AN EXAMINATION OF THE RELATIONSHIP BETWEEN FITTS’ LAW
AND SCHMIDT’S LAW

by

Jonathan McKeeman

A Thesis

Submitted to the Faculty of Purdue University
In Partial Fulfillment of the Requirements for the degree of

Master of Science

Department of Health & Kinesiology
West Lafayette, Indiana
December 2017
THE PURDUE UNIVERSITY GRADUATE SCHOOL
STATEMENT OF COMMITTEE APPROVAL

Dr. Howard Zelaznik, Chair
   Department of Health and Kinesiology
Dr. Shirley Rietdyk
   Department of Health and Kinesiology
Dr. Jeffrey Haddad
   Department of Health and Kinesiology
Dr. Satyajit Ambike
   Department of Health and Kinesiology

Approved by:
   Dr. David Klenosky
   Head of the Graduate Program
To Janae’
ACKNOWLEDGMENTS

This work would not have been possible without the help from many people. First and foremost, thank you to Dr. Howard Zelaznik for his leadership and expertise in the subject matter. Under his guidance, I was able to complete the work in around a calendar year. I am thankful for his direction and ability to challenge me to work harder and think in different ways. It has been a pleasure working with him. Secondly, thank you to my committee members; Dr. Shirley Rietdyk, Dr. Jeffrey Haddad, and Dr. Satyajit Ambike for their thoughtful review of my proposal and final defense. They brought up excellent points that were cause for necessary further revision. Through their assistance, the work became more polished and clear. Lastly, thank you to all the student participants. They took time out of their day to participate in my research and for that I am forever grateful.
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ABSTRACT

Author: McKeeman, Jonathan, D. MS
Institution: Purdue University
Degree Received: December 2017
Title: An Examination of the Relationship Between Fitts’ Law and Schmidt’s Law
Major Professor: Dr. Howard Zelaznik

There are two prominent speed accuracy tradeoff relationships; Fitts’ Law and Schmidt’s Law. The Fitts tasks are considered to be spatially constrained, whereas the Schmidt tasks are temporally constrained. In this experiment the relationship between these two speed accuracy tradeoffs was examined. Previous studies have allowed for the hypothesis that these two laws may be related or share similar control processes. Through analysis of the previous literature on the subject I hypothesized that the two speed accuracy tradeoffs are unrelated and do not share a common control process. To test this hypothesis, participants performed both speed accuracy tradeoff tasks under similar parameters in order to compare the tasks on the same scale. For each task the relation between average velocity and effective target width was examined. The two tasks exhibited unique linear relations between speed and spatial accuracy. Based upon this result it was concluded that these two speed-accuracy relations possess a unique non-shareable control process.
INTRODUCTION

A speed-accuracy tradeoff occurs when a person manipulates speed to achieve the desired level of spatial accuracy. It is seen in the folk lore “haste makes waste.” We demonstrate this when we slow a movement down in order to hit a target. In the study of limb movement speed-accuracy tradeoff, two strong relationships have emerged; Fitts’ Law and Schmidt’s Law. In this thesis an experiment is conducted to compare the two relationships to determine whether they are related or share a common control process.
LITERATURE REVIEW

The Logarithmic Speed Accuracy Tradeoff

Fitts (1954) had participants perform reciprocal tapping, pin transfer, and disc transfer tasks that produced a spatial speed accuracy tradeoff. In the tapping tasks, the participant moves as quickly and as accurately as possible with a stylus to a target of width, W, and distance, D units apart (Fitts, 1954). The task has two goals; move as fast as possible and not miss the target. Thus this is a task with spatial accuracy as a goal and a tradeoff with velocity magnitude. The distance to the target and the width of the target are manipulated in order to observe the effect on movement time. Fitts found that movement time, T, is related logarithmically to the ratio of distance and width, T = a + bLog₂(2D/W) (See Figure 1). Log₂(2D/W) is known as the index of difficulty, or ID. Smaller target widths and longer distances to the target (more difficult ID) result in longer movement times. This relationship is now called Fitts’ Law. The logarithmic description for the spatial speed accuracy tradeoff behavior is generally seen as the best model and most robust. Although other researchers claim to have improved upon Fitts’ equation (Kvalseth, 1980), doing so complicates the equation with more variables. Because science leans toward parsimony, Fitts’ Law remains the preferred description (Passmore 2007, Schmidt, et al 1979, Smits-Engelsman 2007).
The Linear Speed Accuracy Tradeoff

Schmidt, Zelaznik, Hawkins, Frank, and Quinn. (1979) developed a new task different from the Fitts (1954) task. Instead of using a typical Fitts task, Schmidt et. al. (1979) utilized a temporally constrained single aimed movement task. The goal was to hit the target in a specified movement time. Thus this is a task with temporal accuracy as a goal and a tradeoff with spatial magnitude. A participant moved a stylus to a target in a discrete single aimed movement and was informed of their movement time. In the original Schmidt et. al. (1979) experiment the participant, across trials, would then make corrections in speed in order to adhere to the specified movement time. The participant produces a series of endpoints around the target due to the demands of the task. The series of endpoints for a particular task creates an area that surrounds the target which is termed the effective target width. Specifically, the effective target width is defined as one standard deviation in movement endpoints across trials for a participant within a...
particular combination of D and T.

