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EVALUATION AND DEVELOPMENT OF
SYNTHETIC TRIP DISTRIBUTION MODELING
TECHNIQUES AS APPLIED
TO SMALL URBAN AREAS

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Interim Report

EVALUATION AND DEVELOPMENT OF SYNTHETIC TRIP DISTRIBUTION
MODELING TECHNIQUES AS APPLIED TO SMALL URBAN AREAS

TO: Harold L. Michael, Director
Joint Highway Research Project

July 11, 1979

FROM: Kumares C. Sinha, Research Engineer
Joint Highway Research Project

File: 3-7-12

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Attached is the second Interim Report on the HPR Part I Study titled "Use of Synthetic Demand Modelling Techniques in Transportation Planning for Small Urban Areas in Indiana". The title of this report is "Evaluation and Development of Synthetic Trip Distribution Modelling Techniques as Applied to Small Urban Areas". The research has been conducted by Mr. James R. Mekemson, Graduate Instructor in Research of our staff under the direction of Professor K. C. Sinha, Research Engineer.

The Report here is concerned with the evaluation of the available synthetic modelling approaches to trip distribution and the development of a recommended approach which has been tested and evaluated.

This report is forwarded for review, comment and acceptance by the ISHC and FHWA as partial fulfillment of the objectives of the research.

Respectfully submitted,

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MODELLING TECHNIQUES AS APPLIED TO SMALL URBAN AREAS

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16. Abstract As a part of a detailed study of synthetic travel demand modelling techniques that can be applied to small urban areas (population 50,000 to 250,000), this study concentrated solely on the trip distribution phase of the modelling process. The results of this research effort revealed the great variability that exists among the small urban areas. Due to this variability it was concluded that the most commonly suggested and used synthetic trip distribution modelling techniques of borrowing travel time impedance values (friction factors) from similar urban areas or the use of standardized calibration parameters cannot provide consistent and reliable results for all study areas. A trip distribution gravity model therefore was developed which possesses the capability to calibrate itself without the need for home interview survey data, while simultaneously retaining the accuracy of the traditionally calibrated model. The accuracy of the proposed procedure is due to its synthetic calibration at the origin zone level and its use of the origin zone opportunity travel time upon which the model is based. This methodology allows the calibration procedure of the model to be more responsive to the highway network and the spatial distribution of the land use activities.			
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HIGHLIGHT SUMMARY

A major detrimental aspect of the traditional urban transportation planning modelling process, especially for the smaller urban areas, has been its voracity for large amounts of data. The collection and processing of data for urban travel demand modelling is both costly and time consuming, resulting in the final transportation plan being completed years after the initial data collection and often without adequate opportunity for alternative testing. In order to possibly remedy this situation of high costs and slow turnaround time, various synthetic modelling techniques have been suggested and tested in recent years with mixed results. As a part of a detailed study of synthetic travel demand modelling techniques that can be applied to small urban areas (population 50,000 to 250,000) in Indiana, the present study concentrated solely on the trip distribution phase of the modelling process.

The most commonly used trip distribution model is the gravity model in which calibration of the model primarily involves the establishment of the appropriate travel time impedance function. The present study evaluated the performance of some of the various suggested synthetic gravity model calibration techniques and developed an improved procedure for synthetic self-calibration of the gravity model. The primary purpose of this study was to eliminate the need for the home-interview survey

from which calibration data is normally obtained and to do so without sacrificing accuracy of the results.

Data from a wide range of small urban areas were used in determining variability of different characteristics among the small urban areas in order to evaluate the possible application of borrowed travel time impedance factors, or their associated mathematical functions, in urban areas with similar characteristics. Additionally, the complete travel demand modelling data from three small urban areas of Indiana were used as test cases to which various synthetic trip distribution modelling techniques were applied and evaluated. The data from these three urban areas also provided a data source for the development of the recommended synthetic self-calibrating trip distribution model.

The results of this research effort revealed the great variability among small urban areas as well as their individual uniqueness. Due to this variability and uniqueness, it was concluded that the most commonly suggested and used synthetic trip distribution modelling technique of borrowing travel time impedance values from similar urban areas cannot provide consistent and reliable results. The same conclusion was also reached for the technique of using standardized travel time impedance values. In addition, the present study also indicated that small urban areas may differ greatly in terms of their sensitivity to travel time impedance value errors when measured at the traffic assignment level. That is, one small urban area's models may produce acceptable results within a wide travel time impedance value range while another small urban area's models may provide acceptable

results only within a narrow band of travel time impedance values. These findings indicated the need for an improved synthetic trip distribution procedure that would provide consistent and reliable results.

The development of the recommended synthetic self-calibrating gravity model is based primarily on the origin zone opportunity mean travel time, that is, the origin zone mean travel time that would occur if the travel time impedance values were constant. Origin zone opportunity mean travel times are themselves closely related to the spatial location of the zones and the land use activities within the area. These origin zone opportunity mean travel times are then used to estimate the actual origin zone mean travel times and the origin zone travel time impedance function parameter. It is felt that such a model is more responsive to the location of activities than the traditionally calibrated gravity model.

