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ABSTRACT

The heat transfer process that takes place in the interface between the working fluid and the cylinder walls is one of the most relevant effect regarding reciprocating compressor performance. The fast transients and the limitations in terms of space to install transducers imply severe restrictions to the experimental analysis of this phenomenon. On the other hand, it is always desired to develop simple correlations to describe mathematically this heat transfer process, in order to permit its use in simulation programs without computational time penalties.

The main objective of this work is to evaluate existing correlations to the Nusselt number that represent the heat transfer on the above mentioned interface, including complex number formulations. The emphasis is to introduce a new correlation to substitute correlations currently used in compressor simulation programs. The proposed formulation is based on the difference between the gas mean temperature and the cylinder wall temperature, as usual, but it also considers the gas temperature derivative. A simulation program that solves the transient fluid flow and heat transfer inside the cylinder via Finite Volume Method, generated results to corroborate the hypothesis regarding the use of mean gas temperature and pressure and the correlation itself. The validation of the theory is achieved by comparing results of the simulation programs with experimental data, and the evaluation of compressor thermodynamic losses.

INTRODUCTION

Accurate information about heat transfer processes play a significant role in simulation and design of reciprocating machines. In particular, the instantaneous heat transfer between the in-cylinder gas and the solid walls is of crucial importance, due to its paramount influence on the thermodynamic performance of those equipments. This has been recognized since the thirties (Eichelberg, 1939) and a large number of experimental and theoretical studies have been performed in this field. In general, the main objective of all works is to develop simple correlations to predict the Nusselt number for the gas-cylinder interface as a function of global properties. This characteristic is of particular importance when developing compressor simulation programs, because of the requirements that arise from the balance between computational time and accuracy.

In Fagotti et al. (1994), some of the most important correlations proposed in the literature to determine the gas-to-wall heat transfer have been analyzed through a pragmatic approach, although not strictly precise. The correlations were implemented in a compressor simulation code, in order to determine which one best fitted the experimental data relative to a small hermetic reciprocating compressor for domestic appliances. Due to the intrinsic difficulties to install transducers with adequate time response to measure directly the in-cylinder process, only global compressor running characteristics were taken into account. In this aspect, the main restrictions are the narrow spaces available to install probes and the required fast-response time. In this scenario, one should reach a compromise between lack of precision in the measurements and uncertainties due to changes in the characteristics of the set up configuration relative to the real compressor. Both, enlarging the dimensions or diminishing the cycle period would require scaling factors, difficult to predict and to deal with. The correlations proposed by Annand (1963) and Adair et al. (1974) led to the best results, the later with a slightly lower precision.

Both Annand's and Adair's correlations are widely used in reciprocating compressors simulation. Like almost all the models developed up to now, they assume that heat transfer follows Newton's law. However it has long being observed (Pfriem, 1943) that during in-cylinder gas transformations due to reciprocating movement, the instantaneous heat transfer is out of phase with the difference between bulk gas and wall temperatures. Probably the simplification of no phase shift has been widely assumed to take benefit of dealing with simpler correlations and due to the experimental restrictions above mentioned. But assuming heat flux proportional to a temperature difference certainly leads to some degree of imprecision, whatsoever.

The phase lag evinces by a heat transfer during compression larger than during expansion, due to the interaction of gas compression and heat conduction in the boundary layer that evolves adjoining the cylinder wall. Actually, part of the heat transfer is in phase with the work done on the fluid, related to the time derivative of the temperature (*dT/dt*), and part is in phase with the temperature difference between the bulk gas and the cylinder wall (ΔT).

One should also take into account that all the works mentioned hereinafter deal only with the phases of the cycle where the valves are closed; suction and discharge processes are usually too complicated to be analyzed in a straightforward manner. Anyway, in absence of further information, usually the same correlation validated for compression and expansion are used for the whole cycle. This assumption lacks of absolute precision, since turbulence increases when valves are opened, which implies higher heat transfer rates. Another important characteristic assumed in all correlations is that wall temperature is constant throughout the cycle, which is corroborated by some measurements presented by Lawton (1987); this can be rationalized by the large difference between the thermal inertia of the gas and the cylinder.

This present work investigates the application in reciprocating compressors of a new correlation for the gas-to-wall Nusselt number proposed by Lawton (1987) for internal combustion engines. The hypothesis of bulk gas properties as well as the accuracy and consistency of the proposed correlation are assessed using a simulation program specially developed to this end. This program solves the transient and two-dimensional continuity, momentum and energy conservation equations for the in-cylinder gas using the Finite Volume Method (FVM) and is used as benchmarking since its formulation is one order superior to the proposed correlation. The use of a simulation program in this case avoids the need to validate the correlation using experimental data which is very difficult to obtain. Finally, the proposed correlation is applied to an existing reciprocating compressor and the obtained results are compared to experimental data. In spite some of the works related to engines deal with radiation effects, no attempt was made here to include this effect in the analysis, due to the small temperature differences observed in compressors compared to engines.

