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Evaluation of Fin Efficiency and Heat Transfer Coefficient for Fined Tube Heat Exchange

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Evaluation of Fin Efficiency and Heat Transfer Coefficient of Heat Exchanger Having Plate Fins

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ABSTRACT

This study discussed the estimation of the fin efficiency and the pure-heat transfer coefficient in the heat exchanger. One hundred twenty cases of plate fins having known heat transfer coefficients were tested numerically to investigate the validity of the previous classical theory on the fin efficiency. The conventional theory on the fin efficiency was only useful when the value of NTU_f was near zero. However, it was not useful at high NTU_f and low fin efficiency in the heat exchanger. A new definition of fin efficiency and a model for pure-heat transfer coefficient are suggested, which are applicable to the heat exchanger. The present model reduced error greatly than the classical model in the estimation of the pure-heat transfer coefficient at $0 < mL < 2$, $0 < NTU_f < 2.5$.

1. INTRODUCTION

The purpose of the fin is to increase the product of the surface area and the heat transfer coefficient. [Webb (1994)] It is very useful in the heat exchanger design or in the estimation of heat exchanger performance if we know the fin efficiency. Mills (1995) and many textbooks introduced the fin efficiency derived from the following three assumptions: (a) constant fluid temperature, (b) uniform heat transfer coefficient, and (c) one dimensional heat conduction in the fin. However most actual heat exchangers may not satisfy only one of these three assumptions.
A lot of experiments have been performed to measure the heat transfer coefficient of the heat exchanger having fins. Beecher and Fagan (1987), Ali and Ramadhyani (1992) tested their heat exchanger for nearly uniform surface temperature condition. Ito et al. (1977) applied the constant heat flux condition. They directly measured the pure-heat transfer coefficient (h) since the fin efficiency (η) could be assumed as 100%. Goldstein and Sparrow (1976) used the naphthalene sublimation method to get heat transfer coefficient by using the heat and mass transfer analogy. Hatada et al. (1989), and Kang and Kim (1999) tested the actual heat exchangers in a wind tunnel. They measured basically the parameter of the pure-heat transfer coefficient multiplied by the fin efficiency, i.e. ηh. We need clear information for the fin efficiency in the calculation of the heat transfer coefficient.
The present work examined the classical fin efficiency theory in cases where assumptions for the theory were not met. A new definition of fin efficiency and a model to predict the pure-heat transfer coefficient were proposed and compared with the results of numerical simulation.

2. THEORITICAL FIN EFFICIENCY

Figure 1 shows the heat exchanger having a large number of plate fins with constant cross-sectional area. The definition of the theoretical fin efficiency is as below.

$$\eta_{th} = \frac{Q}{Q_{\max}} = \frac{\text{Actual heat transfer}}{\text{Maxium possible heat transfer}} \tag{1}$$

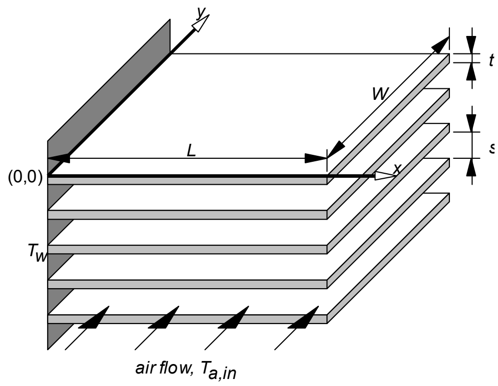


Figure 1: Heat exchanger having plate fins.

If the boundary conditions are $T_f = T_w$ @ $x = 0$, $\partial T_f / \partial x = 0$ @ $x = L$ and under the three assumptions (a) constant fluid temperature, (b) uniform heat transfer coefficient, and (c) one dimensional heat conduction in the fin, the theoretical fin efficiency for the case of Figure 1 is

$$\eta_{th} = \frac{\tanh mL}{mL} \tag{2}$$

The parameter *mL* is $(hA/kA_cL)^{1/2}L$ where *L, h, A, k,* and A_c are length of fin, pure-heat transfer coefficient, fin surface area and thermal conductivity of fin and cross-sectional area of fin respectively. [Mills (1995)]

