SCHOOL OF 1360
CIVIL ENGINEERING

INDIANA

DEPARTMENT OF HIGHWAYS

JOINT HIGHWAY RESEARCH PROJECT

JHRP-85-4

STRUCTURAL ANALYSIS OF RIGID
PAVEMENTS WITH PUMPING

FINAL INFORMATIONAL REPORT

Jesus Larralde

PURDUE UNIVERSITY

JOINT HIGHWAY RESEARCH PROJECT

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PURDUE UNIVERSITY

SCHOOL OF CIVIL ENGINEERING

Summary Informational Report

STRUCTURAL ANALYSIS OF RIGID PAVEMENTS WITH PUMPING

To: R. L. Eskew, Chairman
Joint Highway Research Project
Advisory Board

January 30, 1985

File: 5-10

From: H. L. Michael, Director
Joint Highway Research Project

The attached report is an informational one which describes the new Purdue Method for Analysis of Rigid Pavements (PMARP). This computer model was developed under a FHWA contract with Purdue University. The report is being made available to the IDOH because of its potential value to them.

The analysis is a non-linear finite element one, which can accommodate a number of long-term effects. Included are: fatigue damage and cracking in the concrete; decay in efficiency of transfer devices across joints; and pumping and loss of support in sublayers. Outputs include: pavement deflections and stresses; changes in sublayer moduli; and predictions of quantities of the various kinds of potential pavement damage.

Comments and questions relative to the Report should be directed to Prof. C. W. Lovell, Grissom Hall, Purdue University, phone (317) 494-5034.

Respectfully submitted,

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SUMMARY INFORMATIONAL REPORT

STRUCTURAL ANALYSIS OF
RIGID PAVEMENTS WITH PUMPING

by

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for

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This research was performed by the author under the direction of Professor C.W. Lovell, Principal Investigator, under a research contract between Purdue University and the Federal Highway Administration. The contents do not necessarily reflect the official views or policies of the Federal Highway Administration. The report does not constitute a standard specification or regulation.

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ACKNOWLEDGMENTS

The author wishes to thank his major professor Dr. W.F. Chen, and Dr. C.W. Lovell, Dr. C.D. Sutton, and Dr. C.F. Scholer for their precious guidance, assistance, and understanding during the development of this research. The author also would like to thank Professor E.J. Yoder, deceased, for his guidance in the first stage of this research.

The assistance of the Federal Highway Administration in sponsoring this project, as well as the assistance of CONACYT and IMCYC are gratefully acknowledged.

Finally, the author extends his heartfelt thanks to Dr. Claudia Dolinsky for her moral support and friendship.

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LIST OF SYMBOLS AND ABBREVIATIONS

$\{A\}$	modified nodal displacements
AASHO	American Association of State Highway Officials
AASHTO	American Association of State Highway and Transportation Officials
ACI	American Concrete Institute
ASCE	American Society of Civil Engineers
$\{a\}$	nodal displacement vector
a^e	nodal displacements at element e
$[B]$	strain-displacement matrix
$[C]$	coefficient matrix
CONACYT	Consejo Nacional de Ciencia y Tecnologia
CRCP	Continuously Reinforced Concrete Pavements
$[D]$	matrix of flexural and torsional rigidities
D	flexural rigidity of plate
D_x, D_y, D_l, D_{xy}	flexural and torsional rigidities of plate
DCI	dowel-concrete interaction factor
D_d	dowel diameter
DA	damaged area
d	reinforcing bar diameter
E	modulus of elasticity

ESAL	18K-equivalent single axle load applications
E_x, E_y	moduli of elasticity in x and y directions
{f}	vector of external forces
f_r	relative flexural stress
f_c	compressive strength of concrete
f_t	tensile strength of concrete
f_{rb}	relative load acting on dowel
F	force vector
F^e	nodal forces at element e
G	modulus of dowel support
HRB	Highway Research Board
I	moment of inertia
IMCYC	Instituto Mexicano del Cemento y del Concreto
[K]	stiffness matrix of whole structure
K_1, K_2, K_3, K_4	coefficient matrices
k^e	stiffness matrix of element e
k	modulus of subgrade reaction
k_r	resilient modulus of subgrade reaction
L	embedded length of dowel bar
LTE	load transfer efficiency
l	radius of relative stiffness
{M}	vector of moments per unit length
M_x, M_y	bending moments per unit distance in X and Y directions
M_{xy}	twisting moment per unit distance
MR	modulus of rupture
mils	0.001 in.

NPI	normalized pumping index
P	concentrated force
P_c	crack load on dowel
P_D	concentrated load acting on dowel
PCA	Portland Cement Association
PJP	Plain Jointed Pavements
PCP	Prestressed Concrete Pavement
q	distributed transverse load
RF	reduction factor
R^2	coefficient of correlation
t	thickness
u	displacement vector
w	plate deflection
w_j	joint width
W_{ext}	external work
$W_{int.}$	internal work
α	coefficient of thermal expansion
γ	shear strain
ΔT	temperature gradient
ϵ	normal strain
μ	Poisson's ratio
σ	normal stress
τ	shear stress
p	percentage of reinforcement

ABSTRACT

Larralde, Jesus. Ph.D., Purdue University, December 1984. Structural Analysis of Rigid Pavements with Pumping. Major Professor: Dr. Wai Fah Chen.

Pumping has been considered a major contributor to premature failure in rigid pavements. The traditional design procedures, based on a stress-fatigue criterion, have been unsuccessful in providing pavements which accomplish their intended service lives. In addition to the stress-fatigue criteria, an erosion criteria must be considered. Concrete pavements have been analyzed using the Westergaard's equations, or charts developed from those equations. These methods assume full contact between the slab and subgrade, as well as continuity in the whole pavement structure. Recently, with finite element techniques, the consideration of discontinuities in the pavement structure has been possible. Nevertheless, in the traditional analysis methods, closed form solutions or finite element methods, the reduction of pavement strength resulting from the damage caused by traffic has not been considered. Fatigue damage caused by repetitive load applications reduces the strength and stiffness of the pavement.

In this thesis, a nonlinear finite element method for analysis of rigid pavements which considers the decay of the their stiffness properties due to fatigue damage is presented. The damage caused in the concrete slab, the load transfer devices, and the pavement foundation are considered. The computer scheme has been developed from the previously existing ILLISLAB code. The computer implementation presented herein, called PMARP, has the following main features: nonlinear finite element method; orthotropic plate; consideration of reinforcement in concrete slab; resilient modulus of subgrade reaction; pumping development; determination of states of stresses and deformations at all nodal points; and determination of state of pavement damage. This method can be used for design and for prediction of maintenance requirements of rigid pavements.

CHAPTER 1 INTRODUCTION

1.1 Introduction

Historically, pavements have been divided into two general types [1]: flexible pavements, which generally consist of a relatively thin wearing surface built over a base course and subbase course, which in turn rest upon the compacted subgrade; and rigid pavements made up of a Portland cement concrete slab and sublayers termed bases or subbases between the slab and subgrade (Fig. 1.1). The research reported in this thesis deals with rigid pavements only.

Figure 1.1 Cross section of rigid pavement.

1.2 Statement of the Problem

Since the 1940's, pumping has been considered as a major contributor to failure in concrete pavements. It has been reported [2,3,4,5] that most of the failures in concrete pavements have been preceded by pumping of the material underneath the concrete slab. The traditional design procedures, based on a stress-fatigue criterion, have been unsuccessful in providing pavements which accomplish their intended service lives. Premature failure due to excessive pumping, erosion of sublayers and joint faulting have occurred. Therefore, in addition to the stress-fatigue criteria, erosion criteria must be considered.

As part of the design process, the structural analysis of rigid pavements has been used to determine the stresses produced by certain standard loads at specified critical points of the pavement. Thickness design has been based on limiting these critical stresses at appropriate levels to control fatigue damage of concrete.

Generally, concrete pavements have been analyzed using the Westergaard's equations [6] or charts developed from these equations [7]. These methods assume full contact between the slab and sublayer, as well as continuity in the slab. More recently, the development of finite element analysis has allowed the consideration of discontinuities such as

joints, cracks, and loss of sublayer support. Additionally, variable wheel placement, variable material properties, and variable load transfer at joints can be considered with this method.

Nevertheless, the reduction of strength due to the combined effects of load repetition and erosion has not been considered. Packard and Tayabji [8] have indicated that in addition to the failure of pavements caused by excessive fatigue cracking of the concrete slab, pavements can also fail due to excessive pumping and erosion of the sublayers.

The conventional methods for structural analysis of concrete pavements do not consider the variation in the pavement properties caused by repetitive application of loads. These methods are conservative in the sense that they assume the geometric and mechanical properties to remain at the same values through the life of the pavement.

The major limitation in the analysis method developed by Westergaard, or those based on his equations, is that it is applicable only if the analyzed structure is continuous. A rigid pavement is composed of several elements with different mechanical behaviors. In general, the pavement can be divided into a concrete slab, sublayers and load transfer devices. Each one of these components behaves in a completely different manner, therefore, it is impossible

to use a general differential equation to represent the behavior of the whole structure. Additionally, the existence of joints, cracks, and gaps between slab and sublayers is very common which makes the analysis more complex and the solution in a closed form impossible. Therefore, these methods are used only to check results obtained with other methods using simple, idealized cases.

One of the main causes of pavement failures is the deterioration and gradual weakening of the pavement components. Repetitive loading produces microcracking in the slab as well as erosion and voids in the sublayers. This reduces the resistance of the pavement to the loads applied by traffic. The response of the pavement to a specific load is different throughout its entire service life. Cumulative damage reduces the general stiffness of the structure. Therefore, in the analysis of rigid pavements, especially if erosion and pumping are expected, the pavement components should be considered as having a stress dependent behavior. The response of the slab as well as that of the sublayers should be dependent upon the previous stress history applied on the pavement. Pavement properties should be updated every few years to take into account the reduction of its stiffness caused by exposure to traffic loads. In this way, a more exact calculation of stresses and deflections at different stages of the life of the pavement is accomplished. Therefore, a more precise

estimation of the number of allowable load repetitions (and thereby of service life) can be achieved. Then, the designer has more and better information for an economical analysis.

1.3 Objective

The main objective of the research presented here is to develop a method of structural analysis of concrete pavements which considers the discontinuities in the pavement structure, as well as the long-term effects produced by traffic. The research presented herein is part of project DT FH61-82-C-00035, sponsored by the Federal Highway Administration, to develop a method of design to prevent pumping in rigid pavements. This project is divided into four main tasks: (1) development of a simplified laboratory method for evaluating pump susceptibility of materials; (2) development of methods to detect voids under concrete pavements; (3) development of a theoretical procedure for structural analysis of concrete pavements; and (4) selection of methods of economic trade-off analyses. The research presented herein deals with the development of theoretical procedures for structural analysis of rigid pavements.

To accomplish the objective, a non-linear finite element method for analysis of rigid pavements has been developed. This method of analysis takes into account the

damage produced in the concrete slab, in the load transfer devices, and in the sublayers by the repetitive application of traffic loads. The computer implementation of the method is based on the computer method ILLISLAB previously developed at the University of Illinois [9]. In the method presented herein, the pavement components are assumed to have a stress- or strain-dependent behavior, and the concrete slab is considered as an orthotropic plate.

1.4 Thesis Arrangement

The present thesis is divided into two parts. In the first part, consisting of Chapters 2,3, and 4, the available methods for analysis of concrete pavements are analyzed. Chapter 2 is a literature review of the methods of structural analysis in which closed form solutions, influence charts, and finite element methods are presented. In Chapter 3 a theoretical evaluation of these methods is presented, while Chapter 4 deals with a numerical evaluation of them. In the second part, which consists of Chapters 5, 6, and 7, a finite element method for analysis of rigid pavements with fatigue in the concrete slab, fatigue in the load transfer devices, and pumping is presented. In Chapter 5, the method of analysis and the computer implementation are presented. Numerical results obtained using the proposed method are also given.

CHAPTER 2 LITERATURE REVIEW

2.1 Introduction

For over 50 years, efforts have been made to develop reliable analysis techniques to improve the design and performance of concrete pavements. First attempts to analyze pavements were made by Westergaard in the early 1920's [10]. Later Pickett and Ray in 1951 [7] used the work done by Westergaard to develop influence charts which were used by The Portland Cement Association for the design of highway and airport pavements. Since the 1960's discrete elements and finite element methods have been used. In this chapter, the development of the different methods for structural analysis of rigid pavements is illustrated. Also, a brief description of the most common methods for design of rigid pavements is presented.

2.1.1 Closed Form Solutions.

Closed form solutions of slabs on elastic sublayers were obtained by Westergaard [10,11,12,13], Hogg [14], Volterra [15], Bergstrom [16], and Timoshenko [17].

These solutions were based on the differential equation developed by Lagrange for elastic isotropic thin plates.

This equation is as follows:

$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D} \quad (2.1)$$

Where q is the distributed lateral load, D is the flexural rigidity given by:

$$D = \frac{Et^3}{12(1-\mu^2)} \quad (2.2)$$

w is the deflection at the slab with elastic properties E and μ .

For a plate resting on an elastic sublayers, the differential equation becomes:

$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q-kw}{D} \quad (2.3)$$

Where k is the so-called modulus of subgrade reaction. Note that the term "subgrade" is used regardless of the proper designation of the sublayer in the pavement section.

Even though Westergaard did not provide details of the way this equation was solved, an approximate expression for the displacement in the form of a exponential series was assumed. Using the first two terms of these series, the following equation was obtained

$$w = \frac{P\beta^2}{k} (1.1e^{-\beta x} - 0.88c_1 e^{-2\beta x}) \quad (2.4)$$

Where, $\beta = (k/D)^{1/4}$, and P is the total load. Westergaard obtained expressions for stresses caused by three loading cases: 1) load applied near the corner of a large rectangular slab; 2) load applied near the edge at a considerable distance from the corner; and 3) load applied at the interior of a large slab. These expressions are as follows:

$$\sigma_c = \frac{3P}{t^2} \left[1 - \left(\frac{a\sqrt{2}}{1} \right)^{0.6} \right] \quad (2.5)$$

$$\sigma_e = 0.57 \frac{P}{t^2} \left[4 \log \left(\frac{1}{b} \right) + 0.3593 \right] \quad (2.6)$$

$$\sigma_i = 0.31 \frac{P}{t^2} \left[4 \log \left(\frac{1}{b} \right) + 1.0693 \right] \quad (2.7)$$

where a is the radius of area of load contact (in.). This area is considered circular, in the cases of corner and interior loadings, and semicircular in the case of edge load. The value of b is given by:

$b = \sqrt{1.2a^2 + t^2} - 0.675t$. The quantity 1 , radius of relative stiffness, is given by:

$$1 = \left(\frac{Et^3}{12(1-\mu^2)k} \right)^{1/4} \quad (2.8)$$

The above equations were developed using the following assumptions:

The slab acts as a homogeneous isotropic linear elastic solid with full support from the sublayer.

The reaction of the sublayer at a given point is vertical and proportional to the slab deflection at that specific point. This is called a Winkler foundation, in which the constant of proportionality is called the modulus of subgrade reaction k . The load is distributed uniformly over a circular or semicircular area.

The Winkler foundation assumption was modified in 1938 by Hogg [14], and Holl [18]. They considered the sublayer acting as an elastic solid instead of the Winkler foundation. These two researchers solved the problem of a thin slab of infinite size supported by a semi-infinite elastic solid. However, they only considered symmetrical loading. Pickett et al [19] later obtained influence charts to determine stresses and deflections for any distribution of load. These charts are given in reference [19].

Even though this method can be used for verification purposes in simple problems, the major limitation is that discontinuities and variable properties cannot be considered. It assumes full support conditions; therefore, loss of support or voids cannot be modeled. Variations in the thickness of the slab or in its elastic properties cannot be modeled either. Due to these limitations and with the development of computers, other techniques based on numerical methods began to be studied. Among these, the finite element method has had great acceptance.

2.1.2 Analysis of Joints

Joints in rigid pavements are necessary to facilitate construction and to control the formation of cracks. The need for joints has been recognized since the early 1900's. Initially, joints were simple openings between slabs, load transfer devices appeared around 1917, and from 1912 to 1936 different types of joints with and without load transfer devices were developed and even patented.

One of the first attempts to analyze joints was carried out by Timoshenko and Lessels in 1925 [20]. Considering the case of a semi-infinite beam supported on an elastic sublayers, they obtained mathematical expressions for loads and deflections for a bar encased in concrete. Teller and Sutherland [21], Friberg [22], Sutherland and Cashell [23], Kushing and Fremont [24], Giggin [25], Finney and Fremont [26], Marcus [27], and Teller and Cashell [28], are among the researchers concerned with the behavior of joints. Teller and Sutherland [21] studied the effects caused by variations in joint width, dowel looseness, and dowel stiffness on the performance of joints. The importance of placing the dowel bars correctly, that is, perpendicular to the cross section of the pavement, was studied by Smith and Benham [29]. They concluded that the bars may be distorted or the concrete damaged in the vicinity of the bars if they are not, within certain limits, perpendicular to the joints.

Friberg [22] studied the bearing stress caused by dowel bars, using the equations developed by Timoshenko and Lessels [20] and a Winkler foundation. With that model, stresses produced by the bar on the surrounding concrete of the order of 68.9MPa (10 000 psi) were obtained. Later, Marcus [27], using the same analysis and considering a three dimensional state of stresses, obtained lower stresses. He stated that the primary cause of radial cracks is the principal tensile stress resulting from the tensile and shear stresses present at the bar-concrete interface.

The effect of repetitive loading on dowel joints was studied by Teller and Cashell [28]. They indicated that performance under single loading is no measure of performance under repeated loading. With a test in which an alternating load is applied to either side of the joint of a slab specimen, the effects of dowel diameter, dowel embedment, and width of joint opening were studied. It was indicated that in order to transfer certain percentages of the load across the joint, the dowel diameter should be specified proportionally to the slab thickness. Repeated loading developed dowel looseness which has an important effect on load transfer efficiency.

A clear understanding of the behaviour of joints, as well as of the whole pavement structure, is important in the structural analysis. A mechanistic approach, in which the

pavement structure is divided into pieces which then are modeled by appropriate elements, appears to be the way to proceed. Finite element methods allow such an approach.

2.1.3 Finite Element Methods.

The structural analysis of rigid pavements is a complex problem. Before the development of the numerical methods and the availability of high speed computers, pavements were analyzed by the methods developed by Westergaard or derivations of such methods. These so called "closed form solutions" are subject to severely limiting assumptions. With these assumptions, pavements are considered as continuous with infinite dimensions, and with full contact between slab and subbase. Therefore, the analysis is unrealistic since it is not possible to analyze pavements with cracks, joints, or variable support conditions.

The limitations imposed by these solutions prompted the development of different analysis techniques. Rather than representing pavements as idealized structures they should be represented in a mechanistic way. Newmark [30] proposed a numerical technique for analyzing elastic bodies. Clough [31] and Turner [32] developed a finite element approach to the analysis of complex structures, including plane stress analysis. Finite element analysis of plates and slabs resting on elastic sublayers were developed by Zienkiewicz and Cheung [33], Hudson and Matlock [34], Huang and Wang [35], and Huang [36,37].

In the method developed by Zienkiewicz and Cheung, the concrete slab is divided into rectangular finite elements treated as elastic isotropic plates. These elements are connected to each other at their corners. Each element has three degrees of freedom at each corner; a lateral deflection in the Z direction, and two rotations about perpendicular axes X and Y. These 12 quantities constitute the nodal displacements for the given element ; u. Similarly, at every node there are three forces; two concentrated moments and a lateral force; F . The nodal forces and displacements can be related as:

$$F^e = k^e a^e \quad (2.9)$$

The matrix k^e , composed of 12x12 elements, represents the stiffness matrix for the element e . Putting together the stiffness matrices of all the elements, equilibrium equations at the joints can be written, then:

$$\left\{ \begin{matrix} f \end{matrix} \right\} = [K] \left\{ \begin{matrix} a \end{matrix} \right\} \quad (2.10)$$

$$\left\{ \begin{matrix} f \end{matrix} \right\} = \left\{ \begin{matrix} f_1 \\ f_2 \\ . \\ . \end{matrix} \right\} \quad (2.11)$$

$$a = \begin{Bmatrix} a_1 \\ a_2 \\ \vdots \\ \vdots \end{Bmatrix} \quad (2.12)$$

$$[K] = \sum_{e=1}^{e=n} k^e \quad (2.13)$$

$$e=1, 2, 3, \dots$$

Where $f_1, f_2, f_3, \dots$ are all external forces; $a_1, a_2, \dots$ are all nodal displacements; And n is the number of elements.

If in the set of simultaneous equations, $\begin{Bmatrix} f \end{Bmatrix} = [K] \begin{Bmatrix} a \end{Bmatrix}$, the boundary conditions in terms of forces and displacements are included, the solution of the problem reduces to the solution of such a system of equations.

To determine k^e , Zienkiewicz and Cheung used the principle of virtual work, where for a virtual displacement the external work must be equal to the internal work.

$$W_{\text{ext}} = \left\{ \delta a^e \right\}^T F^e \quad (2.14)$$

Where δa^e is a virtual displacement which can be taken as I, and

$$W_{\text{ext}} = I F^e = F^e \quad I = \text{identity-matrix} \quad (2.15)$$

To compute the internal work, the following expressions developed by Timoshenko are used:

$$M_x = -D_x \frac{\partial^2 w}{\partial x^2} D_1 \frac{\partial^2 w}{\partial y^2} \quad (2.16)$$

$$M_y = -D_y \frac{\partial^2 w}{\partial y^2} D_1 \frac{\partial^2 w}{\partial x^2} \quad (2.17)$$

$$M_{xy} = 2D_{xy} \frac{\partial^2 w}{\partial x \partial y} \quad (2.18)$$

or simply:

$$M = [D] x \quad x = \text{vector-of-curvatures} \quad (2.19)$$

Then:

$$W_{in.} = \iint \left\{ \delta x \right\}^T M \, dx dy \quad (2.20)$$

Where:

$$\left\{ \delta x \right\} = \text{virtual curvatures and twists} = [B][C]^{-1} \left\{ \delta e \right\}^e$$

And:

$$\left\{ \delta x \right\} = [B][C]^{-1} I = [B][C]^{-1}$$

$[B][C]$ relates the curvature to the nodal displacements, therefore:

$$F^e = \iint \left\{ [B][C]^{-1} \right\}^T [D][B][C]^{-1} u^e \, dx dy \quad (2.21)$$

$$F^e = [C]^{-1} \left[\iint [B]^T [D] [B] \, dx dy \right] [C]^{-1} \left\{ a^e \right\}$$

Then:

$$k^e = [C]^{-1} \left[\iint [B]^T [D] [B] \, dx dy \right] [C]^{-1} \quad (2.22)$$

This matrix for a rectangular element is given in the paper by Zienkiewicz and Cheung [33].

Nodal forces are obtained by assigning forces equivalent to the lateral distributed or concentrated loads. This assignment can be done by simple statics, or in such a way that during any virtual displacement, the work done by the actual forces and that done by the equivalent concentrated forces is the same.

To analyze slabs on elastic foundations, an additional element was introduced to represent the support. In a paper published in 1965, Cheung and Zienkiewicz [38] showed that the stiffness matrix coefficients of the foundation are simply added to those of the slab element. They obtained expressions for these coefficients considering the sublayers as a Winkler type or as an isotropic elastic solid. For the Winkler case, nodal forces and displacements are related as:

$$\left\{ f \right\} = ab \, k \begin{bmatrix} \alpha \end{bmatrix} \left\{ w \right\} \quad (2.23)$$

In which a and b are the sides of the element, k is the modulus of subgrade reaction, w represents the deflections at the nodes, and α values are coefficients to account for the area contributing to the nodal forces.

For the isotropic elastic solid, Cheung and Zienkiewicz used the Boussinesq equation, arriving at the following expression:

$$w = \frac{1-\nu^2}{\pi E a} [\phi_t] \{f\} \quad (2.24)$$

where ν and E are the elastic properties of the foundation, and ϕ_t is the flexibility matrix of the foundation which is obtained from:

$$w_{ii} = \frac{1-\nu^2}{a\pi E_o} \phi_{ii} P_i \quad (2.25)$$

ϕ_{ii} values are coefficients which depend on $\frac{b}{a}$.

The sublayers matrix and the plate matrix give a set of simultaneous equations for the whole pavement system. The solution of these equations provides the nodal displacements and with these moments, stresses and reaction pressure can be determined.

In the method developed by Hudson and Matlock [34], in 1966, the pavement slab is defined by a finite-element model consisting of bars, springs, elastic blocks and torsion bars. Combining beams in the X and Y directions, a

grid-beam system is formed which, coupled with torsional bars, represent the concrete slab. The beams are connected with elastic joints and at these joints the lateral loads are applied. The sublayer is represented by spring elements connected with the slab at the beam joints.

Expressing the equilibrium equations for forces and moments at the joints, a set of simultaneous equations for the whole pavement is formed. The solution of these equations provides the deflections at the nodes, and with these the complete structure can be solved.

Huang and Wang [35] considered the presence of doweled joints in a finite element method essentially the same as the one developed by Zienkiewicz and Cheung. For the analysis of doweled concrete pavements, the case of a two slab system connected with dowel bars at the joint was considered. Then, the equations for the system were expressed following the method of Zienkiewicz and Cheung and considering the joint as a discontinuity. This modification destroys the symmetry of the stiffness matrix, and therefore the total matrix has to be stored. In this method by Huang and Wang, the sublayers is considered as a Winkler type.

Later in 1974, Huang [36,37] presented an iterative method for determining the stresses and deflections in pavements in which the sublayer behaves as an elastic solid.

Moreover, the effect of temperature gradients in the slab was considered.

The basic element is the plate element presented by Zienkiewicz and Cheung. Temperature warping is introduced by replacing the nodal displacements, $\{a\}$, by $\{a\} = \{A\}$, in which:

$$\{A\} = \begin{Bmatrix} c_i \\ 0 \\ 0 \end{Bmatrix} \quad (2.26)$$

And:

$$c_i = \alpha \Delta t \frac{d_i^2}{2t} \quad (2.27)$$

Where, c is the amount of warping at node i , α is the coefficient of thermal expansion of concrete, Δt is the temperature differential, between top and bottom of slab, d_i is the distance from node i to center of slab, and t is the slab thickness.

In this model, the resultant set of simultaneous equations is not symmetric. To solve this system, Huang developed an iterative scheme to make the stiffness matrix banded, facilitating its storage and solution.

In the method presented by Huang and Wang, [36,37] the effect of partial contact between the slab and the sub-layer is introduced by simply deleting the reactive forces at the nodes assumed not in contact. However, in

actual conditions a node not in contact with the subbase may come in contact after loading, and vice versa. To take this into account, the same investigators, later in 1974, presented a general method to consider both full and partial contact. Assuming a Winkler foundation, two cases of partial contact were considered. In the first case, there is no pumping or plastic deformation in the sublayer. Therefore, the springs for the sublayer elements are assumed of equal length. In the second case, pumping or plastic deformation can be introduced by using springs of unequal length. In this way, voids can be preassigned to areas where excessive deformation in the sublayer occurs. Temperature is considered in the same way as in the method presented by Huang [36,37].

Another improvement in the finite element modeling of rigid pavements was introduced by Tabatabaie and Barenberg [9]. They developed a two dimensional finite element model, called ILLISLAB, based on a plate element similar to that developed by Zienkiewicz and Cheung. The concrete slab is considered in the same way as in Zienkiewicz's model, but additionally a second sublayer can be integrated into the system. The slab and top sublayer can be considered to be perfectly bonded or not bonded. perfect bond is assumed between the concrete slab and the top sublayer, then an equivalent transformed section is used. This transformed section is treated as the original plate

element. In the case of no bond, the stiffness matrix for the bottom sublayer and slab are simply added to each other.

Dowel bars are modeled by a bar element with two degrees of freedom at each node. Thereby the dowels can transfer a vertical force and a moment. Aggregate interlock and keyway are modeled by spring elements able to transfer shear .

The overall structural stiffness matrix is obtained by superimposing the matrix of the individual elements. Generalized deflections and stresses are calculated by solving the set of simultaneous equations :

$$\{f\} = [K]\{a\} \quad (2.28)$$

where $\{f\}$ represents the equivalent nodal forces for a uniformly distributed load over a rectangular section of the concrete slab. Values of $\{a\}$ are the resultant nodal displacements, and $[K]$ is the overall structural stiffness matrix. In this method, temperature effects are not considered.

Similar models have been presented by Tayabji and Colley [39], and Chou [40]. As in the case of the model developed by Tabatabaie and Barenberg, the basic element is the rectangular thin plate element given by Zienkiewicz and Cheung. The slab is divided into rectangular finite

elements with three degrees of freedom per node. The sublayer reactive forces act at the nodes. The stiffness matrix for the plate and sublayer are simply added to each other. In the model by Tayabji and Colley, called JSLAB, the sublayer is assumed as a Winkler model acting as a series of uniformly distributed spring elements. In this model, dowels are represented by beam elements with two degrees of freedom per node: vertical displacement and rotation. Aggregate interlock and keyways are represented by spring elements by means of which only vertical load is transferred across the joint.

In the JSLAB model, a linear temperature gradient is considered by application of moment along slab edges. The moment per unit width is:

$$E\alpha \Delta T \frac{t^3}{12} \quad (2.29)$$

where t is the slab thickness, α is the coefficient of thermal expansion, ΔT is the temperature gradient, and E_c is the modulus of elasticity of concrete. An iterative analysis is used to establish support conditions. For each iteration, a check of contact between slab and sublayer is accomplished, making the subgrade modulus equal to zero where contact is lost.

The models developed by Chou [40], called WESLIQID and WESLAYER, are also based on the plate element developed by Zienkiewicz and Cheung. WESLAYER assumes that the

sublayer acts as a Winkler foundation, while the WESLAYER considers it as an elastic layered solid. The Winkler foundation is modeled by spring elements. In the case of the elastic foundation (WESLAYER), Burmister's equation is used to compute the stiffness matrix for the sublayer. In this case, the combination of the plate and sublayer matrices gives a non banded matrix. To solve this, Chou applies the same iterative scheme presented by Huang [36,37]. The two programs proceed in an iterative manner, assuming at the beginning full contact between slab and sublayer, except at the nodes where gaps are preassigned. Sublayer contact conditions and convergence of deflections along joints are checked after each iteration. Connections can be considered as doveled, aggregate interlock, or keyway. For dowels, bar elements with shear and moment are used. For aggregate interlock and keyways, only shear is transferred using spring elements for modeling.

2.2 Existing Computer Models

Several computer models for analyzing concrete pavements have been developed since the 1960's. The models ILLISLAB, by Tabatabaie and Barenberg [9] ; JSLAB, by Tayabji and Colley [39] ; and WESLIQID and WESLAYER, by Chou [40] are principal examples. The assumptions, modeling techniques, and applications are briefly illustrated in this chapter. The models ILLISLAB and JSLAB are operational at Purdue University at the present time.

2.2.1 ILLISLAB

Assumptions.- In this model, the pavement system is divided into the following finite elements: rectangular plates for the slab, base or subbase, and overlay; spring elements for the subgrade; spring elements for joints with aggregate interlock and keyway; and bar elements for the dowel bars. The plate element uses the classical theory of medium thick plates, in which lateral forces are supported by the flexural strength of the plate. Lateral stresses and strains, as well as transverse shear deformations are neglected. Both perfect bond and no bond between concrete, base or subbase, and overlay can be assumed. Kirchhoff hypotheses are met for the plate element.

The subgrade is considered a Winkler type. It behaves as a dense liquid for which the deformation at a certain point depends only on the stress at the point. The stress-deformation relation in the subgrade is represented by k , the modulus of subgrade reaction. The values for this parameter can be varied from node to node and can be taken as zero at points with no support.

The dowel bars behave as linearly elastic bars located at the neutral axis of the plates. These bars transfer shear force and bending moment from the slab on one side of the joint to the slab on the opposite side.

For the case of aggregate interlock and keyway, the load is transferred by shear. This transfer is accomplished by a linearly elastic spring element.

The slab is modeled by a plate element with three degrees of freedom per node. These are a vertical force, a rotation around the x axis, and a rotation around the y axis. Corresponding to these degrees of freedom, there are three nodal forces: a vertical force, and two moments around x and y axes. For the case of perfect bond between layers, complete strain compatibility between layers is assumed. The plate considered in the analysis has a transverse section equal to the transformed section resulting from the base, concrete, and overlay sections. For the case of no bond between layers, the stiffness matrices of the concrete, the base or subbase and the overlay are simply superimposed by addition. The properties of the plate can be varied from element to element.

The Winkler foundation is modeled by a series of spring elements distributed uniformly under the slab. The spring elements have one degree of freedom per node: a vertical displacement. Corresponding to this, there is a vertical force. The force at each element is given by the deformation of the element times the spring constant. This spring constant is the modulus of subgrade reaction for the subgrade at the location of the element. The value for this parameter can be changed from node to node.

Dowels are modeled by bars with two degrees of freedom per node. These degrees of freedom are a vertical displacement and a rotation around a horizontal transverse axis. Therefore, the bar is able to transfer a vertical shear force and a moment. In addition, some looseness between the dowel bar and the surrounding concrete is modeled by a spring element whose constant is the so called concrete-dowel interaction factor.

Aggregate interlock and keyway joints are modeled by spring elements. These elements have a vertical displacement as single degree of freedom. Vertical forces are transferred across the joints by the springs.

Applications.- This model can be used in the analysis of cracked and jointed concrete pavements with base or subbase, overlay, and subgrade. Up to nine slabs can be analyzed with one longitudinal joint and two transverse joints. Connections can be with aggregate interlock, with keyway, or with dowel bars. Either perfect bond or no bond can be assumed between the concrete, the overlay, or the sublayers.

It is also possible to analyze concrete shoulders with and without tie bars, as well as concrete pavements with varying thicknesses and modulus of elasticity, and with varying modulus of support for the subgrade.

2.2.2 JSLAB

Assumptions.- This model is similar to the ILLISLAB model. The pavement structure is divided into the same types of elements: the slab, modeled by rectangular plate elements; the joints, modeled by thick beams or spring elements; and the sublayers, modeled by spring elements.

The plate elements for the slab can be composed of one or two layers with or without bond between them. The stiffness matrix for the plate is developed using the classical theory of plates with small deflections, assuming the material to be homogeneous and elastic. Strains in the vertical direction, as well as strains resulting from shear in vertical planes, are neglected. For the case of bond between layers, the transformed section concept is used; for the case of no bond, the stiffness matrices of the layers are superimposed.

With the JSLAB model, it is possible to include the effect of a linear temperature gradient in the concrete slab. This effect is considered by applying, along the slab edges, a moment whose magnitude depends on the temperature gradient, the modulus of elasticity, and the coefficient of thermal expansion. The model uses an iterative process to establish the support conditions due to curling. At locations where loss of support occurs, the subgrade modulus is made zero.

Dowel bars are assumed to behave like thick beams able to transfer shear and moment across the joints. The stiffness matrices for these beams is modified to account for the deformation that occurs in the concrete due to the action of the dowel. This is similar to the concrete-dowel interaction concept in the ILLISLAB. Aggregate interlock and keyway are represented by spring elements transferring only shear force. These elements behave like linearly elastic springs.

As in the ILLISLAB model, the subgrade is assumed to behave as a Winkler sublayers, for which stresses and deflections at different locations are independent. The subgrade is modeled by uniformly distributed springs with equal or different elasticity constants. These springs, with one degree of freedom, can have a spring constant equal to zero at locations where no support occurs.

Applications.— This model can be used in the analysis of jointed concrete pavements with up to nine slabs. Joints can be longitudinal or transverse with dowels, aggregate interlock, or keyway. Any configuration of static vertical loads can be applied as either distributed or concentrated loads. It is also possible to apply vertical deflections or moments at various nodes.

The model can analyze pavements when temperature and moisture gradients exist in the concrete slab. The effect of a moisture gradient is obtained by applying an equivalent temperature differential. The slab can be considered with one or two layers representing the concrete slab and base or subbase. Bond or no bond can be considered. The sublayers can be assumed to have different properties at different locations, making it possible to represent voids underneath the slab by making the subgrade modulus equal to zero at the nodes where no contact occurs.