Effective target width is thought to measure the noise in the motor system. Noise refers to within-participant variability in motor output. Noise is random and may be due to variability in motor unit activation (Schmidt et al., 1979). Noise does not refer to when a participant performs a novel task and variability occurs due to the participant choosing new strategies in order to complete the task effectively. Noise occurs when a participant is trying to repeat the same action over and over, but there is still variability. It is neither good nor bad; it simply exists. By using a temporally constrained task and manipulating the parameters D and T, the spatial variability of a participant can be measured using the effective target width for each task. Thus a participant cannot tradeoff speed for accuracy, but speed produces a level of spatial precision, the effective target width.

Schmidt et al. (1979) predicted and observed that the relationship between speed and accuracy was $W_e = k(D/T)$ with $W_e$ being the effective target width. (See Figure 2). Smaller movement times produce larger effective target widths and longer distances also produce larger effective target widths. The reverse is also true. This relationship is known as the linear speed accuracy tradeoff or Schmidt’s Law as opposed to the logarithmic speed accuracy tradeoff or Fitts’ Law.
The Iterative Correction Model

There are two prominent models to explain Fitts’ Law; Crossman and Goodeve (1983) and Meyer, Abrams, Kornblum, Wright, & Smith (1988). Crossman and Goodeve (1983) examined the trajectories from a Fitts’ Law task. They proposed an intermittently sampled feedback explanation of limb control. In an intermittently sampled control explanation, error is assessed at certain points during the movement which will be the endpoints of submovements. Crossman and Goodeve believe that the participant begins moving toward the target and then uses vision to make corrections in order to be accurate. The idea that participants make corrections based on visual feedback supports multiple submovements as the hand approaches the target because each submovement can be viewed as a correction based on error in movement relative to the target.

Figure 2. Example of an effective target width vs average velocity for the Schmidt et. al. (1979) experiment. From Schmidt et. al. (1979) Figure 9.
The first assumption of Crossman and Goodeve (1983) is that each submovement travels a fixed proportion of the distance remaining. The first submovement brings the participant a fixed proportion closer to the target and the second submovement brings the participant the same fixed proportion of the distance remaining to the target from the endpoint of the first submovement. This continues until the participant reaches the target. The second assumption is that each submovement has a fixed duration. The intermittent process terminates when the distance remaining is less than half the target width. At this point the participant is within the target zone and stops the movement.

Crossman and Goodeve propose a stopping rule for when participants terminate movement. They used the equation for Fitts’ Law to derive this rule. [See Appendix for full derivation.] The stopping rule only allows for participants to terminate the movement up to and including the center of the target, but still within the target zone. Thus, participants would not overshoot the target because when they visually correct intermittently, they only move a fixed proportion of the center of the target. Thus, overshoots of the target are not explained under the Crossman and Goodeve model (p is assumed to be <1 as a fixed proportion of distance remaining). However, overshoots are observed in Fitts’ Law tasks, so the Crossman and Goodeve model or the iterative correction model is incomplete at best (Meyer et. al. 1988).

Wallace and Newell (1983)

One assumption of the iterative correction model that Wallace and Newell (1983) examined is the assumption that vision is essential for Fitts’ Law. They observed that participants still obey Fitts’ Law even when vision is removed. In the Wallace and Newell (1983) experiment vision was removed at the initiation of movement in a Fitts’ law task. Theoretically, when vision is removed there is no information for corrections based on the relative position of
the hand compared to the target, yet participants still obeyed Fitts’ Law. The participants still assess the distance from the first glance at the target before vision was removed and obey Fitts’ law. Crossman and Goodeve (1983) do not explain how the participants could obey Fitts’ Law without vision even though this phenomenon is observed in Wallace and Newell (1983).

**The Stochastic Optimized Submovement Model**

To improve upon the iterative correction model, Meyer et al. (1988) developed the stochastic optimized submovement model. After demonstrating the limitations of the intermittent feedback model, Meyer et al. described some basic assumptions for the new model. First, there exists variability in movement, termed *noise*. Meyer et. al. assumed that in open loop control processes the endpoint variability is the result of noise in the neuromotor execution process. A second assumption is that the submovement endpoints are distributed normally. A normal distribution of endpoints would account for the participant missing the target both long and short when they are aiming for the center of the target. In Fitts (1954) experiment with a light stylus he found that endpoints are distributed about equally long and short of the target, thus creating a centrally located set of endpoints which are relatively normally distributed. Fitts did find that undershoots were more frequent with a heavier stylus however.

A third assumption is that there are either one or two submovements in spatially constrained tasks. This is a significant change from the intermittent feedback model in which multiple submovements are possible. They hypothesize that there is a primary initial adjustment that is aimed at the center of the target. If the initial adjustment will not land in the target zone due to neuromotor noise, there is a secondary submovement termed current control where the participant may use visual feedback in order to correct the trajectory in order to land in the target zone (Meyer et. al. 1988). The two component submovement model is supported by the fact that
most primary submovements travel over 90% of the distance to the target, thus leaving very little room for additional submovements (Langolf et. al. 1976).

The standard deviation for the primary submovement is proportional to the average movement velocity (Schmidt et. al. 1979). This is described by Meyer et. al. in the equation 

\[ S_1 = K \frac{D_1}{T_1} \]

where K is a positive constant and D_1 is the mean distance traveled by the primary submovement and T_1 is the mean duration of the primary submovement. In addition there is a second standard deviation of error associated with the second submovement and can be described by the equation 

\[ S_2 = K \frac{\Delta}{T_2} \]

where K is the same positive constant as before and Δ is distance from the center of the target to the end of the primary submovement. The participant must make the correction and travel distance Δ in a mean duration of T_2. “The desired S_2 is achieved by adjusting the mean duration (T_2) of the second submovements to have a value that depends on Δ, on the target width, and on the required accuracy level” (95%) (Meyer et. al, 1979).

