SCHOOL OF CIVIL ENGINEERING

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THREE-DIMENSIONAL SLOPE STABILITY ANALYSIS

R. H. Chen

PURDUE UNIVERSITY
HIGHWAY COMMISSION
Final Report

THREE-DIMENSIONAL SLOPE STABILITY ANALYSIS

TO: H. L. Michael, Director
    Joint Highway Research Project

FROM: J.-L. Chameau, Research Associate
      Joint Highway Research Project

September 30, 1981
Project: C-36-36N
File: 6-14-14

Attached is a Final Report on the JHRP study titled "Three-Dimensional Slope Stability Analysis for Indiana Highways". The research and report were performed by Mr. R. H. Chen, Graduate Instructor on our staff, under the direction of J.-L. Chameau and C. W. Lovell of our staff. The report is titled "Three-Dimensional Slope Stability Analysis".

This report presents methods of three-dimensional slope stability analysis using limit equilibrium concepts and the finite element method. Two different computer programs based on the limit equilibrium concept, LEMIX and BLOCK 3, were developed to analyze rotational and translational slides, respectively. In addition a 3-D finite element computer program, FESPON, was also generated to analyze rotational slides. Several slope stability analyses were performed using the three-dimensional programs BLOCK 3, LEMIX, and FESPON, for different slope angles, soil parameters, and pore water conditions. The results obtained with these techniques were compared to those given by conventional two-dimensional methods. The 3-D programs are valuable tools for geotechnical engineers in the evaluation of the stability of highway slopes.

The Report is submitted for review, comment and acceptance as fulfillment of the objectives of the approved Study.

Respectfully submitted,

J.-L. Chameau
Research Associate

JLC:ms

Attachment

cc: A. G. Altschaeffl  M. J. Gutzwiller  C. F. Scholer
     J. M. Bell  G. K. Hallock  R. M. Shanteau
     W. L. Dolch  J. F. McLaughlin  K. C. Sinha
     R. L. Eskew  R. D. Miles  C. A. Venable
     J. D. Fricker  P. L. Owens  L. E. Wood
     G. D. Gibson  B. K. Partridge  E. J. Yoder
     W. H. Goetz  G. T. Satterly  S. R. Yoder
Final Report

THREE-DIMENSIONAL SLOPE STABILITY ANALYSIS

by

Rong-Her Chen
Graduate Instructor in Research

Joint Highway Research Project
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Joint Highway Research Project
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Purdue University
in cooperation with the
Indiana Department of Highways

The contents of this report reflect the views of the author who is responsible for the facts and the accuracy of the data presented herein.

Purdue University
West Lafayette, Indiana
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LIST OF ABBREVIATIONS AND SYMBOLS

Abbreviations

JHRP - Joint Highway Research Project
OCR - Overconsolidation Ratio
OMC - Ordinary Method of Columns
OMS - Ordinary Method of Slices
TRB - Transportation Research Board
1-D - One-Dimensional
2-D - Two-Dimensional
3-D - Three-Dimensional

Special Symbols

A prime ("'") indicates that the variable is in terms of effective stress.
Δ (Delta) indicates a change or variation in a variable.
Σ (Sigma) used as summation symbol.
Cₗ Symbol for Center Line.

Symbols

b - The width of the slice.
c,c' - Strength intercept in terms of total (c) and effective (c') stresses, respectively.
cₘ - Mobilized cohesion.
D - Depth to a weak layer, measured from the toe.
e₀ - Initial void ratio.
$E_i, E_t, E_{ur}$ - Initial, tangent, and unloading modulus, respectively.

$F$ - Factor of safety.

$F_2, F_3$ - Two-dimensional and three-dimensional factor of safety, respectively.

$G$ - Shear modulus.

$G$ - Poisson's ratio parameter, value of $v_i$ at $\sigma_3 = P_a$

$H$ - Height of a slope or embankment.

$I_p$ - Plasticity index.

$K, K_e$ - Loading and unloading modulus numbers.

$K_0$ - Coefficient of earth pressure at rest.

$l$ - Length of the base of a slice.

$l_c$ - Length of half of a central cylinder.

$l_s$ - Length of the minor axis of an ellipsoid.

$L$ - Length of a shear surface.

$M_d$ - The driving moment.

$M_o$ - Moment around the center 0.

$M_r$ - The resisting moment.

$n$ - Modulus exponent, relate $E_i$ and $E_{ur}$ to $\sigma_3$

$N, N'$ - Total and effective normal forces acting on the base, respectively.

$P_a$ - Atmospheric pressure.

$Q$ - Resultant of all side and end shear forces acting on a slice (or a column).

$r$ - Radial distance from the center to the failure surface.

$r_u$ - Pore water pressure parameter

$R$ - End shear force.
R - Radius of a circular failure surface.

$R_f$ - Failure ratio.

$T_a$ - Allowable shear strength.

$u$ - Pore water pressure.

$w$ - Water content.

$w_w$ - Subscript designating weak soil layer.

$W$ - Weight of a mass.

$x$ - Horizontal distance from the slice to the center of rotation.

Greek Alphabet

$\alpha$ - Inclination of shear surface with respect to the horizontal.

$\beta$ - Slope inclination.

$\beta_b$ - Inclination of weak soil layer.

$\rho$ - Density of a soil.

$\theta$ - The parallel inclination of all side forces.

$\sigma_1, \sigma_3$ - Maximum and minimum principal stresses.

$\sigma_v, \sigma_h$ - Vertical and horizontal stresses.

$\sigma_N$ - Normal stress on a selected plane.

$\tau$ - Shear stress.

$\tau_N$ - Shear stress on a selected plane.

$\varepsilon_a, \varepsilon_r, \varepsilon_v$ - Axial, radial and volumetric strains.

$\phi, \phi'$ - Strength angle in terms of total ($\phi$) and effective ($\phi'$) stresses, respectively.
\( \phi_m \) – Mobilized strength angle.

\( \nu \) – Poisson’s ratio.

\( \nu_i, \nu_t \) – Initial and tangent Poisson’s ratio.
General methods of three-dimensional slope stability analysis using limit equilibrium concepts and the finite element method are proposed.

Two different computer programs based on the limit equilibrium concept, LEMIX and BLOCK 3, are developed to analyze rotational and translational slides, respectively. For rotational slides, the failure mass is assumed symmetrical and divided into many vertical columns. The interslice forces are assumed to have the same inclination throughout the mass, and the inter-column shear forces are assumed to be parallel to the base of the column and a function of their positions. Force and moment equilibrium are satisfied for each column as well as for the total mass. For translational slides, the critical failure surface is defined according to Rankine's theory and the factor of safety is assumed to be uniform along the total failure surface. The analysis is illustrated for several slope angles, soil parameters, and pore water conditions. The results show that for both translational and rotational slides, the 3-D effect is more significant for cohesive soils with smaller failure lengths. However, a wedge type of failure may result in a smaller factor of safety than that of the 2-D condition. A gently inclined weak layer with lower strength may cause a higher 3-D effect. In rotational slides, the steeper the slope, the less the 3-D effect. Pore water pressures generally cause the 3-D effect to be even more significant.

In addition, a 3-D finite element computer program FESPON is also developed. It uses a hyperbolic stress-strain relationship and an incremental technique to simulate the nonlinear behavior of soils. Isoparametric incompatible elements are used to provide good bending characteristics. The
program can calculate the local factors of safety at selected points on the failure surface as well as the mean factor of safety for a chosen failure mass. The comparison between the limit equilibrium and finite element methods is also conducted for embankments with the same soil conditions and failure surfaces. The agreement is quite good, with the finite element method predictably yielding higher factors of safety.
I. INTRODUCTION

Gravitational, seepage and surcharge loads tend to cause instability in natural and man-made slopes. Stability analysis is an important part of the design of embankments, cut slopes, excavations, and dams. In practice, limit equilibrium methods are used in the analysis of slope stability. It is considered that failure is occurring along an assumed or a known failure surface. The shear strength required to maintain equilibrium is compared with the available shear strength of the soil. This gives an average factor of safety along the failure surface. Most of the stability methods available are two-dimensional and assume plane-strain conditions.

The early limit equilibrium methods were developed for simple failure surfaces such as circular or log-spiral surfaces. Since Fellenius proposed a simple approach in 1936, more than a dozen methods of slices have been proposed. These methods differ in the assumptions made to render the problem determinate and in the statics used in deriving the factor of safety equation. The methods of slices can handle complex geometries and variable soil and water conditions. They are the most commonly used methods of slope stability analysis.

Until now, only a few three-dimensional limit equilibrium methods have been proposed to study the end-effects which occur in actual slides. Relatively little work has been done in this area and these methods are
limited to rather simple problems with uncomplicated geometry and soil and water conditions. They also suffer from the same limitations as the two-dimensional methods: (1) They do not adequately represent the stress-strain characteristics of the soil materials; and (2) They cannot deal with progressive failure in a rational manner.

The work presented in this dissertation is directed at providing the engineers with a general methodology for three-dimensional slope stability analysis. It follows along two lines: (1) Development of general methods of three-dimensional limit equilibrium analysis; and (2) Generation of a finite element computer program to adequately model the stress-strain characteristics of soils.

The most important types of slides which occur in embankments and slopes are rotational and translational slides. Rotational slides occur in slumps which rotate about an axis parallel to the slope. Translational slides are controlled by surfaces of weakness, such as faults, joints, bedding planes, and variations in shear strength between layers of bedded deposits. These different boundary conditions are taken into account in the present study. Two different computer programs based on the limit equilibrium concept, LEMIX and BLOCK3, are developed to analyze rotational and translational slides, respectively.

For rotational slides, a general method is proposed and the simplifying assumptions used by previous investigators are relaxed. The failure mass is assumed symmetrical and divided into many vertical columns. The inclination of the interslice forces are assumed the same throughout the whole failure mass. The intercolumn shear forces (at the two ends of the column) are assumed parallel to the base of the column and to be
a function of their positions. Force and moment equilibrium are satisfied for each column as well as for the total mass. For translational slides, the critical failure surface is assumed according to the Rankine's theory and the factor of safety is applied along the total failure surface. Taken together, the computer programs LEMIX and BLOCK3 can cover a wide range of geometric, soil and water conditions. Typical analyses are presented for several combinations of slope angles, soil parameters and pore water conditions.

In addition, a three-dimensional finite element computer program FESPON is also developed. It uses a hyperbolic stress-strain relationship and an incremental technique to simulate the nonlinear behavior of soils. Isoparametric incompatible elements are used to provide good bending characteristics. The hyperbolic stress-strain parameters are obtained from conventional triaxial and 1-D consolidation test data. This program can calculate the local factors of safety at selected points on the failure surface as well as the mean factor of safety for a chosen failure mass.

Several stability analyses of embankments are performed using existing two-dimensional methods and the programs LEMIX, BLOCK3 and FESPON. The results obtained with these different methods are compared extensively and it is hoped that they will provide the engineers with a better reference in the design and control of embankments.
2.1 Slides

Gravitational, seepage and surcharge loads tend to cause instability in natural or man-made slopes. Under these loads a sloping earth mass has a tendency to move downward and outward. In stability analysis and design of control methods to avoid instability, distinction is made between rotational and translational slides. These two types of slides are illustrated in Figure 2.1 and are briefly described in the following sections.

2.1.1 Rotational Slides

The most common rotational slides are little-deformed slumps along a surface of rupture curving concavely upward. In many slumps the underlying surface of rupture, together with the exposed scarps, is spoon-shaped (Fig. 2.1.a). If the slides extend for a considerable distance along the slope perpendicular to the direction of movement, much of the rupture surface may approach the shape of a cylinder with axis parallel to the slope. In slumps the movement is more or less rotational about an axis parallel to the slope. Rotational slides occur most frequently in fairly homogeneous materials, e.g., in constructed embankments and fills.

2.1.2 Translational Slides

In translational sliding the mass progresses out or down and out along a more or less planar or gently undulatory surface and has little of the rotary movement. If the moving mass of a translational slide
Fig. 2.1 (Cont’d)

(c) Slab Type Failure

Sandstone or Limestone and Shale
consists of a single unit that is not greatly deformed or a few closely related units it may be called a block slide (Fig. 2.1.b and Fig. 2.1.c).

The movement of translational slides is commonly controlled by surfaces of weakness, such as bedding planes and variations in shear strength between layers of bedded deposits.

2.2 Two-Dimensional Slope Stability Analysis by Limit Equilibrium Concept

The stabilities of natural slopes, cut slopes, and fill slopes are commonly analyzed by limit equilibrium methods. These methods take into account the major factors influencing the shearing resistance of a soil.

2.2.1 The $\phi = 0$ Method

Fellenius (1918) proposed what is today commonly known as the '\( \phi = 0 \)' method of stability analysis, a procedure widely used to analyze the short-term stability of slopes.

The shear surface is assumed to be circular. The factor of safety $F$, defined as the ratio of allowable shear strength to mobilized shear strength, can be obtained by summing moments about the center (Fig. 2.2):

$$Wx - \frac{c_u}{F} l_t r = 0$$

in which $W$ is the weight of the soil mass, $x$ the length of the moment arm of $W$ about the center, $c_u$ the undrained strength, $l_t$ the length of the shear surface and $r$ the radius of the circle.
Fig. 2.2 Forces along a Circular Shear Surface
The factor of safety $F$ is derived from equation (2.1):

$$F = \frac{c_a l t}{w x} \quad (2.2)$$

In this method, the normal stresses all act through the center of the circle regardless of their distribution. The shear stresses all act at the same distance from the center of the circle and therefore their moment arm is constant and independent of their distribution. Thus, the use of a circular shear surface results in statical determinacy with respect to moment equilibrium.

2.2.2 The Log Spiral Procedure

When $\phi$ is not equal to zero, a circular shear surface is insufficient to achieve statical determinacy. However, it may be achieved by a log spiral shear surface in the form:

$$r = r_o e^{\theta \tan \phi_m} \quad (2.3)$$

where $r$ is the radial distance from the center point to a point on the spiral, $r_o$ the reference radius, $\theta$ the angle between $r$ and $r_o$, and $\phi_m$ the mobilized friction angle for the shear surface.

This shape has the property that all the resultants of the normal stresses and frictional components of shear strength ($N \tan \phi_m$) pass through the center point of the spiral. Consequently, their contributions to the moments cancel out and the moment equation only involves the weight force and the cohesive resistance of the soil.

Since a value of $\tan \phi_m$ must be assumed in equation (2.3) to define a shear surface, the mobilized cohesion which is calculated may result in a different factor of safety with respect to cohesion than
was assumed in calculating $\phi_m$. Thus, several trials are necessary to obtain a balanced factor of safety which satisfies

$$F = \frac{c}{c_m} = \frac{\tan \phi}{\tan \phi_m} \quad (2.4)$$

2.2.3 The Friction Circle Procedure

For a circular shear surface the resultants of the normal stresses and frictional component of shear resistance will lie tangent to a circle of radius $r \sin \phi'$, called the friction circle (Fig. 2.3). The magnitude and location of this resultant and the factor of safety may be obtained from the three available equilibrium conditions (Taylor, 1937, 1948).

For a reasonable distribution of normal stresses along the shear surface, the resultant force must be less than the scalar sum of its component (Fig. 2.4). Consequently the resultant force must lie tangent to a circle of greater radius than the friction circle. This method thus underestimates the contributions of the moment from the resultant force and therefore the factor of safety obtained is a lower bound solution.

2.2.4 Methods of Slices

During the past three decades approximately one dozen methods of slices have been developed (Wright, 1969). They differ in: (1) the assumptions used to render the problem determinate; and (2) the statics employed in deriving the factor of safety equation. The methods of slices can handle complex geometric and variable soil and water conditions and therefore they are the most commonly used methods. Some of the most significant methods are presented below.
Fig. 2.3 Equivalent Force System for a Circular Shear Surface
Fig. 2.4 Normal and Frictional Shear Forces Acting on a Shear Surface.
2.2.4.1 Ordinary Method

The ordinary method is the simplest of the methods of slices. In this method the interslice forces are neglected (Fellenius, 1936) and the equilibrium of each slice is obtained by summing forces in the vertical and horizontal directions (Fig. 2.5):

\[ \Sigma F_v = 0 \]

\[ W - N \cos \alpha - \frac{T_a}{F} \sin \alpha = 0 \]  

(2.5)

\[ \Sigma F_H = 0 \]

\[ \frac{T_a}{F} \cos \alpha - N \sin \alpha = 0 \]  

(2.6)

where \( W \) is the weight of the slice, \( N \) normal force on the base of the slice, \( \alpha \) angle between the tangent to the center of the base of the slice and the horizontal, and \( T_a \) the allowable shear strength.

Solving for equation (2.5) and (2.6) gives:

\[ N = W \cos \alpha \]  

(2.7)

The factor of safety is derived from the summation of moments about a common point, \( \Sigma M_o = 0 \):

\[ \Sigma W x - \Sigma \frac{T_a}{F} r - \Sigma N f = 0 \]  

(2.8)

where \( x \), \( r \), and \( f \) are the moment arms of \( W \), \( T_a \) and \( N \), respectively.

Introducing the Mohr-Coulomb failure criterion the factor of safety can be obtained as a function of the strength parameters:

\[ F = \frac{\Sigma (c' l r + (N - u_l) r \tan \phi')}{\Sigma W x - \Sigma N f} \]  

(2.9)
Fig. 2.5 Forces System for the Method of Slices
where $c'$ is the effective cohesion intercept, $\phi'$ the effective friction angle, $u$ the pore water pressure, and $\ell$ the area of the base.

2.2.4.2 Simplified Bishop Method

The simplified Bishop method assumes the interslice forces to be horizontal. The normal force on the base of each slice is derived by summing forces in a vertical direction (as in equation (2.5)). Introducing the failure criteria and solving for the normal forces give:

$$N = (w - \frac{c' \ell \sin \alpha}{F} + \frac{u \ell \tan \phi' \sin \alpha}{F})/m_{\alpha}$$

(2.10)

where $m_{\alpha} = \cos \alpha + (\sin \alpha \tan \phi')/F$. The factor of safety is derived from the summation of moments about a common point. This equation is the same as equation (2.8) since the interslice forces cancel out. Therefore, the factor of safety equation is the same as in equation (2.9), with the value of $N$ defined in equation (2.10).

2.2.4.3 Spencer's Method

Spencer's Method assumes there is a constant relationship between the magnitude of the interslice shear and normal forces (Spencer, 1967).

$$\tan \Theta = \frac{X_L}{E_L} = \frac{X_R}{E_R}$$

(2.11)

where $\Theta$ is the angle of the resultant interslice force from the horizontal.

Spencer (1967) summed forces perpendicular to the interslice forces to derive the normal force. The same results can be obtained by summing forces in a vertical and horizontal direction (Fig. 2.5):
\[ \Sigma F_V = 0 \]

\[ W + (X_R - X_L) - N \cos \alpha - \frac{T}{F} \sin \alpha = 0 \] \hspace{1cm} (2.12)

\[ \Sigma F_H = 0 \]

\[ (E_L - E_R) - N \sin \alpha + \frac{T}{F} \cos \alpha = 0 \] \hspace{1cm} (2.13)

Solving equations (2.12) and (2.13):

\[ N = \left\{ W + (E_R - E_L) \tan \Theta - \frac{c' \tan \alpha}{F} + \frac{\mu \tan \Phi' \sin \alpha}{F} \right\} / m_a \] \hspace{1cm} (2.14)

Spencer (1967) derived two factors of safety equations. One is based on the summation of moments about a common point and the other on the summation of forces in a direction parallel to the interslice forces. The moment equation is the same as equation (2.8). The factor of safety equation is the same as equation (2.9).

Spencer's method yields two factors of safety for each angle of side forces. When the two factors of safety are equal for some angle of the interslice forces, both force and moment equilibriums are satisfied.

2.2.4.4 Janbu's Simplified Method

Janbu's simplified method uses a correction factor \( f \) to account for the effect of the interslice shear forces. The correction is related to cohesion, angle of internal friction, and the shape of the failure surface (Janbu et al., 1956).

The normal force can be obtained from equation (2.10). The factor of safety equation is derived from the horizontal equilibrium (Fig. 2.5):
\[ \Sigma F_H = 0 \]

\[ \Sigma (E_L - E_R) - \Sigma N \sin \alpha + \Sigma \frac{T \tan \phi}{F} \cos \alpha = 0 \]  

(2.15)

Since \( \Sigma (E_L - E_R) = 0 \), the factor of safety is:

\[ F_O = \frac{\Sigma \{ c' \ell \cos \alpha + (N - u \ell) \tan \phi' \cos \alpha \}}{\Sigma N \sin \alpha} \]  

(2.16)

The corrected factor of safety is

\[ F = f_o F_O \]  

(2.17)

The correction factors \( F_O \) have been generated by Janbu (1956) for different failure surfaces. For a long flat slip surface the interslice forces are not significant and consequently the correction factor approaches unity.

2.2.4.5 Janbu's Rigorous Method

Janbu's rigorous method assumes that the point of application of the interslice forces can be defined by a 'line of thrust'.

The normal force has a form similar to equation (2.14):

\[ N = \left\{ W + (X_R - X_L) - \frac{c' \ell \sin \alpha}{F} + \frac{u \ell \tan \phi' \sin \alpha}{F} \right\} / m_\alpha \]  

(2.18)

The factor of safety equation is the same as equation (2.13). The difference between simplified and rigorous methods is that the latter takes into account the shear forces in the derivation of the normal force.

To solve for the factor of safety, the shear forces may be set to zero for initial calculations. The factor of safety is obtained by
iterative calculations as in the Bishop's Simplified Method so that an assumed value of $F$ leads to an improved value and so on. The interslice forces then can be computed from the sum of the moments about the midpoint of the base of each slice (Fig. 2.6):

$$\Sigma M_m = 0$$

$$X_L(b/2) + X_R(b/2) + E_L\{t_L - (b/2) \tan \alpha\}$$

$$- E_R\{t_R + (b/2) \tan \alpha - b \tan \alpha_t\} = 0$$

where $t_L, t_R =$ vertical distance from the base of the slice to the line of thrust on the left and right sides of the slice, respectively.

$\alpha_t =$ angle between the line of thrust on the left side of a slice and the horizontal.

After rearranging equation (2.19), several terms can be shown to be negligible. After eliminating these terms, equation (2.19) simplifies to:

$$X_L = E_L \tan \alpha_t + (E_R - E_L) \frac{t_R}{b}$$

(2.20)

with

$$(E_L - E_R) = \{W + (X_R - X_L)\} \tan \alpha - \frac{T}{F \cos \alpha}$$

(2.21)

The horizontal interslice forces are obtained by integration from right to left across the slope. The magnitude of the interslice shear forces then can be obtained from equation (2.21). The factor of safety is recalculated with these computed values of interslice forces. Using these new values of $F$ and interslice forces a new position of
Fig. 2.6 Forces Acting on Each Slice for Janbu's Rigorous Method
the line of thrust is determined. The iterations are stopped when successive values of $F$ are nearly identical.

2.2.4.6 Morgenstern-Price Method

The Morgenstern-Price Method assumes an arbitrary mathematical function to describe the direction of the interslice forces:

$$\frac{X}{E} = \lambda f(x)$$  \hspace{1cm} (2.22)

where $\lambda$ is a constant to be evaluated in solving for the factor of safety and $f(x)$ is a functional variation with respect to $x$.

For a constant function, the Morgenstern-Price method is the same as the Spencer’s method. The normal force is derived from equation (2.18). Two factor of safety equations are computed, one with respect to moment equilibrium and one with respect to force equilibrium. The moment equilibrium equation is taken with respect to a common point. The factor of safety equation is the same as the one derived for Spencer’s method. The computation of interslice shear forces is similar to the derivation presented for Janbu’s rigorous method.

2.2.5 Comparison of Factors of Safety for Example Problem

Fredlund and Krahn (1977) used the methods of slices to solve an example problem in order to assess the effects of the interslice forces assumption. The problem is shown in Fig. 2.7 and the results are presented in Table 2.1. The results in Table 2.1 show that the factor of safety with respect to moment of equilibrium is relatively insensitive to the interslice forces assumption (see also Fig. 2.8). Therefore, the
Fig. 2.7 Example Problem (after Fredlund and Krahn, 1977)

- Condition 2 (weak layer)
- $c_1 = 0$, $\phi = 10^\circ$
- $\gamma = 120$ psf
- $\phi' = 20^\circ$
- $c' = 600$ psf
<table>
<thead>
<tr>
<th>Case no.</th>
<th>Example problem</th>
<th>Ordinary method</th>
<th>Simplified Bishop method</th>
<th>Spencer's method</th>
<th>Janbu's simplified method</th>
<th>Janbu's rigorous method*</th>
<th>Morgenstern-Price method f(x) = constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Simple 2:1 slope, 40 ft (12 m) high, ( \phi' = 20^\circ ), ( c' = 600 ) psf (29 kPa)</td>
<td>1.928</td>
<td>2.080</td>
<td>2.073</td>
<td>14.81</td>
<td>0.237</td>
<td>2.041</td>
</tr>
<tr>
<td>2</td>
<td>Same as 1 with a thin, weak layer with ( \phi' = 10^\circ ), ( c' = 0 )</td>
<td>1.288</td>
<td>1.377</td>
<td>1.373</td>
<td>10.49</td>
<td>0.185</td>
<td>1.448</td>
</tr>
<tr>
<td>3</td>
<td>Same as 1 except with ( r_u = 0.25 )</td>
<td>1.607</td>
<td>1.766</td>
<td>1.761</td>
<td>14.33</td>
<td>0.255</td>
<td>1.735</td>
</tr>
<tr>
<td>4</td>
<td>Same as 2 except with ( r_u = 0.25 ) for both materials</td>
<td>1.029</td>
<td>1.124</td>
<td>1.118</td>
<td>7.93</td>
<td>0.139</td>
<td>1.191</td>
</tr>
<tr>
<td>5</td>
<td>Same as 1 except with a piezometric line</td>
<td>1.693</td>
<td>1.834</td>
<td>1.830</td>
<td>13.87</td>
<td>0.247</td>
<td>1.827</td>
</tr>
<tr>
<td>6</td>
<td>Same as 2 except with a piezometric line for both materials</td>
<td>1.171</td>
<td>1.248</td>
<td>1.245</td>
<td>6.88</td>
<td>0.121</td>
<td>1.333</td>
</tr>
</tbody>
</table>

* The line of thrust is assumed at \( 0.333 \) of height of each slice.
Fig. 2.8 Comparison of Factors of Safety for Case I of Example Problem (after Fredlund and Krahn, 1977)
factors of safety obtained by the Spencer and Morgenstern-Price methods are generally similar to those computed by the simplified Bishop method.

2.3 Three-Dimensional Slope Stability Analysis by Limit Equilibrium Concept

Although there are many two-dimensional methods developed, only a few three-dimensional limit equilibrium methods are available. Until now, the developed 3-D methods are limited to rather simple problems, i.e., simple geometry, uncomplicated soil and water conditions. These methods are summarized below.

2.3.1 Weighted Average Procedure

In Fig. 2.9 consider several parallel cross sections through the slope. For these let \( A_1, A_2, A_3, \) etc. be the areas and \( F_1, F_2, F_3, \) etc. be the limit equilibrium factors of safety calculated for each cross section, respectively (Fig. 2.10). The overall factor of safety may be defined as follows (Sherard et al. 1963; Lambe and Whitman, 1969):

\[
F = \frac{F_1A_1 + F_2A_2 + F_3A_3 + \ldots}{A_1 + A_2 + A_3 + \ldots}
\]  

(2.23)

This weighted average factor of safety will be less than that by the method considering the end resistance.

2.3.2 Inclusion of End Effects Procedure

When the failure mass is long and the cross-sectional area of the potential failure mass is nearly uniform at various sections along its axis, end effects may be directly included in a 2-D analysis. Consider the \( \phi = 0 \) type of analysis for example. In Fig. 2.11, let the failure
Weighted Average Procedure

\[ F = \frac{F_1A_1 + F_2A_2 + F_3A_3}{A_1 + A_2 + A_3 + \ldots} \]

Figure 2.9 Plan View of Landslide
Figure 2.10 Factor of Safety for Different Cross Sections
Figure 2.11 Inclusion of End Effects for $\phi = 0$

\[ F = \frac{c_r \theta L + 2 \sum c \, dA \, r}{W \times L} \]
length be $L$. The resistance will include: (1) that along the cylindrical surface of sliding of length $L$ and radius $r_o$, giving a resisting moment of $c r_o^2 \theta L$; and (2) that at two ends giving a combined resisting moment of $2M_o$. Considering a small element area $dA$ at a distance $r$ from the center of the circle, $M_o$ will be equal to $2\pi c dA r$, where $c$ is the undrained strength. Therefore the new factor of safety is given by:

$$F = \frac{c r_o^2 \theta L + 2\pi c dA r}{W \times L}$$  \hspace{1cm} (2.24)$$

When $L$ is very large in comparison to $M_o$, equation (2.24) reduces to the two-dimensional form. In a similar manner end effects can be taken into account in other problems where $c$ and $\phi$ are included in the analysis or the slip surface cross section is wedge shaped or of arbitrary shape.

Baligh and Azzouz (1975) studied three-dimensional effects on the stability of slopes in cohesive soils. The failure mass was taken as a surface of revolution extending along the ground surface for a finite length $2L$ (Fig. 2.12). Different geometries and shapes were considered to analyze the 'end effects' by attaching either an ellipsoid or a cone at each end of the finite cylinder. Consider the surface of revolution shown in Figure 2.12 which is symmetrical with respect to the plane $z = 0$ and has a generator defined by its radial distance $r$ from the $Z$-axis according to:

$$r = g(z)$$  \hspace{1cm} (2.25)$$

The factor of safety is defined as:

$$F = \frac{M_r}{M_d}$$  \hspace{1cm} (2.26)$$
Figure 2.12 Different Geometries of Failure Surfaces and Their Plan Views
in which the resisting moment $M_r$ is:

$$M_r = \int_0^L M_r^0 \left( \frac{\partial s}{\partial z} \right) ds$$  \hspace{1cm} (2.27)

with

$$\frac{ds}{dz} = \sqrt{1 + \left( \frac{\partial s}{\partial z} \right)^2}$$  \hspace{1cm} (2.28)

and the driving moment $M_d$ is:

$$M_d = \int_0^L M_d^0 dz$$  \hspace{1cm} (2.29)

$M_r^0$ and $M_d^0$ are the resisting and driving moments computed in plane strain problems and are functions of the coordinate $z$.

In general, it is found that $F$ increases from its two-dimensional value. For long shallow failures (in which the ratio of length along axis of slope to depth of failure is greater than eight) the increase is of the order of 5% and can be disregarded. For short deep failures in which this ratio is less than 2 to 4, the increase in factor of safety can exceed 20% ~ 30% and three-dimensional effects must therefore be considered. Baligh and Azzouz also found that the length of failure is difficult to predict since it is very sensitive to slope and material parameters. Finally, the slope angle has little effect on the increase in the factor of safety due to end effects.

2.3.3 General Method

Previous methods are limited to cohesive soils and specific cases. Hovland (1977) proposed a general approach for three-dimensional slope
stability analysis by defining the factor of safety as the ratio of the
total available resistance along a failure surface to the total mobil-
ized stress along it. In order to simplify the analysis, the ordinary
method of slices was used. Thus the inter-column forces can be ignored
and both normal and shear stresses on the base of each column are ob-
tained simply as the component of the weight of the column.

In two-dimensional case, the factor of safety is:

$$F_2 = \frac{\Sigma(c A_2 + W_2 \cos \alpha \tan \phi)}{\Sigma W_2 \sin \alpha}$$

$$= \frac{\Sigma(c \frac{\Delta y}{\cos \alpha} + \rho z \Delta y \cos \alpha \tan \phi)}{\Sigma \rho z \Delta y \sin \alpha}$$

(2.30)

If cohesion $c$, friction angle $\phi$, and density $\rho$ are constants, then:

$$F_2 = (c) \frac{\Sigma \sec \alpha}{\Sigma z \sin \alpha} + (\tan \phi) \frac{\Sigma z \cos \alpha}{\Sigma z \sin \alpha}$$

(2.31a)

or

$$F_2 = \left(\frac{c}{\rho H}\right) G_{c2} + \tan \phi G_{\phi 2}$$

(2.31b)

The $G_{c2}$ and $G_{\phi 2}$ terms are only functions of geometry and $H$ is the
height of the slope.

In three-dimensional case, the factor of safety may be presented
in a similar form by dividing the soil mass above the failure surface
into a number of vertical soil columns. Assume the $XY$ plane to be hori-
zontal, the $Z$ axis to be vertical, and the $Y$ axis to be in the
direction of downslope movement (Fig. 2.13). Let $\Delta x$ and $\Delta y$ define the
Figure 2.13  Plan, Section, and Three-Dimensional Views of One Soil Column
cross-sectional area of vertical soil columns on the $XY$ plane and assume that both $\Delta x$ and $\Delta y$ are constant for all columns. Then:

$$F_3 = \frac{\sum X \sum Y \left\{ \frac{c}{\cos \alpha_{xz}} \cos \alpha_{yz} \sin \theta + \rho \ z \ \Delta x \ \Delta y \ \cos(DIP) \ \tan \phi \right\}}{\sum X \sum Y \ \rho \ z \ \Delta x \ \Delta y \ \sin \alpha_{yz}}$$

(2.32)

in which $\alpha_{xz}$ and $\alpha_{yz}$ are the dip angles in the $XZ$ and $YZ$ planes respectively, and:

$$\cos(DIP) = \left(1 + \tan^2 \alpha_{xz} + \tan^2 \alpha_{yz}\right)^{-1/2}$$

(2.33)

$$\sin \theta = \left(1 - \sin^2 \alpha_{xz} \ \sin^2 \alpha_{yz}\right)^{1/2}$$

(2.34)

If $c$, $\phi$, $\rho$, $\Delta x$, and $\Delta y$ are constant:

$$F_3 = \frac{\sum X \sum Y}{\rho} \ \frac{\sec \alpha_{xz} \ \sec \alpha_{yz} \ \sin \theta}{x \ y \ z \ \sin \alpha_{yz}} + \tan \phi \ \ \frac{\sum X \sum Y \ z \ \cos(DIP)}{x \ y \ z \ \sin \alpha_{yz}}$$

or

$$F_3 = \left(\frac{c}{\rho H}\right) c_3 + \tan \phi \ c_3$$

(2.35)

Hovland reported that every $c - \phi$ soil may have its own critical shear surface and geometry. His studies also suggest that the $F_3/F_2$ ratio is quite sensitive to the soil parameters $c$ and $\phi$, and to the basic shape of the shear surface. However, three-dimensional factors of safety are generally much higher than two-dimensional factors of safety, although in some situations it is not so. His studies also indicated that landslides in cohesive soils may follow a wide shear
surface geometry, approaching a 2-D case. On the other hand, slides in cohesionless soil may follow a 3-D wedge type surface.

2.4 Finite Element Method

Although limit equilibrium methods are widely used, they are subjected to criticism for three main reasons (Wright, 1973): (1) These methods do not consider the stress-strain characteristics of the soil; (2) the factor of safety assumed is the same for every slice, even though there is no reason to expect this to be true except at failure; (3) some of the equilibrium methods do not satisfy all the conditions of equilibrium. However, Wright (1973) concluded that the normal stress distributions determined by linear elastic finite element analyses are very nearly the same as those determined by Bishop's Simplified Method for flat slopes and large values of dimensionless parameters \( \lambda_c \phi \) \((= \frac{\gamma H \tan \phi}{c})\). The average factors of safety determined by the two methods are very nearly the same, varying only by 0% to 8%. However, the material was assumed to have linear elastic behavior which may not be true. In his discussion, Resendiz (1974) used hyperbolic stress-strain relationships proposed by Kondner (1963) to analyze fourteen embankments under end-of-construction conditions. The potential failure line was determined as the locus of \( \epsilon_{\text{max}} \), the maximum principal strain, and the factor of safety was determined as the mean value of the ratio \( \sigma_{\text{df}} / \sigma_d \) along the potential failure line:

\[
F = \frac{\text{the principal stress difference at failure (}\sigma_{\text{df}}\text{)}}{\text{the acting principal stress difference (}\sigma_d\text{)}}
\]  

(2.36)

It was shown that the conventional factors of safety are always lower.
than the ones obtained from this method. The difference may be as large as 30% depending on the magnitude of the factor of safety and on the slope angle. In three-dimensional problems, Lefebre & Duncan (1973) used the finite element method to analyze three dams in V-shaped valleys with three different valley wall slopes equal to 1/1, 3/1, and 6/1. The material was assumed linear elastic. They concluded that: (1) for dams in valleys with wall slope as steep as 1/1 the results will be significantly less accurate, as a result of cross-valley arching; and (2) plane stress analysis of the maximum longitudinal section does not provide accurate results.

2.5 Other Methods of Slope Stability Analysis

An alternative method of slope stability analysis is to investigate the shear stresses by using the theory of elasticity (Perloff and Baron (1976), Romani (1970), Romani, Lovell and Harr (1972)). The factor of safety is defined as the shear strength divided by the shear stress at the point where this ratio is the least, hence it gives the safety at the most critical point.

The method may be useful when dealing with soils where progressive failure is likely to occur. However, it does not take into account the redistribution of stress which occurs when the stress level at a point approaches the strength.

2.6 Summary

1. In dealing with a slope stability problem, the choice of suitable methods should be dependent on the type of failure considered. In this chapter, two kinds of slides, rotational and translational, were defined.
2. Several commonly used two-dimensional slope stability analyses were briefly presented. The derivations are similar. Some methods satisfy determinancy, some do not. Of all the rigorous methods Spencer's method is the simplest and can produce quite accurate results.

3. Three-dimensional limit equilibrium methods developed so far are limited to simple geometry of failure mass, simple soil conditions, and cannot take into account the water conditions. More research on 3-D analysis is worthwhile.

4. Finite element methods are superior to limit equilibrium methods because of their power to handle complicated geometry, many soil parameters, water conditions, and to consider the stress-strain relationships of soils. However, they are much more complicated to use than limit equilibrium methods.

5. Although the results from both limit equilibrium and finite element methods have been compared for 2-D cases, comparisons for 3-D cases are not available.
III. LIMIT EQUILIBRIUM METHODS

3.1 Introduction

At a time when sophisticated approaches had yet to be developed and little was known about the mechanical behavior of earth masses, the limit equilibrium concept played an important role to make possible the use of simple theoretical approaches in solving many problems. In recent years, remarkable progress has been made in the area of stress analysis of continua and discontinua. Development of sophisticated numerical techniques and fast computers have facilitated this progress. However, the limit equilibrium concept has survived and is still considered to be reliable by most practitioners.

In Chapter II, two types of slides (rotational and translational) were defined and, as we mentioned previously, most of the equilibrium methods deal with plane strain conditions. In this Chapter, both types of failure mechanism are considered and three-dimensional solutions are derived. The assumptions in solving these problems and the derivations of equations are presented.

