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ABSTRACT

The internal leakage has a great effect on the performance of a rotary compressor. In this paper a leakage model as a part of a complete simulation model is developed. The leakage model analyzes the oil/refrigerant mixture leakage flow through the roller radial clearance caused by the motion of roller relative to cylinder wall and the pressure difference and considers the effect of the motion of roller and vane on the leakage from oil sump and the solubility of refrigerant to oil. The variations of the leakage loss and the roller radial clearance with the rotating speed are shown. The influence of the tolerance of roller face clearance on the range of performance of compressors mass-produced is discussed.

NOMENCLATURE

e	eccentricity	μ	dynamic viscosity	d	discharge
H	height	v	kinematic viscosity	l	compression chamber
L	length	ρ	density	r	roller
R	radius	ω	angular speed	ri	insider surface of roller
U	velocity			ro	outer surface of roller
V	volume		Subscripts	s	shaft, suction
δ	clearance	b	bearing	t	suction chamber
θ	rotational angle of shaft	c	cylinder	v	vane

INTRODUCTION

The leakage has a great effect on the performance of a rotary compressor due to its structure characteristic. For an inverter rotary compressor or a R410A rotary compressor, the effect is greater. The leakage mechanism, calculation of leakage flow rates and effect on compressor performance of leakage have being an interesting problem. Pandeya assumed that refrigerant gas leaked through the roller radial clearance and the clearances between vane end faces and cylinder head and applied a flow model of convergent-divergent nozzle to the calculation of leakage [1]. Yanagisawa and Shimizu took account of the flow characteristic of oil containing refrigerant through the roller face clearances and the distribution of the clearances [2]. With respect to the leakage through the roller radial clearance, Yanagisawa and Shimizu also assumed that the refrigerant gas leaked through the clearance but considered the friction effect of leakage gas and the dynamic behavior of the clearance [3]. Coste presented a one-dimensional model to evaluate the mass flow rate of oil through the clearance[4]. In order to calculate the leakage flow rate of oil through the larger clearance, Ferrira solved Navier-stokes equations in a bicylindrical coordinate system using the finite volume method[5]. Gache improved the approach by considering the tangential velocity of roller[6]. Rodgers compared the liquid oil/refrigerant mixture leakage model with the gaseous refrigerant leakage model[7]. Fukuta observed the oil film condition in a cylinder of acrylic resin by visualization technique[8].

In this paper, a leakage model as part of a complete computer simulation model is developed in order to select proper major dimensions and clearances to improve efficiency and reliability of compressor. For the leakage of oil/refrigerant mixture from oil sump through the roller axial gaps and the vane side gaps the model analyzes the effects of motions of roller relative to cylinder heads and vane relative to slot as well as pressure differences. Evaluation of the solubility of refrigerant to the oil within sump and the oil having leaked into low-pressure chamber is discussed. With respect to the leakage through roller radial clearance, the Poiseuille flow due to pressure difference is calculated by introducing the effective sealing length of the clearance and the mass flow rate of leakage caused by the relative motion between outer surface of roller and cylinder wall, especially that by the motion of the closest point, is analyzed. The change of roller radial clearance caused by assembly and dynamic behavior of bearing is considered. A Fanno flow model in a channel is applied to the leakage through the axial gaps between vane and bearing end walls. The appropriation of the leakage model presented is indirectly proved. The effect of rotational speed on the minimum of roller radial clearance and the variation of capacity loss by leakage with rotational speed are shown. The effect of the tolerance of roller axial clearance on the consistency of capacities of compressors mass-produced is discussed.

LEAKAGE MODEL

Leakage of oil/refrigerant mixture from oil sump into compression chamber and suction chamber

Because the saturation solubility of refrigerant to oil decreases as the pressure drops, the refrigerant dissolved maybe volatilizes to make the leakage flow to become two phase flow when the oil containing dissolved refrigerant at the bottom of shell chamber leaks through the clearances on roller faces and vane sides. Nevertheless, the leakage flows are assumed as the one-dimensional incompressible viscous laminar flow between parallel plates, one of which is in motion, because the change of the apparent kinematics viscosity is small in the course of leakage [2]. In addition, it is assumed that the mass flow rate caused by pressure difference and that by wall motion of leakage path could be added linearly. The mass flow rate of the flow model is given by:

$$\dot{m} = \frac{1}{2}\left(\frac{\delta^2 \Delta P}{6\nu L} + U\rho\right) H_c \delta \tag{1}$$