3. NUMERICAL SIMULATION

A simplified numerical simulation was conducted to test the validity of the three basic assumptions and to find the fin efficiency for predicting the performance of the heat exchanger. The geometry of the numerical simulation was as shown in Figure 1 in which the fluid flows parallel to the base wall. The heat transfer between fin and fluid is similar to the cross flow heat exchanger. Table 1 shows the dimension and test conditions in the present numerical experiment. This analysis contains: (a) the pure-heat transfer coefficient is known; (b) the pure-heat transfer coefficients are uniform or non-uniform; the non-uniform heat transfer coefficient changes as a function of $y^{-0.5}$ to simulate the laminar boundary layer; (c) the fluid is thermally un-mixed and heat transfer along *x* direction in the fluid is negligible; (d) heat conduction in the fin is two dimensional; (e) properties are constant. The energy equations for the fin and fluid are as below:

$$\frac{\partial^2 T_f}{\partial x^2} + \frac{\partial^2 T_f}{\partial y^2} - 2\frac{h}{kt}\left(T_f - T_a\right) = 0, \tag{3}$$

$$\frac{\partial T_a}{\partial y} = \frac{2h}{\rho s V c_p}\left(T_f - T_a\right), \tag{4}$$

where ρ, *V*, *s*, c_p, and *t* are fluid density, fluid velocity between fins, fin spacing, heat capacity of fluid and fin thickness. The *h* is the known and given pure-heat transfer coefficient to verify the fin efficiency model. The boundary conditions of this calculation are:

$$T_f = T_w = 1 \text{ @ } x = 0,\ T_a = T_{a,in} \text{ @ } y = 0,\ \partial T_f/\partial x = 0 \text{ @ } x = L,\ \partial T_f/\partial y = 0 \text{ @ } y = 0 \text{ and } y = W. \tag{5}$$

The above equations were solved by the finite difference method. The SIMPLE algorithm proposed by Patanker (1980) was used. The grid was non-uniform 60 x 60 in *x* and *y* coordinates. The conversion criteria were that the sum of the residuals is was less than 10^{-6} and the energy difference between fin and fluid was less than 0.01%.

Table 1: Dimensions and test conditions of the heat exchangers in the present work.

Parameter	Value
Fin Length, L	5.0 mm
Fin Width, W	5.0 mm
Fin Thickness, t	0.03, 0.05, 0.085, 0.1$^{(1)}$ mm
Space between Fins, s	1.0 mm
Thermal conductivity of fin, k	40, 80, 160, 200$^{(1)}$ W/m·K
Uniform Heat Transfer Coefficient, h	100 W/m^2·K
Local Heat Transfer Coefficient, h_y	$h_y = 3.54 y^{-0.5}$, $\bar{h}_y = h = 100$ W/m^2·K
Fluid Density, ρ	1.0 kg/m^3
Fluid Velocity, V	0.4, 0.6, 1.0(1), 10.0 m/s
Heat Capacity of Fluid, c_p	1000 J/kg·K
Fin Base Temperature, T_w	1°C
Fluid Inlet Temperature, $T_{a,in}$	0°C

(1) denotes the reference condition.

4. RESULTS AND DISCUSSION

4.1. Average Fin Temperature

Figure 2 shows the fin temperature distribution under the condition of $\mathrm{NTU}_f = 0.1$ and uniform heat transfer coefficient, which satisfies the all assumptions of the classical fin efficiency. Here the number of heat transfer units of the fin NTU_f is the number of heat transfer units for the fin:

$$\mathrm{NTU}_f = Ah / \dot{m}_a c_p \tag{6}$$

where A and $\dot{m}_a c_p$ are the fin surface area and heat capacity of fluid side contacted with fin. The fin temperature shows one-dimensional distribution and the theoretical fin efficiency was the same as the $Q/Q_{\max}$.