3.2 Block Type of Failure

When there is a very soft or loose material beneath a slope, the failure surface usually occurs along this soft or loose layer. The examples may be a slope underlain by a weak contact between colluvium and sloping bedrock, or between sidehill fill and sloping foundation.
The failure is perceived to be that of a relatively intact mass moving above a relatively well defined failure surface.

Mendez (1972) developed a quite general computer program to analyze the stability of a three-plane surface, but the profile was limited to two kinds of soils, i.e., a strong one over a weak one. Mohan (1972) also made simplifying assumptions with respect to the shape of the sliding surface, but his solution is quite versatile with respect to the potential complexity of the subsurface. The 2-D computer program BLOCK or BLOCK2 (Boutrup, 1977) can select the critical surface of very complicated soil conditions and apply the same factor of safety throughout the whole failure surface.

In order to study the 3-D block type of failure, a 3-D computer program BLOCK3 is developed. The assumptions and the derivation of the factor of safety are presented in the following sections.

3.2.1 Assumptions

Fig. 3.1 shows the free body diagram of a block type of failure in three-dimensional space. Boutrup (1977) analyzed the block type of failure by using the method of slices and applied the same factor of safety throughout the most critical failure surface. It was found that the most critical failure surface was close to that selected from Rankine theory, i.e., the shear surface makes \(\left(\frac{\phi}{2}\right)\) and \(\left(\frac{\phi}{2}\right)\) angles with the horizontal in active and passive zones, respectively. Therefore, in this study the ends of the most critical surface will be chosen as that from Rankine theory just for simplicity and convenience in comparison of results.
The assumptions of the method are listed below.

1. The problem is three-dimensional and symmetrical.

2. The ground surface is defined by three slopes and well-defined toe and crest.

3. The soil strata are laterally continuous.

4. The sliding surfaces are plane.

5. The boundaries between (1) active and central blocks, (2) passive and central blocks are vertical. No shear forces along these boundaries.

6. The bottom surfaces are at \((45 + \phi/2)\) and \((45 - \phi/2)\) angles with the horizontal for active and passive zones, respectively.

7. The factor of safety is the same throughout the whole failure surface.

8. The water surface is far below the ground surface.

9. The forces acting at the ends of blocks may be computed by assuming \(K_0\) conditions and linear lateral stress distribution.

3.2.2 Derivation of Equations

The analysis is divided into three parts, namely:

(1) Calculation of the total force acting on the central block from the active block. This force is a function of the factor of safety.

(2) Calculation of the total force acting on the central block from the passive block. This force is also a function of the factor of safety.
(3) Calculation of base, side, and end forces on the central block and of the factor of safety against failure.

3.2.2.1 Active Force

Fig. 3.2 shows the free body diagram of the active block. In Fig. 3.3, consider the force polygon and sum all forces in X and Y coordinate axes:

\[ \Sigma F_x = 0 \]

\[ P_a + 2F_{asm} \sin \phi_m \cos \xi \sin (45 - \phi/2) + c_m (A_{ab} + 2A_{as} \cos \xi) \sin (45 - \phi/2) - F_{ab} \cos (45 - \phi/2 + \phi_m) = 0 \]  

(3.1)

\[ \Sigma F_y = 0 \]

\[ -W_a + 2F_{asm} \sin \phi_m \cos \xi \cos (45 - \phi/2) + c_m (A_{ab} + 2A_{as} \cos \xi) \cos (45 - \phi/2) + F_{ab} \sin (45 - \phi/2 + \phi_m) = 0 \]  

(3.2)

where

- \( P_a \) = the active force
- \( W_a \) = the weight of the active block
- \( F_{asm} \) = the mobilized force acting on the end of the active block
- \( \phi_m \) = the mobilized frictional angle
- \( \xi \) = the angle, on the bottom of the active block, of the intersection of the inclined end with the vertical plane
- \( c_m \) = the mobilized cohesion intercept
- \( A_{ab} \) = the area of the bottom of the active block
Figure 3.2 Free Body Diagram in Active Case
1) $C_m A_{ob}$  
2) $2 \cdot C_m A_{as} \cdot \cos \xi$  
3) $2 \cdot F_{as} \cdot \sin \phi_m \cdot \cos \xi$

Figure 3.3 Force Polygon in Active Case
$A_{as}$ = the area of the end of the active block

$F_{ab}$ = the mobilized force acting on the bottom of the active block

Rearranging equation (3.2):

$$F_{ab} = \csc (45 - \phi/2 + \phi_m) \{W_a - 2F_{asm} \sin \phi_m \cos \xi \cos (45 - \phi/2)$$

$$- c_m (A_{ab} + 2A_{as} \cos \xi) \cos (45 - \phi/2)\}$$

(3.3)

The active force is obtained by substituting equation (3.3) into equation (3.1) and combining the similar terms:

$$P_a = W_a \tan (45 + \phi/2 - \phi_m) - \{c_m (A_{ab} + 2A_{as} \cos \xi)$$

$$+ 2F_{asm} \sin \phi_m \cos \xi \} \cos (45 - \phi/2) \{\tan (45 - \phi/2)$$

$$+ \tan (45 + \phi/2 - \phi_m)\}$$

(3.4)

3.2.2.2 Passive Force

Fig. 3.4 shows the free body diagram of the passive block. In Fig. 3.5, consider the force polygon and sum all forces along X and Y coordinate axes:

$$\Sigma F_x = 0$$

$$- P_p + (2c_m A_{ps} \cos \eta + c_m A_{pb} + 2F_{psm} \sin \phi_m \cos \eta)$$

$$\cos (45 - \phi/2) + F_{pb} \cos (45 + \phi/2 - \phi_m) = 0$$

(3.5)

$$\Sigma F_y = 0$$

$$- W_p - (2c_m A_{ps} \cos \eta + c_m A_{pb} + 2F_{psm} \sin \phi_m \cos \eta)$$
Figure 3.4 Free Body Diagram in Passive Case
1) $C_m \cdot A_{pb}$  
2) $C_m \cdot A_{ps} \cdot \cos \eta$  
3) $F_{psm} \sin \phi_m \cos \eta$

Figure 3.5 Force Polygon in Passive Case
\[
\sin (45 - \phi/2) + F_{pb} \sin (45 + \phi/2 - \phi_m) = 0 \quad (3.6)
\]

where \( P_p \) = the passive force
\( W_p \) = the weight of passive block
\( F_{psm} \) = the mobilized force acting on the end of the passive block
\( F_{pb} \) = the mobilized force acting on the bottom of the passive block
\( A_{pb} \) = the area of the bottom of the passive block
\( A_{ps} \) = the area of the end of the passive block
\( \eta \) = the angle, on the bottom of the active block, of the intersection of the inclined end with vertical plane

Rearranging equation (3.6):

\[
F_{pb} = \csc (45 + \phi/2 - \phi_m) \{W_p + 2c_m A_{ps} \cos \eta + c_m A_{pb} + 2F_{psm} \sin \phi_m \cos \eta \sin (45 - \phi/2)\} \quad (3.7)
\]

The passive force is obtained by substituting equation (3.7) into equation (3.5), and combining the similar terms:

\[
P_p = W_p \tan (45 - \phi/2 + \phi_m) + c_m (2A_{ps} \cos \eta + A_{pb}) + 2F_{psm} \sin \phi_m \cos \eta \cos (45 - \phi/2) \{1 + \tan (45 - \phi/2) \\
\tan (45 - \phi/2 + \phi_m)\} \quad (3.8)
\]
3.2.2.3 Equilibrium of the Central Block and Factor of Safety

Fig. 3.6 shows the free body diagram of the central block. In Fig. 3.7, consider the force polygon and sum all forces along $\beta$ and $\eta$ coordinate axes,

$$\Sigma F_\beta = 0$$

$$\{(2c_m A_s + 2F_{sm} \sin \phi_m) \cos \alpha + c_{bm} A_b + F_b \sin \phi_m\}$$

$$+ (P_p - P_a) \cos \beta - W \sin \beta = 0$$

(3.9)

$$\Sigma F_\eta = 0$$

$$- W \cos \beta - P_p \sin \beta + F_b \cos \phi_{bm} + P_a \sin \beta = 0$$

(3.10)

where $W$ = the weight of the central block

$F_{sm}$ = the mobilized force acting on the end of the central block

$F_b$ = the normal force acting on the bottom of the central block

$A_s$ = the area of the end of the central block

$A_b$ = the area of the bottom of the central block

$c_{bm}$ = the mobilized cohesion intercept of the weak soil

$\alpha$ = the angle, on the bottom of central block, of the intersection of the inclined end with vertical plane

$\beta$ = the angle of inclination of the weak layer

$\eta$ = the direction normal to $\beta$

Rearranging equation (3.10):
Figure 3.6 Free Body Diagram of Central Block
Figure 3.7 Force Polygon of Central Block

1) \( c_{bm} A_b \)  
2) \( 2 \cdot c_m A_s \cos \alpha \)  
3) \( 2 \cdot F_{sm} \sin \phi_m \cos \alpha \)
\[ F_b = \sec \phi_{bm} \{ W \cos \beta + (P_P - P_a) \sin \beta \} \quad (3.11) \]

Substituting equation (3.11) into equation (3.9), and combining the similar terms leads to:

\[
\begin{align*}
(2c_m A_s + 2F_{sm} \sin \phi_m) \cos \alpha + c_{bm} A_b + \tan \phi_{bm} \\
(W \cos \beta + (P_P - P_a) \sin \beta) + (P_P - P_a) \cos \beta \\
-W \sin \beta = 0 
\end{align*}
\]

Equation (3.12) is in terms of the factor of safety \( F \). After substituting the known values listed below, the factor of safety can be calculated by the secant's method (Wolfe, 1959):

\[
\tan (45 - \phi/2 + \phi_m) = \frac{\tan (45 - \phi/2) + \tan \phi/F}{1 - \tan (45 - \phi/2) \tan \phi/F}
\]

\[
\tan (45 + \phi/2 - \phi_m) = \frac{\tan (45 + \phi/2) - \tan \phi/F}{1 + \tan (45 + \phi/2) \tan \phi/F}
\]

\[
\sin \phi_m = 1/(1 + (F/tan \phi)^2)^{1/2}
\]

\[
\tan \xi = \sin (45 + \phi/2)/\tan \gamma
\]

\[
\cos \xi = 1/(1 + (\sin (45 + \phi/2)/\tan \gamma)^2)^{1/2}
\]

\[
\tan \eta = \sin (45 - \phi/2)/\tan \gamma
\]

\[
\cos \eta = 1/(1 + (\sin (45 - \phi/2)/\tan \gamma)^2)^{1/2}
\]

\[
\tan \alpha = \cos \beta \left[ L \left( 1 - a \right)/2 - (H_2 - H_1)/\tan \gamma \right]/B
\]

\[
\cos \alpha = 1/(1 + (\cos \beta \left( L \left( 1 - a \right)/2 - (H_2 - H_1)/\tan \gamma \right)/B)^2)^{1/2}
\]

\[
W = \rho B (B_1 + B_2 + \sqrt{B_1 B_2})/3
\]
where

\[ B_1 = H_2 (L - H_2 \cot \gamma) \]

\[ B_2 = H_1 (a L - H_1 \cot \gamma) \]

\[ A_b = \{0.5 (1 + a) L - \cot \gamma (H_1 + H_2)\} B \sec \beta \]

\[ A_s = \beta (H_1 + H_2)/(2 \cos \alpha \sin \gamma) \]

\[ F_s \sin \phi = k_o \rho \beta \tan \phi (H_1^2 + H_2^2 + H_1 H_2)/(6 \sin \gamma \cos \alpha \cos \beta) \]

\[ W_a = \rho H_2^2 \tan (45 - \phi/2) \{0.5 L - H_2/(3 \tan \gamma)\} \]

\[ A_{as} = H_2^2 \tan (45 - \phi/2)/(2 \sin \gamma) \]

\[ A_{ab} = (L - H_2/\tan \gamma) H_2 \sec (45 - \phi/2) \]

\[ F_a \sin \phi = k_o \rho H_2^3 \tan \phi \tan (45 - \phi/2)/(6 \sin \gamma) \]

\[ W_p = \rho H_1^2 \tan (45 + \phi/2) \{0.5 a L - H_1/(3 \tan \gamma)\} \]

\[ A_{ps} = H_1^2 \tan (45 + \phi/2)/(2 \sin \gamma) \]

\[ F_{ps} \sin \phi = k_o \rho H_1^3 \tan \phi \tan (45 + \phi/2)/(6 \sin \gamma) \]

\[ A_{pb} = (a L - H_1/\tan \gamma) H_1 \sec (45 + \phi/2) \]

\[ \gamma = \text{the\ inclination\ of\ the\ end\ of\ the\ central\ block} \]

\[ L = \text{the\ length\ on\ the\ crest\ of\ the\ central\ block} \]

\[ a = \text{the\ ratio\ between\ the\ length\ of\ the\ central\ block\ at\ the\ toe\ to\ that\ at\ the\ crest} \]

\[ B = \text{the\ width\ of\ the\ central\ block} \]

\[ H_1 = \text{the\ vertical\ height\ of\ the\ passive\ block} \]

\[ H_2 = \text{the\ vertical\ height\ of\ the\ active\ block} \]

\[ k_o = \text{the\ ratio\ of\ horizontal\ principal\ stress\ to\ vertical\ principal\ stress\ at\ rest} \]

\[ \rho = \text{the\ density\ of\ soil\ in\ the\ fill\ or\ foundation} \]
3.3 Rotational Type of Failure

The Hovland's method to analyze rotational slides has been presented in Chapter II. In this method the failure mass is divided into many vertical columns and the factor of safety is defined simply as the ratio of total available strength over total mobilized stress. Several important simplifying assumptions were employed: (1) forces on the vertical sides of each soil column were assumed to be zero; (2) direction of movement is along the X-Y plane only; (3) the bottom forces act at the center of the bottom area; and (4) equilibrium of forces and moments in each column are satisfied. The following method will relax some of these assumptions and present a general approach to the analysis of rotational failures.

3.3.1 General Description

Fig. 3.3 shows the free body diagram of a vertical column taken out from the failure mass. The parameters included are the normal and shear forces acting on four vertical sides and the bottom, the points of application of these forces, and the overall factor of safety F. Table 3.1 presents a comparison of the number of parameters needed in the two-dimensional and three-dimensional analyses. Making the necessary assumptions to reduce the number of these parameters and make the problem determinate is not an easy task. For the two-dimensional case, the number of unknowns is relatively limited and many different assumptions have been proposed to solve the problem (Chapter II). But, if a three-dimensional problem is dealt with, many more parameters are included and the task of making the problem determinate is much more complicated.
Figure 3.8 Free Body Diagram of a Column
### TABLE 3.1 LIST OF UNKNOWNs IN 2-D AND 3-D CASEs

<table>
<thead>
<tr>
<th>Parameters</th>
<th>3-D Unknowns</th>
<th>2-D Unknowns</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Z_{i,j-l} )</td>
<td>((m+1)n)</td>
<td>0</td>
</tr>
<tr>
<td>( Z_{i,j} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( X_{i-l,j} )</td>
<td>((n-1)m)</td>
<td>(n-1)</td>
</tr>
<tr>
<td>( X_{i,j} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( R_{i,j-l} )</td>
<td>((m-1)n)</td>
<td>0</td>
</tr>
<tr>
<td>( R_{i,j} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( P_{i-l,j} )</td>
<td>((n-1)m)</td>
<td>0</td>
</tr>
<tr>
<td>( P_{i,j} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( S_{i,j-l} )</td>
<td>((m-1)n)</td>
<td>0</td>
</tr>
<tr>
<td>( S_{i,j} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( E_{i-l,j} )</td>
<td>((n-1)m)</td>
<td>(n-1)</td>
</tr>
<tr>
<td>( E_{i,j} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( h_{1j} )</td>
<td>((m+1)n)</td>
<td>0</td>
</tr>
<tr>
<td>( h_{3j} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( h_{2i} )</td>
<td>((n-1)m)</td>
<td>(n-1)</td>
</tr>
<tr>
<td>( h_{4i} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( b_{1j} )</td>
<td>((m+1)n)</td>
<td>0</td>
</tr>
<tr>
<td>( b_{3j} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( b_{2i} )</td>
<td>((n-1)m)</td>
<td>(n-1)</td>
</tr>
<tr>
<td>( b_{4i} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( Z_{bij} )</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>( N_{ij} )</td>
<td>(mn)</td>
<td>(n)</td>
</tr>
<tr>
<td>( F )</td>
<td>(1)</td>
<td>1</td>
</tr>
<tr>
<td>( R_{\text{ext}} )</td>
<td>(2n)</td>
<td>0</td>
</tr>
<tr>
<td>( S_{\text{ext}} )</td>
<td>(2n)</td>
<td>(5n-3)</td>
</tr>
</tbody>
</table>

Reduce to \(12mn-5m+5n+1\)
If the mass is divided into 600 vertical columns \((m = 20, n = 30)\), and the geometry is assumed to be symmetrical, the number of the unknowns remaining are

\[
0.5 \cdot 6 \cdot m \cdot n = 0.5 \cdot 6 \cdot 20 \cdot 30 = 1800
\]

which is twenty times that in two-dimensional case \((3n = 90)\). This large number of equations will not only require tremendous storage in the computer but also long computing times. It is therefore necessary to make more assumptions, as listed below, to simplify the problem.

3.3.2 Assumptions

(1) The failure mass is symmetrical

(2) Direction of movement is along the X-Y plane only (no movement in Z-direction), therefore at the instant of failure the shear stresses along the Y-Z plane are assumed to be zero (Fig. 3.8). This assumption makes:

\[
P_{i,j} = P_{i,j-1} = 0
\]

\[
Z_{bi,j} = 0
\]

(3) The length and width of the column is small enough so that it can be assumed that each side force acts along the central vertical line of its side:

\[
b_{1j} = b_{3j} = b/2
\]

\[
b_{2i} = b_{4i} = l/2
\]
(4) Intercolumn shear forces are assumed to be parallel to the bottom (Fig. 3.9). The cohesion part of the mobilized shear force acts at h/2 from the bottom (resultant of the cohesion acts at the center of the side). The cohesionless part of the mobilized shear force acts at h/3 from the bottom (the intercolumn normal stress distribution is assumed to be linear).

The intercolumn shear forces (at the two ends of the column) are assumed to be a function of their positions; they take the largest value at the outmost point and decrease to zero at the central section because of no relative movement in the middle. The outmost shear forces, $R_{\text{ext}}$ and $S_{\text{ext}}$, can be obtained from the following equations, assuming that the $K_o$ condition prevails. These assumptions make:

$$R_{\text{ext}} = (0.5 K_o \rho h \tan \phi + c) b h \cos \alpha$$

$$S_{\text{ext}} = R_{\text{ext}} \tan \alpha$$

$$R_{i,j} = R_{\text{ext}} f(z)$$

$$h_c = h/2$$

$$h_\phi = h/3$$

(5) The interslice forces (on two sides of the column) are assumed to have the same inclination throughout each section ($z = \text{constant}$), then:
Fig. 3.9 The Force System of a Column in Side View
\[
\tan \Theta_i = \frac{E_i}{X_{i,j}} = \frac{E_{i-1,j}}{X_{i-1,j}}
\]  

(3.14)

The resultant of the two interslice forces can be presented as \( Q \).

Table 3.2 lists the unknowns remaining after the above assumptions have been made. The number of the unknowns is reduced from \((12mn - 5m + 5n + 1)\) to \((2mn + 1)\). It is still necessary to have the same number of equations in order to solve for these remaining unknowns. The following procedure will show that the forces, \( X' \)'s and \( N' \)'s (Table 3.2), will not remain in the equations and only the factor of safety \( F \) and the inclined angles \( \Theta' \)'s are left.

In the following sections, three types of failure geometries are discussed: (1) roller type; (2) spoon shape; and (3) the mixed shape of (1) and (2).

3.3.3 Roller Type Failure

In the roller type of failure, the failure mass is of cylindrical shape with two vertical ends. This problem is very similar to the 2-D problem except that the length of the failure mass is not infinitely long. Consequently the intercolumn shear forces should be taken into consideration.

Fig. 3.10 shows the force polygon of a column. The summation of all forces along the \( \alpha \) and \( \eta \) coordinate axes results in:

\[
\Sigma F_\alpha = 0
\]

\[
N' \tan \phi_m + \frac{c'_b}{F} \cdot \ell \cdot b \sec \alpha + \frac{\Delta R}{F} - W \sin \alpha - Q \cos (\alpha - \Theta) = 0
\]  

(3.15)
### TABLE 3.2 LIST OF UNKNOWNS IN 3-D CASE AFTER ASSUMPTIONS

<table>
<thead>
<tr>
<th>Parameters</th>
<th>3-D Unknowns</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_{i,j-1}$</td>
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</tr>
<tr>
<td>$Z_{i,j}$</td>
<td></td>
</tr>
<tr>
<td>$X_{i-1,j}$</td>
<td>$(n-1)m$</td>
</tr>
<tr>
<td>$X_{i,j}$</td>
<td>0</td>
</tr>
<tr>
<td>$R_{i,j-1}$</td>
<td>0</td>
</tr>
<tr>
<td>$R_{i,j}$</td>
<td></td>
</tr>
<tr>
<td>$P_{i-1,j}$</td>
<td>0</td>
</tr>
<tr>
<td>$P_{i,j}$</td>
<td></td>
</tr>
<tr>
<td>$S_{i,j-1}$</td>
<td>0</td>
</tr>
<tr>
<td>$S_{i,j}$</td>
<td></td>
</tr>
<tr>
<td>$E_{i-1,j}$</td>
<td>0</td>
</tr>
<tr>
<td>$E_{i,j}$</td>
<td></td>
</tr>
<tr>
<td>$h_{1j}$</td>
<td>0</td>
</tr>
<tr>
<td>$h_{3j}$</td>
<td></td>
</tr>
<tr>
<td>$b_{2i}$</td>
<td>0</td>
</tr>
<tr>
<td>$b_{4i}$</td>
<td></td>
</tr>
<tr>
<td>$b_{1j}$</td>
<td></td>
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<tr>
<td>$b_{3j}$</td>
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</tr>
<tr>
<td>$Z_{bij}$</td>
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</tr>
<tr>
<td>$N_{ij}$</td>
<td>$mn$</td>
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</tr>
<tr>
<td>$S_{\text{ext}}$</td>
<td>0</td>
</tr>
<tr>
<td>$\Theta_j$</td>
<td>$m$</td>
</tr>
</tbody>
</table>

Total: $2mn + 1$
Figure 3.10  Force Polygon of a Column in Roller Type Failure Mass
\[ \Sigma F \eta = 0 \]

\[ N' + u \ell b \sec \alpha - W \cos \alpha + Q \sin (\alpha - \Theta) = 0 \]  \hspace{1cm} (3.16)

where

- \( N' \) = effective normal force acting on the bottom of the column
- \( W \) = the weight of the column
- \( u \) = the water pressure acting on the bottom of the column
- \( Q \) = the resultant of two interslice forces \( T_n \) and \( T_{n+1} \)
- \( \Delta R \) = the net intercolumn shear force
- \( c'_b \) = the effective cohesion intercept of the soil beneath the bottom of the column
- \( \phi'_m \) = the mobilized effective friction angle
- \( \ell \) = the length of the column
- \( b \) = the width of the column
- \( \alpha \) = the inclination of the bottom of the column
- \( \Theta \) = the inclination of \( Q \)
- \( F \) = the factor of safety

Rearranging equations (3.15) and (3.16) leads to:

\[ Q \cos (\alpha - \Theta) = N' \tan \phi'_m + \frac{c'_b}{F} \ell b \sec \alpha + \frac{\Delta R}{F} - W \sin \alpha \]  \hspace{1cm} (3.17)

and:

\[ N' = - u \ell b \sec \alpha + W \cos \alpha - Q \sin (\alpha - \Theta) \]  \hspace{1cm} (3.18)

Substituting equation (3.18) into equation (3.17) and combining similar terms result in:
\[
Q = \frac{c'b}{F} \sec \alpha + \frac{\tan \phi'}{F} (W \cos \alpha - u \ell \sec \alpha) - W \sin \alpha + \frac{\Delta R}{F} \cos(\alpha - \theta) \left(1 + \frac{\tan \phi'}{F} \tan (\alpha - \theta)\right)
\]

(3.19)

Taking moment at the middle of the base (Fig. 3.9):

\[
Q \cos \theta h_Q - \frac{\Delta R}{F} \cos \alpha \frac{h}{3} - \frac{\Delta R}{F} \cos \alpha \frac{h}{2} = 0
\]

(3.20)

or

\[
h_Q = \frac{h \cos \alpha (2 \Delta R \phi + 3 \Delta R_c)}{6 F Q \cos \theta}
\]

(3.21)

If the whole failure mass is divided into \( m \) sections and if each section is in the state of equilibrium, the sum of all forces in each section must be equal to zero:

\[
\Sigma Q \cos \theta = 0
\]

(3.22)

and

\[
\Sigma Q \sin \theta = 0
\]

(3.23)

Since \( \theta \) is constant, equations (3.22) and (3.23) can be reduced to a unique equation:

\[
\Sigma Q = 0
\]

(3.24)

The whole system is also in equilibrium with respect to moment equilibrium. Thus the overall moment about any point \( O \) much be equal to zero (Fig. 3.11):

\[
\Sigma M_o = 0
\]

\[
\Sigma Q \cos (\theta - \alpha) (r - h_Q \cos \alpha) = 0
\]

(3.25)
Fig. 3.11 Moment Induced by the Resultant Force about a Point O
Substituting $h_q$ from equation (3.21):

$$
\sum r \cos (\theta - \alpha) (Q - \frac{h \cos^2 \alpha (2\Delta R_c + 3\Delta R_e)}{6r \cos \theta}) = 0
$$

(3.26)

If the radius $r$ is constant, then equation (3.26) becomes:

$$
\sum \{Q \cos (\theta - \alpha) - \frac{h \cos^2 \alpha \cos (\theta - \alpha) (2\Delta R_c + 3\Delta R_e)}{6r \cos \theta}\} = 0
$$

(3.27)

For $m$ sections, $m$ equations from the force equilibrium are available (equation (3.24)). One additional equation comes from the overall moment equilibrium (equation (3.26)). The unknowns are $\theta_1, \theta_2, \ldots, \theta_m$ for each section respectively, and the factor of safety $F$. Because there are $(m + 1)$ equations for $(m + 1)$ unknowns, the problem is rendered determinate and can be solved by using the secant's method for nonlinear equations.

3.3.4 Spoon Shape Failure

In most cases, the shape of the failure mass in the embankment is not a roller type failure, but approaches a spoon shape. In this section, the more realistic spoon shape is discussed. The failure mass is assumed to be symmetrical and has an axis of rotation $0-0'$ (Fig. 3.12). The "spoon" shape is mathematically expressed by an ellipsoid:

$$
\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1
$$

(3.28)

For simplicity, each cross section in the $X-Y$ plane is assumed circular (Fig. 3.13), and:

$$
a = b = r_o
$$

$$
c = r_z
$$
Fig. 3.13 2-D and 3-D Views of Spoon Shape
Thus, the equation becomes:

\[
\frac{x^2}{r_0^2} + \frac{y^2}{r_0^2} + \frac{z^2}{r_z^2} = 1
\]

(3.29a)

or:

\[
x^2 + y^2 + m^2z^2 = r_o^2
\]

(3.29b)

where: \( m = r_o/r_z \)

Fig. 3.14 shows the free body diagram of a column. The method allows for different material in the embankment and foundation. The subscript \( E \) represents embankment and \( F \) represents foundation soil.

Fig. 3.15 shows the force system projected on the central plane (\( x-y \) plane) of a column provided that \( dz \) is very small. The resultant \( \Delta R_{cE} \) represents the net sum of two end shear forces \( R_{cE1} \) and \( R_{cE2} \), in which the subscripts \( c, E, 1, 2 \) stand for cohesion, embankment, end 1, and end 2, respectively.

As previously the failure mass is assumed symmetrical, and there is no movement in the \( Z \)-direction. However, all the interslice forces will have the same inclination throughout the whole failure mass. This assumption is different from that assumed in the roller type of failure in which each section had its own inclination of interslice force. Fortunately for the spoon shape of failure, the factor of safety obtained under one \( \theta \) assumption makes hardly any difference with that under
Fig. 3.14: Free Body Diagram of a Column in Spoon Shape Failure
Fig. 3.15 The Force System of a Column Presented in a 2-D View
variable $\theta$'s assumption. Consequently a unique value of $\theta$ can be used throughout the whole failure mass.

Fig. 3.16 shows the force polygon of a column. Considering all forces and summing them in $\alpha$ and $\eta$ coordinate axes, lead to:

$\Sigma F_\alpha = 0$

\[
N' \tan \phi'_m + \frac{c'}{\lambda} A_b - Q \cos (\alpha_{xy} - \theta) - W \sin \alpha_{xy} + R_2 \\
\cos (\alpha_2 - \alpha_{xy}) - R_1 \cos (\alpha_{xy} - \alpha_1) - F_w \cos \alpha_{xy} = 0
\]

$\Sigma F_\eta = 0$

\[
N' + u A_b + Q \sin (\alpha_{xy} - \theta) - W \cos \alpha_{xy} + R_2 \sin (\alpha_2 - \alpha_{xy}) \\
+ R_1 \sin (\alpha_{xy} - \alpha_1) + F_w \sin \alpha_{xy} = 0
\]

where $c'$ = effective cohesion intercept of soil at the base of the column

$A_b$ = the base area of the column

$\alpha_{xy}$ = the inclination of the intersection between the central section ($X$-$Y$ plane) and the base

$R_1, R_2$ = the shear forces acting on two ends 1 and 2

$\alpha_1, \alpha_2$ = the inclination of the intersection between two ends (end 1 and 2) and the base

$F_w$ = the water force existing (if tension crack is considered)

All other symbols $N'$, $\phi'_m$, $F$, $Q$, $\theta$ and $W$ have the same definition as before. From equation (3.31):
Fig. 3.16 Force Polygon of a Column in Spoon Shape Failure
\[ N' = -u A_b - Q \sin (\alpha_{xy} - \Theta) + W \cos \alpha_{xy} - R_2 \sin (\alpha_2 - \alpha_{xy}) \]
\[ - R_1 \sin (\alpha_{xy} - \alpha_1) - F_w \sin \alpha_{xy} \]  
\hspace{1cm} (3.32)

After substituting equation (3.32) into equation (3.30):

\[ Q = \left\{ \frac{a'}{F} A_b - u A_b \tan \phi_m + W \cos \alpha_{xy} \left( \tan \phi_m - \tan \alpha_{xy} \right) \right\} \]
\[ + R_2 \cos (\alpha_2 - \alpha_{xy}) \left\{ 1 - \tan \phi_m \tan (\alpha_2 - \alpha_{xy}) \right\} \]
\[ - R_1 \cos (\alpha_{xy} - \alpha_1) \left\{ 1 + \tan \phi_m \tan (\alpha_{xy} - \alpha_1) \right\} \]
\[ - F_w \cos \alpha_{xy} \left( 1 + \tan \phi_m \tan \alpha_{xy} \right) \}
\[ \{ \cos (\alpha_{xy} - \Theta) \left\{ 1 + \frac{\tan \phi}{F} \tan (\alpha_{xy} - \Theta) \right\} \} \]  
\hspace{1cm} (3.33)

If the whole system is in equilibrium, then the sum of all forces in the system must be equal to zero:

\[ \Sigma Q = 0 \]  
\hspace{1cm} (3.34)

The sum of all moment about any point 0 (Fig. 3.11) must be equal to zero:

\[ \Sigma Q \cos (\Theta - \alpha) (r - h_Q \cos \alpha) = 0 \]  
\hspace{1cm} (3.34a)

or

\[ \Sigma \left\{ Q r \cos (\Theta - \alpha) - Q h_Q \cos \alpha \cos (\Theta - \alpha) \right\} = 0 \]  
\hspace{1cm} (3.34b)

where the value of \( Q h_Q \) can be obtained by summing all moments in a column at the center of the base of that column (Fig. 3.14). This yields:
\[ Q \cos \theta h_Q + \frac{\cos \alpha_1}{F} \left( R_{cE1} \left( h_{F1} + \frac{h_{E1}}{2} - \frac{dz}{2} \tan \alpha_{yz} \right) + R_{\phi E1} \left( h_{F1} + \frac{h_{E1}}{3} - \frac{dz}{2} \tan \alpha_{yz} \right) \right) \]
\[ + R_{\phi F1} \left( y_{F1} - \frac{dz}{2} \tan \alpha_{yz} \right) \} - \frac{\cos \alpha_2}{F} \]
\[ = \frac{1}{6 \cos \theta F} \left( 3R_{cE2} \left( 2h_{F2} + h_{E2} + dz \tan \alpha_{yz} \right) \right) \]
\[ + 3R_{cF2} \left( h_{F2} + dz \tan \alpha_{yz} \right) + R_{\phi F2} \left( h_{F2} + \frac{h_{E2}}{3} + \frac{dz}{2} \tan \alpha_{yz} \right) \]
\[ + 3R_{\phi F2} \left( 2y_{F2} + dz \tan \alpha_{yz} \right) \] - \cos \alpha_1 \left( 3R_{cE1} \left( 2h_{F1} + h_{E1} \right) \right.
\[ - dz \tan \alpha_{yz} \) + 3R_{cF1} \left( h_{F1} - dz \tan \alpha_{yz} \right) + R_{\phi E1} \]
\[ (6h_{F1} + 2h_{E1} - 3dz \tan \alpha_{yz}) + 3R_{\phi F1} \left( 2y_{F1} - dz \tan \alpha_{yz} \right) \} \]

(3.35)

after rearranging the equation:

The parameters appearing in equation (3.36) are defined in Appendix A.

In these equations the only two unknowns are, (1) the inclination of interslice force \( \theta \) and (2) the factor of safety \( F \). Consequently the system of equations can be solved by the secant's method for nonlinear equations.
3.3.5 The Mixed Type Failure

This type of failure is a combination of two kinds of geometry; (1) cylinder in the central portion attached by two cones at two ends (Fig. 2.12a) and (2) cylinder in the central portion attached by two semi-ellipsoids at two ends (Fig. 2.12b). Baligh and Azzouz (1975) examined both cases and found that case (2) is more critical than case (1). In the present study, case (2) is considered and the derivation is the same as those discussed in Sections 3.3.3 and 3.3.4. The computer program is written basically for this mixed type geometry.

3.4 Summary

1. A methodology has been developed to study the block type of failure. The critical failure surface is assumed to make \((45 + \phi/2)\) and \((45 - \phi/2)\) angles with the horizontal in active and passive zones, respectively. The factor of safety is the same along the total failure surface. The active and passive forces are therefore functions of the factor of safety.

2. Similarly a general approach has been proposed to analyze the rotational type of failure. The following assumptions have been made: (1) the failure mass is symmetrical; (2) no movement in the Z-direction; (3) the intercolumn shear forces are parallel to the base; (4) the intercolumn normal stress distribution is linear; (5) the intercolumn shear forces are functions of their positions; and (6) a unique value of \(\theta\), the inclination of the intercolumn shear forces, for the spoon shape of failure or various values of \(\theta\) for the roller type of failure.
3. The mixed type of failure is composed of either two semi-ellipsoids or two cones attached at the two ends of the central cylinder. The roller type of failure or spoon shape of failure is just a special case of the mixed type of failure.
IV. FINITE ELEMENT METHOD

4.1 Introduction

The limit equilibrium methods cannot determine strains and deformations within a potential sliding mass. Though it is possible to determine an approximate stress distribution on an assumed slip surface, each method is based on a different set of assumptions and the stress distributions differ considerably from one method to another. Often the limit equilibrium problem is statically indeterminate and different statically admissible solutions may be found for the stress distribution on the failure surface. Consequently, significantly different values of the factor of safety may result from different assumptions of stress distribution on a given slip surface (Lambe and Whitman, 1969). Thus, the factor of safety depends not only on the method of analysis but also on the assumed or implied stress distribution on the failure surface.

Besides, the limit equilibrium methods allow little or no consideration to be given to the history of slope formation, and the consequent initial stresses. In view of these limitations, it is desirable to supplement
the conventional stability analyses by stress-deformation studies. In
this chapter a three-dimensional finite element computer program is
developed to analyze the stability of slopes and embankments. This
program, FESPON, uses hyperbolic stress-strain relationship and iso-
parametric elements with incompatible displacement modes.

4.2 Basis of the Method

Fig. 4.1 shows a continuum divided into discrete parts called 'elements'. These elements are separated from each other by imaginary
surfaces and are assumed to be interconnected only at a finite number
of nodal points situated on their boundaries. In geotechnical appli-
cations the most convenient formulation of the finite element method
is for a compatible model in which nodal point displacements are
assumed to be the only unknowns. This is generally known as the dis-
placement formulation.

The relationship between generalized displacements \( \{\mathbf{f}\} \) and nodal
displacements \( \{\delta\} \) may be expressed as:

\[
\{\mathbf{f}\} = [\mathbf{N}] \{\delta\} \tag{4.1}
\]

in which the matrix \( [\mathbf{N}] \) depends only on the shapes and sizes of ele-
ments. The strains \( \{\varepsilon\} \) are related to the displacements as follows,
assuming deformations to be small:

\[
\{\varepsilon\} = [\mathbf{B}] \{\delta\} \tag{4.2}
\]

in which the matrix \( [\mathbf{B}] \) depends only on the nodal point coordinates.
The stresses are related to the strains by an appropriate matrix \( [\mathbf{D}] \):
\[ \{\sigma\} = (D) \{\varepsilon\} \]  

(4.3)

For isotropic elastic materials, \((D)\) is dependent only on the modulus of elasticity \(E\) and the Poisson's ratio \(\nu\). In geotechnical problems it is often desirable to express \((D)\) in terms of shear modulus \(G\) and bulk modulus \(K\) which are functions of \(E\) and \(\nu\).

Considering the applied nodal forces and distributed loads, the total potential energy of the system comprising the assemblage of elements and the external loads must be a minimum (from the principle of minimum potential energy). This requirement leads to a relationship between the nodal forces and displacements for each element. Since each node may be common to several elements, these relationships require assembly in an appropriate manner and the complete system of equations may be written as follows:

\[ \{k\} \{\delta\} = \{F\} \]  

(4.4)

in which \(\{\delta\}\) = the nodal displacement matrix  
\(\{F\}\) = the resultant nodal forces  
\(\{k\}\) = the combined stiffness matrix for the assemblage of elements which approximate the continuum

The stiffness matrix \(\{k\}\) is assembled from individual element stiffness matrices \(\{k_e\}\) which depend on matrices \(\{B\}\) and \(\{D\}\) as follows:

\[ \{k_e\} = \int B^T D B \, dV \]  

(4.5)

in which the integration is over the volume of each element in a X, Y, Z-coordinate system. The assemblage and solution of this system of
The finite element computer program solves the simultaneous equations to obtain the displacements at each point and subsequently computes the strains and stresses. The details of formulation, assembly, and solution are discussed in many references (Zienkiewicz, 1971; Cook, 1973; and Desai and Abel, 1972).