Apply it to the specific leakage flow, the leakage mass flow rates of oil/refrigerant mixtures into the compression chamber and to the suction chamber through the vane side clearances and the clearances on roller faces are expressed by:

$$\dot{m}_{lvc} = \frac{1}{2} H_c \delta_{vcl}\left[\frac{\delta_{vcl}^2 (P_d - P_l)}{6\nu L_{vs}} + \rho e \omega_s \sin\theta\right]$$

$$\dot{m}_{tvc} = \frac{1}{2} H_c \delta_{vct} [\frac{\delta_{vct}^2 (P_d - P_t)}{6 \nu L_{vs}} + \rho e \omega_s \sin\theta]$$

$$\dot{m}_{lrb} = \frac{(2\pi - \theta)(\delta_{rbs}^3 + \delta_{rbu}^3)(P_d - P_l)}{12 \nu \ln(R_{ro} / R_{ri})} + \frac{1}{2} \rho \delta_{rb} \frac{\omega_s}{H_c} \frac{dV_l}{d\theta} \tag{2}$$

$$\dot{m}_{trb} = \frac{\theta(\delta_{rbs}^3 + \delta_{rbu}^3)(P_d - P_t)}{12 \nu \ln(R_{ro} / R_{ri})} + \frac{1}{2} \rho \delta_{rb} \frac{\omega_s}{H_c} \frac{dV_t}{d\theta}$$

in which L_{vs} is the effective sealing length of vane sides. The clearances on vane discharge side and suction side δ_{vcl} and δ_{vct} are taken as $\delta_{vc}/3$ and $2\delta_{vc}/3$ respectively. The first terms of above four formulae represent the mass flow rates caused by pressure difference, while the second terms represent those caused by the relative motion between vane and slot or between roller end faces and bearing end faces.

How to select the solubility of refrigerant to oil influences the calculated value of indirect leakage rate of refrigerant greatly. If the discharge temperature and the oil sump temperature of a rotary compressor are 110℃ and 92℃ respectively, the saturation solubility corresponding to the oil sump temperature and discharge pressure of 2.1 MPa is about 21 % whereas that to the discharge temperature is 11 %. Many researchers took the former as the practical solubility of refrigerant to oil within oil sump[2,7,9]. In this paper, it is assumed that when the compressor operates, the oil separated from the refrigerant discharged has not time enough to absorb a great deal refrigerant vapor to reach a new saturation state before the oil is repeatedly supplied to the inside of the cylinder. And the practical solubility in oil sump is evaluated by:

$$S_{os} = C_{sos} S(P_d, T_{os}) + (1 - C_{sos}) S(P_d, T_d) \tag{3}$$

in which S is the saturation solubility as a function of pressure and temperature. In addition it is assumed that a new saturation state is not built up and the liquid oil/refrigerant mixture is supersaturated after it is leaked into the lower pressure chamber. Its solubility is estimated by:

$$S_l = S_{os} - C_{spl} [S_{os} - S(P, T)] \tag{4}$$

in which P and T are the pressure and temperature at the outlet of leakage flow under calculation.

So the mass flow rate of the indirect leakage refrigerant and the liquid part of leakage are obtained by:

$$\dot{m}_{ll} = \begin{cases} \frac{1 - S_{os}}{1 - S_l} \dot{m}_l, & \dot{m}_l > 0 \quad and \quad S_l < S_{os}; \\ \dot{m}_l, & \dot{m}_l < 0 \quad or \quad S_l > S_{os}. \end{cases}, \qquad \dot{m}_{lg} = \dot{m}_l - \dot{m}_{ll} \tag{5}$$

Leakage through the radial clearance between roller and wall

In recent years the flow visualization has shown that the roller radial clearance is filled with oil containing dissolved refrigerant and the liquid oil/refrigerant mixture serves as sealant to prevent refrigerant vapor from leaking directly through the clearance from compression chamber to suction chamber[4,8]. The leakage flow is assumed as an incompressible viscous laminar flow and it is assumed that the Poiseuille flow due to pressure difference and the Couette flow duo to the motion of wall could be added linearly.