2.2.3 WESLIQID and WESLAYER.

Assumptions.- These finite element methods are also based on the classical theory of thin plates with small deformations. The pavement structure is divided into the same elements as those used in ILLISLAB and JSLAB: slab, sublayer or sublayers, and joints. The slab is modeled by the rectangular plate element developed by Zienkiewicz and Cheung. The basic difference is in their modeling of the sublayers, which are considered as an elastic layered solid in the WESLAYER model; however, the WESLIQID model assumes a Winkler sublayers

In the WESLIQID model, the sublayers are considered as a dense liquid material modeled by a series of uniformly distributed elastic springs. The forces due to the reaction at the sublayers are simply added to the forces of the plate at any given node. These reactions have only a

vertical component which is added to the vertical component of the nodal forces in the plate.

In the WESLAYER model, the sublayers are considered as a layered elastic solid, in which the deflection at any given point depends, not only on the forces at the node, but also on the forces and deflections at other nodes. The stiffness matrix for the sublayers is obtained by inverting the flexibility matrix formed using the Burmister equation.

The WESLIQID and WESLAYER models can consider the effect of a linear temperature gradient in the concrete slab. The initial deflection due to the temperature gradient is computed using the coefficient of thermal expansion, the temperature gradient, the thickness, and the distance to the center of the slab where the deflection is zero. The equilibrium equations for the nodes are formed including the temperature deflections. Full contact or partial contact between slab and sublayer can be assumed.

The analysis of the joints is accomplished by combining different amounts of shear and moment transfers. The two models have three options for specifying shear transfer and one for moment. Specification of shear transfer can be accomplished by: a shear transfer efficiency factor, defined by the ratio of the vertical deflections between loaded and unloaded slabs along the joint; a spring -

constant modeled by imaginary springs along the joints; and by diameter and spacing of dowel bars, which is applicable only when steel bars are the only means of shear transfer. It is also possible to specify the conditions of looseness of the dowel bars by a dowel support factor.

Moment transfer across joints or cracks is accomplished in a two step procedure. First, the moments at nodes along the crack or joint are calculated. Then these moments are multiplied by the efficiency of moment transfer, and assigned to each slab as externally applied moments. In a second step the applied moments are included in the analysis. The definition of moment transfer is based on the rotations of the slabs adjacent to the joint or crack.

Applications.- These finite element models can be used in the determination of stresses and deflections in concrete pavements with cracks and joints. Analysis can be carried out for pavements with slabs made up of two layers of materials with different properties. The concrete slab can be either plain concrete or reinforced concrete. The pavement sublayers can be considered either as a Winkler sublayers or as a layered elastic solid. Full or partial support of the concrete slab can be assumed. Voids can be introduced by assuming different values for the properties of the sublayers. The joints can be doweled or connected with other load transfer devices like keyways and aggregate interlock. The WESLAYER model is limited to only two

slabs because of the large computer storage required. The analysis can be done considering external distributed or concentrated loads as well as thermal effects. With all these options, analysis can be performed for different types of concrete pavements and concrete shoulders.

2.3 Brief Summary of Design Methods

Design procedures for concrete pavements have been based on a stress fatigue criterion. Recently, the inclusion of an erosion criteria has been proposed by Packard and Tayabji [8]. In the following, an overview of the stress fatigue design methods is presented. The methods developed by the Portland Cement Association [41], AASHTO [42], and the ACI [43] are discussed as follows:

2.3.1 Portland Cement Association Method

The basic concept that has been used for design of both airfield and highway pavements is the concept of fatigue of concrete. In accordance with this concept, the number of load applications that a concrete beam sustains in flexure depends on the properties of the concrete and on the level of stresses. The Portland Cement Association obtained a table which relates the stress level with the number of load repetitions needed to produce failure [41]. This table shows that if the relative stress (stress divided by modulus of rupture) is less than 0.51, then concrete is able to sustain an unlimited number of stress

repetitions of this or lesser magnitude without failure. As the relative stress increases, the possible number of stress repetitions decreases.

In the design method developed by the Portland Cement Association, the thickness of the concrete slab is dependent upon the magnitude and number of load repetitions, the modulus of rupture of concrete, and the modulus of subgrade reaction. The number of load repetitions that a pavement sustains during its service life is estimated and categorized into axle-load groups. Assuming certain thickness and using Westergaard's equations, the stresses corresponding to each of the load categories are calculated. Each of these stresses is divided by the modulus of rupture of the chosen concrete giving the relative stresses. Using the PCA table, the number of allowable load repetitions is obtained for each category. The percentage of actual load applications with respect to the allowable number of load applications are calculated, and these percentages are summed. Theoretically, the design is adequate if the sum of these percentages is less than 100 percent. The calculation of stresses is made by the Westergaard equations or the Pickett and Ray tables [7]. Continuity and full support conditions are assumed.

2.3.2 AASHTO Method

The AASHTO method, developed after the AASHO Road Test, is also based on the concept of concrete fatigue. Empirical equations were developed from the Road Test to represent the relationship between loss in serviceability, traffic, and pavement thickness. In this method, the expected traffic mix is converted into equivalent 18-kip single axle loads. Given these equivalent load applications, the desired pavement thickness is determined on the basis of the desired working stresses and the value of the modulus of subgrade reaction, using the empirical relations obtained from the Road Test.

The total number of equivalent single-axle load applications is determined by dividing the traffic mix into various weight categories. Each category is multiplied by a load factor (these load factors were developed from the empirical equations of the Road Test to have equivalent effects on the performance of the pavement). Originally, the general Road Test equations were obtained for certain specific values for the modulus of elasticity, modulus of rupture of concrete, modulus of subgrade reaction, and environmental conditions. To account for conditions other than those that existed at the Road Test, the equations were modified by comparing the stresses measured on the test with the stresses calculated with the Westergaard, Spangler, and Pickett equations.

2.3.3 American Concrete Institute Method

The ACI committee 325 [43] presented a design method to determine the pavement thickness and amount of reinforcing steel for continuously reinforced concrete pavements. This method is based on some modifications of the AASHTO method. In continuously reinforced concrete pavements, the slab must resist stresses and deflections produced by applied traffic loads, and the function of the longitudinal steel is to keep the cracks in concrete tightly closed so that there is an effective load transfer across the cracks. In the ACI method a factor for considering the load transfer in the thickness design is included. Additionally, equations to determine the percentage of longitudinal stress are provided. This longitudinal steel is provided to control cracking due to shrinkage and temperature. The results obtained with both methods are comparable, and the difference depends on the value selected for the load transfer factor. The ACI committee 325 recommends a value of 2.2 for the load transfer factor. Assuming this value, the results obtained with the AASHTO and the ACI methods are similar.

The traditional design methods have been based on the assumption of a failure due to concrete fatigue. However, in fact, most of the failures in concrete pavements have occurred due to a different mechanism [2,8].

Most of the failures occurred as a consequence of erosion of the sublayers. Pumping, faulting, and slab breakup are the major problems. Therefore, a realistic design procedure should be developed considering the deterioration of pavements caused by erosion. The analysis method should also reflect this failure mechanism.

2.4 Summary

In the preceding paragraphs, the development of the techniques for structural analysis of concrete pavements has been illustrated. A review of literature published since the 1900's has been presented. This review was done by dividing the available information into three parts: literature related to the analysis with closed form methods; literature related to the analysis of joints; and literature related to finite element methods. Additionally the computer models ILLISLAB, JSLAB, WESLIQID, and WESLAYER were reviewed, and the assumptions and applications of these models were discussed.

CHAPTER 3 THEORETICAL EVALUATION OF EXISTING METHODS OF ANALYSIS

3.1 Introduction

In order to determine the abilities that the existing computer models have for analyzing the structural performance of rigid pavements with pumping, theoretical and numerical evaluations of such models were carried out. At the same time, the characteristics required in the development of an improved model were obtained.

3.2 Criteria of Model Evaluation

The models were studied in accordance with three different criteria as follows:

1. Evaluation in terms of their suitabilities for design.
2. Evaluation in terms of the modeling of the actual behavior of rigid pavements.
3. Numerical Evaluation.

The first two criteria are discussed in this chapter. Numerical evaluation is given in Chapter 4.

3.2.1 Evaluation for Design Purposes

The objective of the design process is to obtain a pavement with such properties, that under given environmental and traffic conditions provides a service life of a specified number of years. This service life is the period of time between the time at which the pavement is opened to traffic until a functional failure occurs. Yoder and Witczack [1] define functional failure as the state of a pavement which is no longer capable of carrying its intended function without causing discomfort to passengers, or without causing high stresses in the vehicles that pass over it. The generally accepted design procedures are based on the principle of limiting the flexural stresses in the concrete slab to safe values. With this, it is intended to avoid fatigue cracking caused by load repetitions. The PCA and ASSHTO design methods provide values for slab thicknesses such that for the predicted traffic, the number of stress repetitions does not exceed the fatigue resistance of concrete. The stress calculations are based upon the assumption of full support at all points, and at all instants in the service lives of the pavements.

In reality, through the service life of a pavement its stiffness properties change. The passage of certain loading over a pavement at the beginning of its life, produces an effect which is different from the one produced by the

same loads applied after several years of service. With time, microcracks can develop in the slab, decreasing its stiffness. Additionally, voids and loss of support can develop increasing stresses and deflections.

The computation of stresses and deflections in the structural analysis should be based on actual conditions. At the beginning of the service life, the assumption of full support is acceptable. However, after several years in service, the assumptions of full support and initial strength and modulus of elasticity are not valid.

The present methods of design have been based on the assumption that failure in rigid pavements occurs due to fatigue in the concrete slab. However, it has been observed [1,2,4] that most of the failures in rigid pavements are preceded by pumping of the material underneath the slab. Packard and Tayabji [8] have proposed a design method which considers the damage in the material under the slab. Generally, an improved method of design should consider fatigue in the slab, as well as the erosion in the sublayers.

As part of the design process, the structural analysis must be capable of determining the response (in terms of stresses, deflections, cracking, and damage in general) of the pavement structure to environmental and traffic actions. With given environment and traffic conditions,

the designer proposes certain geometry and material properties for the pavement. Next, the response of the proposed pavement is evaluated in terms of service life. Calculating the initial and maintenance costs of various alternatives, the designer chooses the best alternative, engaging in a "trade-off" process.

In order to perform the trade-off process with rigid pavements, the designer must be able to consider all the practical types of pavements. These types of pavements are: 1) plain, jointed pavements (PJP); 2) continuously reinforced concrete pavements (CRCP); 3) jointed, reinforced concrete pavements (JRCP); and prestressed concrete pavements (PCP). Yoder and Witczak [1] classify the joints as: 1) contraction joints; 2) expansion joints; 3) construction joints; and 4) warping joints. In accordance with their performance, the joints can be: 1) plain, dummy-groove joints; 2) doweled, dummy-groove joints; 3) doweled joints; 4) keyed joints; and 5) tied joints.

The section of a rigid pavement consists of the concrete slab and several sublayers (Fig. 1.1). These layers, which constitute the sublayers of the slab, can be composed of a stabilized or unstabilized base or subbase, and a subgrade. For these components of the pavement, many different combinations of thicknesses and strengths can be used. In the concrete slab, the thickness and strength can be varied. Values for highway slab thickness vary from

152mm (6 in.) to 305mm (12 in.), and for the compressive strength from 20.7 MPa (3000 psi) to 34.5 MPa (5000 psi) or higher. For the base or subbase, the desired strength can be obtained by varying the gradation of the granular material as well as by varying the compacting energy. Traditionally, the stiffness properties of the sublayers are measured by the value of the modulus of subgrade reaction. Recently however, it has been proposed [44] that a resilient modulus be used. The traditional modulus of subgrade reaction is obtained from a static test. This test does not reproduce the behavior of the sublayer when a moving load (traffic load) is applied. By contrast, the resilient modulus, which is obtained from an impact test and is strain dependent, represents more accurately that behavior. Typical values for the subgrade modulus vary from 27.1 MPa/m (100 pci), for a fine grained material, to 135.5 MPa/m (500 pci) or more for a granular well graded material. For the resilient modulus, values can vary from 81.3 MPa/m (300 pci) for a soft fine material, to 271. MPa/m (1000 pci) or more for a stiff granular material. Most of the time a base or subbase is provided. The modulus of elasticity of a stabilized sublayer may be from 6900. MPa (1×10^6 psi) to 13800 MPa (2×10^6 psi). For nonstabilized sublayers these values may be half of the values for stabilized ones.

In addition to considering the material properties and the geometric dimensions of the pavement, the designer has to consider the climatic and traffic conditions. The amount of rainfall and the daily and yearly temperature variations should be estimated. Traffic can be quantified by the number of vehicles of different weights that are expected to pass over the pavement during its service life. When pumping is expected, certain special conditions should be considered as well. It is important to study the erosion susceptibility and potential for formation of voids in the sublayers. Erosion of sublayers and subsequent formation of voids can modify the support conditions in the slab, causing increase in the stresses and deflections.

Finally, for the trade-off process, the estimation of the possible maintenance costs is important. The economical analysis of the pavement alternatives is on the basis of the total cost, which includes the initial cost plus the maintenance cost. Therefore, although the estimation of the initial cost is relatively simple, since the geometry and material properties are known, the estimation of the maintenance cost is more complex, since the actual maintenance work is not known in advance. The allocation of certain annual amounts of money for maintenance, regardless of the differing service conditions of the pavements is not realistic. Instead, cost should be allocated to each

alternative pavement in accordance with the expected conditions that the specific pavement will achieve after different time periods. Any estimation of the future state of the pavements, given the traffic and climatic conditions, will be of help in the estimation of the maintenance costs. The designer must be able to predict the service life of each of the proposed alternative pavements. Moreover, he has to predict the amount of maintenance that each alternative pavement will need after certain several years in service. To do so, the amount of damage that can occur in each alternative design, given the expected traffic, should be predicted. This damage can be evaluated in terms of permanent deflections, cracking, or faulting.

3.2.2 Evaluation in Terms of Pavement Behavior

Rigid pavements are made up of a Portland cement concrete slab overlying a series of less stiff layers. The essential difference between flexible and rigid pavements is the manner in which they distribute the loads over the subgrade. In rigid pavements, due to the relatively high stiffness of the concrete slab, loads are distributed over a wide area of the subgrade. Therefore, the major structural capacity is provided by the concrete slab. This slab can be a continuous slab, for the case of continuously reinforced concrete pavement, or formed by a series of slabs separated by joints and interconnected by load

transfer devices [1]. Generally, a subbase is placed between the subgrade and the slab to control pumping, frost action, and shrink and swell acting of the subgrade. Additionally, it provides drainage. The subbase is built beyond the edge of the pavement so that loads applied at the edge of the pavement are distributed over a wider area. To prevent pumping, subbases are made up of granular materials, or materials stabilized with lime, portland cement, or asphalt.

The subgrade, usually constructed by compacting the materials existing at the place of construction, provides the ultimate support of the road. Since the subgrade can have relatively low stiffness, the slab and the subbase must distribute the applied forces over a wider area to have low stresses at the subgrade level.

Damage to the pavement structure can be caused by several factors. In general, damage is caused by environmental factors such as presence of water and temperature variations, and traffic factors, such as repetitive stresses and deformations.

Water, either from rainfall or from underground, can enter into the sublayer materials changing their strengths and supporting capacities. Surface infiltration of water through cracks or joints, combined with repetitive application of loads, can cause pumping. Excessive water may

erode the sublayer materials when pressure applied upon the slab by traffic forces water to be ejected at high speed, eroding the subbase and creating voids. Further load repetitions, after some loss of support has occurred, produces cracking in the slab and eventually, functional failure.

Temperature variations can produce uniform longitudinal stresses at the cross section of the pavement, or flexural stresses caused by temperature differentials across the thickness of the slab. Longitudinal tensile stresses develop when the concrete slab is cooled down and its contraction is prevented by the friction with the subbase. Longitudinal compressive stresses occur when the temperature in the slab increases and the expansion of the slab is prevented by friction with the subbase or by adjacent expanded slabs.

Warping stresses result from uneven temperature distribution over the cross section of the slab. The same effect can occur as a result of uneven distribution of moisture in the slab. Differential drying shrinkage can, similarly, cause flexural stresses, but with magnitudes smaller than those due to temperature differential.

One of the major causes of damage in the pavement is the passage of vehicles. Different kinds of traffic loads are applied to concrete pavements during their service lives.

These loads vary in magnitude and in point of application. Consideration of traffic loads for highways is generally based on the determination of the equivalent damage effects of all the vehicle types expressed in terms of the number of load repetitions of a standard vehicle. The relative amount of damage caused by a particular type of vehicle is related to the damage caused by the standard vehicle by using equivalence factors [42],

Loads applied by traffic vary in location and magnitude. A plot of the distribution of load application with respect to the distance to the edge of the pavement, shows that most of the loadings occur within fairly well defined areas. Loads of the same magnitude but applied at different locations produce different effects in the pavement. Therefore, critical load locations should be considered; in general, these are at the corner of the slab, at joints, or at the outer edge of the slab.

Similarly, the magnitudes of the applied loads are variable. Vehicles of different weights and axle configurations transit over highway pavements. Therefore, a classification of vehicles in groups with similar weights and axles is necessary.

When a load is applied, the pavement responds, deflecting and transferring the pressure over its supporting material. Depending on the flexural stiffness of the slab

and the location of the load, the slab acquires certain deflected shapes, producing flexural stresses along its cross section and bearing stresses on the sublayers. The pressure applied on the slab is distributed through a larger area at the slab-subbase interface. The pressure is furthermore reduced as it is distributed through the thicknesses of the sublayers. When a vehicle passes over a point in the pavement, the slab is under flexural and shear stresses, while the sublayers are under the pressure of the deformed slab. The slab and sublayers are relieved of these stresses when the vehicle moves away. Therefore, with vehicles passing, the pavement structure at specific points is subjected to repetitive stressing.

In the slab, the cumulative effect of loads causes micro-cracking and reduction of strength. The concrete strength in general and the flexural strength in particular are reduced. As the strength is reduced, the flexural stiffness of the slab is reduced; thus a higher pressure is transmitted to the sublayers, producing larger deflections.

The traffic loads also damage the sublayers material. The material immediately underneath the loaded slab is subjected to repetitive application of pressure. Each load application produces a small permanent deformation in the subbase and subgrade. Even though compaction and stiffening of the material can occur, the presence of excessive

water may damage the structure of the sublayer material, thereby weakening the sublayers. Additionally, if water accumulates in spaces between the slab and sublayers produced by warping or compression, the sudden application of a load ejects the water. The water moving at high speed causes erosion and additional enlargement of the void. This condition is especially critical at the edge and joints of the slab, where pumping can occur. Moreover, at these locations, the openings allow the penetration of water, and the deflections are larger because of the discontinuity.

Load transfer devices are often provided at the joints to reduce the deflections caused by traffic loads. Dowel bars are placed to transfer part of the load across the joint. These bars cause the adjacent slab to deflect a proportion of the deflection of the loaded slab. These dowel bars, with one end cast in one slab and the other end embedded with a lubricant in the other slab, transfer shear and moment actions across the joint. The efficiency of the bar is not 100%, i.e. deflections at both sides of the joint are not equal. Part of the loss of load transfer is due to the deformation of the bar itself, and part is due to deformations in the concrete-bar interfaces.

In tied joints, the slabs are separated by openings or weak sections but tied together with steel mesh. This mesh does not transfer a substantial part of the load, but

holds the slabs together so that transfer is accomplished by aggregate interlock. Only shear transfer occurs. Similarly, only shear transfer occurs in the keyway type of joints.

The action of repetitive loading reduces the efficiency of the joints. As load transfer is reduced, the slabs behave like two separate entities. In doweled joints, the embedded extreme of the bar becomes loose, allowing larger deflections. The weakening of the concrete-bar zone can produce cracking and eventual breaking of the surrounding concrete. In tied or keyway joints, the separation or destruction of the slab interface can produce a decay in the load transfer efficiency.

3.3 Capabilities and Limitations of Available Methods

Several finite element computer methods for analysis of rigid pavements have been developed in the last twenty years. Herein, the following computer methods will be discussed: 1) ILLISLAB, 2) JSLAB, 3) WESLIQID, and 4) WESLAYER.

In these finite element methods, the pavement structure is divided into three main elements: slab, sublayers, and joints. All these methods are based on the fundamental expressions for thin plates developed by Zienckiewicz and Cheung. The available methods of analysis can provide stresses and deflections caused by a static load,

representative of traffic. The designer chooses a certain thickness so that for the calculated stress level, the pavement is capable of resisting the expected number of load repetitions. However, the load considered in the analysis is a static load, in contrast to the actual load which is a moving load. The response of the pavement to the actual load is stiffer than the response obtained with the assumed static load; the calculated stresses and deflections are higher than the actual values. As a result, a conservative, but more expensive design is obtained.

The present methods give results only for the specific properties of the pavement at the moment considered in the analysis. They cannot provide the stresses and deflections that would occur after the pavement has been in service for certain times. Since the state of the pavement after several years in service cannot be directly determined, the maintenance cost cannot be accurately calculated.

The mechanical behavior of concrete has been described [45] as nonlinear at low stresses, with expansion near failure. Nonlinearity is caused by microcracking due to segregation, shrinkage, or temperature changes. The initial cracks that exist in concrete develop and propagate as loads are applied. Near failure, cracks propagate and join together, causing disruption in the concrete mass.

When reinforcing steel is provided, the cracks distribute in areas larger than those under high stresses. Steel keeps the concrete areas together up to stresses under which disruption would otherwise occur. The available analysis methods assume a linear elastic behavior of concrete at all stress levels. Presence of reinforcing steel is not considered. Therefore, the strains corresponding to certain stresses, especially high stresses, are different from the actual corresponding strains.

In laboratory tests [45] the values of strength and modulus of elasticity obtained for concrete depend upon the rate with which the load is applied. The values obtained in an impact test are higher than those from a static slow-rate test. In the pavement slab, loads are more similar to impact loads than to static loads. Therefore the use of static strength and static modulus of elasticity is somewhat unrealistic.

In the sublayers, the nonlinear behavior is even more pronounced. Deformations occur as a result of plastic deformation of the grains and aggregations of particles. The stiffness properties of the sublayer material are even more sensitive to the rate of load application. Therefore, the assumption of static values with linear behavior is farther away from the actual behavior.

Loads in pavements are applied in a repetitive manner. This produces fatigue in the slab, the sublayers, and the joint devices. Fatigue produces a decrease in the pavement stiffnesses. Actually, a reduction in the pavement life is constantly occurring due to traffic damage. Therefore, in the available methods of analysis, the assumption of constant mechanical properties for the slab, the sublayers, and the joint devices is not realistic.

3.4 Ideal Model for Analysis of Rigid Pavements

From the design point of view, the ideal analysis method should be able to compute the short and long term responses of the pavement. The structural analysis of the pavement with properties equal to those at the beginning of the service life allows the design of the geometry of the pavement. With this, the initial cost can be estimated. During service, the pavement deteriorates and its structural capacity decreases. In order to keep it in good condition, repair is performed. The cost of this maintenance work depends on the state of the pavement at the moment of repair. The analysis method should be able to predict the structural damage that occurs with time; this provides a guidance for the calculation of the maintenance costs.

From the point of view of the mechanical behavior, the ideal method of analysis should reproduce the actual performance of the pavement structure. The stress-strain

relations of the individual components should be accurately represented in the analysis model. Concrete in the pavement slab can be represented by a nonlinear plastic model. The effect caused by the presence of microcracks should be reflected in the stress-strain relationships. Similarly, the presence of reinforcing steel must be taken into account. In the sublayers, the material should be represented by a nonlinear plastic model.

Traffic loads have a repetitive nature. This causes fatigue in all the stressed pavement elements. The concrete properties should be dependent upon the loading history. Decreases in strength and modulus of elasticity should be considered; so should be the formation of cracks and fractures of the slab. In the sublayers, repetitive loading can create areas of reduced support. Excessive water and load repetitions cause erosion and formation of voids underneath the slab. This further loss of support should be considered in the analysis.

The values considered for the mechanical properties should be the values obtained with loads similar in nature to the actual traffic loads. Impact values for the modulus of elasticity of concrete and for the stiffness of the sublayers should be used. These values represent more accurately the performance of the pavement. Load transfer devices at joints are also subjected to repetitive loading. Therefore, the analysis method should consider the

decay in load transfer efficiency. This decay can vary from a quantitative variation of the stiffness values to a qualitative variation of the actual performance. The analysis method should follow this kind of performance. Finally, the combination of traffic loading should also be included.

3.5 Conclusions

The available finite element methods for analysis of rigid pavements provide fairly accurate results of the stresses and deflections produced by static load. With these methods, any static-load distribution can be analyzed in pavements with irregular geometric characteristics. However, the analysis is carried out assuming linearly elastic behavior for the pavement components. Moreover, the long-term effect of the traffic load is not considered. The presence of the reinforcing steel is not taken into account either.

An ideal method of analysis should reproduce the actual behavior of the pavement components. Nonlinear stress-strain relationships are more representative of the slab and sublayer behaviors. The ideal model should also take into account the presence of reinforcement, as well as the long-term effect of the traffic loads. Like fatigue damage in the concrete slab and pumping development.

CHAPTER 4 NUMERICAL EVALUATION OF EXISTING FINITE ELEMENT METHODS

4.1 Introduction

In this chapter, results calculated using two finite element methods are compared with results obtained with Westergaard's equations, and with the Pickett and Ray influence charts. Additionally, deflections measured in a laboratory model are used to determine equivalent composite values of k . For the comparison of deflections, a rectangular plate on an elastic sublayers with full support and with interior, and edge loads is used. To determine the equivalent composite values of k , a small-scale model of slab resting on a soil sublayers is used. Numerical evaluations of the ILLISLAB and JSLAB finite element programs are carried out by comparing the theoretical results obtained with these finite element methods with results obtained using closed form solutions.

4.2 Cases Studied

Two cases are considered: (1) a rectangular concrete slab fully supported by a uniform semi-infinite elastic sublayers, as described in Figure 4.1; and (2) a small-scale model consisting of micro-concrete rectangular slabs

INTERIOR LOAD

EDGE LOAD

$t = 254. \text{ mm}$	(10 in.)
$E = 27,579 \text{ MPa}$	(4,000 ksi)
$u = 0.15$	
$k = 54.2 \text{ MPa/m}$	(200 pci)
$P = 44.48 \text{ kN}$	(10,000 lb)
$a = 254. \text{ mm}$	(10 in.)
$b = 254. \text{ mm}$	(10 in.)

Fig. 4.1 Elastic plate lying on elastic sublayers.

slabs; 673 mm x 406 mm x 25 mm (26.5 in. x 16 in. x 1 in.)

$E_c = 28,400$ MPa (4120 ksi)

$u = 0.18$

transverse joints

dowel diameter = 3.5 mm (0.15 in.)

dowel length = 51 mm (2.0 in.)

dowel spacing = 34 mm (1.33 in.)

$E_s = 200,000$ MPa (29,000 ksi)

Fig. 4.2 Small-scale model of slab resting on layer of crushed limestone.

resting on a layer of crushed limestone, as described in Figure 4.2. For the first case, deflections for given static loads are calculated using Westergaard's equations, Pickett and Ray Influence Charts, and ILLISLAB and JSLAB finite element methods. For the second case, deflections caused by static and impact loads on a laboratory model, are used to determine values of modulus of subgrade reaction using a back-calculation procedure.

4.3 Calculation by Means of Westergaard's Equations

The calculation of deflections in this case is carried out using the following Westergaard's Equations:

For interior load:

$$\begin{aligned}
 w = P/4 \sqrt{\left(\frac{3(1-\mu^2)}{Et^3k}\right)} - & \quad (4.1) \\
 \frac{-0.275(1-\mu^2)P}{Et^3} ((a^2+b^2)/4+x^2+y^2) & \\
 \frac{-0.239(1-\mu^2)P}{Et^3} ((a^2+4ab+b^2)/8+(a-b)/(a+b)(-x^2+y^2)) &
 \end{aligned}$$

For edge load:

$$\begin{aligned}
 w = \frac{P \sqrt{2+1.2\mu}}{\sqrt{Et^3k}} (1-(0.76+0.4\mu)b/l) & \quad (4.2) \\
 (1-(0.76+0.4\mu)y/l)) &
 \end{aligned}$$

Where:

w =deflection at point (x,y)

a,b =semiaxes of an ellipse representing the loaded area.

x,y =horizontal rectangular coordinates.

t =thickness of slab

E =modulus of elasticity of slab.

μ = Poisson ratio of slab.

k = modulus of subgrade reaction

λ =radius of relative stiffness

Deflections are calculated assuming a circular area with uniformly distributed load. For the cases shown in Fig. 4.1, the calculated deflections are:

$$w_{\max} \text{ (Interior load)} = 0.120 \text{ mm (4.73 mils)}$$

$$w_{\max} \text{ (Edge load)} = 0.371 \text{ mm (14.6 mils)}$$

4.4 Calculation by Means of Influence Charts

Calculation of deflections using the Pickett and Ray Influence Charts are carried out with the charts given in reference [7]. The charts developed for the Dense Liquid assumption were used. Scale adjustment is obtained by

determining the radius of relative stiffness. The number of loaded blocks in the charts is counted to the nearest half block. For the case shown in Fig. 4.1, the results are as follows:

$$w_{\max} \text{ (Interior load)} = 0.126 \text{ mm (4.99 mils)}$$

$$w_{\max} \text{ (Edge load)} = 0.360 \text{ mm (14.2 mils)}$$

4.5 Calculation by Means of Finite Element Methods

Deflections for different load conditions were calculated using the JSLAB and ILLISLAB computer schemes. For each loading condition different mesh sizes and mesh configurations were assumed. The loaded area was assumed to be a square of 254.mm x 254.mm (10. in. x 10. in.), therefore the applied pressure was 689. kPa (100 psi) (Fig. 4.1). Two factors were considered for both the ILLISLAB and JSLAB computer schemes: (1) effect of size of slab, and (2) effect of shape of elements. Three different slab sizes were assumed: 5.00 m x 5.00 m (197 in. x 197 in.); 10.00 m x 10.00 m (394 in. x 394 in.); and 20.00 m x 20.00 m (787 in. x 787 in.). For the effect of element shape, three different side ratios were assumed: 1.0, 0.5, and 0.25. Deflection results are summarized in Tables 4.1 and 4.2, and in Figures 4.3 through 4.6.

Table 4.1 Deflections for interior load

Scheme	Slab Size (m)	Elements	Element Size (m)	Deflections (mm)
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Effect of Slab Size

I	5.00x5.00	4x4	1.27x1.27	0.136
I	10.00x10.00	8x8	1.27x1.27	0.124
I	20.00x20.00	16x16	1.27x1.27	0.124
J	5.00x5.00	4x4	1.27x1.27	0.136
J	10.00x10.00	8x8	1.27x1.27	0.124
J	20.00x20.00	16x16	1.27x1.27	0.124

Effect of Element Shape

I	20.00x20.00	16x16	1.27x1.27	0.124
I	20.00x20.00	16x8	1.27x2.54	0.118
I	20.00x20.00	16x4	1.27x5.00	0.083
J	20.00x20.00	16x16	1.27x1.27	0.124
J	20.00x20.00	16x8	1.27x2.52	0.118
J	20.00x20.00	16x4	1.27x5.00	0.084

I=ILLISLAB

J=JSLAB

Table 4.2 Deflections for edge load

Scheme	Slab Size (m)	Elements	Element Size (m)	Deflections (mm)
effect of slab size				
I	5.00x5.00	4x4	1.27x1.27	0.392
I	10.00x10.00	8x8	1.27x1.27	0.369
I	20.00x20.00	16x16	1.27x1.27	0.369
J	5.00x5.00	4x8	1.27x1.27	0.392
J	10.00x10.00	8x8	1.27x1.27	0.369
J	20.00x20.00	16x16	1.27x1.27	0.369
effect of element shape				
I	20.00x20.00	16x16	1.27x1.27	0.369
I	20.00x20.00	16x8	1.27x2.54	0.354
I	20.00x20.00	16x4	1.27x5.00	0.352
J	20.00x20.00	16x16	1.27x1.27	0.353
J	20.00x20.00	16x8	1.27x2.54	0.353
J	20.00x20.00	16x4	1.25x5.00	0.350

I=ILLISLAB

J=JSLAB

Fig. 4.3 Deflections calculated by means of ILLISLAB and JSLAB schemes for different slab sizes (interior load).

Fig. 4.4 Deflections calculated by means of ILLISLAB and JSLAB schemes for different element shapes (interior load).

Fig. 4.5 Deflections calculated by means of ILLISLAB and JSLAB schemes for different slab sizes (edge load).

Fig. 4.6 Deflections calculated by means of ILLISLAB and JSLAB schemes for different element shapes (edge load).

4.6 Results from Laboratory Model

Experimental deflections were obtained by testing a laboratory model of a rigid pavement section. The 1/9 scale model consisted of several slabs of 673. mm x 406. mm x 25. mm (26 1/2 in. x 16 in. x 1 in) connected by doweled transverse joints and an aggregate interlock longitudinal joint (Fig. 4.2). The slabs were made out of micro-concrete of 35,136. kPa (5100 psi) compressive strength; 28,400. MPa (4120 ksi) modulus of elasticity; and $\mu = 0.18$. The slabs were connected at the transverse joints by round-smooth, steel bars 3.5 mm (0.15 in.) in diameter, 51. mm (2. in.) in length, and 34. mm (1.333 in.) in spacing.

Static deflections were obtained from static loads applied gradually by means of a mechanical jack. For the case of impact load, deflections were registered while a weight was dropped in a small-scale model of falling weight deflectometer. Static and impact deflections were measured by means of linear vertical transducers connected to a digital oscilloscope and to an oscillographic recorder. Static loads were applied over two rectangular areas of 28 mm x 17 mm (1.10 in. x 0.667 in.) and 53.6 mm (2.11 in.) at centers. Impact loads were applied over a circular area 33.8 mm (1.333 in.) in diameter. In both cases, loading areas were centered at the middle of the slabs. With the deflections obtained at each load level, values of modulus

of subgrade reaction were obtained by a back-calculation procedure using finite elements. This procedure consists of determining the value of k , such that for a given applied load, the theoretical deflection coincides with the measured deflection. An iteration scheme was implemented in the ILLISLAB computer scheme to calculate automatically an average value of k so that the theoretical deflections are equal to the measured deflections. Results are listed as follows:

Table 4.3 Backcalculated values of k for measured deflections.

Static k			
Load N (lb)	Deflection mm (mils)	k MPa/m (pci)	
Slab No 1			
329 (74)	0.050 (1.99)	34	(126)
507 (114)	0.075 (2.96)	36	(133)
658 (148)	0.099 (3.91)	34	(129)
822 (185)	0.120 (4.73)	37	(136)
987 (222)	0.134 (5.30)	40	(151)
1152 (259)	0.157 (6.20)	40	(150)
Slab No 2			
329 (74)	0.083 (3.27)	16	(60)
507 (114)	0.104 (4.11)	22	(81)
658 (148)	0.130 (5.15)	23	(86)
822 (185)	0.146 (6.23)	27	(101)
987 (222)	0.158 (6.23)	32	(118)
1152 (259)	0.176 (6.94)	34	(127)
Slab No 3			
329 (74)	0.036 (1.44)	55	(206)
507 (114)	0.049 (1.95)	67	(249)
658 (148)	0.061 (2.42)	72	(267)
822 (185)	0.071 (2.78)	82	(303)
987 (222)	0.077 (3.03)	95	(351)
1152 (259)	0.084 (3.31)	105	(388)

Table 4.4 Backcalculated values of k for measured deflections.

Impact k		
Load N (lb)	Deflection mm (mils)	k MPa/m (pci)
Slab No 1		
298 (67)	0.016 (0.642)	170 (629)
618 (139)	0.034 (1.357)	164 (606)
916 (206)	0.052 (2.071)	157 (581)
1205 (271)	0.071 (2.805)	149 (553)
Slab No 2		
293 (66)	0.023 (0.930)	93 (344)
631 (142)	0.048 (1.903)	101 (373)
907 (204)	0.069 (2.723)	101 (375)
1187 (267)	0.096 (3.788)	92 (340)
Slab No 3		
271 (61)	0.020 (0.786)	105 (391)
587 (132)	0.042 (1.664)	111 (410)
929 (209)	0.063 (2.513)	119 (440)
1236 (278)	0.088 (3.467)	112 (415)

The average values of the static modulus of subgrade reaction were obtained by means of the least square error procedure. The static k for a load of 750. N (168 lb) was 47. MPa/m (173 pci); while the impact value of k for the same load was 122.8 MPa/m (453 pci). The difference of the averages is 75.8 MPa/m (280 pci). The least square averages are shown in Fig. 4.7.