4.3 Hyperbolic Strain-Strain Relationship

Konder and his co-workers (1963) have shown that the stress-strain curves for a number of remolded cohesive soils, tested in consolidated-undrained triaxial compression, could be approximated by hyperbolas like the one shown in Fig. 4.2. The equation of this hyperbola is:

$$ (\sigma_1 - \sigma_3) = \frac{1}{E_i} \frac{\varepsilon}{(\sigma_1 - \sigma_3)_{ult}} $$

where $E_i$ is the initial tangent modulus or the initial slope of the stress-strain curve and $(\sigma_1 - \sigma_3)_{ult}$ is the asymptotic value of stress difference which is closely related to the strength of the soil. The value of $(\sigma_1 - \sigma_3)_{ult}$ is always greater than the stress difference at failure for the soil. When triaxial test data are plotted on the transformed plot as in the lower part of Fig. 4.2, the points frequently are found to deviate from the ideal linear relationship. Experience indicates that a good match is usually achieved by selecting straight lines passing through the points where 70% and 95% of the strength are mobilized (Duncan and Chang, 1970; Kulhawy, Duncan, and Seed, 1969; Hansen, 1963; Daniel and Olson, 1974). Thus, in practice, only two points, the 70% and 95% mobilization points, are plotted on the transformed diagram.
Real

\[
(\sigma_1 - \sigma_3) = \frac{\varepsilon}{\frac{1}{E_i} + \frac{\varepsilon}{(\sigma_1 - \sigma_3)_{ult}}}
\]

Transformed

\[
\frac{\varepsilon}{(\sigma_1 - \sigma_3)} = \frac{1}{E_i} + \frac{\varepsilon}{(\sigma_1 - \sigma_3)_{ult}}
\]

Fig. 4.2 Hyperbolic Representation of a Stress-Strain Curve
In order to take into account the increase in strength or a steeper stress-strain curve due to the increase in confining pressure $\sigma_3$, Janbu (1970) suggested the following equations (Fig. 4.3):

$$E_i = K P_a \left(\frac{\sigma_3}{P_a}\right)^n$$

(4.7)

in which $K$ is the modulus number, and $n$ is the modulus exponent. Both are dimensionless numbers. $P_a$ is the atmospheric pressure which is introduced to make conversion from one system of units to another more convenient. The variation of $(\sigma_1 - \sigma_3)_{ult}$ with $\sigma_3$ is accounted for in Fig. 4.4 by relating $(\sigma_1 - \sigma_3)_{ult}$ to the stress difference at failure $(\sigma_1 - \sigma_3)_f$, and then using the Mohr-Coulomb strength equation to relate $(\sigma_1 - \sigma_3)_f$ to $\sigma_3$. The values of $(\sigma_1 - \sigma_3)_{ult}$ and $(\sigma_1 - \sigma_3)_f$ are related by:

$$(\sigma_1 - \sigma_3)_f = R_f (\sigma_1 - \sigma_3)_{ult}$$

(4.8)

in which $R_f$ is the failure ratio. The value of $R_f$ is always smaller than unity, and varies from 0.5 to 0.9 for most soils (Wong and Duncan, 1974). The variation of $(\sigma_1 - \sigma_3)_f$ with $\sigma_3$ is represented by the Mohr-Coulomb strength relationship, which can be expressed as follows:

$$\sigma_1 \sigma_3 = \sigma_3 = \frac{2c \cos \phi + 2\sigma_3 \sin \phi}{1 - \sin \phi}$$

(4.9)

in which $c$ and $\phi$ are the cohesion intercept and the friction angle, as shown in Fig. 4.4.

The tangent modulus $E_t$ is obtained by differentiating equation (4.6) with respect to $\varepsilon$:

$$E_t = \frac{\partial (\sigma_1 - \sigma_3)}{\partial \varepsilon} = E_i (1 - \frac{E_i \varepsilon}{(\sigma_1 - \sigma_3)_{ult} + E_i \varepsilon})^2$$

(4.10)
Fig. 4.3 Variation of Initial Tangent Modulus with Confining Pressure
\[
\frac{(\sigma_1 - \sigma_3)_f}{R_f (\sigma_1 - \sigma_3)_{ult}} = \frac{2c \cos \phi + 2\sigma_3 \sin \phi}{1 - \sin \phi}
\]

Fig. 4.4 Variation of Strength with Confining Pressure
Also, after rearranging equation (4.6):

$$E_i \varepsilon = \frac{\sigma_1 - \sigma_3}{1 - \frac{\sigma_1 - \sigma_3}{(\sigma_1 - \sigma_3)_{ult}}}$$  \hspace{1cm} (4.11)

Substituting equations (4.11), (4.8), (4.9), and (4.7) in equation (4.10) leads to:

$$E_t = E_i \left[ 1 - \frac{\sigma_1 - \sigma_3}{(\sigma_1 - \sigma_3)_{ult}} \right]^2$$

$$= E_i \left[ 1 - \frac{R_f (\sigma_1 - \sigma_3)}{(\sigma_1 - \sigma_3)f} \right]^2$$

$$= E_i \left[ 1 - \frac{R_f (\sigma_1 - \sigma_3)(1 - \sin \phi)}{2c \cos \phi + 2\sigma_3 \sin \phi} \right]$$  \hspace{1cm} (4.12)

$$= K_P \frac{\sigma_3}{P_a} \left[ 1 - \frac{R_f (\sigma_1 - \sigma_3)(1 - \sin \phi)}{2c \cos \phi + 2\sigma_3 \sin \phi} \right]$$

If a triaxial specimen is unloaded at some stage during the test, the stress-strain curve followed during unloading is steeper than the curve followed during primary loading, as shown in Fig. 4.5. During subsequent reloading, the stress-strain curve is also steeper than the curve for primary loading and is quite similar in shape to the unloading curve. It is usually reasonably accurate to assume the same value of unloading-reloading modulus $E_{ur}$ for both unloading and reloading.

Similar to $E_i$, $E_{ur}$ is expressed as:

$$E_{ur} = K_{ur} P_a \frac{\sigma_3}{P_a}$$  \hspace{1cm} (4.13)

The unloading-reloading modulus number $K_{ur}$ may be 20% greater than the primary loading modulus number $K$ for stiff soil such as dense sands. For soft soils, such as loose sand, $K_{ur}$ may be three times as
Fig. 4.5 Unloading - Reloading Modulus

\[ E_{ur} = K_{ur}P_a \left( \frac{\sigma_3}{P_a} \right)^n \]
large as $K$. The value of the exponent $n$ is assumed to be the same for both primary loading and unloading.

If the axial and volumetric strains are measured during the triaxial test, it is convenient to calculate the radial strain $\varepsilon_r$ using:

$$\varepsilon_r = \frac{1}{2} (\varepsilon_v - \varepsilon_a)$$  \hspace{1cm} (4.14)

in which $\varepsilon_v$ and $\varepsilon_a$ are the volumetric and axial strains, respectively. Taking compressive strains as positive, the value of $\varepsilon_a$ is positive and the value of $\varepsilon_r$ is negative, the value of $\varepsilon_v$ may be either positive or negative.

If the variation of $\varepsilon_a$ with $\varepsilon_r$ is plotted as shown in Fig. 4.6, the resulting curve can be reasonably represented by a hyperbolic equation of the form:

$$\varepsilon_a = \frac{-\varepsilon_r}{v_i - d \varepsilon_r}$$

or:

$$-\frac{\varepsilon_r}{\varepsilon_a} = v_i - d \varepsilon_r$$  \hspace{1cm} (4.15)

in which $v_i$ is the initial Poisson's ratio (at zero strain) and $d$ is a parameter representing the change in the value of Poisson's ratio with radial strain. For saturated soils under undrained conditions, there is no volume change and $v_i$ is equal to 0.5 for any value of confining pressure. For most other soils the value of $v_i$ decreases with confining pressures as shown in Fig. 4.7, and this variation of $v_i$ with $\sigma_3$ may be expressed by the equation:
Fig. 4.6 Hyperbolic Axial Strain - Radial Strain Curves

Real
\[ \varepsilon_a = \frac{-\varepsilon_r}{\nu_i - d\varepsilon_r} \]

Transformed
\[ \frac{-\varepsilon_r}{\varepsilon_a} = \nu_i - d\varepsilon_r \]
Fig. 4.7 Variation of Initial Tangent Poisson's Ratio with Confining Pressure

\[ \nu_i = G - F \log \left( \frac{\sigma_3}{P_a} \right) \]
\[ v_1 = G - F \log_{10} \left( \frac{\sigma_3}{P_a} \right) \] 

(4.16)

in which \( G \) is the value of \( v_1 \) at a confining pressure of one atmosphere, and \( F \) is the reduction in \( v_1 \) for a ten-fold increase in \( \sigma_3 \).

The slope of the curve representing the variation of \( \varepsilon_e \) with \( \varepsilon_a \) is \( v_t \). This tangent value of Poisson's ratio is expressed in terms of the stresses as follows (Kulhawy, Duncan, and Seed, 1969):

\[ v_t = \frac{G - F \log \left( \frac{\sigma_3}{P_a} \right)}{1 - \left\{ \frac{\sigma_3}{P_a} \left[ \frac{1}{2} K P_a \left( \frac{\sigma_3}{P_a} \right) n \left[ 1 - \frac{R_f (\sigma_3 - \sigma_1)}{2c \cos \phi + 2 \sigma_3 \sin \phi} \right] \right] \right\} ^2} \] 

(4.17)

The nine parameters of the hyperbolic stress-strain relationships and their functions are summarized in Table 4.1.

Frequently, it is impractical to perform drained triaxial tests on soils of low permeability because of the length of time required. In such cases it is possible to determine the values of \( K \) and \( n \) from consolidation data if the values of \( c' \), \( \phi' \), and \( R_f \) are known. The effective stress parameters \( c' \) and \( \phi' \) may be determined from the results of \( \text{CU} \) tests, and the value of \( R_f \) may be estimated on the basis of values determined for similar soils. Values of \( E_1 \) may be calculated using the following equation (Clough and Duncan, 1969):
Table 4.1 SUMMARY OF THE HYPERBOLIC PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Name</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K, K_{ur}$</td>
<td>Modulus number</td>
<td>Relate $E_i$ and $E_{ur}$ to $\sigma_3$</td>
</tr>
<tr>
<td>$n$</td>
<td>Modulus exponent</td>
<td></td>
</tr>
<tr>
<td>$c$</td>
<td>Cohesion intercept</td>
<td>Relate $(\sigma_1-\sigma_3)_r$ to $\sigma_3$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Friction angle</td>
<td></td>
</tr>
<tr>
<td>$R_f$</td>
<td>Failure ratio</td>
<td>Relates $(\sigma_1-\sigma_3)_\text{ult}$ to $(\sigma_1-\sigma_3)_r$</td>
</tr>
<tr>
<td>$G$</td>
<td>Poisson's ratio parameter</td>
<td>Value of $v_i$ at $\sigma_3 = p_a$</td>
</tr>
<tr>
<td>$F$</td>
<td>Poisson's ratio parameter</td>
<td>Decrease in $v_i$ for ten-fold increase in $\sigma_3$</td>
</tr>
<tr>
<td>$d$</td>
<td>Poisson's ratio parameter</td>
<td>Rate of increase of $v_t$ with strain</td>
</tr>
</tbody>
</table>
\[ E_i = \frac{\Delta p(l + e_o)}{\Delta e} \left\{ 1 - \frac{2K_o^2}{(1 + K_o)} \right\} \]
\[ \left\{ 1 - \frac{p(1 - K_o)^2}{K_o p(tan^2(45 + \phi'/2) - 1) + 2c' tan(45 + \phi'/2)} \right\}^2 \]

in which \( E_i \) = initial tangent modulus, as defined previously

\( \Delta p = \) increment of pressure in consolidation test
\( e_o = \) void ratio at beginning of pressure increment
\( \Delta e = \) decrease in void ratio due to \( \Delta p \)
\( K_o = \) coefficient of earth pressure at rest
\( p = \) average pressure during increment
\( c' = \) cohesion intercept
\( \phi' = \) angle of internal friction
\( R_f = \) failure ratio

The value of \( K_o \) may be estimated from the test results of Brooker and Ireland (1965), which are shown in Fig. 4.6. When values of \( E_i \) have been determined for several different load increments, they are plotted against the corresponding values of \( \sigma_3 \) to determine the value of \( K \) and \( n \) for the soil. The average value of \( \sigma_3 \) during each increment is calculated using the equation:

\[ \sigma_3 = K_o p \] (4.19)

The values of the unloading-reloading modulus number can be determined from the rebound curve in the consolidation test, using the following equation adapted from Clough and Duncan (1969):

\[ \sigma_3 = K_o p \] (4.19)
Fig. 4.8 Relation between $K_o$ and $I_p$ for Various Values of Overconsolidation Ratio (after Brooker and Ireland)
\[ E_{ur} = \frac{\Delta p (1 + e_0)}{\Delta e} \left\{ 1 - \frac{2(K_0^\Delta)^2}{(1 + K_0^\Delta)} \right\} \]  

(4.20)

in which \( K_0^\Delta \) is the ratio of change in lateral stress to change in vertical stress during unloading in a consolidation test. Values of \( K_0^\Delta \) were derived from the data of Brooker and Ireland (1965), and the variation of \( K_0^\Delta \) with the plasticity index \( I_p \) is shown in Fig. 4.9. Clough and Duncan (1969) recommended that \( E_{ur} \) be determined at the point on the curve where the pressure has been reduced to half of its value before unloading. Once a value of \( E_{ur} \) has been defined, the value of \( K_{ur} \) for the soil may be calculated using the equation:

\[ K_{ur} = \frac{E_{ur}}{P_a (\sigma_3^{1/3})^n} \]  

(4.21)

with the value of \( n \) determined from the primary loading data, and the value of \( \sigma_3 \) determined from equation (4.19).

4.4 Three-Dimensional Finite Element Computer Program - FESPON

The three-dimensional finite element computer program, FESPON, developed for the present study has been generated from the two-dimensional program ISBILD (Ozawa, 1973). The program ISBILD itself is an improved version of the older program LSBUILD developed by Kulhawy, Duncan, and Seed (1969). These two programs employed the same hyperbolic stress-strain relationship and accommodated the nonlinear behavior of soil by an incremental procedure. The program ISBILD used isoparametric elements with incompatible displacement modes and a more accurate procedure to assign initial stresses to elements. The program
Fig. 4.9  Correlation Between $K_o$ and $I_p$ for Various Values of Overconsolidation Ratios (after Clough and Duncan)
FESPOW keeps the main features of these two-dimensional programs but is able to perform three-dimensional analyses.

4.4.1 Nonlinear Incremental Finite Element Method

The nonlinear behavior of soil can be simulated by the successive increments procedure, in which the loading is assumed to be linear within each increment. The modulus values for each element are reevaluated during each increment in accordance with the stresses in the element.

The incremental stress-strain relationship for an isotropic material may be expressed in the form:

\[
\begin{align*}
\begin{bmatrix}
\Delta \sigma_x \\
\Delta \sigma_y \\
\Delta \sigma_z \\
\Delta \tau_{xy} \\
\Delta \tau_{yz} \\
\Delta \tau_{xz}
\end{bmatrix}
&= \frac{E_t}{(1+\nu_t)(1-2\nu_t)} \begin{bmatrix}
\nu_t & (1-\nu_t) & \nu_t & 0 & 0 & 0 \\
\nu_t & \nu_t & (1-\nu_t) & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{2}{(1-2\nu_t)} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1-2\nu_t}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1-2\nu_t}{2}
\end{bmatrix}
\begin{bmatrix}
\Delta \varepsilon_x \\
\Delta \varepsilon_y \\
\Delta \varepsilon_z \\
\Delta \gamma_{xy} \\
\Delta \gamma_{yz} \\
\Delta \gamma_{xz}
\end{bmatrix}
\end{align*}
\]

(4.22)

in which \( \Delta \sigma \) and \( \Delta \tau \) are stress increments, \( \Delta \varepsilon \) and \( \Delta \gamma \) strain increments, \( E_t \) the tangent modulus, and \( \nu_t \) the tangent Poisson's ratio. These two parameters are obtained from equations (4.12) and (4.17), respectively.

In order to represent post-failure behavior of soils more accurately, Clough and Woodward (1967) suggested the stress-strain relationship in an alternate form:
in which $M_b = E_t/2(1+\nu_t)(1-2\nu_t)$ and $M_d = E_t/2(1+\nu_t)$. The fact that soils have high resistance to volumetric compression after failure but very low resistance to shearing may be represented by reducing the value of $M_d$ to zero after failure, while $M_b$ is maintained at the value it had in the increment before failure.

It has been found that one of the most effective methods of simulating fill placement is the "average stress" procedure (Ozawa and Duncan, 1973), in which the average stresses during an increment are used for evaluating the modulus and Poisson's ratio. Each increment is analyzed twice, the first time using tangent modulus and Poisson's ratio values based on the stresses at the beginning of the increment, and the second time using tangent modulus and Poisson's ratio values based on the average stresses during the increment. If the stress level decreases during the increment, the unloading-reloading modulus $E_u$ is used in the second evaluation.

4.4.2 Isoparametric Elements

The simplest isoparametric elements are the compatible isoparametric elements which use the same interpolation functions for both the element geometry and the element displacement fields. The geometry
and displacement functions are expressed as:

\[
\begin{bmatrix}
  x \\
  y \\
  z
\end{bmatrix} = \sum_{i=1}^{N} \phi_i(\xi, \eta, \zeta) \begin{bmatrix}
  x_i \\
  y_i \\
  z_i
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
  u_x \\
  u_y \\
  u_z
\end{bmatrix} = \sum_{i=1}^{N} \phi_i(\xi, \eta, \zeta) \begin{bmatrix}
  u_{xi} \\
  u_{yi} \\
  u_{zi}
\end{bmatrix}
\]

in which \(\phi_i\) are interpolation functions in terms of local coordinate \(\xi, \eta, \text{ and } \zeta\), (\(x_i, y_i, z_i\)) global nodal point coordinates, and (\(u_{xi}, u_{yi}, u_{zi}\)) nodal point displacements. It has been shown that compatible isoparametric elements possess poor bending characteristics (Wilson, et al., 1971; Wilson, 1971). Incompatible isoparametric elements use a higher order approximation for the displacements than for the geometry. The additional extra degrees of freedom within the element produce a parabolic incompatibility along the element boundaries. However, the resulting element has good bending characteristics. The displacement functions for the incompatible modes are of the form:

\[
\begin{bmatrix}
  u_x \\
  u_y \\
  u_z
\end{bmatrix} = \sum_{i=1}^{N} \phi_i(\xi, \eta, \zeta) \begin{bmatrix}
  u_{xi} \\
  u_{yi} \\
  u_{zi}
\end{bmatrix} + \sum_{j=1}^{M} \psi_j(\xi, \eta, \zeta) \begin{bmatrix}
  \alpha_{xj} \\
  \alpha_{yj} \\
  \alpha_{zj}
\end{bmatrix}
\]

in which \(\psi_j\) are interpolation functions for the displacement amplitudes \(\alpha_{xj}, \alpha_{yj}, \text{ and } \alpha_{zj}\), which are additional degrees of freedom. For
the eight-node element the displacement approximation may be of the following form:

\[\begin{align*}
\mathbf{u}_x &= \sum_{i=1}^{8} \phi_i u_{x_i} + \psi_1 \alpha x_1 + \psi_2 \alpha x_2 + \psi_3 \alpha x_3 \\
\mathbf{u}_y &= \sum_{i=1}^{8} \phi_i u_{y_i} + \psi_1 \alpha y_1 + \psi_2 \alpha y_2 + \psi_3 \alpha y_3 \\
\mathbf{u}_z &= \sum_{i=1}^{8} \phi_i u_{z_i} + \psi_1 \alpha z_1 + \psi_2 \alpha z_2 + \psi_3 \alpha z_3
\end{align*}\]

\[\begin{align*}
\psi_1 &= 1 - \xi^2 \\
\psi_2 &= 1 - \eta^2 \\
\psi_3 &= 1 - \zeta^2
\end{align*}\]

where

\[\begin{align*}
\phi_1 &= \frac{1}{6} (1+\xi)(1+\eta)(1+\zeta) \\
\phi_2 &= \frac{1}{6} (1-\xi)(1+\eta)(1+\zeta) \\
\phi_3 &= \frac{1}{6} (1-\xi)(1-\eta)(1+\zeta) \\
\phi_4 &= \frac{1}{6} (1+\xi)(1-\eta)(1+\zeta) \\
\phi_5 &= \frac{1}{6} (1+\xi)(1+\eta)(1-\zeta) \\
\phi_6 &= \frac{1}{6} (1-\xi)(1+\eta)(1-\zeta) \\
\phi_7 &= \frac{1}{6} (1-\xi)(1-\eta)(1-\zeta) \\
\phi_8 &= \frac{1}{6} (1+\xi)(1-\eta)(1-\zeta)
\end{align*}\]

The functions \( \psi_1, \psi_2, \) and \( \psi_3 \) must be zero at the eight nodes. Therefore, the resulting element stiffness matrix will be 33x33. However, if the strain energy within the element is minimized with respect to \( \alpha \), the additional displacements can be eliminated and a reduced 24x24 stiffness matrix developed. This is identical to the standard static condensation procedure.

4.4.3 Initial Stresses and Procedure of Analysis

For accurate estimation of stresses and displacements, the analyses are performed by dividing the placement of fill into eight
or more construction layers. The stresses in each layer due to its own weight immediately after placement are assigned rather than calculated. For elements under a horizontal surface the initial vertical stresses are taken to be equal to the overburden pressure. The initial horizontal stresses are taken as $\nu/(1-\nu)$ times the overburden pressure, where $\nu$ is the Poisson's ratio. The shear stresses on horizontal and vertical planes are assumed to be equal to zero. For elements under a sloping surface, estimation of initial stresses is more difficult. The assumptions made by Ozawa and Duncan (1973) in the program ISBILD are used in the present analysis:

\[
\begin{align*}
\sigma_x &= \sigma_z = \frac{\nu}{1-\nu} \rho h \\
\sigma_y &= \rho h \\
\tau_{xy} &= 0.5 \rho h \sin \alpha_{xy} \\
\tau_{yz} &= 0.5 \rho h \sin \alpha_{yz} \\
\tau_{xz} &= 0
\end{align*}
\]

in which $\rho h$ is the overburden pressure at the center of the element, $\nu$ the Poisson's ratio, and $\alpha$ the angle of slope of the surface above the element.

The layer being placed is assigned very small modulus values to simulate the fact that a newly added layer of fill on an embankment has very low stiffness. The nodal points at the top of the newly placed layer are assigned zero displacement, i.e., the positions of these nodal points immediately after placement are taken as the reference
positions for measuring movements due to subsequent loading. The
strains in the newly placed elements are set equal to zero also, thus
taking the condition immediately after placement as the reference state
for strains.

Each increment of loading is analyzed twice. The changes in
stress, strain and displacement during each increment are added to the
stresses, strains and displacements existing at the beginning of the
increment. These resulting values are then used in the next
increment.

The program is capable of handling embankments on rigid or com-
pressible foundations. For a compressible foundation, the initial
stresses are set as:

\[
\begin{align*}
\sigma_y &= \rho h \\
\sigma_x &= \sigma_z = K_0 \rho h \\
\tau_{xy} &= \tau_{yz} = \tau_{xz} = 0
\end{align*}
\]

For more details about the subroutines and their functions, refer
to Appendix B.

4.5 Summary

A three-dimensional computer program FESPON is generated from the
two-dimensional program ISBILD. The hyperbolic stress-strain relation-
ship is combined with an incremental technique to simulate the nonlinear
behavior of soils. Isoparametric incompatible elements are used in order
to provide good bending characteristics. The parameters necessary to
the analysis can be obtained from triaxial and consolidation test data.
If such data are not available, these parameters can be estimated from values and relationships determined for similar soils by previous investigators. The next chapter will present practical applications of the computer program FESPON.
V. RESULTS AND APPLICATIONS

5.1 Introduction

In the previous chapters, several models were developed to analyze the stability of embankments. In Chapter III, three-dimensional limit equilibrium methods were proposed to study both translational and rotational slides. These methods were implemented in the computer programs BLOCK3 and LEMIX for translational and rotational failures, respectively. In Chapter IV, a three-dimensional finite element computer program FESPON was developed to simulate the construction of embankment. This program makes allowance for the nonlinear stress-strain behavior of soils.

This chapter describes typical applications of these three-dimensional models. The factors of safety obtained with the three-dimensional models are compared with the ones obtained with the two-dimensional models. Results obtained with the three-dimensional finite element computer program are also presented and compared to the results obtained with the limit equilibrium methods.

5.2 Analysis of Translational Slides

In this section the computer program BLOCK3 is used to analyze the stability of highway embankments. This program was developed to study three-dimensional translational slides; the derivation of
equilibrium equations and the solution techniques have been discussed in Section 3.2.2.

Translational slides can occur in an embankment when a weak soil layer is present in the foundation soil. This is the problem studied herein. Table 5.1 lists all the geometric and soil parameters necessary to such an analysis. In the following application the ground surface is horizontal and the embankment geometry is assumed as: height of 6.1 m (20 ft), crown width of 12.2 m (40 ft), and slope of 1.5/1. These dimensions are typical for highway embankments in Indiana. The embankment and foundation soils are the same with average density $\rho$ of 1930 kg/m$^3$ (120 pcf). The frictional angle of the weak soil $\phi_w$ is taken as equal to zero. These assumptions are not necessary to the program BLOCK3, but they are made to simplify the discussion of the results. The other parameters used in the study are listed in Table 5.2. Several of these parameters are varied in order to assess their effects on the factor of safety against translational sliding. In particular different values were given to: (1) the strength parameters of the embankment and foundation soils; (2) the strength parameters of the weak layer; (3) the inclination of the weak layer; (4) the depth to weak layer; (5) the inclination of the ends of the central block; and (6) the length ratio ($a$). Factors of safety of the embankment against sliding are computed for several combinations of these parameters, using the program BLOCK3. In all these analyses the stability is investigated to the side of the down-dipping weak seam, which is the most critical case (Boutrup, 1977).
TABLE 5.1 VARIABLES AND SYMBOLS

<table>
<thead>
<tr>
<th>Variables</th>
<th>Symbols*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height and width of an embankment</td>
<td>( H, B )</td>
</tr>
<tr>
<td>The upper and lower length at the top of the central block</td>
<td>( L, aL )</td>
</tr>
<tr>
<td>Depth to weak layer, measured from the toe</td>
<td>( D )</td>
</tr>
<tr>
<td>Inclination of ground surface</td>
<td>( i )</td>
</tr>
<tr>
<td>Inclination of weak soil layer</td>
<td>( \beta )</td>
</tr>
<tr>
<td>Inclination of left and right slope of embankment</td>
<td>( \beta_L, \beta_R )</td>
</tr>
<tr>
<td>Inclination of the ends of the block</td>
<td>( \gamma )</td>
</tr>
<tr>
<td>Strength of embankment soil</td>
<td>( (c, \phi)_E )</td>
</tr>
<tr>
<td>Strength of foundation soil</td>
<td>( (c, \phi)_F )</td>
</tr>
<tr>
<td>Strength of weak soil</td>
<td>( (c, \phi)_w )</td>
</tr>
</tbody>
</table>

* Refer to Fig. 3.1
TABLE 5.2 SYMBOLS AND RANGE OF VARIABLES FOR AN EMBANKMENT BUILT ON A FOUNDATION SOIL WITH A WEAK SOIL LAYER

<table>
<thead>
<tr>
<th>Variables</th>
<th>Symbols</th>
<th>Value or Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Height of embankment</td>
<td>H</td>
<td>6.1 m (20 ft)</td>
</tr>
<tr>
<td>2. Width of embankment</td>
<td>B</td>
<td>12.2 m (40 ft)</td>
</tr>
<tr>
<td>3. The upper length at the top of the central block</td>
<td>L</td>
<td>6.1 - 97.5 m (20-320 ft)</td>
</tr>
<tr>
<td>4. The ratio of the lower length to upper length at the top of the central block</td>
<td>a</td>
<td>0 - 1.0</td>
</tr>
<tr>
<td>5. Depth to weak layer, measured from the toe</td>
<td>D</td>
<td>1.5 - 12.2 m (5 - 40 ft)</td>
</tr>
<tr>
<td>6. Inclination of ground surface</td>
<td>i</td>
<td>0°</td>
</tr>
<tr>
<td>7. Inclination of weak soil layer</td>
<td>β</td>
<td>0° - 11.3° (0 - 5(1))</td>
</tr>
<tr>
<td>8. Inclination of left and right slope of embankment</td>
<td>(\beta_L/\beta_R)</td>
<td>33.7° (1.5(1))</td>
</tr>
<tr>
<td>9. Inclination of the ends of the block</td>
<td>γ</td>
<td>70° - 90°</td>
</tr>
<tr>
<td>10. Average density of soil</td>
<td>(\rho)</td>
<td>1930 kg/m(^3) (120 pcf)</td>
</tr>
<tr>
<td>11. Strength of embankment soil</td>
<td>(c_E, \phi_E)</td>
<td>47.9 kPa (1000 psf), 0°</td>
</tr>
<tr>
<td></td>
<td></td>
<td>24.0 kPa (500 psf), 10°, 0°, 35°</td>
</tr>
<tr>
<td>12. Strength of foundation soil</td>
<td>(c_F, \phi_F)</td>
<td>same as 11</td>
</tr>
<tr>
<td>13. Strength of weak soil</td>
<td>(c_w)</td>
<td>9.6 - 28.7 kPa (200 - 600 psf)</td>
</tr>
</tbody>
</table>
The most significant results of these analyses are presented in Figs. 5.1 to 5.5 and are discussed below. The reader can refer to Appendix C (Tables C.1 to C.9), to obtain a complete description of all the results developed in this study.

In Fig. 5.1 the ratio \( F_3/F_2 \) of the 3-D factor of safety to the 2-D factor of safety is plotted versus the length ratio of the embankment \( L/H \), for several values of the depth ratio \( D/H \). The length ratio \( L/H \) is the ratio of the length of the embankment \( L \) to the height of the embankment \( H \), while the depth ratio \( D/H \) is the ratio of the depth to the weak layer \( D \) to the height of the embankment \( H \). In these analyses, the weak layer is horizontal (\( \beta = 0^\circ \)) and has a cohesion intercept \( c_w \) of 9.6 kPa. This combination of \( \beta \) and \( c_w \) gives the highest \( F_3/F_2 \) ratios (Tables C.1 to C.3). Two sets of strength parameters are considered for the embankment and foundation soil: (1) \( c = 47.9 \) kPa, \( \phi = 0^\circ \) (solid lines); and (2) \( c = 0, \phi = 35^\circ \) (dotted lines). The following conclusions can be drawn from Fig. 5.1:

- The ratio \( F_3/F_2 \) increases with decreasing length ratio \( L/H \).
  
  This three-dimensional effect is more important for cohesive soils than for cohesionless soils.

- For cohesive soils the ratio \( F_3/F_2 \) decreases with the depth ratio \( D/H \). On the contrary, for cohesionless soils, the ratio \( F_3/F_2 \) increases with decreasing depth ratio \( D/H \).

It is obvious that as the length \( L \) gets smaller, the end resistances play a more important role, and consequently a higher factor of safety is obtained with the 3-D method.
Fig. 5.1  $F_3/F_2$ vs. $L/H$ for Various $D/H$ and Soil Parameters
(at $a=1$, $\beta=0^\circ$, $\gamma=90^\circ$, and $c_w=9.6$ kPa)
Fig. 5.2 illustrates the effect of the strength parameter of the weak soil $c_w$ on the $F_3/F_2$ ratio. The solid lines are for a cohesion intercept $c_w$ of 9.6 kPa and the dotted lines for $c_w$ of 28.8 kPa. For two kinds of foundation soil studied, a lower $c_w$ value results in higher $F_3/F_2$ ratios.

Fig. 5.3 presents the effect of the inclination of the weak soil layer $\beta$ on the $F_3/F_2$ ratio. This figure shows that, for any combinations of depth ratio $D/H$ and soil strength, a steeply inclined weak soil layer always yields smaller $F_3/F_2$ ratios.

When the end of the block tilts from an angle $\gamma$ of 90° (vertical ends) to a smaller value (inclined ends), the end area will increase. Hence, the end resistance gets larger and higher $F_3/F_2$ ratios are obtained. This phenomenon is shown in Fig. 5.4 in which $L/H$ is set to unity. As the ratio $L/H$ increases, this increase in the $F_3/F_2$ ratio with decreasing inclination $\gamma$ will certainly be less significant.

It is also predictable that as the front area of the central block gets smaller, which is close to a wedge type of failure, both the passive resistance and the bottom resistance will be reduced. However, the ends area will increase and produce more resistance along the ends of the block. In Fig. 5.5, when $L/H$ ratio is small, the increase of ends resistance may be larger than the decrease of the resistance both from the passive force and the bottom resistance. Therefore, the net resistance is positive and higher $F_3/F_2$ ratios obtained. As $L/H$ ratio approaches a critical value, the net resistance will be negative, and the 3-D factor of safety $F_3$ will be less than the 2-D factor of safety $F_2$, i.e., the $F_3/F_2$ ratio is less than unity.
Fig. 5.2 \( F_3/F_2 \) vs. \( L/H \) for Various \( c_w \) and \( D/H \) (at \( a = 1, \beta = 0^\circ, \) and \( \gamma = 90^\circ \))
Fig. 5.3 $F_3/F_2$ vs. $L/H$ for Various $\beta$ and $D/H$
(at $a = 1$, $y = 90^\circ$, and $c_w = 9.6\text{kPa}$)
Fig. 5.4 $F_3/F_2$ vs. $D/H$ for Various $\gamma$ and $c_w$ (at $L/H = 1$, $a = 1$, and $\beta = 0^\circ$)
Fig. 5.5 $F_3/F_2$ vs. $L/H$ for Various $D/H$ and Soil Parameters
(at $a = 0.8$, $\beta = 0^\circ$, $\gamma = 90^\circ$, and $c_w = 9.6\, \text{kPa}$)
In summary, the most important results obtained from this study are as follows:

1. For translational sliding, the $F_3/F_2$ ratio is usually greater than unity. At small values of $L/H$, this 3-D effect is more significant for cohesive soils than for cohesionless soils.

2. The depth ratio has some effect on the $F_3/F_2$ ratio as shown in Fig. 5.1.

3. For all soils, cohesive or cohesionless, a lower strength of the weak layer may cause a higher three-dimensional effect.

4. A steep weak soil layer always yields smaller $F_3/F_2$ ratios than a gently inclined layer.

5. Reducing the inclination of the ends of the central block cause a higher factor of safety due to the increase in end areas.

6. Wedge type of failure will result in the value of $F_3/F_2$ less than unity, and therefore the stability of a slope needs to be examined carefully when there is potential for such a failure.

5.3 Analysis of Rotational Slides

In this section, the rotational slide will be studied. The soil is assumed to be homogeneous. The 3-D failure surface is composed of a central cylinder attached by two semi-ellipsoids at the two ends. The cross-section of the central cylinder is the most critical circle searched by the 2-D computer program STABL2. After the 2-D critical circle has been determined, the 3-D failure surface then can be generated. The cylinder has a length $2l_c$ and the minor axis of the semi-ellipsoids has a length $l_s$ as shown in Fig. 5.6.
Fig. 5.6 Front View of a Mixed Type of Failure Surface
Five combinations of the strength parameters are considered:

(1) $c' = 0$, $\phi' = 40^0$; (2) $c' = 7.2$ kPa (150 psf), $\phi' = 30^0$; (3) $c' = 14.4$ kPa (300 psf), $\phi' = 25^0$; (4) $c' = 21.6$ kPa (450 psf), $\phi' = 20^0$;

and (5) $c' = 28.7$ kPa (600 psf), $\phi' = 15^0$. The height of the slope is 6.1 m (20 ft) with three different angles, 33.7° (1.5/1), 21.8° (2.5/1), and 16° (3.5/1). Cases with water ($r_u = 0.5$) and without water ($r_u = 0$) conditions are studied. Here, the pore water pressure parameter $r_u$ is defined as:

$$r_u = \frac{u}{\rho h}$$  \hspace{1cm} (5.1)

where $u$ is the mean pore water pressure at the base of the column, $\rho$ the density of soil, and $h$ the mean height of the column.

5.3.1 Pore Water Pressure Parameter $r_u = 0$

For each combination of strength parameters and slope angle, the coordinates of the centers and the radii of the critical circles are listed in Table 5.3. The last two columns in the table list the 2-D factors of safety both from STABL2 and Spencer's method. It can be seen from this table that the 2-D factors of safety obtained by STABL2 are always less than those obtained by Spencer's method. STABL2 is generally conservative (Boutrup, 1977). The most critical circles for different combinations of strength parameters and different slopes are plotted in Fig. 5.7. For low cohesion intercept $c$ and high friction angle $\phi$, the critical circle tends to be shallow and likely to pass through the toe of the slope. On the other hand, for high cohesion intercept $c$ and low frictional angle $\phi$, the critical circle tends to be a deep one and extends beyond the toe.
<table>
<thead>
<tr>
<th>Slope Angle</th>
<th>c (kPa)</th>
<th>φ (degrees)</th>
<th>X₀ (m)</th>
<th>Y₀ (m)</th>
<th>Radius (m)</th>
<th>F₂ (STABLE)</th>
<th>F₂ (SPENCER)</th>
</tr>
</thead>
<tbody>
<tr>
<td>33.7°</td>
<td>0</td>
<td>40</td>
<td>10.1</td>
<td>6.4</td>
<td>12.9</td>
<td>1.557</td>
<td>1.704</td>
</tr>
<tr>
<td></td>
<td>7.2</td>
<td>30</td>
<td>7.3</td>
<td>1.2</td>
<td>8.3</td>
<td>1.755</td>
<td>1.936</td>
</tr>
<tr>
<td></td>
<td>14.4</td>
<td>25</td>
<td>7.0</td>
<td>4.6</td>
<td>11.9</td>
<td>2.124</td>
<td>2.301</td>
</tr>
<tr>
<td></td>
<td>21.6</td>
<td>20</td>
<td>7.0</td>
<td>4.6</td>
<td>11.9</td>
<td>2.370</td>
<td>2.537</td>
</tr>
<tr>
<td></td>
<td>28.7</td>
<td>15</td>
<td>5.6</td>
<td>5.0</td>
<td>13.1</td>
<td>2.611</td>
<td>2.776</td>
</tr>
<tr>
<td>21.8°</td>
<td>0</td>
<td>40</td>
<td>11.3</td>
<td>6.1</td>
<td>12.8</td>
<td>2.334</td>
<td>2.619</td>
</tr>
<tr>
<td></td>
<td>7.2</td>
<td>30</td>
<td>11.3</td>
<td>7.6</td>
<td>14.7</td>
<td>2.315</td>
<td>2.529</td>
</tr>
<tr>
<td></td>
<td>14.4</td>
<td>25</td>
<td>9.3</td>
<td>4.6</td>
<td>12.7</td>
<td>2.566</td>
<td>2.803</td>
</tr>
<tr>
<td></td>
<td>21.6</td>
<td>20</td>
<td>9.3</td>
<td>4.6</td>
<td>12.7</td>
<td>2.750</td>
<td>2.927</td>
</tr>
<tr>
<td></td>
<td>28.7</td>
<td>15</td>
<td>10.2</td>
<td>4.9</td>
<td>14.5</td>
<td>2.935</td>
<td>3.245</td>
</tr>
<tr>
<td>16°</td>
<td>0</td>
<td>40</td>
<td>19.8</td>
<td>30.5</td>
<td>36.6</td>
<td>3.011</td>
<td>3.075</td>
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<td></td>
<td>7.2</td>
<td>30</td>
<td>15.8</td>
<td>13.7</td>
<td>21.3</td>
<td>2.986</td>
<td>3.224</td>
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<td>13.4</td>
<td>7.8</td>
<td>17.2</td>
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<td>13.4</td>
<td>7.8</td>
<td>17.2</td>
<td>3.222</td>
<td>3.592</td>
</tr>
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<td></td>
<td>28.7</td>
<td>15</td>
<td>12.8</td>
<td>7.6</td>
<td>12.7</td>
<td>3.252</td>
<td>3.511</td>
</tr>
</tbody>
</table>

NOTE: X₀ is the horizontal distance between the center and the crest; positive value means the center is on the left side of the crest.