Application of the recommended model to three different urban areas proved successful in generating acceptable traffic assignments when compared to the Origin-Destination survey assignment. For one test case, total volume group percent root-mean-square-error was actually reduced from 67.8 percent to 47.7 percent when compared to the traditionally calibrated gravity model traffic assignment. The recommended model thus appears to be capable of significantly decreasing the time and effort presently expended in trip distribution modelling.

CHAPTER I

INTRODUCTION

Urban areas approaching the 50,000 population level face the formidable task of initiating a comprehensive transportation planning process. Other urban areas, having already completed the initial major transportation planning process, need to continuously update plans as the urban environment and the needs of the community change. Both the initiating and the updating of a comprehensive plan require considerable resources of time and money. Furthermore, the emphasis in urban transportation planning is currently changing from traditional long-range system planning to short-range, quick response improvement programs with the aim of optimizing the utility of the existing facilities (7). The traditional travel forecasting process must be modified if it is to remain a relevant and coordinated phase of both the short-range and long-range planning process.

Consequently, it is necessary to make efforts toward simplifying planning techniques such that savings can be achieved in both the costs and time involved in planning. Additionally, simplified planning techniques would allow greater attention to be given to the more immediate needs of transportation system management planning, where it is imperative that quick response time be inherent to the techniques (16).

In this regard, a recent National Cooperative Highway Research Project (NCHRP) study (11) investigated the ability of various existing

travel estimation procedures to provide information within a quick response framework for analyzing urban transportation policy issues.

The following statement in regards to the results of this investigation was made in the summary of the report:

No existing travel estimation technique was found adequate to respond to all the many questions raised. Because of the widespread use and understanding of the conventional four-step (trip generation, distribution, mode split, and traffic assignment) travel simulation procedure, it was recommended that this procedure continue to form at least a partial basis on which the transportation planning process responds to new needs in the future.

The author of the report further recommended that a set of capabilities be developed for simplified computerized methods for the conventional four-step procedure in order to provide efficient and quick response to transportation planning issues at the regional and subarea level.

Purdue University and the Planning Division of the Indiana State Highway Commission (ISHC) recognized the need for simplified procedures, and therefore, in early 1977, organized a research project to evaluate the use of synthetic demand modelling techniques for transportation planning in the small urban areas of Indiana. The Planning Division of ISHC was very aware of the past resources expended in the transportation planning process for many of the urbanized areas in Indiana and therefore realized that the results derived from such a project could be very beneficial.

The scope of the overall research project included the evaluation of the three transportation modelling steps of trip generation, trip distribution, and traffic assignment. Because the results of this research were to be applicable to small urban areas of population size

50,000 to around 250,000 persons, it was felt that transit planning could be modelled separately, thereby eliminating mode split from the study. This same conclusion was expressed by Grecco, et. al. (6) in a study conducted for the NCHRP.

In the first phase of the research project, conducted by Mahmassani (10), synthetic trip generation techniques as applicable to the small urban areas of Indiana were evaluated. Mahmassani's research indicated that vehicle trip rates per household, as used in the cross-classification technique, were sufficiently stable to be applicable to any of the urbanized areas within Indiana. The cross-classification technique is well documented in the Federal Highway Administration's (FHWA) publication Trip Generation Analysis (17).

Origin-Destination (O-D) surveys of the past had been the cornerstones from which trip generation models were originally developed. The transferrable trip rates per household have eliminated the need for the O-D survey for trip generation. However, O-D surveys had provided other information critical to the transportation modelling process.

Travel time frequency distributions developed from O-D survey trip tables and highway network skim trees (travel time matrix) were used in the calibration of the trip distribution models of the past. Elimination of the O-D survey would remove the information used in trip distribution calibration. Other means of calibrating the model would have to be used in the procedure.

Trip distribution, using synthetic methods, is still in its early development and testing stage. The second phase of the synthetic modelling project therefore focusses on trip distribution, specifically for small urban areas (population size range from 50,000 to about 250,000 persons). The overall objective of the present study was to reduce the quantity of time, money, and manpower consumed in trip distribution modelling without adversely affecting the accuracy of the results.

This research project examined those small urban areas of Indiana where full scale transportation studies have already been conducted, in an effort to make more efficient use of the information already made available by these studies. Data from outside of Indiana was also used so as to increase the sample size. In addition, the complete transportation modelling data sets from three urban areas, Lafayette, Anderson, and Muncie, were used for detailed analysis and testing.

The work plan for this study consisted of five major tasks:

1. Summarize the travel time statistics of the Indiana transportation study areas, and if possible, relate these statistics to other urban area characteristics. Determine if these statistics will be of any use in other urban areas in which the transportation planning process is to be implemented or updated.
2. Investigate the sensitivity of the gravity model with respect to friction factor errors introduced by the use of borrowed or standardized friction factors.