Considering that the change of the channel height in the neighborhood of the closest point between roller outer surface and cylinder wall, where a majority of viscous drag comes into being, is small, the pressure difference on the element is expressed as follows:

$$dp = \frac{12\mu V}{h_c^2} dl = 12 \frac{\dot{m}_1 \nu}{H_c} \frac{1}{h_c^3} dl \tag{6}$$

By introducing the effective sealing length of the leakage path Le and considering the kinematic viscosity could be treated as a constant [2], the pressure difference between two chambers on the leakage flow is given:

$$\Delta P = 12 \frac{\dot{m}_1 \nu L_e}{H_c \delta_{rc}^3} \tag{7a}$$

$$L_e = \delta_{rc}^3 \int \frac{dl}{h_c^3} = \frac{R\delta_{rc}^3}{e^3} \int_{\pi - \Delta\varphi/2}^{\pi + \Delta\varphi/2} \frac{1}{(1+\varepsilon\cos\varphi)^3} d\varphi$$

$$= R \frac{(1-\varepsilon^2)^{\frac{1}{2}}}{(1+\varepsilon)^3} [(2\pi - 2\gamma)(1 + \frac{1}{2}\varepsilon^2) + 4\varepsilon \sin\gamma - \frac{1}{2}\varepsilon^2 \sin(2\gamma)]$$

$$R = \frac{R_c + R_{ro}}{2}, \qquad \varepsilon = 1 - \frac{\delta_{rc}}{e}, \qquad \gamma = \cos^{-1} \frac{\varepsilon - \cos(\Delta\varphi/2)}{1 - \varepsilon\cos(\Delta\varphi/2)}$$

in which $\Delta\varphi$ is the angle range of the clearance sealed by oil film. If the leakage flow velocity is larger, the above equation is improved as follows:

$$\Delta P = \frac{1}{2}\rho (\frac{\dot{m}_1}{H_c \delta_{rc} \rho})^2 + 12 \frac{\dot{m}_1 \nu L_e}{H_c \delta_{rc}^3} \tag{7b}$$

As the crankshaft rotates the closest point between roller outer surface and cylinder wall, which separates the compression chamber from the suction chamber, shifts along the cylinder wall. The oil film adhered to walls of roller and cylinder passes the closest point, the boundary between the compression chamber and the suction chamber, to leak from the compression chamber to the suction chamber. The mass flow rate of oil/refrigerant mixture leaked duo to the motion of the control volume boundary is

$$\dot{m}_{21} = \rho (R_c \omega_s) H_c \delta_{rc} \tag{8}$$

The motion of outer surface of roller relative to the boundary brings about oil film to flow from the suction chamber to the compression chamber. The corresponding mass flow rate of the oil/refrigerant mixture leakage:

$$\dot{m}_{22} = -\frac{1}{2}\rho (e\omega_s + R_{ro}\omega_r) H_c \delta_{rc} \tag{9}$$

So the total mass flow rate of leakage caused by the relative motion between outer surface of roller and cylinder wall is given by:

$$\dot{m}_2 = \rho[(R_{ro} + \frac{1}{2}e)\omega_s - \frac{1}{2}R_{ro}\omega_r] H_c \delta_{rc} \tag{10}$$

The total mass flow rate of oil/refrigerant mixture leakage through the roller radial clearance is:

$$\dot{m}_{rc} = \dot{m}_1 + \dot{m}_2 \tag{11}$$

Most oil/refrigerant mixture leaked through the roller radial clearance comes from the leakage from oil sump and compression chamber during last revolution. The solubility of refrigerant to oil depends on the pressure in compression chamber. By the solubility and the solubility after leakage the mass flow rate of the indirect leakage refrigerant and the liquid part through the radial clearance are obtained by:

$$\dot{m}_{rcg} = \frac{S_{lrc} - S_{trc}}{1 - S_{trc}} \dot{m}_{rc} \qquad \dot{m}_{rcl} = \dot{m}_{rc} - \dot{m}_{rcg} \tag{12}$$