Y₀ is the vertical distance between the center and the crest; positive value means the center is above the crest.
Fig. 5.7 The Most Critical Surfaces for Different Combinations of Strength Parameters in Different Slopes ($r_u = 0$)
Different \( \ell_c/H \) ratios, 0.5, 1, 2, and 4, with different \( l_s/H \) ratios, 0.5, 1, 2, and 4 are studied. Tables D.1 to D.3 show the \( F_3/F_2 \) ratios at various \( \ell_c/H \) and \( l_s/H \) ratios. The results are obtained by both LEMIX and the Ordinary Method of Columns (OMC). The following conclusions can be drawn from this study:

- As the \( l_s/H \) ratio increases, the \( F_3/F_2 \) ratio generally decreases as shown in Fig. 5.8. The reason is that when the width of the failure surface increases the end effects are less in general.

- In certain cases (Figs. D.1b, c, d, e, etc.) there is a minimum \( F_3/F_2 \) ratio. This means that, theoretically, the failure will most likely occur for the ratio \( l_s/H \) corresponding to the minimum \( F_3/F_2 \) ratio. However, these curves are very smooth and it is difficult to predict the exact length of the failure mass. This result was also noted by Baligh and Azzouz (1975).

- For cohesive soils, \( F_3 \) is always greater than \( F_2 \). However, for cohesionless soils, \( F_3 \) may be less than \( F_2 \) (Fig. 5.8a).

- When the \( \ell_c/H \) ratio increases, \( F_3 \) is closer to \( F_2 \). A larger \( \ell_c/H \) ratio means that the problem is closer to the plane strain condition. Hence, the curves corresponding to large \( \ell_c/H \) ratio are closer to the line \( F_3/F_2 = 1 \) (See the difference between Fig. 5.8a and 5.8e).

- The steeper the slope, the less the \( F_3/F_2 \) ratio as shown in Fig. 5.9. This is probably because the volume of the failure
Fig. 5.8 Ratio of $F_3/F_2$ (Slope $1.5/1$, $r_u = 0$)
Fig. 5.8 (Cont'd)

(c) $c = 14.4 \text{ kPa}, \phi = 25^\circ$

(d) $c = 21.6 \text{ kPa}, \phi = 20^\circ$
Fig. 5.8 (Cont'd)

\( c = 28.7 \ \text{kPa}, \ \phi = 15^\circ \)
Fig. 5.9 $F_3/F_2$ vs. $l_s/H$ for Various Slope Angles
($r_u = 0$)

$I_c/H = 0.5$
$c = 28.7kP_0$
$\phi = 15^\circ$
mass is larger in a gentle slope, as shown in Fig. 5.7, and therefore more end effect is produced.

5.3.2 Pore Water Pressure Parameter $r_u = 0.5$

In order to assess the effect of the pore water condition, the analyses presented in the previous section are repeated with a pore pressure coefficient $r_u$ of 0.5. The combinations of strength parameters and slope angles previously described also apply to the following results.

The coordinates of the centers and the radii of the most critical 2-D circles are listed in Table 5.4. The last two columns show the 2-D factors of safety from both STABL2 and Spencer's methods, respectively. Fig. 5.10 shows the most critical failure surfaces for different combinations of strength parameters and slope angles. As mentioned previously for the case with no pore water pressure, deep failure circles are obtained for cohesive soils. On the contrary, failure surfaces are shallow for cohesionless soils. Comparing Figs. 5.7 and 5.10 indicates that the failure circles go deeper into the foundation when pore water pressures are present.

The results of these studies are plotted in Figs. D3 to D5. The conclusions drawn are the same as those obtained with no pore water pressure. In addition, the comparison between Figs. 5.9 and 5.11 shows that pore water pressure can cause the 3-D effect to be even more significant.
TABLE 5.4  THE COORDINATES OF THE CENTERS AND RADIi OF THE MOST CRITICAL 2-D FAILURE CIRCLES AND THE 2-D FACTORS OF SAFETY (r_u = 0.5)

<table>
<thead>
<tr>
<th>Slope Angle</th>
<th>c' (kPa)</th>
<th>φ' (degrees)</th>
<th>x₀ (m)</th>
<th>y₀ (m)</th>
<th>Radius (m)</th>
<th>F₂ (STABL2)</th>
<th>F₃ (SPENCER)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 1.5/1</td>
<td>11.9</td>
<td>15.2</td>
<td>21.6</td>
<td>0.679</td>
<td>1.044</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.2</td>
<td>8.7</td>
<td>6.1</td>
<td>14.5</td>
<td>0.848</td>
<td>1.575</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14.4</td>
<td>8.7</td>
<td>6.1</td>
<td>14.5</td>
<td>1.227</td>
<td>1.998</td>
<td></td>
<td></td>
</tr>
<tr>
<td>21.6</td>
<td>8.7</td>
<td>6.1</td>
<td>14.5</td>
<td>1.657</td>
<td>2.272</td>
<td></td>
<td></td>
</tr>
<tr>
<td>28.7</td>
<td>8.7</td>
<td>5.5</td>
<td>17.7</td>
<td>1.999</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.5/1</td>
<td>14.4</td>
<td>6.4</td>
<td>2.1</td>
<td>9.9</td>
<td>1.505</td>
<td>1.933</td>
<td></td>
</tr>
<tr>
<td>21.6</td>
<td>5.3</td>
<td>4.0</td>
<td>13.0</td>
<td>1.877</td>
<td>2.251</td>
<td></td>
<td></td>
</tr>
<tr>
<td>28.7</td>
<td>5.3</td>
<td>4.0</td>
<td>13.0</td>
<td>2.163</td>
<td>2.586</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0 3.5/1</td>
<td>19.2</td>
<td>28.3</td>
<td>34.5</td>
<td>0.968</td>
<td>1.440</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.2</td>
<td>13.6</td>
<td>8.2</td>
<td>17.5</td>
<td>1.396</td>
<td>1.970</td>
<td></td>
<td></td>
</tr>
<tr>
<td>21.6</td>
<td>11.3</td>
<td>4.9</td>
<td>14.9</td>
<td>1.749</td>
<td>1.006</td>
<td></td>
<td></td>
</tr>
<tr>
<td>28.7</td>
<td>12.2</td>
<td>7.9</td>
<td>21.4</td>
<td>2.316</td>
<td>2.813</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: X₀ is the horizontal distance between the center and the crest; positive value means the center is on the left side of the crest.

Y₀ is the vertical distance between the center and the crest; positive values mean the center is above the crest.
1: $c' = 0$, $\phi' = 40^\circ$
2: $c' = 7.2$ kPa, $\phi' = 30^\circ$
3: $c' = 14.4$ kPa, $\phi' = 25^\circ$
4: $c' = 21.6$ kPa, $\phi' = 20^\circ$
5: $c' = 28.7$ kPa, $\phi' = 15^\circ$

Fig. 5.10 The Most Critical Surfaces for Different Combinations of Strength Parameters in Different Slopes ($r_u = 0.5$)
Fig. 5.11 $F_3/F_2$ vs. $l_s/H$ for Various Slope Angles ($r_u=0.5$)
5.3.3 Comparison of Interslice Angles between 2-D and 3-D Cases

This section compares the 2-D interslice angle $\theta_2$ with the 3-D interslice angle $\theta_3$. This study is of interest because the interslice angle represents the magnitude of the interslice shear forces which are related to the factor of safety.

The 2-D interslice inclinations corresponding to the critical shear surfaces analyzed by the Spencer's method are shown in Table 5.5 for values of $r_u$ equal to 0 and 0.5, respectively. Although the interslice inclinations are slightly flatter for $r_u$ equal to 0.5, the variations are similar, regardless of the value of $r_u$. For soils of low cohesion intercept, high frictional angle, and steep slope, the side forces are inclined more steeply.

The comparison between $\theta_2$ and $\theta_3$ is presented in Tables 5.6 and 5.7 and in Fig. 5.12. Several conclusions can be drawn from these results:

- For soil of high cohesion intercept and low frictional angle, $\theta_3$ is less than $\theta_2$. This phenomenon is more significant at smaller $l_s/H$ ratio (See Table 5.6 and the lower part of Fig. 5.12). Therefore, the $F_3/F_2$ ratio is higher than unity as stated in Sections 5.3.1 and 5.3.2.

- For soil of low cohesion intercept and high frictional angle, $\theta_3$ is larger than $\theta_2$, and consequently $F_3$ is less than $F_2$ (See Table 5.7 and the upper part of Fig. 5.12).

- For soils of high cohesion intercept and low friction angle, the interslice angles obtained with a pore pressure parameter of
TABLE 5.5  2-D INTERSLICE ANGLES FOR $r_u = 0$ AND $r_u = 0.5$

<table>
<thead>
<tr>
<th>Slope Angle</th>
<th>$c'$ (kPa)</th>
<th>$\phi'$ (degrees)</th>
<th>Inclination (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$r_u = 0$</td>
</tr>
<tr>
<td>1.5/1</td>
<td>0</td>
<td>40</td>
<td>24.4</td>
</tr>
<tr>
<td></td>
<td>7.2</td>
<td>30</td>
<td>21.0</td>
</tr>
<tr>
<td></td>
<td>14.4</td>
<td>25</td>
<td>15.2</td>
</tr>
<tr>
<td></td>
<td>21.6</td>
<td>20</td>
<td>13.3</td>
</tr>
<tr>
<td></td>
<td>28.7</td>
<td>15</td>
<td>9.7</td>
</tr>
<tr>
<td>2.5/1</td>
<td>0</td>
<td>40</td>
<td>19.3</td>
</tr>
<tr>
<td></td>
<td>7.2</td>
<td>30</td>
<td>16.4</td>
</tr>
<tr>
<td></td>
<td>14.4</td>
<td>25</td>
<td>14.0</td>
</tr>
<tr>
<td></td>
<td>21.6</td>
<td>20</td>
<td>12.7</td>
</tr>
<tr>
<td></td>
<td>28.7</td>
<td>15</td>
<td>8.6</td>
</tr>
<tr>
<td>3.5/1</td>
<td>0</td>
<td>40</td>
<td>15.5</td>
</tr>
<tr>
<td></td>
<td>7.2</td>
<td>30</td>
<td>12.6</td>
</tr>
<tr>
<td></td>
<td>14.4</td>
<td>25</td>
<td>10.5</td>
</tr>
<tr>
<td></td>
<td>21.6</td>
<td>20</td>
<td>9.7</td>
</tr>
<tr>
<td></td>
<td>28.7</td>
<td>15</td>
<td>7.9</td>
</tr>
</tbody>
</table>
TABLE 5.6  THE RATIO OF $\tan \theta_3/\tan \theta_2$ FOR SOIL OF $c' = 28.7$ kPa AND $\phi' = 15^\circ$ IN SLOPE OF 1.5/1 ($\theta_2 = 9.7^\circ$ AND $\theta_2' = 7.2^\circ$)

<table>
<thead>
<tr>
<th>$\frac{l_s}{H}$</th>
<th>$\frac{l_c}{H}$</th>
<th>0.5</th>
<th>1</th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>$a^*$</td>
<td>0.811</td>
<td>0.874</td>
<td>0.927</td>
<td>0.958</td>
</tr>
<tr>
<td></td>
<td>$b^*$</td>
<td>0.916</td>
<td>0.956</td>
<td>0.980</td>
<td>0.986</td>
</tr>
<tr>
<td>1</td>
<td>$a$</td>
<td>0.833</td>
<td>0.906</td>
<td>0.958</td>
<td>0.979</td>
</tr>
<tr>
<td></td>
<td>$b$</td>
<td>0.888</td>
<td>0.972</td>
<td>1.000</td>
<td>1.014</td>
</tr>
<tr>
<td>2</td>
<td>$a$</td>
<td>0.885</td>
<td>0.937</td>
<td>0.979</td>
<td>0.989</td>
</tr>
<tr>
<td></td>
<td>$b$</td>
<td>0.916</td>
<td>0.972</td>
<td>1.014</td>
<td>1.026</td>
</tr>
<tr>
<td>4</td>
<td>$a$</td>
<td>0.916</td>
<td>0.948</td>
<td>0.979</td>
<td>0.990</td>
</tr>
<tr>
<td></td>
<td>$b$</td>
<td>0.944</td>
<td>0.972</td>
<td>1.014</td>
<td>1.028</td>
</tr>
</tbody>
</table>

TABLE 5.7  THE RATIO OF $\tan \theta_3/\tan \theta_2$ FOR SOIL OF $c' = 0$ AND $\phi' = 40^\circ$ IN SLOPE OF 1.5/1 ($\theta_2 = 24.4^\circ$ AND $\theta_2' = 19.7^\circ$)

<table>
<thead>
<tr>
<th>$\frac{l_s}{H}$</th>
<th>$\frac{l_c}{H}$</th>
<th>0.5</th>
<th>1</th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>$a$</td>
<td>1.037</td>
<td>1.023</td>
<td>1.009</td>
<td>1.004</td>
</tr>
<tr>
<td></td>
<td>$b$</td>
<td>1.033</td>
<td>1.017</td>
<td>1.011</td>
<td>1.006</td>
</tr>
<tr>
<td>1</td>
<td>$a$</td>
<td>1.056</td>
<td>1.033</td>
<td>1.019</td>
<td>1.009</td>
</tr>
<tr>
<td></td>
<td>$b$</td>
<td>1.039</td>
<td>1.028</td>
<td>1.017</td>
<td>1.011</td>
</tr>
<tr>
<td>2</td>
<td>$a$</td>
<td>1.075</td>
<td>1.052</td>
<td>1.033</td>
<td>1.019</td>
</tr>
<tr>
<td></td>
<td>$b$</td>
<td>1.050</td>
<td>1.033</td>
<td>1.022</td>
<td>1.016</td>
</tr>
<tr>
<td>4</td>
<td>$a$</td>
<td>1.099</td>
<td>1.075</td>
<td>1.052</td>
<td>1.033</td>
</tr>
<tr>
<td></td>
<td>$b$</td>
<td>1.055</td>
<td>1.044</td>
<td>1.033</td>
<td>1.022</td>
</tr>
</tbody>
</table>

*a: $r_u = 0$,  b: $r_u = 0.5$
Curves Concave Upward
\[
\begin{align*}
C' &= 0 \\
\phi &= 40^\circ
\end{align*}
\]

Curves Concave Downward
\[
\begin{align*}
C' &= 28.7 \text{ kPa} \\
\phi &= 15^\circ
\end{align*}
\]

Fig. 5.12 \( \tan \theta_3 / \tan \theta_2 \) vs. \( l_s / H \) for Various Soils at Various \( l_s / H \) (Slope 1.5/1)
0.5 are larger than those obtained with no pore pressure. The effect is opposite for soil of low cohesion intercept and high friction angle.

- As the \( \ell_0/H \) ratio increases, the \( \tan \theta_3/\tan \theta_2 \) ratio gets closer to unity (Fig. 5.12). This corresponds to the plane strain condition.

- Steeper slopes show higher \( \tan \theta_3/\tan \theta_2 \) ratios. This can also be explained by the smaller values of \( F_3/F_2 \) for steep slopes (Sections 5.3.1 and 5.3.2).

5.3.4 Comparison of Results (LEMIX and Ordinary Method of Columns)

In the 2-D case, the ordinary method of slices (OMS) usually produces lower values of factor of safety than other more rigorous methods of slices. Therefore, the OMS is generally considered as a more conservative method. In the 3-D case, the results from both LEMIX and the Ordinary Method of Columns (OMC) for no water condition are presented in Tables D.1 to D.3. The results for \( r_u \) equal to 0.5 are in Tables D.4 to D.6. The conclusions are as follows:

- For no water condition, the OMC usually produces lower factors of safety. The differences are less than 10\% in most cases.

- When pore water pressures exist, the OMC gives higher values of factor of safety for steep slope (Table D.4). For gentle slope, the OMC may produce both higher or lower values of factor of safety (Table D.5 and D.6). Similarly, the difference in results between the two methods is less than 10\%.

It is therefore concluded that the OMC also produces satisfactory results for homogeneous soils.
5.4 Finite Element Analysis

In this section the finite element computer program FESPON is used to analyze spoon shape failure surfaces. The results are compared to those obtained with the limit equilibrium method. The hyperbolic parameters used in FESPON are generated from the results of conventional triaxial and consolidation tests on highly plastic Saint Croix clay.

5.4.1 Evaluation of the Values of Hyperbolic Parameters

The values of the hyperbolic parameters can be determined using data from conventional triaxial tests. Weitzel (1979) studied the 'short-term' or as-compacted laboratory strength of a highly plastic Saint Croix clay. The 'short term' refers to the fill material immediately after compaction and before environmental factors have an opportunity to alter the as-compacted condition of the soil. Weitzel measured the as-compacted strength in unconsolidated-undrained triaxial tests. The samples were prepared by kneading compaction to densities that fit on three impact energy curves: low energy, standard, and modified Proctor, with four water contents on each. The samples were then sheared at four levels of confining pressure to simulate a variety of embankment depths.

Johnson (1979) evaluated the effective stress strength parameters for analysis of long term stability. These parameters were evaluated for various compaction conditions through consolidated undrained triaxial tests with pore water pressure measurements. These were run at a constant rate of strain on kneading compacted samples of the same highly plastic clay used by Weitzel. The long term environmental effects
were approximated by back pressure saturation and consolidation under states of stress representing the body forces at different positions in the embankment.

The results both from Weitzel's and Johnson's study are used to generate the values of hyperbolic parameters. These hyperbolic parameters then can be used in the computer program FESPON to examine the stability of an embankment of the highly plastic St. Croix clay, both for short-term and long-term conditions.

5.4.1.1 Parameters for Short-Term Condition

The procedure to determine the hyperbolic parameters has been presented in Section 4.3. Wong (1974) developed a computer program SP-1 to evaluate the hyperbolic parameters $c$, $\phi$, $K$, $n$, and $R_f$ using stress-strain data. Value of $G_f$, $F$, and $d$ were obtained using volumetric strain data from conventional triaxial compression tests. Least-square curve-fitting procedures are used in determining the parameters. The data required for the program are confining pressure $\sigma_3$, stress difference at failure $(\sigma_1 - \sigma_3)_f$, axial strains at 70% and 95% stress levels, and volumetric strains at 70% and 95% stress levels.

These data can be obtained from Appendix C of Weitzel (1979). The hyperbolic parameters are computed for each energy level (or dry density $\rho_d$) and water content $w$. Equations of these parameters as functions of energy level and water content can then be generated using regression techniques. The resulting equations are listed below:

$$c = -740 + 0.755 \rho_d - 14.5 w$$

(5.2)

$$\phi = 63.4 - 0.00180 \rho_d + 0.0234 w^2$$

(5.3)

$$K = 870 - 0.157 \rho_d + 4.300 w^2 + 0.00108 \rho_d^2$$

(5.4)
\[ n = -4.37 + 0.00226 \rho_d + 0.0348 w \]  
\[ G = -1.63 + 0.000798 \rho_d + 0.0305 w \]  
\[ d = 14.8 - 0.000384 w \rho_d + 0.00214 w^2 \]  
\[ F = 0.916 - 0.0000363 w \rho_d + 0.00085 w^2 \]

where \( c \) is in kPa, \( \rho_d \) in kg/m\(^3\), and \( w \) in per cent. The contours of each parameter are plotted in Fig. 5.13. It is necessary to note that these contours may be inappropriate for Modified Proctor energy level because the stress-strain curve of this energy level behaves differently from a hyperbola.

5.4.1.2 Parameters for Long-Term Condition

As we mentioned in Section 4.3, if the long-term stability needs to be examined, the hyperbolic parameters may be obtained from drained triaxial test data. However, it is very often too time consuming to run the drained triaxial tests. Clough and Duncan (1969) developed an approach which used data from ordinary 1-D consolidation tests. The details of this approach was presented in Section 4.3. In the following, the generation of the effective hyperbolic parameters from both Johnson's (1979) \( \text{CU} \) and DiBernardo's (1979) consolidation data for St. Croix clay is explained.

Johnson (1979) found that the effective stress friction angles ranged only from 18.9 to 21.4 degrees. This measured variation of 2.5 degrees (21.4 - 18.9 = 2.5) was not statistically significant. Therefore, for the range of compaction and consolidation conditions investigated, the effective stress friction angle could be taken as a constant value of 20 degrees. Johnson also generated an equation for the effective stress cohesion intercept \( c' \) as follows:
Fig. 5.13 Contours of Hyperbolic Parameters In As-Compacted Condition
Fig. 5.13 (Cont'd)
Fig. 5.13 (Cont'd)
Fig. 5.13 (Cont'd)
\[ c' = 1.71 - 3.83 \, w \, \log e_0 \]  

(5.9)

in which \( c' \) = the estimated value of the effective stress intercept (kPa)

\( w \) = compaction moisture content (%)

\( e_0 \) = initial void ratio

With the effective strength parameters \( c' \) and \( \phi' \) available, and assuming the failure ratio equal to 0.8 (as obtained from the similar soils), the initial elastic modulus constants, \( K \) and \( n \), can be estimated from the consolidation test data. The following example will present the procedure to obtain \( K \) and \( n \) from the data of DiBernardo's (1979):

**Example**

For sample number LOA, \( w = 25.63\% \) and \( e_0 = 0.8206 \), \( c' \) is obtained from equation (5.8) as follows:

\[ c' = 1.71 - 3.83 \, (0.2566) \, \log (0.8206) = 10.2 \, (\text{kPa}) \]

From Table B2 (DiBernardo, 1979), and considering the normal consolidation range, Table 5.8 is developed.

Let \( R_f \) be equal to 0.8, \( \phi' \) equal to 20 degrees, and \( K_0 \) to 0.6 (as obtained from Fig. 4.8 for OCR equal to one). Take the atmospheric pressure \( P_a \) equal to 101.4 kPa, the values in columns 12 and 14 are drawn in the log-log plot of Fig. 5.14. The slope of the curve is \( n \) and the intercept at \( \sigma_3/P_a \) equals to one is \( K \). From the figure the values of \( n = 0.53 \) and \( K = 95 \) are obtained.

If all samples are used to get the mean values of \( K \) and \( n \) of these samples, the results are as presented in Table 5.9. These data are from
<table>
<thead>
<tr>
<th>1 Applied Load ( p_1 ) (kPa)</th>
<th>2 ( e_0 )</th>
<th>3 Mean Load ( p ) (kPa)</th>
<th>4 Increment ( \Delta p ) (kPa)</th>
<th>5 ( \sigma_3 )</th>
<th>6 ( \Delta p (1 + e_0) \Delta e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>171.52</td>
<td>0.7761</td>
<td>214.41</td>
<td>85.78</td>
<td>0.0228</td>
<td>6682</td>
</tr>
<tr>
<td>257.30</td>
<td>0.7533</td>
<td>321.70</td>
<td>128.80</td>
<td>0.0305</td>
<td>7404</td>
</tr>
<tr>
<td>386.10</td>
<td>0.7228</td>
<td>482.60</td>
<td>193.0</td>
<td>0.0381</td>
<td>8727</td>
</tr>
<tr>
<td>579.10</td>
<td>0.6847</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
7 \quad p(1 - K_o) R_f \\
8 \quad k_o p (\tan^2 (45 + \phi'/2) - 1) \\
9 \quad 2c' \tan (45 + \phi'/2) \\
10 \quad \frac{2K_o^2}{(1 + K_o)} \\

\begin{align*}
68.6 & \quad 133.7 & \quad 29.1 & \quad 0.55 \\
102.9 & \quad 200.7 & \quad 29.1 & \quad 0.55 \\
154.4 & \quad 301.0 & \quad 29.1 & \quad 0.55 \\
\end{align*}

<table>
<thead>
<tr>
<th>11 ( E_1 ) (kPa)</th>
<th>12 ( E_1/P_a )</th>
<th>13 ( \sigma_3 ) (kPa)</th>
<th>14 ( \sigma_3/P_a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10970</td>
<td>108.2</td>
<td>128.7</td>
<td>1.269</td>
</tr>
<tr>
<td>13351</td>
<td>131.7</td>
<td>193.0</td>
<td>1.904</td>
</tr>
<tr>
<td>16961</td>
<td>167.3</td>
<td>289.6</td>
<td>2.856</td>
</tr>
</tbody>
</table>

TABLE 5.8 PROCEDURE TO OBTAIN \( K \) AND \( n \) FOR SAMPLE LOA
Fig. 5.14 $E_i/P_a$ vs. $\sigma_3/P_a$.

Sample LOA

$n = 0.53$

$K = 9.5$
<table>
<thead>
<tr>
<th>Sample Group</th>
<th>Average Water Content ( w (%) )</th>
<th>Average Dry Density ( \rho_d ) (kg/m(^3))</th>
<th>( K )</th>
<th>( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>LD</td>
<td>20.51</td>
<td>1456.8</td>
<td>118</td>
<td>0.64</td>
</tr>
<tr>
<td>LO</td>
<td>24.24</td>
<td>1513.0</td>
<td>95</td>
<td>0.53</td>
</tr>
<tr>
<td>LW</td>
<td>25.68</td>
<td>1540.5</td>
<td>126</td>
<td>0.58</td>
</tr>
<tr>
<td>SD</td>
<td>18.74</td>
<td>1532.4</td>
<td>54</td>
<td>1.11</td>
</tr>
<tr>
<td>SO</td>
<td>22.45</td>
<td>1636.5</td>
<td>133</td>
<td>0.62</td>
</tr>
<tr>
<td>SW</td>
<td>24.54</td>
<td>1603.8</td>
<td>160</td>
<td>0.50</td>
</tr>
<tr>
<td>MD</td>
<td>13.96</td>
<td>1796.8</td>
<td>95</td>
<td>0.87</td>
</tr>
<tr>
<td>MO</td>
<td>15.66</td>
<td>1845.8</td>
<td>215</td>
<td>0.65</td>
</tr>
<tr>
<td>MW</td>
<td>18.86</td>
<td>1776.4</td>
<td>113</td>
<td>0.84</td>
</tr>
</tbody>
</table>

TABLE 5.9 VALUES OF \( K \) AND \( n \)
DiBernardo (1979). In this table water content \( w \) and dry density \( \rho_d \) are mean values; \( K \) and \( n \) are in terms of these mean values. The results are also plotted in Fig. 5.15, in which the first number in parenthesis is the value of \( K \) and the second number is the value of \( n \).

5.4.2 Finite Element Method Results

In Section 5.4.1 the hyperbolic stress-strain parameters were evaluated. These parameters are plotted in Figs. 5.13 and 5.15. They are now introduced in the finite element computer program FESPON to analyze the stability of an embankment under as-compacted and long-term conditions.

5.4.2.1 As-Compacted Condition

The soil parameters for the as-compacted condition are shown in Table 5.10. These parameters are obtained from Fig. 5.13 for a water content \( w \) of 26.8\% and a dry density \( \rho_d \) of 1540 kg/m\(^3\) (low energy level). The soil is assumed homogeneous in both embankment and foundation.

The contours of major and minor principal stresses \((\sigma_1 \text{ and } \sigma_3)\) generated by FESPON are presented in Figs. 5.16 and Fig. 5.17. The \( \sigma_1 \) values are related to the overburden pressure \((\rho h)\). These contours have similar shape and are parallel to each other. Figs. 5.18 and 5.19 gives the contours of maximum shear stress \( \tau_{max} \) and stress levels \((\sigma_1 - \sigma_3)/ (\sigma_1 - \sigma_3)_f \). These contours have similar shape; high values of maximum shear stresses correspond to high values of stress levels. These figures can be compared to Fig. 5.20 which shows the critical failure circle as searched by the program STABL2. This critical circle has the
Fig. 5.15 Values of $K$ and $n$ at Various Dry Densities and Water Contents for Long-Term Condition
**TABLE 5.10 HYPERBOLIC PARAMETERS FOR AS-COMPACTED CONDITION**

<table>
<thead>
<tr>
<th>c(kPa)</th>
<th>$\phi$(degrees)</th>
<th>K</th>
<th>$K_u$</th>
<th>n</th>
<th>G</th>
<th>d</th>
<th>F</th>
<th>$R_f$</th>
<th>$K_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td>34.5</td>
<td>6</td>
<td>36</td>
<td>110</td>
<td>0.048</td>
<td>0.42</td>
<td>0.52</td>
<td>0.028</td>
<td>0.8</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Note: $G_s = 2.79$, $w = 26.8\%$, $\rho_d = 1540$ kg/m³

**TABLE 5.11 HYPERBOLIC PARAMETERS FOR LONG-TERM CONDITION**

<table>
<thead>
<tr>
<th>c(kPa)</th>
<th>$\phi$(degrees)</th>
<th>K</th>
<th>$K_u$</th>
<th>n</th>
<th>G</th>
<th>d</th>
<th>F</th>
<th>$R_f$</th>
<th>$K_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.5</td>
<td>20</td>
<td>125</td>
<td>375</td>
<td>0.55</td>
<td>0.42</td>
<td>0.0</td>
<td>0.0</td>
<td>0.8</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Note: $\rho = 1990$ kg/m³

**TABLE 5.12 COMPARISON OF $F_2$ AND $F_3$ FOR AS-COMPACTED CONDITION ($R_z = 12.2$ m)**

<table>
<thead>
<tr>
<th></th>
<th>$F_2$</th>
<th>$F_3$</th>
<th>$F_3/F_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEM</td>
<td>1.59</td>
<td>1.90</td>
<td>1.20</td>
</tr>
<tr>
<td>FEM</td>
<td>1.62</td>
<td>2.01</td>
<td>1.24</td>
</tr>
<tr>
<td>FEM-LEM/FEM x 100%</td>
<td>1.8%</td>
<td>5.5%</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE 5.13 COMPARISON OF $F_2$ AND $F_3$ FOR LONG-TERM CONDITION ($R_z = 12.2$ m)**

<table>
<thead>
<tr>
<th></th>
<th>$F_2$</th>
<th>$F_3$</th>
<th>$F_3/F_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEM</td>
<td>0.778</td>
<td>0.893</td>
<td>1.15</td>
</tr>
<tr>
<td>FEM</td>
<td>0.877</td>
<td>0.981</td>
<td>1.12</td>
</tr>
<tr>
<td>FEM-LEM/FEM x 100%</td>
<td>11.3%</td>
<td>9.0%</td>
<td></td>
</tr>
</tbody>
</table>
The numbers in the parentheses are in kPa.

Fig. 5.16 Contours of $\sigma_1$ (tsf) (As-Compacteted Condition)
Fig. 5.17 Contours of $\sigma_3$ (tsf) (As-Compacted Condition)

The numbers in the parentheses are in kPa:

- 0.4 (38.3)
- 0.8 (76.6)
- 1.2 (115.0)
- 1.6 (153.2)
The numbers in the parentheses are in kPa.

0.1 (9.6)
0.2 (19.2)
0.3 (28.7)
0.4 (38.3)
0.3 (28.7)
0.2 (19.2)
0.1 (9.6)

Fig. 5.18 Contours of $\tau_{\text{max}}$ (tsf) (As-Compacted Condition)
Fig. 5.19. Contours of Stress Level (As-Compacted Condition)
Fig. 5.20 The Critical Failure Surface for Embankment in As-Compacted Condition

\[ c = 34.5 \text{kPa} \]
\[ \phi = 6^\circ \]
\[ \rho = 1949 \text{kg/m}^3 \]
maximum value of radius of the spoon shape failure mass (Fig. 3.12).

It is obvious that the critical failure surface follows the zone of
highest maximum shear stress.

The contours of horizontal \( u_x \) and vertical \( u_y \) movements are shown in
Figs. 5.21 and 5.22. The maximum horizontal displacements occur close
to the toe. The maximum vertical displacements occur about one third of
the height \( (H/3) \), from the top of the embankment and near the center
line. These values of displacements are relative displacements to the
top of the embankment. Near the toe, the soil may have positive (or
upward) vertical movements.

Local factors of safety are computed along a spoon shape failure
surface defined by the critical circle obtained by STABL2 and a minor
axis of length 12.2 m (40 ft). The local factor of safety \( F_N \) is defined
as:

\[
F_N = \frac{c + \sigma_N \tan \phi}{\tau_N} = \frac{c}{\sigma_N} + \frac{\sigma_N}{\tau_N} \tan \phi \quad (5.10)
\]

where \( \sigma_N \) is normal stress and \( \tau_N \) the shear stress. The normal stress,
shear stress, and local factor of safety are given in Fig. 5.23 for
different sections of the failure surface (as a function of the Z-
coordinate). The arrow in Fig. 5.23 shows the position of the toe.
These figures show that the normal stress is higher in the central por-
tion of the embankment and is very small at the two ends. The shear
stress distribution is similar to the normal stress distribution. The
maximum shear stresses are only about 20% of the maximum normal
stresses. As the section is farther away from the center line, both
the normal and shear stresses decrease at the same rate and the local
factor of safety increases.
Fig. 5.21 Contours of $u_x$ (ft) (As-Compacted Condition)

The numbers in the parentheses are in kPa.
The numbers in the parentheses are in kPa.

Fig. 5.22 Contours of u_y (As-Compacted Condition)
Fig. 5.23  Normal Stress, Shear Stress, and Local Factor of Safety vs. x (As-Compacted)
Fig. 5.23 (Cont'd)

(b) $z = 2.3 \text{ m}$
(c) $z = 3.8$ m

Fig. 5.23 (Cont'd)
Fig. 5.23 (Cont'd)

(d) \( z = 5.3 \text{ m} \)
Fig. 5.23 (Cont'd)
Fig. 5.23 (Cont'd)

(f) $z = 8.4$ m
Fig. 5.23 (Cont'd)

(g) \( z = 9.9 \text{ m} \)
5.4.2.2 Long-Term Condition

In this section an effective stress analysis of the embankment is performed for long-term condition. The time effects on the long-term behavior of an embankment are very complex. A more versatile soil model than the one used in the present work would be needed to take into account these effects. Such a model is not available and consequently effects such as change in pore pressure, creep, etc. are disregarded in this analysis.

The soil parameters are listed in Table 5.11. The cohesion intercept is obtained from equation (5.9) with the initial water content and initial void ratio known. The hyperbolic parameters $K$ and $n$ are obtained from consolidation tests on the same soil at the same initial water content (refer to Fig. 5.15). The unloading value of $K (K_{ur})$ is taken as three times $K$. The density of soil may change with time due to saturation, settlement, etc. In this example, the final density of soil is taken as 1990 kg/m$^3$. The pore pressure parameter $r_u$ is equal to 0.5.

Fig. 5.24 shows the contours of stress level obtained with FESPON. The highest stress level is in a zone close to the toe. The critical 2-D circle given by STABL2 is shown in Fig. 5.25. The circle passes through the zone of the highest stress level and indicates that a toe failure may happen in the long-term condition. The curves of normal stress, shear stress, pore water pressure, and local factor of safety along the section of Z-coordinate equal to 2.5 m are shown in Fig. 5.26. The smaller local factors of safety occur in the zone of highest pore water pressure. Conversely, the higher local factors of safety occur near the toe and crest due to low pore water pressure.
$SL = \frac{1}{2}(\sigma_1 - \sigma_3)$
Fig. 5.25 The Critical Failure Surface for Embankment in Long-Term Condition
Fig. 5.26 Normal Stress, Shear Stress, Water Pressure, and Local Factor of Safety vs. $x$ at $z = 2.5$ m (Long-Term)
5.4.3 Comparison between Finite Element Method and Limit Equilibrium Method

Although comparisons of the results between finite element and limit equilibrium methods are given in a few papers (Wright, 1973, Résendiz, 1972) for 2-D cases, there is no comparison for 3-D cases. In this section, comparisons of the results for as-compacted and long-term conditions are presented.

The mean factor of safety used in the comparison is defined as

$$\text{Mean } F = \frac{\Sigma(c + \sigma_N \tan \phi) \, dA}{\Sigma \tau_N \, dA}$$  \hspace{1cm} (5.11)

where the summation \( \Sigma \) is over the whole failure surface and \( dA \) is the bottom area of a vertical column.

The results for the as-compacted condition are presented in Table 5.12. The limit equilibrium methods, Spencer’s method and LEMIX, yield factors of safety \( F_2 \) and \( F_3 \) of 1.59 and 1.90, respectively. The two-dimensional finite element computer program ISBILD (Ozawa, 1974) gives a mean factor of safety \( F_2 \) of 1.62, while FESPON leads to a mean factor of safety \( F_3 \) of 2.01. The ratio \( F_3/F_2 \) is 1.20 for the limit equilibrium methods. It is 1.24 for the finite element solutions. The factors of safety obtained from limit equilibrium analysis are smaller than those from finite element analysis. The agreement is quite good in this case with differences of 1.8% and 5.5% in 2-D and 3-D cases, respectively.

Table 5.13 shows the comparison of \( F_2 \) and \( F_3 \) for the long-term condition. The 3-D factor of safety obtained with the finite element method is 9.0% larger than the one given by the limit equilibrium method. The \( F_3/F_2 \) ratios are very close, 1.15 and 1.12 for the limit
equilibrium and finite element methods, respectively. Comparing Tables 5.12 and 5.13 indicates that the long-term stability is more critical than the as-compacted stability.

Finally it should be recognized that the strength parameters selected in these examples are for low energy level (wet side). The strength parameters of actual embankments are higher than the ones selected. It is only for the purpose of illustrating that the factors of safety are of the order of 1.5 for the as-compacted condition. This results in low factor of safety for the long-term condition. Actual embankments will show much higher factors of safety than those computed for this example.

5.4.4 Other Applications

The discussion of the results obtained in the previous section was simplified by assuming the embankment and foundation soils to be the same. In fact the finite element computer program FESPON can handle problems with complex soil conditions and/or geometries. This will be illustrated by the following applications.