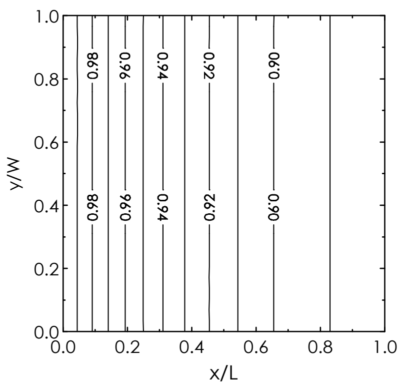
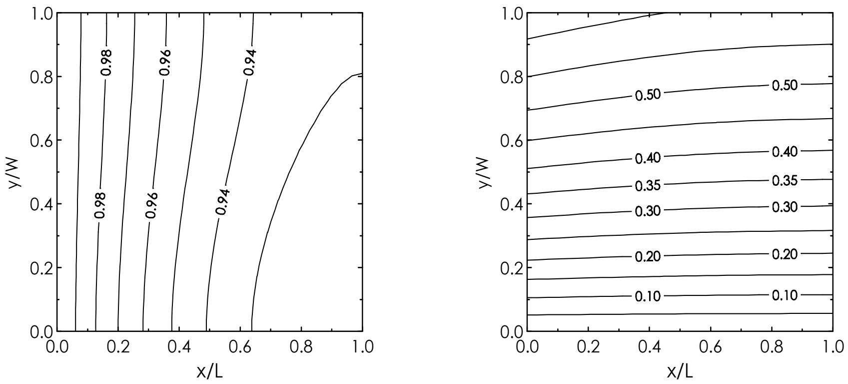


Figure 2: Fin temperature distribution of external flow condition, mL= 0.5, NTU_f= 0.1, ($\eta_{th} = T_f^* = 0.924$).



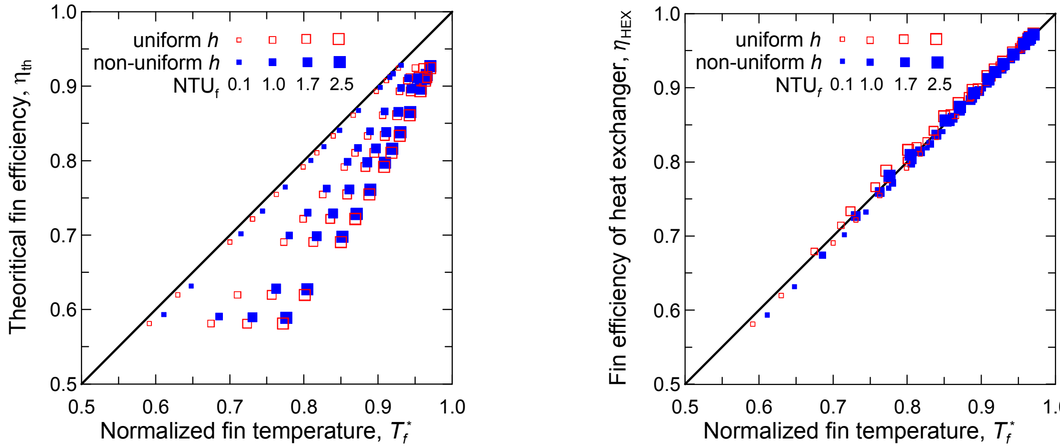
(a) Fin temperature (b) Fluid temperature

Figure 3: Fin and fluid temperature distribution in the reference condition on Table 1, $mL = 0.5$, NTU$_f$=1, uniform heat transfer coefficient (η_{th} = 0.924, $\varepsilon/\varepsilon_{fa}$ = 0.948, T_f^* = 0.952).

Figure 3 shows the fin and fluid temperature distributions for the reference condition of Table 1 at $\mathrm{NTU}_f = 1.0$, $mL = 0.5$ and non-uniform heat transfer coefficient. The temperature distributions are two-dimensional. The non-dimensional fin temperature, T_f^*, is defined as follow.

$$T_f^* = \frac{\dfrac{\int T_f dA}{A} - T_{a,in}}{T_w - T_{a,in}} = \frac{T_f - T_{a,in}}{T_w - T_{a,in}} \tag{7}$$

The theoretical fin efficiencies, η_{th} by equation (2) were 0.924 in the both cases of Figure 2 and Figure 3. The two values, η_{th} and T_f^*, must be the same in order that the classical theory is available in the heat exchanger. Those were the same at NTU$_f$=0 shown in Figure 2. However the theoretical fin efficiency was lower than the non-dimensional fin temperature at NTU$_f$=1 as shown in Figure 3.
Figure 4 (a) shows the comparison of η_{th} and T_f^* in the heat exchanger the three assumptions are not valid. The two values agree well at low and the difference increases as the NTU$_f$ increases. Therefore the theoretical fin efficiency is applicable in the actual heat exchanger only when NTU$_f$→0.



(a) Theoretical fin efficiency, η_{th} (b) Fin efficiency of heat exchanger, η_{HEX}

Figure 4: Comparison of the fin efficiencies and the normalized fin temperature for the uniform and non-uniform heat transfer coefficients.

