Shape Optimization of a Compressor Supporting Plate Based on Vibration Modes

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Introduction

- Supporting plate
Problem Definition

- The support base-plate of a refrigerator is responsible for the structural connection between the cabinet and the compressor, which is its primary function.

- However, the plate must attend some requirements, among them, to have low dynamic response during operation of the compressor. The correct dynamic response of the plate is important for filtering the vibrations of the compressor, reducing the energy transmitted to the cabinet, as well as for not allowing excessive noise radiation.

- The project of a base-plate with desirable vibration characteristics consists on allocating its structural modes as far as possible from the operational frequency and first harmonics.
Problem Definition

Simplified FE model

Model Validation (base-plate only)

\[(K - \lambda_i M)\Phi_i = 0\]
Problem Definition

Parameterization of the problem

- Mesh parameters = optimization variables.

\[
J = \begin{cases} 
J_j, & \text{for } j = 1, 2, 3, 4 \\
J_{j-4} + \text{thk}, & \text{for } j = 5, 6, 7, 8 
\end{cases}
\]

- Number of variables = number of the nodes on the plate bottom face.

\[
z_{\text{low}} \leq J_j \leq z_{\text{upper}}
\]
Problem Definition

First modes of the simplified FE model.

1. 11.75 Hz
2. 16.64 Hz
3. 44.29 Hz
4. 140.71 Hz
Problem Definition

Optimization problem: objective function

\[ \min f(z) \]
\[ \text{subject to } g(z) \leq 0 \]
\[ z_{low} \leq z_i \leq z_{upper} \]

The 3\(^{rd}\) natural frequency of the system is 44,3Hz, very close to the fundamental frequency of excitation. The authors suggest tuning this structural mode to 75Hz, avoiding the resonances near 50Hz. Consequently, due to the target value of this tuning, the 3\(^{rd}\) mode will be also far from 100Hz, the first excitation harmonic.

\[ f(z) = (\lambda_3 - 2\pi 75)^2 \]
Problem Definition

Optimization problem: constraint

- modification of twin nodes coordinates are enabled only for Z-direction and the plate thickness is considered constant;
- elementary volumes do not change along the process. So, a mass constraint, widely used in optimization procedures, cannot be used;
- this work proposes a measure of mesh distortion as a constraint for the optimization problem, defined as:

\[
g(z) = \frac{1}{N_e} \sum_{e=1}^{N_e} \frac{L_e}{L_e} = \frac{|n_2 - n_1| + |n_3 - n_2| + |n_4 - n_3| + |n_1 - n_4|}{4}
\]
Problem Definition

Optimization problem: sensitivity analysis

- Gradient Method: due to large number of variables.
  \[
  \frac{\partial \lambda_i}{\partial z_j} = \Phi_i^T \left( \frac{\partial K}{\partial z_j} - \lambda_i \frac{\partial M}{\partial z_j} \right) \Phi_i, \quad \text{(Haftka and Gurdal, 1992)}
  \]

- Node coordinates modification: only elements in the vicinity are distorted.
  \[
  \frac{\partial \lambda_i}{\partial z_j} = \sum_{e=1}^{N_v} \left( \Phi_{ie}^T \left( \frac{\partial K_e}{\partial z_j} - \lambda_i \frac{\partial M_e}{\partial z_j} \right) \Phi_{ie} \right)
  \]

- For SOLID45 non conforming:
  \[
  \frac{\partial M_e}{\partial z_j} = 0
  \]

\[
\frac{\partial K_e}{\partial z_j} = \sum_{p=1}^{N_p} \left( \frac{\partial B_e^T}{\partial z_j} C B_e(p) \det(J_e) + B_e^T C \frac{\partial B_e(p)}{\partial z_j} \det(J_e) + B_e(p)^T C B_e(p) \frac{\det(J_e)}{\partial z_j} \right) \alpha_p
\]
Problem Definition

Optimization problem: sensitivity analysis

Objective function:

\[
\frac{\partial f(z)}{\partial z_j} = 2(\lambda_3 - 2\pi 75) \frac{\partial \lambda_3}{\partial z_j}
\]

Constraint:

\[
\frac{\partial g(z)}{\partial z_j} = \frac{1}{N_e} \sum_{e=1}^{N_e} \frac{\partial \bar{L}_e}{\partial z_j}
\]

\[
\frac{\partial \bar{L}_e}{\partial z_j} = \frac{1}{4} \left( \frac{1}{|n_2-n_j|} (z_2 - z_j) + \frac{1}{|n_j-n_4|} (z_j - z_4) \right)
\]
Numerical Procedure

Large number of variables: gradient-based optimization algorithm (Method of Moving Asymptotes – MMA from Svanberg, 1987)

i) input initial design values;
ii) enter the optimization loop. It calls the FEM program (after writing the current iteration values of design variables to a file) to run a modal analysis in batch mode and return to optimization procedure the eigenvalues and eigenvectors;
iii) calculate the sensitivities in the interface module and send these information to MMA;
iv) if attended the stopping criteria, stop the optimization and plot the optimized design. Otherwise, repeat from (ii).
Numerical Procedure

**Numerical instabilities:** this problem occurs due to the symmetry of the geometry in Z-direction, since a node movement in negative or positive sense leads to the same values on derivatives of the objective function.

<table>
<thead>
<tr>
<th>Initial design: flat plate</th>
<th>Initial design: proportional to a mode shape</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Flat plate" /></td>
<td><img src="image2" alt="Mode shape" /></td>
</tr>
<tr>
<td>Zero initial deformation</td>
<td>Slight initial deformation based on a mode shape (graphically amplified)</td>
</tr>
<tr>
<td>Calculated derivatives of objective function</td>
<td>Calculated derivatives of objective function</td>
</tr>
</tbody>
</table>

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Numerical Procedure

**Numerical instabilities:** in order to avoid abrupt variation on the objective function derivatives along the design domain, this work suggests the use of a filter applied to obtain a weighted local average of the derivative values:

\[
\frac{\partial f(z)}{\partial z_j} = \frac{1}{\sum_{i=1}^{N_{\text{filt}}} H_i} \sum_{i=1}^{N_{\text{filt}}} H_i \frac{\partial f(z)}{\partial z_j}
\]
Results

\[ z_{\text{lower}} = -10\text{mm} \]
\[ z_{\text{upper}} = 10\text{mm} \]

\[ g(z) < 100,1\% \text{ of its initial value} \]

The 3\textsuperscript{th} mode is used to create the initial deformation.
Results

Final design of the base-plate (amplified scale).

Final design of the base-plate (FE model).
Harmonic Analysis

\( F = 1 \text{N on the mass} \)
\( \xi = 0.03\% \)
Material: Steel

The great advantage of methods based on the movement of nodes is the possibility of creating unconventional geometries. The application of this method, however, is not a simple task, and varies depending on the problem studied.
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