5.4.4.1 Stability of a Non-homogeneous Embankment

The construction of an embankment in rolled lifts frequently results in non-homogeneous soil properties. The strength characteristics may vary from layer to layer and be different from the foundation soil strength characteristics. Such an embankment is shown in Fig. 5.27. The foundation and compacted fill are composed of two and eight different layers, respectively. The hyperbolic strength parameters of each layer are listed in Table 5.14. Using these data the finite element
Fig. 5.27 Compacted Fill on a Foundation
<table>
<thead>
<tr>
<th>Layer No.</th>
<th>Density (kg/m³)</th>
<th>c (kPa)</th>
<th>$\phi$ (degrees)</th>
<th>K</th>
<th>$K_{ur}$</th>
<th>n</th>
<th>d</th>
<th>G</th>
<th>F</th>
<th>$R_f$</th>
<th>$k_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1949</td>
<td>39.3</td>
<td>0</td>
<td>80</td>
<td>240</td>
<td>0.2</td>
<td>0.0</td>
<td>0.495</td>
<td>0.0</td>
<td>0.8</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>1949</td>
<td>39.3</td>
<td>0</td>
<td>80</td>
<td>240</td>
<td>0.2</td>
<td>0.0</td>
<td>0.495</td>
<td>0.0</td>
<td>0.8</td>
<td>0.5</td>
</tr>
<tr>
<td>3</td>
<td>1965</td>
<td>41.4</td>
<td>5</td>
<td>40</td>
<td>120</td>
<td>0.03</td>
<td>0.7</td>
<td>0.45</td>
<td>0.035</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1949</td>
<td>41.2</td>
<td>5</td>
<td>40</td>
<td>120</td>
<td>0.03</td>
<td>0.7</td>
<td>0.45</td>
<td>0.035</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1933</td>
<td>41.0</td>
<td>5</td>
<td>40</td>
<td>120</td>
<td>0.03</td>
<td>0.7</td>
<td>0.45</td>
<td>0.035</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1917</td>
<td>40.7</td>
<td>5</td>
<td>40</td>
<td>120</td>
<td>0.03</td>
<td>0.7</td>
<td>0.45</td>
<td>0.035</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>1901</td>
<td>40.5</td>
<td>5</td>
<td>40</td>
<td>120</td>
<td>0.03</td>
<td>0.7</td>
<td>0.45</td>
<td>0.035</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>1885</td>
<td>40.2</td>
<td>5</td>
<td>40</td>
<td>120</td>
<td>0.03</td>
<td>0.7</td>
<td>0.45</td>
<td>0.035</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>1869</td>
<td>40.0</td>
<td>5</td>
<td>40</td>
<td>120</td>
<td>0.03</td>
<td>0.7</td>
<td>0.45</td>
<td>0.035</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>1853</td>
<td>39.8</td>
<td>5</td>
<td>40</td>
<td>120</td>
<td>0.03</td>
<td>0.7</td>
<td>0.45</td>
<td>0.035</td>
<td>0.8</td>
<td></td>
</tr>
</tbody>
</table>
analysis of the embankment is performed similarly to the analyses described in the previous section.

The 3-D limit equilibrium program LEMIX can also be used to analyze the stability of this problem. In this case, since the program LEMIX can only handle one material in the foundation and one in the embankment, it is necessary to use mean values of strength parameters for the foundation and embankment soils. These mean values are given in Fig. 5.28.

The contours of stress level generated by the finite element analysis are shown in Fig. 5.29. Table 5.15 gives the 2-D and 3-D factors of safety obtained with the limit equilibrium and finite element methods. The mean factors of safety obtained by the finite element method on 2-D and 3-D failure surfaces are almost identical to the factor of safety obtained by the limit equilibrium method on the same surfaces (difference of the order of 2%). The methods also result in very consistent $F_3/F_2$ ratios, 1.31 for the limit equilibrium method and 1.33 for the finite element method.

<table>
<thead>
<tr>
<th></th>
<th>Mean $F_2$</th>
<th>Mean $F_3$</th>
<th>$F_3/F_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEM</td>
<td>1.527</td>
<td>2.00</td>
<td>1.31</td>
</tr>
<tr>
<td>FEM</td>
<td>1.532</td>
<td>2.04</td>
<td>1.33</td>
</tr>
<tr>
<td>FEM-LEM</td>
<td>$0.4%$ x 100</td>
<td>$2.0%$</td>
<td></td>
</tr>
</tbody>
</table>

TABLE 5.15 COMPARISON OF $F_2$ AND $F_3$ FOR COMPACTED FILL ON A FOUNDATION IN TOTAL STRESS ANALYSIS ($R_z = 12.2$ m)
Fig. 5.28 The Critical Failure Surface for Compacted Fill on a Foundation
Fig. 5.29 Contours of Stress Level (Layered)

\[ SL = \frac{\sigma_1 - \sigma_3}{\sigma_1 - \sigma_3} \]
5.4.4.2 A Pavement Analysis

Since the program FESPON can simulate the construction of an embankment, it is also capable of analyzing similar problems such as the construction of a pavement. The profile of a pavement section is shown in Fig. 5.30. This problem was studied by Palmerton (1972) who used the 3-D finite element computer program SOLSAP to study the deflection of the pavement. SOLSAP also uses hyperbolic stress-strain relationships, but employs compatible modes for element displacements. This pavement is analyzed using FESPON, and the results are compared to those obtained with the program SOLSAP.

The pavement section is composed of 0.076 m (3 in) of asphaltic pavement, 0.53 m (21 in) of crushed limestone base, and 2.74 m (9 ft) of selected clays. The values of hyperbolic parameters for each layer are given in Figure 5.31. A lateral earth pressure coefficient of 0.5 is assumed. It is also assumed that the stress-strain behavior of the asphaltic pavement is linear; thus the Young's modulus E and Poisson's ratio ν are constant values. This pavement is subjected to a 12-wheel load. Each wheel produces a 113 kN (30 k) vertical force.

The finite element mesh used for the analysis is shown in Fig. 5.32. It is only necessary to grid one-half of the problem since the problem is symmetrical with respect to the center line of the loading. The system is composed of four layers of elements. The wheel loads are applied as point loads, acting at the nodal points. The load is applied in one step for simplicity. Vertical deflections along the section A, B, and C are shown in Figs. 5.33 to 5.35.
Fig. 5.30 Loading and Foundation Conditions
<table>
<thead>
<tr>
<th>Depth, (m)</th>
<th>Section Profile</th>
<th>Material Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Asphalitic Concrete</td>
<td>( \rho, (kg/m^3) = 2077 )</td>
</tr>
<tr>
<td>0.61</td>
<td>Crushed Limestone</td>
<td>( c, (kN/m) = 300 )</td>
</tr>
<tr>
<td>1.52</td>
<td>4 CBR Buckshot Clay</td>
<td>( \phi = 47^\circ )</td>
</tr>
<tr>
<td>3.35</td>
<td>4 CBR Processed Lean Clay</td>
<td>( R_f = 0.70 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( K = 2500 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( n = 0.30 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( d = 6.4 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( G = 0.32 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( F = 0.14 )</td>
</tr>
</tbody>
</table>

Fig. 5.31 Section Profile and Material Properties

*(Linear Behavior Assumed for Asphalitic Concrete
\( E = 2.04 \times 10^8 \) Pa, \( \nu = 0.45 \))
Fig. 5.32 Finite Element Mesh
Fig. 5.33  Vertical Deflections—Section A
Fig. 5.34 Vertical Deflections—Section B
Fig. 5.35 Vertical Deflections—Section C
a. Surface Deflections

b. Deflections at 0.61m Depth

c. Deflections at 1.52m Depth

Fig. 5.36 Vertical Deflections—Section D
The dotted lines are results from SOLSAP, while the solid lines are results from FESPON. Field deflections measured at the surface are also given in these figures. Larger deflections are computed by FESPON than by SOLSAP. The comparison between computed values and measured data shows that the program FESPON with incompatible displacement mode produced better agreement with the measured values than the program SOLSAP.

Fig. 5.36 shows similar results in the transverse direction, section D. Again FESPON produces larger vertical deflections and closer agreement with the measured values.

5.5 Summary

Several slope stability analyses were performed using two-dimensional limit equilibrium methods and the three-dimensional programs BLOCK3, LEMIX and FESPON. The main findings of these analyses are as follows:

1. For both translational and rotational slides, the three-dimensional effect is most significant for cohesive soils and small failure lengths.

2. In the case of translational slides, the 3-D effect will increase with decreasing inclination of the weak layer and with lower strength of weak layer.

3. Wedge types of failure should be given particular attention because, in this case, the 3-D factor of safety may be less than the 2-D factor of safety.

4. It is difficult to predict the failure length of a rotational slide.
5. The steeper the slopes, the less important the 3-D effect.

6. Pore water pressures may cause the 3-D effect to be more significant.

7. The agreement between the finite element and limit equilibrium methods is quite good. The average factor of safety given by FESPON on a given failure surface is close to the factor of safety obtained by limit equilibrium on the same failure surface.
VI. CONCLUSIONS

This study is directed at developing techniques of three-dimensional slope stability analysis and comparing the results obtained with these techniques to those given by conventional two-dimensional methods. Computer programs based upon the limit equilibrium method are developed to assess the stability of both translational and rotational slides. A finite element technique is also proposed to perform the analysis of rotational slides.

In studying the stability of translational slides, attention is focused upon the most important controlling factor, the existence of a weak soil layer. The computer program BLOCK3 is generated to perform such an analysis. The ends of the critical surface is assumed according to Rankine's theory and the factory of safety is applied along the total failure surface. The study of translational slides yields several conclusions as follow:

(1) The 3-D factor of safety is usually greater than that of 2-D. However, a wedge type of failure may produce a $F_3/F_2$ ratio less than unity, and therefore should be examined carefully.

(2) The 3-D effect is more significant for cohesive soils than for cohesionless soils. This is also true for rotational slides.
(3) The lower the strength in the weak soil stratum, the more profound the 3-D effect.

(4) A steeply dipping weak soil always yields smaller ratios of $F_3/F_2$ than gently dipping layers.

A methodology is developed to study rotational slides, and a computer program LEMIX using the limit equilibrium method is generated. The failure mass is assumed symmetrical and divided into many vertical columns. The inclinations of the interslice forces is assumed the same throughout the whole failure mass. The intercolumn shear forces are assumed parallel to the base of the column and to be a function of their positions. Force and moment equilibrium are satisfied for each column as well as for the total mass. This method can handle different slopes, soil parameters, and pore water conditions and is considered a rather general method. The main conclusions of the analyses of rotational failures are summarized below:

(1) The 3-D effects are more significant at smaller lengths of the failure mass.

(2) For gentle slopes, the 3-D effects are most significant for soils of high cohesion intercept and low friction angle.

(3) For soils of low cohesion intercept and high friction angle, the 3-D factor of safety may be slightly less than that for the 2-D case. Pending more research, the 3-D stability analysis on this type of soil should be examined carefully.

(4) Pore water pressures may cause the 3-D effects to be even greater.
(5) The interslice angle influences the factor of safety. For soils of high cohesion intercept and low friction angle, \( \theta_3 \) is less than \( \theta_2 \) and thus \( F_3 \) is higher than \( F_2 \). The interslice angles \( \theta_3 \) obtained with a pore pressure parameter of 0.5 are larger than those obtained with no pore pressure \( (r_u = 0) \). On the contrary, for soils of low cohesion intercept and high friction angle, \( \theta_3 \) may be higher than \( \theta_2 \) and hence \( F_3 \) is less than \( F_2 \). In this case, \( \theta_3 \) for \( r_u \) equal to 0.5 is less than \( \theta_3 \) for \( r_u \) equal to 0.

(6) It is difficult to predict the length of the failure mass.

A finite element computer program FESPON is developed to perform the analysis of spoon shape failures. It uses a hyperbolic stress-strain relationship and an incremental technique to simulate the nonlinear behavior of soils. Isoparametric incompatible elements are used to provide good bending characteristics. The comparison of the results from both limit equilibrium method and finite element method are made for highly plastic St. Croix clay for which the stress-strain relationship is assumed to be hyperbolic. The hyperbolic parameters can be generated from conventional traxial test data or consolidation test data. Both the as-compacted condition and long-term condition are studied. The soil conditions and failure surface are assumed to be the same for both limit equilibrium and finite element methods. The results are quite similar, with the finite element method predictably yielding slightly higher factors of safety.

Though the proposed methods provide better techniques to analyze the 3-D slope stability, they still have shortcomings and in particular it is recommended to devote more research to the following points:
1. Development of searching techniques to find 3-D failure surfaces is worthwhile.

2. The assumptions of the angles of inclination and the distribution of the ends shear stress should be carefully studied. This is especially important when the soil conditions are complex.

3. More research on translational slides considering more complicated soil conditions (such as joints, faults, and anisotropy) and water conditions is needed.

4. The 3-D models presented in this dissertation should be applied to actual case studies in order to assess their prediction capabilities.
LIST OF REFERENCES


Boutrup, Eva (1977), "Computerized Slope Stability Analysis for Indiana Highways", JHRP Report No. 77-25 and 77-26, Purdue University, December, 1977. (Also MSCE Thesis, Purdue University, May, 512 pp.).


APPENDIX A

End Shear Forces of a Column

The end shear forces can be calculated using the following equations:

\[ R_{cE1} = c \, dx \, h_{E1} \quad (A.1) \]

\[ R_{cF1} = c_F \, dx \, h_{F1} \quad (A.2) \]

\[ R_{\phi E1} = \frac{1}{2} k_o (\rho - \rho_w) \, h_{E1}^2 \, dx \, \tan \phi_E \quad (A.3) \]

\[ R_{\phi F1} = k_o \left\{ (\rho_F - \rho_w) \, h_{E1} h_{F1} + \frac{1}{2} (\rho_F - \rho_w) h_{F1}^2 \right\} \, dx \, \tan \phi_F \quad (A.4) \]

Similarly,

\[ R_{cE2} = c \, dx \, h_{E2} \quad (A.5) \]

\[ R_{cF2} = c_F \, dx \, h_{F2} \quad (A.6) \]

\[ R_{\phi E2} = \frac{1}{2} k_o (\rho - \rho_w) \, h_{E2}^2 \, dx \, \tan \phi_E \quad (A.7) \]

\[ R_{\phi F2} = k_o \left\{ (\rho_F - \rho_w) \, h_{E2} h_{F2} + \frac{1}{2} (\rho_F - \rho_w) h_{F2}^2 \right\} \, dx \, \tan \phi_F \quad (A.8) \]

where \( h_{E} \) and \( h_{F} \) are shown in Fig. A.1. The resultant of horizontal forces acting in the foundations part, \( F_F \), and its position, \( y_F \), can be calculated using the following equations,

\[ F_F = \rho_E' \, h_E \, h_F + \frac{1}{2} \rho_F' \, h_F^2 \quad (A.9) \]

\[ y_F = \frac{\frac{1}{2} \rho_E' \, h_E \, h_F^2 + \frac{1}{6} \rho_F' \, h_F^3}{\rho_E' \, h_E \, h_F + \frac{1}{2} \rho_F' \, h_F^2} = \frac{(m h_E + \frac{1}{3} h_F)}{2 m h_E + h_F} \quad (A.10) \]

where \( m = \rho_E' / \rho_F' \).
Fig. A.1  Linear Distribution of Horizontal Stress Acting on the End of a Column
This Appendix describes the subroutines of the computer program FESPON and their functions (see Fig. B.1):

(a) Subroutine SETUP reads and prints input data, calculates the equation number according to the nodal points degrees of freedom, calculates the band width, and computes and prints the initial stresses and the initial values of modulus and Poisson's ratio for the elements.

(b) Subroutine RSEIG calculates the principal stresses and their directions in three-dimensional space.

(c) Subroutine CONTPAR looks for the major principle stresses and strains, and the minor principal stresses and strains.

(d) Subroutine MODU calculates the modulus values for the elements in accordance with the magnitudes of the stresses in the elements.

(e) Subroutine FOMING calls subroutine RELATE to establish strain-displacement matrices for elements.

(f) Subroutine RELATE forms the strain-displacement matrix.

(g) Subroutine CALNEQ determines the number of elements and nodal points for the problem to be analyzed, the number of equations, the number of equations in each block, and the number of blocks for each construction layer increment or load increment.
Fig. B.1 The Flow Chart of Computer Program—FESPON

IT = 2
NO

LN = LN+1

SPOON
SETUP
FORMING
LN=1
CALREQ
FORCE
IT=1
BILDUP
ADDSF
SYMBAN
RESULT
RSEIG
COMPAR
MODU
RELATE

IT=2
YES
LN = NUMLD
YES
FACTXY or FACTYZ
END

LN: Increment Number
IT: Iteration Number
NUMLD: Number of Load Increments
(h) Subroutine FORCE calculates nodal point forces due to weights of added elements (each node takes one-eighth of the weight of the element), reads concentrated load data, prints nodal points forces, sets up a force vector.

(i) Subroutine BILDUP formulates the constitutive equations, forms the element stiffness matrix and the strain-displacement matrix for each element.

(j) Subroutine ADDSTF forms the total stiffness matrix, two blocks at a time, by making a pass through the element stiffness matrices and adding the appropriate coefficients.

(k) Subroutine SYMBM solves the simultaneous equations representing the structural matrix and the structural load vector for nodal point displacements using the Gaussian elimination technique.

(l) Subroutine RESULT calculates stress increments and average stresses and evaluates the modulus for each element after the first iteration. After the second iteration it calculates the incremental and cumulative displacements for each nodal point, incremental and cumulative stresses and strains for each element, and modulus values for each element to be used in the next increment.

(m) Subroutine FACTXY, assuming the axis of rotation is parallel to the Z-axis and the movement is along the X-Y plane only, selects points on a well defined critical surface, and calculates the six components of stresses at these points. Thus, the local factors of safety can be calculated. After
the local factors of safety are obtained, the mean factor of safety may be calculated subsequently.

(n) Subroutine FACTYZ assuming the axis of rotation is parallel to X-axis and the movement is along Y-Z plane only. The functions of FACTXY and FACTYZ are the same.
APPENDIX C

TABLES RELATED TO TRANSLATIONAL SLIDES
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**TABLE C.2** $F_3/F_2$ FOR VARIOUS COMBINATIONS OF L, D, $c_w$, AND $\beta$, AT $c = 24.0$ kPa (500 psf), $\phi = 10^\circ$, $a = 1$ AND $\gamma = 90^\circ$
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TABLE C.4 \( F_3/F_2 \) FOR VARIOUS COMBINATIONS OF L, D, \( c_w \), AND \( \gamma \), AT 
\( c^2 = 47.9 \text{ kPa (1000 psf)} \), \( \phi = 0^\circ \), \( a = 1 \), AND \( \beta = 11.3^\circ \)

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APPENDIX D

FIGURES AND TABLES RELATED TO ROTATIONAL SLIDES
Fig. D.1 Ratio of $F_3/F_2$ (Slope 2.5/1, $r_u = 0$)
Fig. D.1 (Cont'd)
(e) $c'=28.7 \text{ kPa}, \phi'=15^\circ$

Fig. D.1 (Cont'd)
Fig. D.2 Ratio of $F_3/F_2$ (Slope $3.5/1$, $r_u = 0$)
Fig. D.2 (Cont'd)
\( F_3/F_2 \)

\( I_s/H \)

\[ c' = 28.7 \text{ kPa}, \ \phi' = 15^\circ \]

Fig. D.2 (Cont'd)
Fig. D.3 Ratio of $F_3/F_2$ (Slope 1.5/1, $r_u = 0.5$)
(c) $c' = 14.4 \text{kPa}$, $\phi' = 25^\circ$

(d) $c' = 21.6 \text{kPa}$, $\phi' = 20^\circ$

Fig. D.3 (Cont'd)
\( c' = 28.7 \text{ kPa}, \phi' = 15^\circ \)

Fig. D.3 (Cont'd)
Fig. D.4 Ratio of $F_3/F_2$ (Slope 2.5/1, $r_u = 0.5$)
Fig. D.4 (Cont'd)

(c) $c = 14.4 \text{ kPa, } \phi = 25^\circ$

(d) $c = 21.6 \text{ kPa, } \phi = 20^\circ$
Fig. D.4 (Cont'd)
Fig. D.5 Ratio of $F_3/F_2$ (Slope 3.5/1, $r_u = 0.5$)
Fig. D.5 (Cont'd)
Fig. D.5  (Cont'd)
TABLE D.1  COMPARISON OF $F_3$ BETWEEN ORDINARY METHOD OF COLUMNS (OMC) AND LEMIX (Slope 1.5/1, $r_u = 0$)

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*a: $c = 0$, $\phi = 40^\circ$; b: $c = 7.2$ kPa, $\phi = 30^\circ$; c: $c = 14.4$ kPa, $\phi = 25^\circ$; d: $c = 21.6$ kPa, $\phi = 20^\circ$; e: $c = 28.7$ kPa, $\phi = 15^\circ$
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TABLE D.2 COMPARISON OF F₃ BETWEEN ORDINARY METHOD OF COLUMNS AND LEMIX (Slope 2.5/1, \( r_u = 0 \))
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TABLE D.4 COMPARISON OF F3 BETWEEN ORDINARY METHOD OF COLUMNS AND LEMIX (Slope 1.5/1, r_u = 0.5)

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* a: \( c' = 0, \phi' = 40^\circ \) ; b: \( c' = 7.2 \text{ kPa}, \phi' = 30^\circ \)

c: \( c' = 14.4 \text{ kPa}, \phi' = 25^\circ \) ; d: \( c' = 21.6 \text{ kPa}; \phi' = 20^\circ \)
d: \( c' = 28.7 \text{ kPa}, \phi' = 15^\circ \)
### TABLE D.5 COMPARISON OF $P_3$ BETWEEN ORDINARY METHOD OF COLUMNS AND LEMIX (Slope 2.5/1, $r_u = 0.5$)

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TABLE D.6 COMPARISON OF $F_3$ BETWEEN ORDINARY METHOD OF COLUMNS AND LEMIX (Slope 3.5/1, $r_u = 0.5$)

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<td>4.4</td>
<td>4.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>- 5.6</td>
<td>- 5.8</td>
<td>- 5.9</td>
<td>- 6.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>10.0</td>
<td>10.4</td>
<td>10.5</td>
<td>10.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>- 2.9</td>
<td>- 2.9</td>
<td>- 3.0</td>
<td>- 3.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>e</td>
<td>- 2.2</td>
<td>- 2.5</td>
<td>- 2.8</td>
<td>- 2.7</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX E

User's Guide for Computer Programs BLOCK3, LEMIX, and FESPON

User's guide for computer programs BLOCK3, LEMIX, and FESPON is presented in this section. For each program there is an example to show how the input data are prepared and to provide output which can be used to check the operation of the computer programs:

Example Problem 1 - BLOCK3

Example Problem 2 - LEMIX

Example Problem 3 - FESPON

It should be noted that the meshes used in example 3 are too coarse to give accurate results. For accurate values of stress and displacement within an embankment, eight or more layers of elements should be used, and the number of elements should be larger than the ones in this example.

In all three samples, the units are in metric system.
E.1 User's Guide for Program BLOCK3

1. **Strength Parameter Card (6F10.0)**
   - 1-10 C  - Cohesion of foundation or embankment soil
   - 11-20 FI - Friction angle of foundation or embankment soil, degrees
   - 21-30 G  - Unit weight or density of foundation or embankment soil
   - 31-40 CB - Cohesion of weak layer
   - 41-50 FIB - Friction angle of weak layer, degrees
   - 51-60 UK - Earth pressure coefficient in the foundation or embankment

2. **Geometric Data Card (7F10.0)**
   - 1-10 TL  - The upper length at the top of the central block
   - 11-20 H  - Height of embankment
   - 21-30 SLOPE - Slope angle of embankment, degrees
   - 31-40 A  - The ratio of the lower length to upper length at the top of the central block
   - 41-50 D  - Depth ratio; depth (to weak layer) to height of embankment
   - 51-60 BETA - Inclination of weak soil layer, degrees
   - 61-70 GAMA - Unit weight or density of weak layer

3. **Surcharge Card (2F10.0)**
   - 1-10 SURA - Surcharge on active block
   - 11-20 SURP - Surcharge on passive block

4. **Initial Guess Value Card (F10.0)**
   - 1-10 X(1) - Initial guess value of the factor of safety
Fig. E.1 Example Problem for Program BLOCK3

- $L = 30.5 \text{ m}$
- $\alpha = 1$
- $\gamma = 90^\circ$
- $d = 0.67$

- $c = 40 \text{kPa}$
- $\phi = 10^\circ$
- $\rho = 1920 \text{kg/m}^3$

$\phi_w = 0^\circ$

9m 1.5
6m 20
<table>
<thead>
<tr>
<th>UNIT WT</th>
<th>C</th>
<th>FI</th>
<th>CB</th>
<th>FIB</th>
</tr>
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<tr>
<td>18.81</td>
<td>40.00</td>
<td>10.00</td>
<td>25.00</td>
<td>0</td>
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</table>

<table>
<thead>
<tr>
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<th>HT</th>
<th>SLOPE</th>
<th>A</th>
<th>D</th>
<th>BETA</th>
<th>GAMMA</th>
<th>ALFA</th>
<th>SF</th>
</tr>
</thead>
<tbody>
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<td>30.50</td>
<td>9.00</td>
<td>33.70</td>
<td>1.00</td>
<td>.67</td>
<td>2.90</td>
<td>90.00</td>
<td>0</td>
<td>2.01</td>
</tr>
</tbody>
</table>

Fig. E.2: Output Data for Program BLOCK3
PROGRAM BLOCK3(INPUT,OUTPUT,TAPES=INPUT,TAPEG=OUTPUT)
DIMENSION X(1)
COMMON/MATL/ TF,TFB,TANA,TANP
COMMON/FORS/ FAS,FS,FPS,WA,W,WP,CAS,CAB,CCS,CCB,CP,S,CPB
COMMON/GEOM/ COSA,COSB,COSP,SEINB,COSKSI,COSETA
EXTERNAL FCN

C

PROGRAM BLOCK3(INPUT,OUTPUT,TAPES=INPUT,TAPEG=OUTPUT)
DIMENSION X(1)
COMMON/MATL/ TF,TFB,TANA,TANP
COMMON/FORS/ FAS,FS,FPS,WA,W,WP,CAS,CAB,CCS,CCB,CP,S,CPB
COMMON/GEOM/ COSA,COSB,COSP,SEINB,COSKSI,COSETA
EXTERNAL FCN

C

READ AND WRITE INPUT DATA

C

READ(5,2000) C,FI,CI,FB,FI,RK
READ(5,2010) TL,H,SLOPE,A,D,BETA,GAMA
READ(5,2020) SURA,SURP
READ(5,2040) XINITI

PI=3.1415926/180.

SLOPE=SLOPE*PI
FI=FI*PI
TF=TAN(FI)
FIB=FIB*PI
TFB=TAN(FIB)
Q1=45.*PI/2.
TANP=TAN(Q1)
COSP=COS(Q1)
Q2=45.*PI/2.
TANA=TAN(Q2)
Q5=FI
GAMA=GAMA*PI
BETA=BETA*PI
Q6=BETA
COSB=COS(Q6)
SINB=SIN(Q6)
Q7=GAMA
H1=D*H
B=H*TAN(SLOPE)
H2=H1+B*TAN(Q5)
COSA=1./SQRT(1.+(COSB*(TL*(1.-A)/2.-(H2-H1)/TAN(GAMA)))**2)
COSTA=1./SQRT(1.+(SIN(PI/4.+FI/2.)/TAN(GAMA)))**2)
COSKSI=1./SQRT(1.+(SIN(PI/4.-FI/2.)/TAN(GAMA)))**2)

C

ACTIVE BLOCK

C

SURA=SURA*TL*H2*TAN(Q1)
WA=G*H2*H2*TAN(Q1)*(0.5*TL-H2/TAN(Q7))/3.+SURA
CAS=0.5*C*H2*H2*TAN(Q1)/SIN(Q7)
CAB=C*(TL-H2/TAN(Q7))*H2/COS(Q1)
FAS=UK*G*H2*H2*H2*TAN(Q1)*TAN(Q5)/(6.*SIN(Q7))

C

PASSIVE BLOCK

C

SURP=SURP*A*TL*H1*TAN(Q2)
WA=G*H2*H2*TAN(Q1)*(0.5*TL-H2/TAN(Q7))/3.+SURP
CAS=0.5*C*H2*H2*TAN(Q1)/SIN(Q7)
CAB=C*(TL-H2/TAN(Q7))/H2/COS(Q1)
FAS=UK *G*H2*H2*H2*TAN(Q1)*TAN(Q5)/(6.*SIN(Q7))

C ***********************************************************************
C PASSIVE BLOCK
C ***********************************************************************
SURP=SURP*A*TL*H1*TAN(Q2)
WP=G*H1*H1*TAN(Q2)*(0.5*A*TL-H1/TAN(Q7))/3.+SURP
CPB=0.5*G*H1*H1*TAN(Q2)/SIN(Q7)
CPB=C*(A*TL-H1/TAN(Q7))*H1/COS(Q2)
FPS=UK *G*H1*H1*TAN(Q2)*TAN(Q5)/(6.*SIN(Q7))

C ***********************************************************************
C CENTRAL BLOCK
C ***********************************************************************
B1=H2*(TL-H2/TAN(Q7))
B2=H1*(A*TL-H1/TAN(Q7))
Bm=(H1+H2)*((1.+A)*TL-(H1+H2)/TAN(Q7))
CCS=0.5*C*B1/(H1*TAN(Q7)*COS(ALFA))
CCB=CB*0.5*(1.+A)*TL-(H1+H2)/TAN(Q7))*B/COS(Q6)
FS=UK *G*B1*(H1+H2)*/TAN(Q5)/(6.*SIN(Q7)*COS(ALFA)*
   * COS(Q6))

C ***********************************************************************
C CALCULATE THE FACTOR OF SAFETY
C ***********************************************************************
X(1)=XINITI
N=1
NDIGIT=7
RNORM=0.
CALL SECANT(X,N,FCN,NDIGIT,RNORM)
SF=X(1)

PFI=FI/PI
PFIB=FI/PI
PSLOPE=SLOPE/PI
PBETA=BETA/PI
PGAMA=GAMA/PI
PLFA=ALFA/PI
WRITE(6,1000) G,C,PFIB,TL,HP,PSLOPE,ALFA,BETA,GAMA,ALFA,SF

10 FORMAT(132H1,UNIT,WHT,C,FI,CB,FIB,LENGTH
   * HT,SLOPE,AR,DR,BETA,GAMA,ALFA
   * SF)
1000 FORMAT(8F9.1,2F9.2,2F9.1,FS.2,F10.2)
2000 FORMAT(F8.10.0)
2010 FORMAT(7F10.0)
STOP
END

SUBROUTINE FCN(X,F,UDF)
DIMENSION X(1),F(1)
COMMON/MATL/ TF,TFB,TANA,TANP
COMMON/FORS/ FAS,FS,FPS,WA,W,WP,CAS,CAB,CCS,CCB,CPS,CPB
COMMON/GEOM/ COSA,COSB,COSP,SINB,COSKSI,COSETA

F(1)=(2.*(CCS+FS/SORT(1.+(X(1)/TF)**2))*COSA+CCB+TFB*W*COSB)/X(1)
1 -WA=(TANA-TF/X(1))/(1.+TANA*TF/X(1)+COSP*(1./X(1))*)((2.*CPS*
3 COSKSI+CPB+2.*FPS*COSKSI/SORT(1.+(X(1)/TF)**2))*(1.+TANP*(
4 TANP+TF/X(1))/(1.-TANP*TF/X(1)))+(2.*CAS*COSETA+CAB+2.*FAS
5 *COSETA/SORT(1.+(X(1)/TF)**2))*(TANP+(TANA-TF/X(1))/
6 (1.+TANA*TF/X(1))))

RETURN
END
1. **Embankment Information Card (8F10.2)**

   1-10 C  - Cohesion of embankment soil  
   11-20 FI  - Friction angle of embankment soil, degrees  
   21-30 GAMA  - Unit weight or density of embankment soil  
   31-40 RU  - Pore water pressure parameter, \( r_u \)  
   41-50 BETA  - Slope angle of embankment, degrees  
   51-60 H  - Height of embankment  
   61-70 EK  - Earth pressure coefficient in embankment  
   71-80 GW  - Unit weight or density of water  

2. **Foundation Information Card (5F10.2)**

   1-10 CF  - Cohesion of foundation soil  
   11-20 FIF  - Friction angle of foundation soil, degrees  
   21-30 GAMAF  - Unit weight or density of foundation soil  
   31-40 HTF  - The distance between the crest and foundation  
   41-50 FK  - Earth pressure coefficient in foundation  

3. **Critical Circle Information Card (4F10.2, 3I5)**

   1-10 RXY  - The radius of the 2-D critical circle, \( R_{xy} \)  
   11-20 RZ  - The length of minor axis of the semi-ellipsoid  
   21-30 CXI  - X-distance, from center to crest  
   31-40 Y  - Y-distance, from center to crest  
   41-45 NCOLUMN  - Number of columns along Z-direction  
   46-50 NSLICE  - Number of slices along X-Y plane  
   51-55 IFTC  - Zero, if tension crack is not considered; otherwise punch one
4. **Miscellaneous Card (4F10.2, 4I5)**

1-10 TWOD - The ratio between half of the length of the central cylinder to the height of the slope

11-20 EX - The exponential number of X-coordinate. One means the shear stress distribution is linear, two means the distribution is hyperbolic, etc.

21-30 FACTS - The ratio between the subsequent length to former length of the minor axis of the spoon

31-40 FACTR - The ratio between the subsequent length to former length of the central cylinder

41-45 NSP - Number of various spoons investigated

46-50 NRL - Number of various cylinders investigated

51-55 ICOND - One, if the results from the Ordinary Method of Columns need to be printed out; otherwise, punch zero

56-60 IPRINT - One, if the information of width, height, area, and weight of the columns need to be printed out; Otherwise, punch zero

5. **Initial Guess Value Cards (2F10.2)**

1-10 X(1) - The initial guess value of the factor of safety

11-20 X(2) - The initial guess value of the angle of inclination, degrees

These cards must be read for as many times as the number of NSP x NRL.

**REMARKS:**

The value of RNORM in the output indicates

- If RNORM = 0.0, then X is a root of the given system of equations to machine accuracy.

- If RNORM .GT. 0.0 then the relative convergence criterion was satisfied. In this case \( RNORM = F(1)\times2 + \ldots + F(N)\times2 \) where \( F \) contains the function values at \( X \), \( N \) the number of nonlinear equations to be solved.
- If RNORM = -1.0, then SECANT was unable to find a better approximation than the current X. If this approximation is not good enough the user may try a new initial guess.

- If RNORM = -2.0 then the maximum number of iterations was exceeded. The user may try a new initial guess value.