Fig. 4.7 Modulus of subgrade reaction obtained from impact and static deflections measured in laboratory model.

4.7 Summary

Deflections in a square plate supported by an elastic sublayers caused by interior and edge loads were calculated using the ILLISLAB and JSLAB computer schemes. The results obtained by means of these two finite element methods were compared with results obtained by Westergaard equations and the Pickett and Ray influence charts. Different slab sizes, number of elements, and shape of elements were used. For both the interior and edge loads deflections calculated by means of the finite element methods were closer to the Westergaard and Pickett and Ray solutions, when larger slab sizes or more elements were used. For different element shapes, the two finite element schemes give the same results for the case of interior load. A specific trend is not defined however. For the case of edge load, the JSLAB scheme gives more consistent results for all element shapes than ILLISLAB does.

As reported by Thompson et al [44], the backcalculated values of k are higher for an impact load than for a static load. For the static load, the k values appear to increase as the load increases. This may be due to an increase in the composite value of k resulting from adjustments of the slab-sublayer interface and to an increase of the area of contact as load increases. Initial

support may be low due to curling. The value of k at different load levels would be expected to decrease as the magnitude of the load increases. This trend is not observed very markedly in Fig. 4.7, but the value of k decreases as the magnitude of the load increases.

CHAPTER 5 COMPUTER METHOD FOR ANALYSIS OF RIGID PAVEMENTS

5.1 Introduction

In this chapter, a Nonlinear Finite Element Method for analysis of rigid pavements with fatigue is presented. It is termed the Purdue Method for Analysis of Rigid Pavements (PMARP). Fatigue damage caused by traffic deteriorates the pavement components so that their stiffness properties decay with time. The modulus of elasticity and strength of concrete decrease as traffic imposes load repetitions stressing and straining the concrete. Micro-cracking takes place reducing the strength and modulus of elasticity of the concrete slab. Voids are also produced due to pumping action created by repetitive traffic loads. The method of analysis proposed herein takes into account the damaging effect caused by load repetitions on the slab as well as on the sublayers. The damage is quantified as decay in the stiffness of concrete, amount of cracking, decay in load transfer efficiency, and amount of damage due to pumping.

The computer implementation is based on the computer method ILLISLAB previously developed at the University of

Illinois [9]. In the method presented herein, the pavement components are assumed to have a stress- or strain-dependent behavior, and the concrete slab is considered as an orthotropic plate. Methods for including damage in the concrete slab, damage in the load transfer devices, and formation of voids are described. In the first part, an outline of the behavior of rigid pavements under repetitive loading is presented. A method for analysis of rigid pavements with fatigue is given at the end of the chapter.

5.2 Structural Behavior of Rigid Pavements with Pumping

The pavement structure, consisting of a series of layers made up of different materials with different stiffnesses, provides a smooth surface to ride on, and a way to transfer the highly concentrated loads to the lower layers at successively lower stresses. The concrete slab transfers the vertical stresses into a larger area in the subbase which, in turn, transfers them to even a larger area in the subgrade (Fig. 5.1). The slab is under the action of bearing and, more important, flexural stresses, while the subbase and subgrade are under vertical and confinement stresses. The level of stresses occurring at each pavement component depends on the relative values of stiffnesses of the individual structural elements. For example, a relatively stiff slab (high concrete strength or large thickness) supports the majority of the load,

- (1) TIRE PRESSURE
- (2) BEARING STRESSES CAUSED BY TIRE
- (3) FLEXURAL STRESSES (COMPRESSION)
- (4) FLEXURAL STRESSES (TENSION)
- (5) STRESSES AT THE SLAB-SUBBASE INTERFACE
- (6) VERTICAL AND CONFINEMENT STRESSES IN SUBBASE
- (7) VERTICAL AND CONFINEMENT STRESSES IN SUBGRADE
- (8) STRESSES AT CONCRETE-BAR INTERFACE

Figure 5.1 Idealized vertical cut of pavement structure.

transferring relatively low stresses to the subbase and subgrade (the load is distributed over a large area in the subbase and subgrade).

In the conventional analysis of rigid pavements, the variation of the pavement properties caused by repetitive application of traffic loads is not taken into account. The traditional premise underlying the design of rigid pavements is that full contact exists between the slab and the supporting materials. However, several factors can make the concrete slab lose contact with the subbase including warping, curling, and pumping. In addition, fatigue damage caused by a repetitive load application reduces the strength and stiffness of concrete pavements. Therefore, the response of a pavement to a certain load application depends on its structural soundness, which varies with the service life of the pavement.

Pumping can be analyzed from several viewpoints. Herein, only the structural aspect of pumping is considered. The main effects produced by pumping, from the structural viewpoint, are the development of zones which create low bearing capacity, or even voids. In the traditional methods of analysis, the pavement structure is assumed to be in a full and perfect contact at all points of the slab-sublayers interface. However, this condition, if it ever occurs, exists only at the beginning of the service life of a pavement. After a pavement is opened to service,

the environment and traffic actions cause changes in its structural conditions.

The effect of pumping is the ejection of water with sub-layer material from under the slab producing a void. Free water can accumulate in an initial void, from where it can be ejected through cracks and joints when the slab is deflected down by a passing load. Muddy water is ejected indicating erosion of the subbase. The initial voids or spaces underneath the slab can be produced by temperature curling of the slab or by plastic deformation of the subbase. The continuing application of loads increases the size of the initial voids until a relatively large space is formed. The change in the support conditions when voids are formed can augment the critical stresses in the slab. Cracking and faulting may occur as a result of this increase of stresses.

In the slab, the strength of the concrete increases with time as the hydration process develops. At the same time, however, seasonal and daily temperature changes produce differential dilatations in the components of the concrete which causes microcracking. This microcracking reduces the strength of concrete. It can be assumed that these two effects can, approximately, cancel each other.

Additionally, pavement structures are subjected to the action of traffic. The repetitive application of traffic

loads causes stressing and fatigue damage in the concrete slab. This is the type of damage considered in the method presented herein.

5.3 Proposed Method for Analysis of Rigid Pavements

In the following, a Non-linear Finite Element method for analysis of rigid pavements (PMARP) considering the decay of the pavement stiffness properties due to traffic damage is presented. This method can be used to calculate the stresses and deflections of rigid pavements subjected to fatigue damage caused by traffic. It can also be used to estimate the current state of damage of rigid pavements that have been under traffic for certain time. Thus, the method can be applied in design as well as in maintenance.

In the first part of this section, the bases for the development of the nonlinear method for structural analysis of rigid pavements are presented. The pavement structure is divided into several well defined components whose behaviors reflect the behavior of the whole pavement structure. Since the behaviors of these pavement components are stress- or strain- dependent, the solution is obtained through an iterative process. In the second part, the computer implementation of the method is illustrated.

5.3.1 Finite Element Discretization

The method presented herein is based on a finite element technique in which the whole structure is divided into specific discrete elements. Two different methods for dividing the pavement structure have been generally used. In the first method, the pavement is divided into three basic elements: sublayer elements, load transfer elements, and slab elements. In the first case, the slab is formed by a combination of beam and torsion elements. (i.e. Hudson and Matlock) [34] In the second technique, the pavement structure is also divided into three elements, but the slab is divided into plate elements [33,38]. In the method presented herein, the second approach is used; the slab elements are treated as thin plate elements. See Figure 5.2.

Slab Elements.— The concrete slab is assumed to behave like a thin plate with small deformations. Kirchhoff hypotheses are applicable:

- (i).—Deflections at midsurface are small compared with thickness of plate.
- (ii).—The midplane remains unstrained during bending.
- (iii).—Plane sections initially normal to the midsurface remain plane and normal to that surface, after bending.

Figure 5.2 Decomposition of pavement section into finite elements.

(iv).-The stress normal to the midplane, σ_z , is small compared with other stress components and therefore can be neglected.

From hypothesis (i), the slope of the deflected surface is assumed to be small and the square of the slope can be neglected when comparing with unity. Hypothesis (iii) implies that the vertical shear strains, γ_{xz} and γ_{yz} , are negligible. The deflection of the plate is therefore associated principally with bending strains. The normal strain, ϵ_z , resulting from transverse loading may also be omitted. Hypothesis (iv) is acceptable if transverse loads are not highly concentrated.

If an overlay or stabilized subbase are present, such layers are also considered as thin-plate elements. No bond is assumed at the interfaces, thus, the stiffness of the whole group of layers is that of three superimposed plates. No composite action is considered.

The finite element formulation of the plate elements is as given by Zienkiewicz and Cheung [33,38]. In this formulation, three displacements are considered as nodal parameters: transverse displacements, w ; rotation about x axis, θ_{xi} ; and rotation about Y axis, θ_{yi} , as shown in Fig. 5.3. Each element has twelve degrees of freedom.

Figure 5.3 Isotropic and orthotropic thin plate elements.

Corresponding with these displacements, there are three nodal forces: transverse force, f_i ; a couple about x axis, f_{Xi} ; and a couple about y axis, f_{Yi} . Therefore:

nodal displacements

$$a_i = \begin{Bmatrix} w_i \\ \theta_{Xi} \\ \theta_{Yi} \end{Bmatrix} \quad (5.1)$$

nodal forces

$$f_i = \begin{Bmatrix} f_i \\ f_{Xi} \\ f_{Yi} \end{Bmatrix} \quad (5.2)$$

For an element e , the nodal forces and displacements are related by:

$$k^e a^e = f^e \quad (5.3)$$

For the entire slab composed of n plate elements, we have:

$$[K] \{a\} = \{f\} \quad (5.4)$$

where $[K]$ is the stiffness matrix of the whole slab;

$$[K] = \sum_{i=1}^{i=n} [k]^e \quad (5.5)$$

$\{a\}$ and $\{f\}$ are the nodal displacements and nodal forces for the whole structure.

In the present method, there are two options to represent the concrete slab: (1) as an isotropic slab, and (2) as an orthotropic slab. The orthotropic case is to take into

account the presence of reinforcement in both the X and Y directions. For the orthotropic case, only one plate layer can be considered. The stiffness matrices for the orthotropic and isotropic cases are as follows:

$$k^e = \iint_V B^t D B dx dy \quad (5.6)$$

The result of this integration is given by Zienkiewicz [46] (p 237),

$$k^e = \frac{1}{60ab} (D_x K_1 + D_y K_2 + D_1 K_3 + D_{xy} K_4) L \quad (5.7)$$

where K_1 , K_2 , K_3 , and K_4 have constant values depending on the dimensions, a and b , of the element. L is also a constant [46].

The terms D_x , D_y , D_1 , and D_{xy} represent the flexural and torsional rigidities of the orthotropic plate and are calculated as follows:

$$\begin{aligned} D_x &= \frac{E_x t^3}{12(1-\mu_x \mu_y)} \\ D_y &= \frac{E_y t^3}{12(1-\mu_x \mu_y)} \\ D_1 &= \frac{E_x \mu_y t^3}{12(1-\mu_x \mu_y)} \\ D_{xy} &= \frac{E_x t^3}{24(1+\mu_y)} \end{aligned} \quad (5.8)$$

Where E_x and E_y , and μ_x and μ_y are the moduli of elasticity and Poisson ratios in the X and Y directions.

Stresses are computed by:

$$\sigma_x = \frac{E_x}{1 - \mu_x \mu_y} (\epsilon_x + \mu_x \epsilon_y) \quad (5.9)$$

$$\sigma_y = \frac{E_y}{1 - \mu_x \mu_y} (\epsilon_y + \mu_y \epsilon_x)$$

$$\tau_{xy} = \frac{E_x}{12(1 + \mu_y)} \gamma_{xy}$$

Where the strains depend on the nodal displacements, the displacement function, and the nodal coordinates;

$$\{\epsilon\} = [B] \{a\} \quad (5.10)$$

In the above, $\{a\}$ is obtained from the solution of the F.E. equations and $[B] = QC^{-1}$. Q depends on the displacement function and C^{-1} on the nodal coordinates. Expressions for Q and C can be found in reference [46] p 236, and $\mu_x = \mu_y = \mu$.

Foundation Elements.— The subgrade is considered as a Winkler sublayers in which the deflection at a point is assumed to depend only on the pressure acting at that point. Therefore, the subgrade can be represented by a series of discrete vertical elements with only one degree of freedom.

In the method presented herein, the vertical elements are assumed to have a nonlinear, deflection-dependent behavior. The values of the coefficient of subgrade

reaction are resilient values obtained from impact tests [44]. These coefficients are the ratio between the applied pressure and the resulting deflection. The vertical elements are placed at the nodal points of the slab. The displacement component at each nodal point is a vertical displacement, w_i , in the Z direction. The corresponding force component is the vertical force f_i . The force and displacement are related by:

$$[k] \{a\} = \{f\} \quad (5.11)$$

or:

$$\begin{bmatrix} k & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} w_i \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} f_i \\ f_{Xi} \\ f_{Yi} \end{Bmatrix} \quad (5.12)$$

Load Transfer Elements.— The concrete slab is divided into small sections by longitudinal and transverse joints. At these joints, devices to transfer portions of the load among adjacent slabs are provided. In the present method, joints can be considered as doweled, aggregate interlock, and keyed joints [9] (Fig. 5.4). Doweled joints are represented by bar elements which are capable of transferring shear and moment forces across the opening of the joint. These elements have two degrees of freedom: a vertical displacement w_i , and a rotation about a horizontal axis transverse to the bar θ_{Yi} . Corresponding, there are two nodal forces: a shear force f_i , and a

Figure 5.4 Finite element representation of pavement joints.

moment about a horizontal axis f_{y1} . The displacements and forces are related by the stiffness matrix of the bar. Aggregate interlock and keyed joints are represented by vertical elements with only one degree of freedom. These elements have a vertical displacement and a correspondent vertical force (Fig. 5.4). The value of the stiffness of this element depends upon the type of joint and the particular conditions of the joint.

5.3.2 Fatigue Damage in Concrete Pavements

Highway pavements are subjected to many repetitions of traffic loads during their service lives. The importance of fatigue damage in the performance and design of rigid pavements is self-evident. Each time a vehicle travels over a road its weight is subsequently supported by different zones of the road. For certain critical positions of the load, its effects, like stresses and deflections, reach maximum values. Generally, these maximum, or critical, values are used in the design of the entire pavement. Since the traffic consists of vehicles of different configurations and sizes, the magnitude of the traffic load depends on the particular vehicle traveling over the road at a given time. Heavy trucks impose heavy loads with corresponding high stresses. A complete representation of loads imposed on a road can be obtained from the traffic serviced by the road. This traffic is quantified by the number of occurrences of vehicles of each different type

and size. Since it would be very complex to analyze a particular pavement considering the loading of each vehicle, it is necessary to introduce some simplifications. First, it is assumed that the total traffic can be classified in a finite number of vehicle types. Second, the Palmegran-Miner hypothesis is assumed to be valid, that is: the damage caused by several vehicles of different types is cumulative and independent of the order in which the vehicles travel over the pavement [47]. The supporting of loads is carried out by all the interconnected elements of the structure: the concrete slab, the subbase, and subgrade together resisting the load. When a vehicle passes over the pavement all the components of the structure suffer certain fatigue damage. The fatigue damage suffered by each structural element depends upon the relative magnitude of stress or strain occurring in that element, and on the fatigue properties of the particular element. In the following, the fatigue behavior of the pavement components is treated from the viewpoint of a pavement structure.

Fatigue in Slab.— The concrete slab constitutes a very important structural element in rigid pavements. For the concrete slab, as for any other concrete structure subjected to repetitive loading, it is important to consider the possibility of fatigue failure. The fact that concrete can suffer from fatigue under repeated stresses has been

recognized since the early 1900's . Since then, a great deal of research has been conducted on the behavior of concrete under fatigue loading. Yet, at the present time, a perfect model of the fatigue behavior of concrete is not available.

In rigid pavements the concrete slab is constantly subjected to stresses of different natures. Traffic loads produce bending stresses as well as impact and horizontal stresses. Environmental actions, such as temperature and moisture changes, are also continuously affecting the concrete of the slab. All these actions produce repetitive stresses and strains in the structure of the concrete causing damage and ,eventually, failure. In the method of analysis considered herein, only the damage produced by repetitive loading is taken into account.

Fatigue performance is usually expressed in terms of an endurance curve in which the mean value of the number of cycles to failure under a particular loading condition is plotted together with the magnitudes of the cyclic stress applied normalized with respect to the flexural strength (Fig. 5.5). Various forms of fatigue tests have been used to investigate the fatigue behavior of plain concrete, for example: direct compression, direct tension, indirect tension, and flexure. For rigid pavements, the fatigue behavior in flexure is of particular interest.

Figure 5.5 Typical flexural fatigue performance of concrete.

Several factors affect the fatigue performance of concrete in flexure: stress-history, loading amplitude, rate of loading, rest periods, age of testing, and moisture conditions. In the following , a brief discussion of these factors is presented.

In practice, service loading is not applied in regular cyclic patterns over periods of hours or days as in a conventional fatigue test. Traffic loading is constantly varying in amplitude and frequency. There are few published results of the behavior of concrete under variable amplitude loading, in spite of the fact that service loading histories on any concrete structure are certainly not represented by constant amplitude loadings. In a real structure, under a given loading the stress distribution may change with time as the modulus of elasticity and strength vary with age and with moisture movements. The only true form of representative loading would be to reproduce service loading histories in real time together with realistic moisture movements and temperature changes. This, however, would be impracticable and would not constitute an accelerated-test procedure. In view of this, a simple constant amplitude type of loading is used and a simple cumulative fatigue damage is assumed for the estimation of the fatigue behavior of concrete [48].

Although it has been known for a long time that the compressive and flexural strengths of concrete increase

with the increase in rate of loading, the effect on fatigue performance is less marked [48]. In the same manner, rest periods between loading cycles seem unlikely to have a very significant effect of fatigue performance of concrete [49].

For a given stress level, there is a marked increase in cycles to failure as the age of concrete increases [49]. At the same time there is a corresponding increase in the static strength so that the applied fatigue loading represents a diminishing proportion of the static strength at the time of testing. If, however, all the cycles to failure are plotted against the appropriate flexural strengths at different ages, all the points tend to lie close to a single endurance curve. However, the changes in temperature and moisture produce differential stresses in the concrete mass. These differential stresses may produce microcracking in the pavements and thus the strength and modulus of elasticity of concrete may be reduced. For simplicity, the gain in strength after 28 days is assumed to be reduced by microcracking caused by temperature and moisture changes. Therefore, the variation of strength of concrete due to maturity can be neglected if no well defined variation of this strength is known.

Decay in Load Transfer Efficiency.— The occurrence of pumping is greatly affected by the efficiency of the load transfer devices. Unfortunately, this is an area where

considerable research remains to be done in order to have a clear understanding of the phenomena occurring at the joints. What is clear now is that through the service life of a pavement the load transfer efficiency is modified by the action of traffic. The application of extended repetitive loading decreases the initial ability of a given system to transfer load.

From the little research that has been conducted, [21,22,23,24,25] some conclusions can be drawn: the ability to transfer loads depends on many variables, such as joint width, dowel diameter, dowel length, dowel spacing, initial dowel looseness, and concrete strength and modulus of elasticity. Teller and Cashell [28] found that a definite exponential relation exists between dowel diameter and load-transfer capacity, other variables being constant. For a given dowel diameter and condition of loading, decreasing the width of the joint opening decreases the bending stresses in the dowel. It also decreases the dowel deflections and hence increases the percentage of load transfer both initially and after extended repetitive loading. The condition of dowel looseness has an important effect on the structural performance of the dowel, since it can function at full efficiency only after this looseness is taken up by load induced deflection. This is true for both the initial looseness and the subsequent looseness developed during the course of repetitive loading

Mainly, the decrease in the load transfer efficiency is due to damage that occurs in the concrete surrounding the dowel bars. As reported in reference [28], loss in initial capacity to transfer load depends on the number of load cycles and on the magnitude of the load (Fig. 5.6). From the data reported by Cashell and Teller [28], a regression equation can be found to relate the values of modulus of dowel support with the number of load cycles and the magnitude of the stress at the bar-concrete interface.

5.3.3 Pumping Development

From the structural point of view, pumping is important whenever voids are formed under the slab and the support conditions of the pavement are modified. In the method of analysis presented in this chapter, formation of voids resulting from pumping is included. Support conditions in the pavement are modified in accordance with the magnitude of the voids formed during pumping action. Estimation of void sizes is achieved by means of an analytical method which has been developed in part from statistical analysis of the pumping indices reported in the AASHO Road Test [2], and in part from the structural analysis of the pavement sections used in that test. Pumping index is defined as the volume of material pumped out from under the slab, per unit length of pavement.

Figure 5.6 Increase in dowel looseness as a function of load cycles.

The material pumped out from underneath the concrete slab leaves a space which modifies the support conditions of the slab. The voids formed during the pumping action offer an area of low or no support for the slab. The identification of the shape and location of the voids allows for a more accurate analysis of the pavement structure. In the following paragraphs, a model to define the development of voids in concrete pavements is presented.

The amount of material pumped in a rigid pavement depends on the following factors:

- 1.-Structural properties of pavement.
- 2.-Pumping susceptibility of subbase material.
- 3.-Magnitude and number of load applications
- 4.-Climatic conditions.

The analytical model presented herein consists of two parts: (1) a qualitative description of location and shape of voids, and (2) a quantitative description of the size of the voids.

Qualitative Description. The qualitative description of the location and shape of voids is based on field observations and on analysis of deflection patterns of rigid pavements. It has been observed that pumping occurs at the edges of the pavement slabs, and at the sides of the

transverse joints, with the most significant pumping occurring at the leave side of the joint [50,1].

Pumping occurs when water is ejected at high speed from underneath the slab when it is deflected downward by a passing load. Therefore, ejection of water takes place where free water accumulates and where high deflections occur. Water accumulates at those places where initial space exists between the slab and the supporting material. The initial spaces, or voids, can be caused by negative temperature gradient in the slab (the top of the slab has a lower temperature with respect to the bottom of the slab), or by plastic deformation of the sublayers. From the structural analysis of pavement sections under negative temperature differential, it can be inferred that the slabs adopt a bowl shape, leaving spaces at the corners of the slabs. At the corners also, specially at the exterior corner, the maximum deflections take place.

For the pumping model described here, it is assumed that pumping occurs mainly at two places: at the joints, and along the exterior edge of the pavements. It is assumed that voids start to develop at the exterior corner of the slab and propagate towards the centerline of the pavement. The shapes of the areas of the voids are assumed to be rectangular at the edge and triangular at the corner. For purposes of structural analysis and for allocation of voids with no support in the finite element system, void

areas will be provided to the nodes with the highest deflections, as described later in the computer implementation.

Quantitative Description. A mathematical model to predict the amount of pumping has been developed from the AASHO Road Test data. This model considers only the structural features of the pavement and the amount of traffic. With this model a so called "pumping potential" can be determined for a certain pavement section and volume of traffic given in ESAL. The pumping potential is the pumping index that would occur for the conditions which were present in the AASHO Road Test; mainly, a non-treated subbase and the rainfall conditions of that location. The pumping potential can be modified by multiplying it by factors which depend on the erosion susceptibility of the subbase (type of subbase treatment), and on the amount of rainfall.

An equation to predict pumping potential was obtained from a regression analysis of the pumping indices of the AASHO Road Test and of the amount of energy imposed on the pavement by traffic. The amounts of deformation energy imposed by one application of a 80.1 KN (18K) -single axle load on a limited deflection basin were determined for all the sections of the AASHO Road Test. These energies were computed using a finite element technique. The AASHO pumping indices were normalized to account for the fact that different slab lengths were used. This was accomplished by

dividing the pumping indices by the number of joints per 30.5 m (100 feet) ; this gives the normalized pumping index. The deformation energy applied on a deflection basin, with deflections larger than 20 mils, was computed as:

$$\text{energy} = \sum_{i=1}^{i=n} A_i k_i w_i^2 \quad (5.13)$$

where:

A_i = nodal area

k_i = nodal subgrade modulus

w_i = nodal deflection

n = number of nodes with deflections larger than 20 mils

The values of deformation energy for different thicknesses were as follows:

thickness	deformation energy
89. mm (3.5 in.)	27.370 j (242.25 in.-lb)
127. mm (5.0 in.)	10.072 j (89.15 in.-lb)
165. mm (6.5 in.)	2.728 j (24.15 in.-lb)
203. mm (8.0 in.)	0.986 j (8.73 in.-lb)
241. mm (9.5 in.)	0.309 j (2.74 in.-lb)
279. mm(11.0 in.)	0.095 j (0.84 in.-lb)
318. mm(12.5 in.)	0.028 j (0.25 in.-lb)

With these values and using a least square approximation, a regression equation between the values of $\log(\text{total energy})$ and the values of $\ln(\text{NPI})$ was obtained for 202 cases. The resulting coefficient of correlation was $R = 0.78$. The correlation equation can be expressed as follows:

$$\text{NPI} = \exp [-2.884 + 1.652 \log(\text{total}/10000)] \quad (5.14)$$

where:

$\text{NPI} = \text{normalized pumping index (in}^2 \text{)}$

$\text{total} = (\text{energy per application}) (\text{ESAL}) (\text{in} \cdot \text{lb})$

This equation is implemented in the computer method of analysis, to determine the value of pumping index for a pavement section given certain traffic volume.

5.3.4 Computer Implementation

In this section, the computer implementation of the non-linear method for analysis of rigid pavements is presented. The computer implementation is illustrated with the flow chart shown in Fig. 5.7. In general, the computer scheme is an iterative process in which the properties of the pavement section are modified during each iteration, in accordance with the level of stresses and number of load repetitions that occur at the critical section of the pavement. In the first iteration, stresses, strains and deflections are calculated for all elements

assuming the initial properties of the pavement. In subsequent iterations, the properties of each individual element are modified given the stresses and deflections that have taken place in that particular element and in accordance with the fatigue behavior of the element.

The computer scheme presented herein, called PMARP, is based on modifications of the computer code ILLISLAB developed previously at the University of Illinois [9]. The computer method presented herein, is a Fortran program with a main subroutine for memory allocation, and eight subroutines with the following functions:

Subroutine Slab. This subroutine is to read all the pavement properties and geometry, to call all the other subroutines and to carry out the iterative process.

Subroutine Stiff; calculation of all stiffness matrices given the updated pavement properties.

Subroutine Uptri; inversion of stiffness matrices.

Subroutine Load; allocation of boundary conditions and applied forces.

Subroutine Disp; solution of finite element equations and calculation of strains and displacements.

Subroutine Stress; calculation of stresses.

Figure 5.7 Flow chart for computer scheme of nonlinear finite element method for analysis of rigid pavements.

Subroutine Check; modification of pavement properties.

Subroutine Pump; calculation of pumping index and allocation of support conditions. A detailed description of the method of finite elements can be found elsewhere [46,33,38]. In this section, detailed descriptions of the iterative process, the modification of pavement properties, modification of load transfer efficiency, development of voids, and final output are presented.

Iterative Process. Referring to the flow chart in Fig. 5.7, the computation process consists of a specified number of iterations during which the following operations are executed:

- a. Determination of stiffness matrices using updated pavement properties
- b. Inversion of updated stiffness matrices.
- c. Specification of boundary conditions and application of nodal forces.
- d. Solution of F.E. equations and calculation of strains and nodal displacements.
- e. Determination of pumping index in third iteration and allocation of support conditions.

- f. Calculation of stresses.
- g. Calculation of amount of damage to pavement, decay in pavement properties, and determination of updated pavement properties.
- h. Print results and stop.

The nonlinear analysis is carried out by means of an iterative process. The stress- and strain-dependent properties of structural elements are updated in each iteration. In the computer scheme, the total number of load repetitions is applied by steps. For each step, several iterations are performed to obtain convergence between the assumed values of stiffnesses and the values calculated after the fatigue damage. The total number of iterations, is equal to the number of loading steps multiplied by the number of iterations per step. During each iteration, stresses, strains, and deflections are calculated assuming the current stiffness properties and the current number of load repetitions. The stiffness matrices of the slab, load transfer elements, and sublayer elements are updated correspondingly.

Modification of Concrete Slab Properties.- The concrete slab is divided into rectangular plate elements. Two options to analyze these elements are provided: (1) isotropic and (2) orthotropic elastic thin plate elements.

Only the orthotropic case is explained herein; the isotropic case is a special case of the orthotropic one for which the mechanical properties are the same in all the horizontal directions.

The capability to analyze orthotropic plates has been developed to take into account the presence of different amounts of reinforcement in the X and Y directions. The stiffness matrix of each plate element is calculated as shown in equation (5.7), which is a function of dimensions and mechanical properties of the element. The mechanical properties are defined in terms of E_x and E_y . These values are the equivalent values of the moduli of elasticity of the composite section. If the sections subjected to bending in X and Y directions are composed of concrete and steel bars, the equivalent moduli are obtained as follows:

Equivalent Modulus of Elasticity:

$$E = E_c + E_s \frac{I_2}{I_1} \quad (5.15)$$

For the X and Y directions:

$$\begin{aligned} E_x &= E_c + 0.75 p_x \left(\frac{d}{t} \right)^2 E_s \\ E_y &= E_c + 0.75 p_y \left(\frac{d}{t} \right)^2 E_s \end{aligned}$$

Where

E_x = Modulus of elasticity in X direction

E_y = Modulus of elasticity in Y direction

E_c = Modulus of elasticity of concrete
 E_s = Modulus of elasticity of reinforcing bars
 p_x = Percentage of reinforcement in X direction
 p_y = Percentage of reinforcement in Y direction
 I_1 = Moment of inertia of the concrete section
 I_2 = Moment of inertia of one reinforcing bar
 d = Diameter of reinforcing bar
 t = Thickness of slab

The values of moduli of elasticity of concrete are modified as the loads are imposed. The modification is carried out in accordance with the level of stresses and the number of load repetitions, making the following assumptions:

(i).-The moduli of elasticity of slab elements in the X and Y directions are equal to the values of the composite sections in the X and Y directions as given in equations 5.8.

(ii).-Decay in strength and modulus of elasticity in the composite section takes place in the concrete only.

(iii).-A flexural endurance curve is known for the particular, or similar, concrete.

iv.-The Palmgren-Miner linear cumulative damage hypothesis is applicable.

(v).-Rest periods between loading cycles have no effect on the fatigue behavior of concrete.

(vi).-The relations among the modulus of elasticity, the compressive strength, and the flexural strength of concrete are known.

Under these assumptions, the strength and modulus of elasticity of a slab element subjected to certain number of stress repetitions can be calculated by determining the remaining fatigue life of the concrete. Three conditions may occur:

if $f_r > 1.0 \Rightarrow$ cracking takes place

if $f_{min} < f_r < 1.0 \Rightarrow$ fatigue damage takes place

if $f_r < f_{min} \Rightarrow$ no damage occurs

Where:

$$f_r = \text{relative stress} = \frac{\sigma_{max}}{MR}$$

σ_{max} = maximum principal stress

MR = modulus of rupture of concrete

f_{min} = minimum stress necessary-to-produce damage

Depending on the value of f_r , the properties of the slab element are modified because cracking occurs ($f_r > 1.0$), or because of fatigue decay ($f_{min} < f_r < 1.0$). When cracking occurs, the thickness of the composite section is modified so that moment equilibrium in the cracked section is satisfied. This is accomplished in the computer program by means of a trial-

and-error process to find the position of the neutral axis in the composite section. The number of iterations required depends upon the approximation error desired. After the neutral axis is found, the new thickness of the slab element is calculated as the initial thickness minus the depth of the crack, which is assumed to extend through the entire tension zone.

When only fatigue damage occurs the properties of the concrete slab, at the particular point, are calculated using the endurance curve. For example, taking a concrete element with initial MR = 3447 kPa (500 psi) and subjected to 1000,000 load repetitions a relative stress $f_r = 0.7$ ($\sigma_{\max} = 2413 \text{ kPa (350 psi)}$) is produced. If the endurance curve of this concrete is the one shown in Fig. 5.5, the number of load repetitions to failure is 500,000. Therefore, the remaining fatigue life is 400,000 repetitions, which corresponds to $f_r = 0.75$. Thus, the updated MR is $2413./0.75 = 3217 \text{ kPa (467 psi)}$. With this value of the modulus of rupture, the strength and the modulus of elasticity are recalculated knowing the relations among these parameters. Then, the properties of the slab at the particular point are recalculated using the composite section as given in equations (5.8).

Modification of Load Transfer Efficiency.— Unfortunately, very little experimental research has been conducted to investigate the behavior of dowel bars under the action of

repetitive loading. Therefore, the method proposed herein to modify the load transfer efficiency as a function of the number of load cycles, is a simplified but practical approach based on the available experimental data. It is intended that the model proposed herein shall be considered as a guide which can be modified or adjusted later, especially if experimental data are available for a particular case.

The method for modifying the values of load transfer efficiency is based on the experimental results presented by Teller and Cashell [28]. These results are used to obtain a trend of behavior for the variation of load transfer efficiency as a function of load repetitions and level of stresses. In the computer implementation, the value of load transfer efficiency is calculated as a function of the geometry of the dowels and the concrete properties by means of a parameter called Dowel-Concrete Interaction Factor. The value of this factor is given in [22]

$$DCI = \frac{G^{0.75} D_d^{2.5}}{0.041 D_d^{0.75} + 0.0004 G^{0.25} w_j} \quad (5.16)$$

where:

G = modulus of dowel support

D_d = dowel diameter

w_j = joint width

The initial value of DCI is changed by multiplying it by a reduction factor, RF, obtained from a regression equation obtained from data reported in reference [28]. The value of RF is calculated as:

$$RF = 0.268 - 0.0457 \log(n) + 1.123 f_{rb} \quad (5.17)$$

where

RF = reduction factor

n = number of load repetitions

f_{rb} = relative load acting on dowel

$$f_{rb} = \frac{P_D}{P_c}$$

P_D = concentrated load acting on dowel

$$P_c = \text{crack load} = 3/2 \frac{D_d (t - D_d)}{(1 + w/L)} f_t$$

t = thickness of slab

L = embedded length of dowel

$$f_t = 10 (f_c)^{1/2}$$

The expression for crack load is given by Marcus [27]. The values of DCI are calculated in each iteration in the computer process depending on the number of load cycles and the level of loads applied to the dowels. If more experimental data become available, a more accurate expression for the value of RF can be implemented in the computer scheme.

Pumping Development. Pumping development and allocation of support conditions are included in this method of

analysis. Pumping index, determined by means of equation 5.14, is used to determine the volume of pumped material and to determine the area of voids. The area of voids is allocated in the pavement structure by removing the support at several nodes to cover the calculated void area. Allocation of nodes with zero support is provided considering the nodes with the highest deflections. The procedure is as follows:

1. Compute amount of deformation energy per one ESAL application.

$$\text{energy} = \sum_{i=1}^{i=n} A_i k_i w_i^2 \quad (5.18)$$

where:

A_i = nodal area

k_i = nodal subgrade modulus

w_i = nodal deflection

2. Compute total applied energy.

$$\text{total} = (\text{energy per one ESAL}) \times (\text{ESAL})$$

3. Compute normalized pumping index.

$$\text{NPI} = \exp(-2.8847 + 1.6521 \log(\text{total}/10000))$$

4. Compute total volume of pumped material per slab.

$vol = (NPI)(slab\ length)(number\ of\ joints\ per\ 100\ ft)$

5. Obtain void area.

$area = vol / (average\ void\ depth)$

6. Allocate nodes with zero support to cover void area.

Allocation of nodes with zero support is done by sorting and arranging the nodes in descending order in accordance with their deflections. Given the void area, nodes with zero support, with their corresponding area, are assigned until the total area covered by the nodes is equal to that of the void. After zero support conditions are allocated to certain nodes, these nodes remain with no support through all the iterative procedure.