3. Investigate the possibility of using areawide travel time statistics and other variables in a self-calibrating gravity model.
4. Investigate the possibility of using origin zone specific travel time statistics in a synthetically self-calibrating gravity model.
5. Evaluate and make recommendations as to what method of synthetic trip distribution modelling should be used in the transportation planning process for small urban areas.

CHAPTER II

TRAVEL TIME CHARACTERISTICS

An urban area's travel time characteristics reflect the nature of the activity system of the urban area as represented by the land use spatial distribution and the socio-economic background of the area. On the basis of a given activity system and an associated transportation system, decisions are made by individual trip makers as to how often, at what time, to what place, by what mode, and by which route a trip is to be made. The resulting cumulative travel pattern is generally quantified by trip purpose in terms of the mean travel time, travel time variance, and by the corresponding travel time frequency distribution.

Literature Review

Relationships between travel time characteristics and the size and structure of urban areas have been the subject of several studies with the general conclusion that mean travel times are positively related to population and the developed land area. The more prominent research efforts in this subject were performed by Chan (2) and Voorhees et.al.(19)

Voorhees, et. al. extensively studied travel time (trip length) characteristics for work trips, shopping trips, socio-recreational trips, non-home based trips, and truck trips from origin-destination data collected from 34 transportation study areas. Study area

population ranged from a minimum of 33,000 to a maximum of 6,489,000 persons.

Regression analysis was used by Voorhees et. al. to test the hypothesis that trip length (trip duration) increases with increasing population size. Equations 1 and 2 show the results of this analysis for the home based work (HBW) and non-home based (NHB) trips, respectively.

$$\begin{aligned} \text{Home Based Work} \quad \bar{t} &= 0.98 \text{ POP}^{0.19} \\ R^2 &= 0.71 \end{aligned} \tag{1}$$

$$\begin{aligned} \text{Non-Home Based} \quad \bar{t} &= 0.63 \text{ POP}^{0.20} \\ R^2 &= 0.67 \end{aligned} \tag{2}$$

where

$\bar{t}$ = mean travel times in minutes

POP = urban area population.

Voorhees et.al. also compared travel time variance with population. A linear relationship involving travel time variance as a function of population was determined. However the sample size was small and within a narrow population range, and therefore the statistical strength of the relationship was severely limited.

The main objective of Chan's research was to synthesize the demand-forecasting experiences of the past and to come up with an estimation procedure which would allow the determination of passenger travel on an urbanized-area-level.

Chan analyzed the mean travel time for the home based work trips with data from 86 cities with population ranging from 55,000 to 4,042,000.

He classified the urban areas by population and structural form. For those urban areas with a population between 50,000 and 800,000, Equations 3 and 4 were developed by regression analysis for a core-concentrated urban structure and a multinucleated urban structure classification.

$$\begin{array}{ll} \text{Core-concentrated} & \bar{t} = 6.715 + 0.017 \text{ POP} \\ & R^2 = 0.48 \end{array} \quad (3)$$

$$\begin{array}{ll} \text{Multinucleated} & \bar{t} = -3.748 + 2.134 \ln \text{ POP} \\ & R^2 = 0.83 \end{array} \quad (4)$$

where

$\bar{t}$ = average travel time in minutes

POP = population size in thousands

$\ln$ = natural log function

Summary of Travel Time Statistics for the Study Areas in Indiana

In Table 1 are given the location, base year population, and average travel time and variance by trip purpose for each of the study areas. This Table clearly demonstrates the wide range over which the areawide travel time statistics can vary. The minimum and maximum average travel times for HBW trips, excluding Indianapolis, are 8.69 and 19.71 minutes. The minimum and maximum average travel times for the HBO trips, excluding Indianapolis, are 7.25 and 16.28 minutes. Average travel times are 7.37 and 17.00 minutes for the minimum and maximum values for NHB trips, excluding Indianapolis. Minimum and maximum travel time variances for the HBW, HBO and NHB trips, excluding

Table 1. Indiana Study Areas' Travel Time Statistics by Trip Purpose (in minutes)

City	Base Year Population	HBW		HBO		NHB	
		Average	Variance	Average	Variance	Average	Variance
Columbus	38000	9.34	13.88	8.33	11.60	8.84	11.30
Lafayette	79000	10.57	20.54	8.56	16.08	9.12	12.56
Anderson	89000	11.44	9.09	8.53	10.04	9.50	6.51
Terre Haute	90000	19.71	42.70	16.28	40.48	17.00	50.43
Muncie	100000	8.95	10.53	7.39*	9.27*	6.84	9.83
Evansville	176000	13.30	35.44	9.90*	25.98*	10.84	23.86
Fort Wayne	204000	8.69	10.60	7.25*	11.55*	7.37	8.71
South Bend	242000	15.27	41.79	10.99*	38.67*	12.92	41.22
Indianapolis	745000	20.19	80.99	11.06*	48.76*	13.00	69.16**

* Weighted combination of various home based trips.