- If RNORM = -3.0 then SECANT was forced to stop because it was unable to improve the approximation to the root. The user may try a new initial guess.
Fig. E.3  Example Problem for Program LEMIX

Rz = 12 m

$c_e^i = 30$ kPa
$\phi_e^i = 5^\circ$
$\rho_e = 1920 \text{ kg/m}^3$

$c_f^i = 0$
$\phi_f^i = 40^\circ$
$\rho_f = 1926 \text{ kg/m}^3$
EMBANKMENT HEIGHT
SLOPE ANGLE
NUMBER OF SLICES
NUMBER OF COLUMNS
BIG RADIUS
SMALL RADIUS
X-DIST. FROM CENTER TO CREST
Y-DIST. FROM CENTER TO CREST
TENSION CRACK
EXPONENTIAL NUMBER

6.000
33.690
30
10
11.180
12.000
6.000
4.000
0
1.000

********** EMBANKMENT PARAMETERS **********

<table>
<thead>
<tr>
<th>C</th>
<th>FI</th>
<th>GAMA</th>
<th>RU</th>
</tr>
</thead>
<tbody>
<tr>
<td>30.00</td>
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<td>18.81</td>
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</tbody>
</table>

********** FOUNDATION PARAMETERS **********

<table>
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<tr>
<th>C</th>
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<th>GAMA</th>
<th>HTF</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>40.00</td>
<td>18.86</td>
<td>6.00</td>
</tr>
</tbody>
</table>

THE WIDTH OF CURVE SHAPE IS: 9.541
HALF WIDTH OF UNIFORM CROSS SECTION IS: 0.006

FS  THET  RNORM
2-D (SPENCER)  2.375  14.839  .000
3-D  2.408  11.612  .000

3-D (ORDINARY)  2.420

Fig. E.4  Output Data for Program LEMIX
PROGRAM LEMIX(INPUT, OUTPUT, TAPES=INPUT, TAPEB=OUTPUT)
COMMON/MLFL/GAMA, GAMAF, C, CF, TF, TFF, RU, W2, W3, GW
COMMON/GEMD1/AXY, AY2, SLFA, PLFA, YS, YF, HE, HF, SHE, SHF
COMMON/GEOM2/R, RAXA, BAREA, DX, DZ, DTL, EX, FK
COMMON/MISL/H, NCOL2, ITER, FH2, FH3, DISTRI
DIMENSION AXY(50,20), AY2(50,20), SLFA(50,20), PLFA(50), ALFA(50)
DIMENSION BAREA(50,20), THET(20)
DIMENSION RAD(20), YS(50), YP(50), SHE(50,20), SHF(50,20)
DIMENSION HE(50,20), HF(50,20), YE(50), YF(50), DISTRI(20)
DIMENSION X2(20), XX(50), YY(50), ZZ(50), DX(50), Z(50,20), DZ(50,20)
DIMENSION FW2(50), FW3(50,20), WH(50,20), W2(50), W3(50,20)
EXTERNAL FCN

C ******************************************************************************
C INPUT PARAMETERS
C ******************************************************************************
READ 1000, C, FI, GAMA, RU, BETA, H, EK, GW
READ 1010, CF, FIF, GAMAF, HIF, FK
READ 1020, RXY, RZ, CX1, Y, NCOLUMN, NSLICE, IFTC
READ 1030, THOD, EX, FACTS, FACTR, NSP, NRE, ICOND, IPRINT
TC1=1.33*C*SQRT((1. + SIN(FI/57.295775))/(-SINCFI/SF.SS1FSS1))
*C
GAMA
IF(IFTC.EQ.0) TC1=0.
PRINT 2000, H, BETA, NSLICE, NCOL2, RXY, RZ, CX1, Y, TC1, EX
PRINT 2010 PRINT 2020, C, FI, GAMA, RU
PRINT 2030 PRINT 2040, CF, FIF, GAMAF, HIF
R=RXY
ENA=R*RX/(RZ*RZ)
FI=FI/57.2957751
FIF=FIF/57.2957751
TF=TAN(FI)
TFF=TAN(FIF)
BETA=BETA/57.2957751
TB=TAN(BETA)
RU=RU/GAMA

C ******************************************************************************
C GEOMETRY OF THE SLOPE
C ******************************************************************************

THE1=ASIN(Y/R)
XC=R*COS(THE1)
HX=XC-CX1
THE2=ATAN(Y/(XC-HX)) - THE1
EF=H/SIN(BETA)
OE=EF*THE1 + THE2
OF=SQRT(EF**2 + OE**2 - 2*EF*OE*COS(THET1+THET2+BETA))
THE3=ASIN((H*EF)/(OE*THE1+THE2+BETA)/(SIN(BETA)*OF))
TOTHE1+THE2+THE3+BETA
RSIN=R*SIN(THE1+THE2+THE3)
IF(RSIN-(Y+H)) 6, 5
5
THE4=3.1415926-ASIN(OF*SIN(THE1+THE2+THE3)/R)-THE1-THE2
* -THE3
GO TO 7
6
THE4=3.1415926-ASIN(OE*SIN(THE1+THE2+BETA)/R)-TOTHE1-THE2
Y0=OE*SIN(3.1415326-THE1-THE2+BETA)/COS(BETA)
\( CX_2 = HC - H / TB \)
\( CX_3 = R \cdot \cos(THET_1 + THET_2 + THET_3 + THET_4) \)
\( TT_1 = THET_1 + 57.29577951 \)
\( TT_2 = THET_2 + 57.29577951 \)
\( TT_3 = THET_3 + 57.29577951 \)
\( TT_4 = THET_4 + 57.29577951 \)
\( \text{ANGLE} = \text{ANGLE} / \text{FLOAT} (NSLICE) \)

C READ INITIAL GUESS VALUES FOR SECANT METHOD

\[ \text{ITER} = 1 \]
\[ \text{THODI} = \text{THOD} \]
\[ \text{DO } 620 \text{ IS} = 1, \text{NSP} \]
\[ \text{READ } 1040. \text{ X(1), X(2)} \]
\[ \text{X(2)} = \text{X(2)} / 57.29577951 \]

C CALCULATE THE HEIGHT, THE WIDTH, AND THE DIP FOR EACH SLICE

\[ \text{ET}_1 = \text{ANG}_1 + 0.5 \cdot \text{DANGLE} \]

10 \( K = 0 \)
20 \( K = K + 1 \)
30 \( I = K \)
40 \( \text{XX}(K) = R \cdot \cos(\text{ET}_1) \)
50 \( \text{CK} = \text{XX}(K) - \text{CX}_1 \)
60 \( \text{ZZ}(K) = R \cdot \sin((\sin(\text{ET}_1) \cdot 2 - \sin(\text{THET}_1) \cdot 2) / \text{ENA}) \)
70 \( \text{IF} (\text{ITER} \cdot \text{EQ.} \cdot 2) \text{ GO TO 40} \)
80 \( \text{YY}(K) = R \cdot \sin(\text{ET}_1) \)
90 \( \text{YE}(K) = \text{YY}(K) - \text{Y} \)

C CALCULATE THE HEIGHT, THE WIDTH, AND THE DIP FOR EACH SLICE

\[ \text{ET}_1 = \text{ANG}_1 + 0.5 \cdot \text{DANGLE} \]

10 \( K = 0 \)
20 \( K = K + 1 \)
30 \( I = K \)
40 \( \text{XX}(K) = R \cdot \cos(\text{ET}_1) \)
50 \( \text{CK} = \text{XX}(K) - \text{CX}_1 \)
60 \( \text{IF} (\text{CK} \cdot \text{LT.} \cdot 0.) \text{ GO TO 50} \)
70 \( \text{ZZ}(K) = R \cdot \sqrt{((\sin(\text{ET}_1) \cdot 2 - \sin(\text{THET}_1) \cdot 2) / \text{ENA})} \)
80 \( \text{IF} (\text{ITER} \cdot \text{EQ.} \cdot 2) \text{ GO TO 40} \)
90 \( \text{YY}(K) = R \cdot \sin(\text{ET}_1) \)
100 \( \text{YE}(K) = \text{YY}(K) - \text{Y} \)

C CALCULATE THE HEIGHT, THE WIDTH, AND THE DIP FOR EACH SLICE

\[ \text{ET}_1 = \text{ANG}_1 + 0.5 \cdot \text{DANGLE} \]

10 \( K = 0 \)
20 \( K = K + 1 \)
30 \( I = K \)
40 \( \text{XX}(K) = R \cdot \cos(\text{ET}_1) \)
50 \( \text{CK} = \text{XX}(K) - \text{CX}_1 \)
60 \( \text{IF} (\text{CK} \cdot \text{LT.} \cdot 0.) \text{ GO TO 50} \)
70 \( \text{ZZ}(K) = R \cdot \sqrt{((\sin(\text{ET}_1) \cdot 2 - \sin(\text{THET}_1) \cdot 2) / \text{ENA})} \)
80 \( \text{IF} (\text{ITER} \cdot \text{EQ.} \cdot 2) \text{ GO TO 40} \)
90 \( \text{YY}(K) = R \cdot \sin(\text{ET}_1) \)
100 \( \text{YE}(K) = \text{YY}(K) - \text{Y} \)

C CALCULATE THE HEIGHT, THE WIDTH, AND THE DIP FOR EACH SLICE

\[ \text{ET}_1 = \text{ANG}_1 + 0.5 \cdot \text{DANGLE} \]
GO TO 60
100 IF(THET4 .LT. DANGLE) GO TO 150
   L=L-1
   ET3=ET2
110 L=L+1
   XX(L)=R*COS(ET3)
   IF(XX(L) .LT. CX3) GO TO 150
   IF(L .GT. NSLICE) GO TO 150
   ZZ(L)=SORT((R*R-XX(L)**2-(Y+H)**2)/ENA)
   IF(ITER .EQ. 2) GO TO 140
   DX(L)=R*DANGLE*SIN(ET3)
   YY(L)=R*SIN(ET3)
   YS(L)=YY(L)-R*SIN(ANGLE+ANG1)
   IF(YY(L) .LT. (Y+HTF)) GO TO 120
   YE(L)=YS(L)
   YF(L)=0.
   GO TO 130
120 YF(L)=YY(L)-Y-HTF
   YE(L)=YS(L)-YF(L)
130 ALFA(L)=ET3
   GO TO 110
150 IF(ITER .EQ. 2) GO TO 200
   DO 160 I=1,(NSLICE-1)
      YP(I+1)=(YS(I)+YS(I+1))/2.
   160 CONTINUE
   WP(I)=TC1
   DO 170 I=1,NSLICE
      PLFA(I)=1.5707953-ALFA(I)
   170 CONTINUE
C CALCULATE THE WEIGHT OF EACH SLICE AND WATER PRESSURE IN
C TENSION CRACK
   DTL=1.
   FW2(I)=0.5*TC1**2*GW*DTL
   DO 180 I=2,NSLICE
      FW2(I)=0.
   180 CONTINUE
   DO 190 I=1,NSLICE
      W2(I)=(YE(I)*GAMA+YF(I)*GAMAF)*DX(I)*DTL
   190 CONTINUE
C *************************************************************************
C SOLVE 2-D FACTOR OF SAFETY
C *************************************************************************
   RNORM=0.
   CALL SECANT(XC2,FNC,7,RNORM)
   FS=XC1)
   SETA=XC2)
   57.29577951
   RNOR1=RNORM
C *************************************************************************
C GENERATE 3-D COORDINATES
C *************************************************************************
C FIND THE MAXIMUM WIDTH OF SPOON SHAPE FAILURE SURFACE
200 BIG=ZZ(1)
DO 310 I=2,NSLICE  
   ABI=Z(I)  
   IF(BIG-ABI) 300,310,310  
300 BIG=AB  
310 CONTINUE  
   DTL=BIG/(FLOAT(NCOLUM)-1.)  
   TCX=R*COS(ANG1)  
   SENSE=1-((R*COS(ANG1))**2-(SIN(THET1))**2)/ENA  
   IF(SENSE .LT. 0.) SENSE=0.  
   TCZ=R*SORT(SENSE)  

C CALCULATE THE HEIGHT, SIDE HEIGHT, WIDTH, DIP, AND BOTTOM AREA FOR  
C EACH COLUMN  
   RAD(1)=R  
   DO 350 J=2,NCOLUM  
      RAD(J)=SORT(R-((J-1.5)*DTL)**2*ENA)  
330 CONTINUE  
   DO 340 I=1,NSLICE  
      IF(ZZ(I) .LT. (J-2.)*DTL) GO TO 330  
      IF(J .EQ. NCOLUM) GO TO 320  
      IF(ZZ(I) .GT. (J-2.)*DTL .AND. ZZ(I) .LE. (J-1.)*DTL) GO TO 320  
      DZ(I,J)=DTL  
      Z(I,J)=DZ(I,J)*0.5+(J-2.)*DTL  
      YHC=SORT(R*R-XX(I)**2-ENA*((J-1.5)*DTL)**2)  
      HF(I,J)=YHC-Y-HTF  
      IF(HF(I,J) .LT. 0.) HF(I,J)=0.  
      HE(I,J)=YHC-(YY(I)-YS(I))-HF(I,J)  
      IF(HE(I,J) .LE. 0.) HE(I,J)=0.  
      SH=SORT(R*R-XX(I)**2-ENA*((J-2.)*DTL)**2)  
      SHF(I,J)=SH-Y-HTF  
      IF(SHF(I,J) .LT. 0.) SHF(I,J)=0.  
      SHE(I,J)=SH-(YY(I)-YS(I))-SHF(I,J)  
      IF(SHE(I,J) .LE. 0.) SHE(I,J)=0.  
      AKY(I,J)=ATAN(XX(I)/SORT(R*R-XX(I)**2-ENA*Z(I,J)**2))  
      SLF(I,J)=ATAN(XX(I)/SORT(R*R-XX(I)**2-ENA*((J-2.)*DTL)**2))  
      AYZ(I,J)=ATAN(ENA*Z(I,J)/SORT(R*R-XX(I)**2-ENA*Z(I,J)**2))  
      W3(I,J)=DX(I)*DZ(I,J)*GAMA*HE(I,J)+GAMAF*HF(I,J))  
      BAREA(I,J)=DX(I)*DZ(I,J)*(SORT(1.-SINCAYZ(I,J))*SINCAYZ(I,J))  
      I **2))/(COS(AYZ(I,J)))*COS(AKY(I,J)))  
   1 **2))  
   GO TO 340  

320 DZ(I,J)=2*ZZ(I)-2*(J-1.5)*DTL  
   Z(I,J)=DZ(I,J)/2.+(J-2.)*DTL  
   YHC=SORT(R*R-XX(I)**2-ENA*(Z(I,J)**2))  
   HF(I,J)=YHC-Y-HTF  
   IF(HF(I,J) .LT. 0.) HF(I,J)=0.  
   HE(I,J)=YHC-(YY(I)-YS(I))-HF(I,J)  
   IF(HE(I,J) .LE. 0.) HE(I,J)=0.  
   SH=SORT(R*R-XX(I)**2-ENA*((J-2.)*DTL)**2)  
   SHF(I,J)=SH-Y-HTF  
   IF(SHF(I,J) .LT. 0.) SHF(I,J)=0.  
   SHE(I,J)=SH-(YY(I)-YS(I))-SHF(I,J)  
   IF(SHE(I,J) .LE. 0.) SHE(I,J)=0.  
   AKY(I,J)=ATAN(XX(I)/SORT(R*R-XX(I)**2-ENA*Z(I,J)**2))  
   SLF(I,J)=ATAN(XX(I)/SORT(R*R-XX(I)**2-ENA*((J-2.)*DTL)**2))  
   AYZ(I,J)=ATAN(ENA*Z(I,J)/SORT(R*R-XX(I)**2-ENA*Z(I,J)**2))  
   W3(I,J)=DX(I)*DZ(I,J)*GAMA*HE(I,J)+GAMAF*HF(I,J))  
   BAREA(I,J)=DX(I)*DZ(I,J)*(SORT(1.-SINCAYZ(I,J))*SINCAYZ(I,J))  
   (COS(AYZ(I,J)))*COS(AKY(I,J)))  
   IF(HE(I,J) .LE. 0. .AND. HF(I,J) .LE. 0.) BAREA(I,J)=0.  
   GO TO 340  

330 DZ(I,J)=0.
Z(I,J)=0.
HE(I,J)=0.
HF(I,J)=0.
SHE(I,J)=0.
SHF(I,J)=0.
AXY(I,J)=0.
AYZ(I,J)=0.
SLFA(I,J)=0.
W3(I,J)=0.
BAREA(I,J)=0.

CONTINUE

CONTINUE

C THIS PART DEALS WITH UNIFORM CROSS SECTIONS

TWOI=TWOI
DO 610 IU=1,NRL
C2D=TWOI
DO 360 I=1,NSLICE
DZ(I,1)=C2D
AXY(I,1)=PLFA(I)
BAREA(I,1)=DX(I)*C2D/COS(AXY(I,1))
W3(I,1)=W2(I)*C2D
HF(I,1)=YF(I)
HE(I,1)=YE(I)
SHE(I,1)=YE(I)
SHF(I,1)=YF(I)
SLFA(I,1)=PLFA(I)

C CALCULATE WATER PRESSURE IN 3-D TENSION CRACK

CONTINUE

DO 390 J=2,NCOLUMN
IF(TCZ .LE. (J-1.)*DTL) GO TO 380
IF(TCZ .GT. (J-1.)*DTL .AND. TCZ .LE. J*DTL) GO TO 370
WH(1,J)=SQRT(R*R-TCX*TCX-ENA*((J*DTL)**2./4.)-Y
FW3(I,J)=0.5*GW*WH(1,J)**2
GO TO 390

WH(1,J)=SQRT(R*R-TCX*TCX-ENA*((TCZ+(J-1.)*DTL)**2./4.)-Y
FW3(I,J)=0.5*GW*(TCZ-(J-1.)*DTL)*WH(1,J)**2
GO TO 390

380 FW3(I,J)=0.
390 CONTINUE

WH(1,1)=TC1
FW3(I,1)=0.5*GW*C2D*TC1**2.

1000 FORMAT(8F10.2)
1010 FORMAT(5F10.2)
DO 400 I=1,NSLICE
SHE(I,NCOLUMN+1)=0.
SHF(I,NCOLUMN+1)=0.
SLFA(I,NCOLUMN+1)=0.
400 CONTINUE

DO 410 I=2,NSLICE
DO 420 J=1,NCOLUMN
WH(I,J)=0.
FW3(I,J)=0.
410 CONTINUE

IF(IPRINT .EQ. 0) GO TO 440
PRINT 2050
DO 430 J=1,NCOLUMN
DO 440 I=1,NSLICE
XYA=AXY(I,J)*57.29577951
YZA=AYZ(I,J)*57.29577951
**SOLVE 3-D FACTOR OF SAFETY**

**ASSUMING INTER-COLUMN SHEAR STRESSES DISTRIBUTION**

```plaintext
DISTRI(1)=0,
DO 500 J=2,(NCOLUMN+1)
  DISTRI(J)=((C2D+(J-2.))/((NCOLUMN-1.)*DTL+C2D))**EX
500 CONTINUE

ITER=3
RNORM=0.
X(1)=FS
X(2)=SETA/57.29577951
CALL SECANT(X,2,FCN,7,RNORM)
DEGREE=X(2)*57.29577951
RNOR2=RNORM
PRINT 3020, BIG
PRINT 3030, C2D
PRINT 3040
PRINT 3050, FS,SEMA,RNOR1
PRINT 3050, X(1),DEGREE,RNOR2
IF(ICOND.EQ.0) GO TO 610
```

**CALCULATE FACTOR OF SAFETY FROM ORDINARY METHOD OF COLUMNS**

```plaintext
FST=0.
FSB=0.
DO 550 J=1,NCOLUMN
  DO 540 I=1,MSLICE
    IF(HE(I,J).EQ.0. .AND. HF(I,J).EQ.0.) GO TO 540
    HRE=HE(I,J)/(HE(I,J)+HF(I,J))
    HRF=1.-HRE
    IF(HE(I,J)) 510,510,520
510 PA=C
  PB=TFF
  GO TO 530
520 PA=TFF
  PB=TF
  GO TO 530
530 FST=FST+PA*BAREA(I,J)+W3(I,J)*PB*(1.-RU*(HRE/GAMA+HRF/GAMAF))
  FSB=FSB+H3(I,J)*SIN(AXY(I,J))+FW3(I,J)*(Y+2.*WH(I,J)/3.)/RAD(J)
540 CONTINUE
550 CONTINUE
FSORD=FST/FSB
PRINT 3070,FSORD
610 THOD=THOD*FACTR
ITER=2
620 ENA=ENA/(FACTS*FACTS)
```

**FORMAT**

```plaintext
1020 FORMAT(4F10.2,3I5)
```
SUBROUTINE FCNX,X,F,UDF)

COMMON/MATL/GAMA,GAMAF,C,CF,TF,TFF,RU,W2,W3,GW
COMMON/GEOM1/AXY,AYZ,SLFA,PLFA,YS,YF,HE,HF,SHE,SHF
COMMON/GEOM2/R,RAD,BAREA,DX,DZ,DTL,EK,FK
COMMON/MISL-NSLICE,NCOLUMN,ITER,FH2,FH3,DISTRI

DIMENSION HE(50,20),HF(50,20),YS(50),YF(50,20),DX(50)
DIMENSION AXY(50,20),AYZ(50,20),SLFA(50,20),PLFA(50),YF(50)
DIMENSION SHE(50,20),SHF(50,20),DISTRI(20)
DIMENSION FH2(50),W3(50,20),W2(50),W3(50,20),BAREA(50,20)
DIMENSION RAD(20),DZ(50,20),X(20),F(20)

F(1)=0.
F(2)=0.

RG=GAMA/GAMAF
IF(ITER.EQ.3) GO TO 740

DO 730 I=1,NSLICE

IF(YF(I)) 700,700,710

700 PA=C
PB=TF
GO TO 720

710 PA=CF
PB=TFF

720 F(I)=F(I)+
1 (PA*DX(I)*DTL/(X(I)*COS(PLFA(I)))+PB*(W2(I)*COS(PLFA(I)))
2 -RU*YS(I)*DX(I)*DTL/COS(PLFA(I)))/X(I)-W2(I)*
3 SIN(PLFA(I))*FH2(I)*(COS(PLFA(I)))+SIN(PLFA(I)))*PB/X(I))

STOP
740 DO 790 J=1,NCOLUMN
    DO 790 I=1,NSLICE
       WEIGHT=W3(I,J)*COS(AXY(I,J))
       FWC=FW3(I,J)*COS(AXY(I,J))
       DZT=DZ(I,J)*TAN(AYZ(I,J))
       TF1=TAN(AXY(I,J)-SLFA(I,J))
       TF2=TAN(SLFA(I,J+1)-AXY(I,J))
       IF(SHE(I,J).EQ.0. .AND. SHF(I,J) .EQ. 0.) GO TO 750
       YYF(I,J)=(RG*/*SHE(I,J)+SHF(I,J)/3.)*SHF(I,J)/(2.*RG*/*SHE(I,J)+*
       * SHE(I,J))
       RCE1=C*DX(I)*SHE(I,J)
       RCF1=CF*DX(I)*SHF(I,J)
       RSE1=0.5*EB*(GAMA-RU)*SHE(I,J)**2*DZ(I)*TF
       RSEF=FK*((GAMA-RU)*SHE(I,J)+SHF(I,J)+0.5*(GAMAF-RU)*SHF(I,J)**2)
       * DX(I)*TFF
       RCE2=C*DX(I)*SHE(I,J+1)
       RCF2=CF*DX(I)*SHF(I,J+1)
       RSE2=0.5*EB*(GAMA-RU)*SHE(I,J+1)**2*DZ(I)*TF
       RSEF=FK*((GAMA-RU)*SHE(I,J+1)+SHF(I,J+1)+0.5*(GAMAF-RU)*SHF(I,J+1)
       * **2)*DX(I)*TFF
       R1=(RCE1+RCF1+RSE1+RSF1)*DISTRI(J)
       R2=(RCE2+RCF2+RSE2+RSF2)*DISTRI(J+1)
       T0TR1=R1*COS(SLFAC(I,J)-AXY(I,J))
       T0TR2=R2*COS(AXY(I,J)-SLFA(I,J+1))
       RS1=RCE1*(6.*SHF(I,J)+3.*SHE(I,J)-3.*DZT)+RCF1*(3.*SHF(I,J)-3.*DZT)
       *+RSE1*(6.*SHE(I,J)+2.*SHE(I,J)-3.*DZT)+RSF1*(6.*YYF(I,J)-3.*DZT)
       RS2=RCE2*(6.*SHF(I,J+1)+3.*SHE(I,J+1)+3.*DZT)+RCF2*(3.*SHF(I,J+1)+*
       * 3.*DZT)+RSE2*(6.*SHF(I,J+1)+2.*SHE(I,J+1)+3.*DZT)+RSF2*(6.*
       * YYF(I,J+1)+3.*DZT)
       RR1=RS1/DISTRI(J)
       RR2=RS2/DISTRI(J+1)
       IF(HF(I,J)) 760,760,770
       PA=C
       PB=TF
       GO TO 780
       PA=CF
       PB=TFF
    780 COHESN=PA*BAREA(I,J)
       PORPRE=RU*BAREA(I,J)+HE(I,J)+HF(I,J))*PB/(COS(AXY(I,J))*SQRT
       * (1.+TAN(AYZ(I,J))***2+TAN(AXY(I,J)**2))
       F(I)=F(1)+
       (COHESN/X(1)-PORPRE/X(1)+HEIGHT*(PB/X(1)-TAN(AXY(I,J)))+
       (TOTR2*(1.-PB*TF2/X(1)))/TOTR1*(1.+PB*TF1/X(1)))/X(1)-FHC
       *(1.+PB*TAN(AYZ(I,J)))*X(1))/X(1))
       TAN(AXY(I,J))=X(2)*/X(1)))
F(2) = F(2) + RAD(J) * 
1 (COHESN/X(1) - PORPRE/X(1) + WEIGHT*(PB/X(1) - TAN(AXY(I,J)))) + 
2 (TOTR2*(1 - PB*TF2/X(1)) - TOTR1*(1 + PB*TF1/X(1)))/X(1) - FWC 
3 *(1 + PB*TAN(AXY(I,J))/X(1)))/(1 + PB*TAN(AXY(I,J) - X(2)) 
4 /X(1)) 
5 -COS(X(2) - AXY(I,J)) + COS(AXY(I,J)) * (COS(SLFA(I,J+1)) - RR2 - 
6 COS(SLFA(I,J)) + RR1) / (6 * COS(X(2)) * X(1)) 

790 CONTINUE 
800 CONTINUE 
RETURN 
END
1. Control Cards

a) Heading Card (12A6)

2-72 HED — Title card for program identification

b) Control Data Card (9I5)

1-5 NUMELT — Total number of elements in the complete structure
6-10 NUMNPT — Total number of nodal points in the complete structure
11-15 NFEL — Number of elements in the foundation part
16-20 NFNP — Number of nodal points in the foundation part
21-25 NUMCEL — Number of elements in the preexisting part
26-30 NUMCNP — Number of nodal points in the preexisting part
31-35 NUMMAT — Number of different material types
36-40 NLAY — Number of construction layer increments
41-45 NFORCE — Number of load increments after construction

2. Material Property Cards

a) Units Conversion Card (F10.0)

1-10 PATM — Atmospheric pressure expressed in the system of units used in the problem.

For example:

<table>
<thead>
<tr>
<th>Length</th>
<th>Unit Weight</th>
<th>Cohesion</th>
<th>Atmospheric Pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>ft</td>
<td>ton/ft³</td>
<td>ton/ft²</td>
<td>1.058</td>
</tr>
<tr>
<td>ft</td>
<td>kip/ft³</td>
<td>kip/ft²</td>
<td>2.116</td>
</tr>
<tr>
<td>ft</td>
<td>lb/ft³</td>
<td>lb/ft²</td>
<td>211.62</td>
</tr>
<tr>
<td>m</td>
<td>ton/m³</td>
<td>ton/m²</td>
<td>10.35</td>
</tr>
<tr>
<td>m</td>
<td>kN/m³</td>
<td>kN/m²</td>
<td>101.4</td>
</tr>
</tbody>
</table>

b) Material Properties (I5,7F10.0/4F10.0)

The first and second cards must be specified for each material.
First Card

1-5 M - Material type number
6-15 EMPR(M, 1) - Unit weight
16-25 EMPR(M, 2) - Modulus number K
26-35 EMPR(M, 3) - Unloading-reloading modulus number $K_{ur}$
36-45 EMPR(M, 4) - Modulus exponent n
46-55 EMPR(M, 5) - Poisson's ratio parameter $d$
56-65 EMPR(M, 6) - Poisson's ratio parameter $G$
66-75 EMPR(M, 7) - Poisson's ratio parameter $F$

Second Card

1-10 EMPR(M, 8) - Cohesion $c$
11-20 EMPR(M, 9) - Friction angle (degrees)
21-30 EMPR(M,10) - Failure ratio $R_f$
31-40 EMPR(M,11) - Earth pressure coefficient in the foundation $K_0$ (zero or blank if the material is not in the foundation).

3. Nodal Point and Boundary Condition Cards (I5,3F10.0,3I5)

One card for each nodal point.

1-5 N - Nodal point number
6-15 X(N) - X-coordinate (+ to right)
16-25 Y(N) - Y-coordinate (+ up)
26-30 Z(N) - Z-coordinate (left-hand rule)
31-35 ID(N, 1) - Boundary condition code for X-direction
36-40 ID(N, 2) - Boundary condition code for Y-direction
41-45 ID(N, 3) - Boundary condition code for Z-direction
Nodal points must be read in sequence. If nodal points cards are omitted, the nodal point data for a series of nodal points are generated automatically at equal spacing between those specified. The boundary condition codes for the generated nodal point are set equal to the boundary condition codes for the previous nodal point. The first and the last nodal points must be specified.

Boundary condition code:

Zero or blank indicates that the nodal point is free to move in that direction and loads may be applied.

One indicates that the nodal point is fixed in that direction.

4. **Element Cards (1015)**

One card for each element.

1-5 N - Element number

6-10 INP(N,1) - Number of nodal point I

11-15 INP(N,2) - Number of nodal point J

16-20 INP(N,3) - Number of nodal point K

21-25 INP(N,4) - Number of nodal point L

26-30 INP(N,5) - Number of nodal point M

31-35 INP(N,6) - Number of nodal point N

36-40 INP(N,7) - Number of nodal point O

41-45 INP(N,8) - Number of nodal point P

46-50 INP(N,9) - Material number

Elements must be read in sequence. The nodal point numbers must be specified proceeding counterclockwise around each element in the order I, J, K, L, M, N, O, P as shown in Fig. E.1. If element cards are omitted, the element data for a series of elements are generated automatically by increasing the preceding values of I, J,
Fig. E.5 Eight Point Three-Dimensional Element
K, L, M, N, O, P by one. The material number for the generated element is set equal to the material number for the previous element. The first and last elements must be specified.

The center of the element is calculated by

\[
\begin{align*}
X_{CP} &= \frac{1}{8} \{X(I) + X(J) + X(K) + X(L) + X(M) + X(N) + X(O) + X(P)\} \\
Y_{CP} &= \frac{1}{8} \{Y(I) + Y(J) + Y(K) + Y(L) + Y(M) + Y(N) + Y(O) + Y(P)\} \\
Z_{CP} &= \frac{1}{8} \{Z(I) + Z(J) + Z(K) + Z(L) + Z(M) + Z(N) + Z(O) + Z(P)\}
\end{align*}
\]

5. Construction Layer Element and Nodal Point Cards (915)

If NLAY = 0, these cards are omitted.

One card for each construction layer.

1-5 LN - Number of the construction layer, increasing upward from the bottom

6-10 NOMEL(LN,1) - Smallest element number of the newly placed elements in this layer

11-15 NOMEL(LN,2) - Largest element number of the newly placed elements in this layer

16-20 NOMP(LN,1) - Smallest nodal point number of the newly placed nodal points in this layer

21-25 NOMP(LN,2) - Largest nodal point number of the newly placed nodal points in this layer

26-30 NPHUMP(LN,1) - The first nodal point on the humped surface

31-35 NPHUMP(LN,2) - The second nodal point on the humped surface

36-40 NPHUMP(LN,3) - The third nodal point on the humped surface

41-45 NPHUMP(LN,4) - The fourth nodal point on the humped surface
For simplicity, the position of the "humped surface" is defined by the coordinates of the four nodal points on the central section \( z = 0 \). To the left of the first nodal point and to the right of the fourth nodal point the surface is assumed to be horizontal.

6. **Foundation Cards**

If \( NFEL = 0 \), these cards are omitted.

a) Control Card \((I5,F10.0)\)

1- 5 NFLAY  - Number of layers of elements in foundation

The maximum number of foundation layers is \( 10 \).

6-15 HFLEV  - Elevation of rigid base at bottom of foundation.

b) Layer Information Cards \((I5,F10.0)\)

1- 5 I     - Foundation layer number (Number from bottom upward)

6-10 MATNO(I)  - Material property number for this layer

11-15 NLEL(I)  - The first element number of this layer

16-20 NREL(I)  - The last element number of this layer

21-30 HL(I)   - Elevation of the top of this layer

7. **Force Cards**

If \( NFORCE = 0 \), these cards are omitted.

If \( NFORCE \geq 1 \), \( NFORCE \) sets of cards, each set consisting of types (a) through (b) below, are required.

**Number of Nodal Point Force Cards to be Used \((I5)\)**

1- 5 NUMFC  - Number of nodal point force cards for this load case
b) Nodal Point Force Cards \((I5, 3F10.0)\)

If \(\text{NUMFC} = 0\), these cards are omitted. Otherwise need \(\text{NUMFC}\) cards.

1-5 MM  - Nodal point number where force is applied
6-15 FX(MM) - X-component of force applied at MM (+ to right)
16-25 FY(MM) - Y-component of force applied at MM (+ up)
26-30 FZ(MM) - Z-component of force applied at MM (right-hand rule)

8. Geometry Cards

a) The Direction of the Movement Card \((2I5)\)

1-5 IFXY  - One, if the movement of the failure mass is along X-Y plane; otherwise zero
6-10 IFYZ  - One, if the movement of the failure mass is along Y-Z plane; otherwise zero

b) Number of Layers Card \((2I5)\)

1-5 LAYSUM - Total number of layers
6-10 MFLAY - Number of layers in the foundation

c) Elevation Information Cards \((8F10.0)\)

1-5 HEIGHT(1) - Elevation at the top of layer 1
6-10 HEIGHT(2) - Elevation at the top of layer 2

Elevation must be read in sequence from the lowest value to the highest value. They are read in the same card.

d) Foundation Element Number Cards \((2I5)\)

1-5 MEL(M) - The first element number of foundation layer M
6-10 MREL(M) - The last element number of foundation layer M
The number must be read from the lowest layer to the highest layer of foundation. The number of these cards are equal to the number of layers in the foundation.

e) Embankment Element Number Cards (2I5)

1-5 MOMEL(LP,1) - The first element number of embankment layer LP

6-10 MOMEL(LP,2) - The last element number of embankment layer LP

The number must be read from the lowest layer to the highest layer of embankment. The number of these cards are equal to the number of layers in the embankments.

9. Factor of Safety Cards

A. If IFXY = 0, these cards are omitted

a) 2-D Critical Circle Information Card (6F10.3,I5)

1-10 XO - X-coordinate of the toe

11-20 YO - Y-coordinate of the toe

21-30 BETA - The angle of the slope on X-Y plane in degrees

31-40 RU - Pore pressure parameter

41-50 GAMAE - Mean unit weight or density of embankment soil

51-60 GAMAF - Mean unit weight or density of foundation soil

61-65 NTIME - Number of critical surfaces selected
b) 3-D Critical Surface Information Cards (7F10.3,215/15)

1-10 RADIUS - The radius of the critical circle, $R_{xy}$
11-20 RZ - The length of minor axis, $R_z$, of the
semi-ellipsoid

21-30 DANGLE - $\Delta \theta$, the spacing of selecting the points
on the failure circles along X-Y plane

31-40 XR - X-coordinate of the center of the
ellipsoid

41-50 YR - Y-coordinate of the center of the
ellipsoid

51-60 ZR - Z-coordinate of the center of the
ellipsoid

61-70 DZ - $\Delta z$, the spacing of selecting the points
interested along Z-direction

71-75 NUMBER - The number of the sections divided along
Z-direction in the embankment

76-80 NUMBF - The number of the sections divided along
Z-direction in the foundation

Next Card

1- 5 ISIGN - +1, the semi-ellipsoid on the right side
of the central plane is chosen; -1, the
left side is chosen. This choice provides
the convenience to calculate the factor
of safety if the failure mass is not
symmetrical
These cards are repeated for as many times as the number of failure surfaces selected.

B. If $IFYZ = 0$, these cards are omitted

a) 2-D Critical Circle Information Card ($6\text{F10.3,I5}$)

1-10  $YO$  - Y-coordinate of the toe  
11-20  $ZO$  - Z-coordinate of the toe  
21-30  $BETA$  - The angle of the slope on Y-Z plane, in degrees  
31-40  $RU$  - Pore water pressure parameter  
41-50  $GAMAE$  - Mean unit weight or density of embankment soil  
51-60  $GAMAF$  - Mean unit weight or density of foundation soil  
61-65  $NTIME$  - Number of critical surfaces selected

b) 3-D Critical Surface Information Cards ($7\text{F10.3,2I5/I5}$)

1-10  $RADIUS$  - The radius of the critical circle, $R_{yz}$  
11-20  $RX$  - The length of minor axis, $R_x$, of the semi-ellipsoid  
21-30  $DANGLE$  - $\Delta\theta$, the spacing of selecting the points on the failure circles along Y-Z plane  
31-40  $XR$  - X-coordinate of the center of the ellipsoid  
41-50  $YR$  - Y-coordinate of the center of the ellipsoid  
51-60  $ZR$  - Z-coordinate of the center of the ellipsoid
61-70 DX  - $\Delta x$, the spacing of selecting the points interested along $X$-direction

71-75 NUMBER  - The number of the sections divided along $X$-direction in the embankment

76-80 NUMBF  - The number of the sections divided along $X$-direction in the embankment

Next Card

1- 5 ISIGW  - +1, the semi-ellipsoid on the right side of the central plane is chosen; -1, the left side is chosen. This choice provides the convenience to calculate the factor of safety if the failure mass is not symmetrical.