The determination of the area without support (area of void) is accomplished by dividing the volume of the void by an average void depth. The size of the area with no support is also a critical parameter in the analysis. Unfortunately, there is only minimal detailed experimental evidence concerning the actual size and configuration of the voids. Therefore, the assumption of an average void depth represents an attempt not to describe the geometrical properties of a void, but to provide a parameter to obtain an unsupported area, which is in accordance with measured values obtained by means of deflection-based experimental methods. In the computer code, the value of

the average void depth is a necessary part of the input. In the absence of experimental evidence, the analysis must necessarily be based on a reasonable assumption for this parameter. More experimental evidence of the size of voids, or of functionally equivalent sizes, is urgently needed.

Resilient Modulus of Subgrade Reaction.— The structural analysis of rigid pavement generally has been carried out assuming a static constant value for the stiffness of the subbase. This type of analysis assumes a linearly elastic behavior of the subbase. In the finite element analysis of rigid pavements, this assumption is represented by a Winkler sublayers consisting of a series of spring elements. However, even though this assumption provides fairly accurate results, the actual behavior of the subbase is complex and the nature of the traffic load is not static. A better representation can be obtained by using an impact resilient modulus of subgrade reaction, k_r [44] In accordance with this assumption, the value for the stiffness at a specific point in the sublayer depends on the value of the deflection at that point. This results in a non-linear, load-deflection relationship.

The method of analysis proposed herein has the capability for considering a composite resilient value for the stiffness of the sublayers. In Figure 5.8, a typical curve relating the values of resilient modulus versus the

deflections is shown [44]. In the computer implementation, first, the values of k_r are introduced as part of the input, by specifying the k_r values corresponding to several deflections. In the iterative process, the value of the modulus of subgrade reaction assumed at each point is calculated from the deflection occurring at that point. In the iterations, the initially assumed value of k_r is adjusted, as the iterations proceed in accordance with the resulting deflection. Thus, a nonlinear pressure-deflection ratio is obtained.

5.4 Conclusions and Recommendations

A Non-Linear Finite Element Method, called PMARP, for analysis of rigid pavements subjected to fatigue damage caused by traffic has been presented. The method is implemented in an iterative computer scheme in which the pavement properties are modified in accordance with the fatigue damage produced by repetitive loading.

Figure 5.8 Deflection-dependent behavior of resilient composite modulus of subgrade reaction.

In a rigid pavement, most of the structural support is provided by the concrete slab. This slab, then, under the action of traffic is subjected to many number of load repetitions along the service life of the pavement. The load repetitions produce gradual damage in all the pavement components until eventually cracking and faulting occurs producing the failure of the structure. In jointed concrete pavements, the devices which transfer the load across the joints, specially transverse joints, also lose their efficiency along the life of the pavement. Fatigue damage imposed to the concrete slab reduces its strength and modulus of elasticity because of the occurrence of microcracks. Similarly, fatigue damage caused by repetitive loading reduces the load transfer efficiency of the dowel bars by increasing the degree of looseness of the bars. These decays in the properties of the pavement components modify the response of the pavement to a given load. Stresses and deflections calculated assuming the initial constant stiffness values differ from those obtained by using reduced values which account for fatigue damage. Assuming fatigue damage, results closer to actual field conditions are obtained.

In the computer implementation of the method of analysis presented herein, the modifications to the properties of the concrete slab are based on the flexural behavior of a particular concrete specified as part of the input data.

Similarly, the modification of the values of the resilient modulus of subgrade reaction is made on the basis of values of k_r specified in the input data. For the modification of the load transfer efficiency, however, only the initial value can be specified in the input data. The fatigue behavior is controlled by an expression built into the computer scheme. Nevertheless, this expression can be modified by changing the values of the coefficients used in the computer implementation. In any event, accuracy of the absolute values obtained with the method proposed herein, depends on the accuracy of the information provided in the input; especially the information concerning the fatigue behavior of the different pavement components. The relative results, however, are more accurate than those obtained by less comprehensive methods, and are especially useful in comparative studies.

The support conditions are modified as a consequence of pumping. Pumping of material from under the concrete slab produces areas of low or no support. A pumping model developed using the AASHO Road Test data has been implemented in the method of analysis presented here. By means of this pumping model, the support conditions, of a pavement section being analyzed, are modified automatically by means of a subroutine called "PUMP". The modification of the support conditions depends upon the amount of pumping which is also calculated by means of this subroutine.

CHAPTER 6 NUMERICAL STUDY

6.1 Introduction

In this chapter, numerical results obtained by means of the method of analysis described in the previous chapter are presented. These numerical results are analyzed for the following purposes: (1) to verify the sensitivity of the proposed method to the variations of different pavement parameters, and (2) to provide results for estimation of pavement performance.

First, a sensitivity study is presented in this chapter by evaluating, for the pavement section shown in Fig. 6.1, deflections and amount of fatigue damage caused by variations in the following parameters:

- 1.-Slab thickness.
- 2.-Amount of traffic.
- 3.-Amount of reinforcement.
- 4.-Type of sublayer.
- 5.-Endurance behavior of concrete.

In the second part, results in terms of amount of pumping, amount of fatigue damage in the slab, size of damaged area, and maximum deflections are obtained for different

Figure 6.1 Two-slab pavement section with doweled joint.

values of the following pavement parameters:

- 1.-Slab thickness.
- 2.-Amount of traffic.
- 3.-Type of sublayer.

In this chapter, the term sublayer refers to the layers (subbase and subgrade) below the concrete slab. The composite resilient modulus of subgrade reaction is an equivalent value which corresponds to the stiffness of the sublayers (subbase and subgrade). Therefore, the term type of sublayer refers to the sublayers whose composite stiffnesses are given in Fig. 6.2.

6.2 Sensitivity Study

The effects of several pavement parameters on the results obtained using the method of analysis presented in Chapter 5 are shown here. These effects are quantified in terms of maximum deflection and amount of damage. In this case, full contact is assumed to remain at all load repetitions. The pavement section shown in Fig. 6.1 is analyzed assuming the following pavement parameters:

- 1.-Slab thickness (t).
- t=150. mm (6. in.)
- t=178. mm (7. in.)
- t=203. mm (8. in.)
- t=254. mm (10. in.)

Figure 6.2 Composite, deflection dependent resilient moduli of subgrade reaction.

Figure 6.3 Endurance curves for concrete under flexure.

2.-Equivalent single-axle load applications (ESAL).

ESAL=100,000

ESAL=1,000,000

ESAL=10,000,000

3.-Amount of reinforcement (p).

p=0.%

p=0.05%

p=0.25%

4.-Type of sublayer (Fig. 6.2).

Soft.

Medium.

Stiff.

5.-Endurance behavior of concrete (Fig. 6.3).

Type 1.

Type 2.

6.3 Pavement Performance

Results in terms of pumping index, amount of fatigue damage in slab, decay in load transfer efficiency, and deflections are obtained for the following parameters:

1.-Slab thickness (t)

t=150 mm (6 in.)

t=203 mm (8 in.)

t=254 mm (10 in.)

t=305 mm (12 in.)

2.-Equivalent single-axle load applications (ESAL)

ESAL=100,000.

ESAL=1,000,000.

ESAL=10,000,000.

3.-Type of sublayer

Soft

Medium

Stiff

4.-Average void depth = 6.3 mm (0.25 in.)

6.4 Results

Results in terms of amount of damage and deflections are shown in Tables 6.1 to 6.10. These results were obtained for the case in which fatigue damage in the load transfer devices and pumping are not considered. Deflections are considered at corner of the loaded slab (node 57 in finite element mesh) produced by a 177.9 kN (40-K) single axle load applied after the pavement has been under the given ESAL. Amount of damage in the pavement is quantified in terms of total damaged area, and in terms of fatigue damage calculated as follows:

$$\text{Damage} = \sum_{i=1}^{i=n} A_i D_i \quad (6.1)$$

where, A_i is the partial damaged area, and D_i is the decay of either the modulus of elasticity or the effective thickness of the slab given by:

$$D_i = \frac{Ec_o - Ec_f}{Ec_o} \quad (6.2)$$

or

$$D_i = \frac{t_o - t_f}{t_o} \quad (6.3)$$

Ec_o and Ec_f are the initial and final values of the modulus of elasticity of the slab. Similarly, t_o and t_f are the initial and final effective slab thicknesses. Tables 6.11 to 6.16 show the results in terms of amount of fatigue damage in the slab and deflections for a 177.9 KN (40-K) single axle load. In this case, fatigue damage in the load transfer devices is included. Additionally, the change in load transfer efficiency is also presented. Load transfer efficiency is quantified as:

$$LTE = \frac{1}{n} \sum_{i=1}^{i=n} \frac{\delta_2}{\delta_1} \quad (6.4)$$

where: LTE is the value of load transfer efficiency, and δ_1 and δ_2 are the values of the deflections at the loaded and opposite slabs respectively. The quantity n is the number of dowels considered. The LTE results shown in Tables 6.11 to 6.16 correspond to nodes 57 and 65.

Table 6.1 Effects of thickness and ESAL on damaged area and amount of fatigue damage (p=0.05%, soft sublayer, and concrete type 1).

ESAL		THICKNESS			
		150. mm	178. mm	203. mm	254. mm
100,000.	area $m^2(in^2)$	2.70 (4200)	1.80 (2800)	0.90 (1400)	0.
	damage	1006.63	13.57	2.69	0.
1,000,000.	area $m^2(in^2)$	2.70 (4200)	1.93 (3000)	0.90 (1400)	0.
	damage	1009.10	75.76	26.70	0.
10,000,000.	area $m^2(in^2)$	2.70 (4200)	1.93 (3000)	0.90 (1400)	0.
	damage	1270.16	597.20	257.00	0.

Table 6.2 Effects of thickness and ESAL on deflections (p=0.05%, soft sublayer, and concrete type 1)

ESAL	THICKNESS			
	150. mm	178. mm	203. mm	254. mm
100,000.	2.69	1.38	1.16	0.88
	(106.0)	(54.4)	(45.8)	(35.0)
1,000,000.	2.70	1.38	1.16	0.88
	(106.4)	(54.5)	(45.9)	(35.0)
10,000,000.	2.76	1.41	1.18	0.88
	(109.0)	(55.7)	(46.4)	(35.0)
deflections in mm (mils)				

Table 6.3 Effects of ESAL and percentage of reinforcement (p) on damaged area and amount of fatigue damage (t=178 mm, soft sublayer, and type 1 concrete)

p%		ESAL		
		100,000	1,000,000	10,000,000
0%	area m ² (in ²)	1.80 (2800)	1.93 (3000)	1.93 (3000)
	damage	13.58	75.88	598.1
0.05%	area m ² (in ²)	1.80 (2800)	1.93 (3000)	1.93 (3000)
	damage	13.57	75.76	597.2
0.25%	area m ² (in ²)	1.80 (2800)	1.93 (3000)	1.93 (3000)
	damage	13.52	75.46	597.1

Table 6.4 Effects of ESAL and percentage of reinforcement (p) on deflections (t=178 mm, soft sublayer, and type 1 concrete)

p%	ESAL		
	100,000	1,000,000	10,000,000
0%	1.38 (54.4)	1.38 (54.5)	1.41 (55.7)
	1.38 (54.4)	1.38 (54.5)	1.41 (55.7)
0.05%	1.38 (54.4)	1.38 (54.5)	1.41 (55.7)
	1.38 (54.4)	1.38 (54.5)	1.41 (55.6)
deflections in mm (mil)			

Table 6.5 Effects of ESAL and percentage of reinforcement (p) on damaged area and amount of fatigue damage (t=152 mm, soft sublayer, and type 1 concrete).

p%		ESAL		
		100,000	1,000,000	10,000,000
0%	area $m^2(in^2)$	3.22 (5000)	3.22 (5000)	3.48 (5400)
	damage	2755.6	2840.00	3428.0
0.05%	area $m^2(in^2)$	2.70 (4200)	2.70 (4200)	2.70 (4200)
	damage	1006.6	1009.1	1270.10
0.25%	area $m^2(in^2)$	2.70 (4200)	2.7 (4200)	2.7 (4200)
	damage	991.6	1003.3	1073.7

Table 6.6 Effects of ESAL and percentage of reinforcement (p) on deflections (t=152 mm, soft sublayer, and type 1 concrete)

p%	ESAL		
	100,000	1,000,000	10,000,000
0%	4.31 (169.7)	4.30 (169.4)	4.51 (177.7)
	2.69 (106.0)	2.70 (106.4)	2.77 (109.0)
0.05%	6.69 (106.0)	2.70 (106.4)	2.71 (106.7)
deflections in mm (mil)			

Table 6.7 Effects of ESAL and type of sublayer on damaged area and amount of fatigue damage ($t=178$ mm, $p=0.05\%$, and type 1 concrete).

Sublayer		ESAL		
		100,000	1,000,000	10,000,000
STIFF	area $m^2(in^2)$	1.03 (1600)	1.03 (1600)	1.03 (1600)
	damage	13.46	43.63	243.3
MEDIUM	area $m^2(in^2)$	1.29 (2000)	1.29 (2000)	1.41 (2200)
	damage	13.50	47.34	474.2
SOFT	area $m^2(in^2)$	1.80 (2800)	1.93 (3000)	1.93 (3000)
	damage	13.57	75.76	597.2

Table 6.8 Effects of type of sublayer and ESAL on deflections ($t=178$ mm, $p=0.05\%$, and type 1 concrete).

Sublayer	ESAL		
	100,000	1,000,000	10,000,000
STIFF	0.82 (32.5)	0.82 (32.5)	0.83 (33.0)
MEDIUM	1.11 (43.8)	1.11 (43.8)	1.13 (44.6)
SOFT	1.38 (54.4)	1.38 (54.5)	1.41 (55.7)
deflections in mm (mil)			

Table 6.9 Effects of ESAL and type of concrete on damaged area and amount of fatigue damage ($t=178$ mm, $p=0.05\%$, and soft sublayer).

TYPE OF CONCRETE	ESAL		
	100,000	1,000,000	10,000,000
Type 1 area m^2 (in^2)	1.80 (2800)	1.93 (3000)	1.93 (3000)
damage	13.57	75.76	597.2
Type 2 area m^2 (in^2)	1.80 (2800)	1.80 (2800)	1.80 (2800)
damage	7.97	18.63	105.7

Table 6.10 Effects of type of concrete and ESAL on deflections ($t=178$ mm, $p=0.05\%$, and soft sublayer).

TYPE OF CONCRETE	ESAL		
	100,000	1,000,000	10,000,000
Type 1	1.38 (54.4)	1.38 (54.5)	1.41 (55.7)
Type 2	1.38 (54.4)	1.38 (54.4)	1.38 (54.5)
deflections in mm (mil)			

The amount of damage is very sensitive to the variation of thickness and ESAL (Table 6.1). A change of fatigue damage from an average of 1095 to an average of 95 occurs when the thickness varies from 150 mm (6 in.) to 203 mm (8 in.). The variation of amount of fatigue damage as a function of ESAL is more pronounced for a thickness of 203 mm (8 in.) than for 150 mm (6 in.). The values of fatigue damage for $t=150$ mm (6 in.) are much higher however. The deflection values, as expected, are very sensitive to variations in thickness. Deflections remain almost constant for different ESAL (Table 6.2).

The flexural resistance of the slab is controlled in the computer scheme by specifying the percentage of reinforcement, and the fatigue flexural behavior of the particular concrete. The effects of percentage of reinforcement and flexural behavior of concrete are given on Tables 6.3, 6.5, and 6.9. The percentage of reinforcement has a small effect on the amount of fatigue damage, while the flexural resistance has a severe effect. Therefore, when reinforcement is provided, not only should the amount of reinforcement be specified as input in the computer method of analysis, but also the fatigue flexural behavior should be modified. A concrete with higher flexural resistance should be assumed for higher reinforcement percentages even for the same concrete strength. The percentage of

Table 6.11 Amount of fatigue damage and damaged area caused in pavement section with decay in load transfer efficiency ($p=0.05\%$, soft sublayer, concrete type 1).

ESAL		Thickness (mm)			
		178	203	228	254
100,000.	area $m^2 (in^2)$	1.68(2600)	0.90(1400)	0.26(400)	0.13(200)
	damage (in^2)	16.85	2.34	0.83	0.
1,000,000.	area $m^2 (in^2)$	1.68(2600)	0.90(1400)	0.64(1000)	0.13(200)
	damage (in^2)	155.01	47.05	19.06	3.78
10,000,000.	area $m^2 (in^2)$	1.68(2600)	1.03(1600)	0.64(1000)	0.13(200)
	damage (in^2)	551.44	314.52	118.34	31.65

Table 6.12 Amount of fatigue damage and damaged area caused in pavement section with decay in load transfer efficiency ($p=0.05\%$, medium sublayer, concrete type 1).

ESAL		Thickness (mm)			
		178	203	228	254
100,000.	area $m^2 (in^2)$	1.03(1600)	0.64(1000)	0.26(400)	0.13(200)
	damage (in^2)	13.19	1.51	0.	0.
1,000,000.	area $m^2 (in^2)$	1.16(1800)	0.64(1000)	0.38(600)	0.13(200)
	damage (in^2)	142.02	30.58	12.49	3.84
10,000,000.	area $m^2 (in^2)$	1.29(2000)	0.77(1200)	0.38(600)	0.13(200)
	damage (in^2)	480.45	246.90	127.04	32.27

Table 6.13 Amount of fatigue damage and damaged area caused in pavement section with decay in load transfer efficiency ($p=0.05\%$, stiff sublayer, concrete type 1).

ESAL	Thickness (mm)				
	178	203	228	254	
100,000.	area $\pm^2$ (in 2)	0.77(1200)	0.64(1000)	0.26(400)	0.13(200)
	damage (in 2)	13.45	1.32	0.35	0.35
1,000,000.	area $\pm^2$ (in 2)	0.77(1200)	0.64(1000)	0.26(400)	0.13(200)
	damage (in 2)	23.50	26.70	9.12	3.84
10,000,000.	area $\pm^2$ (in 2)	0.90(1400)	0.64(1000)	0.39(600)	0.13(200)
	damage (in 2)	341.22	186.50	111.45	32.43

Table 6.14 Effect of thickness and ESAL on deflections and on load transfer efficiency ($p=0.05\%$, soft sublayer, concrete type 1).

ESAL	Thickness (mm)			
	178	203	228	254
100,000.	1.11(55)	1.20(47)	1.04(41)	0.92(36)
	36.1%	83.9%	81.8%	79.9%
1,000,000.	1.34(56)	1.22(48)	1.06(42)	0.94(37)
	83.8%	81.3%	78.3%	75.8%
10,000,000.	1.19(59)	1.26(49)	1.10(43)	0.99(39)
	81.3%	77.8%	73.1%	68.8%
deflections in mm (mils)				

Table 6.15 Effect of thickness and ESAL on deflections and load transfer efficiency ($p=0.05\%$, medium sublayer, concrete type 1).

ESAL	Thickness (mm)			
	178	203	228	254
100,000.	1.04(41) 82.0%	0.89(35) 79.2%	0.78(30) 76.4%	0.69(27) 73.9%
1,000,000.	1.06(42) 79.5%	0.91(36) 75.9%	0.79(31) 72.3%	0.71(28) 68.9%
10,000,000.	1.10(43) 76.5%	0.94(37) 71.7%	0.83(33) 66.2%	0.75(29) 60.9%
deflections in mm (mils)				

Table 6.16 Effect of thickness and ESAL on deflections and on load transfer efficiency ($p=0.05\%$, stiff sublayer, concrete type 1).

ESAL	Thickness (mm)			
	178	203	228	254
100,000.	0.85(33) 78.9%	0.73(29) 74.6%	0.64(25) 72.4%	0.58(23) 69.5%
1,000,000.	0.87(34) 75.8%	0.75(30) 71.8%	0.66(26) 67.8%	0.60(24) 64.0%
10,000,000.	0.90(35) 72.5%	0.78(31) 66.9%	0.69(27) 61.1%	0.63(25) 55.2%
deflections in mm (mils)				

reinforcement has a greater effect on deflections when thinner pavement slabs are considered (Tables 6.4 and 6.6).

The sensitivity of the amount of damage to type of sublayer is higher for higher values of ESAL (Table 6.7). Moreover, the variation of the amount of damage as a function of ESAL is higher for lower values of subgrade modulus. The values of the deflections are not very sensitive to the ESAL for each value of subgrade modulus, but, as expected, deflection values are very sensitive to values of subgrade modulus (Table 6.8). The small variation of deflections as a function of ESAL is due to the fact that in this case full support was assumed to exist at all levels of ESAL.

In general, higher variations of the amount of damage occur when decay in load transfer efficiency is considered (Tables 6.1 and 6.11). When this decay is included, the type of sublayer affects the deflection values at all levels of ESAL (Table 6.14, 6.15, and 6.16). The values of ESAL have a noticeable effect on the load transfer efficiency. The load transfer efficiency decreases as the number of ESAL increases. For the same number of ESAL, the thicker the slab, and the stiffer the sublayers, the lower the values of load transfer efficiency.

Numerical results were obtained assuming several pavement parameters , to evaluate the performance of the pavement section shown in Fig. 6.1 when pumping development is included. As described before, four different thicknesses, three different traffic levels, and three different values of subgrade modulus were used.

First, pumping indices were determined for the twelve different cases. The results are shown in Tables 6.17 to 6.20. The same results are plotted in Fig. 6.4 to 6.6. These pumping indices are given in in^3 / in , as in the AASHO Road Test, and in dm^3 / dm . The areas with no support were obtained assuming an average void depth of 6.3 mm (0.25 in.). The curves were drawn by graphically fitting lines through the pumping index values obtained from the analysis. As shown in the Figures, the pumping index varies seriously with the number of ESAL, the thickness of the slab, and the stiffness of the sublayer. Since the state of cracking of the concrete slab varies from element to element, the stiffness properties of the slab vary in a discrete manner. Therefore, the variations of deflections, amount of damage, and damaged area are also discrete. To compensate for this, regression equations are obtained to fit the numerical values.

Table 6.17 Pumping index for different levels of ESAL and different types of sublayers. $t=152$ mm.

ESAL	Sublayer type		
	type 1	type 2	type 3
100000.	7.560	4.200	3.000
	48.774	27.097	19.355
1000000.	39.760	22.020	15.750
	256.516	142.064	101.613
10000000.	211.660	134.500	83.390
	1365.546	867.740	537.999
sq.in. sq.cm			

Table 6.18 Pumping index for different levels of ESAL and different types of sublayers. $t=203$ mm.

ESAL	Sublayer type		
	type 1	type 2	type 3
100000.	2.900	1.210	0.620
	18.710	7.806	4.000
1000000.	15.180	6.360	3.260
	97.935	41.032	21.032
10000000.	81.580	34.600	18.260
	526.322	223.225	117.806
sq.in. sq.cm			

Table 6.19 Pumping index for different levels of ESAL and different types of sublayers. $t=254$ mm.

ESAL	Sublayer type		
	type 1	type 2	type 3
100000.	1.180	0.310	0.060
	7.613	2.000	0.387
1000000.	6.200	1.670	0.350
	40.000	10.774	2.258
10000000.	32.500	8.950	2.000
	209.677	57.742	12.903
sq.in. sq.cm			

Table 6.20 Pumping index for different levels of ESAL and different types of sublayer. $t=305$ mm.

ESAL	Sublayer type		
	type 1	type 2	type 3
100000.	0.410	0.040	0.
	2.645	0.258	0.
1000000.	2.170	0.220	0.
	14.000	1.419	0.
10000000.	11.570	1.300	0.
	74.645	8.387	0.
sq.in. sq.cm

Figure 6.4 Pumping indices for different slab thicknesses and number of load applications. Sublayer type 1.

Figure 6.5 Pumping indices for different slab thicknesses and number of load applications. Sublayer type 2.

Figure 6.6 Pumping indices for different slab thicknesses and number of load applications. Sublayer type 3.

Deflections at node 57 were determined for the twelve cases. From these values, a regression equation, in terms of ESAL, thickness (t in.), and modulus of subgrade reaction (k pci) was obtained as follows:

$$w_{57} = \exp(a_1 + a_2 \log(\text{ESAL}) + a_3 t + a_4 k) \quad (6.5)$$

w_{57} = deflection at node 57 (mil)

coefficient	95% confidence interval	
$a_1 = -0.6714$	-1.556	0.213
$a_2 = 0.339$	0.216	0.463
$a_3 = -0.422$	-0.467	-0.377
$a_4 = -0.0022$	-0.00283	-0.00160

$$R^2 = 0.93$$

The numerical results are given in Tables 6.21 to 6.24, and graphical representations of the regression equation are given in Figures 6.7 to 6.9. These Figures show the deflections caused by an 80.1 kN (18-K) single load. This load is applied after the given number of ESAL repetitions has been imposed on the pavement.

Table 6.21 Deflections produced by a 18-K single axle load, for different levels of ESAL and different types of sublayers. $t=152$ mm.

ESAL	Sublayer type		
	type 1	type 2	type 3
100000.	203.900	102.300	74.210
	5.179	2.598	1.885
1000000.	215.000	134.900	115.400
	5.461	3.426	2.931
10000000.	261.400	168.900	129.100
	6.640	4.290	3.279
mils			
mm			

Table 6.22 Deflections produced by 18-K single axle load, for different levels of ESAL and different types of sublayer. $t=203$ mm.

ESAL	Sublayer type		
	type 1	type 2	type 3
100000.	36.050	22.430	16.470
	0.916	0.570	0.418
1000000.	114.300	71.010	24.390
	2.903	1.804	0.620
10000000.	124.600	97.790	71.210
	3.165	2.484	1.809
mils			
mm			

Table 6.23 Deflections produced by 18-K single axle load, for different levels of ESAL and different types of sub-layer. $t=254$ mm.

ESAL	Sublayer type		
	type 1	type 2	type 3
100000.	20.620	13.540	10.210
	0.524	0.344	0.259
1000000.	36.680	17.270	11.390
	0.932	0.439	0.289
10000000.	38.190	28.320	14.480
	0.970	0.719	0.368
mils			
mm			

Table 6.24 Deflections produced by 18-K single axle load, for different levels of ESAL and different types of sub-layer. $t=305$ mm.

ESAL	Sublayer type		
	type 1	type 2	type 3
100000.	14.360	9.960	8.290
	0.365	0.253	0.211
1000000.	18.360	10.280	8.360
	0.466	0.261	0.212
10000000.	29.150	12.570	8.430
	0.740	0.319	0.214
mils			
mm

Figure 6.7 Deflections caused by a 80.1 kN (18-K) single axle load, for different levels of ESAL repetitions and for different thicknesses. Sublayer type 1

Figure 6.8 Deflections caused by a 80.1 KN (18-K) single axle load, for different levels of ESAL repetitions and for different thicknesses. Sublayer type 2.

Figure 6.9 Deflections caused by a 80.1 KN (18-K) single axle load, for different levels of ESAL repetitions and for different thicknesses. Sublayers type 3.

A better fitting of the regression-calculated deflections with deflections calculated by means of the structural analysis could have been obtained if individual regression lines were obtained for particular cases of thickness or subgrade modulus. The values of subgrade modulus for the regression equation are the static modulus of subgrade reaction. A conversion factor of 0.6 was used to obtain these static values given the average value of resilient modulus for a range of deflections from 0. to 40. mils. These values were as follows:

type of sublayer	static k
type 1 (soft)	54.1 MPa/m (200. pci)
type 2 (medium)	108.2 MPa/m (400. pci)
type 3 (stiff)	162.3 MPa/m (600. pci)

The amount of damage, quantified as explained in Chapter 5 and expressed in dm^2 (in.^2), was determined for the twelve cases. The numerical results obtained from the analyses (shown in Tables 6.25 to 6.28) were used to calculate a regression equation for amount of damage in terms of ESAL, thickness, and type of sublayer. This equation is as follows:

$$D = \exp((\text{ATAN}(a_1 + a_2 \log(\text{ESAL}) + a_3 t + a_4 k) 5.5) \quad (6.6)$$

D=fatigue damage (in.²)

t=slab thickness (in.)

k=static subgrade modulus (pci)

coefficient	95% confidence interval	
a1= 11.985	-15.1093	39.0794
a2= 4.718	1.1204	8.3190
a3= -3.183	-4.9828	-1.3835
a4= -0.0214	-0.0394	-0.0034

$$R^2 = 0.54$$

Table 6.25 Amount of damage caused at different levels of ESAL and for different types of sublayer. $t=152$ mm.

ESAL	Sublayer type		
	type 1	type 2	type 3
100000.	3481.000	1791.000	1531.300
	224.559	115.537	98.784
1000000.	3906.000	2827.000	1937.000
	251.976	182.370	124.956
10000000.	5083.000	3780.000	3010.000
	327.904	243.848	194.175
sq.in.			
sq.dm			

Table 6.26 Amount of damage caused at different levels of ESAL and for different types of sublayer. $t=203$ mm.

ESAL	Sublayer type		
	type 1	type 2	type 3
100000.	13.400	5.900	2.810
	0.864	0.381	0.181
1000000.	2589.500	1462.800	89.610
	167.049	94.365	5.781
10000000.	3677.000	2523.000	2129.000
	237.203	162.759	137.342
sq.in.			
sq.dm			

Table 6.27 Amount of damage caused at different levels of ESAL and for different types of sublayer. $t=254$ mm.

ESAL	Sublayer type		
	type 1	type 2	type 3
100000.	0.700	0.400	0.300
	0.045	0.026	0.019
1000000.	73.200	30.000	6.500
	4.722	1.935	0.419
10000000.	1180.000	620.700	53.000
	76.122	40.041	3.419
sq.in. sq.dm			

Table 6.28 Amount of damage caused at different levels of ESAL and for different types of sublayer. $t=305$ mm.

ESAL	Sublayer type		
	type 1	type 2	type 3
100000.	0.	0.	0.
	0.	0.	0.
1000000.	0.	0.	0.
	0.	0.	0.
10000000.	0.	0.	0.
	0.	0.	0.
sq.in. sq.dm

Figure 6.10 Amount of damage caused by different levels of ESAL repetitions and for different thicknesses. Sublayer type 1.

Figure 6.11 Amount of damage caused by different levels of ESAL repetitions and for different thicknesses. Sublayer type 2.

Figure 6.12 Amount of damage caused by different levels of ESAL repetitions and for different thicknesses. Sublayer type 3.

The areas damaged under fatigue in the concrete slab were determined for the twelve cases (Tables 6.29 to 6.32). With the numerical results obtained from the computer method of analysis, a regression line was obtained to fit all the cases. This regression equation is as follows:

$$DA = \exp(ATAN(a_1 + a_2 \log(ESAL) + a_3 t + a_4 k)6.) \quad (6.7)$$

where:

DA=damaged area (in.²)

t=slab thickness (in.)

k=static subgrade modulus (pci)

coefficient	95% confidence interval	
a1= 39.006	15.928	63.271
a2= 3.941	0.346	6.636
a3= -4.387	-5.959	-2.815
a4= -0.036	-0.521	-0.021

$$R^2 = 0.73$$

Graphical representation of this equation is given in Figures 6.13 through 6.15.

Table 6.29 Size of area damaged by different numbers of ESAL applications and for different types of sublayers.
 $t=152$ mm.

ESAL	Sublayer type		
	type 1	type 2	type 3
100000.	10400.000 670.904	5400.000 348.354	4000.000 258.040
1000000.	10600.000 683.806	8200.000 528.982	6400.000 412.864
10000000.	10800.000 696.708	8800.000 567.688	7800.000 503.178
sq.in. sq.dm			

Table 6.30 Size of area damaged by different numbers of ESAL applications and for different types of sublayers.
 $t=203$ mm.

ESAL	Sublayer type		
	type 1	type 2	type 3
100000.	5000.000 322.550	2600.000 167.726	1400.000 90.314
1000000.	9000.000 580.590	6000.000 387.060	3600.000 232.236
10000000.	9600.000 619.296	6800.000 438.668	5800.000 374.158
sq.in. sq.dm			

Table 6.31 Size of area damaged by different numbers of ESAL and for different types of sublayers.
t=254 mm.

ESAL	Sublayer type		
	type 1	type 2	type 3
100000.	1000.000	200.000	200.000
	64.510	12.902	12.902
1000000.	3400.000	1000.000	400.000
	219.334	64.510	25.804
10000000.	4800.000	2600.000	500.000
	309.648	167.726	32.255
sq in. sq dm			

Table 6.32 Size of area damaged by different numbers of ESAL applications and for different types of sublayers.
t=305 mm.

ESAL	Sublayer type		
	type 1	type 2	type 3
100000.	0.	0.	0.
	0.	0.	0.
1000000.	0.	0.	0.
	0.	0.	0.
10000000.	0.	0.	0.
	0.	0.	0.
sq.in. sq.dm

Figure 6.13 Area of slab damaged under flexural fatigue; for different levels of ESAL and different slab thicknesses. Sublayer type 1.

Figure 6.14 Area of slab damaged under flexural fatigue; for different levels of ESAL and different slab thicknesses. Sublayer type 2.

Figure 6.15 Area of slab damaged under flexural fatigue; for different levels of ESAL and different slab thicknesses. Sublayer type 3.

6.5 Summary

A given pavement section was used to obtain numerical results of pavement response using the method of analysis presented in Chapter 5. These results were used to study and demonstrate the sensitivity of the method of analysis to variations in several pavement parameters. Additionally, numerical results were also obtained to determine the response of a given pavement section to different traffic levels. Pavement response was quantified by means of amount of fatigue damage caused in the concrete slab, decay in load transfer efficiency, and maximum deflections. The parameters which cause the largest variations in pavement response are: slab thickness, number of equivalent-single-axle-load applications, modulus of subgrade reaction, and flexural behavior of concrete. The numerical values are given in Tables 6.1 to 6.10 for the case in which only fatigue damage in the concrete slab is considered. Tables 6.11 to 6.16 show the same type of results but for the case in which decay in load transfer efficiency is also considered. Higher variations of the amount of damage occur when decay in load transfer efficiency is included. The number of load repetitions has a noticeable effect on the load transfer efficiency. Pavements with thicker slab or stiffer sublayers have lower values of load transfer efficiency after a given number of

load repetitions. Pavement performance was determined for different traffic levels assuming that pumping takes place. In this case, in addition to considering fatigue damage in the slab and decay in the load transfer efficiency, pumping of the material under the slab was also considered. Results of pavement performance were presented in Tables 6.17 to 6.32. Additionally, to compensate the discrete variation of these results, regression equations were obtained for values of deflections, amount of fatigue damage, and damaged area, as a function of slab thickness, number of ESAL, and type of sublayer.

The computer scheme presented in Chapter 5 was used to obtain the numerical results presented in this Chapter. The pavement geometries as well as the pavement properties presented at the beginning of this Chapter were used as input data. Loads were specified in terms of number of 18-K ESAL applications. Additionally, a specified number of iterations of 15 to achieve convergence was used. A complete listing of the computer scheme is given in the Appendix. Additionally, in this Appendix an example with the input and output data is also provided.

CHAPTER 7 SUMMARY CONCLUSIONS AND
RECOMMENDATIONS FOR FURTHER STUDY:

Since the 1940's, pumping has been considered as a major contributor to failure in concrete pavements. It has been reported [2,3,4,51] that most of the failures in concrete pavements have been preceded by pumping of the material underneath the concrete slab. Traditional design procedures, based on a stress-fatigue criteria, have been unsuccessful in providing pavements which accomplish their intended service lives. Premature failures due to excessive pumping, erosion of sublayers and joint faulting have occurred. Therefore, in addition to the stress-fatigue criterion, an erosion criterion must be considered.

The conventional methods for structural analysis of concrete pavements do not consider the variation in the pavement properties caused by repetitive application of loads. In these methods, it is assumed that the geometric and mechanical properties retain the same values through the life of the pavement.

The major limitation in the analysis method developed by Westergaard, or any method based on his equations, is that

it is applicable only if the analyzed structure is continuous. A rigid pavement is composed of several elements with different mechanical behaviors. In general, the pavement can be divided into a concrete slab, sublayers and load transfer devices. Each one of these components behaves in a completely different manner, therefore, it is impossible to use a general differential equation to represent the behavior of the whole structure. Additionally, the existence of joints, cracks, and gaps between slab and sublayers are very common, which makes the analysis more complex and the solution in a closed form impossible. Therefore, these methods are used only to check results obtained with other methods using simple, idealized cases.