4.2. Fin Efficiency Model for Heat Exchanger

The possible heat transfer ($Q_{k\to\infty}$) through the fin would be maximized when the thermal conductivity of the fin is infinite ($k \to \infty$) in the heat exchanger. The average fin temperature T_f is close to the wall temperature T_w as increase of the thermal conductivity. The maximum thermal effectiveness is independent of the fin configurations such as the maximum effectiveness of heat exchanger for fluid flow types, i.e. parallel, counter and cross flows. The actual and maximum thermal effectivenesses in the heat exchanger between wall and fluid and the ratio are as below:

$$\varepsilon = Q/Q_{\max} = \frac{T_{a,ex} - T_{a,in}}{T_w - T_{a,in}} \tag{8}$$

$$\varepsilon_{k\to\infty} = Q_{k\to\infty}/Q_{\max,\mathrm{fin}} = \frac{T_{a,ex} - T_{a,in}}{T_f - T_{a,in}} = 1 - e^{-\mathrm{NTU}_f} \tag{9}$$

$$\frac{\varepsilon}{\varepsilon_{k\to\infty}} = \frac{T_f - T_{a,in}}{T_w - T_{a,in}} = \frac{\varepsilon}{1 - e^{-\mathrm{NTU}_f}} = T_f^* \tag{10}$$

The fin efficiency for the heat exchanger is defined as the ratio of the effectivenesses in the present work. This means as:

$$\eta_{HEX} = \frac{\text{Actual heat transfer}}{\text{Heat transfer when the thermal conductivity of the fin is infinite}}. \tag{11}$$

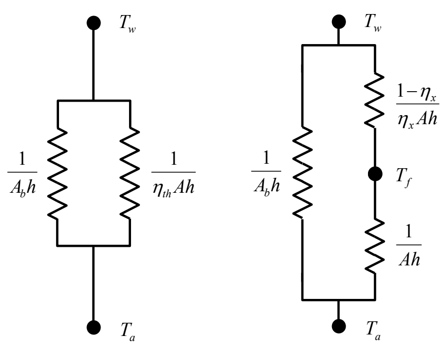
Figure 4 (b) shows the comparison of the fin efficiency and the non-dimensional fin temperature for the various conditions. These two data agree well. The standard deviations are 0.5% and 0.9% for the cases of uniform and non-uniform heat transfer coefficient respectively. It is concluded that the non-dimensional fin temperature in equation (7) is nearly the same as the fin efficiency of the heat exchanger as defined in equation (11)

4.3. Estimation of Heat Transfer Coefficient in the Heat Exchanger

In the evaluation of the pure-heat transfer coefficient h, we often use the measured values the heat transfer rate Q, wall temperature T_w, inlet and exit temperatures $T_{a,in}$ and $T_{a,ex}$ from the experiment. The thermal resistance models for the heat transfer from the fin to fluid are shown in Figure 5. The classical model [Mills (1995)] of Figure 5 (a) expressed that the heat transfers through the base and fin surfaces as a parallel circuit:

$$Q = Q_b + Q_f = (A_b + \eta_{th} A) h \Delta T_{\ln,w} \tag{12}$$

$$\Delta T_{\ln,w} = \frac{T_{a,ex} - T_{a,in}}{\ln \dfrac{T_w - T_{a,in}}{T_w - T_{a,ex}}} \tag{13}$$



(a) Previous model (b) Present model

Figure 5: Thermal resistance models for the heat exchanger.

where $\Delta T_{\ln,w}$ is the mean temperature difference between the wall and fluid temperatures. The total thermal resistance is $1/\left(A_b + \eta_{th} A\right)h$.

Figure 6 (a) shows the comparison of errors in the prediction of the pure heat transfer coefficient h by using the classical model in equations (12-13). The h_{true} and h_{cal} are the true value (given pure-heat transfer coefficient in the present work) and the calculated value by using the model respectively. The results show that the classical model could underestimate the heat transfer coefficient up to 25% in the present test range. The product of the number of heat transfer unit of fin and fin efficiency parameter $\text{NTU}_f\, mL$ related on the deviation from the ideal conditions in the previous classical fin efficiency theory.