These cards are repeated for as many times as the number of failure surfaces selected.
Assuming no movement in Z-direction...
EXAMPLE PROBLEM

TOTAL NUMBER OF ELEMENTS********** 18
TOTAL NUMBER OF NODES********** 58
NUMBER OF ELEMENTS IN FOUNDATION******** 12
NUMBER OF NODES IN FOUNDATION******** 42
NUMBER OF PREEXISTING ELEMENTS******** 0
NUMBER OF PREEXISTING NODES******** 0
NUMBER OF DIFF. MATERIALS********** 2
NUMBER OF CONSTRUCTION LAYERS******** 2
NUMBER OF LOAD CASES ********** 0
FINAL RESULTS ARE NOT PUNCHED OUT

MATERIAL PROPERTY DATA

ATMOSPHERIC PRESSURE = 101.4000

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STRESSES AND STRESS LEVELS FOR FINAL CONDITION AT END OF INCREMENT

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**Fig. E.7 (Cont'd)**

OVERALL FACTOR OF SAFETY = 3.453
PROGRAM FESPO (INPUT, OUTPUT, TAPE1, TAPE2, TAPE3, TAPE4, TAPE7, TAPE8,
1 TAPE9, TAPE10, TAPE11, PUNCH)

COMMON / ISOP/ E1, E2, E3, PR(8), ZZ(8), QQ(8), LM(24), P(24), S(33, 33),
1 STR(6, 33), STS(6, 24), UJAC
DIMENSION HED(12), T(10)

PROGRAM CAPACITY CONTROLLED BY THE FOLLOWING TWO STATEMENTS

COMMON A(12000)
MTOTAL=12000

PROGRAM CONTROL DATA

100 CALL SECOND (T(1))
READ 1000, HED, NUMELT, NUMNPT, NFEL, NFNP, NUMCEL, NUMCNP, NUMMAT,
1 NLAY, NFORCE, NPUNCH
IF (NUMELT .EQ. 0) STOP
PRINT 2000, HED
PRINT 2010, NUMELT, NUMNPT, NFEL, NFNP, NUMCEL, NUMCNP, NUMMAT, NLAY,
1 NFORCE
NUMLD=NLAY+NFORCE
IF(NPUNCH.EQ.0) GO TO 110
PRINT 2020
GO TO 120
PRINT 2030
CONTINUE

C BLOCK OUT VARIABLES IN A-VECTOR

N1=1
N2=N1+13*NUMMAT
N3=N2+3*NUMNPT
N4=N3+4*NUMNPT
N5=N4+NUMNPT
N6=N5+NUMNPT
N7=N6+9*NUMELT
N8=N7+NUMELT
N9=N8+NUMELT
N10=N9+NUMNPT
N11=N10+NUMELT
N12=N11+NUMCEL+1
N13=N12+4*NUMCNP+1
N14=N13+2*NUMLD
N15=N14+2*NUMLD
N16=N15+4*NUMELT
N17=N16+NPUNCH
N18=N17+NUMNPT
N19=N18+NUMNPT
N20=N19+NUMNPT
N21=N20+5*NUMELT
N22=N21+6*NUMELT
N23=N22+6*NUMELT
N1=24=N23+6*NUMELT
N31=NN1+NUMNPT
MTMN1G=MTOTAL-N17
NN2=N22+3*NUMELT
IF(NN2.GT.N23) N23=NN2
IF(N23.LT.MTN1G) GO TO 130
PRINT 5000
CALL EXIT

READ AND PRINT INPUT DATA AND SET UP INITIAL CONDITIONS

130 CALL SETUP (A(N1), A(N2), A(N3), A(N4), A(N5), A(N6), A(N7), A(N8),
1 A(N9), A(N10), A(N11), A(N12), A(N13), A(N14), A(N15), A(N16), A(N17),
CALL SECONDT(2)
N24=N23+TSEQ
N25=Tx24+ISEQB
IF(N25.LT.I1T0TAL)GO TO 140
PRINT 5000
CALL EXIT

FORM STRAIN-DISPLACEMENT MATRIX FOR ALL ELEMENTS, STORE ON TAPE 7

CALL FORMING(A(N3),A(N4),A(N5),A(NS),NUMELT)
CALL SECOND(T(3))
T(1)=T(2)-T(1)
T(2)=T(3)-T(2)
TIME=T(1)+T(2)
DO 400 LN=1,NUMLD
      T(10)=0.
CALL SECOND(T(3))
PRINT 2000,HED

DETERMINE CONTROL DATA FOR EACH LAYER

CALL CALNEQ(A(N2),A(N11),A(N12),A(N13),A(N14),A(N15),NUMELT,
1 NUMNPT,NUMCEL,NUMCNP,NUMMAT,NUMLD,NUML,MBAND,NUMNP,
2 NELCAL,NELRED,NMNPRED,NEQ,NEOB,NSB,NNMEOB)
CALL SECOND(T(4))
TN1=N20+NEQ

SET UP LOAD VECTOR

CALL FORCE(A(N1),A(N2),A(N3),A(N4),A(N5),A(NS),A(N11),A(N13),
1 A(N17),A(N18),A(N19),A(N20),A(NNN1),NUMELT,NUMNPT,NUMCEL,NUMMAT,
2 NUMLD,NUML,NEQ,NEOB,NUMNP)
CALL SECOND(T(5))
T(3)=T(4)-T(3)
T(4)=T(5)-T(4)
DO 300 IT=1,2
      CALL SECOND(T(5))

CALCULATE ELEMENT STIFFNESS MATRIX FOR ALL ELEMENTS, STORE
ON TAPE 2
CALCULATE STRESS-DISPLACEMENT MATRIX FOR ALL ELEMENTS, STORE
ON TAPE 11

CALL BILDUP(A(N7),A(N8),A(N11),NUMCEL,NUMEL,
1 NELCAL,NELRED)
CALL SECOND(T(6))
NE2B=2*NEOB
NN1=N17+NE2B*MBAND
NN2=NN1+NE2B

FORM TOTAL STIFFNESS MATRIX, STORE ON TAPE 4
CALL ADDSTF(A(N17),A(NNN1),A(NNN2),NUMEL,NEOB,NE2B,NBLOCK,MBAND)
CALL SECOND(T(7))
NSB=(MBAND+1)*NEQB
NNN1=N17+NSB
NNN2=NNN1+NSB

SOLVE FOR DISPLACEMENT UKNOWNS

CALL SYMBANC(A(N17),A(NNN1),A(NNN2),NEOB,MBAND,NBLOCK,NSB,
1 4,3,1,2,2)
CALL SECOND (T(8))

EVALUATE RESULTS

CALL RESULT(A(N1),A(N2),A(N3),A(N4),A(N5),A(N6),A(N7),A(N8),
1 A(N9),A(N10),A(N11),A(N12),A(N13),A(N14),A(N15),A(N16),A(N17),A(N18),A(N19),
2 A(N20),A(N21),A(N22),A(N23),A(N24),PATH,NUMELT,NUMNP,NUMCEL,NUMCNP,
3 NUMMAT,NUNLDT,NLAY,LN,IT,NPUNCH,NUNLDT,NUMNP,NUMCEL,NUMCNP,
4 NNPRE,NEQ,NEOB,NBLOCK)

CALL SECOND (T(9))

DO 250 I=5,8

250 T(I)=T(I+1)-T(I)

T(9)=T(5)+T(6)+T(7)+T(8)

PRINT 2110,T(5),T(6),T(7),T(8),T(9)

IF (IT.LT.2) GO TO 280

T(10)=T(10)+T(3)+T(4)+T(9)

PRINT 2120,T(3),T(4),T(10)

T(I)=T(I)+T(9)

GO TO 300

280 CONTINUE

400 CONTINUE

PRINT 2100, T(1),T(2),TIME

GO TO 100

1000 FORMAT(12A6/10I5)

2000 FORMAT (1H1,12A6)

2100 FORMAT (/,

13SHOTOTAL NUMBER OF ELEMENTS*********** I3/
2 3SHOTOTAL NUMBER OF NODES************ I3/
335H0NUMBER OF ELEMENTS IN FOUNDATION*** I3/
435H0NUMBER OF NODES IN FOUNDATION***** I3/
535H0NUMBER OF PREEXISTING ELEMENTS*** I3/
635H0NUMBER OF PREEXISTING NODES****** I3/
735H0NUMBER OF DIFF. MATERIALS********* I3/
835H0NUMBER OF CONSTRUCTION LAYERS**** I3/
9 35H0NUMBER OF LOAD CASES ************* I3)

2020 FORMAT(4SHORESULTS ARE PUNCHED OUT FOR FOLLOWING LOAD CASES /)

2030 FORMAT(34H0FINAL RESULTS ARE NOT PUNCHED OUT /)

5000 FORMAT (1H1,17H STORAGE EXCEEDED)

2100 FORMAT (14H0SOLUTION TIME /)

2110 FORMAT (/,

1 35H0ELEMENT STIFFNESSES********** F8.2 /
2 35H0TOTAL STIFFNESS*************** F8.2 /
3 35H0EQUATION SOLVING*************** F8.2 /
4 35H0CALCULATE STRESSES AND STRAINS**** F8.2 /
5 35H0SOLUTION TIME FOR THIS ITERATION** F8.2)

2120 FORMAT(/,

1 35H0DETERMINE CONTROL DATA************ F8.2 /
2 35H0LOAD VECTOR********************* F8.2 /
3 35H0TOTAL TIME FOR THIS LOAD CASE***** F8.2)

2130 FORMAT(12H0OVERALL LOG /,

1 35H0DATA INPUT*********************** F8.2 /
2 35H0STRAIN-DISPLACEMENT MATRIX*** F8.2 /
3 35H0TOTAL SOLUTION TIME************ F8.2)

END

SUBROUTINE SETUP (EMPR, ID, X, Y, Z, INP, BULK, SHEAR, POIS, SLMAX, NCEL,
1 NCNP, NOMEL, NOMNP, NPHUMP, NLDP, DISP, STRESS, STRAIN, XCP, VCP, ZCP,
2 NUMELT, NUMNP, NUMCEL, NUMCNP, NFEL, NUMMAT, NUMLDT, NUMNP, NEQ, NEOB,
3 MBAND, PATH, MTMNIG, NMXEQB, NPUNCH)
READ AND PRINT MATERIAL PROPERTY DATA
READ 1010, M, (EMPR(M,I), I=1,11)
PRINT 2010, M, (EMPR(M,I), I=1,11)
PHI=EMPR(M,9)/57.23577951
CONST=2.0/(EMPR(M,10)**(1.0-SIN(PHI)))
EMPRCM,12)=CONST*EMPR(M,8)*COS(PHI)
EMPRCM,13)=CONST*SIN(PHI)
IF (CM.LT.NUMMAT) GO TO 50
LL=0
READ AND PRINT NODAL POINT DATA AND BOUNDARY CONDITIONS
100 READ 1020, MM.XCMM),YCMM),ZCMM),CIDCMM, I=1,3)
DO 110 I=1,3
DIFNP=MM-LL
DX=(X(MM)-X(LL))/DIFNP
DY=(Y(MM)-Y(LL))/DIFNP
DZ=(Z(MM)-Z(LL))/DIFNP
110 LL=LL+1
IF(MM-LL)150,140,120
120 X(LL)=X(LL-1)+DX
Y(LL)=Y(LL-1)+DY
Z(LL)=Z(LL-1)+DZ
130 ID(LL,I)=ID(LL-1,I)
GO TO 110
140 IF(HNUMNPNT-MM)150,160,100
150 PRINT 5000,MM
CALL EXIT
160 IF(NUMNPNT.MM.MM)150,160,100
170 N=N+1
PRINT 2040, N,X(N),Y(N),Z(N),(ID(N,I),I=1,3)
IF(N.LT.NUMNPNT) GO TO 170
IF(N,LL.150,200,100
PRINT 2030
N=0
200 READ 1030, N, (INP(N,I), I=1,9)
210 NN=NN+1
IF(N,NN.MM) GO TO 230
DO 220 K=1,8
220 INP(NN,K)=INP(NN-1,K)+1
INP(NN,9)=INP(NN-1,9)
230 IF(N.GT.NN) GO TO 210
   IF(NUMELT.GT.NN) GO TO 200
   PRINT 2050
   N=0
250 N=N+1
   XCP(N)=0.
   YCP(N)=0.
   ZCP(N)=0.
   DO 255 I=1,8
      II=INP(N.I)
      XCP(N)=XCP(N)+X(II)/8.
      YCP(N)=YCP(N)+Y(II)/8.
   255 CONTINUE
   PRINT 2060, N, (INP(N,M),M=1,9), XCP(N), YCP(N), ZCP(N)
   IF (N.LT.NUMELT) GO TO 250
   SET UP EQUATION NUMBERS

   NEQ=0
   DO 330 N=1,NUMNPT
      DO 310 I=1,3
         IF(ID(N,I).LE.1000) GO TO 310
         NN=ID(N,I)-1000
         ID(N,I)=ID(NN,I)
      310 CONTINUE
      GO TO 330
   320 CONTINUE
   PRINT 2070
   PRINT 2080, (N, (ID(N,I),I=1,3), N=1, NUMNPT)

   DETERMINE BAND WIDTH

   MBAND=0
   DO 430 N=1,NUMELT
      MIN=100000
      MAX=0
      DO 410 I=1,8
         II=INP(N,I)
         DO 410 J=1,3
            IJ=IJ+1
            LM(IJ)=ID(II,J)
         410 CONTINUE
      WRITE(4) (LM(I),I=1,24)
      DD 420 L=1,24
      IF(LM(L).EQ.0) GO TO 420
      IF(LM(L).GT.MAX) MAX=LM(L)
      IF(LM(L).LT.MIN) MIN=LM(L)
   420 CONTINUE
   NDIF=MAX-MIN+1
   IF(NDIF.GT.MBAND) MBAND=NDIF
   PRINT 2085, MBAND, NEQ, NEQB
   READ AND PRINT CONSTRUCTION SEQUENCE INFORMATION

   PRINT 2090
   READ 1040, (LN, (NOMEL(LN,I),I=1,2), (NOMNP(LN,J),J=1,2),
C \text{READ AND PRINT DATA FOR PREEXISTING ELEMENTS AND NODAL POINTS}\nC \text{CONTINUE}\n\text{IF (NUMCEL \text{.EQ. 0}) GO TO 450}\n\text{PRINT 2110}\n\text{READ 1050, (NCEL(N), N=1, NUMCEL)}\n\text{PRINT 1050, (NCEL(N), N=1, NUMCEL)}\n\text{PRINT 2120}\n\text{READ 1050, (NCNP(N), N=1, NUMCNP)}\n\text{PRINT 1050, (NCNP(N), N=1, NUMCNP)}\n\text{CONTINUE}\n\text{INITIALIZATION OF STRESSES, STRAINS, AND STRESS LEVELS}\n\text{IN ALL ELEMENTS AND DISPLACEMENTS OF ALL NODAL POINTS}\n\text{DO 500 I=1, NUMELT}\n\text{SLMAX(I)=0.}\n\text{DO 500 J=1, 6}\n\text{STRESS(I, J)=0.}\n500 \text{CONTINUE}\n\text{DO 510 I=1, NUMNPT}\n\text{DO 510 J=1, 3}\n\text{DISP(I, J)=0.}\n510 \text{CONTINUE}\n\text{IF (NUMCEL \text{.EQ. 0}) GO TO 550}\n\text{READ 505, NMODL}\n\text{READ 1100, (N, (STRESS(N, M), M=1, 6), J=1, NUMCEL)}\n\text{IF(NMODL \text{.EQ. 0}) GO TO 520}\n\text{READ 1100, (N, (STRAIN(N, M), M=1, 6), J=1, NUMCEL)}\n\text{READ 1110, (N, (DISP(N, M), M=1, 3), J=1, NUMCNP)}\n\text{IF(NMODL \text{.EQ. 1}) GO TO 520}\n\text{READ 1120, ((N, BULK(N), SHEAR(N), POIS(N), SLMAX(N)), J=1, NUMCEL)}\n\text{GO TO 550}\n\text{CONTINUE}\n\text{READ STRESSES, STRAINS AND DISPLACEMENTS AND CALCULATE MODULUS VALUES FOR PREEXISTING PART}\n\text{DO 530 I=1, NUMCEL}\n\text{N=NCEL(I)}\n\text{NH=3}\n\text{NM=3}\n\text{IND=0}\n\text{A(1,1)=STRESS(N, 1)}\n\text{A(2,2)=STRESS(N, 2)}\n\text{A(3,3)=STRESS(N, 3)}\n\text{A(1,2)=STRESS(N, 4)}\n\text{A(2,3)=STRESS(N, 5)}\n\text{A(1,3)=STRESS(N, 6)}\n\text{A(2,1)=A(1,2)}\n\text{A(3,2)=A(2,3)}\n\text{CALL RSEIG(NH, NM, A, IND, D, Z1)}\n\text{CALL COMPAR(D)}\n\text{DO 523 JP=1, 3}\n\text{PRS(JP)=D(JP)}\n523 \text{CONTINUE}\n\text{MTYPE=INP(N, 9)}\n\text{CALL MODU(EMPR, BULK, SHEAR, POIS, SLMAX, PRS, PATM, NUMMAT, N, 1 MTYPE, STRLEU, 1)}\n\text{SLMAX(N)=STRLEU}\n530 \text{CONTINUE}
CONTINUE
IF(NFEL.EQ.0) GO TO 700

READ AND PRINT DATA FOR FOUNDATION LAYERS USED IN CALCULATING
INITIAL STRESSES

PRINT 2200
READ 1200,NFLAY,HFLEU
PRINT 2210, NFLAY,HFLEU
PRINT 2220
I=0
600 I=I+1
READ 1210, I,MATNO(I),NLEL(I),NREL(I),HL(I)
PRINT 2230,I,MATNO(I),NLEL(I),NREL(I),HL(I)
IF(I.GT.1) GO TO 610
HH(I)=HL(I)-HFLEU
GO TO 620
610 HH(I)=HL(I)-HL(I-1)
620 CONTINUE
IF(I.LT.NFLAY) GO TO 600

CALCULATE INITIAL STRESSES AND MODULI FOR FOUNDATION ELEMENTS

DO 670 I=1,NFLAY
SINITY(I)=0.
IF( I.EQ.NFLAY) GO TO 640
NN=I+1
DO 630 J=NN,NFLAY
M Type=MATNO(J)
630 SINITY(I)=SINITY(I)+EMPR(M Type,1)*HH(J)
640 M Type=MATNO(I)
SINITY(I)=SINITY(I)+EMPR(M Type,1)*HH(I)/2.
SINITX(I)=EMPR(M Type,11)*SINITY(I)
SINITZ(I)=SINITX(I)
N N L=N L E L( I )
N N R=N R E L( I )
DO 650 N=N N L,N N R
STRESS(N,1)=SINITX(I)
STRESS(N,2)=SINITY(I)
STRESS(N,3)=SINITZ(I)
650 N N L=I N N L+1
N=NNL
M Type=INP(NNL,9)
PRS(1)=STRESS(NNL,2)
PRS(2)=STRESS(NNL,1)
PRS(3)=STRESS(NNL,3)
CALL MODU (EMPR,BULK,SHEAR,POIS,SLMAX,PRS,PATH,NUMMAT,N,
M Type, STRLEU,1)
SLMAX(NNL)=STRLEU
DO 660 N=NNL1,NNR
BULK(N)=BULK(NNL)
SHEAR(N)=SHEAR(NNL)
POIS(N)=POIS(NNL)
660 N N S=M A X(N N L)
670 CONTINUE
700 CONTINUE

CALCULATE INITIAL STRESSES AND MODULI FOR LAYERS TO BE ADDED

PRINT 2310
IF(NLAY.EQ.0) GO TO 790
DO 780 LN=1,NLAY
NL=NOMEL(LN,1)
N R=N O M E L(LN,2)
II=MPHUMP(LN,1)
II2=NPHUMP(LN,2)
II3=NPHUMP(LN,3)
II4=NPHUMP(LM,4)
DO 770 N=NNL,NNR
IF(NUMCEL.EQ.0) GO TO 720
DO 710 M=1,NUMCEL
IF(N.EQ.0) NCCEL(M)) GO TO 770
CONTINUE
720 CONTINUE
MTYPE=INP(N,9)
IF(XCP(N).LE.X(I11)) GO TO 731
IF(XCP(N).LE.X(I12)) GO TO 732
IF(XCP(N).LE.X(I13)) GO TO 733
IF(XCP(N).LE.X(I14)) GO TO 734
SLOPE=0.
HT=Y(I14)-YCP(N)
GO TO 740
731 SLOPE=0.
HT=Y(I11)-YCP(N)
GO TO 740
SLOPE=(Y(I12)-Y(I11))/(X(I12)-X(I11))
HT=Y(I11)+(XCP(N)-X(I11))*SLOPE-YCP(N)
GO TO 740
733 SLOPE=(Y(I13)-Y(I12))/(X(I13)-X(I12))
HT=Y(I12)+(XCP(N)-X(I12))*SLOPE-YCP(N)
GO TO 740
734 SLOPE=(Y(I14)-Y(I13))/(X(I14)-X(I13))
HT=Y(I13)+(XCP(N)-X(I13))*SLOPE-YCP(N)
740 BETA=ATAN(SLOPE)
IF(ZCP(N).LT.Z(I11).AND.ZCP(N).GT.Z(I12)) GO TO 741
CETA=0.
GO TO 743
CETA=(Y(I12)-Y(I11))/(Z(I11)-Z(I12))
HT1=Y(I11)+(ZCP(N)-Z(I11))*CETA-YCP(N)
IF(HT-HT1) 742,743,743
742 HT=HT1
GO TO 744
743 HT=HT
744 STRESS(N,2)=HT*EMPR(MTYPE,1)
STRESS(N,4)=0.5*STRESS(N,2)*SIN(BETA)
STRESS(N,5)=0.5*STRESS(N,2)*SIN(ATAN(CETA))
STRESS(N,6)=0.
SIGAVE(2)=STRESS(N,2)/2.
SIGAVE(4)=STRESS(N,4)/2.
SIGAVE(5)=STRESS(N,5)/2.
SIGAVE(6)=0.
POISI=EMPR(MTYPE,6)
750 IF(POISI .GT.0.49) POISI=0.49
STRESS(N,1)=STRESS(N,2)/POISI/(1.-POISI)
SIGAVE(1)=STRESS(N,1)/2.
STRESS(N,3)=STRESS(N,1)
SIGAVE(3)=SIGAVE(1)
MN=3
MM=3
IND=0
A(1,1)=SIGAVE(1)
A(2,2)=SIGAVE(2)
A(3,3)=SIGAVE(3)
A(1,2)=SIGAVE(4)
A(2,3)=SIGAVE(5)
A(1,3)=SIGAVE(6)
A(2,1)=A(1,2)
A(3,2)=A(2,3)
A(3,1)=A(1,3)
CALL RSEIGCNM(MM,NN,A,IND,D,Z1)
CALL COMPAR(I)
DO 753 I=1,3
PRS(I)=DI(I)
753 CONTINUE
CALL MODU (EMPR, BULK, SHEAR, POIS, SMLAX, PRS, PATM, NUMMAT, N,
1 MTYPE, STRLEV, I)
POIS=POIS(N)
IF (ABS(POIS1 - POIST) .LT. 0.0001) GO TO 760
POISI=POISI+(POIST-POIS1)/10.
GO TO 750
760 CONTINUE
SMLAX(N)=STRLEV
770 CONTINUE
780 CONTINUE
C
PRINT INITIAL MODULI AND STRESSES FOR ALL ELEMENTS
C
DO 800 N=1,NUMELT
EMOD=2. * SHEAR (N) *(1. + POIS(N))
PRINT 2320, N, XCP(N), YCP(N), ZCP(N), EMOD, BULK(N), SHEAR(N), POIS(N),
1 (STRESS(N,N,M), M=1,6)
800 CONTINUE
REWIND 8
REWIND 9
WRITE(8) ((STRESS(I,J), J=1,6), I=1,NUMELT)
WRITE(8) ((DISP(N,M), N=1,13), M=1,NUMNPT)
WRITE(8) ((STRAIN(N,M), M=1,6), N=1,NUMELT)
RETURN
1000 FORMAT(F10.0)
1010 FORMAT(IS,7F10.0/4F10.0)
1020 FORMAT(IS,3F10.0,3I5)
1030 FORMAT(10IS)
1040 FORMAT(IS)
1050 FORMAT(IS)
1100 FORMAT(IS,6F10.0)
1110 FORMAT(IS,3F10.0)
1120 FORMAT(IS,4F10.0)
1200 FORMAT(IS,5F10.0)
1210 FORMAT(4IS,5F10.0)
2000 FORMAT(/,,23H MATERIAL PROPERTY DATA //,
1 22H ATMOSPHERIC PRESSURE=.F10.4//)
2010 FORMAT(28X,8H MODULUS, 18X, 14H POISSON RATIO /,
1 51H MAT UNIWT K KUR N D,9X,1HG,
2 9X,1HF,9X,1HC,9X,3HPhi,5X,10FAIL, RATIO, 5X,2HKO /)
2020 FORMAT(IS,F10.4,2F10.1,8F10.4)
2030 FORMAT(23H NODE INPUT DATA//, SH NODE, 5X,
1 23H NODE COORDINATES, 19X, SHB. C. CODE//, 7H NUMBER,
2 5SH X-ORD Y-ORD Z-ORD XX YY ZZ /)
2040 FORMAT(17,3F10.3,10X,3I5)
2050 FORMAT(31H EIGHT NODES SOLID ELEMENT DATA//,
1 5H ELET, 5X, 15H CONNECTED NODES, 21X, SH MATL, 4X,
2 28H ELEMENT CENTER COORDINATES/,
3 52H NO. I J K L M N O P NO.,
435H X-ORD Y-ORD Z-ORD /)
2060 FORMAT(10IS,3F12.3)
2070 FORMAT(17HEQUATION NUMBERS//, 20H N X Y Z/)
2080 FORMAT(4IS)
2085 FORMAT(/,,13H BAND WIDTH ************I4 /,
2 3SH NUMBER OF EQUATIONS************I4 /,
3 3SH NUMBER OF EQUATIONS IN BLOCK************I4)
2090 FORMAT(31H CONSTRUCTION LAYER INFORMATION //, 6H LAYER,
1 23H ADDED ELEMENTS, 12H ADDED NODES, 5X,
2 40H NODES OF HUMPED SURFACE/)
SUBROUTINE MODU(EMPR,BULK,SHAEAR,POISS,SLMAX,PRST,PATM,
NNUMAT,N,MTYPE,STRLEU,KK)
DIMENSION EMPR(NNUMAT,13),BULK(1),SHAEAR(1),POISS(1),SLMAX(1),PRS(3)

C CALCULATE SHEAR MODULUS, BULK MODULUS AND POISSON RATIO VALUES
C
DEUSTR=PRS(1)-PRS(3)
DEUFH=EMPR(NTYPE,12)+EMPR(NTYPE,13)*PRS(3)
IF(DEUFH.GT.0.0) GO TO 100
STREU=0.
DEULEU=0.
GO TO 110

100 DEULEU=DEUSTR/DEUFH
STREU=DEULEU-EMPR(MTYPE,10)
CONTINUE

GO TO 110

110 IF(KK.EQ.1) GO TO 140
IF(PRS(3).LT.0.0) GO TO 120
POISS(N)=EMPR(MTYPE,6)
IF(POISS(N).LT.0.49) POISS(N)=0.49
GO TO 130

120 IF(STRLEU.LT.1.0.AND.SHAER(N).LT.0.0001) GO TO 140
SHEAR(N)=0.0001
GO TO 200

140 CONTINUE
IF(PRS(3).LT.0.01) PRS(3)=0.01
IF(KK.EQ.3.AND. STRLEU. LT. SLMAX(N)) GO TO 150
EINIT=PATM*EMPR(MTYPE,2)+(PRS(3)/PATH)**EMPR(MTYPE,4)
EMOD=EINIT*(1.-DEULEU)**2.
GO TO 160

150 EMOD=PATM*EMPR(MTYPE,3)+(PRS(3)/PATH)**EMPR(MTYPE,4)
CONTINUE
POISS=EMPR(MTYPE,6)-EMPR(MTYPE,7)*ALOG10(PRS(3)/PATH)
EPSAX=DEUSTR/(EINIT*(1.-DEULEU))
POISS(N)=POISS/(1.-EMPR(MTYPE,5)*EPSAX)**2.
IF(POISS(N).LT.0.49) POISS(N)=0.49
SHEAR(N)=EMOD/(2.*(1.+POISS(N)))
BULK(N)=SHEAR(N)*(1.-2.*POISS(N))
IF(KK.NE.1) GO TO 200
IF(STRLEU.GE.1.0.AND.PRS(3).LT.0.0) SHEAR(N)=0.0001

200 CONTINUE
RETURN
END
SUBROUTINE FORMING(X,Y,Z,INP,NUMELT)
COMMON /ISOP/ E1,E2,E3,RR(8),ZZ(8),QQ(8),LM(24),P(24),S(33,33),
1 STR(6,33),STS(6,24),UJAC
DIMENSION X(1),Y(1),Z(1),INP(NUMELT,9)
DIMENSION SSS(2),TTT(2),QSS(2)
DATA SSS /-0.57735026918963,0.57735026918963/
DATA TTT /-0.57735026918963,0.57735026918963/
DATA QSS /-0.57735026918963,0.57735026918963/
FORM STRAIN-DISPLACEMENT MATRIX
REWIND 4
REWIND 7
DO 300 N=1,NUMELT
DO 50 I=1,6
DO 50 J=1,33
50 STR(I,J)=0.
READ(4) (LM(I),I=1,24)
WRITE(7) (LM(I),I=1,24)
DO 100 I=1,8
II=INP(N,I)
RR(I)=X(II)
ZZ(I)=Y(II)
100 QQ(I)=Z(II)
DO 200 II=1,2
EI=SSS(II)
DO 200 JJ=1,2
EZ=TTT(JJ)
DO 200 KK=1,2
E3=QSS(KK)
CALL RELATE
WRITE(7) UJAC,((STR(I,J),J=1,33),I=1,6)
200 CONTINUE
EI=0.
EZ=0.
E3=0.
CALL RELATE
WRITE(7) ((STR(I,J),J=1,24),I=1,6)
300 CONTINUE
RETURN
END

SUBROUTINE RELATE
COMMON /ISOP/ SX,TY,QZ,RR(8),ZZ(8),QQ(8),LM(24),P(24),S(33,33),
1 STR(6,33),STS(6,24),UJAC
DIMENSION HR(11),HZ(11),HQ(11),AC(3,11),BC(3,3),XX(8,3)
DIMENSION II(11),JJ(11),KK(11),DC(3,3),IPERM(3)
DATA II/1,4,7,10,13,16,19,22,25,28,31/
DATA JJ/2,5,8,11,14,17,20,23,26,29,32/
DATA IPERM/2,3,1/
DATA KK/3,6,9,12,15,18,21,24,27,30,33/
MATRIX OF DERIVATIVES
SP=1.+SX
SM=1.-SX
TP=1.+TY
TM=1.-TY
QP=1.+QZ
QM=1.-QZ
A(1,1)=-TM*QM/8.
A(1,2)=A(1,1)
A(1,3)=TP*QM/8.
A(1,4)=-A(1,3)
A(1,5)=-TM*QP/8.
A(1,6)=-A(1,5)
A(1,7)=TP*QP/8.
A(1,8)=A(1,7)
A(1,9)=-2.*SX
A(1,10)=0.
A(1,11)=0.
A(2,1)=-SM*QM/8.
A(2,2)=-SP*QM/8.
A(2,3)=-A(2,2)
A(2,4)=-A(2,1)
A(2,5)=-SM*QP/8.
A(2,6)=-SP*QP/8.
A(2,7)=-A(2,6)
A(2,8)=A(2,5)
A(2,9)=0.
A(2,10)=-2.*TY
A(2,11)=0.
A(3,1)=-SM*TM/8.
A(3,2)=-SP*TM/8.
A(3,3)=-SP*TP/8.
A(3,4)=-SM*TP/8.
A(3,5)=-A(3,1)
A(3,6)=-A(3,2)
A(3,7)=-A(3,3)
A(3,8)=-A(3,4)
A(3,9)=0.
A(3,10)=0.
A(3,11)=-2.*QZ

JACOBIAN
DO 40 I=1,8
XX(I,1)=RR(I)
XX(I,2)=ZZ(I)
XX(I,3)=QQ(I)
40 CONTINUE
DO 60 I=1,3
DO 60 J=1,3
C=0.
DO 50 L=1,8
C=C+A(I,L)*XX(L,J)
50 CONTINUE
DO 70 L=1,8
D(I,J)=C
UJAC=D(I,1)*D(2,2)+D(3,3)+D(1,2)*D(2,3)*D(3,1)+D(1,3)*D(2,1)*
1 D(3,2)-D(1,3)*D(2,2)*D(3,1)-D(1,1)*D(2,3)*D(3,2)+D(1,2)*D(2,1)*
2 D(3,3)
70 CONTINUE
DO 100 I=1,11
HR(I)=0.
HZ(I)=0.
HQ(I)=0.
DO 100 J=1,3
FORM STRIN DISPLACEMENT MATRIX

DO 200 K3=1,11
I=II(K3)
J=JJ(K3)
K=KK(K3)
STR(1,I)=HR(K3)
STR(2,J)=HZ(K3)
STR(3,K)=HQ(K3)
STR(4,I)=HR(K3)
STR(5,J)=HQ(K3)
STR(6,K)=HR(K3)
RETURN
END

SUBROUTINE CALNEQC.ID. NCEL. NCNP. NOMEL. NOMNP. NPHUMP. NUMELT. NUMNPT, 
1 NUMCEL. NUMCNP. NUMLID. NLAY. LN. MBAND. NUMEL. NUMNP. NELCAL 
1. NNPCEL. NELRED. NNPRED. NEQ. NEOB. NBLOCK. NMXEQB)
DIMENSION ID(NUMNPT,3).NCEL(1).NCNP(1)
DIMENSION N0MEL(NUMLD,2).N0MNP(NUMLD,2).NPHUMP(NUMLD,4)

C C DETERMINE CONTROL DATA
C IF(LN. GT. NLAY) GO TO 10
PRINT 2000. LN
PRINT 2010. (N0MEL(LN,M),M=1,2),(N0MNP(LN,M),M=1,2).
1 (NPHUMP(LN,L),L=1,4)
GO TO 20
10 LNMLAY=LN-NLAY
PRINT 2020. LNMLAY
20 CONTINUE
IF(LN.GT.NLAY) GO TO 80
IF (NUMCEL .EQ. 0) GO TO 50
NUMEL=MAX0(NOMEL(LN,2).NCEL(NUMCEL))
NUMNP=MAX0(N0MNP(LN,2).NCNP(NUMCEL))
GO TO 60
50 NUMEL=NOMEL(LN,2)
NUMNP=N0MNP(LN,2)
60 NELCAL=NOMEL(LN,2)
NELRED=N0MEL(LN,1)
NMPCAL=N0MNP(LN,2)
NNPRED=N0MNP(LN,1)
GO TO 100
80 NUMEL=NUMELT
NUMNP=NUMNPT
NELCAL=NUMELT
NMPCAL=NUMNPT
NELRED=NUMELT+1
NNPRED=NUMNPT+1
100 CONTINUE

C C DETERMINE NUMBER OF EQUATIONS AND BLOCKS
DO 120 N=1,NUMNP
   DO 120 I=1,3
      IF(ID(N,I).GT. MAX) MAX=ID(N,I)
   CONTINUE
   NEO=MAX
   IF(NEOB .GT. NEO) NEOB=NEO
   NBLOCK=(NEO-1)/NEOB+1
   PRINT 2100,MBAND,NEO,NEOB,NBLOCK
   RETURN
2000 FORMAT(/,
      1 57H *************************************************************** /,
      2 57H  *************************************************************** /,
      3 57H  *                                                       /,
      4 45H  * LAYER NUMBER***************************************,I4,8H * /,
      5 57H  *                                                       /,
      6 57H  *************************************************************** /,
      7 57H  *************************************************************** )
2010 FORMAT(/,
      1 35H ADDED ELEMENTS******************************************,I4,8H /
      2 35H ADDED NODAL POINTS***********************************,I5,5H /
      3 35H NODAL POINTS OF HUMPED SURFACE**** 4IS */
2020 FORMAT(/,
      1 35H BAND WIDTH************************** I4 /
      2 35H TOTAL NUMBER OF EQUATIONS******** I4 /
      3 35H NUMBER OF EQUATIONS IN BLOCK****** I4 /
      4 35H NUMBER OF BLOCKS*************** I4 */
END

SUBROUTINE FORCE(EMPR,ID,X,Y,Z,INP,NCEL,NOMEL,FX,FY,FZ,B,R,
1 NUMELT,NUMNPT,NUMCEL,NUMMAT,NUMLD,LN,NEQ,NEOB,NUMNP)
   DIMENSION EMPR(NUMMAT,13),ID(NUMNPT,3),X(1),Y(1),Z(1)
   DIMENSION INP(NUMELT,9),NOMEL(NUMLD,2 ),FX(1),FY(1),FZ(1),B(1)
   DIMENSION R(NEOB),NCEL(1)
C
C CALCULATE NODAL POINT FORCES DUE TO WEIGHTS OF ADDED ELEMENTS
C
DO 50 I=1,NUMNP
   FX(I)=0.
   FY(I)=0.
   FZ(I)=0.
   IF(LN.GT. NLAY) GO TO 400
   NELS=NOMEL(LN,1)
   NELL=NOMEL(LN,2)
   DO 300 N=NELS,NELL
      IF(NUMCEL (EQ. 0) GO TO 100
      DO 80 M=1,NUMCEL
         IF(N(M,EQ. NCEL(M)) GO TO 300
50   CONTINUE
100   CONTINUE
   FG=0.
   MTYPE=INP(N,9)
KK=1
I=INP(N,1)
J=INP(N,2)
K=INP(N,4)
L=INP(N,5)

120 UOL=((X(J)-X(I))*Z(L)-Y(L)*Z(K))+(Y(J)-Y(I))*Z(L))+(Z(J)-Z(I))*X(L)
121 Y(J)=Z(I)+X(I)*Z(J)+(Y(I)+Z(I))+(L(I)+Z(I))+(X(L)-X(I))
122 Z(Y(J))*Z(K)+Z(J)*X(I)+(X(J)-X(I))+(Y(J)-Y(I))*X(L)
123 (X(K)-X(I))*Z(I))+(Y(I)-Z(L)*Z(I))
124 VOL=((X(J)-X(I))*Z(L)-Y(L)*Z(K))+(Y(J)-Y(I))*Z(L))+(Z(J)-Z(I))*X(L)

FG=FG-EMPR(MTYPE,1)*VOL

IF(KK.EQ.5) GO TO 140
IF(KK.EQ.2) GO TO 137
IF(KK.EQ.3) GO TO 138
IF(KK.EQ.4) GO TO 139

KK=KK+1
I=INP(N,2)
J=INP(N,3)
K=INP(N,4)
L=INP(N,7)
GO TO 120

137 KK=KK+1
I=INP(N,2)
J=INP(N,3)
K=INP(N,4)
L=INP(N,7)
GO TO 120

138 KK=KK+1
I=INP(N,4)
J=INP(N,5)
K=INP(N,7)
L=INP(N,8)
GO TO 120

140 CONTINUE
FG=FG/8.

DO 250 I=1,KL
II=INP(N,I)
250 FY(II)=FY(II)+FG

300 CONTINUE
GO TO 650

400 CONTINUE

C
READ NODAL POINT FORCE DATA AND DISTRIBUTED LOAD DATA
FOR LOAD CASE AFTER CONSTRUCTION

C
READ 1000,NUMFC
PRINT 2000,NUMFC
IF(NUMFC.EQ.0) GO TO 550
DO 520 I =1,NUMFC

520 READ 1010,MM,FX(MM),FY(MM),FZ(MM)

550 CONTINUE

C
SET UP FORCE VECTOR

C
DO 700 I=1,NEQ
B(I)=0.
DO 720 N=1,NUMNP
DO 720 I=1,3
II=ID(N,I)
IF(II.LT.1) GO TO 720
IF(I.EQ.2) GO TO 710
IF(I.EQ.3) GO TO 715
B(II)=B(II)+FX(N)
GO TO 720
710 B(II)=B(II)+FY(N)
GO TO 720
715 B(II)=B(II)+FZ(N)
720 CONTINUE
REWIND 10
KSHIFT=0
DO 730 I=1,NEOB
730 R(I)=0.
DO 750 N=1,NEQ
II=N-KSHIFT
R(II)=B(N)
IF(II.NE.NEOB) GO TO 750
WRITE(10) R
KSHIFT=KSHIFT+NEOB
DD 740 I=1,NEOB
740 R(I)=0.
750 CONTINUE
WRITE(10) R
C
C PRINT NODAL POINT FORCES
C
IF(LN.LE.NLAY) GO TO 760
PRINT 2030
GO TO 770
760 PRINT 2035
770 CONTINUE
DO 800 N=1,NUMNP
800 PRINT 2040, N,FX(N),FY(N),FZ(N)
1000 FORMAT(15,3F10.0)
2000 FORMAT(/,
1 35HNUMBER OF N.P. FORCE CARDS********* ,1X,I3)
2030 FORMAT(15H1NODAL POINT FORCES // 35H NP X-FORCE Y-FORCE Z-
1FORCE/) 
2035 FORMAT(47H1NODAL POINT FORCES (WEIGHT OF ADDED ELEMENTS ) //,
1 35H NP X-FORCE Y-FORCE Z-FORCE/) 
2040 FORMAT(I5,3F10.3)
RETURN
END
C
C INITIALIZATION:
C
DO 300 N=1,NUMEL
KEY=0
DO 20 I=1,33
DO 20 J=1,33

SUBROUTINE BILDUP (BULK,SHEAR,NCEL,NUMCEL,NUMEL,NELCAL,NELRED)
COMMON /ISOP/ E1,E2,E3,RR(8),ZZ(8),QQ(8),LM(24),P(24),S(33,33),
1 STR(6,33),STS(6,24),UJAC
DIMENSION BULK(1),SHEAR(1),POIS(1),NCEL(1)
REWIND 2
REWIND 11
REWIND 7
S(I,J)=0.