During the operation of the rotary compressor the roller radial clearance changes because of the dynamic behavior of main bearing and eccentric bearing and the eccentricity between cylinder and bearings. By the eccentricities $c_{mb}\varepsilon_{mb}, c_{eb}\varepsilon_{eb}$ and the eccentric directions δ_{mb}, δ_{eb} of the two bearing obtained from the dynamics simulation of the rotary compressor[10], the instantaneous value of the radial clearance is given by:

$$\delta_{rc} = \delta_0 + c_{mb}\varepsilon_{mb}\cos(\theta - \delta_{mb}) - c_{eb}\varepsilon_{eb}\cos(\theta - \delta_{eb}) - \delta_a \cos(\theta - \theta_a) \qquad (13)$$

in which δ_0 is the concentric radial clearance and θ_a and δ_a are the eccentric direction angle and eccentricity of main bearing relative to cylinder respectively.

Leakage through the clearance between vane end faces and bearing end faces

Unlike above leakage paths the leakage clearance is not effectively sealed by oil film. The leakage flow through the clearance is assumed as a Fanno flow of refrigerant containing a little oil. The mass ratio of oil to refrigerant vapor is proportional to that in compression chamber. The velocity and mass rate of leakage flow is obtained by the equations of Fanno flow.

The above leakage models developed as a part of a complete simulation model of a rotary compressor need be coupled to the differential equations of energy conservation and mass conservation of control volume and the other models. The calculating accuracy of the effect of leakage on the performance of compressor is related to the equations and models. The leaked refrigerant and oil at high temperature heat not only the refrigerant within low-pressure chamber but also oil within low-pressure chamber. When a rotary compressor operates at low speed, the heating effect of leakage influences greatly the performance of compressor. Therefore, the differential equation of energy conservation of control volume needs to include the term of internal energy of oil within control volume and the mass conservation of oil is required [11].

RESLUTS AND DISSCUSIONS

It is impossible to verify the leakage model proposed directly. In this paper the appropriation of the leakage model proposed is proved indirectly by the comparison of calculated results of cooling capacity and power with measured data with various dimensions, clearances, discharge pressures and rotating speeds. Table 1 shows the measured and calculated results of cooling capacity and power of seven rotary compressors with various dimensions. The clearances within these compressors are very different. Table 2 shows those of the Compressor II among them operating at various discharge pressures. The measured and calculated results of performance of the Compressor I, a single cylinder inverter compressor, and the Compressor V, a dual cylinder inverter compressor, operating at various rotating speeds are shown in the related paper[11]. The differences between the measured and calculated results are small. Most relative errors is less than 2.5%.

Ferreira performed precise measurements of the lubrication oil flow through the radial clearance between stationary roller and cylinder wall [5]. In this paper the oil flow rate caused by pressure difference is calculated by Equation (7) using the dimensions and oil properties given out by Ferreira. When the minimal clearance is 80 μ m, the relative errors between the results calculated and the experimental data given out are close to 8%. However, as the clearance is 47 μ m and the pressure difference is greater than $0.3MP_a$, the relative errors are less than 2%. However, the practical clearance of rotary compressors is smaller and unable be 80 μ m.

Besides the leakage created by pressure difference, that by relative motion between the parts constituting leakage path walls is also considered. Table 3 shows the flow velocity components of oil/refrigerant mixture leakage by pressure difference and by relative motion between parts through the roller axial and radial clearances within the Compressor I operating at the shaft rotating angle of 200° , when the pressure in the compression chamber is close to the discharge pressure. It is seen that the flow rate component through the roller radial clearance caused by the relative motion between roller and cylinder wall is comparable with that caused by the pressure difference at 60HZ but is larger at higher rotating speed. For the leakage through the roller axial clearances, the leakage flow rates by the motion of roller are smaller than those by the pressure difference but should not be negligible.

Figure 1 shows the variation of roller radial clearance with the rotating angle of crankshaft at various operating frequencies. It is seen that the minimum of radial clearance decreases as the rotating speed rises up. So the larger radial clearance is required for an inverter controlled rotary compressor. This is able to aggravate the leakage loss through the radial clearance as the compressor operates at low speed.