**Trucks not included.

Indianapolis, are 9.09 and 42.7 minutes, 9.27 and 40.48 minutes, and 6.51 and 50.43 minutes, respectively.

Evaluation of Travel Time Data for Small Urban Areas

The emphasis of the present research project was on small urban areas. For this reason, data from Voorhees' study and Chan's study were combined into a single data set along with the information collected for this current study for only those urban areas with a population between 38,000 and 400,000 persons. For the sake of uniformity this combined data set included only home based work trips.

Figure 1 is a plot of work trip mean travel times versus population size. It can be clearly seen that trip duration is positively correlated with population, but there is also a wide dispersion of the mean travel times at any population level. It is interesting to note that the data points for seven out of the eight Indiana urban areas considered fall within a relatively narrow band. It is even more noteworthy that one of the Indiana urban areas, Terre Haute, is very much an outlier. Linear regression analysis for the combined home based work data set resulted in Equation 5. The regression equation is also plotted on Figure 1. An equation was also fitted to the seven Indiana urban areas that appear to fall approximately within a narrow band and is given in Equation 6 for the purpose of comparison study.

Figure 1. Home Based Work Average Travel Time Versus Urban Area Population Size

$$\text{Combined Data} \quad \bar{t}=8.41+0.017\text{POP} \quad R^2=0.23 \quad (5)$$

$$\text{Seven Indiana Areas} \quad \bar{t}=8.61+0.018\text{POP} \quad R^2=0.33 \quad (6)$$

where

$\bar{t}$ = average travel time in minutes

POP = population size in thousands

The most important aspect of Figure 1 and the regression equations is not the positive correlation of trip duration with population size, but the large spread of the data points with respect to the regression line. Considering the above seven Indiana urban areas only and their regression equation, average travel time percentage error ranged from 0.03 percent to 42.81 percent, with the average percentage error being 14.20 percent. This average value is much greater than FHWA's suggested ± 3.0 percent limit (13). Prediction of trip duration for home based work trips based on population size alone would be unreliable due to this wide variance in the data.

Scattergrams and regression analysis were also performed using the data collected in the present study for home based other (HBO) and non-home based (NHB) trips. Positive correlation between mean travel time and population size was observed, but a very wide dispersion of the data points was present. The large spread of the data observed severely limits the usefulness of any relationship developed for trip duration prediction as was shown for the above HBW trip data.

Comparison of Travel Time Frequency Distributions

In Figures 2 and 3 are shown the travel time frequency distributions for the home based work trips for the eight Indiana study areas. The division of the eight areas into two groups is for reasons of clarity only. The Figures do illustrate how differently the travel time

Figure 2. Indiana Transportation Study Areas' Travel Time Frequency Distributions. Population Range: 38,000 to 100,000.

Figure 3. Indiana Transportation Study Areas' Travel Time Frequency Distribution. Population Range: 176,000 to 242,000.

characteristics can vary within a given population range, even though home based work (HBW) trips are themselves the least sensitive with respect to travel time impedances as compared to home based other (HBO) and non-home based (NHB) trips.

Population range for Figure 2 is from 38,000 persons (Columbus, Indiana) to 100,000 persons (Muncie, Indiana). In this range, Terre Haute has travel time characteristics much different from the other urban areas. The population range for Figure 3 is 176,000 (Evansville, Indiana) to 242,000 persons (South Bend, Indiana). It can be seen that the average travel time for Fort Wayne, Indiana is much lower than the other two urban areas.

A close examination of the travel time frequency distributions in Figures 2 and 3 indicated that the high average travel time for Terre Haute and the low average travel time for Fort Wayne might be explained by the magnitude of the zonal terminal times for these areas.

Chapter Conclusions

Travel time analysis conducted in this study revealed that the prediction of average travel time based on the population size of the urban area alone would be of little practical use. This conclusion was based on the fact that even though a positive correlation existed between average travel time and population size, the variance of the data about the regression line was substantial as measured by the low R-squared values. This results in a regression equation with a confidence interval too large to be of practical use, as was evidenced by the large average travel time percentage error observed for the Indiana study areas.

A comparison of home based work travel time frequency distributions for the Indiana study areas suggested that more careful selection of zonal terminal times might enable better comparisons to be made between the various study areas of Indiana. However, travel time frequency distributions can be compared only in terms of general shape. Even though two distributions may look similar, the average travel times can differ significantly. A difference of only one minute can easily result in large discrepancies in traffic assignment. Using travel time frequency distributions of one urban area to compare with the distributions generated from another urban area's synthetic trip distribution model is therefore of little benefit, unless it is previously known that the average travel time in the two areas are very close.