The adjustment of T_2 is referred to as time minimization of submovements. Meyer et. al. hypothesize that submovements are optimized to cope with noise. The primary submovement and secondary submovement are assumed to have programmed average velocities in order to minimize the total movement time. They are determined by the particular participant’s neuromotor noise. Thus each person attempts to cope with noise by optimizing average movement velocity. There is an optimal first submovement duration and an optimal second submovement duration rather than a fixed proportion for both as described in the intermittent feedback model. Thus there is an optimal total movement duration for a given set of parameters. The Crossman and Goodeve model also has fixed proportions for submovement distance in addition to movement time. The stochastic optimized model does not have constant values for
submovement durations. Instead the participant adjusts the duration of submovement one and two to control neuromotor noise in order to be accurate and fast. There is a compromise between submovement one and submovement two. In order to minimize total movement time, the duration of submovement one cannot be too short or the standard deviation, $S_1$, will be too large, resulting in a longer movement time for the second submovement in order to make the larger correction. Submovement one cannot be too long either. Though a longer first submovement will decrease the movement time of submovement two because there is less error to correct for, there will still be an overall increase in the total movement time. Therefore, primary submovements durations cannot be too long or too short and secondary submovements cannot be too frequent or infrequent (Meyer et. al. 1988). There is an optimal duration for each submovement and a compromise between the two submovements dependent on factors such as target width, distance, and neuromotor noise.

Meyer et. al. also make an assumption that there is preparatory processing for the first submovement based on the targets perceived location and size. The second movement in the model utilizes feedback, but the first is viewed as a preprogrammed movement. It is assumed that the participant makes adjustments and there is no stop period where the participant is deciding or correcting. If the first submovement is preprogrammed based on the width and distance, the first submovement could get the participant relatively close to the target. This would explain the Wallace and Newell finding that participants still obey Fitts’ law when concurrent vision is removed. The first glance at the target may give the subject enough information to get relatively close. The Meyer et. al. model would still allow for Fitts’ law to be followed in no vision tasks and allow for a two submovement model as well whereas the Crossman and Goodeve model falls short in this aspect.

The stochastic optimized correction model makes a few important changes to the intermittent feedback model. First, it recognizes that noise exists in the human neuromotor system. Second, it has normally distributed endpoints that allow for both long and short misses based on noise. Thirdly, it reduces the maximum number of submovements to two. The first submovement is seen as a large, preprogrammed movement of varying duration based on the specifics of the task and the second submovement is called current control or a homing in on the target possibly using visual feedback. These two submovements work together to minimize total movement duration and cope with neuromotor noise by having optimal submovement durations that are neither too long nor too short.

Meyer et. al. (1988) proposed equation for modeling Fitts’ Law

The stochastic optimized submovement model predicts movement time as

\[ T = a + b \sqrt{D/W} \]

for Fitts’ Law. \( T \) is the average total movement time and \( a \) and \( b \) are constants. The square root function of this model and the logarithmic function from the original Fitts’ equation both increase at decreasing rates and produce similar looking curves on a graph and thus should be comparable models for Fitts’ Law. The square root function, \( \sqrt{D/W} \), is highly correlated with the original Fitts’ logarithmic function, \( \log_2(2D/W) \). Fitts’ original equation is so well documented and robust that the need for a change has not been truly considered. Meyer et. al. do state that even in Fitts’ original data some of the conditions were more highly correlated with the square root function than the logarithmic function.
Meyer et al. (1988) theory on relationship of temporal and spatial speed accuracy tradeoffs

Meyer et al. state that the form of speed-accuracy tradeoff equation also depends on the task demands. There is a distinction made between spatially constrained tasks and temporally constrained tasks. Spatially constrained tasks require the participant hit a target accurately and temporally constrained tasks require the participant to produce an accurate movement duration. For temporally constrained tasks, a linear relationship fits better than a logarithmic relationship (Schmidt et. al. 1979). Meyer et. al. then claim that by varying emphasis on spatial or temporal accuracy, the participants will shift from producing one speed accuracy tradeoff to the other. This reasoning allows for the idea that the two tradeoffs may have some relation. The two tradeoffs may be part of the same speed accuracy control process. Meyer et. al. also believe that the square root function should be able to describe other speed accuracy tradeoffs such as Schmidt’s law. If both Fitts’ law and Schmidt’s law can be described by the square root function they could be considered part of the same speed accuracy tradeoff.

Zelaznik et. al. (1988)

A study done by Zelaznik et. al. (1988) partially supported the Meyer et.al. theory that changing the emphasis of temporal or spatial accuracy changes which relationship will be observed. In the experiment, participants performed a Schmidt’s Law task with varied temporal constraints ranging from plus or minus 10% to plus or minus 40% of the movement time goal. By relaxing the temporal constraint, the strength of the linear speed accuracy tradeoff should decrease. Zelaznik et. al. (1988) did find that at more relaxed temporal constraints the strength of the linear speed accuracy tradeoff relation was weaker. They suggested that the temporal constraints determine the nature of the speed accuracy tradeoff (Zelaznik et. al., 1988). When the temporal constraints were relaxed, the participants shifted to Fitts’ Law behavior as the strength
of the linear speed accuracy tradeoff was weaker. Because the linear speed accuracy tradeoff relationship deteriorated at relaxed temporal constraints, there could be a common set of principles for the logarithmic speed accuracy tradeoff and the linear speed accuracy tradeoff according to Zelaznik et. al. (1988).