The main motivation for of this work is the substitution of Annand's and Adair's correlations commonly used in compressors simulation codes, with a new correlation that can account for the existing phase lag between heat flux and temperature difference, which actually occurs in reciprocating compressors.

PROPOSED CORRRELATION

Energy conservation for an ideal gas during expansion and compression requires that,

$$\rho c_v \frac{dT}{dt} = k\nabla^2 T - p\vec{\nabla}\cdot\vec{u} + \mu\Phi \tag{1}$$

where *dT/dt* is the substantial derivative of the instantaneous temperature, and Φ is the viscous dissipation function (Burmeister, 1983).

From the continuity equation, $\vec{\nabla}\cdot\vec{u} = (1/\rho)d\rho/dt$, and the perfect gas law, $p = \rho RT$, equation (1) can be written as

$$\frac{dT}{dt} = \frac{k}{\rho c_v}\nabla^2 T - \frac{(\gamma - 1)}{V}\frac{dV}{dt}T + \frac{\mu\Phi}{\rho c_v} \tag{2}$$

where $\gamma = cp/cv$ and V is the instantaneous cylinder volume. Neglecting the viscous dissipation, equation (2) can be written in dimensionless form as,

$$\frac{d\theta}{d\tau}=\frac{\gamma}{Re\,Pr}\overline{\nabla}^2\theta-\frac{(\gamma-1)}{V}\frac{dV}{d\tau}\theta \tag{3}$$

where,

$$\theta=T/Tw\ ,\ \tau=\overline{U}\,t/D\ ,\ Re=\rho\overline{U}\,D/\mu\ ,\ Pr=\mu cp/k \tag{4}$$

and $\overline{\nabla}^2$ is the laplacean operator made dimensionless through the cylinder bore D, and $\overline{U}$ is the mean piston speed.

The dimensionless temperature, θ, as given by equation (3) is function of six dimensionless quantities,

$$\theta=f_1\left(\tau,\frac{r}{D},\frac{z}{D},Re,\frac{Pr}{\gamma},\frac{\gamma-1}{V}\frac{dV}{d\tau}\right) \tag{5}$$

where r and z, are the radial and axial coordinates, respectively. From equation (5) the wall heat flux ($q_w=-k\partial T/\partial n|_w$, n is the normal to the wall), can be written as,

$$\frac{q_w D}{kT_w}=f_2\left(\tau,Re,\frac{Pr}{\gamma},\frac{\gamma-1}{V}\frac{dV}{d\tau}\right) \tag{6}$$

In equation (6) the compression and expansion work is associated to the quantity $[(\gamma-1)/V]dV/d\tau$. Usually a quasi-steady equation is sought for qw and the two non-steady groups appearing in equation (6) are neglected yielding,

$$\frac{q_w D}{kT_w}=f_3\left(Re,\frac{Pr}{\gamma}\right) \tag{7}$$

In the literature, the correlations based on equation (7) are usually obtained for a particular gas (Pr/γ constant). Furthermore the wall temperature is replaced by the gas-to-wall temperature difference resulting on expressions of the following form,

$$Nu=\frac{q_w D}{k(T_g-T_w)}=A\,Re^B \tag{8}$$

That is the case of Adair (1974) and Annand (1963) correlations, for example. In order to take into account the work done on the fluid by the piston reciprocating motion, equation (8) should be corrected to incorporate the omitted term, that is, $[(\gamma-1)/V]dV/d\tau$. Because equation (8) underestimates the heat flux during compression and overestimates it during expansion, a possibility for a new correlation is simply to add the omitted term to the right hand side of equation (8),

$$Nu=\frac{q_w D}{k(T_g-T_w)}=A\,Re^B+f_4\left(\frac{\gamma-1}{V}\frac{dV}{d\tau}\right)\frac{T_w}{T_g-T_w} \tag{9}$$

where the $Tw/(Tg-Tw)$ factor was included to comply with equation (6) since in equation (8) Tw was replaced by $(Tg-Tw)$.