The above conclusions point out the important fact that travel time statistics of an urban area are a unique result of the area's characteristics and are therefore difficult to compare with, or use for prediction in another study area. Analyses conducted in Chapters III and IV further support this conclusion. This uniqueness between the urban areas is best exemplified by the descriptions and discussions of the spatial land use distributions and network configurations of the Lafayette, Anderson, and Muncie study areas given in Appendix A. At first appearance, these three urban areas of approximately the same population size are very similar. All three urban areas are divided by a river and have a college or university within their boundary. However, a close inspection reveals that significant differences do

exist in the land use pattern and the associated network configuration, and therefore the respective travel demand models respond differently to changes in the calibration parameters.

CHAPTER III

GRAVITY MODEL SENSITIVITY ANALYSIS

The gravity model has been the most widely used trip distribution technique mainly because it is simple in concept and the procedure is well documented. Mathematically, the gravity model is stated as follows in Equation 7:

$$T_{ij} = \frac{P_i A_j F_{ij} K_{ij}}{\sum_{j=1}^m A_j F_{ij} K_{ij}} \quad (7)$$

where:

T_{ij} = trips produced in zone i and attracted to zone j,

P_i = trips produced by zone i,

A_j = trips attracted by zone j, and

F_{ij} = friction factor which expresses the effect of spatial separation on trip interchanges.

K_{ij} = zone-to-zone adjustment factors. (These factors are ignored when synthetic techniques are used because origin-destination data is needed to determine these values.)

Friction factors can be represented by a set of values at one minute time increments or by an equation, such as Equation 8.

$$F(t) = Ct^\alpha e^{\beta t} \quad (8)$$

From equation 8, two alternative forms, Equations 9 and 10 can also be formulated:

$$F(t) = C t^{-\alpha} \quad (9)$$

and

$$F(t) = C e^{-\beta t} \quad (10)$$

where:

$F(t)$ = friction factor at time t ,

C = constant of proportionality (in gravity model operation, this constant is not needed due to the nature of the model form),

α = Alpha, calibration parameter,

β = Beta, calibration parameter, and

e = base of the natural logarithms.

Equations 8, 9 and 10 are generally referred to as the gamma function, inverse travel time function, and the negative exponential function, respectively. Although in most transportation studies a set of friction factor values has been used instead of mathematical equations, the parameters in the equations enable transportation planning researchers to relate model sensitivity with respect to various population sizes, urban structural forms, trip purposes, and travel time characteristics of various study areas.

The calibration of a gravity model basically involves repeated adjustments of a set of friction factors until the generated gravity model travel time frequency distribution replicates the O-D survey travel time distribution within a predetermined error limit. Allowable error limit is usually specified as a ± 3 percent error in the gravity

model mean travel time when compared to the O-D survey mean travel time (13). Other gravity model accuracy tests include screenline checks, topographical barrier checks and spiderweb network checks. The calibration procedure, however, requires that an O-D survey has been performed in order to conduct the accuracy tests and the calibration itself.

In order to eliminate the cost and time involved in conducting an O-D survey, researchers and transportation planners have occasionally borrowed friction factors from other urban areas possessing similar characteristics and then applying them to a new study area. The PLANPAC/BACKPAC General Information (14) manual suggests borrowing friction factors for travel simulation in small urban areas. However, not much work has been done to examine the sensitivity of a gravity model to any possible errors that might be introduced by the borrowing of friction factors from a similar urban area or by the use of standardized friction factors or other synthetic techniques. Because this is an important aspect associated with synthetic modelling of transportation demand, an analysis was performed in the present study to evaluate the performance of the gravity model utilizing various synthetic techniques.

The transportation study areas of Lafayette and Anderson were used for the purpose of this evaluation. Descriptions of these two study areas are given in Appendix A. The population sizes of these two study areas are approximately equal, but spatial activity distributions and network impedances are quite different from each other. These differences can possibly have a profound effect on trip distribution modelling.

The research discussed in this chapter consisted of four parts. First, tests were conducted to measure the sensitivity of the gravity model with respect to small errors in the friction factors. Simultaneously, these tests revealed the sensitivity of the gravity model with respect to the network characteristics and land use activity distributions of the individual study areas. Second, the variation of friction factors with respect to population size was investigated. This investigation involved the examination of the concept of similar urban areas from which friction factors may be borrowed. Third, an evaluation was performed using the standardized α parameters, developed by the Comsis Corporation (3), in the gravity models for the Lafayette and Anderson urban areas. Fourth, the friction factors of Lafayette and Anderson, Indiana were exchanged or borrowed from each other in order to test how borrowed friction factors may affect trip distribution modelling accuracies.

The Effect of Friction Factors

Before discussing the sensitivity of gravity models with respect to error introduced by synthetically determined or borrowed friction factors, it is important to examine whether or not friction factors are necessary in trip distribution modelling in small urban areas. This was accomplished by applying to the gravity model a set of friction factors of constant value at all time values. This results in a trip distribution in which relative travel times are not a determinant.

