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NONLINEAR VIBRATION ANALYSIS OF REED VALVES

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ABSTRACT

The present paper deals with a nonlinear valve modeling and uses the assumed-modes method to simulate the dynamic behavior of reed valves. To incorporate the geometric nonlinearity due to large deflection of reed valves into the valve dynamic model, the large-deflection strain-displacement relation is considered. As example of use, a parametric analysis of an actual suction valve is presented and the discrepancy between the linear and nonlinear models is numerical compared. The simulation results show that the nonlinear strain term plays an important role on the dynamic behavior of reed valve systems if the thickness of the valve is small.

NOMENCLATURE

\( A(x) \) cross-sectional area of the valve
\( A_e \) effective force area
\( A_v \) flow area of the suction valve, \( \pi dh \)
\( C_r \) reflecting coefficient of the valve
\( E \) Young's modulus of the valve
\( f(t) \) generalized force
\( I(x) \) moment of inertia of the valve cross-sectional area
\( I \) length of the valve
\( I_1 \) distance indicated in Fig. 2
\( p \) pressure inside the cylinder
\( p_s \) pressure in the suction chamber
\( q(t) \) time-dependent generalized coordinate
\( r_s \) radius of the suction orifice
\( R \) gas constant
\( t \) time variable
\( T \) kinetic energy of the valve
\( T_s \) suction temperature
\( u, w \) translations of neutral axis of the valve in \( x \)- and \( z \)-directions respectively
\( u_x, u_z \) translations of the valve in \( x \)- and \( z \)-directions respectively
\( U \) strain energy of the valve
\( V \) instantaneous volume of the cylinder
\( W \) external work done by the pressure difference
\( x, y, z \) reference coordinate of the valve
\( \alpha \) valve flow coefficient
\( \beta \) pushing force coefficient of the valve
\( \varepsilon_{xx} \) longitudinal strain
\( \phi(x) \) admissible function of the cantilever beam
\( \eta \) coefficient of the nonlinear term defined in Eq. (12)
\( \kappa \) isentropic exponent
\( \rho \) density of the valve
\( \omega_s \) natural frequency of the valve
\( \xi \) damping ratio
\( (\cdot) \) differentiation with respect to time
\( (') \) differentiation with respect to \( x \)

INTRODUCTION

Reed valves in many hermetic reciprocating compressors are usually acted automatically by pressure differences across the valve. They are the most important element for controlling suction and discharge gas flow. When the valve motion is abnormal, compressor efficiency drops dramatically. The highly unsteady characteristic of the reed valves' vibration exacts a proper design in order to achieve higher standards of compressor efficiency, noise, and reliability. Generally, too stiff of the reed valve causes over-compression and too flexible causes unnecessary fluctuation of the reed valve.

The reed valve models currently employed usually consider a linear (small amplitude) model [1-3]. Linearity is a conceptual ideal which is never completely realized in the dynamic behavior of a real structure. Although it is
known that linearized equations provide no more a first approximation of an actual situation, they are sufficient for many practical and engineering purposes. For many phenomena, predictions based on linear models are qualitatively correct and only slightly wrong, quantitatively. This is particularly true for the small amplitudes of motion involved in many vibration problems. Linearized theory is inadequate, however, if the vibration of a reed valve involves amplitude that is not very small, as is assumed in linear theory. Since the displacement of valve is several times to the thickness of valve for typical reciprocating compressors, nonlinear theory should be used to obtain the more accurate results.

The present paper considers the large-deflection strain-displacement relation [4] to capture the geometric nonlinearity due to large deflection of reed valves. The assumed-modes method [5] is used to reasonably simplify the governing equation of motion. By means of the Runge-Kutta method [6], the dynamic displacement of a typical suction valve is simulated. Numerical results show that the nonlinear strain term has a considerable influence on the dynamic behavior of the reed valve systems.

**MATHEMATICAL FORMULATION**

The Lagrangian approach is used to derive the governing equation of motion for a cantilever type valve. The kinetic energy $T$, potential energy $U$, and the work of external load $W$ are written in terms of a time-dependent generalized coordinates with subsequent application of Lagrange's equation.

A schematic representation of a hermetic reciprocating compressor is shown in Fig. 1. Figure 2 depicts the geometry of the suction reed valve. According to the geometry of the valve shown in Fig. 2, the suction reed valve can be regarded as a cantilever beam and reasonably modeled by the Bernoulli-Euler beam theory. Under Kirchhoff’s hypothesis [4] and referring to the Nomenclature, the displacement of the valve can be written as

$$u_x = u(x,t) - z \frac{\partial w(x,t)}{\partial x}, \quad u_z = w(x,t).$$

Since the valve is assumed to be free to move axially, the effect of the midplane stretching strains can be considered approximately to be zero. Assuming that the transverse displacement is not small compared to the thickness of the valve, the strain components can be computed from the large-deflection strain-displacement relation [4] as

$$\varepsilon_{ex} = -2 \frac{\partial^2 w}{\partial x^2} + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2,$$

where the second term in the right hand side is nonlinear.