**Carlton (1994)**

Zelaznik et. al. (1988) suggest that there could be a common set of principles for the two speed accuracy tradeoffs and Meyer et. al. reasoned that changing the emphasis on spatial or temporal accuracy will cause participants to shift from one speed accuracy tradeoff to the other. They postulated that this is shift along a common control process for a single speed accuracy tradeoff. Thus, the Meyer et. al. theory leads to the inference that Schmidt’s Law and Fitts’ Law, are related.

Carlton (1994) attempted to understand the relationship between the two laws. Carlton had participants perform one task similar to the Schmidt task where participants moved 12 inches in 400 ms to a target. Carlton computed the effective target width from the Schmidt task and used it to create a Fitts task. He hypothesized that if the two tasks had the same control processes, the movement time of the Fitts task would be 400 ms. However, movement time was not the same, thus contradicting the idea that both tasks share the same control process. Carlton also showed that effective target width for the Fitts task and the Schmidt task were not the same. Carlton’s work supported a one submovement model (one movement with no corrections) for the temporal accuracy task and a two submovement model for the time minimization task. He concluded that the tasks were indeed distinct from one another and shared no common processing.
Carlton (1994) was the first to analyze differences between movement time and accuracy between these two tasks, but a more recent work by Hsieh et. al. (2016) expands upon this work by examining whether or not the speed-accuracy tradeoff properties are the same independent of the task when the spatial and temporal conditions are the same. Like Carlton (1994) they used the effective target width from a Schmidt task as the target width in a Fitts task. Then they also used the movement time from a Fitts task as the movement time in a Schmidt task. If the effective target width taken from a Schmidt task is placed as the target width in a Fitts task, the tasks should have the same movement time if they are related. Similarly, the movement time taken from a Fitts task with a particular target width should produce the same effective target width in a Schmidt task of the same movement time if they are related.

Hsieh et. al. (2016) used a discrete single aimed movement of 20 cm to the target. Participants started with a Fitts task on the first day with a one centimeter target width. From that Fitts task, the average movement time was taken to create a Schmidt task for the next day. The same participant performed a Schmidt task based on the average movement time from the first Fitts task. The effective target width from the day two Schmidt task was taken to create a target width for a new Fitts task for day three. The same participant performed the new Fitts task based on the day two Schmidt task effective target width and a new average movement time was taken to create a Schmidt task for day four. The same participant then performed the new Schmidt task based on the day three Fitts task.

The results of the experiment were contradictory to Carlton’s (1994) work. Under the same spatial and temporal constraints, the Fitts tasks and Schmidt tasks had similar accuracy and movement time variability. This result would support the idea that both tasks share a similar
control process. Hsieh et. al. (2016) claim that Carlton (1994) has the limitation that the temporal constraints were not controlled equally for both tasks. However, in the Hsieh et. al. (2016) results, the average Fitts movement time between day one and day three were not the same. The effective target width in the Schmidt tasks was not the same between day two and day four. Both Fitts tasks should have the same movement time because the target width and distance should be the same. The Schmidt tasks should also have the same effective target width because the movement times should be the same. Hsieh et. al. (2016) explain this difference through the practice effect, stating that the participant improved in two days and that accounts for the differences between both Fitts tasks. After several trials of practice participants are usually considered proficient at the task and likely won’t improve too much in a single session. There were only 100 trials of practice at the Schmidt task in between the last day one Fitts task and the first day three Fitts task. It seems unlikely that improvements were made in such a small time. While improvement due practice over the course of many sessions could explain a difference such as seen in the Hsieh et. al. (2016) study, it is unlikely that differences are due to such a small amount of practice. It is possible that when creating a Schmidt task from a Fitts task, the properties will be the same, but when creating a Fitts task from a Schmidt task, the properties are different. This would explain both results from Carlton (1994) and Hsieh et. al. (2016). It does not however provide any conclusive evidence that these tasks are related or not related. This experiment analyzes this possible relationship in greater detail and across a wider range of movement times and distance/target width parameters.
PRELIMINARY WORK

Unpublished Zelaznik Study

The basis for the current experiment is an unpublished experiment by Zelaznik (2016, unpublished). This experiment further examined Carlton’s claim that the tasks are governed by different control processes and the Hsieh et. al. claim that they are governed by a central control process. The study utilized three Schmidt’s Law repetitive tasks with a 25 cm distance and metronome period of 325, 400, and 475 ms. The study also utilized a Fitts’ Law task with two levels of distance; 12.5 cm and 25 cm, as well as two levels of target width; 1.00 cm and .50 cm.

To compare the Fitts task and a Schmidt task on the same scale, an effective target width versus average velocity plot was made for each task (See Figure 3). This makes the Fitts task resemble a linear speed accuracy task on the graph. The Fitts task and the Schmidt task are plotted using linear functions. They are highly correlated at .99 for the Schmidt task and .88 for the Fitts task. The observation that there were two distinct linear functions indicates that the two tasks are unrelated. If there are two separate functions, there are separate control processes as Carlton hypothesizes.

However, an argument could be made for a single curvilinear function. Examining Figure 3 shows the points are arranged in a way that a curvilinear function might fit the data better. If there is a single function, then the Schmidt task and Fitts task lie on the same curve and could be part of the same control process. An underlying control process for both tasks is supported by the Hsieh et. al. work.