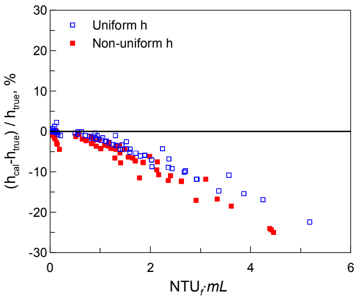
The present work modified the classical model as shown in Figure 5 (b). The thermal resistance related the fin in the previous classical model is divided into two resistances: the conduction resistance between wall and fin and the convection resistance between fin and fluid. The total heat transfer is:

$$Q = Q_b + Q_f = hA_b \Delta T_{\ln,w} + hA\Delta T_{\ln,f} \tag{14}$$

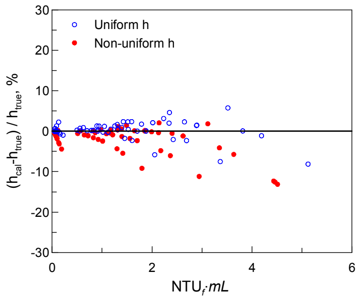
$$\Delta T_{\ln,f} = \frac{T_{a,ex} - T_{a,in}}{\ln \dfrac{T_f - T_{a,in}}{T_f - T_{a,ex}}} \tag{15}$$

where $\Delta T_{\ln,f}$ is the mean temperature difference between the average fin temperature and fluid temperatures. The heat transfer relation between wall and the fin is as below:

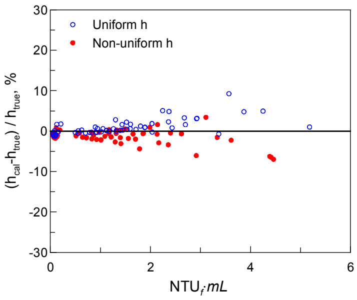
$$Q_f = \frac{\eta_x A h}{1-\eta_x}\left(T_w - T_f\right) \tag{16}$$



(a) Classical model



(b) Present model



(c) Present model with correction coefficients

Figure 6: Comparison of errors in the prediction of the pure-heat transfer coefficient according to the models and the uniform and non-uniform heat transfer coefficients. (a) Classical model, (b) Present model using coefficients c_1=1 and c_2=0 in the modified fin efficiency η_x, (c) Present model using coefficients c_1=1.05 and c_2=0.008 in the modified fin efficiency η_x

where η_x is the modified fin efficiency. In this work, the theoretical fin efficiency is modified to reduce the errors as below:

$$\eta_x = \frac{\tanh\left(mL^{c1}\mathrm{NTU}_f^{c2}\right)}{mL^{c1}\mathrm{NTU}_f^{c2}} \tag{17}$$

$$c_1 = 1.05, c_2 = 0.008 \tag{18}$$

$$0 \le mL < 2\,,\ 0 \le \mathrm{NTU}_f < 2.5 \tag{19}$$

Figure 6 (b) and Figure 6 (c) show a comparison of errors in the prediction of the heat transfer coefficient by using the present model. Substituting the theoretical fin efficiency for the modified fin efficiency, $\eta_x = \eta_{th}$, for c_1=1 and c_2=0, reducing the error to half that of the previous classical model, as shown in Figure 6(a) and Figure 6 (b). Figure 6 (c) compares the errors of the present model using the equations (17-19) for the modified fin efficiency. The present model with correction coefficient predicted the pure heat transfer coefficient well; the standard deviations were 1.70 and 1.65% respectively in the 120 uniform and non-uniform cases. The error in the prediction increased as the value NTU_f mL increased. The exponent c_1 for mL and c_2 for NTU_f related with the additional heat transfer effects by two dimensional heat conduction and by the fluid temperature respectively. The exponent c_1 and c_2 are more effective for the case of non-uniform heat transfer efficient as shown in Figure 6 (b) and (c)

Figure 7 shows the procedure to obtain the pure heat transfer coefficient from the experimental or numerical data. The heat transfer rate and geometric data such as $Q, \dot{m}c_p, T_w, T_{a,in}, T_{a,ex}, A, A_b$ and are obtained from the experiment. Assuming the value for the average fin temperature T_f, the logarithmic mean temperature differences $\Delta T_{\ln,w}$ and $\Delta T_{\ln,f}$ are calculated from equations (13) and (15). The heat transfer coefficient h and heat transfer rate through fin Q_f are calculated from equation (14). The average fin temperature T_f in equation (16) can be calculated by using the modified fin efficiency η_x in equation (17). Iterative calculations update the fin temperature and converge on the solution.

