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MODELLING OF REFRIGERATION PISTON COMPRESSORS

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Abstract

A model for piston compressors working in vapor compression systems is presented, including a comparison between calculated and measured results for a commercial compressor of nominal capacity of 23 kW. This model has been validated via an experimental installation constructed in the authors' laboratory. This installation has enough accuracy in the measurement of the volumetric and isentropic efficiency to compare its experimental results with the output of the model. The model is capable to predict the performance of the mentioned hermetic compressor with a mean error of 3% and a maximum error of 8% in the calculation of the volumetric and isentropic efficiency.

INTRODUCTION

The purpose of this paper is to present a model for piston compressors. The model is based on the Purdue model [7] and [2] complementing the hydraulic problem with the solution of the thermal problem. McGovern [4] and Liu [3] solved the two problems simultaneously using a lumped heat capacity method. The former used ideal gas equations in the integration of a first law model and Liu solved the two problems in a rolling piston rotary compressor. On the other hand,Padhy [6] and Cavallini [1] solved only the thermal problem using an energy balance based in the first law assuming the circulating mass flow rate known.

The results of Liu and McGovern indicates that the wall temperatures are nearly constant during a crankshaft revolution in the steady performance of the compressor.Then, the approach used in the model presented is to assume the wall temperatures constant.

The solution to the thermal and hydraulic problem simultaneously together with a modular approach and an experimental support for the phenomena non-well modelled like the flow through the valve makes this model a valuable tool to simulate any type of piston compressors.

DESCRIPTION OF THE MODEL

The compressor is divided into a number of elements. Each of these elements has different governing equations, and therefore they need different approaches for the simulation. These elements are

Volumes The velocity of the refrigerant inside these elements is negligible and therefore the properties of the refrigerant inside them can be considered homogeneous. This assumption of homogeneous properties does not imply steady conditions, the properties vary. Examples: case volume and cylinder.

Orifices These elements connect volumes and allow the mass transfer between them. The refrigerant mass flow rate through them is calculated using the Saint-Venant equation for compressible flow. The mass flow rate through the valves is modeled considering them as orifices with a variable value of the effective area.

Valves The movement of the valves must be included in the model to calculate their effective areas.

Walls These elements are the solid bodies of the compressor. There is a heat transfer among these elements and the refrigerant in the volumes. Each of these elements has a homogeneous and constant temperature in the steady performance of the compressor.

Thermal connections They allow the heat transfer among elements by radiation, convection or conduction. The connections can be: volume-wall like the heat transfer between the refrigerant inside the cylinder and the wall of the cylinder or wall-wall like the radiation heat transfer between electric motor wall and the case wall. Also, there are heat sources like the heat produced by the friction and by the electrical losses.

These models are linked together in order to produce the global model of a piston compressor. The details of every model and the main algorithm of the compressor´s model are described below.

Calculation of the volumes

The governing equations in these elements are the mass conservation equation and the energy conservation equation invoked by the first law of thermodynamics. These equations are:

$$\frac{dm}{dt} = \sum_{i=1}^{i=N} \dot{m}_i \tag{1}$$

$$\frac{dU}{dt} = \sum_{i=1}^{i=N} \left(\dot{m}h_i + \dot{Q} - \dot{W}\right) \tag{2}$$

where m is the mass inside the volume,$\dot{m}_i$ is the refrigerant mass flow rate that flow through the i connection, U is the internal energy, h_i is the enthalpy of the refrigerant that enters of exit from the volume,$\dot{Q}$ is the heat rate transferred with the connecting walls and $\dot{W}$ is the power made by the refrigerant inside the volume.

The first equation shows that the mass variation in a volume is due to the different refrigerant mass flow rates that pass through the orifices. These mass flow rates must be obtained by the model of orifices and valves.The second equation shows that the variation of the internal energy inside the volume is due to the energy transported by the inlet and outlet refrigerant, by the heat transferred to the connecting walls, and the power made by the refrigerant when the volume changes. The first term is calculated again by the model of the orifices, the heat transfer is obtained by the thermal connection model and the power done in the volume is calculated as $\dot{W} = p\frac{dV}{dt}$. The last term is only applicable in a variable volume such as the compressor cylinders. The volume in a cylinder of a reciprocating compressor depends on the crankcase angle as:

$$V = A_p R \left[1 + \frac{1}{\lambda} - \left(cos\theta + \frac{1}{\lambda}\sqrt{1 - \lambda^2 sen^2\theta}\right)\right] \tag{3}$$

where A_p is the piston area,R is the length of the crank,λ is the crank an connecting rod ratio and θ is the crankshaft angle.The value of null angle is taken when the volume is minimum.