One of the main causes of pavement failures is the deterioration and gradual weakening of the pavement components. Repetitive loading produces microcracking in the slab as well as erosion and voids in the sublayers. This reduces the resistance of the pavement to the loads applied by traffic. The response of the pavement to a specific load is different throughout its entire service life. Cumulative damage reduces the general stiffness of the structure. Therefore, in the analysis of rigid pavements, especially if erosion and pumping are expected, the pavement components should be considered as having a stress dependent behavior. The response of the slab as

well as that of the sublayers should be dependent upon the previous stress history applied on the pavement. Pavements properties should be revised every few years to take into account the reduction of stiffness caused by exposure to traffic loads. In this way, a more exact calculation of stresses and deflections in the pavement at different stages of its life is accomplished. Therefore, a more precise estimation of the number of allowable load repetitions (and thereby of service life) can be achieved. Then, the designer has more and better information to carry out an economical analysis.

The present methods only give results for specific properties that the pavement has at the moment considered in the analysis. They cannot provide the stresses and deflections that occur after the pavement has been in service a certain time. The state of the pavement after several years in service cannot be determined, and thus, the maintenance cost cannot be accurately estimated.

The mechanical behavior of concrete has been described [45] as nonlinear at low stresses, and with expansion near failure. Nonlinearity is caused by microcracking due to segregation, shrinkage, or temperature changes. The initial cracks that exist in concrete develop and propagate as stresses are applied. Near failure, cracks propagate and bridge together causing disruption in the concrete mass. When reinforcing steel is provided, the cracks

distribute in areas larger than those under high stresses. Steel keeps the concrete areas together up to stresses under which disruption would otherwise occur. The available analysis methods assume a linear elastic behavior of concrete at all stress levels. Presence of reinforcing steel is not considered. Therefore, the strains corresponding to certain stresses, especially high stresses, are different from the actual corresponding strains.

In laboratory tests [45] the values of strength and modulus of elasticity obtained for concrete depend upon the rate with which the load is applied. The values obtained in an impact test are higher than those from a static slow-rate test. In the pavement slab, loads are more similar to impact loads than to static loads. Therefore the use of static strength and static modulus of elasticity is unrealistic.

In the sublayers, the nonlinear behavior of the layers is even more pronounced. Deformations in the sublayers occur as a result of relative movements of aggregates and grains. Consolidation can occur. The stiffness properties of the sublayers may be even more sensitive to the rate of load application. Therefore, the assumption of static values with linear behavior may be far from reality.

Loads in pavements are applied in a repetitive manner.

This produces fatigue in the slab, the sublayers, and the joint devices. Fatigue produces a decrease in the pavement stiffnesses. Therefore, in the available methods of analysis, the assumption of constant mechanical properties for the slab, the sublayers, and the joint devices is not realistic.

From the point of view of the mechanical behavior, an ideal method of analysis should reproduce the actual performance of the pavement structure. The stress-strain relations of the individual components should be accurately represented in the analytical model. Concrete in the pavement slab can be represented by a nonlinear plastic model. The effect caused by the presence of micro-cracks should be reflected in the stress-strain relationships. Similarly, the presence of reinforcing steel must be taken into account. In the sublayers, the sublayers should be represented by a nonlinear plastic model.

Traffic loads are repetitive in nature. This causes fatigue of all the stressed pavement elements. The concrete properties should be dependent upon the loading history. Decreases in strength and modulus of elasticity should be considered, so should the formation of cracks and break-ups in the slab. In the sublayers, repetitive loading, and excess water may create erosion and pumping leading to the formation of voids underneath the slab. This loss of support should be considered in the analysis.

The values considered for the mechanical properties should be the values obtained with loads similar in nature to the actual traffic loads. Impact values for the modulus of elasticity of concrete and for the stiffness of the sub-layers should be used. These values more accurately represent the performance of the pavement.

Load transfer devices at joints are also subjected to repetitive loading. Therefore, the method of analysis should consider the decay in load transfer efficiency. This decay can vary from a quantitative variation of the stiffness values to a qualitative variation of the actual performance. The analysis method should follow this kind of performance.

A non-linear finite element method for analysis of rigid pavements subjected to fatigue damage caused by traffic has been presented. The method is implemented in an iterative computer scheme called PMARP in which the pavement properties are modified in accordance with the fatigue damage produced by repetitive loading.

In a rigid pavement, most of the structural support is provided by the concrete slab. This slab, then, under the action of traffic is subjected to many load repetitions through the service life of the pavement. The load repetitions produce gradual damage in all the pavement components until, eventually, cracking and faulting

occurs, producing failure of the structure. In jointed concrete pavements, the devices needed to transfer the load across the joints, especially transverse joints, also lose their efficiency along the life of the pavement. Fatigue damage imposed to the concrete slab reduces its strength and modulus of elasticity because of the occurrence of microcracks. Similarly, fatigue damage caused by repetitive loading reduces the load transfer efficiency of the dowel bars by increasing the degree of looseness of the bars. These decays in the properties of the pavement components modify the response of the pavement to a given load. Stresses and deflections calculated assuming the initial

stiffness values are different from those obtained assuming the values reduced in accordance with fatigue damage. Assuming fatigue damage, results closer to actual field conditions are obtained.

In the computer implementation of the method of analysis presented herein, the modifications of the properties of the concrete slab are based on the flexural behavior of a particular concrete, specified as part of the input data. Similarly, the modification of the values of the resilient modulus of subgrade reaction is made on the basis of values of k_r specified in the input data. For the modification of the load transfer efficiency, however, only the initial value can be specified in the input data.

The fatigue behavior is controlled by an expression built into the computer scheme. Nevertheless, this expression can be modified by changing the values of the coefficients used in the computer implementation. In any event, accuracy of the absolute values obtained with the method proposed herein, depends on the accuracy of the information provided in the input; especially the information concerning the fatigue behavior of the different pavement components. The relative results, however, are more accurate and especially useful in comparative studies.

Additionally, a pumping model has been developed based on data of the AASHO Road Test. This pumping model has been implemented in the method of analysis presented. With this pumping model, implemented in a subroutine called "pump", the pumping index, as well as the support conditions occurring in a pavement section can be determined for any number of load applications.

The method of analysis presented herein has the following features:

1. Non-linear finite element method for analysis of cracked and jointed concrete pavements.

2. Finite element discretization:

- 2.1 Slab; orthotropic cracked and uncracked thin plates, with or without reinforcement

2.2 Sublayers: Winkler sublayers with resilient subgrade modulus

2.3 Joints: load transfer devices as beam elements

3. Long term effect of load applications.

3.1 Fatigue damage and cracking can occur in concrete slab

3.2 Fatigue decay in load transfer efficiency of dowel bars can occur

3.3 Pumping development and loss of support can occur

4. Results from analysis:

4.1 Deflections at all nodal points

4.2 Final values of subgrade moduli at all nodes

4.3 State of stresses at all nodal points

4.4 Amount of damage caused to concrete slab

4.5 Damaged area in concrete slab

4.6 Total decay in load transfer efficiency

4.7 Potential of pumping

4.8 Area with no support caused by pumping

The computer implementation of the proposed method for analysis of rigid pavements considering fatigue is not a very critical task. A computer scheme of the method of analysis presented in Chapter 5 as well as an example are given in the Appendix. It is more difficult to obtain the appropriate experimental information to use the analytical model. Experimental and field results are required to develop the following tasks: (1) accurate representation of behavior of plain and reinforced concrete in flexural fatigue; (2) Behavior of load transfer devices under repetitive loading; and (3) accurate pumping model which considers the following factors: structural, climatic, traffic, and erosion resistance.

For a typical pavement section, the parameters which provide the largest effects on pavement response are: slab thickness; number of ESAL repetitions, modulus of subgrade reaction, and flexural behavior of concrete. The numerical results obtained by means of the method of analysis presented here vary in a discrete manner due to the fact that cracking and void formation are allocated in such a manner. This discrete variation can be solved in two ways: by refining the finite element mesh so that the discrete changes are small, or by using regression equations to represent the numerical values obtained from the analysis. Refining the finite element mesh would require large computer memory which in some cases could not be provided.

The results obtained in Chapter 6 are represented by regression equations. These regression equations are plotted to provide an easy way to determine the pavement performance given the pavement features and the traffic conditions. The results provided in Chapter 6 are obtained for the assumptions explained in Chapter 5. Pavement performance is very sensitive to the support conditions which are defined by the degree of pumping. The pumping model provided in Chapter 5 considers only the structural features of the pavement and the volume of traffic. Any information concerning other factors, such as amount of rainfall, erosion susceptibility of the sub-base, or joint sealing, should be included. This information can be included by using factors which modify the potential volume of pumped material to obtain the actual volume of pumped material.

Further research to develop an appropriate method of analysis of rigid pavements is required in the following areas:

- 1 Development of an appropriate nondestructive method to obtain the configuration of voids under concrete pavements, so that a pumping model can be verified by means of actual field data.

- 2 Development of accurate methods to predict amount of pumping based on: material erodability, drainage condi

tions, amount of rainfall, structural conditions of pavement, and volume of traffic.

3 Development of procedures to relate pumping index with actual loss of support of concrete slab.

4 Development of a more accurate representation of behavior of reinforced concrete under flexural stresses.

5 Development of a more accurate model to represent the behavior of dowel bars under repetitive loading.

REFERENCES

1. Yoder, E.J. and Witczak, M.W., Principles of Pavement Design, John Wiley & Sons Inc. 1975.
2. AASHTO, The AASHTO Road Test, Report 7, Summary Report, HRB Special Report 61G, 1962.
3. Yoder, E.J., Pumping of Highway and Airfield Pavements, Joint Highway Research Project, 1946. Purdue University, W. Lafayette IN.
4. Cedergren, H.R., Methodology and Effectiveness of Draining Systems for Airport Pavements, U.S. Army Construction Engineering Research Laboratory, 1974. Champaign IL, Tech Report C-13
5. Highway, Research Board, HRB Special Report 95, Highway Research Board, Washington, D.C., 1968, pp 1-33.
6. Westergaard, H.M., New Formulas For Stresses in Concrete Pavements of Airfields, Proc. ASCE Vol. 73 (5) May, 1947.
7. Pickett, G. and Ray, G.K., Influence Charts for Concrete Pavements, Transactions, ASCE Vol. 116 1951. pp 49-73.
8. Packard, R.G. and Tayabji, S.D., Mechanistic Design of Concrete Pavements to Control Joint Faulting and Subbase Erosion., International Seminar on Drainage and Erodability, Paris, France., pp 38.
9. Tabatabaie, A. and Barenberg, E.J., Finite Element Analysis of Jointed or Cracked Concrete Pavements, Transportation Research Board, Transportation Research Record, (676) 1978. pp 11-19
10. Westergaard, H.M., stresses in concrete pavements computed by theoretical analysis, Public Roads, April, 1926. pp 25-35
11. Westergaard, H.M., Analytical Tools For Judging Results of Structural Tests of Concrete Pavements, Public Roads, Dec., 1933. pp 185-188
12. Westergaard, H.M., Stresses in Concrete Runways of Airports, Proc. Highway Research Board, National Research Council Vol. 19 1939. pp 197-202

REFERENCES

13. Westergaard, H.M., Stress Concentrations in Plates Loaded Over Small Areas, Transactions ASCE Vol. 108 1943. pp 831-856
14. Hogg, A.H., Equilibrium of a Thin Plate, Symmetrically Loaded, Resting on an Elastic Foundation of Infinite Depth, Philosophical Magazine Vol. 25 1938. Series 7, pp 576-582.
15. Volterra, E., Sul Problema Generale della Piastra Poggiata su Suolo Elastico, Istituto per le Applicazioni del Calcolo, (201) 1947. Consiglio Nazionale Delle Ricerche, Roma
16. Bergstrom, S.G., Circular Plates with Concentrated Load on an Elastic Foundation, Swedish Cement and Concrete Institute, (6) 1946. Stockholm.
17. Timoshenko, S. and Woinowsky, S., Theory of Plates and Shells, McGraw Hill Book Company, 2nd. Ed., 1959.
18. Holl, D.L., Thin Plates on Elastic Foundations, 5th International Conf. for Applied Mechanics, 1938. Cambridge Mass.
19. Pickett, G., Raville, M.E., Janes, W.C., and McCormick, F.J., Deflections, Moments and Reactive Pressure for Concrete Pavements, Bulletin 65, October 1951. Kansas State College Bulletin.
20. Timoshenko, S. and Lessels, J., Applied Elasticity , Westinghouse Technical Night School Press, 1925.
21. Teller, L.W. and Sutherland, E.C., A Study of the Structural Action of Several Types of Transverse and Longitudinal Joint Designs, Public Roads Vol. 17 (7 and 8) Oct. 1936. pp 143-192
22. Friberg, B.F., Design of Dowels in Transverse Joints of Concrete Pavements, Transactions, ASCE, 1938.
23. Sutherland, E.C. and Cashell, H.D., Structural Efficiency of Transverse Weakened Plane Joints, Public Roads Vol. 24 (4) 1945. pp 88-97
24. Kushing, J.W. and Fremont, W.O., Design of Load Transfer Joints in Concrete Pavements, Highway Research Board, 1940.
25. Giggin, H.W., Transverse Joints in The Design of Heavy Concrete Pavements, Proc. HRB, 1943.

26. Finney, E.A. and Fremont, W.O., Progress Report on Load Deflection Tests Dealing with Length and Size of Dowels, Proc. HRB, 1947.
27. Marcus, H., Load Carrying Capacity of Dowels at Transverse Pavements Joints, ACI Proc. Vol. 48 1951. pp 169-184
28. Teller, L.W. and Cashell, H.D., Performance of Doweled Joints Under Repetitive Loading, Public Roads, 1955.
29. Smith, A.R. and Benham, S.W., Effect of Dowel-Bar Misalignment Across Concrete Pavement Joints, Transactions, ASCE, 1938.
30. Newmark, N.M., Numerical Methods of Analysis of Bars, Plates, and Elastic Bodies, Numerical Methods of Analysis in Engineering, McMillan Co., 1949. N.Y. pp 139-168
31. Clough, R., The Finite Element Method in Plane Stress Analysis, Proc. 2nd Conf. on Electronic Computations, ASCE, Sept. 1960.
32. Turner, M., Stiffness and Deflection Analysis of Complex Structures, Aero. Sci. Vol. 23 (9) 1956. pp 805-823
33. Zienkiewicz, O.C. and Cheung, Y.K., The Finite Element Method for the Analysis of Elastic Isotropic and Orthotropic Slabs, Proc. J. Inst. Civ. Engrs., (28,471) 1964.
34. Hudson, W.R. and Matlock, H., Analysis of Discontinuous Orthotropic Pavement Slabs Subjected to Combined Loads, Highway Research Record, (137) 1966.
35. Huang, Y.H. and Wang, S.T., Finite Element Analysis of Concrete Slabs and Its Implications for Rigid Pavement Design, Highway Research Record, (466) 1973. pp 59-69
36. Huang, Y.H., Finite Element Analysis of Slabs on Elastic Foundations, ASCE, Transp. Engineering Journal, May 1974. pp 403-416
37. Huang, Y.H., Analysis of Symmetrically Loaded Slabs on Elastic Solid, ASCE, Transp. Engineering Journal, May, 1974. pp 537-541

38. Cheung, Y.K. and Zienkiewicz, O.C., Plates and Tanks on Elastic Foundations- An Application of Finite Element Method, International Journal of Soils and Structures Vol. 1 1965. pp 451-461
39. Tayabji, S.P. and Colley, B.E., Analysis of Jointed Concrete Pavements, Federal Highway Administration, National Technical Information Service, 1981.
40. Chou, Y.T., Structural Analysis Computer Programs for Rigid Multicomponent Pavement Structures with Discontinuities, WESLIQID and WESLAYER, Technical Reports 1,2,and 3, U.S. Army Engineering Waterways Experiment Station, C.V. Vicksburg Miss., May 1981.
41. Portland, Cement Association, Thickness Design for Concrete Pavements, 1966.
42. AASHTO, Interim Guide For Design of Pavement Structures, American Association of State Highway and Transportation Officials, Chapter III revised, 1981.
43. ACI, Committee 325 5T Design Procedure for Continuously Reinforced Concrete Pavements for Highways, ACI Journal, June, 1972, pp 309-319.
44. Thompson, M.R., Barenberg, E.J., Fischer, J.A., and Ioannides, A.M., The Resilient Modulus of Subgrade Reaction, Annual Meeting, TRB., Jan., 1984.
45. Chen, W.F., Plasticity in Reinforced Concrete, McGraw-Hill, 1982.
46. Zienkiewicz, O.C., The Finite Element Method Third Edition, McGraw Hill Book Co., New York, 1977.
47. Miner, M., Cumulative Damage in Fatigue , ASCE Transactions, vol. 67, 1945, pp A159-A164
48. Galloway, J.W. and Raithby, K.D., Effects of Rate of Loading in Flexural Strength and fatigue Performance of Concrete, Transport and Road Research Laboratory, Report LR 547, 1973.
49. Raithby, K.D. and Galloway, J.W., Effects of Moisture Condition, Age, and Rate of Loading on Fatigue of Plain Concrete., Abeles Symposium on Fatigue of Concrete, 1974.

50. Gulden Wouter, P.E., Experience in Georgia with Drainage of Jointed Concrete Pavements, Georgia Department of Transportation Office of Materials and Research, March, 1983 Paris France. International Seminar on Drainage and Erodability

APPENDIX

INPUT GUIDE AND EXAMPLE

Input Guide

The "PMARP" iterative, nonlinear finite element program can be used to analyze reinforced/unreinforced concrete pavements with joints. Different amounts of reinforcement may be used in the longitudinal and transverse directions. Joint types may be: doweled, aggregate interlock, or keyed. Only one layer, constituted by the concrete slab, can be handled. The stiffness of the sublayers can be represented by a composite resilient impact modulus. Loads may consist of one application with any configuration, or of several repetitions with constant configuration. Stresses and deflections at all nodal points, after any number of ESAL load repetitions, can be obtained. Additionally, fatigue damage in the concrete slab, and in the doweled load transfer devices can be quantified. Pumping potential can also be determined as well as loss of support due to pumping.

The pavement section may consist of 1, 2, 3, 4, or 6 slabs. with nodes numbered from left to right and from bottom to top (Fig. A.1).

Figure A.1 Slab configuration and numbering of nodes.

Format

CARD	FORMAT	VARIABLE
1	(815)	NUMBER OF NODES X DIRECTION; SLABS 1,4 NUMBER OF NODES X DIRECTION; SLABS 2,5 NUMBER OF NODES X DIRECTION; SLABS 3,6 NUMBER OF NODES Y DIRECTION; SLABS 1,2,3 NUMBER OF NODES Y DIRECTION; SLABS 4,5,6 NUMBER OF LOADED ELEMENTS NUMBER OF ITERATIONS PER LOAD INCREMENT NUMBER OF LOAD INCREMENTS
2	(8F10.3)	X COORDINATES OF NODES IN X DIRECTION X COORDINATES OF NODES IN Y DIRECTION (AS MANY CARDS AS NEEDED)
3	(3I5,F10.3)	NUMBER OF SLABS NUMBER OF LAYERS (1) COMPOSITE ACTION (0) SATIATIC SURGRADE MODULUS
4	(F10.3,E10.3,6F10.3)	THICKNESS OF SLAB (IF CONSTANT) MODULUS OF ELASTICITY OF CONCRETE (IF CONSTANT) POISSON'S RATIO OF CONCRETE TENSILE STRENGTH OF CONCRETE Z OF REINFORCEMENT IN X DIRECTION DIAMETER OF BARS IN X DIRECTION Z OF REINFORCEMENT IN Y DIRECTION DIAMETER OF BARS IN Y DIRECTION
5	(8F10.3)	THICKNESSES AT NODES IF THICKNESS OF SLAB=0
6	(RE10.3)	MODULI OF ELASTICITY AT NODES IF E=0
7	(RF10.3)	SURGRADE MODULI AT NODES IF K=0
8	(15)	TYPE OF LOAD TRANSFER DEVICE IN X-DIR. DOWEL => 1 AGGREG. INT. => 0 KEYED => 2

9	(2F10.3,E10.3,4F10.33F10.3)	IF DOWELS IN X DIR. INSIDE DIAMETER OF DOWEL BARS X-DIR. OUTSIDE DIAMETER OF DOWEL BARS X-DIR. MODULUS OF ELASTICITY OF BAR X-DIR. LENGTH OF DOWEL BARS X-DIR. JOINT OPENING POISSON'S RATIO DOWELS X-DIR. DOWEL CONCRETE INTERACTION FACTOR
10	(E10.3)	IF AGGREG. INT. IN IN X-DIR. USE A LARGE VALUE; 10E+9.
11	(15)	TYPE OF LOAD TRANSFER DEVICE IN Y-DIR. (SAME AS IN X-DIR)
12	USE ADDITIONAL CARDS FOR TYPE OF LOAD TRANSFER DEVICE AS IN X-DIR	
13	(15,5F10.3)	NUMBER OF LOADED ELEMENT PRESSURE (1 PER ELEMENT) X1 COORDINATE (LOCAL COORDINATE) X2 COORDINATE (LOCAL COORDINATE) } LOADED Y1 COORDINATE (LOCAL COORDINATE) } AREA Y2 COORDINATE (LOCAL COORDINATE)
14	(3F5.3)	RELATIVE STRENGTHS OF CONCRETE (FLEXURAL ENDURANCE CURVE)
15	(3F10.1)	NUMBER OF REPETITIONS AT FAILURE (CORRESPONDING WITH PREVIOUS STRENGTHS)
16	(F10.0)	TOTAL NUMBER OF LOAD REPETITIONS
17	(FREE)	POWER OF CURVE TO FIT RESILIENT K (1 OR 2)
18	(FREE)	K SUB R, DEFLECTION
19	(FREE)	K SUB R, DEFLECTION
20	(FREE)	K SUB R, DEFLECTION

Example

The pavement section shown in Fig. 6.1 is used herein for illustration purposes. The pavement properties are as follows:

$$t = 203 \text{ mm (8 in.)}$$

$$E_c = 20.7 \text{ MPa (3000. ksi)}$$

$$u = 0.15$$

$$p_x = p_y = 0.25 \text{ z}$$

$$\text{Diameter of reinforcement} = 6.3 \text{ mm (0.25 in.)}$$

Dowel bars:

$$D_d = 31.8 \text{ mm (1.25 in.)}$$

$$\text{Length} = 457 \text{ mm (18 in.)}$$

$$\text{Spacing} = 305 \text{ mm (12 in.)}$$

$$E_m = 199,949. \text{ MPa (29,000 ksi)}$$

$$\text{Joint width} = 19. \text{ mm (0.75 in.)}$$

$$\text{PCI factor} = 11,030 \text{ MPa (1,600 ksi)}$$

Load

$$80.1 \text{ kN (18-K) equivalent single axle load.}$$

$$\text{number of applications} = 1,000,000. \text{ ESAL}$$

$$\text{Loaded area} = \text{two- } 203 \text{ mm} \times 254 \text{ mm (8 in.} \times 10 \text{ in.) areas.}$$

$$\text{Tire pressure} = 775 \text{ kPa (112.5 psi)}$$

$$\text{Number of load increments} = 1$$

$$\text{Number of iteration per load increment} = 15$$

Flexural behavior

relative stress	1.0	0.8	0.5
load repetitions	1.0	200,000.	1,000,000

Resilient modulus of subgrade

deflection (in.)	0.01	0.025	0.050
resilient (pct)	710.	610.	510.

Input Data

	1	2	3	4	5	6	7	8
	1234567890123456789012345678901234567890123456789012345678901234567890							
8	8	0	8	0	2	15	1	500.0
	0.0	60.0	80.0	100.0	120.0	140.0	160.0	180.0
	180.0	200.0	220.0	240.0	260.0	280.0	300.0	360.0
	0.0	20.0	40.0	60.0	80.0	100.0	120.0	140.0
2	1	0	400.0					
	8.0	0.300e+07	0.15	500.	0.0025	0.25	0.0025	0.25
1								
	0.000	1.25	0.290e+08	12.0	18.0	0.75	0.29	0.16e+07
43	112.5	10.0	20.0	0.0				
46	112.5	10.0	20.0	12.0	20.0			
1.0	0.8	0.50						
	1.0	200000.0	1000000.0					
1000000.0								
2								
	710.0	0.010						
	610.0	0.025						
	510.0	0.050						

Output

```

*****
*
*   finite element analysis of conc. pav.
*
*   with fatigue and pumping
*
*   PURDUE UNIVERSITY
*
*   Based on method developed at:
*
*   university of illinois
*****

```

total
28196

```

no. of nodes in x-direction slabs  1,4=  8
no. of nodes in x-direction slabs  2,5=  8
no. of nodes in x-direction slabs  3,6=  0
no. of nodes in y-direction slabs  1,2,3=  8
no. of nodes in y-direction slabs  4,5,6=  0

```

x-coordinates are:

0.	60.000	80.000	100.000	120.000	140.000	160.000	180.000	180.0
220.000	240.000	260.000	280.000	300.000	360.000			

y-coordinates are:

0.	20.000	40.000	60.000	80.000	100.000	120.000	140.000
----	--------	--------	--------	--------	---------	---------	---------

```

no. of slabs= 2
no. of layers= 1
comp. action= 0

properties of the top layer is:
poisson ratio of top layer= 0.150
slab thickness= 8.000
% reinforcement in X = 0.250 bar diameter in X = 0.250
% reinforcement in Y = 0.250 bar diameter in Y = 0.250
modulus of top layer= 0.300e+07
subgrade modulus= 400.000

```

***** joints in xdirection *****

type of load transfer is steel bars

```

properties of the dowel bars are:
inside dia.= 0.
outside dia.= 1.250
modulus of elasticity= 0.290e+08
spacing= 12.000
length= 18.000
poisson ratio= 0.290
dowel-concrete interaction= 0.160e+07

```

joint width= 0.750

element	press	x1-coor.	x2-coor.	y1-coor.	y2-coor.
43	112.500	10.000	20.000	0.	8.000
46	112.500	10.000	20.000	12.000	20.000

pumping index= 6.36

void depth= 0.250000

Iteration # 10									
Number of Load Applications= 1.000000e+06									
node	type of damage	updated properties		y#rotation		x#rotation		y#rotation	
33	Decay in T	New Thickness=	4.428						
34	Decay in Ec	Ec=	0.298e+07	Ec=	0.298e+07				
41	Decay in T	New Thickness=	4.425						
42	Decay in T	New Thickness=	4.420						
43	Decay in Ec	Ec=	0.293e+07	Ec=	0.293e+07				
50	Decay in T	New Thickness=	4.530						
51	Decay in Ec	Ec=	0.273e+07	Ec=	0.273e+07				
58	Decay in Ec	Ec=	0.293e+07	Ec=	0.293e+07				
59	Decay in T	New Thickness=	4.437						
60	Decay in Ec	Ec=	0.293e+07	Ec=	0.293e+07				
61	Decay in Ec	Ec=	0.292e+07	Ec=	0.292e+07				
67	Decay in T	New Thickness=	4.421						
68	Decay in Ec	Ec=	0.296e+07	Ec=	0.296e+07				
74	Decay in T	New Thickness=	4.434						
75	Decay in T	New Thickness=	4.426						
76	Decay in Ec	Ec=	0.293e+07	Ec=	0.293e+07				
77	Decay in Ec	Ec=	0.294e+07	Ec=	0.294e+07				
81	Decay in T	New Thickness=	4.424						
82	Decay in T	New Thickness=	4.420						
83	Decay in Ec	Ec=	0.297e+07	Ec=	0.297e+07				
84	Decay in Ec	Ec=	0.293e+07	Ec=	0.293e+07				
89	Decay in Ec	Ec=	0.291e+07	Ec=	0.291e+07				
DAMAGED AREA=6800.00									
AMOUNT OF DAMAGE=1462.									
error in k= 8.23452e-03									
% decay in load transfer= 51.0638									
node	deflection	x#rotation		y#rotation		subgrade stress		damaged area	
1	0.73703e-04	0.326844e-06	-0.172635e-05			0.061		200.000	791.477
2	0.743722e-04	0.110739e-06	-0.246379e-05			0.059		400.000	791.549
3	0.688227e-04	0.458670e-06	-0.224513e-05			0.054		200.000	791.641
4	0.585266e-04	0.726870e-06	-0.137196e-05			0.045		400.000	791.870
5	0.419726e-04	0.672138e-06	-0.403305e-06			0.033		200.000	792.163
6	0.315475e-04	0.347543e-06	0.186619e-06			0.025		400.000	792.390
7	0.288431e-04	-0.821781e-07	0.226233e-06			0.023		200.000	792.433
8	0.336078e-04	-0.308332e-06	-0.253386e-06			0.027		400.000	792.258
9	-0.208329e-03	0.291602e-05	-0.929799e-05			0.		200.000	799.112
10	-0.256671e-03	0.136693e-05	-0.118634e-04			0.		400.000	799.475
11	-0.248801e-03	-0.212932e-05	-0.125507e-04			0.		200.000	799.635
12	-0.180043e-03	-0.435518e-05	-0.103229e-04			0.		400.000	798.352
13	-0.925166e-04	-0.399345e-05	-0.688405e-05			0.		200.000	796.457
14	-0.329601e-04	-0.181057e-05	-0.455046e-05			0.		400.000	794.950
15	-0.204996e-04	0.433466e-06	-0.433875e-05			0.		200.000	794.329
16	-0.387587e-04	0.977373e-06	-0.556221e-05			0.		400.000	794.340
17	-0.335601e-03	0.802524e-05	0.715229e-06			0.		200.000	799.367
18	-0.506080e-03	0.699420e-05	-0.104663e-04			0.		400.000	803.662

19	-0.559372e-03	-0.196521e-05	-0.169104e-04	0.	805.719
20	-0.468716e-03	-0.812967e-05	-0.157554e-04	0.	804.299
21	-0.27457e-03	-0.789363e-05	-0.111827e-04	0.	801.092
22	-0.166571e-03	-0.289040e-05	-0.865610e-05	0.	798.483
23	-0.161935e-03	0.223992e-05	-0.994788e-05	0.	797.761
24	-0.234951e-03	0.416013e-05	-0.145609e-04	0.144	798.710
25	-0.182644e-03	0.276559e-04	0.642213e-04	0.	786.288
26	-0.408766e-03	0.258080e-04	0.297387e-04	0.	798.692
27	-0.757892e-03	0.724084e-05	0.297562e-05	0.	807.288
28	-0.719507e-03	-0.830963e-05	0.727528e-05	0.	808.523
29	-0.498789e-03	-0.110527e-04	0.840696e-05	0.	805.348
30	-0.356933e-03	-0.197908e-05	-0.821031e-05	0.	802.422
31	-0.413830e-03	0.778692e-05	-0.137414e-04	0.	802.775
32	-0.637838e-03	0.132693e-04	-0.247158e-04	0.	806.534
33	0.353730e-02	0.111677e-03	0.306469e-03	2.564	724.936
34	0.135573e-02	0.858003e-04	0.164573e-03	1.032	761.437
35	0.397830e-04	0.428453e-04	0.868211e-04	0.031	785.954
36	-0.358305e-03	0.507349e-05	0.522908e-04	0.	795.326
37	-0.282419e-03	-0.591066e-05	0.420169e-04	0.	797.053
38	-0.284335e-03	0.809902e-05	0.240669e-04	0.	796.965
39	-0.587070e-03	0.218939e-04	0.124528e-05	0.	805.029
40	-0.113245e-02	0.311905e-04	-0.209123e-04	0.	815.494
41	0.160501e-01	0.429409e-03	0.988331e-03	0.	0.
42	0.797080e-02	0.310256e-03	0.507852e-03	5.175	649.280
43	0.383183e-02	0.119322e-03	0.286453e-03	2.733	713.349
44	0.253583e-02	0.315323e-04	0.255630e-03	1.871	737.648
45	0.190144e-02	0.408055e-04	0.198561e-03	1.432	753.086
46	0.899139e-03	0.550966e-04	0.105396e-03	0.696	773.972
47	-0.201592e-03	0.547665e-04	0.409706e-04	0.	796.257
48	-0.134736e-02	0.400603e-04	0.362185e-05	0.	819.394
49	0.430120e-01	0.875198e-03	0.158214e-02	0.	0.
50	0.240191e-01	0.874211e-03	0.101241e-02	0.	0.
51	0.159010e-01	0.278769e-03	0.545932e-03	0.	0.
52	0.922070e-02	0.902859e-04	0.393507e-03	0.	0.
53	0.711519e-02	0.143197e-03	0.326941e-03	0.	0.
54	0.382218e-02	0.162362e-03	0.176451e-03	2.760	721.982
55	0.109086e-02	0.112154e-03	0.884345e-04	0.842	771.573
56	-0.890209e-03	0.942337e-04	0.437243e-04	0.	810.424
57	0.710102e-01	0.112811e-02	0.111753e-02	0.	0.
58	0.442525e-01	0.140096e-02	0.100174e-02	0.	0.
59	0.237187e-01	0.533918e-03	0.536265e-03	0.	0.
60	0.169704e-01	0.150994e-03	0.375014e-03	0.	0.
61	0.135151e-01	0.272257e-03	0.284312e-03	0.	0.
62	0.744847e-02	0.272666e-03	0.202584e-03	0.	0.
63	0.325726e-02	0.175986e-03	0.116688e-03	2.388	733.087
64	0.399436e-03	0.130152e-03	0.755176e-04	0.314	785.886
65	0.666305e-01	0.116928e-02	-0.176953e-02	0.	0.
66	0.432959e-01	0.116951e-02	-0.131355e-02	0.	0.
67	0.235822e-01	0.635853e-03	-0.746388e-03	0.	0.
68	0.155256e-01	0.234352e-03	-0.502332e-03	0.	0.
69	0.114691e-01	0.203473e-03	-0.384592e-03	0.	0.
70	0.712896e-02	0.213616e-03	-0.245947e-03	0.	0.
71	0.311604e-02	0.160238e-03	-0.132051e-03	2.428	732.103

72	0.626504e-03	0.123054e-03	-0.732538e-04	0.490	781.475
73	0.727585e-01	0.716169e-03	-0.155039e-02	0.	0.
74	0.187958e-01	0.633028e-03	-0.104485e-02	0.	0.
75	0.981153e-02	0.258635e-03	-0.549856e-03	6.203	632.230
76	0.636512e-02	0.104181e-03	-0.323536e-03	4.334	680.921
77	0.451907e-02	0.829299e-04	-0.283176e-03	3.212	710.863
78	0.268934e-02	0.925626e-04	-0.128966e-03	1.998	242.861
79	0.940221e-03	0.811299e-04	-0.932198e-04	0.729	775.328
80	-0.630786e-03	0.782474e-04	-0.438070e-04	0.	806.040
81	0.969160e-02	0.254855e-03	-0.696518e-03	6.235	643.357
82	0.469629e-02	0.197500e-03	-0.324708e-03	3.353	714.029
83	0.204593e-02	0.728115e-04	-0.215628e-03	1.547	725.912
84	0.103060e-02	0.310239e-04	-0.161272e-03	0.797	773.057
85	0.566664e-03	0.190914e-04	-0.120157e-03	0.443	781.785
86	0.170100e-03	0.221055e-04	-0.751077e-04	0.134	789.802
87	-0.356808e-03	0.315048e-04	-0.361254e-04	0.	800.424
88	-0.109797e-02	0.414502e-04	-0.425517e-05	0.	815.441
89	0.154895e-02	0.631860e-04	-0.167412e-03	1.191	769.011
90	0.234921e-03	0.564300e-04	-0.930700e-04	0.186	790.782
91	0.559282e-03	0.224933e-04	-0.563116e-04	0.	804.237
92	0.788205e-03	0.234722e-05	-0.355199e-04	0.	808.142
93	0.720551e-03	-0.620656e-05	-0.212008e-04	0.	806.999
94	-0.599861e-03	-0.311197e-05	-0.102200e-04	0.	804.933
95	-0.644900e-03	0.822942e-05	0.234600e-05	0.	806.073
96	-0.913560e-03	0.170145e-04	0.184372e-04	0.	811.662
97	-0.731720e-03	0.209309e-04	-0.342974e-04	0.	799.299
98	-0.677571e-03	0.198329e-04	-0.895072e-05	0.	806.212
99	-0.922424e-03	0.504311e-05	0.742308e-05	0.	810.924
100	-0.899923e-03	-0.648104e-05	0.138145e-04	0.	810.353
101	-0.713916e-03	-0.106986e-04	0.137732e-04	0.	806.969
102	-0.522989e-03	-0.737463e-05	0.123211e-04	0.	803.463
103	-0.445604e-03	-0.766110e-07	0.118961e-04	0.	802.064
104	-0.501286e-03	0.439619e-05	0.202434e-04	0.	803.339
105	-0.406565e-03	0.693097e-05	0.664481e-05	0.	801.409
106	-0.547065e-03	0.569695e-05	0.152662e-04	0.	803.698
107	-0.594719e-03	-0.876938e-06	0.201164e-04	0.	804.454
108	-0.525811e-03	-0.594067e-05	0.197371e-04	0.	803.089
109	-0.383421e-03	-0.229180e-05	0.161984e-04	0.	800.574
110	-0.253472e-03	-0.525263e-05	0.124615e-04	0.	798.197
111	-0.182624e-03	-0.181340e-05	0.110179e-04	0.	796.908
112	-0.167009e-03	-0.438312e-06	0.126908e-04	0.	796.695
113	-0.169842e-03	0.286556e-05	0.108906e-04	0.	797.092
114	-0.243639e-03	0.143362e-05	0.111042e-04	0.	792.823
115	-0.244844e-03	-0.133393e-05	0.138739e-04	0.	792.784
116	-0.195681e-03	-0.327122e-05	0.122233e-04	0.	796.905
117	-0.124678e-03	-0.355491e-05	0.933762e-05	0.	795.636
118	-0.619246e-04	-0.258702e-05	0.656901e-05	0.	794.502
119	-0.224708e-04	-0.144514e-05	0.506703e-05	0.	793.790
120	0.380501e-05	-0.166134e-05	0.681965e-05	0.	793.327
121	0.841789e-04	0.383809e-06	0.140696e-05	0.003	791.724
122	0.787103e-04	0.241847e-06	0.201339e-05	0.062	791.882
123	0.720129e-04	0.454718e-06	0.180934e-05	0.057	792.005
124	0.604362e-04	0.626959e-06	0.113221e-05	0.048	792.211

node	depth	x#stress	y#stress	xy#stress	max#stress	min#stress
125	0.463160e-04	0.690616e-06	0.376150e-06	0.037	792.467	
126	0.349735e-04	0.388949e-06	-0.163308e-06	0.028	792.677	
127	0.326533e-04	-0.176557e-06	-0.326820e-06	0.026	792.721	
128	0.402236e-04	-0.449064e-06	-0.348712e-06	0.032	792.577	
1	0.	0.	0.	0.	0.	0.
	8.000	0.	0.	0.	0.	0.
	8.000	0.	0.	0.	0.	0.
	8.000	0.	0.	0.	0.	0.
2	0.	0.	0.	0.	0.	0.
	8.000	0.	0.	0.	0.	0.
	8.000	0.	0.	0.	0.	0.
	8.000	0.	0.	0.	0.	0.
3	0.	0.	0.	0.	0.	0.
	8.000	0.	0.	0.	0.	0.
	8.000	0.	0.	0.	0.	0.
	8.000	0.	0.	0.	0.	0.
4	0.	0.	0.	0.	0.	0.
	8.000	0.	0.	0.	0.	0.
	8.000	0.	0.	0.	0.	0.
	8.000	0.	0.	0.	0.	0.
5	0.	0.	0.	0.	0.	0.
	8.000	0.	0.	0.	0.	0.
	8.000	0.	0.	0.	0.	0.
	8.000	0.	0.	0.	0.	0.
6	0.	0.	0.	0.	0.	0.
	8.000	0.	0.	0.	0.	0.
	8.000	0.	0.	0.	0.	0.
	8.000	0.	0.	0.	0.	0.
7	0.	0.	0.	0.	0.	0.
	8.000	0.	0.	0.	0.	0.
	8.000	0.	0.	0.	0.	0.
	8.000	0.	0.	0.	0.	0.
8	0.	0.	0.	0.	0.	0.
	8.000	0.	0.	0.	0.	0.
	8.000	0.	0.	0.	0.	0.
	8.000	0.	0.	0.	0.	0.
9	0.	0.	0.	0.	0.	0.
	8.000	0.	0.	0.	0.	0.
	8.000	0.	0.	0.	0.	0.
	8.000	0.	0.	0.	0.	0.