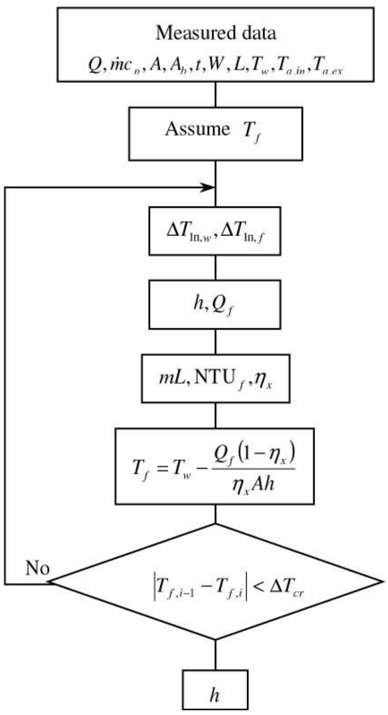


Figure 7: Procedure to obtain the pure heat transfer coefficient for the heat exchanger.

The present work suggests a new definition of fin efficiency and a new method of predicting the pure-heat transfer coefficient in the actual heat exchanger. The present model reasonably agrees with the present numerical experiment, that is, the simplest heat exchanger. The author recommends that we need more precise consideration and study to extend this theory to heat exchangers generally.

5. CONCLUDING REMARKS

This study was performed to investigate the validity of the fin efficiency estimation and the evaluation method of the pure-heat transfer coefficient for the plate-fin heat exchanger. A numerical experiment was conducted on a simple heat exchanger having constant cross-sectional area. The 120 cases that the fluid flowed across the fin were tested in the range of $0 < mL < 2$, $0 < \mathrm{NTU}_f < 2.5$. Conclusions are as follow.

- The previous classical model on the fin efficiency was the same as the non-dimensional average fin temperature only when the value of NTU_f was near zero.
- The fin efficiency in the actual heat exchanger is proposed as the ratio of the real heat transfer to the maximum heat transfer of the fin as the thermal conductivity of fin approaches infinite. The fin efficiency was nearly the same as the non-dimensional fin temperature normalized by the inlet and wall temperatures.
- A model was suggested for evaluating the pure-heat transfer coefficient in the heat exchanger. The error in the present model was reduced to about a quarter of that in the classical model; however, the error increased as the product of NTU_f and mL increased.

More detailed consideration and testing will be needed to extend these to general heat exchangers.

NOMENCLATURE

A	fin surface area	(m^2)
A_b	base surface area, m^2	
c_p	heat capacity of fluid	(J/kg·K)
h	average pure heat transfer coefficient	($\mathrm{W/m^2 \cdot K}$)
h_y	local heat transfer coefficient	($\mathrm{W/m^2 \cdot K}$)
k	thermal conductivity of fin	(W/m·K)
L	fin length	(m)
m	parameter in theoretical fin efficiency	(1/m)
$\dot{m}_a$	mass flow rate of fluid	(kg/s)
NTU_f	number of heat transfer unit for fin	
Q	total heat transfer rate	(W)
Q_b	heat transfer rate from base surface	(W)
Q_f	heat transfer rate through fin	(W)
$Q_{\max}$	maximum heat transfer rate from wall to fluid	(W)
$Q_{\max,\mathrm{fin}}$	maximum heat transfer rate from fin to fluid	(W)
t	fin thickness	(m)
T_a	fluid temperature	(K)
$T_{a,ex}$	outlet fluid temperature	(K)
$T_{a,in}$	inlet fluid temperature	(K)
T_f	fin temperature	(K)
T_f^*	fin temperature normalized by wall and inlet fluid temperatures	
T_w	base wall temperature	(K)
$\Delta T_{\ln,w}$	logarithmic mean temperature difference between wall and fluid	(K)
$\Delta T_{\ln,f}$	logarithmic mean temperature difference	

Greek symbols

ε	thermal effectiveness
$\varepsilon_{k\to\infty}$	thermal effectiveness when thermal conductivity of fin is infinite
η_{HEX}	fin efficiency of heat exchanger
η_{th}	theoretical fin efficiency
η_x	modified fin efficiency

	between fin and fluid	(K)
V	fluid velocity between fins	(m/s)
W	fin width	(m)
x	coordinate in fin length direction	(m)
y	coordinate in fluid flow direction	(m)

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