Furthermore, on the graph there is only limited overlap between the Fitts task and the Schmidt task in the average velocity values. The lack of overlap between the tasks makes it difficult to assess whether the graph is one function or two functions. In order to make
distinctions between the two tasks there needs to be greater overlap in the average velocity range by using faster Fitts’ Law tasks and slower Schmidt’s Law tasks.

**Possible functions for the speed-accuracy tradeoff**

From the current graph, it is possible that there is a single curvilinear function, rather than two separate linear functions. Hancock and Newell (1985) claim that the Schmidt et al. (1979) data would be better represented with a curvilinear function than a linear function. They also argue that even though Fitts’ law is robust, it does not account for a complete range of IDs (Hancock and Newell, 1985). Assuming that Schmidt’s Law could be better represented by a curvilinear function, if Fitts’ Law is related to Schmidt’s law, on the graph below (See Figure 3), Hancock and Newells’ claim would be represented by a single curvilinear function for both Schmidt and Fitts tasks. However, this claim, along with the claim that the relationships are distinct as shown by the linear fitted lines to each task on the graph, needs to be examined in greater detail. Because it is quite possible that there is one function, I provide a more in depth examination. The simplest way to do so was to create a greater overlap in the average velocity range of the two tasks by increasing the speed of the Fitts’ Law tasks and decreasing the speed of the Schmidt’s Law tasks. The Fitts tasks are not fast enough and the Schmidt tasks are not slow enough in Zelaznik (2016, unpublished). I hypothesize that the results will support two separate functions.

Secondly, because the Meyer et. al. (1988) model is the most accepted model for describing Fitts’ law behavior, I analyzed whether the square root function equation predicts this behavior as well as or better than Fitts’ original logarithmic function. The square root function is simple and comparable with the logarithmic equation without adding additional variables. I hypothesize that the square root function will perform as well as the original logarithmic function
for modeling Fitts’ Law.

Figure 3. Effective target width vs average velocity for three Schmidt tasks and four Fitts tasks from Zelaznik (2016, unpublished).
METHOD

Participants

Participants were twenty-nine students at Purdue University. Participants did not have prior knowledge of the hypotheses being tested. The participants had no known neurological impairments and had normal or corrected to normal vision. They were paid $15 for their participation.

Apparatus

The lab contained a 79 cm high table for the participant to sit at. The participant sat at a standard classroom desktop chair with a foam pad to rest their arm on. The participant used a 2 mm wooden mechanical pencil with 2H hardness graphite. The task paper was 8 ½ by 11 in and was in a plastic sheet protector, in a file holder. The Fitts task papers had targets of three distances apart; 6.25 cm, 12.50 cm, and 25.00 cm. The Fitts task used three widths; 1.00 cm, 0.50 cm, 0.25 cm. The targets were unfilled rectangles 10 cm from the lower edge of the paper. The index of difficulty ranged from 3.64 to 7.64 bits for the Fitts task. The Schmidt task papers had the same target distance as the Fitts task papers; 25.00 cm, 12.50 cm, and 6.25 cm. The Schmidt task had four metronome periods; 325, 400, 475, and 550 ms. The targets were two filled .25 cm squares. The trajectory of the receiver attached to the pencil was captured by a Liberty Polhemus motion capture system in order to analyze movement time and accuracy.

Fitts task

In the nine Fitts’ law tasks there are two equally important goals for the participant; to move as fast as possible and not to miss the target. The distance between targets and the target width was manipulated. In this task the participant starts with the pencil upright inside the rightmost target and when the system beeps he or she begins to lift the pencil and move it to the
opposite target and back to the starting target repetitively for a duration of 25 seconds, at which time a tone sounds to signal the end of the trial. The motion capture system records the location of the receiver attached to the pencil 240 times per second and transmits it to Matlab. There were five trials for each condition.

**Schmidt task**

In the 12 Schmidt’s law tasks the participant moves in a repetitive motion from one target to the other starting with the pencil on the rightmost target. The targets were very small black squares (.25 cm$^2$) instead of unfilled squares like in the Fitts task. The goal of this task is to stay “on time” with the metronome. The participant hears beeps at a rate specified by the metronome and moves from one target to the other coincident with each individual beep of the metronome. The metronome engaged for 16 beeps at which time it disengaged, but the participant continues to move between targets as though the metronome were still going for 25 more movements. After an additional 25 movements a series of short beeps signals the end of the trial. There were five trials for each condition.

**Procedure**

The experiment began by first showing the participant to the standard desktop chair and having them remove any jewelry or metal that could interfere with the motion capture system. After the participant was comfortably seated, the tasks, starting with the Fitts task were described. After the experiment was explained and the participant gave informed consent, the experiment began. During the experiment odd numbered participants performed the Fitts task first and even numbered participants performed the Schmidt task first.

