., & McKee, C. F. 1977, ApJ, 211, 135

Denton, R. E., Anderson, B. J., Gary, S. P., & Fuselier, S. A. 1994, J. Geophys. Res., 99, 11225

Gary, S. P., Li, H., O'Rourke, S., & Winske, D. 1998, J. Geophys. Res., 103, 14567

Gary, S. P., McKean, M. E., Winske, D., Anderson, B. J., Denton, R. E., & Fuselier, S. A. 1994, J. Geophys. Res., 99, 5903

Hollweg, J. V. 1985, J. Geophys. Res., 90, 7620

Kennel, C. F., & Petscheck, H. E. 1966, J. Geophys. Res., 71, 1

Kulsrud, R. M., Bernstein, I. B., Krusdal, M., Fanucci, J., & Ness, N. 1965, ApJ, 142, 491

Landau, L. D., & Lifshitz, E. M. 1975, Fluid Dynamics (Oxford: Pergamon Press)

———. 1982, The Electrodynamics of Continuous Media (Oxford: Pergamon Press)

Levinson, A., & Eichler, D. 1992, ApJ, 387, 212

Lloyd-Davies, E. J., Ponman, T. J., & Cannon, D. B. 2000, MNRAS, 315, 689

Manheimer, W. M., & Boris, J. P. 1977, Comments Plasma Phys. Controlled Fusion, 3, 15

McNamara, B. R., et al. 2000, ApJ, 534, L135

Peterson, J. R., & Fabian, A. C. 2006, Phys. Rep., 427, 1

Quest, K. B., & Shapiro, V. D. 1996, J. Geophys. Res., 101, 24457

Schekochihin, A. A., & Cowley, S. C. 2006, Phys. Plasmas, 13, 6501

Sharma, P., Hammett, G. W., Quataert, E., & Stone, J. M. 2006, ApJ, 637, 952

Voit, G. M., & Bryan, G. L. 2001, Nature, 414, 425