The derivative of the volume needed in the evaluation of the power is:

$$\frac{dV}{dt} = A_p R\Theta \left(sen\theta + \lambda sen\theta cos\theta \frac{1}{\sqrt{1 - \lambda^2 sen^2\theta}}\right) \tag{4}$$

where Θ is the angular velocity in rad/s

Calculation of the mass flow rates among volumes

The Saint-Venant equation is used for calculating the mass flow rate through an orifice when the flow is unchoked.

$$\dot{m} = A_{eff}\rho_u \left(\frac{p_d}{p_u}\right)^{\frac{1}{\gamma}} \sqrt{2\frac{\gamma-1}{\gamma}\left[1-\left(\frac{p_d}{p_u}\right)^{\frac{\gamma-1}{\gamma}}\right]} \tag{5}$$

where A_{eff} is the effective area,ρ_u is the upstream density, γ is the specific heat ratio and p_d,p_u are the downstream and upstream pressure respectively. The pressure ratio p_d/p_u when the flow becomes critical is defined by:

$$\frac{p_{crit}}{p_u} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \tag{6}$$

and the pressure ratio in equation 5 must be substituted by this critical pressure ratio when $p_d/p_u < p_{crit}/p_u$.

The derivation of equations 5 and 6 is made under the assumptions of one-dimensional steady flow, isentropic flow upstream of the orifice and isobaric flow downstream. Strictly, this equation is applicable only for a perfect gas, in a real gas the mass flow rate should be calculated using the first law of thermodynamics assuming stagnation conditions upstream by the following equation:

$$\dot{m} = \rho_{is} A_{eff}\sqrt{2\left(h_u - h_{is}\right)} \tag{7}$$

where ρ_{is} and h_{is} are the conditions of a isentropic evolution from the upstream pressure and density to the downstream pressure.

However, the use of equation 5 instead of equation 7 has been tested by the authors for the refrigerants R22 and propane in practical situations and it gives a relative error of less than 2%. Equation 5 requires less computational effort and therefore the mass flow rate is calculated with this equation.

Two different connections can be distinguished among volumes, orifices and valves. Equation 5 is a good approximation to the real process in an orifice and the effective area of orifices can be consulted in the literature.The flow through the valves is three-dimensional and there are effects like the turbulence that make an accurate calculation of the circulating mass flow rate difficult. Also, the valve is moving and therefore the boundary conditions of the problems change. One solution to this problem is to use CFD techniques to solve the flow through the valves. This approach is difficult to program and needs a huge amount of time to obtain a solution. Also, these techniques can not assure a reliable solution because of the non-well modeled phenomena like turbulence. Therefore, the right approach used in the calculation of the mass flow rate through the valves is to use equation 5 with a measured value of the valve effective area. This effective area is measured in a steady flow rig for every lift. The lift is calculated by the dynamic model of the valves. This approach combines a high accuracy with a little computational effort.

Calculation of the valves dynamics

The model can simulate the motion of two types of valves: spring loaded ring valves and flexible reed valves, if the pressure at both sides of the valve is known. The lift of the valve is used to interpolate an effective area from previous measurements obtained in the steady flow rig and this effective area is used to calculate the mass flow rate through the valve using equation 5.

The first type of valves are calculated using a one degree of freedom model,

$$M\ddot{w}(t) + C\dot{w}(t) + Kw(t) = F(t) \tag{8}$$

where w is the valve lift, M is the valve mass,C is the damping coefficient,K is the spring stiffness and F is the actuating force on the valve.