At this point it should be noted that the dimensionless time, τ, introduced in equation (4) employed the piston speed and the cylinder bore to determine the characteristic time, $D/\overline{U}$. However, as recognized by Lawton (1987), there are two time constants for this problem. In addition to $D/\overline{U}$, which represents the speed of compression and expansion, the speed of diffusion, D^2/α (where α is a reference gas diffusivity), should also play an important role in the gas-to-wall heat transfer. It was then proposed by Lawton (1987) that the two characteristic times should be combined in a geometric mean to yield,

$$\tau = \frac{t}{t_o} \quad , \quad t_o^{1/2} = \left(\frac{D}{\overline{U}}\right)\left(\frac{D^2}{\alpha}\right) = \frac{D^3}{\alpha\overline{U}} \tag{10}$$

The resulting correlation according to equation (9) emerges as (Lawton, 1987),

$$Nu = \frac{q_w D}{k[T_g(t) - T_w]} = A\,Re(t)^B + CL(t)\frac{T_w}{T_g(t) - T_w} \tag{11}$$

where A, B and C are constants to be determined for each situation, and $L(t)$ is the compressibility number,

$$L(t) = \frac{\gamma - 1}{V(t)}\frac{dV(t)}{dt}\sqrt{\frac{D^3}{\alpha\overline{U}}} \tag{12}$$

It should be noted that the density and viscosity appearing in the Reynolds number should be evaluated instantaneously using the current value of pressure and temperature; to emphasize that the Reynolds number was written as a function of time. However, in evaluating $L(t)$, α is a fixed reference value of the gas diffusivity. In the present work the values of A, B and C appearing in equation (11) are determined theoretically as will be explained further.

For completeness, and to provide an alternative for comparison, a new correlation introduced by Kornhauser & Smith (1994) will also be investigated. Their correlation also consider the phase lag between heat flux and gas-to-wall temperature difference.

Kornhauser & Smith (1994) proposed a complex number approach as,

$$q_w'' = \frac{k}{Dh}\left\{Nu_r[T(t) - T_w] + \frac{Nu_i}{\omega}\frac{dT}{dt}\right\} \tag{13}$$

where Nur and Nui are the real and imaginary parts of the Nusselt number, respectively, evaluated by

$$Nu = Nu_r + iNu_i, \qquad Nu_r = APe_\omega^a, \qquad Nu_i = BPe_w^b \tag{14}$$

Parameter Pe is the Peclet number ($\omega D2/4\alpha$) and ω is the crankshaft angular velocity. Values of constants in both parts of the Nusselt number were obtained by best fit to a number of experimental data relative to a gas-spring apparatus, which plays the role of a valveless cylinder. It was observed by Kornhauser & Smith (1994) that the smaller the Peclet number, the lower the influence of the imaginary part in the global heat transfer, which means heat flux in phase with temperature difference. The range suggested for neglecting the imaginary part of the Nusselt number was $Pe<10$. On the other hand, at high Peclet number ($Pe>100$) the real and imaginary parts tend to assume equal values, which leads to a phase lag of $\pi/4$. For the class of compressors in consideration here, instantaneous Peclet number ranges typically from 10 to 10000, yielding expectations that phase lag is important.

NUMERICAL SOLUTION AND VALIDATION

The complete problem formulation is presented elsewhere (Catto and Prata, 1997) and will be just outlined here for brevity. In resume, the fluid mechanics problem is treated as two-dimensional, unsteady and compressible. Pressure, velocity and temperature fields throughout the cylinder where obtained by the simultaneous solution of the continuity, the Navier-Stokes and the energy equations. The governing differential equations are integrated over each control volume throughout the computational domain, yielding a set of linear equations which is solved iteractively. Staggered volumes and a fully implicit formulation were employed. As the piston reciprocates, the mesh is based on a moving coordinate system. Gas-to-wall heat flux is evaluated by the gas temperature derivative normal to the wall; this value is used to evaluate the heat transfer coefficient. Since the properties are calculated locally throughout the domain, one can easily evaluate the bulk properties. Several cycles of the crankshaft were required until convergence is established. The simulations were performed for a gas-spring configuration and the conclusions were generalized for an actual compressor.

Heat flux obtained using the numerical procedure were used to adjust the constants of both Lawton's and Kornhauser's correlations. The values obtained for Lawton's approach were $A=0.28$, $B=0.65$ and $C=0.25$; for Kornhauser's model the values were $A=6.6$, $a=0.28$, $B=6.45$, $b=0.088$.

RESULTS AND DISCUSSION

The model presented in the previous section was used to simulate a small reciprocating hermetic compressor operating wit R134a in ASHRAE check-point condition. Figure (1) depicts the gas-to wall heat flux and figure (2) the in-cylinder gas bulk temperature; both explore the influence of different correlations on the compressor performance evaluation.