There were 21 conditions; nine Fitts and 12 Schmidt. There were five trials for each condition, thus a total of 105 trials. The experiment lasted approximately 80 minutes. In each
task the participant moves the pencil from one target to the other in a repetitive fashion until the trial ends. The participants were given a debriefing sheet describing the experiment and $15 for compensation for their time.
DATA ANALYSIS

Speed Accuracy Tradeoff Relationship

For the analysis, only trials three through five were analyzed as trials one and two were considered practice. In this experiment the type of task, either Fitts or Schmidt, is the explanatory variable, a binary variable of either zero or one. It will follow a linear regression model of

\[ Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \epsilon \]

\( Y = \) effective target width

\( X_1 = \) type of task (Fitts or Schmidt)

\( X_2 = \) average velocity

\( X_3 = X_1 \times X_2 \) interaction term of average velocity and type of task

For Schmidt (\( X_1 = 0 \)) The intercept is \( \beta_0 \) and the slope of \( X_2 \) (average velocity) is \( \beta_2 \).

\[ Y = \beta_0 + \beta_2 X_2 + \epsilon \]

For Fitts (\( X_1 = 1 \)) The intercept is \( \beta_0 + \beta_1 \) and the slope of \( X_2 \) (average velocity) is \( \beta_2 + \beta_3 \).

\[ Y = (\beta_0 + \beta_1) + (\beta_2 + \beta_3) X_2 + \epsilon \]

By using a binary variable for the type of task the two tasks can be plotted using the same \( X \) variable average velocity. The interaction term of average velocity and the type of task allows for analyzing whether the effective target width is dependent on the type of task or independent of the type of task. If the effective target width is independent of the type of task the tasks would produce the same effective target width at the same average velocities. If it is dependent on the type of task, the effective target width will be different at the same average velocities. The simplest way to do this is to conduct a same line test based on the linear regression model above.
A sameline test will show whether the effective target width is dependent on the type of task based on whether or not the slopes and intercepts of two linear functions are the same. The null hypothesis is that both lines have the same slope and intercept. Assuming that the lines are the same, the terms \( X_1 \) and \( X_3 \) are not necessary for the model because the regression lines are the same regardless of the type of task \( (X_1) \) and there would be no interaction between average velocity and type \( (X_3) \) either. To test this assumption set the slopes of those respective terms equal to zero. If the terms \( \beta_1 \) and \( \beta_3 \) are equal to zero then the Fitts regression line equation, 

\[
Y = (\beta_0 + \beta_1) + (\beta_2 + \beta_3)X_2 + \epsilon,
\]

becomes identical to the Schmidt’s regression line equation, 

\[
Y = \beta_0 + \beta_2X_2 + \epsilon.
\]

\( H_0: \beta_1 = \beta_3 = 0 \) the regression lines are the same \( (Y = \beta_0 + \beta_2X_2 + \epsilon) \)

\( H_1: \beta_1 \neq 0 \ or \ \beta_3 \neq 0 \) the regression lines are not the same \( (Y = \beta_0 + \beta_2X_2 + \epsilon \) and \( Y = (\beta_0 + \beta_1) + (\beta_2 + \beta_3)X_2 + \epsilon) \)

An F-test is done and a p-value of <.05 indicates a rejection of the null hypothesis and that the lines are not the same. A rejection of the null hypothesis indicates that the effective target width is dependent of the type of task at the same average velocities. This result would support the work of Carlton (1994) and that the two tasks operate under separate control processes. A failure to reject the null hypothesis supports the notion that the lines are the same, meaning the effective target width is the same for each task at the same average velocities. This result would support the work of Hsieh et. al (2016) and Hancock and Newell (1985) and that the two task are governed by a single control process.

**Model of Fitts’ Law Equation**

The second part of the data analysis is determining which model, Fitts’ original equation or the Meyer et. al. (1988) equation performs better at modeling Fitts’ Law. In order to determine
which is better I used the coefficient of determination, $R^2$, to see which model is a better fit. I also used the same line test like before to test whether the slopes are the same. The intercepts are clearly different, but the same slope would indicate that neither equation is better than the other at predicting spatial speed accuracy tradeoff tasks. The model was the same but the variables changed.

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \epsilon$$

$Y$= movement time

$X_1$ = type of index of difficulty (square root or log$_2$)

$X_2$= index of difficulty

$X_3 = X_1 * X_2$ interaction term of index of difficulty and type of index of difficulty

For Log$_2$ ($X_1=0$) The intercept is $\beta_0$ and the slope is $\beta_2$.

$$Y = \beta_0 + \beta_2 X_2 + \epsilon$$

For Square Root ($X_1=1$) The intercept is $\beta_0 + \beta_1$ and the slope is $\beta_2 + \beta_3$

$$Y = (\beta_0 + \beta_1) + (\beta_2 + \beta_3) X_2 + \epsilon$$

This will test whether the effect of the index of difficulty of the task on movement time is dependent on the type function used to model spatial speed accuracy tradeoff tasks. If movement time is dependent on the type of function, one model is likely better than the other, but if movement time is independent of the type of function used, then both models are adequate for describing Fitts’ law behavior. The null hypothesis is that both lines have the same slope. Setting $\beta_3$ equal to zero results in two regressions equation with the same slope, $\beta_2$, but different intercepts. The Log$_2$ equation remains the same, $Y = \beta_0 + \beta_2 X_2 + \epsilon$, but the square root equation becomes $Y = (\beta_0 + \beta_1) + (\beta_2) X_2 + \epsilon$. The equations have different intercepts but the same slope ($\beta_2$). To test this assume $\beta_3$ is equal to zero.
\( H_0: \beta_3 = 0 \) the regression line slopes are the same

\( H_a: \beta_3 \neq 0 \) the regression line slopes are not the same

An F-test is done like before for the same line test. This will result in a large F-value and the rejection of the hypothesis that the lines are the same. To analyze if the slopes are the same, a t-test is done for the interaction term \( X_3 \). If the p-value for the t-test is \(<.05\) it is considered a significant result and the null hypothesis is rejected. The slopes would not be the same in this case. If the p-value is \(>.05\) for the t-test it is considered an insignificant result and the null hypothesis is not rejected. This would indicate that the slopes are the same. Insignificant results also mean that the model would be improved without the interaction term because the slopes are the same.
RESULTS

Speed Accuracy Tradeoff Relationship

The graph below shows the effective target width versus average velocity graph across for the average across trials 3-5 for each participant. The unfilled triangles are the Fitts task and the unfilled circles are the Schmidt task.