21	8.000	0.	8.000	0.	e+00	0.	e+00	0.	e+00	0.	e+00	0.	e+00
	0.	0.264000e+01	0.246853e+01	0.246853e+01	0.205513e+01	0.205513e+01	0.450946e+01	0.339086e+00					
	8.000	-0.264000e+01	-0.246853e+01	-0.246853e+01	-0.205513e+01	-0.205513e+01	-0.399086e+00	-0.450946e+01					
	8.000	0.	0.	0.	e+00	e+00	0.	e+00	0.	e+00	0.	e+00	0.
	8.000	0.	0.	0.	e+00	e+00	0.	e+00	0.	e+00	0.	e+00	0.
22	0.	0.277738e+01	0.329198e+01	0.329198e+01	0.221993e+00	0.221993e+00	0.337451e+01	0.269485e+01					
	8.000	-0.277738e+01	-0.329198e+01	-0.329198e+01	-0.221993e+00	-0.221993e+00	-0.269485e+01	-0.337451e+01					
	8.000	0.	0.	0.	e+00	e+00	0.	e+00	0.	e+00	0.	e+00	0.
	8.000	0.	0.	0.	e+00	e+00	0.	e+00	0.	e+00	0.	e+00	0.
23	0.	0.469480e+01	0.331445e+01	0.331445e+01	0.155155e+01	0.155155e+01	0.556463e+01	0.224462e+01					
	8.000	-0.469480e+01	-0.331445e+01	-0.331445e+01	-0.155155e+01	-0.155155e+01	-0.556463e+01	-0.224462e+01					
	8.000	0.	0.	0.	e+00	e+00	0.	e+00	0.	e+00	0.	e+00	0.
	8.000	0.	0.	0.	e+00	e+00	0.	e+00	0.	e+00	0.	e+00	0.
24	0.	0.636132e+01	0.	0.	e+00	0.	0.636132e+01	0.	e+00	0.	e+00	0.	e+00
	8.000	-0.636132e+01	0.	0.	e+00	0.	0.	-0.636132e+01	0.	e+00	0.	e+00	0.
	8.000	0.	0.	0.	e+00	0.	0.	0.	e+00	0.	e+00	0.	e+00
	8.000	0.	0.	0.	e+00	0.	0.	0.	e+00	0.	e+00	0.	e+00
25	0.	-0.519291e+02	0.	0.	e+00	0.	-0.190735e-03	-0.519291e+02					
	8.000	0.519291e+02	0.	0.	e+00	0.	0.519291e+02	0.190735e-05					
	8.000	0.	0.	0.	e+00	0.	0.	0.	e+00	0.	e+00	0.	e+00
	8.000	0.	0.	0.	e+00	0.	0.	0.	e+00	0.	e+00	0.	e+00
26	0.	-0.364471e+02	-0.126047e+02	-0.126047e+02	-0.106656e+02	-0.106656e+02	-0.826067e+01	-0.387911e+02					
	8.000	0.364471e+02	0.126047e+02	0.126047e+02	0.106656e+02	0.106656e+02	0.826067e+01	0.387911e+02					
	8.000	0.	0.	0.	e+00	e+00	0.	e+00	0.	e+00	0.	e+00	0.
	8.000	0.	0.	0.	e+00	e+00	0.	e+00	0.	e+00	0.	e+00	0.
27	0.	-0.259835e+02	-0.168760e+02	-0.168760e+02	-0.840926e+01	-0.840926e+01	-0.118667e+02	-0.309928e+02					
	8.000	0.259835e+02	0.168760e+02	0.168760e+02	0.840926e+01	0.840926e+01	0.118667e+02	0.309928e+02					
	8.000	0.	0.	0.	e+00	e+00	0.	e+00	0.	e+00	0.	e+00	0.
	8.000	0.	0.	0.	e+00	e+00	0.	e+00	0.	e+00	0.	e+00	0.
28	0.	-0.155798e+02	-0.748307e+01	-0.748307e+01	-0.265962e+01	-0.265962e+01	-0.668759e+01	-0.163753e+02					
	8.000	0.155798e+02	0.748307e+01	0.748307e+01	0.265962e+01	0.265962e+01	0.668759e+01	0.163753e+02					
	8.000	0.	0.	0.	e+00	e+00	0.	e+00	0.	e+00	0.	e+00	0.
	8.000	0.	0.	0.	e+00	e+00	0.	e+00	0.	e+00	0.	e+00	0.
29	0.	-0.487150e+01	0.154829e+01	0.154829e+01	0.408342e+00	0.408342e+00	0.157415e+01	-0.489737e+01					
	8.000	0.487150e+01	-0.154829e+01	-0.154829e+01	-0.408342e+00	-0.408342e+00	-0.157415e+01	0.489737e+01					
	8.000	0.	0.	0.	e+00	e+00	0.	e+00	0.	e+00	0.	e+00	0.
	8.000	0.	0.	0.	e+00	e+00	0.	e+00	0.	e+00	0.	e+00	0.
30	0.	-0.258152e+01	0.511870e+01	0.511870e+01	-0.125702e+01	-0.125702e+01	0.531871e+01	-0.278153e+01					
	8.000	0.258152e+01	-0.511870e+01	-0.511870e+01	0.125702e+01	0.125702e+01	-0.531871e+01	0.278153e+01					
	8.000	0.	0.	0.	e+00	e+00	0.	e+00	0.	e+00	0.	e+00	0.
	8.000	0.	0.	0.	e+00	e+00	0.	e+00	0.	e+00	0.	e+00	0.
31	0.	0.386953e+00	0.448127e+01	0.448127e+01	-0.315894e+01	-0.315894e+01	0.619839e+01	-0.133016e+01					

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32	8.000	-0.386953e+00	-0.448127e+01	0.315894e+01	0.133016e+01	-0.619839e+01
	8.000	0.	0.	0.	0.	0.
	8.000	0.	0.	0.	0.	0.
33	0.	0.294385e+01	0.	0.	0.294385e+01	0.119209e-06
	8.000	-0.294385e+01	0.	0.	-0.119209e-06	-0.294385e+01
	8.000	0.	0.	0.	0.	0.
34	0.	-0.210325e+03	0.	0.	0.	-0.210325e+03
	8.000	0.210325e+03	0.	0.	0.210325e+03	0.
	8.000	0.	0.	0.	0.	0.
35	0.	-0.146486e+03	-0.569610e+02	-0.534766e+02	-0.319853e+02	-0.171462e+03
	8.000	0.146486e+03	0.569610e+02	0.534766e+02	0.319853e+02	0.171462e+03
	8.000	0.	0.	0.	0.	0.
36	0.	-0.104407e+03	-0.485224e+02	-0.242634e+02	-0.394569e+02	-0.113472e+03
	8.000	0.104407e+03	0.485224e+02	0.242634e+02	0.394569e+02	0.113472e+03
	8.000	0.	0.	0.	0.	0.
37	0.	-0.471663e+02	-0.159212e+02	-0.498014e+01	-0.151466e+02	-0.479409e+02
	8.000	0.471663e+02	0.159212e+02	0.498014e+01	0.151466e+02	0.479409e+02
	8.000	0.	0.	0.	0.	0.
38	0.	-0.303114e+02	-0.167336e+01	-0.446435e+01	-0.993559e+00	-0.309912e+02
	8.000	0.303114e+02	0.167336e+01	0.446435e+01	0.993559e+00	0.309912e+02
	8.000	0.	0.	0.	0.	0.
39	0.	-0.213392e+02	0.416269e+01	-0.775993e+01	0.633836e+01	-0.235149e+02
	8.000	0.213392e+02	-0.416269e+01	0.775993e+01	-0.633836e+01	0.235149e+02
	8.000	0.	0.	0.	0.	0.
40	0.	-0.135097e+02	0.461640e+01	-0.907199e+01	0.837675e+01	-0.172701e+02
	8.000	0.135097e+02	-0.461640e+01	0.907199e+01	-0.837675e+01	0.172701e+02
	8.000	0.	0.	0.	0.	0.
41	0.	-0.731671e+01	0.	0.	-0.238419e-06	-0.731671e+01
	8.000	0.731671e+01	0.	0.	0.238419e-06	0.731671e+01
	8.000	0.	0.	0.	0.	0.
42	0.	-0.535639e+03	0.	0.	0.	-0.535639e+03
	8.000	0.535639e+03	0.	0.	0.	0.535639e+03
	8.000	0.	0.	0.	0.	0.

42	0. 8.000 8.000 8.000	-0.368788e+03 0.368788e+03 0. e+00 0. e+00	-0.220204e+03 0.220204e+03 0. e+00 0. e+00	-0.210211e+03 0.210211e+03 0. e+00 0. e+00	-0.715432e+02 0.517448e+03 0. e+00 0. e+00	-0.517448e+03 0.715432e+02 0. e+00 0. e+00
43	0. 8.000 8.000 8.000	-0.129566e+03 0.129566e+03 0. e+00 0. e+00	-0.803517e+02 0.803517e+02 0. e+00 0. e+00	-0.287931e+02 0.287931e+02 0. e+00 0. e+00	-0.670836e+02 0.142836e+03 0. e+00 0. e+00	-0.142836e+03 0.670836e+02 0. e+00 0. e+00
44	0. 8.000 8.000 8.000	-0.742370e+02 0.742370e+02 0. e+00 0. e+00	-0.143544e+02 0.143544e+02 0. e+00 0. e+00	-0.606195e+01 0.606195e+01 0. e+00 0. e+00	-0.177044e+02 0.748870e+02 0. e+00 0. e+00	-0.748870e+02 0.177044e+02 0. e+00 0. e+00
45	0. 8.000 8.000 8.000	-0.753636e+02 0.753636e+02 0. e+00 0. e+00	-0.293085e+01 0.293085e+01 0. e+00 0. e+00	-0.315849e+02 0.315849e+02 0. e+00 0. e+00	0.140838e+02 0.865166e+02 0. e+00 0. e+00	-0.865166e+02 0.140838e+02 0. e+00 0. e+00
46	0. 8.000 8.000 8.000	-0.657145e+02 0.657145e+02 0. e+00 0. e+00	-0.950113e+01 0.950113e+01 0. e+00 0. e+00	-0.434942e+02 0.434942e+02 0. e+00 0. e+00	0.141776e+02 0.893932e+02 0. e+00 0. e+00	-0.893932e+02 0.141776e+02 0. e+00 0. e+00
47	0. 8.000 8.000 8.000	-0.292208e+02 0.292208e+02 0. e+00 0. e+00	-0.335426e+01 0.335426e+01 0. e+00 0. e+00	-0.243904e+02 0.243904e+02 0. e+00 0. e+00	0.113198e+02 0.438948e+02 0. e+00 0. e+00	-0.438948e+02 0.113198e+02 0. e+00 0. e+00
48	0. 8.000 8.000 8.000	-0.209131e+02 0.209131e+02 0. e+00 0. e+00	0. e+00 0. e+00 0. e+00 0. e+00	0. e+00 0. e+00 0. e+00 0. e+00	-0.953674e-06 0.209131e+02 0. e+00 0. e+00	-0.209131e+02 0.953674e-06 0. e+00 0. e+00
49	0. 8.000 8.000 8.000	-0.104803e+02 0.104803e+02 0. e+00 0. e+00	0. e+00 0. e+00 0. e+00 0. e+00	0. e+00 0. e+00 0. e+00 0. e+00	-0.476837e-06 0.104803e+02 0. e+00 0. e+00	-0.104803e+02 0.476837e-06 0. e+00 0. e+00
50	0. 8.000 8.000 8.000	-0.612976e+02 0.612976e+02 0. e+00 0. e+00	-0.125614e+03 0.125614e+03 0. e+00 0. e+00	-0.144413e+03 0.144413e+03 0. e+00 0. e+00	0.544941e+02 0.241406e+03 0. e+00 0. e+00	-0.241406e+03 0.544941e+02 0. e+00 0. e+00
51	0. 8.000 8.000 8.000	-0.515905e+02 0.515905e+02 0. e+00 0. e+00	-0.134188e+03 0.134188e+03 0. e+00 0. e+00	-0.624778e+02 0.624778e+02 0. e+00 0. e+00	-0.179956e+02 0.167783e+03 0. e+00 0. e+00	-0.167783e+03 0.179956e+02 0. e+00 0. e+00
52	0. 8.000 8.000 8.000	-0.297372e+02 0.297372e+02 0. e+00 0. e+00	-0.368659e+02 0.368659e+02 0. e+00 0. e+00	-0.343363e+02 0.343363e+02 0. e+00 0. e+00	0.121928e+01 0.678224e+02 0. e+00 0. e+00	-0.678224e+02 0.121928e+01 0. e+00 0. e+00

64	8.000	0.	e+00	0.489481e+02	0.	e+00	0.489481e+02	0.	e+00
	8.000	0.	e+00	0.	0.	e+00	0.	0.	e+00
	8.000	0.	e+00	0.	0.	e+00	0.	0.	e+00
65	0.	0.	e+00	0.	0.	e+00	0.	0.	e+00
	8.000	0.	e+00	0.	0.	e+00	0.	0.	e+00
	8.000	0.	e+00	0.	0.	e+00	0.	0.	e+00
66	0.	0.	e+00	0.	0.	e+00	0.	0.	e+00
	8.000	0.	e+00	0.	0.	e+00	0.	0.	e+00
	8.000	0.	e+00	0.	0.	e+00	0.	0.	e+00
67	0.	0.	e+00	0.483397e+01	0.	e+00	0.483397e+01	-0.483397e+01	0.
	8.000	0.	e+00	0.483397e+01	0.	e+00	0.483397e+01	0.	e+00
	8.000	0.	e+00	0.	0.	e+00	0.	0.	e+00
68	0.	0.	e+00	-0.469549e+03	0.	e+00	-0.469549e+03	-0.469549e+03	0.
	8.000	0.	e+00	0.469549e+03	0.	e+00	0.469549e+03	0.152588e-04	0.
	8.000	0.	e+00	0.	0.	e+00	0.	0.	e+00
69	0.	0.	e+00	-0.102257e+03	0.	e+00	-0.102257e+03	-0.102257e+03	0.
	8.000	0.	e+00	0.102257e+03	0.	e+00	0.102257e+03	0.	e+00
	8.000	0.	e+00	0.	0.	e+00	0.	0.	e+00
70	0.	0.	e+00	0.370753e+02	0.	e+00	0.370753e+02	0.	e+00
	8.000	0.	e+00	-0.370753e+02	0.	e+00	-0.370753e+02	-0.370753e+02	0.
	8.000	0.	e+00	0.	0.	e+00	0.	0.	e+00
71	0.	0.	e+00	-0.229092e+02	0.	e+00	-0.229092e+02	-0.229092e+02	0.
	8.000	0.	e+00	0.229092e+02	0.	e+00	0.229092e+02	0.933674e-06	0.
	8.000	0.	e+00	0.	0.	e+00	0.	0.	e+00
72	0.	0.	e+00	-0.384060e+02	0.	e+00	-0.384060e+02	-0.384060e+02	0.
	8.000	0.	e+00	0.384060e+02	0.	e+00	0.384060e+02	0.	e+00
	8.000	0.	e+00	0.	0.	e+00	0.	0.	e+00
73	0.	0.	e+00	0.	0.	e+00	0.	0.	e+00
	8.000	0.	e+00	0.	0.	e+00	0.	0.	e+00
	8.000	0.	e+00	0.	0.	e+00	0.	0.	e+00

P

74	0. 8.000 8.000 8.000	-0.406384e+03 0.406384e+03 0. 0.	-0.228336e+03 0.228336e+03 0. 0.	0.282592e+03 -0.282592e+03 0. 0.	-0.210767e+02 0.210767e+02 0. 0.	-0.613643e+03 0.613643e+03 0. 0.
75	0. 8.000 8.000 8.000	-0.254902e+03 0.254902e+03 0. 0.	-0.188649e+03 0.188649e+03 0. 0.	0.164804e+03 -0.164804e+03 0. 0.	-0.536757e+02 0.536757e+02 0. 0.	-0.389876e+03 0.389876e+03 0. 0.
76	0. 8.000 8.000 8.000	-0.149307e+03 0.149307e+03 0. 0.	-0.625175e+02 0.625175e+02 0. 0.	0.532604e+02 -0.532604e+02 0. 0.	-0.372116e+02 0.372116e+02 0. 0.	-0.174613e+03 0.174613e+03 0. 0.
77	0. 8.000 8.000 8.000	-0.112841e+03 0.112841e+03 0. 0.	-0.112084e+02 0.112084e+02 0. 0.	0.497095e+02 -0.497095e+02 0. 0.	0.906216e+01 -0.906216e+01 0. 0.	-0.133111e+03 0.133111e+03 0. 0.
78	0. 8.000 8.000 8.000	-0.719526e+02 0.719526e+02 0. 0.	-0.144079e+02 0.144079e+02 0. 0.	0.519751e+02 -0.519751e+02 0. 0.	0.162273e+02 -0.162273e+02 0. 0.	-0.102588e+03 0.102588e+03 0. 0.
79	0. 8.000 8.000 8.000	-0.415882e+02 0.415882e+02 0. 0.	-0.136796e+02 0.136796e+02 0. 0.	0.347892e+02 -0.347892e+02 0. 0.	0.984953e+01 -0.984953e+01 0. 0.	-0.651174e+02 0.651174e+02 0. 0.
80	0. 8.000 8.000 8.000	-0.299213e+02 0.299213e+02 0. 0.	0. 0. 0. 0.	0. 0. 0. 0.	-0.953674e-06 0.953674e-06 0. 0.	-0.299213e+02 0.299213e+02 0. 0.
81	0. 8.000 8.000 8.000	-0.517477e+03 0.517477e+03 0. 0.	0. 0. 0. 0.	0. 0. 0. 0.	-0.305176e-04 0.305176e-04 0. 0.	-0.517477e+03 0.517477e+03 0. 0.
82	0. 8.000 8.000 8.000	-0.318818e+03 0.318818e+03 0. 0.	-0.147241e+03 0.147241e+03 0. 0.	0.139964e+03 -0.139964e+03 0. 0.	-0.688666e+02 0.688666e+02 0. 0.	-0.397193e+03 0.397193e+03 0. 0.
83	0. 8.000 8.000 8.000	-0.179440e+03 0.179440e+03 0. 0.	-0.733690e+02 0.733690e+02 0. 0.	0.428844e+02 -0.428844e+02 0. 0.	-0.581998e+02 0.581998e+02 0. 0.	-0.194609e+03 0.194609e+03 0. 0.
84	0. 8.000 8.000	-0.118890e+03 0.118890e+03 0.	-0.346794e+02 0.346794e+02 0.	0.180067e+02 -0.180067e+02 0.	-0.309907e+02 0.309907e+02 0.	-0.122579e+03 0.122579e+03 0.

[illegible]

96	8.000	0.129358e+02	-0.611020e+01	-0.730302e+01	0.154137e+02	±0.858810e+01
	8.000	0. e+00	0. e+00	0. e+00	0. e+00	0. e+00
	8.000	0. e+00	0. e+00	0. e+00	0. e+00	0. e+00
97	0. e+00	-0.558191e+01	0. e+00	0. e+00	0. e+00	-0.558191e+01
	8.000	0.558191e+01	0. e+00	0. e+00	0.558191e+01	0. e+00
	8.000	0. e+00	0. e+00	0. e+00	0. e+00	0. e+00
98	0. e+00	-0.402429e+02	0. e+00	0. e+00	-0.190735e-05	-0.402429e+02
	8.000	0.402429e+02	0. e+00	0. e+00	0.402429e+02	0.190735e-05
	8.000	0. e+00	0. e+00	0. e+00	0. e+00	0. e+00
99	0. e+00	-0.307807e+02	-0.127409e+02	-0.116354e+02	-0.703867e+01	-0.364829e+02
	8.000	0.307807e+02	0.127409e+02	-0.116354e+02	0.364829e+02	0.703867e+01
	8.000	0. e+00	0. e+00	0. e+00	0. e+00	0. e+00
100	0. e+00	-0.188008e+02	-0.119610e+02	-0.520319e+01	±0.915442e+01	-0.216074e+02
	8.000	0.188008e+02	0.119610e+02	-0.520319e+01	0.216074e+02	0.915442e+01
	8.000	0. e+00	0. e+00	0. e+00	0. e+00	0. e+00
101	0. e+00	-0.118446e+02	-0.661370e+01	0.137324e+01	-0.627510e+01	±0.121832e+02
	8.000	0.118446e+02	0.661370e+01	-0.137324e+01	0.171812e+02	0.627510e+01
	8.000	0. e+00	0. e+00	0. e+00	0. e+00	0. e+00
102	0. e+00	-0.684802e+01	-0.933827e+00	-0.750618e+00	-0.840047e+00	±0.694180e+01
	8.000	0.684802e+01	0.933827e+00	0.750618e+00	0.694180e+01	0.840047e+00
	8.000	0. e+00	0. e+00	0. e+00	0. e+00	0. e+00
103	0. e+00	-0.322387e+01	0.336208e+01	-0.225691e+00	0.336981e+01	-0.323160e+01
	8.000	0.322387e+01	-0.336208e+01	0.225691e+00	0.323160e+01	0.336981e+01
	8.000	0. e+00	0. e+00	0. e+00	0. e+00	0. e+00
104	0. e+00	0.213518e+00	0.494571e+01	0.206383e+01	0.571933e+01	-0.560097e+00
	8.000	-0.213518e+00	-0.494571e+01	-0.206383e+01	0.560097e+01	-0.571933e+01
	8.000	0. e+00	0. e+00	0. e+00	0. e+00	0. e+00
105	0. e+00	0.372105e+01	0. e+00	0. e+00	0.372105e+01	0.119209e-06
	8.000	-0.372105e+01	0. e+00	0. e+00	-0.119209e-06	-0.372105e+01
	8.000	0. e+00	0. e+00	0. e+00	0. e+00	0. e+00
106	0. e+00	-0.696827e+01	0. e+00	0. e+00	-0.238619e-06	-0.696827e+01
	8.000	0.696827e+01	0. e+00	0. e+00	0.238619e-06	0.696827e+01
	8.000	0. e+00	0. e+00	0. e+00	0. e+00	0. e+00

106	0. 8.000 8.000 8.000	-0.294689e+01 0.294689e+01 0. 8.000	-0.411352e+01 0.411352e+01 0. e+00	0.408889e+01 -0.408889e+01 0. e+00	0.600080e+00 0.766049e+01 0. e+00	-0.766049e+01 -0.600080e+00 0. e+00
107	0. 8.000 8.000 8.000	0.129541e+01 -0.129541e+01 0. 8.000	-0.394042e+01 0.394042e+01 0. e+00	0.107123e+01 -0.107123e+01 0. e+00	0.150611e+01 0.415111e+01 0. e+00	-0.415111e+01 -0.150611e+01 0. e+00
108	0. 8.000 8.000 8.000	0.139088e+01 -0.139088e+01 0. 8.000	-0.149438e+01 0.149438e+01 0. e+00	-0.124952e+01 0.124952e+01 0. e+00	0.369191e+01 0.179562e+01 0. e+00	-0.179562e+01 -0.369191e+01 0. e+00
109	0. 8.000 8.000 8.000	0.394409e+01 -0.394409e+01 0. 8.000	0.984313e+00 -0.984313e+00 0. e+00	-0.217944e+01 0.217944e+01 0. e+00	0.509859e+01 0.170191e+00 0. e+00	-0.170191e+00 -0.509859e+01 0. e+00
110	0. 8.000 8.000 8.000	0.392845e+01 -0.392845e+01 0. 8.000	0.262043e+01 -0.262043e+01 0. e+00	-0.151711e+01 0.151711e+01 0. e+00	0.492632e+01 -0.162236e+01 0. e+00	0.162236e+01 -0.492632e+01 0. e+00
111	0. 8.000 8.000 8.000	0.435533e+01 -0.435533e+01 0. 8.000	0.273625e+01 -0.273625e+01 0. e+00	0.129681e+00 -0.129681e+00 0. e+00	0.436585e+01 -0.272593e+01 0. e+00	0.272593e+01 -0.436585e+01 0. e+00
112	0. 8.000 8.000 8.000	0.551909e+01 -0.551909e+01 0. 8.000	0. e+00	0. e+00	0.551909e+01 0. e+00	0. e+00
113	0. 8.000 8.000 8.000	0.351994e+01 -0.351994e+01 0. 8.000	0. e+00	0. e+00	0.351994e+01 -0.119209e-06 0. e+00	0.119209e-06 -0.351994e+01 0. e+00
114	0. 8.000 8.000 8.000	0.470897e+01 -0.470897e+01 0. 8.000	-0.909652e+00 0.909652e+00 0. e+00	0.974474e+00 -0.974474e+00 0. e+00	0.487318e+01 0.107386e+01 0. e+00	-0.107386e+01 -0.487318e+01 0. e+00
115	0. 8.000 8.000 8.000	0.560854e+01 -0.560854e+01 0. 8.000	-0.804145e+00 0.804145e+00 0. e+00	-0.262193e+00 0.262193e+00 0. e+00	0.561925e+01 0.814847e+00 0. e+00	-0.814847e+00 -0.561925e+01 0. e+00
116	0. 8.000 8.000 8.000	0.543940e+01 -0.543940e+01 0. 8.000	0.150565e+00 -0.150565e+00 0. e+00	-0.130750e+01 0.130750e+01 0. e+00	0.574698e+01 0.155037e+00 0. e+00	-0.155037e+00 -0.574698e+01 0. e+00

117	8.000	0.	0.	e+00	0.	e+00	0.	e+00	0.	e+00	0.	e+00
	0.	0.461670e+01	0.	0.102455e+01	-0.160329e+01	0.528280e+01	0.	0.413049e+00	0.	0.413049e+00	0.	0.413049e+00
	8.000	-0.461670e+01	0.	0.102455e+01	0.160329e+01	-0.528280e+01	0.	-0.413049e+00	0.	-0.413049e+00	0.	-0.413049e+00
	8.000	0.	0.	e+00	0.	e+00	0.	e+00	0.	e+00	0.	e+00
	8.000	0.	0.	e+00	0.	e+00	0.	e+00	0.	e+00	0.	e+00
118	0.	0.371398e+01	0.	0.138791e+01	-0.117859e+01	0.420676e+01	0.	0.895126e+00	0.	0.895126e+00	0.	0.895126e+00
	8.000	-0.371398e+01	0.	0.138791e+01	0.117859e+01	-0.420676e+01	0.	-0.895126e+00	0.	-0.895126e+00	0.	-0.895126e+00
	8.000	0.	0.	e+00	0.	e+00	0.	e+00	0.	e+00	0.	e+00
	8.000	0.	0.	e+00	0.	e+00	0.	e+00	0.	e+00	0.	e+00
119	0.	0.329967e+01	0.	0.100573e+01	-0.396026e+00	0.336611e+01	0.	0.939288e+00	0.	0.939288e+00	0.	0.939288e+00
	8.000	-0.329967e+01	0.	0.100573e+01	0.396026e+00	-0.336611e+01	0.	-0.939288e+00	0.	-0.939288e+00	0.	-0.939288e+00
	8.000	0.	0.	e+00	0.	e+00	0.	e+00	0.	e+00	0.	e+00
	8.000	0.	0.	e+00	0.	e+00	0.	e+00	0.	e+00	0.	e+00
120	0.	0.347050e+01	0.	e+00	0.	e+00	0.	0.347050e+01	0.	0.347050e+01	0.	0.347050e+01
	8.000	-0.347050e+01	0.	e+00	0.	e+00	0.	-0.347050e+01	0.	-0.347050e+01	0.	-0.347050e+01
	8.000	0.	0.	e+00	0.	e+00	0.	e+00	0.	e+00	0.	e+00
	8.000	0.	0.	e+00	0.	e+00	0.	e+00	0.	e+00	0.	e+00
121	0.	0.	0.	e+00	0.	e+00	0.	e+00	0.	e+00	0.	e+00
	8.000	0.	0.	e+00	0.	e+00	0.	e+00	0.	e+00	0.	e+00
	8.000	0.	0.	e+00	0.	e+00	0.	e+00	0.	e+00	0.	e+00
	8.000	0.	0.	e+00	0.	e+00	0.	e+00	0.	e+00	0.	e+00
122	0.	0.	0.	e+00	0.	e+00	0.	0.740808e+02	0.	0.740808e+02	0.	0.740808e+02
	8.000	0.	0.	e+00	0.	e+00	0.	-0.740808e+02	0.	-0.740808e+02	0.	-0.740808e+02
	8.000	0.	0.	e+00	0.	e+00	0.	e+00	0.	e+00	0.	e+00
	8.000	0.	0.	e+00	0.	e+00	0.	e+00	0.	e+00	0.	e+00
123	0.	0.	0.	e+00	0.	e+00	0.	0.797847e+01	0.	0.797847e+01	0.	0.797847e+01
	8.000	0.	0.	e+00	0.	e+00	0.	-0.797847e+01	0.	-0.797847e+01	0.	-0.797847e+01
	8.000	0.	0.	e+00	0.	e+00	0.	e+00	0.	e+00	0.	e+00
	8.000	0.	0.	e+00	0.	e+00	0.	e+00	0.	e+00	0.	e+00
124	0.	0.	0.	e+00	0.	e+00	0.	0.169777e+01	0.	0.169777e+01	0.	0.169777e+01
	8.000	0.	0.	e+00	0.	e+00	0.	-0.169777e+01	0.	-0.169777e+01	0.	-0.169777e+01
	8.000	0.	0.	e+00	0.	e+00	0.	e+00	0.	e+00	0.	e+00
	8.000	0.	0.	e+00	0.	e+00	0.	e+00	0.	e+00	0.	e+00
125	0.	0.	0.	e+00	0.	e+00	0.	0.173498e+00	0.	0.173498e+00	0.	0.173498e+00
	8.000	0.	0.	e+00	0.	e+00	0.	-0.173498e+00	0.	-0.173498e+00	0.	-0.173498e+00
	8.000	0.	0.	e+00	0.	e+00	0.	e+00	0.	e+00	0.	e+00
	8.000	0.	0.	e+00	0.	e+00	0.	e+00	0.	e+00	0.	e+00
126	0.	0.	0.	e+00	0.	e+00	0.	0.384207e+00	0.	0.384207e+00	0.	0.384207e+00
	8.000	0.	0.	e+00	0.	e+00	0.	-0.384207e+00	0.	-0.384207e+00	0.	-0.384207e+00
	8.000	0.	0.	e+00	0.	e+00	0.	e+00	0.	e+00	0.	e+00
	8.000	0.	0.	e+00	0.	e+00	0.	e+00	0.	e+00	0.	e+00
127	0.	0.	0.	e+00	0.	e+00	0.	0.496757e+00	0.	0.496757e+00	0.	0.496757e+00
	8.000	0.	0.	e+00	0.	e+00	0.	-0.496757e+00	0.	-0.496757e+00	0.	-0.496757e+00

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      8.000      0.      e+00      0.496757e+00      0.      e+00
      8.000      0.      e+00      0.      e+00      0.      e+00
      8.000      0.      e+00      0.      e+00      0.      e+00

