SOME SHORT-CUTS ON HORIZONTAL AND VERTICAL CURVES

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The center line of a highway is a series of straight lines in the horizontal and the vertical planes, connected together by curves. These curves may be simple, compound, or reversed. The curve may have a uniform rate of change or a variable rate of change.

Horizontal Curves

The horizontal curves used are usually designated by the degree values, designated D. This D, the degree of the curve, may be defined in two different ways. If we use the arc definition, then we say that D is the angle at the center which is subtended by an arc of 100 feet. Or we may use the chord definition, then we say that D is the angle at the center which is subtended by a chord of 100 feet.

The radius of the curve is called R, and the value of this R will be slightly different for the two definitions. If we are using the arc definition, we have this direct relation in proportion:

$$100 : 2\pi R :: D : 360$$

$$R = \frac{5729.58}{D}$$

For $D = 1^\circ$ $\qquad R = 5729.58'$

If we are using chord definition, then we have this exact mathematical condition

$$R = \frac{50}{\sin D/2}$$

For $D = 1^\circ$ $\qquad R = 5729.65'$

The difference in R is small and negligible for flat curves of small D value, but it is appreciable for sharp curves of large D value.

L represents the length of the curve; but do you mean the exact length of the circular arc, or a certain number of chord measurements? If the curve is flat, the chord and the arc will be close together and there is no appreciable error in the use of either; but if the curve is sharp, then there is a considerable error in the use of the chord measurement, unless the length of the chord used is reduced so that the measured chord does not differ from the arc by an amount sufficient to cause error in the layout work.

The following chord lengths may be used without appreciable error from arc measurements:

If D lies between 0°— 5°, use 100′ chords.
If D lies between 5°—14°, use 50′ chords.
If D lies between 14°—25°, use 25′ chords.
If D used is 25° and up, use 10′ chords.

Should you desire to compute mathematically the functions of the curve, the calculations should be based upon the length of R, where R has these values:

$$\text{For } 100' \text{ chords} \quad R = \frac{50}{\sin D/2}$$

$$\text{For } 50' \text{ chords} \quad R = \frac{25}{\sin D/4}$$

$$\text{For } 25' \text{ chords} \quad R = \frac{12.5}{\sin D/8}$$

$$\text{For } 10' \text{ chords} \quad R = \frac{5}{\sin D/20}$$

Using these values of R, the line values in the curve are computed. The R used must be the one for the chord length to be used in the field, and all curve functions computed from that R.

I, the deflection angle; measured in field, or computed from field measurements.
D, the degree of curve; assumed, or computed.
R, the radius; computed, based on chord to be used.
L, the length of curve; $100 \times I/D$, or $I \times R \times \text{Arc } 1°$.
T, the tangent length; R tan I/2, true for all curves.

Setting up the curve data for use in the field, we know that the deflection angle, from the tangent to the curve at the P.C. measured to the station stake being set, is one-half the angle at the center that lies between the radius lines, one to the P.C. and one to the point to be set. For setting the P.T. stake this will give a deflection of I/2, which is the check angle for curve setting using a transit.

However, if the center line is being stationed with respect to the beginning of the line as Station 0+00, then hardly ever will a curve begin or end on a regular +00 station; but the curve will begin with and end with a subchord, a length less than the regular chord for the curve. The small deflection angle for the first point on the curve can be easily determined by the direct ratio

d/2, the small deflection angle,
D/2, unit deflection angle for a 100′ chord.
l, any length less than a 100′ chord,
L, a 100′ chord.

$$d/2 \; : \; D/2 \; :: 1 \; : \; L$$

$$\frac{d}{2} = \frac{1D}{2L} \text{ in degrees;}$$

but the subangle is usually small; reduce this d/2 to degrees by multiplying by 60 minutes,

$$\frac{d}{2} = \frac{1 \times 60 \times D}{200} = 1 \times 0.3 \times D, \text{ in minutes}$$

and this product will give the deflection angle for any length of chord in minutes of angle.

Vertical Curves

The curve that connects the straight lines in the vertical alignment is a parabola and the calculations are based upon the properties of the parabola. We know that the curve of the parabola must pass through a point which is half way between the point of intersection of the tangent lines and the chord connecting the P.C. and the P.T. of the vertical curve. This vertical distance we designate as e, the middle offset. The curve is symmetrical with respect to the center. Another function of the parabola is that the offset at any point, from the tangent line to the curve, is equal to the square of the distance from the P.C. or the P.T. to the point under consideration, divided by the square of the half-length of the curve, and this number multiplied by the middle offset.

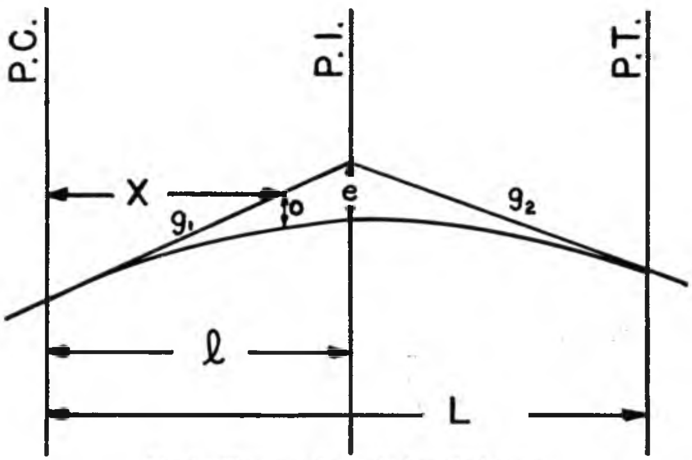

Fig. 1. Diagram of a typical vertical curve.

The length of the curve in stations, designated L, is a function of the rates of grade and the sight distance;

$$L = \frac{1^2 G}{8h}, \text{ in stations}$$

l = sight distance in stations,
G = total change in grade in percent,
h = height of line of sight above roadway at the point of observation in feet.

In the vertical alignment we are concerned only with the horizontal and vertical distances. If the P.C. and the P.T. points are assumed as L stations of distance apart, with the P.I. point in the middle position, we have an infinite number of possible vertical curves, for the P.C., P.I., P.T. points may take any position whatsoever along the vertical lines. But the curve connecting the P.C. and the P.T. is always a parabola.

The middle offset e may be determined in several different ways. Using known grade percents and known station elevations for the three control points, e is determined by numerical application of these values. However, we do not need to know these elevation values to find e, for e is a function of the total change in grade and the length of the curve;

$$e = \frac{(g_1 - g_2) L}{8}$$

Knowing the value of e, and the length L, we can apply the equation of the parabola and determine the amount of the offset at any point,

$$0 = \frac{x^2}{l^2} e$$

o = vertical offset in feet.
x = distance in feet to point under consideration.
l = half length of curve in feet.
e = middle offset in feet.

This value of 0 is added or subtracted to the straight-line elevation of the station point, according to whether the grade curve is above or below the straight-line tangents, to get the elevation of the grade line at the point under consideration.

If the points on the curve are at irregular intervals or chord lengths, then there is no short cut in the reduction of the mathematics of the parabola; but if the conditions are such that the curve is divided into chords of equal lengths, the mathematical work may be shortened by recognizing the relation existing in the progression through the half-length of the curve.

Keep in mind that the value of the work in dollars and cents is the ultimate end; as the price of land and unit costs go up, greater accuracy in calculations is desirable and it is not always advisable to apply short cuts by approximate methods; close, accurate mathematics is required.