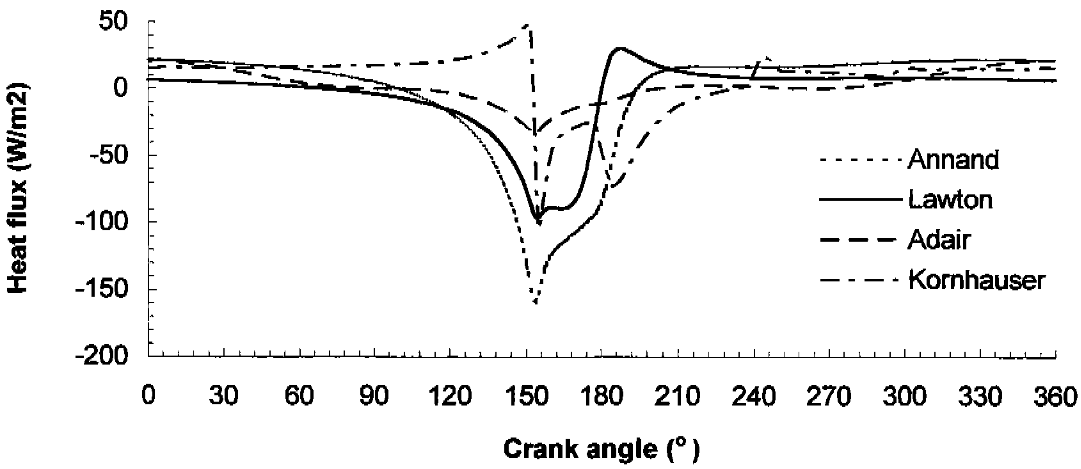

Figure (1) - Heat flux in the gas-cylinder interface

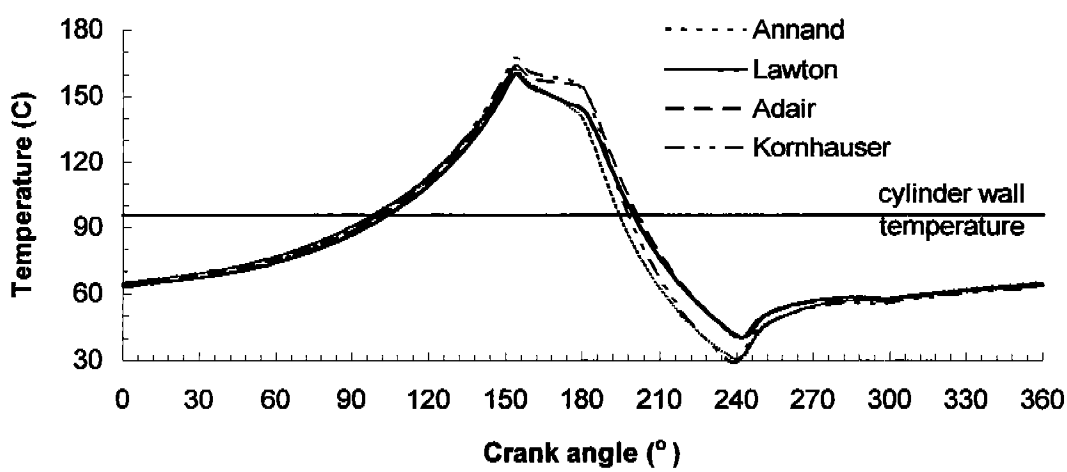

Figure (2) - Bulk gas and cylinder wall temperatures

No comparison has been made regarding the heat transfer coefficient, since the phase lag between heat flux and temperature for Lawton an Kornhauser models leads to infinite values in two points. Lawton's model results are more physically consistent. The heat flux has no sharp edge, like Kornhauser's, and one should expect a smooth curve, considering the diffusivity characteristic of the mains phenomena. It also result in a positive heat flux at the end of expansion, which is also expected and no other model present. This superiority is also obvious from the comparison presented at table (1).

Table (1) - Overall results of the compressor simulation program compared to experimental data (deviation, %)

model	Adair	Annand	Lawton	Kornhauser
refrigerating capacity	+2.6	+1.5	+1.7	+3.4
EER	-0.7	-2.0	-0.5	-1.1

CONCLUSIONS

The instantaneous heat flux at the surface of the cylinder was successfully correlated to the gas-to-wall temperature difference and the time derivative of the averaged instantaneous gas temperature, using an approach derived from the energy equation without neglecting the compression and expansion work as is usually the case in the literature. The resulting correlation was then fitted to results for a valveless piston-cylinder assembly obtained from the numerical solution of the mass, momentum and energy differential equations. The following correlation was obtained,

$$Nu = \frac{q_w D}{k[T_g(t) - T_w]} = 0.28\, Re(t)^{0.65} + 0.25 L(t) \frac{T_w}{T_g(t) - T_w} \tag{15}$$

where $L(t)$ is the compressibility factor defined in equation (12). The proposed correlation where incorporated in a compressor simulation code and very good agreement prevailed between computation and experimental results. It was found that the major part of the thermodynamic losses in reciprocating compressors are due to real gas behavior, although some amount is consequence of the hystheresis in the heat transfer throughout the cycle.

One should be aware that the values of the constants presented are valid only for the compressor model under study; different compressor designs lead to different values of the constants. In this work a numerical simulation program has been used to evaluate them, in spite of an experimental approach could also be used.

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