Figure 2 shows the variation of the volumetric efficiency loss caused by leakage with the operating frequency. The roller axial, vane axial and vane side clearances are taken as 12 μ , 14 μ and 29 μ respectively. The roller radial clearance is taken as shown Figure 1. It is seen that the leakages through the roller radial clearance, roller face clearance and the vane axial clearance produce significant impacts on the performance of compressor while the effect of the leakage through the vane side clearances is small on condition that they are sealed by oil film. If a smaller magnitude of roller radial clearance were selected, the impact of the leakage through the clearance would decrease but would be great still. In addition, as the rotating speed rises the effects of the leakages decrease gradually whereas as the speed falls the losses increase quickly.

Figure 3 and Figure 4 show the variation of cooling capacity, power, volumetric efficiency and COP of the Compressor I with the clearance on roller faces. The performance of compressor descents to a great extent as the clearance enlarges. Generally, the leakage of oil/refrigerant mixture through the clearance into the compression chamber enhances power. The reason of the power declines with the clearance enlarging from 6 μ to 22 μ is that the leakage elevates the temperature of refrigerant vapor and produces the power loss caused by the heating of cylinder wall on refrigerant vapor to reduce. When the compressors are commercially manufactured, the consistency of performance is an important problem. A proper simulation model may be applied to analyze the range of performance produced by the tolerance of clearance or manufacturing tolerance and grouping of parts. Figure 5 shows that the variation of the range of capacity and COP with the tolerance of the clearance on roller faces with the lower limit of the clearance is 8 μ . When the clearance tolerance is 10 μ , that is, its upper limit and lower limit are 18 μ and 8 μ , the amplitude of capacity variation is 2.6%. If it is expected that the variation of capacity caused by the clearance tolerance is limited within 1%, the clearance tolerance should be taken as 6 μ .

CONCLUSIONS

A leakage model as a part of a complete computer simulation model is developed and verified. The results calculated by the model show:

1. The leakages through the roller radial, roller axial and vane axial clearances have great impacts on the

performance of compressor. As the rotating speed rises the effects of leakage decrease gradually whereas as the speed falls the losses increase quickly.

2. The oil leakage flow rate through the roller radial clearance caused by the motion roller relative to cylinder wall, which produces the motion of boundary point and the relative sliding at the point, is comparable with that caused by the pressure difference. The accuracy of the simple model for calculating the oil leakage flow rate through the clearance caused by the pressure difference is satisfactory. The oil leakage rate through vane side and roller face clearances by the motion of vane or roller should not be negligible
3. The performance of compressor descents to a great extent as the roller face clearance enlarges. The clearance tolerance has a great influence on the variation of capacity and COP of compressors mass-manufactured and need be controlled accurately.

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Table 1 Comparison of results calculated with data measured for various compressors

Comp.	Dc (mm)	e (mm)	Vh (cm^3)	Qmea (w)	Qcal (w)	Nemea (w)	Necal (w)
I	44	4.2	13.1	2080	2112	762	761
II	44	4.7	16.8	2758	2750	943	967
III	54	5.55	21.1	3546	3544	1297	1308
IV	54	5.1	21.4	3558	3554	1220	1213
V	54	3.6	10.55×2	3342	3372	1142	1131
VI	57.2	3.74	16.0	2694	2730	929	935
VII	57.2	5.7	23.5	3952	4010	1372	1383

Table 3 Leakage flow velocity (m/s)

operating frequency	Through roller axial clearance to suction chamber			through roller radial clearance		
	pressure	motion	total	pressure	motion	total
30	0.47	0.18	0.65	5.74	2.72	8.46
60	0.47	0.43	0.90	6.32	6.41	12.7
90	0.46	0.67	1.13	6.81	10.2	17.0

Table 2 Comparison of results at various pressure

t_k (℃)	t_3 (℃)	Q_{mea} (w)	Q_{cal} (w)	N_{emea} (w)	N_{ecal} (w)
45	40	2778	2789	783	792
55	50	2467	2441	942	972
65	60	2041	2068	1089	1129

in which t_k is condensing temperature, t_3 is liquid temperature. Evaporating temperature is 7.2℃, suction temperature is 35℃