      128
      0.      0.      e+00      0.      e+00      0.      e+00
      8.000      0.      e+00      0.      e+00      0.      e+00
      8.000      0.      e+00      0.      e+00      0.      e+00
      8.000      0.      e+00      0.      e+00      0.      e+00

```

***** joints in xdirection ***** { 1,

node	transferred load	node	transferred load
57	713.712	65	-713.712
58	-804.616	66	804.616
59	-953.801	67	953.801
60	-61.988	68	61.988
61	1760.635	69	-1760.635
62	-19.050	70	19.050
63	-114.013	71	114.013
64	-134.026	72	134.026

Listing

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*****
c Structural Analysis of Rigid Pavements
c With fatigue behavior and pumping
c
c *****
c orthotropic concrete slab
c *****
c fatigue damage in concrete slab
c *****
c fatigue damage in load transfer devices
c *****
c pumping development
c *****
c
c *****
c resilient modulus of subgrade reaction
c *****
c
c Developed by J. Larralde (1984)
c updated Oct. 1984
c
c Based on Method by A.M. Tahatahale (1977) and
c K.Y. Wong (1980)
c *****

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+ a(9*ldm9+1), a(10*ldm9+1), a(11*ldm9+1), a(11*ldm9+ldm6+1),
+ a(second+ldm4+1), a(second+ldm4+1), a(second+2*ldm4+1),
+ a(third+1), a(third+3*ldm9+1), a(third+2*ldm9+1),
+ a(fourth+1), a(fourth+5*ldm9+1), a(fourth+2*ldm9+1),
+ a(fourth+3*ldm9+1),
+ a(fifth+1), a(fifth+nx+1), a(fifth+nx+1+ldm4),
+ a(fifth+nx+1+ldm4+ldm9), a(fifth+nx+1+ldm4+2*ldm9),
+ a(fifth+nx+1+ldm4+3*ldm9), a(fifth+nx+1+ldm4+4*ldm9),
+ a(fifth+nx+1+ldm4+5*ldm9), a(fifth+nx+1+ldm4+6*ldm9),
+ a(fifth+nx+1+ldm4+7*ldm9), a(fifth+nx+1+ldm4+8*ldm9),
+ a(fifth+nx+1+ldm4+9*ldm9), a(fifth+nx+1+ldm4+10*ldm9),
+ fsbnt,rox,roy,coef1,coef2,a1,a2,a3,deck,dect,doutx,douty,cycls,
+ dsx,dlx,djwx,dlx,dprx,fyx,fyxl,
+ nlx,n2x,n3x,nly,n2y,nfor,nx,ny,
+ ldm9,ldm8,ldm7,ldm6,ldm5,ldm4,ltter,nincl,ite )
C
stop
end
C
C *****
subroutine slah(x,t1,t2,e1,e2,sub,fc,tt,xf,yf,st,su,
+ p,atr1,atr2,
+ atr1,atr1,atr17,atr17,
+ xc,yc,pr1,
+ telx,ely,
+ tulx,uly,
+ theta,thick,
+ a2x,e2y,
+ thicl,subg2,
+ fsbnt,rox,roy,coef1,coef2,a1,a2,a3,deck,dect,doutx,douty,cycls,
+ dsx,dlx,djwx,dlx,dprx,fyx,fyxl,
+ nlx,n2x,n3x,nly,n2y,nfor,nx,ny,
+ ldm9,ldm8,ldm7,ldm6,ldm5,ldm4,ltter,nincl,ite )
C
C
integer comp
dimension x(ldm9),y(ldm9),v(2),t1(ldm9),t2(ldm9),e1(ldm9),e2(
+ ldm9),sub(ldm9),n(2),r(2),n(4),nf(90),r(90,2),s(ldm6),s1(
+ ncl(ldm4),pr1(ldm4),xl(ldm4),yl(ldm4),fc(ldm9),p(ldm8),s1(
+ ldm6,3),atr2(ldm9,3),scr1(ldm9,5),atr1(ldm9,5),atr2(ldm9,5),
+ atr2(ldm9,5),tc(ldm9),xl(ldm9),yf(ldm9),xc(nx),yc(ny),x2(ldm4),
+ y2(ldm4),pr1(ldm4),
+ elx(ldm9),ely(ldm9),
+ tulx(ldm9),uly(ldm9),
+ theta(ldm9),thick(ldm9),
+ a2x(ldm9),e2y(ldm9),thicl(ldm9),
+ subg2(ldm9),
+ dc-1(30),dc12(10)
C
C *****

```

Initialize constants


```

dex=0.
dsx=0.
dlx=0.
dlxx=0.
dlrx=0.
fxx=0.
fxy=0.
dxy=0.
dly=0.
dlwy=0.
dpry=0.
fyy=0.
dcix=0.
dcly=0.

C*****
2 format(8f10.3)
3 format(//,10x,'no. of nodes in x-direction slabs 1,4=',15,/,
+10x,'no. of nodes in x#direction slabs 2,5=',15,/,10x,
+no. of nodes in x#direction slabs 3,6=',15,/,10x,
+no. of nodes in y#direction slabs 1,2,3=',15,/,10x,
+no. of nodes in y#direction slabs 4,5,6=',15)
4 format(//,10x,'x-coordinates are:',/,10(3x,f10.3))
5 format(//,10x,y-coordinates are:',/,10(3x,f10.3))
8 format(3i5,f10.3)
9 format(//,10x,'no. of slabs=',15,/,10x,'no. of layers=',15,/,
+10x,'comp. action=',15)
11 format(//,10x,'properties of the top layer is:')
12 format(f10.3,e10.3,6f10.3)
13 format(//,10x,'poisson ratio of top layer=',f10.3)
16 format(//,10x,'slab thickness=',f10.3)
250 format(//,10x,'reinforcement in X =',f10.3,5x,
+'bar diameter in X =',f10.3)
251 format(//,10x,'reinforcement in Y =',f10.3,5x,
+'bar diameter in Y =',f10.3)
18 format(8f10.3)
19 format(//,10x,'thickness of top layer at the nodes is:',/,8(15,
+f10.3))
22 format(//,10x,'modulus of top layer=',e10.3)
24 format(8e10.3)
25 format(//,10x,'modulus of top layer at the nodes is:',/,8(15,e10.
+3))
27 format(//,10x,'properties of the bottom layer is:')
28 format(//,10x,'poisson ratio of bottom layer is:',f10.3)
31 format(//,10x,'thickness of bottom layer=',f10.3)
33 format(//,10x,'thickness of bottom layer at the nodes is:',/,8(
+15,f10.3))
36 format(//,10x,'modulus of bottom layer=',e10.3)
38 format(//,10x,'modulus of bottom layer at the nodes is:',/,8(15,
+e10.3))
42 format(//,10x,'subgrade modulus=',f10.3)
64 format(8f10.3)
45 format(//,10x,'subgrade modulus at the nodes is:',/,8(15,f10.3))

```

```

6 format(2f10.3,e10.3,4f10.3,e10.3)
7 format(//,10x,'properties of the dove! bars are:',//,10x,
+ 'inside dia.=' ,f10.3,/,10x,'outside dia.=' ,f10.3,/,10x,
+ 'modulus of elasticity=' ,e10.3,/,10x,'spacing=' ,f10.3,/,10x,
+ 'length=' ,f10.3,/,10x,'polegon ratio=' ,f10.3,/,10x,
+ 'dove!-concrete interaction=' ,e10.3,/,10x,'joint width=' ,f10.3)
61 format(15,5f10.3)
62 format(//,11x,'element',5x,'press',2x,'x1-coor.',2x,'x2-coor.',
+ 2x,'y1-coor.',2x,'y2-coor. ')
64 format(//,10x,15,5x,5f10.3)
70 format(//,11x,'node',5x,'deflection',5x,'x#rotation',5x,
+ 'y#rotation',5x,'subgrade stress',/)
71 format(10x,15,3(2x,e13.6),10x,2f10.3) !
79 format(//,11x,'node',10x,'depth',6x,'x#stress',8x,'y#stress',8x,
+ 'xy#stress',7x,'max#stress',6x,'min#stress',/)
81 format(//,10x,15,5x,f10.3,5(3x,e13.6))
82 format(20x,f10.3,5(3x,e13.6))
83 format(//,11x,'node',9x,'transferred load',21x,'node',9x,
+ 'transferred load',/)
85 format(10x,15,10x,f10.3,20x,15,10x,f10.3)
190 format(//,10x,'***** joints in x#direction *****')
191 format(//,10x,'***** joints in y#direction *****')
200 format(15)
211 format(//,10x,'type of load transfer is aggregate interlock')
212 format(//,10x,'type of load transfer is steel bars')
213 format(//,10x,
+ 'type of load transfer is comb. of agg. interlock and steel bars')
202 format(e10.3)
203 format(//,10x,'aggregate interlock factor',e10.3)
c *****
c defining spaces for the properties of each node
c *****
do 11=1,1dm9
x(1)=0.
y(1)=0.
t1(1)=0.
t2(1)=0.
e1(1)=0.
e2(1)=0.
1 sub(1)=0.
c
c
c read(5,2)(xc(1),1=1,nx)
read(5,2)(yc(1),1=1,ny)
write(6,3)n1x,n2x,n3x,n1y,n2y
write(6,4)(xc(1),1=1,nx)
write(6,5)(yc(1),1=1,ny)
c *****
c determine coordinates for each node
c *****
do 101=1,1dm9
j=(i-1)/ny
x(1)=xc(j+1)

```



```

c
c      if(ct2.eq.0.)goto29
      do 30i=1,ldm9
      t2(i)=ct2
      write(6,31)ct2
      goto 32
c
c      29 read(5,18)(t2(i),i=1,ldm9)
      write(6,33)(t,t2(i),i=1,ldm9)
c
c      32 if(ce2.eq.0.)goto34
      do 33i=1,ldm9
      e2(i)=ce2
      write(6,36)ce2
      goto 37
c
c      34 read(5,24)(e2(i),i=1,ldm9)
      write(6,38)(t,e2(i),i=1,ldm9)
      goto 37
c
c      26 v(2)=0.
      do 39i=1,ldm9
      t2(i)=0.
      37 if(ck.eq.0.)goto40
      do 41i=1,ldm9
      41 sub(i)=ck
      write(6,42)ck
      goto 43
c
c      40 read(5,44)(sub(i),i=1,ldm9)
      write(6,45)(t,sub(i),i=1,ldm9)
      43 continue
      if(nslab.eq.1)goto49
      if(n2x.eq.0)goto999
c
c      read(5,200)ltdx
      write(6,190)
      if(ltdx.eq.1)write(6,212)
      if(ltdx.eq.0)write(6,211)
      if(ltdx.eq.2)write(6,213)
      if(ltdx.eq.0)goto201
c
c      read(5,6)dlnx,doutx,dex,dax,dlx,djwx,dprx,dclx
      write(6,7)dlnx,doutx,dex,dax,dlx,djwx,dprx,dclx,djwx
c*****

```

211

```

c      dix is moment of inertia of dowel bar section in x direction
c      shax is the effective cross sectional area of dowel bar for shear
c      and is equal to 0.9 times the actual dowel bar cross sectional area
c      (in x direction)
c      fyx is a constant
c      *****
c      dix=0.049087*(doutx**4-dinx**4)
c      if(dinx.eq.0.)goto149
c      shax=0.350617*(doutx-dinx)*((doutx**2+dinx**2)**2)/((doutx+dinx)
c      +**3)
c      goto 150
149 shax=0.706858*(doutx**2)
150 fyx=24.*dix*(1.+dprx)/(shax*djy**2) {
c      fyx1=fyx
c      if(ltdx.eq.1)goto999
c
c      201 read(5,202)aggx
c      write(6,203)aggx
c      999 if(n2y.eq.0)goto49
c
c      read(5,200)ltdy
c      write(6,191)
c      if(ltdy.eq.1)write(6,212)
c      if(ltdy.eq.0)write(6,211)
c      if(ltdy.eq.-2)write(6,213)
c      if(ltdy.eq.-0)goto107
c
c      read(5,6)diny,douty,dey,dxy,dly,djy,dpry,dely
c      write(6,7)diny,douty,dey,dxy,dly,djy,dpry,dely,djy
c      *****
c      diy is the moment of inertia of dowel bar in y direction
c      shay is the effective cross sectional area of dowel bar for shear
c      and is equal to 0.9 times the actual dowel bar cross sectional area
c      (in y direction)
c      fyy is a constant
c      *****
c      diy=0.049087*(douty**4-diny**4)
c      if(diny.eq.0.)goto349
c      shay=0.350617*(douty-diny)*((douty**2+diny**2)**2)/((douty+diny)*
c      +**3)
c      goto 350
349 shay=0.706858*(douty**2)
350 fyy=24.*diy*(1.+dpry)/(shay*djy**2)
c      fyy1=fyy
c      if(ltdy.eq.1)goto49
c
c      307 read(5,202)aggy
c      write(6,203)aggy
c

```

```

49 ite=0
   asub1=0.
   asub2=0.
   asub3=0.
   asub4=0.
   asub7=0.
   asub8=0.
   asub11=0.
   asub12=0.
   bsub1=0.
   bsub2=0.
   bsub3=0.
   bsub4=0.
   bsub8=0.
   csub1=0.
   csub2=0.
   csub3=0.
*****
c allocate initial values of dowel-concrete interaction factors
c dcl1 and dcl2 are values of dowel-concrete interaction factors
c for individual bars in x and y directions
*****
do 347 i=1,30
  dcl1(i)=dclx
  dcl2(i)=dcly
347
  kappa=nlnc1*ltter+1
c
c start iterative process
c
do 5999 kappa=1,kapa
  ite=ite+1
c
do 353 i=1,ldm9
  xf(i)=0.
353 yf(i)=0.
c
c
  u(1)=0.5*(1.-v(1))
  u(2)=0.5*(1.-v(2))
  do 501 i=1,2
    50 r(i)=0.
  do 511 i=1,90
    xf(i)=0.
  do 521 i=1,4
    51 a(1,1)=0.
    52 n(i)=0
    do 531 i=1,ldm6
      53 su(i)=0.
      do 541 i=1,ldm9
        fc(i)=0.
        xf(i)=0.

```

```

54 yf(1)=0.
do 55 i=1,ldm8
55 p(1)=0.

c
c save initial constants fyx and fyy
c fyx=fx1
c fyy=fy1
c
c
c call stiff(ldm5,nx,nly,n2y,ny,x,y,ldm9,nalab,nlx,n2x,n3x,
+ldm6,nlayer,comp,
+tf,ebh,t2,e2,u,r,a,n,ldm7,at,thick,elx,ely,ulx,uly,
+tl,el,v,t2,e2,u,r,a,n,ldm7,at,thick,elx,ely,ulx,uly,
+dcil,dcil2,
+dtms,doutx,dex,
+dsx,dlx,djux,dlx,dprx,fyx,ltcx,aggr,diny,douty,dey,dly,djuy,
+dlx,dpry,fyy,ltcy,aggr,dcilx,dcily)

c
c call uptrt(ldm7,ldm8,su,at,ldm6)
c
c
c
c
c if((ite-gt.1)goto 77
do 60 i=1,ldm4
60 read(5,61)nel(1),prs(1),xl(1),x2(1),yl(1),y2(1)
c
c use the heaviest load for computation of fatigue damage
c
c do 65 i=1,ldm4
pral(1)=prs(1)
65 pral(1)=prs(1)*40./18.
c*****
c read endurance data (concrete)
c*****
coef1=0.
coef2=0.
read(5,72)fr1,fr2,fr3
read(5,73)cycl1,cycl2,cycl3
read(5,74)cyclc

c use modified number of load applications (asash factor)
c
c cyclc=cyclc/31.
72 format(3f5.3)
73 format(3f10.1)
74 format(f10.0)
cycl1=log10(cycl1)
cycl2=log10(cycl2)
cycl3=log10(cycl3)
c define two straight lines for endurance curve

```

```

cnef1=(fr3-fr1)/(cycl2-cycl1)
coef2=(fr3-fr2)/(cycl3-cycl2)
cycl1=10.*acycl1
cycl2=10.*acycl2
cycl3=10.*acycl3
write(6,62)
do 63 i=1,ldm4
63 write(6,64)nel(1),pral(1),x1(1),x2(1),y1(1),y2(1)
c *****
c read Resilient k *****
c *****
c read*,optin
if(optio.eq.1)goto 67
read*,subgel,defn1
read*,subge2,defn2
read*,subge3,defn3
tsum=(subge2-subge3)*(defn1-defn2)-(subgel-subge2)*(defn2-defn3)
den=(defn3**2.-defn3**2.)*(defn1-defn2)-(defn1**2.-defn2**2.)*
+(defn2-defn3)
a3=tsum/den
a2=((subgel-subge2)-a3*(defn1**2.-defn2**2.))/(defn1-defn2)
a1=subgel-a2*(defn1)-a3*(defn1**2.)
goto 77
67 read*,subgel,defn1
read*,subge2,defn2
a1=subgel-((subge2-subgel)/(defn2-defn1))*defn1
a2=((subge2-subgel)/(defn2-defn1))
n3=0.0
c 77 if((lte.eq.lttter*inlcl+1)goto 78
cycal=cycl8*((lte-1)/ltter+1)/inlcl
if((lte-ltter*((lte-1)/ltter).eq.lttter)then
print*,-----
print*, iteration #, lte-ltter*((lte-1)/ltter)
print*, Number of Load Applications=, cycl8*31.
end if
c
c for last iteration use actual load
c
78 if((lte.eq.lttter*inlcl+1)then
do 393 i=1,ldm4
393 prs(i)=pral(i)
end if
call load(ldm4,nel,prs,x1,y1,ny,x,y,ldm9,p,ldm8,x2,v2)
c
c call disp(ldm7,ldm8,p,su,st,ldm6)
c
c if((lte.ne.lttter*inlcl+1)goto 99
c
c write(6,70)

```



```

do 981-1,ldm9
  fc(1)=p((i-1)*3+1)*sub(1)
  if((fc(1)).le.0.)fc(1)=0.
98 write(6,71)f,p((i-1)*3+1),p((i-1)*3+2),p((i-1)*3+3),fc(1),subh(1)
c
99 continue
c
  fyx=fxyl
  fyy=fyyl
  call atreas(ldm9,nlx,n2x,n3x,n4x,n5x,nly,n2y,n3y,n4y,n5y,atrl,atrl2,atrl3,
+atrl4,atrl5,atrl6,ldm8,x,y,v,u,nlayer,e1,e2,cl,t1,t2,tt,
+xf,yf,elx,ely,ulx,uly,theta,thick,dcl1,dcl2,
+comp,nalab,
+dex,dex,dix,djwx,dprx,fyx,ltdx,aggx,dev,day,dly,
+djwy,dpry,fyy,ltdy,aggy,dclx,dcly)
c
c
  if((ite.ge.lttcr*nincl+1)goto 5998
c
  if((ite-lttcr*((ite-1)/lttcr).eq.lttcr)then
    write(6,220)
    end if
220 format(/,6x,'node',2x,'type of damage',10x,'updated properties',
+11x,'damaged area')
c
c
  call check(ldm9,ldm8,ldm5,nlx,nx,ny,ite,ltter,
+inlah,nlx,n7x,n3x,n2y,ltdx,ltdy,
+atrl,p,
+elx,ely,ulx,uly,theta,thick,
+e2x,e2y,thicl,
+sub,subg2,
+xx,y,
+dcl1,dcl2,xf,yf,
+fsubt,rox,roy,fr1,fr2,fr3,coef1,coef2,cyclal,
+tcyl1,cyl2,cyl3,relnfx,relnfy,cel,
+nl,a2,a3,deck,dect,
+doutx,dlx,djwx,douty,dly,djwy,dclx,dcly)
c
c
c obtain least square approximation of decay values
c
  asub1=asub1+1
  asub2=asub2+ite
  asub3=asub3+(ite)**2.
  asub4=asub4+deck
  asub7=asub7+(ite)**3.
  asub8=asub8+ite*deck
  asub11=asub11+(ite)**4.
  asub12=asub12+deck*(ite)**2.
  bsub1=bsub1+1
  bsub2=bsub2+ite
  bsub3=bsub3+(ite)**2.

```

```

      bsub4=bsub4*dect
      hsub8=hsub8+lte*dect
c *****
c 5998 if((lte.eq.n1ncl*ittr))then
      csb2=asub8-(asub2*asub6/asub1)
      csb2=csb2/(asub1-(asub2**2.)/asub1)
      csb1=(asub4-csub2*asub2)/asub1
      print*, 'extrapolated decay=', csb1+csb2*lte
      tempor=csb1+csb2*lte
      csb2=asub8-(bsub2*asub4/asub1)
      csb2=csb2/(bsub1-(bsub2**2.)/bsub1)
      csb1=(asub4-csub2*asub2)/bsub1
      print*, 'extrapolated thickness decay=', csb1+csb2*lte
      print*, 'AMOUNT OF DAMAGE=', tempor+csb1+csb2*lte
      end if
c
c determine pumping during third iteration
c
c to have stable values of resilient k
c
      if((ite.ne.3))goto 5999
      call pump(idm9, idm8, idm5, nx, ny, n1x,
      +x, y, sub, p, subg2)
c
c 5999 continue
c
      tq=0.
      write(6,79)
      do 801=1, idm9
      write(6,81)l, es, (strtl(i,j), j=1,5)
      write(6,82)l1(i), (strb1(i,j), j=1,5)
      write(6,82)l1(i), (strb2(i,j), j=1,5)
      write(6,82)l1(i), (strb2(i,j), j=1,5)
      if(nslab.eq.1)goto890
      if(n2x.eq.0)goto890
      if(ltdx.eq.0)goto890
      write(6,190)
      write(6,83)
c *****
c giving out the transferred load at the loading edge in x-direction
c *****
      j11=(n1x-1)*ny+1
      j33=n1x*ny
      do 84k=j11, j33
      j1=k
      j2=k+ny
      84 write(6,85)j1,xf(j1),j2,xf(j2)
      if(n3x.eq.0)goto890
c *****
c giving out the transferred load at the loading edge in y-direction

```

```

c *****
      l11=(n1x+n2x-1)*ny+1
      l33=(n1x+n2x)*ny
      do 86k=111,133
        l1=k
        l2=k+ny
        86 write(6,85)l1,xf(l1),l2,xf(l2)
      890 if((l1dy.eq.0).go to 90
        if(n2y.eq.0).go to 90
        if(l1dy.eq.0).go to 90
        write(6,191)
        write(6,83)
        nx=n1x+n2x+n3x
        j11=(nx-1)*ny+ny
        do 886k=n1y,j11,ny
          l11=k
          l12=k+1
          886 write(6,85)l11,yf(l11),l12,yf(l12)
        90 continue
      return
    end
  c
c *****
      subroutine stiff(idm5,nx,nly,n2y,ny,x,y,idm9,nlab,n1x,n2x,n3x,
      +idm6,nlayer,comp,
      +sf,sub,
      +t1,e1,v,t2,e2,u,r,s,n,idm7,at,thick,e1x,e1y,u1x,u1y,
      +dcil,dcf2,
      +dinx,doutx,dex,
      +dax,d1x,djyx,d1x,dpx,fyx,ltdx,aggr,d1ny,douty,dex,d1y,djwy,
      +d1y,dpry,fyy,ltdy,aggy,dcfx,dcfy)
  c
  c      integer comp
      dimension x(idm9),y(idm9),sf(90),sub(idm9),t1(idm9),e1(idm9),v(
      +2),t2(idm9),e2(idm9),u(2),r(2),s(90),n(4),st(idm6),
      +thick(idm9),e1x(idm9),e1y(idm9),u1x(idm9),u1y(idm9),
      +dcil(30),dcf2(30)
  c
  c      do 1011-1,idm5
c *****
c      nel,ne2,ne3,+ ne4 are node numbers belonging to a specific
c      element
c *****
      nel=1+(i-1)/(ny-1)
      ne2=nel+1
      ne3=nel+ny
      ne6=nel+ny+1
      a=(x(ne3)-x(nel))/2.
      b=(y(ne2)-y(nel))/2.
      do 1911jk=1,90

```

```

1911 sf(jk)=0.
      if(nalab.eq.1)goto102
      if(a.eq.0..and.b.eq.0.)goto101
      if(a.eq.0.)goto600
      if(b.eq.0.)goto601
      goto 102
600  if(ltdx.eq.0)goto510
      dx=dx+dx*b/dx
      if(djwx.le.0.01)djwx=0.01
c
c      allocate the appropriate dcfix nt element l
c
      if(nel.ge.(nlx-1)*ny+1..and.ne2.le.(nlx*ny))then
        jji=nel-(nlx-1)*ny
      end if
      if(nel.ge.(nlx+n2x-1)*ny.and.ne2.le.(nlx+n2x)*ny)then
        jji=ny+nel-(nlx+n2x-1)*ny
      end if
      dx=dcil(jji)*b/(dax)
      d5x=12.*ddx/(djwx**3*(1.+fxx))
      d4x=(d4x/djwx)*(1./(1.-dpx))*0.5
      sf=(1./((1./dxx)+(1./d5x)))
      tf=dxx*0.0981747*(doutx**4.-dinx**4.)
c*****
c      assembly of the dovle bar stiffness matrix to the overall structural
c      stiffness matrix (in x-direction)
c*****
      sf(1)=sfxx
      sf(5)=tfxx
      sf(19)=-sfxx
      sf(23)=-tfxx
      sf(37)=sfxx
      sf(41)=tfxx
      sf(55)=-sfxx
      sf(59)=-tfxx
      sf(64)=sfxx
      sf(68)=tfxx
      sf(82)=sfxx
      sf(86)=tfxx
      if(ltdx.eq.1)goto303
510  agx=agxx*b
c*****
c      assembly of the aggregate interlock stiffness matrix to the overall
c      structural stiffness matrix (in x-direction)
c*****
      sf(1)=sf(1)+agx
      sf(19)=sf(19)-agx
      sf(37)=sf(37)+agx
      sf(55)=sf(55)-agx
      sf(64)=sf(64)+agx
      sf(82)=sf(82)+agx
      goto 303
601  if(ltdy.eq.0)goto610

```

```

ddy=dey*dly*a/dsy
if(djwy.le.0.01)djwy=0.01
c
c allocate the appropriate dcy at element i
c
j1=(i+2)/(ny-1)
dcy=(dc12((i-1)/(ny-1)+1)+dc12((i-1)/(ny-1)+2))*a/(2.*dsy)
dcy=dcly*a/dsy
dxy=djwy*a/dsy
dxy=12.*ddy/(dwy**3*(1.+fy))
dxy=(ddy/djwy)*(1./((1.-dprg))**0.5)
sfy=(1./((1.-dcy)*(1.-d5y)))
ctfy=d4y*0.0981747*(dout**4.-dly**4.)
c*****
c assembly of the dowel bar stiffness matrix to the overall structural
c stiffness matrix (in y-direction)
c*****
sf(1)=sfy
sf(9)=tfy
sf(10)=sfy
sf(18)=tfy
sf(37)=sfy
sf(45)=tfy
sf(64)=sfy
sf(72)=tfy
sf(77)=sfy
sf(81)=tfy
sf(82)=sfy
sf(90)=tfy
if(1ldy.eq.1)goto303
610 ngy=ngy*as
c*****
c assembly of the aggregate interlock stiffness matrix to the overall
c structural stiffness matrix (in y-direction)
c*****
sf(1)=sf(1)+ngy
sf(10)=sf(10)+ngy
sf(37)=sf(37)+ngy
sf(64)=sf(64)+ngy
sf(73)=sf(73)+ngy
sf(82)=sf(82)+ngy
goto 303
c
c
102 elx(1)=(elx(nel)+elx(ne2)+elx(ne3)+elx(n66))/4.
thick(1)=(thick(nel)+thick(ne2)+thick(ne3)+thick(ne4))/4.
avsub=(sub(nel)+sub(ne2)+sub(ne3)+sub(ne4))/4.
r1=thick(1)**3.
if(nlayer.eq.2)goto8003
r2=0.
v(2)=0.
goto 8004
c
c

```

```

8003  avt2=(t2(ne1)+t2(ne2)+t2(ne3)+t2(ne4))/4.
      ave2=(e2(ne1)+e2(ne2)+e2(ne3)+e2(ne4))/4.
      r2=ave2*avt2*#3/(12.*(1.-v(2)**2))
c
c      8004  h2=2.*h
      a2=2.*a
      r(1)=r1/(60.*a*h)
      r(2)=r2/(60.*a*g*b)
      bs=b**2.
      as=a**2.
      g=bs/as
      h=as/bs
      ab=a*b
c*****
c      formation of stiffness matrix for concrete slab, stabilized base, and
c      overlay
c*****
      do 300 j=1, 2
        if (j.eq.2) goto 8010
        g=8.*elx(1)/(12.*(1.-ulx(1)*uly(1)))
        h=h*ely(1)/(12.*(1.-ulx(1)*uly(1)))
        v(1)=ely(1)*ulx(1)/(12.*(1.-ulx(1)*uly(1)))
        u(1)=ely(1)/(24.*(1.+ulx(1)))
        x1=60.*g+60.*h+30.*v(j)+84.*u(j)
        x2=(20.*h+8.*u(j))*4.*bs
        x3=(20.*g+8.*u(j))*4.*as
        x4=(-30.*h-15.*v(j)-6.*u(j))*h2
        x5=(30.*g+15.*v(j)+6.*u(j))*a2
        x6=(-15.*v(j))*4.*ah
        x7=30.*g-60.*h-30.*v(j)-84.*u(j)
        x8=-60.*g+30.*h-30.*v(j)-84.*u(j)
        x9=(30.*h+6.*u(j))*h2
        x10=(10.*h-2.*u(j))*4.*bs
        x11=(15.*g-15.*v(j)-6.*u(j))*a2
        x12=(10.*g-8.*u(j))*4.*as
        x13=(-15.*h+15.*v(j)+6.*u(j))*h2
        x14=(-30.*g-6.*u(j))*a2
        x15=-30.*g-30.*h+30.*v(j)+84.*u(j)
        x16=(15.*h-6.*u(j))*h2
        x17=(-15.*g+6.*u(j))*a2
        x18=(10.*h-8.*u(j))*4.*bs
        x19=(30.*h+6.*u(j))*h2
        x20=(5.*h+2.*u(j))*4.*bs
        x21=(10.*g-2.*u(j))*4.*as
        x22=(5.*g+2.*u(j))*4.*as
        x23=0.
        x24=0.
        x25=0.
        s(1,1)=r(1)*x1
        s(2,1)=r(1)*x4
        s(3,1)=r(1)*x5
        s(4,1)=0.

```

```

s(5,j)=r(j)*x2
s(6,j)=r(j)*x6
s(7,j)=0.
s(8,j)=0.
s(9,j)=r(j)*x3
s(10,j)=r(j)*x7
s(11,j)=r(j)*(-x9)
s(12,j)=r(j)*x11
s(13,j)=r(j)*x9
s(14,j)=r(j)*x10
s(15,j)=r(j)*(-x23)
s(16,j)=r(j)*x11
s(17,j)=r(j)*x23
s(18,j)=r(j)*x12
s(19,j)=r(j)*x8
s(20,j)=r(j)*x13
s(21,j)=r(j)*(-x14)
s(22,j)=r(j)*x13
s(23,j)=r(j)*x18
s(24,j)=r(j)*(-x24)
s(25,j)=r(j)*x16
s(26,j)=r(j)*x24
s(27,j)=r(j)*x21
s(28,j)=r(j)*x15
s(29,j)=r(j)*(-x16)
s(30,j)=r(j)*(-x17)
s(31,j)=r(j)*x16
s(32,j)=r(j)*x20
s(33,j)=r(j)*x25
s(34,j)=r(j)*x17
s(35,j)=r(j)*(-x25)
s(36,j)=r(j)*x22
s(37,j)=r(j)*x1
s(38,j)=r(j)*(-x4)
s(39,j)=r(j)*x5
s(40,j)=0.
s(41,j)=r(j)*x2
s(42,j)=r(j)*(-x6)
s(43,j)=0.
s(44,j)=0.
s(45,j)=r(j)*x3
s(46,j)=r(j)*x15
s(47,j)=r(j)*x16
s(48,j)=r(j)*(-x17)
s(49,j)=r(j)*(-x16)
s(50,j)=r(j)*x20
s(51,j)=r(j)*(-x25)
s(52,j)=r(j)*x17
s(53,j)=r(j)*x25
s(54,j)=r(j)*x22
s(55,j)=r(j)*x6
s(56,j)=r(j)*(-x13)
s(57,j)=r(j)*(-x14)

```

```

s(58,j)=r(j)*(-x13)
s(59,j)=r(j)*(x18)
s(60,j)=r(j)*x26
s(61,j)=r(j)*x16
s(62,j)=r(j)*x26
s(63,j)=r(j)*x21
s(64,j)=r(j)*x1
s(65,j)=r(j)*(x4)
s(66,j)=r(j)*(-x5)
s(67,j)=0.
s(68,j)=r(j)*x2
s(69,j)=r(j)*(-x6)
s(70,j)=0.
s(71,j)=0.
s(72,j)=r(j)*x3
s(73,j)=r(j)*x7
s(74,j)=r(j)*(-x19)
s(75,j)=r(j)*x19
s(76,j)=r(j)*x19
s(77,j)=r(j)*(x10)
s(78,j)=r(j)*(-x23)
s(79,j)=r(j)*(-x11)
s(80,j)=r(j)*x23
s(81,j)=r(j)*x12
s(82,j)=r(j)*x1
s(83,j)=r(j)*(-x4)
s(84,j)=r(j)*(-x5)
s(85,j)=0.
s(86,j)=r(j)*x2
s(87,j)=r(j)*x6
s(88,j)=0.
s(89,j)=0.
300 s(90,j)=r(j)*x3
c*****
c formation of stiffness matrix for subgrade
c*****
do 301 k=1,90
301 sf(k)=s(k,1)+s(k,2)
q=(sb*avsub)/4100.
q1=a*q
q2=b*q
q3=c*a*q
q4=b*b*q
q5=a*b*q
sf(1)=sf(1)+24178.*q
sf(2)=sf(2)-6454.*q2
sf(3)=sf(3)+6454.*q1
sf(5)=sf(5)+2240.*q4
sf(6)=sf(6)-1766.*q5
sf(9)=sf(9)+2240.*q3
sf(10)=sf(10)+8582.*q
sf(11)=sf(11)+3836.*q2
sf(12)=sf(12)+2786.*q1