The governing equation for the flexible reed valves in two dimensions is,

$$D\nabla^4 w(x,y,t) + \rho h\ddot{w}(x,y,t) = p(x,y,t) \tag{9}$$

where

$D = \frac{Eb^3}{12(1-\mu^2)}$ E is the Young's modulus,b the width of the valve and μ is the Poisson's ratio

$$\nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}$$

This partial differential equation is transformed in a set of ordinary differential equations using a modal analysis as,

$$\ddot{q}_m(t) + 2\zeta\omega_m\dot{q}_m(t) + \omega_m^2 q_m(t) = \frac{\int_s \int \phi_m(x,y)p(x,y,t)\mathrm{d}\,S}{\rho h \int_s \int \phi_m^2(x,y)\,\mathrm{d}S} \tag{10}$$

Equations 8 and 10 are integrated using a fourth order Runge Kutta method. This procedure gives us the position of the valves in the next time step if we know the pressure on each side of the valves and the initial conditions. The values of natural frequencies $\omega_m$and mode shapes $\phi_m(x,y)$ are calculated previously using a commercial code of finite element analysis.

Thermal connections

There are two type of thermal connections in a compressor, volume-wall and wall-wall. The first one calculates the heat transfer between the fluid inside a volume and a wall by convection as,

$$\dot{Q} = \alpha_{i,j}A_{i,j}\left(Tw_i - Tv_j\right) \tag{11}$$

where $h_{i,j}$ and $A_{i,j}$ are the corresponding heat transfer coefficients and area between the wall i and the volume j,$\alpha_{i,j}$ is the heat transfer coefficient,Tw_i is the wall temperature and Tv_j is the volume temperature.

The other possibility is a thermal connection between two walls. The heat transfer is calculated by radiation between the walls i and j using an equivalent convection coefficient as:

$$\dot{Q} = A_i\alpha r\left(T_i - T_j\right) \tag{12}$$

where αr is the equivalent convection coefficient.

In fact, almost all the radiative heat transfer is neglected in the walls, except the heat transfer between the stator and the shell in a hermetic compressor because of their big areas.

Calculation of the walls' temperature

The temperature of the walls is assumed to be constant during one crankshaft revolution due to its high thermal inertia compared with the compressor's speed.

This temperature can be calculated posing a thermal balance in each wall. The sum of the convective, radiative and sources of heat transfer must be equal to zero. This thermal balance is made with mean values of the heat rate throughout a crankshaft revolution. Then for a wall this balance has the following form for a wall

$$\sum_{volumes}\overline{\dot{Q}_c} + \sum_{walls}\overline{\dot{Q}_r} + \sum\overline{\dot{Q}_s} = 0 \tag{13}$$

Where the convective heat transfer $\dot{Q}_c$is given by equation 11 and the radiative heat rate $\dot{Q}r$ is given by by equation 12. The source term $\dot{Q}_s$ includes the heat produced by friction between the cylinder and the piston and the heat released by the electric motor. Their mean values are the same as the instantaneous ones due to the assumption of constant wall temperatures and heat sources throughout a revolution.The mean convective heat transfer $\dot{Q}_c$ between a wall i and a volume j has to be calculated as

$$\overline{\dot{Q}_c} = f\int_0^{\frac{1}{f}} \alpha_{i,j}A_{i,j}\left(Tv_j - Tw_i\right)\mathrm{d}\,t \tag{14}$$

where f is the speed of the compressor in cycles/s

Rearranging equation 13 a linear equation for each wall is obtained. Each wall is connected with J volumes by convective thermal connections and with K walls by radiative thermal connections. The equation for each wall i is

$$\left(\sum_{j=1,J}\overline{\alpha_{i,j}A_{i,j}} + \sum_{k=1,K;k\neq i}\alpha r_{i,k}Ar_{i,k}\right)Tw_j + \sum_{k=1,K;k\neq i}\alpha r_{i,k}Ar_{i,k}Tw_k = \sum_{j=1,J}\overline{\alpha_{i,j}A_{i,j}T_j} + \sum\dot{Q}_s \qquad (15)$$

These equations (one for each wall) form a system of linear equations with the walls' temperatures as unknowns. The system can be solved by a direct method like the Gauss elimination.

Convergence algorithm

All the submodels described above have to be linked together in order to obtain a periodic solution of the compressor modeled. The model obtain a periodic solution by iteration.