Expressions for the kinetic energy $T$, potential energy $U$, and work of external load $W$ are given in terms of the transverse displacement $w$ as follows:

$$T = \frac{1}{2} \int pA(x) \dot{w}^2 dx,$$

$$U = \frac{1}{2} \int EI(x) w^2 dx + \frac{1}{8} \int E A(x) w^4 dx,$$

$$W = \int \beta A_e (p_r - p) w \delta(x - l_r) dx,$$

where $\beta$ is the pushing force coefficient of the valve, $A_e = \pi r_c^2 + \{w(l_r)^2\}$ is the effective force area, $p$ is the pressure field inside the cylinder, and $\delta(x - l_r)$ is the Dirac $\delta$-function. Generally, the term $\beta$ has to be obtained by experiment. The assumptions and derivation of the pressure field inside the cylinder have been described elsewhere [7], therefore they will not be shown in detail here. The pressure field inside the cylinder can be expressed as

$$\frac{dp}{dt} = \frac{\alpha A_e k_p}{V} \left( \frac{p}{p_r} \right)^\frac{\gamma}{\kappa} \sqrt{\frac{2kRT}{\kappa - 1} \left[ 1 - \left( \frac{p}{p_r} \right)^\frac{\kappa - 1}{\kappa} \right]} \frac{k_p dV}{V dt},$$
where the valve flow coefficient $\alpha$ is determined from experiment and is a function of the displacement of the valve.

According to the assumed-modes method, the displacement, $w$, of the valve at any point $x$ can be expressed as

$$w(x,t) = \phi(x)q(t),$$

(8)

where $\phi(x)$ is the admissible function (or mode shape of the reed valve) satisfying the boundary conditions and $q(t)$ is the time-dependent generalized coordinate. There exists infinite number of combinations of mode shapes, but in many cases especially for the reed valve, the first mode consideration alone may be sufficient. From the boundary conditions of the cantilever beam with a length $l$,

$$\phi(x) = x^4 - 4lx^3 + 6l^2x^2.$$  

(9)

Substituting the expression for $w$ in the energy expressions, applying Lagrange’s equation [6], and adding the damping effect yields the governing motion equation of the reed valve

$$\ddot{q} + 2\xi \omega_n \dot{q} + \omega_n^2 q + \eta q^3 = f(t),$$

(10)

where

$$\omega_n^2 = k/m, \quad \eta = h/m, \quad f(t) = f^*(t)/m,$$

(11-13)

and

$$m = \int \rho A(x)\phi^2 dx, \quad k = \int EI(x)\phi^{*2} dx,$$

(14, 15)

$$h = \frac{1}{2} \int EA(x)\phi^{*2} dx, \quad f^*(t) = \pi \beta \phi(l_t)(p_s - p)(r_t^2 + [\phi(l_t)]^2 q^2).$$

(16, 17)

The equation of motion (10) is no longer valid when the valve impacts against the seat. The impact between valves and seats is assumed to occur over a period short enough to assume that the valves do not vary their position and only their velocity are affected by the collision. Seats are considered as rigid since the stiffness of the seats is much higher than that of the valves. The velocities of the valve before and after impacts against the seat can be described by the following equation.

$$\dot{q}_{\text{after}} = -C_s \dot{q}_{\text{before}}.$$  

(18)

The displacement of valve can be simulated by simultaneously solving Equations (7) and (10) with appreciate initial conditions via the Runge-Kutta method.

RESULTS AND DISCUSSIONS

Before the valve model was incorporated with the compressor model to predict the valve flow characteristics, the accuracy of natural frequency of the reed valve was checked against a finite element result. This procedure was required because that the present analysis employs only the fundamental mode shape. For a typical suction valve, good agreements were obtained from two models on the natural frequencies of the valve fundamental mode ($\omega_n = 260.5$ Hz obtained from ANSYS, and $\omega_n = 271.0$ Hz calculated by the assumed-modes method).

The results presented hereafter were obtained using the parameters relative to a small hermetic reciprocating compressor for R134a. Figure 3 presents a comparison between the results obtained from the linear and nonlinear models with different thickness of the valve. The displacements of the valve evaluated through the nonlinear model in Fig. 3 are higher than those obtained through the linear model. This simulation result shows that the nonlinearity is of the hardening type. Another observation in Fig. 3 is that the nonlinear effect is pronounced when the thickness of the valve decreases. This result can be expected since the linear strain component in Eq. (3) decreases, while the nonlinear one remains constant although it is a high order term, with the decreasing of the thickness. In addition, the bending rigidity of the valve diminishes with the decreasing valve thickness; therefore, the displacement of the valve could be several times to its thickness, which enhances the significance of the geometric nonlinearity. Consequently, it can be concluded that the nonlinear strain term plays an important role and should be retained if the thickness of the valve is small.

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CONCLUSIONS

In this work, the impact of the geometric nonlinearity on the dynamic behavior of the reed valves has been studied. The present paper investigates the geometric nonlinearity of the valve by employing the large-deflection strain-displacement relation. The simulation results illustrate that the nonlinear strain component is prominent and should not be omitted when the thickness of valves is small.

The present modeling studies the reed valve as a cantilever beam which is the case of some suction valves without stop. Nevertheless, with some additional efforts, it could be implemented to other more complex arrangement.

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REFERENCES


Fig. 1 Scheme of a hermetic reciprocating compressor.  
Fig. 2 Geometry of the suction reed valve.
Fig. 3 Valve motion during suction process. (a) thickness = 0.5 mm; (b) thickness = 0.4 mm; (c) thickness = 0.3 mm; (d) thickness = 0.2 mm. (--- linear; ------ nonlinear)