Figure 4. Effective target width versus average velocity for all participants for all conditions
From looking at this graph there appear to be two separate lines, but in order to make a clearer distinction the average value of all the participants for each condition was taken. This will also give a plot with less possibility of outliers. The graph of the effective target width versus average velocity for the averages of all participants for each condition is below. Again, circles represent Schmidt tasks and triangles represent Fitts tasks.

![Graph](image)

Figure 5. *Effective target width versus average velocity for the average of all participants for each condition.* This graph gives a much clearer picture of what appears to be two separate linear functions. The $R^2$ values are relatively high which could indicate that these linear functions are adequate models for each task. The results of the same line test from the statistical software program SAS are below.
Table 1. Sameline F test for Fitts’ and Schmidt’s task.

Test sameline Results for Dependent Variable We

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F(2,605) = 276, p<.0001. There are two separate linear functions and effective target width is dependent on the task type, Fitts or Schmidt.

Model of Fitts’ Law Equation

In Figure 6 the movement time versus index of difficulty graph across all trials for all participants is depicted. This represents the average of trials 3-5 for each condition for each participant. The unfilled circles represent the square root function model and the unfilled triangles represent the logarithmic function model.

Figure 6 Movement time versus index of difficulty for square root model and logarithmic model.
From this graph there are clear linear functions that are highly correlated with $R^2$ values almost identical to each other at .71 and .71 for the square root function and the logarithmic function respectively. The correlation for both is also .84, relatively high. The lines appear to be almost parallel as well. To make statistical calculation easier and to make the graph clearer, the average of all participants was taken for each condition. This graph is below with the same markers, circles for square root and triangles for logarithmic.

This graph is much clearer than the previous graph and the correlation is much higher, indicating that the models are performing well. The coefficient of determination is .93 for the square root model and .93 for the logarithmic model. The correlation is .96 for both models, also relatively high. The slopes of the lines are slightly different at 141.7 and 136.63 respectively, but a same slope test can determine whether it is significant or not. To conduct the same slope test, first a same line test is done. The same line test gave results as expected.
Table 2. Sameline F test for Fitts’ and Meyer model of Fitts’ Law.

Test sameline Results for Dependent Variable MT

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F(2,518) =631, p<.0001. The two lines are not the same. It is clear from Figure 7 they have different intercepts. Second, the same slope test is conducted. The t-test for the same slope is below.

Table 3. t-test for Fitts’ and Meyer model of Fitts’ Law

Parameter Estimates

| Variable                        | DF | Parameter Estimate | Standard Error | t Value | Pr > |t| |
|---------------------------------|----|--------------------|----------------|---------|------|---|
| Intercept                       | 1  | -144.79285         | 22.75699       | -6.36   | <.0001|
| Index of Difficulty             | 1  | 150.41963          | 5.99073        | 25.11   | <.0001|
| √D/W or Log₂(2D/W)              | 1  | 150.71773          | 28.33617       | 5.32    | <.0001|
| Interaction                     | 1  | 2.83003            | 8.54300        | 0.33    | 0.7406|

This table gives estimates for the β values for the regression equation. In addition it shows a t-value for each β term. The t-value for the interaction term, β₃, for type of index of difficulty and index of difficulty was .33 which gave a p-value of .7406. This p-value is much greater than .05 and is therefore not significant. The slopes of the lines are the same.
DISCUSSION

In this experiment the relationship between two speed accuracy tradeoffs, Fitts’ Law and Schmidt’s Law was examined. From the results of the experiment the theory that the two tasks are governed by a common central control process was not supported. At low average velocities the effective target width is similar between the two tasks but the difference increases with increasing average velocity. It is clear that at the same average velocity, the speed accuracy tradeoff properties are different between the two tasks. This leads to the idea that these two speed accuracy tradeoffs are governed by separate control processes. This further supports the work of Carlton (1994). It is possible that since only one target distance was used in the Hsieh et. al. (2016) work it was difficult to see a difference in the two speed accuracy tradeoffs. The average velocity for the Hsieh et. al for all tasks was around 45-50cm/s. Looking at (Figure 5) there is a significant difference in the effective target width at those average velocities. If the average velocity was considerably low for that experiment, the effective target width might appear to be the same for both tasks. Unfortunately with only one set of target parameters it is difficult to claim that the two tasks are related. By using several varied conditions, this allowed me to create a graph of many indices of difficulty and make conclusions for the relationships as a whole, rather than an isolated condition like both Carlton (1994) and Hsieh et. al. (2016).

This experiment also improved upon the previous Zelaznik (2016, unpublished) work by creating a greater overlap in the average velocity values across the two task types. The statistical analysis resulted in a very clear rejection of the null hypothesis that the regression lines were equivalent. Effective target width in reciprocal tapping speed accuracy tradeoff tasks is dependent on the type of task. This result does not support the idea of shifting along a single control process like Hsieh et. al. (2016) suggest. With this result, Meyer et. al. (1988) is not
supported either because they hypothesized that changing the emphasis on spatial or temporal accuracy would create a shift from one relationship to the other along a single control process. Meyer et. al (1988) state that a separate set of principles for other speed accuracy tradeoffs are not necessary and the stochastic optimized submovement model is sufficient for both Fitts’ law and Schmidt’s law. If the Meyer et. al. (1988) principles are correct and their assumption that the square root function is the right model for describing Fitts’ law behavior, then the square root function could possibly describe other speed accuracy tradeoffs such as Schmidt’s law. The results found in this study do not support that notion. There is a clear difference between the two tasks that indicate that they are not related by a single control process. The result support two different control processes and therefore a different function to describe Schmidt’s law behavior. The Meyer et. al (1988) square root function is a worthy equation for describing Fitts’ law behavior but not Schmidt’s law.