First, the conditions in all the volumes are initialized with arbitrary values as well as the wall temperatures. The valves are initially closed.

Then, the value of the internal energy and mass inside the volumes is calculated in the next time step integrating equation 1 and 2 using a fourth order Runge-Kutta method.The value of the time step is given by the user of the model.

The idea of the Runge-Kutta methods is to begin with an initial value of the unknown variables(in this case,m_i^n and U_i^n) and to calculate the final values m_i^{n+1} and U_i^{n+1} evaluating the derivatives dm/dt and dU/dt at intermediate time steps. The derivative at the initial time can be evaluated using equations 1 and 2. First, the lift of the valves is calculated integrating equation 8 or 10 depending on the type of valve with the values of the pressure at both sides of the valves. The effective area of the valves is interpolated between the previous measurements in the steady flow rig. Then, the circulating mass flow rates between volumes can be calculated using equation 5 and the effective areas of the orifices and the valves. The specific enthalpy of the inlet flows of a volume is used as the enthalpy of the upstream connecting volume. The heat rate can be calculated using equation 11 with the values of the wall and volume temperatures. The power is calculated using expression 4. Knowing all these variables, it is possible to evaluate the derivatives at initial time $dm/dt|_n$ and $dU/dt|_n$ using expressions 1 and 2 respectively and therefore the value of mass and internal energy in the next time step or intermediate time steps. Proceeding again as described before, it is possible to evaluate different derivatives at intermediate time steps. Finally, depending on the type of the method used, the value at the next time step ,m_i^{n+1} U_i^{n+1}, is calculated.

Then, it is possible to calculate the density in every volume as $\rho_i^{n+1} = m_i^{n+1}/V_i n + 1$, where the volume in a cylinder is computed using equation 3 or the equivalent, if the compressor mechanism is not a reciprocating one. Finally, every thermodynamic property as temperature, enthalpy, specific heat, ... can be calculated using these two independent variables (density and internal energy) with the Refprop routines [5].

The loop starts again and the algorithm calculates the conditions in each volume every finite time step.

When a crankshaft revolution is completed, the wall temperatures are updated solving the system of equations 15. The whole main loop is repeated until a periodic solution of the variables is obtained.

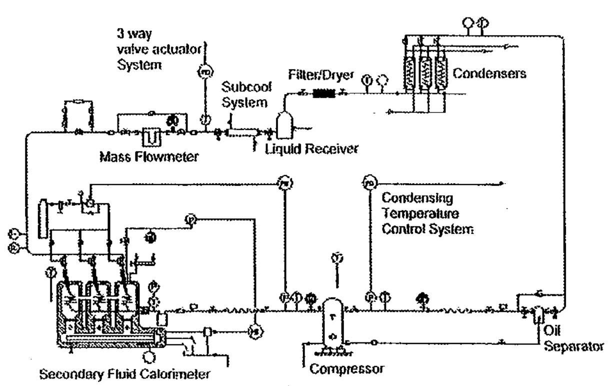

Figure 1: Experimental set-up

DESCRIPTION OF THE COMPRESSOR TEST RIG

The compressor rating procedure was performed according to the relevant standards in the field such as the ISO-917 and American ANSI ASHRAE 23-1993. According to the aforementioned standards, the circulating refrigerant mass flow is the determining parameter to be measured. For this, primary and confirming tests have to be used. The primary test procedure chosen is the secondary refrigerant calorimeter method. A Coriolis mass flow meter was used as the confirming test method. Confirming tests were always carried out simultaneously with the primary mass flow rate determination with an agreement within $\pm$ 1% in the worst case. In figure 1, a general scheme of the experimental set-up is shown.

4 PID control loops (compressor inlet and outlet pressure, superheat and subcooling controls) were incorporated to allow a precise adjustment of the refrigerant conditions at compressor inlet (evaporating temperature and superheat) and outlet (condensing temperature). The rig was thus fully automated, making it possible to reach any allowable test conditions without manual adjustments.