In order to have the friction factors equal a constant value, the β parameter of the negative exponential friction factor function, Equation 10 can be set to zero. This results in friction factors being equal to 1.0 at all time values. This analysis was conducted with the data from both the study areas of Lafayette and Anderson.

The resulting average travel times as generated by the gravity models with constant friction factors are shown in Tables 2 and 3 for the two urban areas considered. Also shown in the Tables are the O-D survey mean travel times and percentage error by trip purpose. For Lafayette the gravity model generated average travel time was 30 percent greater than the O-D survey value, while the generated average travel time for Anderson was 19 percent greater than the corresponding O-D survey value. This relative difference in average travel time percentage error of 11 percent between the two urban areas may be attributable to differing spatial land use distributions and network characteristics and how the individual gravity model responds to these differences.

The effect of a constant friction factor was further evaluated by comparing the travel time frequency distributions as determined by the O-D survey and as generated by the gravity model by trip purpose. Figures 4 through 9 compare the gravity model and O-D survey travel time frequency distributions for Lafayette and Anderson by trip purpose. As can be readily seen from the Figures, the travel time frequency distributions of the gravity model and O-D survey differ greatly, especially for the home based other and non-home based trips.

The next phase of the evaluation of the results of a gravity model with constant friction factors involved assignment of the model generated trips to the highway network and subsequent comparison of the assigned trips to those obtained by assigning the O-D survey trips to the same highway network. The traffic assignment was based on an all-or-nothing assignment technique where the trips between two zones

Table 2. Results of the Lafayette Gravity Model With Constant Friction Factor Values

Trip Purpose	<u>Average Travel Time</u>		Percent Difference
	Survey	Model	
HBW	10.57	12.32	+16.56
HBO	8.56	11.78	+37.62
NHB	9.12	11.42	+25.22
Total	9.05	11.76	+29.94

Table 3. Results of the Anderson Gravity Model With Constant Friction Factor Values

Trip Purpose	<u>Average Travel Time</u>		Percent Difference
	Survey	Model	
HBW	11.44	12.10	+ 5.77
HBO	10.04	12.30	+22.51
NHB	9.50	11.72	+15.58
Total	10.14	12.07	+19.03

Figure 4. Greater Lafayette Travel Time Frequency Comparison for Home Based Work Trips; Origin-Destination Survey Versus Gravity Model with Constant Friction Factor Values

Figure 5. Greater Lafayette Travel Time Frequency Comparison for Home Based Other Trips; Origin-Destination Survey Versus Gravity Model with Constant Friction Factor Values

Figure 6. Greater Lafayette Travel Time Frequency Comparison for Non-home Based Trips; Origin-Destination Survey Versus Gravity Model with Constant Friction Factor Values

Figure 7. Anderson Travel Time Frequency Comparison for Home Based Work Trips; Origin-Destination Survey Versus Gravity Model with Constant Friction Factor Values

Figure 8. Anderson Travel Time Frequency Comparison for Home Based Other Trips; Origin-Destination Survey Versus Gravity Model with Constant Friction Factor Values

Figure 9 . Anderson Travel Time Frequency Comparison for Non-Home Based Trips; Origin-Destination Survey Versus Gravity Model with Constant Friction Factor Values

are assigned to the minimum time path without regard to the capacities of the individual highway network links along the path. An all-or-nothing assignment was used in order to eliminate possible additional differences attributable to a capacity-restrained assignment alone and not to the gravity model.

The statistical test for comparing two different traffic assignments is based upon grouping the individual links into specific volume groups and computing various statistics within each volume group. The statistic most widely used is the Percent Root-Mean-Square-Error (PRMSE) as defined in Equation 11.

$$\text{PRMSE (by volume group)} = \frac{\sqrt{\frac{\sum (\text{test volume} - \text{base volume})^2}{N}}}{\frac{\sum (\text{base volume})}{N}} \times 100.0 \quad (11)$$

where:

N = number of links within the base volume group.

An equation which estimates what the expected PRMSE should be for a given volume and O-D survey dwelling unit sample rate was developed by A. Sosslau and G. Brokke (13). A comparison can then be made between the expected and observed values.

Volume group link comparisons are given in Tables 4 and 5 for Lafayette and Anderson, respectively. The O-D survey sampling rate for both urban areas was 10 percent. It can be seen that all of the observed PRMSE's are much larger than the expected PRMSE values for the 10 percent sampling rate. This comparison is also presented graphically in Figures 10 and 11 for the two urban areas considered.

Table 4. All-or-Nothing Traffic Assignment Comparison for Lafayette Gravity Model with Constant Friction Factors Versus Origin-Destination Survey

Volume Group	No. of Links	Mean Difference	Expected PRMSE	Observed PRMSE
0- 500	216	1.48	34.63	75.13
500- 1000	116	95.32	20.25	51.78
1000- 2000	197	523.15	14.43	55.33
2000- 3000	140	930.19	11.25	58.22
3000- 5000	172	2023.06	8.94	77.39
5000-10000	163	5441.60	6.54	98.81
10000-15000	31	10710.32	5.12	103.92
15000-20000	1	11281.00	4.35	64.90
Total	1036	1759.57	--	129.83

Excludes centroid links.