FORM STIFFNESS MATRIX AND WRITE ON TAPE 2

IF(N .LE. NELCAL .OR. NUMCEL .EQ. 0) GO TO 40
DO 30 M=1,NUMCEL
   IF (N.EQ. NCEL(M)) GO TO 40
30 CONTINUE
GO TO 50

KEY=1
50 CONTINUE
IF(KEY .EQ. 0) GO TO 80
IF(N.GE.NELRED.AND.N.LE.NELCAL) GO TO 60

LL=1
FMAG=1.
GO TO 80

IF (NUMCEL .EQ. 0) GO TO 75
DO 70 M=1,NUMCEL
   IF (N.EQ. NCEL(M)) GO TO 55
70 CONTINUE

LL=0
FMAG=0.00001
80 CONTINUE
READ(7) (LM(I),I=1,24)
C1=FMAG*(BULK(N)+SHEAR(N))
C2=FMAG*(BULK(N)-SHEAR(N))
C3=FMAG*SHEAR(N)
DO 100 LX=1,2
   DO 100 LY=1,2
      DO 100 LZ=1,2
         READ(7) UJAC,((STR(I,J),J=1,33),I=1,6)
         IF(KEY . EQ. 0) GO TO 100
         DO 90 J=1,33
            D1=UJAC*C1*STR(1,J)+C2*STR(2,J)+C2*STR(3,J))
            D2=UJAC*C2*STR(1,J)+C1*STR(2,J)+C2*STR(3,J))
            D3=UJAC*C3*STR(1,J)+C2*STR(2,J)+C1*STR(3,J))
            D4=UJAC*C3*STR(4,J)
            D5=UJAC*C3*STR(5,J)
            D6=UJAC*C3*STR(6,J)
         90 CONTINUE
         S(I,J)=S(I,J)+STR(1,I)*D1+STRC2,I)»D2+STR(3.I)»D3+STR(4.1)»D4
            +STR(5.I)»D5+STR(6.I)»D6
         S(J,I)=S(I,J)
         IFCKEY . EQ. 0) GO TO 100
         DO 100 N=1,33
         S(I,J)=S(I,J)-C*S(K.J)
      100 CONTINUE
      WRITE(2) (LM(I),I=1,24),((S(I,J),J=1,24),I=1,24)
   100 CONTINUE

ELIMINATE EXTRA DEGREES OF FREEDOM

DO 150 NN=1,9
   L=33-NN
   K=L+1
   IF(S(K,K).EQ.0.) GO TO 139
   GO TO 141
139 PRINT 140,K,S(K,K)
140 FORMAT(5X.^K=^,,F13.3)
141 DO 150 I=1,L
   C=S(I,K)/S(K,K)
   DO 150 J=1,L
      S(I,J)=S(I,J)-C*S(K,J)
   150 CONTINUE
WRITE(2) (LM(I),I=1,24),((S(I,J),J=1,24),I=1,24)

FORM STRAIN-DISPLACEMENT MATRIX AND WRITE ON TAPE 11

READ(7) ((STR(I,J),J=1,24),I=1,6)
SUBROUTINE ADDSTFC(A,B,NUMEL,NEOB,NE2B,NBLOCK,MBAND)

C FORM EQUILIBRIM EQUATIONS IN BLOCKS
COMMON /ISOP/ E1,E2,E3,RR(8),ZZ(8),QQ(8),LM(24),P(24),S(33,33),
1 STR(6,33),STS(6,24),UJAC
DIMENSION A(NE2B,MBAND),B(NE2B)
K=NEOB+1
X=NBLOCK
MB=MB+1
NEBB=MB*NE2B
MM=1

C NSHIFT=0
REIND 10
REIND 4
C C FORM EQUATIONS IN BLOCKS (2 BLOCKS AT A TIME)
C DO 500 M=1,NBLOCK,2
DO 100 I=1,NE2B
DO 100 J=1,MBAND
100 A(I,J)=0.
READ (10) (B(I),I=1,NEOB)
IF(M .EQ. NBLOCK) GO TO 120
READ (10) (B(I),I=K,NE2B)
120 CONTINUE
C REIND 2
REIND 3
MA=3
NUME=NUM3
IF(MM .NE. 1) GO TO 150
MA=2
NUME=NUMEL
NUM3=0
150 DO 300 N=1,NUME
READ(NA) (LM(I),I=1,24),((S(I,J),J=1,24),I=1,24)
DO 220 I=1,24
LMN= 1-LM(I)
II=LM(I)-NSHIFT
IFII .LE. 0 .OR. II .GT. NE2B) GO TO 220
DO 200 J=1,24
JJ=LM(J)+LMN
IF(JJ.LE.0) GO TO 220
A(II,JJ)=A(II,JJ)+S(I,J)
200 CONTINUE
220 CONTINUE
C C DETERMINE IF STIFFNESS IS TO BE PLACED ON TAPE 3
C IF(MM.GT. 1 )GO TO 300
DO 250 I=1,24
II=LM(I)-NSHIFT
IF(II .GT. NE2B .AND. II .LE. NEBB) GO TO 260
250 CONTINUE
GO TO 300
WRITE(3) ((LM(I), I=1, 24), ((S(I, J), J=1, 24), I=1, 24)
NUM3=NUM3+1
CONTINUE
WRITE(4) ((A(I, J), I=1, NEQB), J=1, MBAND), (B(I), I=1, NEQB)
IF(M .EQ. MBLOCK) GO TO 500
WRITE(4) ((A(I, J), I=K, NE2B), J=1, MBAND), (B(I), I=K, NE2B)
IF(MM .EQ. MB) MM=0
MM=MM+1
NSHIFT=NSHIFT+NE2B
CONTINUE
RETURN
END

SUBROUTINE COMPAR(VB)
DIMENSION PRS(3), VB(3)
IF(VB(1)-VB(2)) 10, 11, 11
10 PRS(1)=VB(1)
PRS(3)=VB(2)
GO TO 12
11 PRS(1)=VB(1)
PRS(3)=VB(2)
12 IF(PRST(1)-VB(3)) 13, 14, 15
13 PRS(2)=PRS(1)
PRS(3)=VB(3)
GO TO 18
14 IF(PRST(3)-VB(3)) 16, 15, 15
15 PRS(2)=PRS(3)
PRS(3)=VB(3)
GO TO 18
16 PRS(2)=VB(3)
18 CONTINUE
VB(1)=PRS(1)
VB(2)=PRS(2)
VB(3)=PRS(3)
RETURN
END

SUBROUTINE SYMBANC(A, B, MAXB, NEQB, MB, MBLOCK, NSB, NORG, NBKS, NT1,
1 NT2, NRST)
DIMENSION A(NSB), B(NSB), MAXB(NEQB)
C
NC=MB+1
NBR=(MB-1)/NEQB+1
INC=NEQB-1
NMB=NEQB*MB
N2=NT2
N1=NT1
REWIND NORG
REWIND NBKS
C
REDUCE EQUATIONS BLOCK-BY-BLOCK
DO 900 N=1, MBLOCK
IF(N .GT. 1 .AND. NBR .EQ. 1) GO TO 110
IF(NBR .EQ. 1) GO TO 105
REWIND N1
C
REPLACE N1, N2

105 MI=N1
IF(N.EQ.1) NI=MORG
READ(N1) A

110 DO 300 I=1, NEOB
    D=A(I)
    IF(D .LT. 0.0) GO TO 300
    M=NEOB*(N-1)*I
    PRINT 116, M, D

116 FORMAT(33HSET OF EQUATIONS MAY BE SINGULAR / 1 26H DIAGONAL TERM OF EQUATION IS, 8H EQUALS 1PE12.4) STOP

120 C II=I
   DO 125 J=2, NC
   II=II+NEOB
   A(II)=A(II)/D

125 C DO 130 J=I, NMB, NEOB
   IF(A(J), NE.0.) MAXB(I)=J
   CONTINUE

130 C JL=I+1
   IF(JL.GT. NEOB) GO TO 300
   II=I
   DO 200 J=JL, NEOB
      II=II+NEOB
      IF(II.GT. NMB) GO TO 200
      C=A(II)
      IF(C.EQ.0.0) GO TO 200
      C=C*A(I)
   C KK=J-II
   MAX=MAXB(I)
   DO 150 JJ=II, MAX, NEOB
   C(JJ+KK)=A(JJ+KK)-C*A(JJ)

150 C KK=J+NMB
   JJ=I+NMB
   A(KK)=A(KK)-C*A(JJ)
   CONTINUE

300 C WRITE(NBKS) A, MAXB
   CONTINUE

C SUBSTITUTE INTO REMAINING EQUATIONS

C DO 800 NN=1, NBR
   IF(NN.GT. NBLOCK) GO TO 800
   NI=N1
   IF(N.EQ.1) NI=MORG
   IF(NN.EQ. NBR) NI=MORG
   READ(NN) B
   IL=I+NN*NEOB*NEOB
   DO 700 I=1, NEOB
      II=IL
      DO 690 K=1, NEOB
         IF(II.GT. NMB) GO TO 680
         C=A(II)
         IF(C.EQ.0.0) GO TO 680
         C=C*A(K)
         MAX=MAXB(K)
      690 F(JJ+KK)=B(JJ+KK)-C*A(JJ)
      CONTINUE
   700 C
   800 CONTINUE

C KK=I-II
   DO 640 JJ=II, MAX, NEOB
   B(JJ+KK)=B(JJ+KK)-C*A(JJ)
C     KK=I+NMB
     JJ=K+NMB
     B(KK)=B(KK)-C*A(JJ)

C
890  II=II-INC
700  IL=IL-NEQB
C
740  A(I)=B(I)
     GO TO 800
750  WRITE(N2) B
800  CONTINUE
C
810  M=M1
     N2=M
900  M=N1
        N1=N2
C
C     BACKSUBSTITUTION- RESULTS ON TAPE NRST
C
C
905  C
N1=NEQB*(NBR+1)
NUM=NBR*NEQB
DO 905 I=1,NEQB
905  B(I)=0.
C
REWIND NRST
C
DO 1000 N=1,NBLOCK
BACKSPACE NBKS
READ(NBKS) A,MAXB
BACKSPACE NBKS
K=NEB
DO 910 J=1,NUM
   I=K-NEQB
   B(K)=B(I)
910  K=K-1
C
   I=NMB
   K=0
   DO 920 J=1,NEQB
      I=I+1
      K=K+1
920  B(K)=A(I)
C
   DO 950 I=1,NEQB
      J=NEQB+1-I
      MAX=MAXB(J)
      IF(A(J).EQ.0.0) GO TO 950
      KK=J
      JJ=KK+1
      IL=J+NEQB
      C=C-A(II)*B(JJ)
      DO 940 II=IL,MAX,NEQB
      C=C-A(II)*B(JJ)
940  JJ=J+1
      B(KK)=C
950  CONTINUE
C
   I=0
   K=0
   DO 960 J=1,NEQB
      K=K+1
      I=I+1
960  A(I)=B(KK)
SUBROUTINE RESULTCEMPR, ID, X, Y, Z, INP, BULK, SHEAR, POIS, SLMAX, NCNP, NLDP, DISP, STRESS, STRAIN, SNEW, DELD, B, R, PATM, NUMELT, NUMNPT, NUMCNP, NUMMAT, NUMLID, NLAY, LN, IT, NPUNCH, NUMEL, NUMNP, NELCAL, NNPRED, NEQ, NEOB, NBLOCK
COMMON /ISOP/ E1, E2, E3, RR(8), ZZ(8), QQ(3), LM(24), P(24), S(33, 33), STR(G, 33), UJAC
COMMON /JSOP/ LAYSUM, MFLAY, MREL, MOMEL, HEIGHT
DIMENSION EMPR(NUMMAT, 13), ID(NUMNPT, 3), BULK(1), SHEAR(1), POIS(1)
DIMENSION INP(NUMELT, 9), SLMAX(1), NLDP(1), NCNP(1)
DIMENSION DISP(NUMNPT, 3), STRESS(NUMELT, 6), STRAIN(NUMELT, 6)
DIMENSION SNEW(NUMELT, 6), DELD(NUMNPT, 3), B(1), R(1)
DIMENSION SIG(6), EPS(6), PRS(14), A(3, 3), ZI(3, 3), D(3)
DIMENSION X(1), Y(1), Z(1)
DIMENSION HEIGHT(20), MLEL(20), MREL(20), MOMEL(20, 2)
REWIND 2
REWIND 6
REWIND 11

MOVE DISPLACEMENTS INTO CORE
NO=NEOB*NBLOCK
DO 10 NN=1, NBLOCK
READ(2) (R(I), I=1, NEOB)
N=NEOB
IF(NN.EQ.1) N=NEOB-NO+NEOB
NO=NO-NEOB
DO 10 J=1, N
I=NO+J
10 B(I)=R(J)
IF(LN.GT.NLAY) GO TO 15
PRINT 2000, LN, IT
GO TO 16
15 LNMLAY=LN-NLAY
PRINT 2005, LNMLAY, IT
GO TO 16
16 CONTINUE
IF(IT.LT.2) GO TO 110

ADD INCREMENTAL DISPLACEMENTS AND PRINT INCREMENTAL AND TOTAL DISPLACEMENTS
PRINT 2010
REWIND 3
READ(3) ((DISP(N, M), M=1, 3), N=1, NUMNPT)
READ(3) ((STRAIN(N, M), M=1, 6), N=1, NUMELT)
DO 20 N=1, NUMNP
DO 20 I=1, 3
20 DELD(N, I)=0.
DO 70 N=1, NUMNP
IF(N .LE. NNPRED .OR. NUMCNP .EQ. 0) GO TO 40
DO 30 M=1, NUMCNP
IF(N.EQ. NCNP(M)) GO TO 40
30 CONTINUE
GO TO 70
40 CONTINUE
IF(N.LT.NNPRED .OR. N.GT.NNPRED) GO TO 45
IF(NUMCNP .EQ. 0) GO TO 70
DO 42 M=1,NUMCNP
IF(N .EQ. NCNP(M)) GO TO 45
42 CONTINUE
GO TO 70
45 CONTINUE
DO 50 I=1,3
II=ID(N,I)
IF(II.LT.1) GO TO 50
DELD(N,I)=B(II)
50 CONTINUE
DO 60 J=1,3
60 DISP(N,J)=DISP(N,J)+DELD(N,J)
70 CONTINUE
DO 100 N=1,NUMNP
IF(N .LE. NNPCAL .OR. NUMCNP .EQ. 0) GO TO 90
DO 80 M=1,NUMCNP
IF(N .EQ. NCNP(M)) GO TO 90
80 CONTINUE
GO TO 100
90 CONTINUE
TD=SQRT(DISP(N,1)**2+DISP(N,2)**2+DISP(N,3)**2)
PRINT 2050, N, (DELD(N,I),I=1,3), (DISP(N,M),M=1,3), TD, N
100 CONTINUE
110 CONTINUE
C CALCULATE INCREMENTAL STRESSES AND STRAINS. ADD INCREMENTAL
C STRESSES AND STRAINS AND PRINT STRAINS AND MODULUS VALUES
C
READ(8) ((STRESS(I,J),J=1,6),I=1,NUMEL)
DO 120 N=1,NUMEL
DO 120 I=1,6
120 SNEW(N,I)=STRESS(N,I)
DO 300 N=1,NUMEL
IF(N .LE. NCEL .OR. NUMCEL .EQ. 0) GO TO 150
DO 140 M=1,NUMCEL
IF(N .EQ. NCEL(M)) GO TO 150
140 CONTINUE
GO TO 300
150 CONTINUE
READ(11) (LM(I),I=1,24),((STR(I,J),J=1,24),I=1,6),LL
IF(LL.EQ.0) GO TO 222
C FORM STRESS-DISPLACEMENT MATRIX
C
C1=BULK(N)+SHEAR(N)
C2=BULK(N)-SHEAR(N)
C3=SHEAR(N)
DO 200 K=1,24
STS(1,K)=C1*STR(1,K)+C2*STR(2,K)+C2*STR(3,K)
STS(2,K)=C2*STR(1,K)+C1*STR(2,K)+C2*STR(3,K)
STS(3,K)=C2*STR(1,K)+C2*STR(2,K)+C1*STR(3,K)
STS(4,K)=C3*STR(4,K)
STS(5,K)=C3*STR(5,K)
STS(6,K)=C3*STR(6,K)
KK=LM(K)
IF(KK.EQ.0) GO TO 180
P(K)=B(KK)
GO TO 200
180 P(K)=0.
200 CONTINUE
DO 220 I=1,6
SIG(I)=0.
DO 220 K=1,24
220 SIG(I)=SIG(I)+STS(I,K)*P(K)
GO TO 228
222 CONTINUE
   IF(IT.EQ.1) GO TO 225
   DO 224 I=1,6
   224 SIG(I)=0.
   GO TO 228
225 CONTINUE
   DO 226 I=1,6
   226 SIG(I)=STRESS(N,I)
228 CONTINUE
   IF(IT.EQ.2) GO TO 240
   DO 230 I=1,6
   230 SNEW(N,I)=STRESS(N,I)-0.5*SIG(I)
   GO TO 300
240 CONTINUE
   DO 250 I=1,6
   SNEW(N,I)=STRESS(N,I)-SIG(I)
250 STRESS(N,I)=SNEW(N,I)
   DO 270 I=1,6
   EPS(I)=0.
   IF(LL.EQ.0) GO TO 270
   DO 260 K=1,24
   260 EPS(I)=EPS(I)+STR(I,K)*P(K)
270 STRAIN(N,I)=STRAIN(N,I)-EPS(I)*100.
300 CONTINUE
   IF(IT.LT.2) GO TO 400
   PRINT 2100
   DO 360 N=1,NUMEL
   IF(N LE. NELCAL .OR. NUMCEL .EQ. 0) GO TO 340
   DO 320 M=1,NUMCEL
   IF(N EQ. NCEL(M)) GO TO 340
   320 CONTINUE
   GO TO 360
340 CONTINUE
C CALCULATE PRINCIPAL STRAINS
   IND=0
   NM=3
   NI=3
   A(1,1)=STRAIN(N,1)
   A(2,2)=STRAIN(N,2)
   A(3,3)=STRAIN(N,3)
   A(1,2)=STRAIN(N,4)
   A(2,3)=STRAIN(N,5)
   A(1,3)=STRAIN(N,6)
   A(2,1)=A(1,2)
   A(3,2)=A(2,3)
   A(3,1)=A(1,3)
   CALL RSEIG(MN,N1,A,IND,DI,21)
   CALL COMPAR(D)
   PRS(1)=D(1)
   PRS(2)=D(3)
   PRS(3)=(PRS(1)-PRS(2))/2.
   EMOD=2.*BULK(N)*(1.+POIS(N))*(1.-2.*POIS(N))
   PRINT 2150, N, EMOD, BULK(N), SHEAR(N), POIS(N), (STRAIN(N,M), M=1,6), 1 PRS(3), N
360 CONTINUE
   PRINT 2200
   PRINT 2160
400 CONTINUE
C CALCULATE PRINCIPAL STRESSES AND NEW MODULUS VALUES
   DO 500 N=1,NUMEL
IF(N .LE. MELCAL .OR. NUMCEL .EQ. 0) GO TO 440
DO 420 M=1,NUMCEL
IF (N .LE. NCCEL(N)) GO TO 440
420 CONTINUE
GO TO 500
440 CONTINUE
IND=0
N=3
M=3
A(1,1)=SNEW(N,1)
A(2,2)=SNEW(N,2)
A(3,3)=SNEW(N,3)
A(1,2)=SNEW(N,4)
A(2,3)=SNEW(N,5)
A(1,3)=SNEW(N,6)
A(2,1)=A(1,2)
A(3,2)=A(2,3)
A(3,1)=A(1,3)
CALL RSEIGNM,N1,A,IND,D,Z1)
CALL COMPAR(D)
DO 443 I=1,3
PRS(I)=D(I)
443 CONTINUE
MTYPE=INP(N,9)
KL=2
IF(IT.EQ.1) KL=3
CALL MODU (EMPR,BULK,SHEAR,POIS,SLMAX,PRS,ATML,NM,MTYPE,STRLEU,KL)
IF(IT.LT.2) GO TO 500
IF(STRLEU.GT.SLMAX(N)) SLMAX(N)=STRLEU
PRS(4)=(PRS(1)-PRS(3))/2.
C CALCULATE THREE DIRECTIONS OF STRESSES
JJ1=1
DO 451 I=1,3
PRS(5+(JJ1-1)*I)=57.29577961*Z1(I,1)
PRS(6+(JJ1-1)*I)=57.29577961*Z1(I,2)
PRS(7+(JJ1-1)*I)=57.29577961*Z1(I,3)
JJ1=JJ1+1
451 CONTINUE
IF(PRST(3).NE.0.) GO TO 460
PRS(14)=999.99
GO TO 470
460 PRS(14)=PRS(1)/PRS(3)
470 CONTINUE
C PRINT STRESSES
C PRINT 2250, N,(STRESS(N,M),M=1,6), (PRS(I),I=1,3),PRS(14),
ISLMAX(N),STRLEU
500 CONTINUE
IF(IT.EQ.2) GO TO 530
REIND 9
WRITE(9) ((DISP(N,M),M=1,3),N=1,NUMNPT)
WRITE(9) ((STRAIN(N,M),M=1,6),N=1,NUMELT)
IF(LN.EQ.0) GO TO 501
GO TO 508
501 READ 2203, IFXY,IFYZ
IF(IFXY.EQ.0 .AND. IFYZ.EQ.0) GO TO 504
READ 1000, LAYSUM,MFLAY
READ 1010, ((HEIGHT(I),I=1,LAYSUM)
READ 1020, ((M0MEL(I),M0MEL(I),I=1,MFLAY)
MELAY=LAYSUM-MFLAY
READ 1020, ((MOMEL(I,1),MOMEL(I,2)),I=1,MELAY)
IF(IFXY.EQ.0) GO TO 502
CALL FACTXY(EMPR,XY,Z,INP,SHEAR,STRESS,NUMELT,NUMMAT)
SUBROUTINE FACTXYCEMPR, X, Y, Z, IMP, SHEAR, STRESS, NUMELT, NUMMAT

COMMON /JSOP/ LAYSUM, NFLAY, NREL, NOMEL, HEIGHT
DIMENSION EMPR(13), X(1), Y(1), Z(1), IMP(9), STRESS(8), SHEAR(1)
DIMENSION HEIGHT(20), NLEL(20), NREL(20), NOMEL(20,2)

READ 2000, XO, YO, BETA, RU, GAMAE, GAMAF, NTIME
TB=TAN(BETA/57.29577951)
VT=HEIGHT(LAYSUM)
XT=XO+(YT-YO)/TB

DO 280 NUM=1,NTIME
2000 FORMAT(ISH, 'STRESSES AND STRESS LEVELS FOR FINAL CONDITION AT,'/ 1 1STH END OF INCREMENT,"
2 30H STRESSES DURING THE INCREMENT,"
3 51H ELE ELAS MOD BULK MOD SHEAR MOD POIS EPS-X,"
4 51H EPS-Y EPS-Z GAM-XY GAM-YZ GAM-ZX GAMMAX ELE"
2150 FORMAT(IS, 'NP DELTA-X DELTA-Y DELTA-Z X-DISP Y-DISP Z-DISP TOTAL NP')
2160 FORMAT(132H SIG-X SIG-Y SIG-Z TAU-XY TAU-YZ 2 TAU-2X SIG-1 SIG-2 SIG1/SIG3 SLMAX S 3LPRES')
2200 FORMAT(IS, 'TAUMX T1-12 T1-23 T1-13 T2-12 T2-13 T3-12 T3-23 T3-13')
2201 FORMAT(IS, 'TAUMX T1-12 T1-23 T1-13 T2-12 T2-13 T3-12 T3-23 T3-13')
2202 FORMAT(IS, 'TAUMX T1-12 T1-23 T1-13 T2-12 T2-13 T3-12 T3-23 T3-13')
2250 FORMAT(IS, 'TAUMX T1-12 T1-23 T1-13 T2-12 T2-13 T3-12 T3-23 T3-13')
2300 FORMAT(IS, 'TAUMX T1-12 T1-23 T1-13 T2-12 T2-13 T3-12 T3-23 T3-13')
2350 FORMAT(IS, 'TAUMX T1-12 T1-23 T1-13 T2-12 T2-13 T3-12 T3-23 T3-13')
2400 FORMAT(IS, 'TAUMX T1-12 T1-23 T1-13 T2-12 T2-13 T3-12 T3-23 T3-13')
2450 FORMAT(IS, 'TAUMX T1-12 T1-23 T1-13 T2-12 T2-13 T3-12 T3-23 T3-13')
2500 FORMAT(IS, 'TAUMX T1-12 T1-23 T1-13 T2-12 T2-13 T3-12 T3-23 T3-13')
2550 FORMAT(IS, 'TAUMX T1-12 T1-23 T1-13 T2-12 T2-13 T3-12 T3-23 T3-13')
2600 FORMAT(IS, 'TAUMX T1-12 T1-23 T1-13 T2-12 T2-13 T3-12 T3-23 T3-13')

READ 2001, RADIUS, RZ, DANGLE, XR, YR, ZR, D2, NUMBER, NUMBF, ISIGN
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PRINT 3000
PRINT 3001. RADIUS, R2, XR, YR, ZR, BETA, DANGLE
PRINT 2002
RTOP=SORT((XR-XT)**2.+(YR-YT)**2.)
DIST=ABS(XR-TB-YR+YO-XO+TB)/SORT(1.+TB+TB)
TSIGN=0.
TTAUN=0.
TCOHES=0.

I=1
50 ZP=ZR+(FLOAT(I)-0.5)*DZ*ISIGN
DET=1.-((FLOAT(I)-0.5)*DZ/RZ)**2.
IF(DET .LT. 0.) GO TO 280
RXY=RADIUS*SORT(DET)
IF(RXY .LE. DIST) GO TO 275
ALFA0=ASIN((YR-YT)/RXY)
ANGLE=ALFAO+DANGLE/(2.*SIN(2.5729577951))
IF(RXY .GE. RTOP) GO TO 80
60 YL=YT-YR-RXY*SIN(ANGLE)
YU=(XR-XO+RXY*COS(ANGLE))*TB
IF(YL .GE. YU) GO TO 70
GO TO 80
70 ANGLE=ANGLE+DANGLE/(2.*SIN(2.5729577951))
GO TO 60

80 YP=YR-RXY*SIN(ANGLE)
ITER=1
90 NI=LAYSUM
IF(NI .EQ. 1) GO TO 100
GO TO 110
100 IF(VP .LE. HEIGHT(1) .AND. YP .GE. 0.) GO TO 120
GO TO 300
110 IF(VP .LE. HEIGHT(NI) .AND. VP .GT. HEIGHT(NI-1)) GO TO 120
NI=NI-1
GO TO 90

120 XP=XR+RXY*COS(ANGLE)
CKX=XP-XO
CKY=YP-YO
IF(CKX .LE. 0. .AND. CKY .GE. 0.) GO TO 270
IF(CKX .GE. 0. .AND. CKY .GT. 0.) GO TO 130
GO TO 140
130 SXY=CKY/CKX
IF(SXY .GE. TB) GO TO 270
140 IF(NI .LE. NFLAY) GO TO 150
NF=NI-NFLAY
NS=HOMEL(NF,1)
GO TO 160
150 NS=MLEL(NI)
160 NI=INP(NS,1)
N2=INP(NS,2)
N3=INP(NS,3)
N4=INP(NS,4)
N5=INP(NS,5)
N8=INP(NS,8)

IF(ITER .EQ. 2) GO TO 210
IF(X(N1) .EQ. X(N4)) GO TO 170
CK1=X(N1)+(X(N4)-X(N1))/(Y(N4)-Y(N1))*(YP-Y(N1))
GO TO 180
170CK1=X(N1)
180CK2=X(N2)+((X(N3)-X(N2))/(Y(N3)-Y(N2)))*(YP-Y(N2))
GO TO 200
190CK2=X(N2)
200 IF(XP .GE. CK1 .AND. XP .LT. CK2) GO TO 210
   NS=NS+1
   GO TO 160
210 IF(Z(N1) .EQ. Z(N4)) GO TO 220
   CK3=(Z(N1)-(Z(N1)-Z(N4)))*(YP-Y(N1))/(Y(N4)-Y(N1))
   GO TO 230
220 CK3=Z(N1)
230 IF(Z(NS) .EQ. Z(NS)) GO TO 240
   CK4=(Z(NS)-Z(NS))*(YP-Y(NS))/(Y(NS)-Y(NS))
   GO TO 250
240 CK4=Z(NS)
250 IF(ZP .GE. CK3 .AND. ZP .LT. CK4) GO TO 260
   IF(HI .GE. NFAIL .AND. NS .GE. NREL(N1)) GO TO 290
   IF(HI .LE. NFAIL .AND. NS .GE. NREL(N1)) GO TO 290
   IF(Z(N1)-NFAIL) 252,252,254
   MS=NS+NUMBER
   GO TO 256
254 NS=NS+NUMBER
256 ITER=2
   GO TO 160
260 AXY=1.5707963-ANGLE
   DX=RXY*(DANGLE/57.29577951)*COS(AXY)
   AYZ=ATAN((RADIUS/R2)**2.*ABS(ZP-ZR)/(YR-YP))
   ANXY=AXY+57.29577951
   AYZ=AYZ+57.29577951
   AREA=DX*D2/SORT(1-((SIN(AYZ)*SIN(AXY))*2.)/(COS(AXY)*COS(AYZ))
   UT=SORT(1.+TAN(AXY)**2.+TAN(AYZ)**2.)
   U1=TAN(AXY)/UT
   U2=1./UT
   U3=TAN(AYZ)/UT
   T1=STRESS(NS,1)*U1-STRESS(MS,4)*U2-STRESS(MS,5)*U3
   T2=STRESS(NS,4)*U1-STRESS(NS,2)*U2-STRESS(NS,3)*U3
   T3=STRESS(NS,6)*U1-STRESS(NS,5)*U2+STRESS(NS,3)*U3
   SIGN=T1*T1+T2*T2+T3*T3
   IF(SIGM .GE. 0.) SIGM=0.
   TAUN=SORT(ABS(T1*T1+T2*T2+T3*T3-SIGN*SIGN))
   MTYPE=INP(MS,9)
   IF(SHEAR(MS) .LE. 0.0001) GO TO 261
   TAMFI=TAMCEMPR(I1TYPE,9)/57.29577951
   COHES=EMPR(MTYPE,8)
   GO TO 262
261 TAMFI=0.
   COHES=0.
262 IF(XP .GE. XT) GO TO 265
   IF(XP .LT. XO) GO TO 263
   WP=RU+GAMAF*(YD-YP)
   GO TO 267
263 IF(YP .GE. YO) GO TO 264
   WP=RU+(GAMAE*(XP-XO)+TB+GAMAF*(YD-YP))
   GO TO 267
264 WP=RU+GAMAE*(YP+YD)*TB-YP)
   GO TO 267
265 IF(YP .GE. YO) GO TO 266
   WP=RU+(GAMAE*(YT-YD)+GAMAF*(YD-YP))
   GO TO 267
266 WP=RU+GAMAE*(YT-YP)
267 FS=((-SIGN-WP)*TAMFI*COHES)/TAUN
   PRINT 2003,NS,XP,YP,ZP,SIGN,WP,TAUN,AAXY,AAYZ,AREA,FS
TSGN=TSIGN+(SIGN-WP)*TANF1*AREA
TTAUN=TTAUN+TAUN*AREA
TCOHES=TCOHES+COHES*AREA
ANGLE=ANGLE+DANGLE/57.29577951
GO TO 80
270 I=I+1
GO TO 290
275 TFS=(TSGN+TCOHES)/TTAUN
PRINT 2004, TFS
280 CONTINUE
GO TO 310
290 PRINT 2005
GO TO 310
300 PRINT 2006
310 CONTINUE
RETURN
2000 FORMAT(F10.3, IS)
2001 FORMAT(F10.3, 3F15.3)
2002 FORMAT(/, 11TH ELE X Y Z SIGN
1 TAUN AXY AYZ AREA LOCAL FS)
2003 FORMAT(I10, 10F10.3)
2004 FORMAT(/, SX, OVERALL FACTOR OF SAFETY = #*F10.3)
2005 FORMAT(SX, ######WARNING### THE FAILURE SURFACE OUTSIDE THE EMBANKMENT
1 #/)
2006 FORMAT(SX, ######WARNING### THE FAILURE SURFACE BELOW THE RIGID FOUNDATION
1 !/#)
3000 FORMAT(SOH1 RXY RYZ XR YR ZR, 1 33H BETA DANGLE)
3001 FORMAT(F10.3)
END

SUBROUTINE FACTYZ(ENPR, X, Y, Z, INP, SHEAR, STRESS, NUMELT, NUMMAT)
COMMON/JOSP/, LAYSUM, NFLAY, NREL, NOMEL, HEIGHT
DIMENSION EMPR(NUMMAT, 13), X(1), Y(1), Z(1), INP(NUMELT, 9)
DIMENSION STRESS(NUMELT, 6), SHEAR(1)
DIMENSION HEIGHT(20), NREL(20), NOMEL(20, 2)
READ 2000, YO, ZO, BETA, RADIUS, GAMMA, NTIME
TB=TAN(BETA/57.29577951)
YT=HEIGHT(LAYSUM)
RTOP=SQRT((ZT-ZT)*2+(YR-YT)*2)
DO 280 INUM=1, NTIME
READ 2001, RADIUS, RX, DANGLE, XR, YR, ZR, DX, NUMBER, NUMBF, ISIGN
PRINT 3000
PRINT 3001, RX, RADIUS, XR, YR, ZR, BETA, DANGLE
PRINT 3002
RTOP=SQRT((ZR-ZT)**2+(YR-YT)**2)
DIST=ABS(-ZR*TB-YR+YO+ZO/TB)/SQRT(1. +TB*TB)
IF CRYZ LE. DIST GO TO 275
ANGLE=ALFAO+DANGLE/(2.*57.29577951)
IF (RYZ .GE. RTOP) GO TO 80
60   YL=YL-YR-RYZ=SIN(ANGLE)
     YU=(20-2R+RYZ=COS(ANGLE))*TB
     IF(YL .GE. YU) GO TO 70
     GO TO 60
70   ANGLE=ANGLE+ANGLE/(2.*S7.29577951)
     GO TO 60
80   YP=YL-RYZ=SIN(ANGLE)
     ITER=1
     NI=LAYSUM
90   IF(NI .EQ. 1) GO TO 100
     GO TO 110
100  IF(YP .LE. HEIGHT(1) .AND. YP. GE. 0.) GO TO 120
     GO TO 300
110  IF(YP .LE. HEIGHT(NI) .AND. YP. GT. HEIGHT(NI-1)) GO TO 120
     NI=NI-1
     GO TO 90
120  ZP=ZR-RYZ-COS(ANGLE)
     CKZ=ZP-ZP
     CKY=YP-YO
     IF(CKZ .LE. 0. .AND. CKY .GE. 0.) GO TO 270
     IF(CKZ .GT. 0. .AND. CKY .GT. 0.) GO TO 130
     GO TO 140
130  SYZ=CKY/CKZ
     IF(SYZ .GE. TB) GO TO 270
140  IF(NI .LE. NFLAY) GO TO 150
     NF=NI-NFLAY
     NS=NOMEL(NF,1)
     GO TO 160
150  NS=NLEL(NI)
160  N1=INP(NS,1)
     N2=INP(NS,2)
     N3=INP(NS,3)
     N4=INP(NS,4)
     N5=INP(NS,5)
     N6=INP(NS,8)
     IF(ITER .EQ. 2) GO TO 210
     IF(X(N1) .EQ. X(N4)) GO TO 170
     CK1=X(N1)*(X(N4)-X(N1))/(Y(N4)-Y(N1))*(YP-Y(N1))
     GO TO 180
170  CK1=X(N1)
180  IF(X(N2) .EQ. X(N3)) GO TO 190
     CK2=X(N2)*(X(N3)-X(N2))/(Y(N3)-Y(N2))*(YP-Y(N2))
     GO TO 200
190  CK2=X(N2)
200  IF(CK1 .GE. CK2) GO TO 210
     NS=NS+1
     GO TO 160
210  IF(Z(N1) .EQ. Z(N4)) GO TO 220
     CK3=Z(N1)-Z(N1)-Z(N4))*(YP-Y(N1))/(Y(N4)-Y(N1))
     GO TO 230
220  CK3=Z(N1)
230  IF(Z(N5) .EQ. Z(N8)) GO TO 240
     CK4=Z(N5)-Z(N5)-Z(N8))*(YP-Y(N5))/(Y(N8)-Y(N5))
     GO TO 250
240  CK4=Z(N5)
250  IF(2P .GE. CK3 .AND. ZP .LT. CK4) GO TO 260
     IF(NI .GT. NFLAY .AND. NS .GE. NOMEL(NF,2)) GO TO 280
     IF(NI .LE. NFLAY .AND. NS. GE. NRELC(NI)) GO TO 290
     IF(NI-NFLAY) 252,252,254
252  NS=NS+NUMBF
     GO TO 256
254  NS=NS+NUMBER
256  ITR=2
260  GO TO 160
260  AYZ=1.5707963-ANGLE
260  AZY=ATAN((DANGLE/S7./29577951)*COS(AYZ))
260  AXY=ATAN((RADIUS/RX)**2.*ABS(XP-XP)/(YR-YP))
260  AXZ=AYZ*S7./29577951
260  AYZ=AYZ*S7./29577951
260  UT=SORT(1.+TAN(AXY)**2.+TAN(AYZ)**2.)
260  U1=TAN(AXY)/UT
260  U2=1./UT
260  U3=TAN(AYZ)/UT
260  T1=STRESS(NS,1)*U1-STRESS(NS,4)*U2-STRESS(NS,6)*U3
260  T2=STRESS(NS,4)*U1+STRESS(NS,2)*U2-STRESS(NS,5)*U3
260  T3=STRESS(NS,6)*U1-STRESS(NS,5)*U2+STRESS(NS,3)*U3
260  SIGN=T1+U1+T2+U2+T3+U3
260  IF(SIGN .LE. 0.) SIGN=0.
260  TAUN=SORT(ABS(T1*+T2+T3*T3-SIGN*SIGN))
260  MTYP=INTP(9)
260  IF(SHEAR(NS).LE.0.0001) GO TO 261
260  COHES=EMFR(MTYP,8)
262  GO TO 262
261  TANFI=0.
261  COHES=0.
262  IF(ZP .GE. ZT) GO TO 265
262  IF(ZP .GT. ZO) GO TO 263
262  WP=RU*GAMAF*(YO-YP)
262  GO TO 267
264  IF(YP .GE. YO) GO TO 266
264  IF(YO .GT. YO) GO TO 263
264  WP=RU*GAMAE*(ZP-ZO)*TB+GAMAF*(YO-YP)
264  GO TO 267
265  WP=RU*GAMAE*(YO-20)*TB-YP
265  GO TO 267
266  WP=RU*GAMAE*(YT-YP)
267  FS=((SIGN-WP)*TANFI+COHES)/TAUN
267  PRINT 0003,NS,XP,YP,ZP,SIGN,WP,TAUN,AAXY,AAYZ,AREA,FS
267  TSIGN=TSIGN+(SIGN-UP)*TANFI*AREA
267  TTAUN=TTAUN+TAUN*AREA
267  TCOHES=TCOHES+COHES*AREA
267  ANGLE=ANGLE+DANGLE/S7./29577951
269  CONTINUE
270  I=I+1
275  TFS=TSIGN+TCOHES)/TTAUN
275  PRINT 0004, TFS
280  CONTINUE
280  GO TO 310
290  CONTINUE
290  PRINT 0005
290  CONTINUE
300  CONTINUE
300  RETURN
300  FORMAT(7F10.3,15)
300  CONTINUE
300  RETURN
300  FORMAT(7F10.3,215/15)
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2002 FORMAT(//,110H ELE X Y Z SIGN
1 WP TAUN AXY AYZ AREA LOCAL FS)

2003 FORMAT(I10,10F10.3)

2004 FORMAT(//,5X,OVERALL FACTOR OF SAFETY = F10.3)

2005 FORMAT(5X,#***WARING*** THE FAILURE SURFACE OUTSIDE THE EMBANKMENT
1 X/,

2006 FORMAT(5X,#***WARING*** THE FAILURE SURFACE BELOW THE RIGID FOUNDATION?

3000 FORMAT(50H1 RXY RYZ XR YR ZR,
1 33H BETA DANGLE )

3001 FORMAT(7F10.3)

END