COMPARISON WITH MEASUREMENTS

The compressor modeled is a Maneurop model MT 100 HS,its main features are summarized in table 1. The geometrical model is shown in Figure 2. The compressor takes refrigerant from the evaporator and pumps it to the condenser being both treated as boundary conditions. The refrigerant flows through the volumes defined and transfers heat with the walls defined in the thermal model. These walls include the cylinder wall, the external case, the electric motor, and the pipes.

Cylinders	Displacement	Capacity	Power
4	29.8 m^3/h	23.425 Kw	8.16 Kw

Table 1: Characteristics of the compressor MT 100 HS working with R22 (Ari Conditions)

There are two types of valves in this compressor. The suction valve is a reed one and the discharge valve is a spring loaded ring valve. The effective areas of the valves have been measured in a steady flow rig for air. The valves were mounted in the rig and the lift was varied with a set of tools designed specifically for each valve. The lift of the suction valve is taken as the maximum lift. The results are presented in figure 3. The natural frequencies and modes of the suction valve, which are needed to solve equation 10. These values have been obtained using a commercial finite element code. The results of the model have been compared with measurements in the compressor test rig for two different refrigerants, R22 and propane. This comparison in terms of the volumetric efficiency and isentropic efficiency is presented in figure 4.

The mean error in the prediction of the volumetric efficiency is 3% for R22 and 2% for propane with maximum errors of 9% and 4% respectively. Concerning the prediction of the isentropic efficiency the mean error are 1.81% for R22 and 7% in propane, and the maximum errors are 4% and 11% respectively. The model gives similar values of volumetric and isentropic efficiency for R22 and propane although the difference in isentropic efficiency for propane with respect to R22 is not fully predicted by the model. The model takes into account the heating of the gas in the motor windings, the pressure drop in the orifices and valves, the heat losses to the ambient and the different thermodynamic properties of the refrigerants. Thus,the possible reason of the different isentropic efficiency for propane can be the different performance of the electric motor. The torque needed in the compression is lower for propane for the same pressure ratio because of its lower specific heat ratio. This fact means that the electric motor has higher velocity working with propane than with R22 and therefore the electric efficiency is higher in propane than in R22. Now,the electric motor is modeled with a constant value of the electric efficiency equal to 90%.This validation shows us the need of a better modeling of the electric motor that have to be accomplished in a near future.

CONCLUSIONS

A model for calculating the performance of piston compressors has been developed. The model includes the following features:

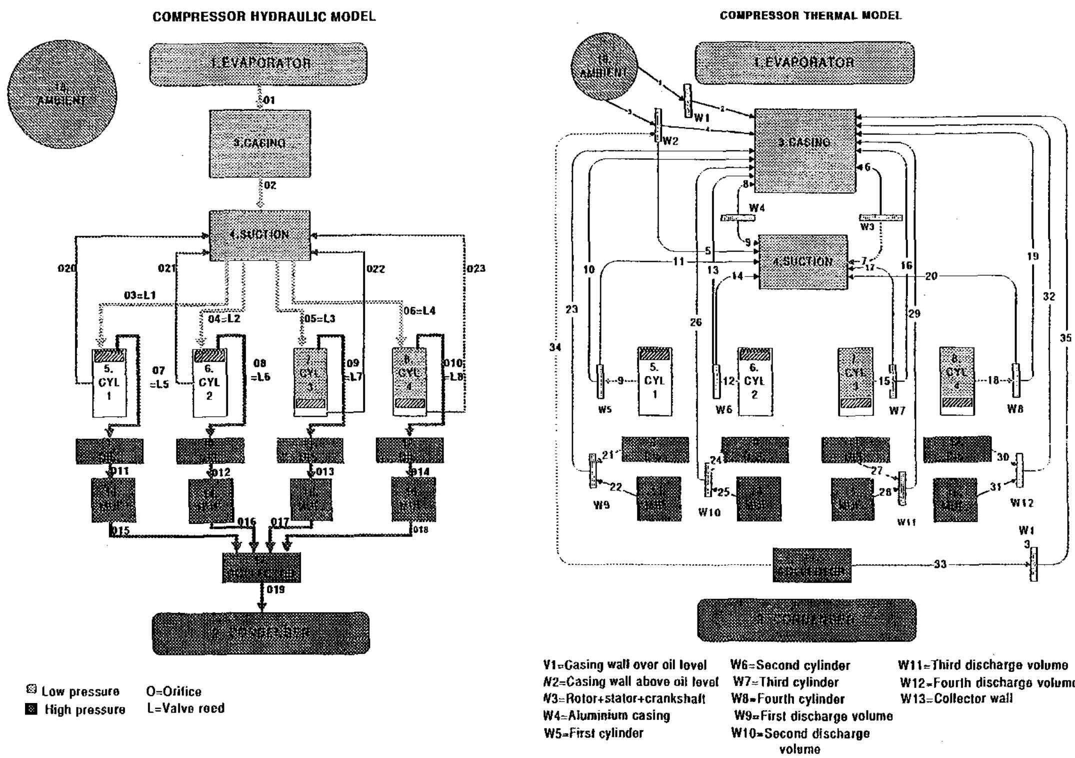

Figure 2: Hydraulic and thermal model of the compressor MT100HS

1. The model can calculate any type of piston compressors and it is not constrained to a given geometry. The user defines the topology of the modeled compressor.
2. The hydraulic and thermal problem are solved simultaneously with the described algorithm.
3. The model can take into account the heat released by the electric motor, the heat losses of the compressor and the leakage in the cylinder.
4. The simulation can be performed for any refrigerant defined in the NIST Refprop Database.
5. The CPU time of the model is low. A complete simulation takes about four minutes. Studies made by the authors show that the program takes 90% of the total time to calculate the thermodynamic properties.The substitution of that routines with interpolation tables will make that a simulation cost about 30 seconds.

This model has been used to calculate the compressor Maneurop MT100HS and the results are compared with measurements made in a compressor test rig. The effective areas of the valves have been measured in a steady flow rig for air and their natural frequencies and modes have been calculated with a commercial finite element code.

The comparison between the model and the experiments are good and the predicted tendencies over the whole range are similar. A better modeling of the electric motor is planned for the future.

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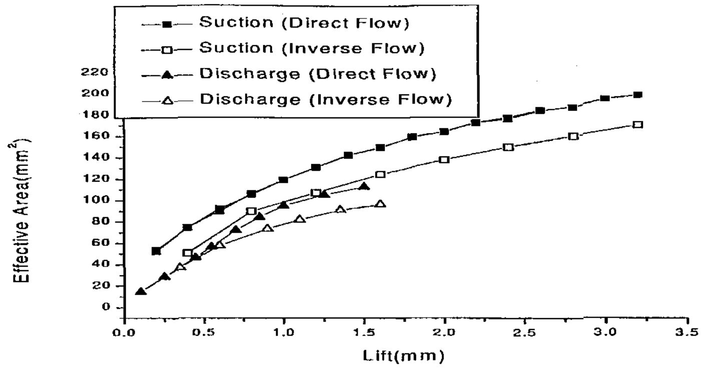

Figure 3: Effective area of the valves

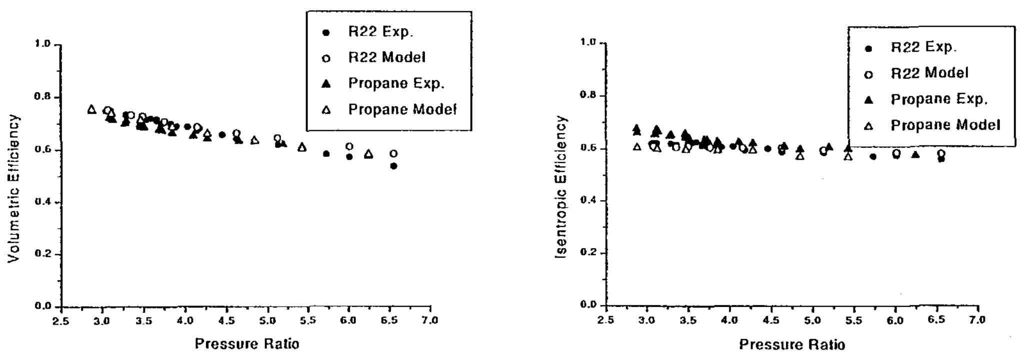

Figure 4: Comparison of the model vs. measurements

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