The second result from the experiment supported the idea that both models for Fitts’ Law performed equally well. Neither model was better at predicting spatial speed accuracy tradeoff behavior than the other. Both had high and nearly identical coefficients of determination and slopes that were not statistically different. Unlike some other models for Fitts’ law, the Meyer et. al. square root function is fairly simple and does not complicate the equation with more variables. Both are acceptable models, and though they have limitations such as the logarithmic function not performing as well at low indices of difficulty and the square root function not performing as well at high indices of difficulty, they are both strong and robust equations that can be used to describe Fitts’ law behavior. The square root function performed well, just not better than the logarithmic function.
There is a limitation to the current study. Although greater overlap was created in the average velocities, the number of conditions between the two tasks was unequal. The Schmidt tasks had twelve conditions and the Fitts tasks had nine conditions. This made comparing conditions at each average velocity difficult and the graph shows three Schmidt task points that are no longer overlapped or near the Fitts task points. Although these may have been unnecessary to include, having three more Fitts conditions similar to the average velocities of those Schmidt condition would have been more convenient and provide an even clearer result.
CONCLUSION

This experiment provided a greater examination of the relationship between the two speed accuracy tradeoffs, Fitts’ Law and Schmidt’s Law. The experiment supports the theory that the two laws are unrelated and do not share a common central control process. The two tasks are likely governed by separate control processes and emphasizing spatial or temporal accuracy changes which control process is used. Lastly, the Meyer et. al. square root function is an adequate equation for describing spatial speed accuracy tradeoff behavior and though it will likely not replace Fitts’ original equation, neither is better than the other at describing Fitts’ Law.
APPENDIX

Fitts' Law Derivation from Crossman and Goodeve Model

\[ D = \text{Distance Remaining} \]
\[ p = \text{fixed proportion} \]
\[ N = \text{number of sub movements} \]
\[ t = \text{fixed duration of sub movement} \]
\[ T = \text{movement time} \]
\[ \frac{W}{2} = \text{error allowed for participant to be considered in the target zone} \]
\[ pD = \text{Distance traveled in 1st sub movement} \]
\[ D - pD = \text{Distance remaining after 1st sub movement} \]
\[ p(D - pD) = \text{distance traveled in 2nd sub movement} \]
\[ p(D(1 - p)^1) = \text{distance traveled in 2nd sub movement} \]
\[ (D - pD) - p(D(1 - p)^1) = \text{distance remaining after 2nd sub movement} \]
\[ D(1 - p) - p(D(1 - p)^1) = \text{distance remaining after 2nd sub movement} \]
\[ (1 - p)(D - pD) = \text{distance remaining after 2nd sub movement} \]
\[ D(1 - p)(1 - p) = \text{distance remaining after 2nd sub movement} \]
\[ D(1 - p)^2 = \text{distance remaining after 2nd sub movement} \]
\[ D(1 - p)^N = \text{distance remaining after } N\text{th sub movement} \]
\[ D(1 - p)^N \leq \frac{W}{2} = \text{The stopping Rule from Crossman and Goodeve (1983)} \]
\[ T = N \times t = \text{fixed duration } (t) \times \# \text{ of submovements } (N) = \text{Movement time} \]
\[ N = \frac{T}{t} \]
\[ D(1 - p)^{\frac{T}{t}} \leq \frac{W}{2} \]

\[ 2D(1 - p)^{\frac{T}{t}} \leq W \]

\[ \log_2 2D(1 - p)^{\frac{T}{t}} \leq \log_2 W \]

\[ \log_2 2D + \log_2 (1 - p)^{\frac{T}{t}} \leq \log_2 W \log_2 \frac{2D}{W} \]

\[ \log_2 W \log_2 \frac{2D}{W} \leq -\log_2 (1 - p)^{\frac{T}{t}} \]

\[ \log_2 \frac{2D}{W} \leq -\frac{T}{t} \log_2 (1 - p) \]

\[ \frac{\log_2 \frac{2D}{W}}{-t \log_2 (1 - p)} \leq T \]

\[ T \leq \frac{\log_2 \frac{2D}{W}}{-t \log_2 (1 - p)} \]

\[ b = \frac{1}{-t \log_2 (1 - p)} \]

*\(-t \log_2 (1 - p)\) can be considered a constant (b) under the Crossman and Goodeve Model because \((t)\) is considered the constant time for each submovement and \((p)\) is the fixed proportion of the distance remaining.

\[ T \leq b \log_2 \frac{2D}{W} \]

*Adding the constant \((a)\) to the derived inequality above resembles Fitts’ Original equation. \(a\) is assumed to be negative.

\[ T \leq a + b \log_2 \frac{2D}{W} \]

*The Crossman and Goodeve stopping rule is based on the idea that the endpoint of the movement lands somewhere in the target zone up to the center, but not beyond. This inequality resembles Fitts’ original equation \(T = a + b \log_2 \frac{2D}{W}\).
REFERENCES