Table 5. All-or-Nothing Traffic Assignment Comparison for Anderson Gravity Model with Constant Friction Factors Versus Origin-Destination Survey

Volume Group	No. of Links	Mean Difference	Expected PRMSE	Observed PRMSE
0- 500	604	35.80	34.63	84.96
500- 1000	211	240.55	20.25	50.63
1000- 2000	228	675.49	14.43	56.40
2000- 3000	169	1102.44	11.25	53.48
3000- 5000	185	1924.28	8.94	54.35
5000-10000	108	3545.44	6.54	59.66
10000-15000	2	5440.50	5.12	61.60
Total	1507	771.39	--	86.00

Excludes centroid links.

Figure 10. Volume Group Link Comparison for Lafayette All-or-Nothing Traffic Assignment: Gravity Model with Constant Friction Factor Versus OD Survey

Figure 11. Volume Group Link Comparison for Anderson All-or-Nothing Traffic Assignment: Gravity Model with Constant Friction Factor Versus OD Survey

The equation for estimating the expected PRSME for a given household sampling rate is also shown in the Figures. It can be seen from the Figures that a wide divergence exists between the expected and observed PRMSE values for both Lafayette and Anderson.

A final test, a link by link model/OD load ratio check was also performed. This test was performed on the basis of the research done by Creighton and Hamburg on the effect of assignment inaccuracy on the design process in relation to the number of lanes needed to serve a given hourly volume (4). These load ratio ranges, as referenced in the FHWA publication, Traffic Assignment (15), were used to find the number of links with unacceptable load ratios. The acceptable load ratio ranges used are given in Table 6.

Table 6. Acceptable Model/OD Traffic Assignment Link Load Ratio Ranges

ADT Volume	Acceptable Model/OD Link Load Ratio Range
1,000 - 5,000	0.55 - 1.45
5,000 - 10,000	0.60 - 1.40
10,000 - 15,000	0.65 - 1.35
15,000 - 20,000	0.70 - 1.30
20,000 - 30,000	0.75 - 1.25

Source: U.S. Department of Transportation, Federal Highway Administration, Urban Planning Division, Traffic Assignment, Washington, D.C., August 1973.

It should be noted that the link load ratios are a result of a gravity model and an O-D survey all-or-nothing assignment and are not the loaded link volume/ground count ratio normally used for the test. Because the purpose of the present study is to investigate the

sensitivity of synthetic trip distribution modelling, additional errors that may be associated with a capacity-restrained assignment and errors in ground counts were considered undesirable.

The number of unacceptable model/OD survey link load ratios found for Lafayette and Anderson, are respectively 363 and 435 links. These links represent 35.1 and 28.9 percent of all loaded links, respectively.

These results clearly point out the importance of friction factors in gravity model trip distribution simulation in small urban areas and therefore also the importance of travel time in the trip making process. It is imperative therefore that friction factor values be calibrated as accurately as possible and that it may be necessary to exercise great care when using synthetic methods in trip distribution modelling.

Calibration of the Gravity Models for Lafayette and Anderson, Indiana

The purpose of this present study was to evaluate various synthetic trip distribution techniques and to ultimately recommend one procedure. Because the traditional gravity model calibration procedure is an accepted methodology, the trip distribution results from such an accepted procedure were used to serve as the data base to which synthetic methods were to be compared. The objective was to choose or develop a technique which can replicate the O-D survey trips almost as accurately as the traditional method of calibration. It was therefore necessary to calibrate the Lafayette and Anderson gravity models for use in the forthcoming analysis.

Because comparisons were to be made between gravity models, calibration of the gravity models for Lafayette and Anderson using the O-D data were performed on the basis of a negative exponential friction factor function. This would allow comparisons to be made between various synthetic techniques using the single calibration parameter β instead of a set of friction factor data points which are difficult to compare quantitatively. Additionally, the travel time matrices (skim trees) developed from the minimum path trees and used in the gravity model calibration did not include turn penalties. Since the objective of the present research was to simplify the trip distribution process for small urban areas, it was felt that the inclusion of turn penalties would not appreciably improve the model results.

Average travel times and travel time variances, along with the β parameters obtained from the traditionally calibrated gravity models are given in Tables 7 and 8 for the study areas of Lafayette and Anderson, respectively. Volume group link comparisons, where gravity model traffic assignments are compared to O-D survey traffic assignments, were conducted for both of the urban areas. Results of this comparison are presented in Tables 9 and 10 for Lafayette and Anderson respectively. The Tables indicate that the traditionally calibrated gravity models resulted in very acceptable traffic assignments for both urban areas. Graphical presentation of the volume group link comparisons for Lafayette and Anderson, are shown in Figures 12 and 13, respectively. A final model/O-D survey link load ratio test revealed that there were no links with an unacceptable load ratio for Lafayette, while only two links were found to have unacceptable link load ratios for Anderson.