```


$\text{sf}(13) = \text{sf}(13) - 3836 \cdot q^2$
 $\text{sf}(14) = \text{sf}(14) - 1680 \cdot q^4$
 $\text{sf}(15) = \text{sf}(15) - 1176 \cdot q^5$
 $\text{sf}(16) = \text{sf}(16) + 2786 \cdot q^1$
 $\text{sf}(17) = \text{sf}(17) + 1176 \cdot q^5$
 $\text{sf}(18) = \text{sf}(18) + 1120 \cdot q^3$
 $\text{sf}(19) = \text{sf}(19) + 8582 \cdot q^1$
 $\text{sf}(20) = \text{sf}(20) - 2786 \cdot q^2$
 $\text{sf}(21) = \text{sf}(21) - 3836 \cdot q^1$
 $\text{sf}(22) = \text{sf}(22) - 2786 \cdot q^2$
 $\text{sf}(23) = \text{sf}(23) + 1120 \cdot q^4$
 $\text{sf}(24) = \text{sf}(24) + 1176 \cdot q^5$
 $\text{sf}(25) = \text{sf}(25) + 3836 \cdot q^1$
 $\text{sf}(26) = \text{sf}(26) - 1176 \cdot q^5$
 $\text{sf}(27) = \text{sf}(27) - 1680 \cdot q^3$
 $\text{sf}(28) = \text{sf}(28) + 2758 \cdot q^1$
 $\text{sf}(29) = \text{sf}(29) + 1624 \cdot q^2$
 $\text{sf}(30) = \text{sf}(30) - 1624 \cdot q^1$
 $\text{sf}(31) = \text{sf}(31) - 1624 \cdot q^2$
 $\text{sf}(32) = \text{sf}(32) - 840 \cdot q^4$
 $\text{sf}(33) = \text{sf}(33) + 784 \cdot q^5$
 $\text{sf}(34) = \text{sf}(34) + 1624 \cdot q^1$
 $\text{sf}(35) = \text{sf}(35) + 784 \cdot q^5$
 $\text{sf}(36) = \text{sf}(36) - 840 \cdot q^3$
 $\text{sf}(37) = \text{sf}(37) + 24178 \cdot q^1$
 $\text{sf}(38) = \text{sf}(38) + 6454 \cdot q^2$
 $\text{sf}(39) = \text{sf}(39) + 6454 \cdot q^1$
 $\text{sf}(41) = \text{sf}(41) + 2240 \cdot q^4$
 $\text{sf}(42) = \text{sf}(42) + 1764 \cdot q^5$
 $\text{sf}(45) = \text{sf}(45) + 2240 \cdot q^3$
 $\text{sf}(46) = \text{sf}(46) + 2758 \cdot q^1$
 $\text{sf}(47) = \text{sf}(47) - 1624 \cdot q^2$
 $\text{sf}(48) = \text{sf}(48) - 1624 \cdot q^1$
 $\text{sf}(49) = \text{sf}(49) + 1624 \cdot q^2$
 $\text{sf}(50) = \text{sf}(50) - 840 \cdot q^4$
 $\text{sf}(51) = \text{sf}(51) - 784 \cdot q^5$
 $\text{sf}(52) = \text{sf}(52) + 1624 \cdot q^1$
 $\text{sf}(53) = \text{sf}(53) - 784 \cdot q^5$
 $\text{sf}(54) = \text{sf}(54) - 840 \cdot q^3$
 $\text{sf}(55) = \text{sf}(55) + 8582 \cdot q^1$
 $\text{sf}(56) = \text{sf}(56) + 2786 \cdot q^2$
 $\text{sf}(57) = \text{sf}(57) - 3836 \cdot q^1$
 $\text{sf}(58) = \text{sf}(58) + 2786 \cdot q^2$
 $\text{sf}(59) = \text{sf}(59) + 1120 \cdot q^4$
 $\text{sf}(60) = \text{sf}(60) - 1176 \cdot q^5$
 $\text{sf}(61) = \text{sf}(61) + 3836 \cdot q^1$
 $\text{sf}(62) = \text{sf}(62) + 1176 \cdot q^5$
 $\text{sf}(63) = \text{sf}(63) - 1680 \cdot q^3$
 $\text{sf}(64) = \text{sf}(64) + 24178 \cdot q^1$
 $\text{sf}(65) = \text{sf}(65) - 6454 \cdot q^2$
 $\text{sf}(66) = \text{sf}(66) - 6454 \cdot q^1$
 $\text{sf}(68) = \text{sf}(68) + 2240 \cdot q^4$
 $\text{sf}(69) = \text{sf}(69) + 1764 \cdot q^5$

```

      sf(72)=sf(72)+2240.*q3
      sf(73)=sf(73)+8582.*q
      sf(74)=sf(74)-3836.*2
      sf(75)=sf(75)-2786.*q1
      sf(76)=sf(76)+3836.*q2
      sf(77)=sf(77)-1680.*q4
      sf(78)=sf(78)-1176.*q5
      sf(79)=sf(79)-2786.*q1
      sf(80)=sf(80)+1176.*q5
      sf(81)=sf(81)+1120.*q3
      sf(82)=sf(82)+24178.*q
      sf(83)=sf(83)+6454.*q2
      sf(84)=sf(84)-6454.*q1
      sf(86)=sf(86)+2240.*q4
      sf(87)=sf(87)-1768.*q5
      sf(90)=sf(90)+2240.*q3
c*****
c      formation of the overall structural stiffness matrix
c*****
      303 n(1)=nel
      n(2)=nc2
      n(3)=nc3
      n(4)=nc4
      i1=0
      do 302 i1=1,4
      do 302 i2=1,4
      i1=i1+1
      do 302 i3=1,3
      do 302 i4=1,3
      j1=i4*(i3-1)+3*(i1-1)+9
      if(i1.eq.i2.and.i4.lt.i3) goto 302
      kk=(n(i1)-1)*idm7*3+i4-i3+1*(i3-1)*idm7+(n(i2)-n(i1))*3
      st(kk)=st(kk)+sf(jj)
302 continue
101 continue
      return
      end
c
c
c*****
c      subroutine uptrf(idm7,idm8,su,st,idm6)
c*****
c
c      dimension su(idm6),st(idm6)
c
c
c      i1=idm7-1
c      i2=idm8-idm7+1
c      do 1000 i1,i2
c      j=(i1-1)*idm7+1
c      su(j)=st(j)
c      do 1001 k=i1,i1
c      su(j+k)=st(j+k)

```

```

1001 st(j+k)=st(j+k)/su(j)
13=11
do 1000 i=1,11
  m=j+1*idm7-1
  do 1002 k=1,13
    1002 st(m+k)=st(m+k)-st(j+k+11-13)*su(j+1)
  1000 i3=i1-1
  12=12+1
  14=11-1
  do 1003 i=12,idm8
    j=(i-1)*idm7+1
    su(j)=st(j)
    if(i-14)1004,1004,1005
  1004 do 1006 k=1,14
    1006 su(j+k)=st(j+k)
  1005 do 1007 k=1,11
    1007 st(j+k)=st(j+k)/su(j)
  13=11-1
  if(i-14)1008,1008,1008
1008 do 1010 i=1,14
  m=j+1*idm7-1
  do 1011 k=1,13
    1011 st(m+k)=st(m+k)-st(j+k+11-13-1)*su(j+1)
  13=13-1
  if(i-13)1010,1010,1012
1010 continue
1012 i4=i4-1
1003 continue
return
end
c
c
c*****
c subroutine load(idm4,nel,prs,x1,y1,ny,x,y,idm2,p,idm8,x2,y2)
c
c
c dimension nel(idm4),prs(idm4),xl(idm4),yl(idm4),x(idm9),y(idm9),
+  tp(idm8),x2(idm4),y2(idm4)
c
c do 2006 i=1,idm4
  nel=nel(i)+(nel(i)-1)/(ny-1)
  nel2=nel+1
  nel3=nel+ny
  nel4=nel+ny+1
  a=(x(nel2)-x(nel))/2.
  b=(y(nel2)-y(nel))/2.
  f1=prs(i)*(x2(i)-x1(i))*(y2(i)-y1(i))/2.
  f2=prs(i)*(x2(i)*x2(i)-x1(i)*x1(i))*x2(i)*y2(i)-y1(i)*y1(i))/2.
  f3=prs(i)*(x2(i)-x1(i))*(y2(i)-y1(i))*x2(i)*y2(i)-y1(i)*y1(i))/2.
  f4=prs(i)*(x2(i)*x2(i)-x1(i)*x1(i))*x2(i)*y2(i)-y1(i)*y1(i))/3.
  f5=prs(i)*(x2(i)*x2(i)-x1(i)*x1(i))*x2(i)*y2(i)*x2(i)*y2(i)-y1(i)*y1(i)*x2(i)*y2(i)/4.
  f6=prs(i)*(x2(i)-x1(i))*(y2(i)-y1(i))*x2(i)*y2(i)*x2(i)*y2(i)/3.

```



```

      n(i)=p(i)/su(j)
      do 3000 i=1,41
      3000 p(i+1)=p(i+1)-p(i)*su(j+1)
      12=12+1
      14=14-1
      do 3001 i=12,ldm8
      j=(i-1)*ldm7+1
      p(i)=p(i)/su(j)
      13=13-1
      14=14-1
      3003 do 3004 i=1,14
      p(i+1)=p(i+1)-p(i)*su(j+1)
      13=13-1
      14=14-1
      3004 continue
      3005 i4=i4-1
      3001 continue
      3002 do 3006 m=1,ldm8
      i=ldm8-m+1
      j=(i-1)*ldm7+2
      do 3006 k=1,i1
      if(1+k.gt.ldm8)goto3006
      p(i)=p(i)-p(1+k)*at(1+k-1)
      3006 continue
      return
      end
      c *****
      c
      subroutine stress(ldm9,nlx,n2x,n3x,nx,nly,n2y,ny,nc2,xt,xtf,
      +str1,scrbl,str2,scrh2,p,ldm8,x,y,v,u,nlayer,el,e2,t1,t2,tt,xf,
      +xf,elx,ely,ulx,uly,theta,thick,dcl1,dcl2,
      +comp,nslab,dex,dex,dix,djwx,dprx,fyx,ltdx,aggx,dev,day,dly,
      +djwy,dpry,fyy,ltdy,aggy,dclx,dely)
      c
      c
      integer comp
      dimension str1(ldm9,3),str2(ldm9,3),scrbl(ldm9,5),scrh2(ldm9,5),
      +str2(ldm9,5),scrh2(ldm9,5),p(ldm8),x(ldm9),y(ldm9),v(2),u(2),
      +el(ldm9),e2(ldm9),t1(ldm9),t2(ldm9),tt(ldm9),xf(ldm9),yf(ldm9),
      +elx(ldm9),ely(ldm9),ulx(ldm9),uly(ldm9),theta(ldm9),thick(ldm9),
      +dcl1(30),dcl2(30)
      c
      c
      nlxy=nlx*ny
      n2xy=n2x*ny+nlxy
      do 4000 i=1,ldm9
      if(1-le.nlxy)goto4001
      if(1-le.n2xy)goto4002
      i1=ny
      i2=n3x
      i3=n2xy
      i4=ny-n2y

```

```

      goto 4003
4002  i1=ny
      i2=n2x
      i3=n1x
      i4=ny-n2y
      goto 4003
4001  i1=ny
      i2=n1x
      i3=0
      i4=ny-n2y
      do 4004 j=1,3
        atrl(i,j)=0.
      4004  do 4005 j=1,5
        atrl(i,j)=0.
        strbl(i,j)=0.
        atrl2(i,j)=0.
        atrl2(i,j)=0.
      4005  atrl2(i,j)=0.
            if((i.eq.i3+.or.i.eq.i1+i3.or.i.eq.i1*(i2-1)+i3+.or.i.eq.i1*(i2+
            +i3)goto4006
            if((i.eq.i3+i4+.or.i.eq.i3+i4+i1.or.i.eq.i1*(i2-1)+nly+i3.or.i.eq.
            +i1*(i2-1)+nly+i3+i1)goto4006
            if(i.eq.(i-13-1)/i1+i1+i1+i3)goto4007
            if(i.eq.(i-13-1)/i1+i1+i1+i3-n2y)goto4007
            if(i.eq.i1*(i2-1)+i3)goto4008
            a=(x(i+1)-x(i))/2.
            b=(y(i+1)-y(i))/2.
            al=a/b
            bl=b/a
            ab4=4*a*b
            j=i+3
            i=i+3
            atrl(i,j)=(6*(bl+ulx(i)*al)*p(j-2)-8*a*ulx(i)*p(j-1)+8*b*p(j)-
            +ulx(i)*p(j+1))-4*a*ulx(i)*p(j+2)-6*bl*p(j+1-2)+4*b*p(j+1))/ab4
            atrl(i,j)=(6*(al+uly(i)*bl)*p(j-2)-8*a*p(j-1)+8*b*uly(i)*p(j)-
            +6*al*
            +p(j+1))-4*a*p(j+2)-6*bl*uly(i)*p(j+1-2)+4*b*uly(i)*p(j+1))/ab4
            atrl(i,j)=0.5*(1.-ulx(i))*(-2*p(j-2)+4*b*p(j-1)-4*a*p(j)+2*p(j+1)+
            +a*p(j+2)+2*p(j+1-2)-4*b*p(j+1)-2*p(j+1))/ab4
            if(nlayer.eq.-1)goto4009
            atrl(i,j)=(6*(bl+v2)*al)*p(j-2)-8*a*v2)*p(j-1)+8*b*p(j)-6*al*
            +v2*p(j+1))-4*a*v2)*p(j+2)-6*bl*p(j+1-2)+4*b*p(j+1))/ab4
            atrl2(i,j)=(6*(al+v2)*bl)*p(j-2)-8*a*p(j-1)+8*b*v2)*p(j)-6*al*
            +p(j+1))-4*a*p(j+2)-6*bl*v2)*p(j+1-2)+4*b*v2)*p(j+1))/ab4
            atrl2(i,j)=u2)*(-2*p(j-2)+4*b*p(j-1)-4*a*p(j)+2*p(j+1)+4*a*p(j+
            +j)+2*p(j+1-2)-4*b*p(j+1-2)*p(j+1))/ab4
            if(i.eq.(i-13-1)/i1+i1+i3)goto4008
            if(i.eq.(i-13-1)/i1+i1+i3+ny-n2y)goto4008
      4007  a=(x(i+1)-x(i))/2.
            b=(y(i+1)-y(i))/2.
            al=a/b
            bl=b/a

```

```

ab4=4*a*b
j=1+3
str1(1,1)=scr1(1,1)+(-6*a1*ulx(1)*p(j-5)+4*a*ulx(1)*p(j-6)+6*
+(bl*ulx(1)*a1)*p(j-2)+8*a*ulx(1)*p(j-1)+8*b*p(j)-6*bl*p(j+1-2)+
+4*b*p(j+1))/ab4
str1(1,2)=scr1(1,2)+(-6*a1*p(j-5)+4*a*p(j-4)+6*(al+uly(1)*bl)*
+p(j-2)+8*a*p(j-1)+8*b*uly(1)*p(j)-6*bl*uly(1)*p(j+1-2)+4*b*uly(1)*
+p(j+1))/ab4
str1(1,3)=scr1(1,3)+0.5*(1.-ulx(1))*(-2*p(j-5)-4*a*p(j-3)+2*p(j-
+2)+4*b*p(j-1)+4*a*p(j)+2*p(j+1-5)-2*p(j+1-2)-4*b*p(j+1-1))/ab4
if(nlayer.eq.1)goto4010
str2(1,1)=scr2(1,1)+(-6*a1*v(2)*p(j-5)+4*a*v(2)*p(j-4)+6*(bl+v(
+2)*a1)*p(j-2)+8*a*v(2)*p(j-1)+8*b*p(j)-6*bl*p(j+1-2)+4*b*p(j+1))/
+/ab4
str2(1,2)=scr2(1,2)+(-6*a1*p(j-5)+4*a*p(j-4)+6*(al+v(2)*bl)*p(j-
+2)+8*a*p(j-1)+8*b*v(2)*p(j)-6*bl*v(2)*p(j+1-2)+4*b*v(2)*p(j+1))/
+ab4
str2(1,3)=scr2(1,3)*u(2)*(-2*p(j-5)-4*a*p(j-3)+2*p(j-2)+4*b*p(j-
+1)+4*a*p(j)+2*p(j+1-5)-2*p(j+1-2)-4*b*p(j+1-1))/ab4
4010 if(1.eq.(1-13-1)/11*11+11+13)goto4011
if(1.eq.(1-13-1)/11*11+11+13-n2y)goto4011
if(1.le.11+13)goto4012
4008 a=(x(1)-x(1-11))/2.
b=(y(1+1)-y(1))/2.
al=a/b
bl=b/a
ab4=4*a*b
j=1+3
l=11+3
str1(1,1)=scr1(1,1)+(-6*bl*p(j-1-2)-4*b*p(j-1)+6*(bl+uly(1)*al)*
+p(j-2)-8*a*ulx(1)*p(j-1)-8*b*p(j)-6*a*ulx(1)*p(j+1)-4*a*ulx(1)*
+p(j+2))/ab4
str1(1,2)=scr1(1,2)+(-6*bl*uly(1)*p(j-1-2)-4*b*uly(1)*p(j-1)+
+6*(al+uly(1)*bl)*p(j-2)-8*a*p(j-1)-8*b*uly(1)*p(j)-4*a*uly(1)*p(j+1)-
+4*a*p(j+2))/ab4
str1(1,3)=scr1(1,3)+0.5*(1.-ulx(1))*(-2*p(j-1-2)+4*b*p(j-1-1)+
+2*p(j-1)+2*a*p(j-2)-4*b*p(j-1)-4*a*p(j)-2*p(j+1)+4*a*p(j+2))/ab4
if(nlayer.eq.1)goto4013
str2(1,1)=scr2(1,1)+(-6*bl*p(j-1-2)-4*b*p(j-1)+6*(bl+v(2)*al)*p(
+j-2)-8*a*v(2)*p(j-1)-8*b*p(j)-6*a1*v(2)*p(j+1)-4*a*v(2)*p(j+2))/
+ab4
str2(1,2)=scr2(1,2)+(-6*bl*v(2)*p(j-1-2)-4*b*v(2)*p(j-1)+6*(al+
+v(2)*bl)*p(j-2)-8*a*p(j-1)-8*b*v(2)*p(j)-6*a1*p(j+1)-4*a*p(j+2))/
+/ab4
str2(1,3)=scr2(1,3)+u(2)*(-2*p(j-1-2)+4*b*p(j-1-1)+2*
+p(j-2)-4*b*p(j-1)-4*a*p(j)-2*p(j+1)+4*a*p(j+2))/ab4
4013 if(1.eq.(1-13-1)/11*11+11+13)goto4012
if(1.eq.(1-13-1)/11*11+11+13+ny-n2y)goto4012
goto 4011
4012 c=2.
goto 4014
4011 a=(x(1)-x(1-11))/2.

```

```

b=(y(i)-y(i-1))/2.
a1=a/b
b1=b/a
ab4=4*a*b
j=1*3
i=1*3
str1(i,1)=str1(i,1)+(-6*b1*p(j-1-2)-4*b*p(j-1)-6*a1*ulx(i)*
+p(j-5)+4*a*ulx(i)*p(j-4)+6*(b1+ulx(i)*a1)*p(j-2)+8*a*ulx(i)*
+p(j-1)-8*b*p(j))/ab4
str1(i,2)=str1(i,2)+(-6*b1*uly(i)*p(j-1-2)-4*b*uly(i)*p(j-1)-
+6*a1*p(j-5)+4*a*p(j-4)+6*(a1+uly(i)*b1)*p(j-2)+8*a*p(j-1)-
+8*b*uly(i)*p(j))/ab4
str1(i,3)=str1(i,3)+0.5*(1.-ulx(i))*(-2*p(j-1-5)+2*p(j-1-2)+
+4*b*p(j-1-1)+2*p(j-5)-2*a*p(j-2)-4*b*p(j-1)+4*a*p(j-5)+
if(nlayer.eq.1)goto4015
str2(i,1)=str2(i,1)+(-6*b1*p(j-1-2)-4*b*p(j-1)-6*a1*v(2)*p(j-5)+
+4*a*v(2)*p(j-4)+6*(b1+v(2)*a1)*p(j-2)+8*a*v(2)*p(j-1)-8*b*v(2)*p(j))/
+ab4
str2(i,2)=str2(i,2)+(-6*b1*v(2)*p(j-1-2)-4*b*v(2)*p(j-1)-6*a1*p(
+j-5)+4*a*p(j-4)+6*(a1+v(2)*b1)*p(j-2)+8*a*p(j-1)-8*b*v(2)*p(j))/
+ab4
str2(i,3)=str2(i,3)+u(2)*(-2*p(j-1-5)+2*p(j-1-2)+4*b*p(j-1-1)+2*
+p(j-5)-4*a*p(j-3)-2*p(j-2)-4*b*p(j-1)+4*a*p(j))/ab4
4015 if(i.eq.(1-13-1))/i1+i1+i3.or.i.ge.i1+(i2-1)+i1+i3)goto4012
if(i.eq.(1-13-1))/i1+i1+i3-n2y)goto4012
c=4.
4014 if(nlayer.eq.2)goto4016
h1=(chick(i)*3)/(i2.*(1.-ulx(i)*uly(i)))
str1(i,1)=h1*6.*elx(i)*str1(i,1)/(c*(chick(i)*2))
str1(i,2)=h1*6.*ely(i)*str1(i,2)/(c*(chick(i)*2))
str1(i,3)=h1*6.*a1x(i)*str1(i,3)/(c*(chick(i)*2))
do 4017 j=1,3
strb1(i,j)=str1(i,j)
str2(i,j)=0.
strb2(i,j)=0.
4017 strb2(i,j)=0.
goto 4018
4016 if(comp.eq.1)goto4019
h1=(chick(i)*3)/(i2.*(1.-ulx(i)*uly(i)))
h2=e2(i)*e2(i)*3/(i2.*(1.-v(2)*2))
str1(i,1)=h1*6.*elx(i)*str1(i,1)/(c*(chick(i)*2))
str1(i,2)=h1*6.*ely(i)*str1(i,2)/(c*(chick(i)*2))
str1(i,3)=h1*6.*a1x(i)*str1(i,3)/(c*(chick(i)*2))
do 4020 j=1,3
strb1(i,j)=str1(i,j)
str2(i,j)=h2*6.*str2(i,j)/(c*e2(i)*2)
strb2(i,j)=str2(i,j)
goto 4018
4019 ena1=0.5*(el1(i)+e2(i))/(el1(i)+e2(i)+e1(i))
ena1=0.5*(el1(i)+e2(i))-ena2
h1=e1(i)/(i2.*(1.-v(1)*2))
h2=e2(i)/(i2.*(1.-v(2)*2))
do 4021 j=1,3
str1(i,j)=h1*6.*str1(i,j)*(el1(i)+2.*ena1)/c

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      strbl(1,j)=hl*6.*strbl(1,j)*(-tl(4)+2.*enal)/c
      str2(1,j)=h2*6.*str2(1,j)*(c2(1)-2.*ena2)/c
4021  strb2(1,j)=h2*6.*str2(1,j)*(-c2(1)-2.*ena2)/c
4018  if(1.eq.(1-13-1)/11*11+1+13.or.1.eq.(1-13-1)/11*11+1+13)
      +goto4022
      if(1.eq.(1-13-1)/11*11+1+13+ny-n2y.or.1.eq.(1-13-1)/11*11+1+13-
      +n2y)goto4022
      goto 4023
4022  strtl(1,2)=0.
      strbl(1,2)=0.
      str2(1,2)=0.
      strb2(1,2)=0.
4023  if(1.le.41+13.or.1.gt.11*(12-1)+13)goto4024
      goto 4025
4024  strtl(1,1)=0.
      strbl(1,1)=0.
      str2(1,1)=0.
      strb2(1,1)=0.
4025  if(strtl(1,1).eq.0..or.strtl(1,2).eq.0.)goto4026
      goto 4006
4026  strtl(1,3)=0.
      strbl(1,3)=0.
      str2(1,3)=0.
      strb2(1,3)=0.
4006  continue
      strtl(1,4)=(strtl(1,1)+strtl(1,2))/2.+sqrt(0.25*(strtl(1,1)-
      +strtl(1,2))*2+strtl(1,3)**2)
      strbl(1,4)=(strbl(1,1)+strbl(1,2))/2.+sqrt(0.25*(strbl(1,1)-
      +strbl(1,2))*2+strbl(1,3)**2)
      str2(1,4)=(str2(1,1)+str2(1,2))/2.+sqrt(0.25*(str2(1,1)-
      +str2(1,2))*2+str2(1,3)**2)
      strb2(1,4)=(strb2(1,1)+strb2(1,2))/2.+sqrt(0.25*(strb2(1,1)-
      +strb2(1,2))*2+strb2(1,3)**2)
      strtl(1,5)=(strtl(1,1)+strtl(1,2))/2.-sqrt(0.25*(strtl(1,1)-
      +strtl(1,2))*2+strtl(1,3)**2)
      strbl(1,5)=(strbl(1,1)+strbl(1,2))/2.-sqrt(0.25*(strbl(1,1)-
      +strbl(1,2))*2+strbl(1,3)**2)
      str2(1,5)=(str2(1,1)+str2(1,2))/2.-sqrt(0.25*(str2(1,1)-
      +str2(1,2))*2+str2(1,3)**2)
      strb2(1,5)=(strb2(1,1)+strb2(1,2))/2.-sqrt(0.25*(strb2(1,1)-
      +strb2(1,2))*2+strb2(1,3)**2)
      tt(1)=tl(1)+t2(1)
4000  continue
      if(nslab.eq.1)goto6100
      if(1/dxx.eq.0.or.n2.eq.0)goto5101
      j1l=(nlx-1)*ny+1
      j3=nlx*ny
      ddx=dx*dix
      if(djwx.le.0.01)djwx=0.01
      d5x=12.*ddx/(djwx**3*(1.+fyx))
      d2x=6.*ddx/(djwx**2*(1.+fyx))
      do 4101k=j1l,j3
      dxx=del(k-j1l+1)

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dlx=(1./dcx)+(1./d5x))
jl=k
j2=k+ny
xf(j1)=dlx*p(j1+3-2)+d2x*p(j1+3)-dlx*p(j2+3-2)+d2x*p(j2+3)
4101 xf(j2)=-xf(j1)
      if(n3x.eq.0)goto5101
      l1l=(n1x+n2x-1)*ny+1
      l33=(n1x+n2x)*ny
      do 4111k=l11,133
        dcx=dc1(k-l1l+ny+1)
        dlx=(1./dcx)+(1./d5x))
      l1=k
      l2=k+ny
4111 xf(l1)=dlx*p(l1+3-2)+d2x*p(l1+3)-dlx*p(l2+3-2)+d2x*p(l2+3)
5101 if((l1dy.eq.0.or.n2y.eq.0)goto4100
      nx=n1x+n2x+n3x
      j1l=(nx-1)*ny+nly
      ddy=dey*dly
      if(djwy.le.0.01)djwy=0.01
      dcy=dcly
      d5y=12.*ddy/(djwy**3*(1.+fyy))
      d2y=6.*ddy/(djwy**2*(1.+fyy))
      dly=(1./((1./dcy)+(1./d5y)))
      do 5107k=nly,j1l,ny
        l1l=k
        l12=k+1
        yf(l1l)=dly*p(l1l+3-2)-d2y*p(l1l+3-1)-dly*p(l12+3-2)-d2y*p(l12+
+3-1)
5107 yf(l12)=-yf(l1l)
4100 continue
      return
      end
c
c*****
c
      subroutine check(ldm9,ldm8,ldm5,nx,ny,lte,ltdv,
+nsalab,nlx,n2x,n3x,n2y,lcdx,lcdv,
+stctrl,p,
+telx,ely,ulx,uly,theta,thick,
+e2x,e2y,thicl,
+sub,subg2,
+xx,y,
+dc1,dc12,xf,yf,
+tsuhc,tox,roy,fcl,fr2,fr3,coef1,coef2,cyclsl,
+ccyl1,cycl2,cycl3,relnfx,relnfy,cel,
+al,n2,a3,deck,dect,
+tdout,x,dlx,djwx,douty,dly,djwy,dc1x,dc1y)
      dimension stctrl(ldm9,5),p(ldm8),elx(ldm9),ely(ldm9),
+ulx(ldm9),uly(ldm9),theta(ldm9),thick(ldm9),
+e2x(ldm9),e2y(ldm9),thicl(ldm9),
+sub(ldm9),subg2(ldm9),
+tx(ldm9),y(ldm9),

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```

+dcf1(30),dcf2(30),xf(1dm9),yf(1dm9)
c
c *****
c subroutine to check the level of stresses *****
c *****
c
areak=0.
areat=0.
erkr=0.
decayk=0.
docont=0.
do 5001 i=1,1dm9
c faubr is the relative stress
c faubr is the compressive strength of concrete
c esubr and esubrl are the moduli in direction theta
c remain is the remaining life of section
c
    if(i.gt.ny)then
        distx1=x(i)-x(1-ny)
    end if
    distx2=x(1+ny)-x(i)
    disty1=y(i)-y(1-ny)
    disty2=y(i+1)-y(i)
c
    if(i.eq.1)then
        distx1=x(1+ny)-x(i)
        distx2=0.
        disty1=y(2)-y(i)
        disty2=0.
        goto 5065
    end if
    if(i.le.ny)then
        distx1=x(1+ny)-x(i)
        distx2=0.
        goto 5060
    end if
    if(i.gt.(nx-1)*ny)then
        distx1=x(i)-x(1-ny)
        distx2=0.
        goto 5060
    end if
    if(x(1+ny)-x(i).eq.0.)then
        distx1=x(i)-x(1-ny)
        distx2=0.
        goto 5060
    end if
    if(x(i)-x(1-ny).eq.0.)then
        distx1=x(1+ny)-x(i)
        distx2=0.
        goto 5060
    end if
c

```



```

      faubr=1.
      goto 5055
c      obtain modified modull of elasticity
c
5050 if(remain.le.cycl2)then
      faubr=frc1*coef1*log10(remain)
      goto 5055
end if
if(remain.le.cycl3)then
      faubr=frc2+coef2*(log10(remain)-log10(cycl2))
end if
5055 faubr=atrt1(1,4)/faubr
      esub1=57000.*faubr/9.
      ely(1)=2x(1)*esub1/esubt+21750000.*rox*(reinfy/thick(1))*2.
      ely(1)=2y(1)*esub1/esubt+21750000.*roy*(reinfy/thick(1))*2.
      area=0.25*(dintx1+dintx2)*(dinty1+dinty2)
      areak=areak+area
      decayt=decayt+area*(e2x(1)+e2y(1)-elx(1)-ely(1))/(e2x(1)+e2y(1))
      if(ite-itter*((ite-1)/itter).eq.itter)then
          write(6,230)1,ely(1),ely(1),area
      end if
230 format(5x,i5,2x,'Decay in Ec',7x,'Ecx=',e10.3,2x,'Ecy=',e10.3,4x,
+fl0.3)
      goto 5000
c
5100 i11=0
c
c      cracking occurs
c
c      if there is not reinforcement, reduce thickness to a minimum.
c
      if(rox.eq.0. .and. roy.eq.0.)then
          if(ite-itter*((ite-1)/itter).eq.itter)then
              write(6,241)1
          end if
          epsubc=0.0001
          ax1sn=0.086
          esubt=100000.
          tmont=1.0
          goto 5153
      end if
241 format(5x,"Crack develops through all thickness at node",2x,i5)
c
c      determine cracked section
c
      asubs=(rox*cos(orient)+roy*sin(orient))*thick(1)
      tmont=atrt1(1,4)*(thick(1)*2.)/6.
      i11=i11+1
      ax1su=0.5*thick(1)-.01*i11
      epsubc=12.*tmont/((3*thick(1)-2*ax1sn)*esubc*ax1sn)
      epsubs=tmont*(ax1sn**2.)*epsubc*esubt/6.-.0000005*(ax1sn**2.)*
+esubt/epsubc-.33333*(ax1sn**2.)*esubt/(1000000000.*epsubc**2.)

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epsubs=epsubs/(14500000.*asubs*thick(1))
error1=abs(0.5*epsubc*esubt*axian-asubs*290000000.*
+epsubs-.0000005*axian*esubt/epsubc)
error1=error1/(0.5*epsubc*esubt*axian)*100.
if(error1.ge.2.5)gotn 5150
area=0.25*(diatx1+diatx2)*(diaty1+diaty2)
areat=areat+area
decay=decny*(thick(1)-(epsubc+.0008)*axian/epsubc)*area/thick(1)

c
c calculate equivalent thickness for stiffness matrix
c
thick(1)=sqrt(6.*tmont/(epsubc*esubt))
if(1ite-1tter*((1ite-1)/1tter).eq.1tter)then
write(6,240)1,(epsubc+.0008)*axian/epsubc,areat
end if
240 format(5x,i5.2x,'Decay in T',8x,'New Thickness=',f10.3,
+10x,f10.3)

c
c modify resilient modulus of subgrade reaction
c
5000 subg2(1)=sub(1)
if(sub(1).ne.0.)then
sub(1)=.01*a2*(p((1-1)*3+1))+3*(p((1-1)*3+1))*2
end if
erck=erck+(sub(1)-subg2(1))*2.
erck=sqrt(erck/1dm9)
5001 continue
deck=decay
deck=decayt
if(1ite-1tter*((1ite-1)/1tter).eq.1tter)then
print*, 'DAMAGED AREA=',areat+areat
print*, 'error in k=', erck
end if

c
c if (1tdx.ne.1.and.1tdy.ne.1)gotn 7010
c compute decay in dowel-concrete interaction factor
c
do 7005 l=1,1dm5
fscr=0.
a=0.
b=0.
fact=relative load at dowel bar.
use absolute values of loads at dowels
nel=1+(1-1)/(ny-1)
xf(nel)=abs(xf(nel))
yf(nel)=abs(yf(nel))
yf(nel)=abs(yf(nel))
a=x(nel+ny)-x(nel)
b=y(nel+1)-y(nel)
if(a.ne.0.or.1tdx.ne.1)gotn 7000
c
c dowels in x-direction.

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```

farb=yf(nel+1)/(1.5*douty*(thick(nel+1)-douty)*
tely(nel+1)/5700.)
farb=farb*(1.+dlyy/dly)
farb=(0.268-0.0657*log10(cyclsl)+1.123*fsrb)/(0.268+1.123*farb)
if(farb.lt.0.)farb=0.
del2=((1-l)/(ny-1)+2)-farb*dcty
sum3=sum3+(1.-farb)
sum4=sum4+1.
c
c end of joint with dnwels in y-direction
c
c 7005 continue
c
if(ite-itter*((ite-1)/itter).eq.itter)then
print*, 'Z decay in load transfer=', sum3*100./sum4
end if
7010 return
end
c
c
c subroutine pump(idm9,idm8,idm5,nx,ny,nlx,
+cyclsl,
+x,y,sub,p,subg2)
c
c dimension x(idm9),y(idm9),sub(idm9),p(idm8),subg2(idm9),
+node(30),area(30)
c
c compute amount of energy imposed on deflected basin
c
ener=0.
do 8000 i=1,idm9
c
c determine semiarea of nodal area surrounding node i
c
if(i.lt.ny)then
distx1=x(i)-x(i-ny)
end if
distx2=x(i+ny)-x(i)
disty1=y(i)-y(i-1)
disty2=y(i+1)-y(i)
if(i.eq.1)then
distx1=x(i+ny)-x(i)
distx2=0.
disty1=y(2)-y(1)
disty2=0.
goto 8065
end if
if(i.le.ny)then
distx1=x(i+ny)-x(i)
distx2=0.
goto 8060

```



```

end if
if (t.gt.(nx-1)*ny) then
  distx1=x(t)-x(t-ny)
  distx2=0.
  goto 8060
end if
if (x(t+ny)-x(t).eq.0.) then
  distx1=x(t)-x(t-ny)
  distx2=0.
  goto 8060
end if
if (x(t)-x(t-ny).eq.0.) then
  distx1=x(t+ny)-x(t)
  distx2=0.
  goto 8060
end if

c
8060 if (y(t)-y(t-1).le.0.) then
  disty1=y(t+1)-y(t)
  disty2=0.
  goto 8065
end if
if (y(t+1)-y(t).le.0.) then
  disty1=y(t)-y(t-1)
  disty2=0.
end if
8065 if (p((t-1)*3+1)-g*.0.02) then
  ener=ener+0.25*(distx1+distx2)*(disty1+disty2)*nuh(t)*
  +((p((t-1)*3+1)-0.02)**2.)
end if
8000 continue
c
c determine normalized pumping index (pi/(# of joints @ 100 ft))
pi=**(-2.884+1.652*log10(total/10000))
pump1=2.7183**((1.652*log10(ener*cyclal/10000.))-2.884)
pump1=pump1*(1200./(s(nlx*ny)-x(1)))
write(6,8040)pump1
8040 format(///,10x,'pumping index=',f10.2,///)
c
c pump1 is the pumping index obtained from aasho road test
c The total volume is:
c
c pump1=pump1*(x(nlx*ny)-x(1))
c this volume can be modified to account for other factors
c introduce appropriate factors
c
c define nodes with potential for pumping
c
do 8050 k=1,30
8050 node(k)=0
do 8100 j=2,21
  def1=0.
do 8100 i=1,1dm9

```

```

c      define nodal semiaxis
c
      if (i.gt.ny) then
        distx1=x(i)-x(i-ny)
      end if
      distx2=x(i+ny)-x(i)
      disty1=y(i)-y(i-1)
      disty2=y(i+1)-y(i)
c
      if (i.eq.1) then
        distx1=x(i+ny)-x(1)
        distx2=0.
        disty1=y(2)-y(1)
        disty2=0.
        goto 8365
      end if
      if (i.le.ny) then
        distx1=x(i+ny)-x(i)
        distx2=0.
        goto 8360
      end if
      if (i.gt.(nx-1)*ny) then
        distx1=x(i)-x(i-ny)
        distx2=0.
        goto 8360
      end if
      if (x(i+ny)-x(i).eq.0.) then
        distx1=x(i)-x(i-ny)
        distx2=0.
        goto 8360
      end if
      if (x(i)-x(i-ny).eq.0.) then
        distx1=x(i+ny)-x(i)
        distx2=0.
        goto 8360
      end if
c
      8360 if (y(i)-y(i-1).le.0.) then
        disty1=y(i+1)-y(i)
        disty2=0.
        goto 8365
      end if
      if (y(i+1)-y(i).le.0.) then
        disty1=y(i)-y(i-1)
        disty2=0.
        goto 8365
      end if
c
      8365 continue
c
c      sort the nodes in accordance with deflection values
c
      do 8367 m=2, j

```

```

      if (node(m).eq.1) goto 8100
8167 continue
      if (p((i-1)*3+1).gt.def1) then
        def1=p((i-1)*3+1)
        node(j)=i
        area(j)=0.25*(distx1+distx2)*(disty1+disty2)
      end if
8100 continue
c
c      obtain area of void
c
      pump1=pump1/0.25
      print*, 'void depth=', 0.25
c
c
      do 8150 k=2,30
        sub(node(k))=0.
        subg2(node(k))=0.
        pump1=pump1-area(k)
8150 if (pump1.le.0.) goto 8155
8155 return
      end

```


COVER DESIGN BY ALDO GIORGINI