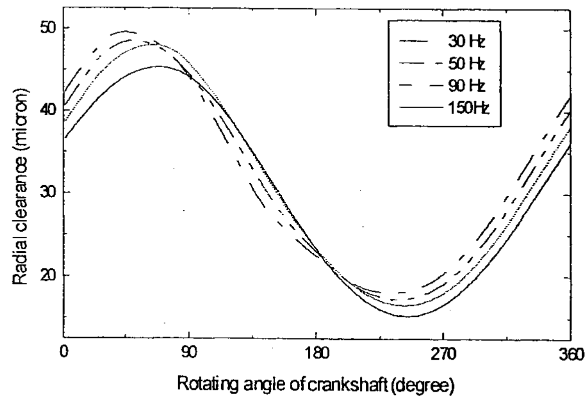

Fig 1 Roller radial clearances at various speeds

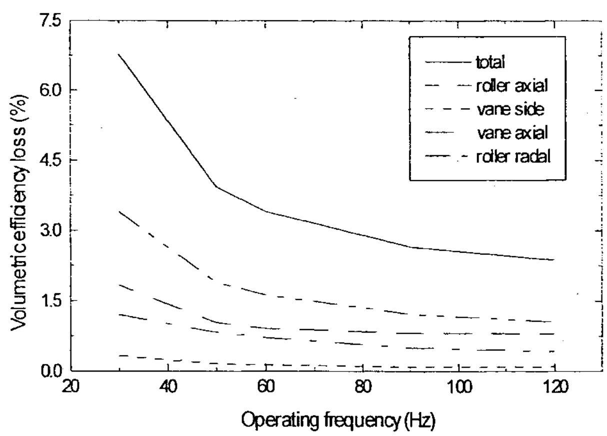

Fig 2 Volumetric efficiency loss due to leakage

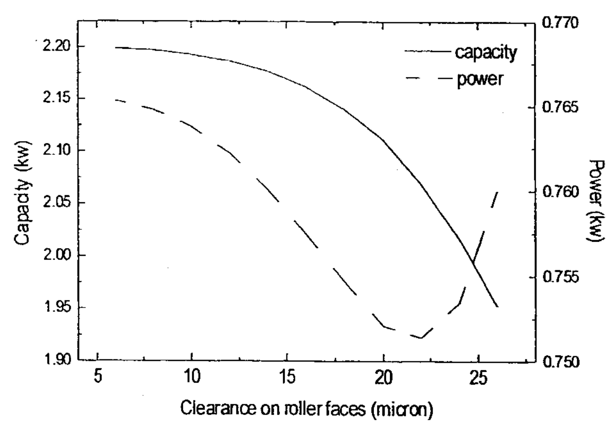

Fig 3 Variation of Q_0 and Ne with roller axial clearance

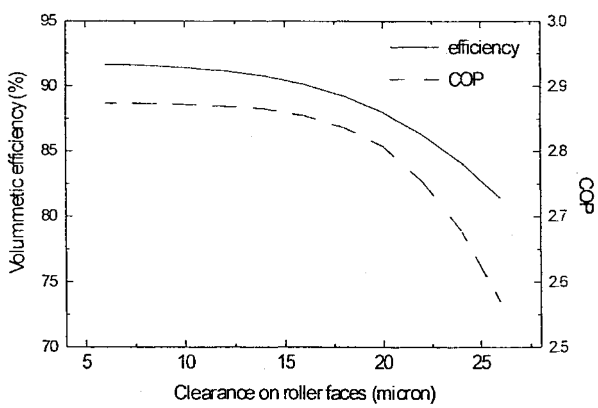

Fig 4 Variation of η_v and COP with roller axial clearance

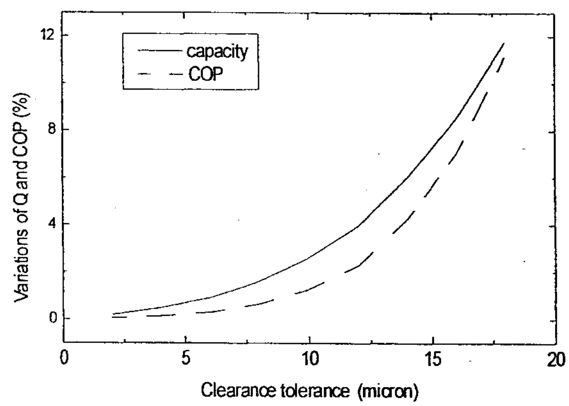

Fig 5 Effect of clearance tolerance on performance