Table 7. Traditionally Calibrated Gravity Model Statistics for Lafayette

Trip Purpose	<u>Average Travel Time</u>		Model Travel Time Variance	β Parameter
	O-D Survey	Model		
HBW	10.60	10.58	20.54	0.1679
HBO	8.57	8.57	16.11	0.2856
NHB	9.12	9.12	12.56	0.2647
Total	9.05	9.04	16.18	--

Table 8. Traditionally Calibrated Gravity Model Statistics for Anderson

Trip Purpose	<u>Average Travel Time</u>		Model Travel Time Variance	β Parameter
	O-D Survey	Model		
HBW	11.44	11.44	9.09	0.1327
HBO	10.05	10.04	8.53	0.2961
NHB	9.50	9.50	6.51	0.3516
Total	10.14	10.14	8.46	--

Table 9. All-or-Nothing Traffic Assignment Comparison for Lafayette with Traditionally Calibrated Gravity Model Versus Origin-Destination Survey

Volume Group	No. of Links	Mean Difference	Expected PRMSE	Observed PRMSE
0- 500	216	10.4	34.63	49.54
500- 1000	116	32.2	20.25	17.13
1000- 2000	192	48.4	14.43	13.74
2000- 3000	140	122.0	11.25	12.48
3000- 5000	172	176.5	8.94	11.40
5000-10000	163	109.0	6.54	8.93
10000-15000	31	297.5	5.12	4.65
15000-20000	1	-320.0	4.35	1.84
Total	1036	86.5	--	12.48

Excludes centroid links.

Table 10. All-or-Nothing Traffic Assignment Comparison for Anderson with Traditionally Calibrated Gravity Model Versus Origin-Destination Survey

Volume Group	No. of Links	Mean Difference	Expected PRMSE	Observed PRMSE
0- 500	604	7.8	34.63	33.43
500- 1000	211	22.6	20.35	15.50
1000- 2000	228	54.6	14.43	12.11
2000- 3000	169	-16.3	11.25	10.31
3000- 5000	185	-36.2	8.94	8.99
5000-10000	108	-63.9	6.54	7.31
10000-15000	2	-717.0	5.12	8.24
Total	1507	3.0	--	13.28

Excludes centroid links.

Figure 12. Volume Group Link Comparison for Lafayette All-or-Nothing Traffic Assignment: Traditionally Calibrated Gravity Model Versus OD Survey

Figure 13. Volume Group Link Comparison Plot for Anderson All-or-Nothing Traffic Assignment: Traditionally Calibrated Gravity Model Versus OD Survey

The calibration of the gravity models for Lafayette and Anderson hence provided standards to which synthetic trip distribution models could be easily compared. It should be pointed out that though both of the urban areas are of a similar population size, there are some land use and network characteristics which are distinctively unique to each area.

These unique characteristics resulted in dissimilar travel patterns for the two study areas. When constant valued friction factors were previously used on the Lafayette and Anderson study areas so as to emphasize the importance of friction factors, the resulting gravity model travel patterns when compared to their respective O-D survey results demonstrated significant differences between the two study areas. These differences between Lafayette and Anderson do provide, however, a wider statistical inference space over which to test the validity of synthetic trip distribution modelling techniques.

Previous Studies on the Use of Borrowed Friction Factors

Various studies have been performed to test the feasibility of using borrowed friction factors. The Federal Highway Administration in a publication entitled Urban Trip Distribution Factors (18), summarized friction factor curve data from a number of transportation studies, specifically for the purpose of gathering the information so that friction factor curves could easily be borrowed. Friction factor curve data was categorized by trip purpose and urban area population size in the FHWA report. In addition, each category of friction factor curves was normalized into one comprehensive curve.

In order to test the feasibility of borrowing friction factor curves from the report, an experiment was conducted by the FHWA using friction factors developed for Tallahassee, Florida and the normalized friction factor values in a test application to the Sioux Falls, South Dakota study area. Both Tallahassee and Sioux Falls are in the same population range. On the basis of a comparison of travel time frequency distributions and total vehicle-miles traveled, it was concluded that the use of borrowed friction factors produced adequate results. However, the available information did not specify any evaluation of traffic assignment error. An analysis of the traffic assignment would have been a more reliable and revealing testing methodology.

In another study, the Comsis Corporation (3) used regression techniques to fit an inverse travel time friction factor equation to the FHWA normalized friction factor curves. The α parameter is the calibration parameter of the inverse travel time function. Much like the FHWA, Comsis Corporation classified the α parameters by trip purpose and population size. A final report, in which the test case results of these standardized α parameters are reported, is not published as of this writing. However, these standardized α parameters were applied to the Lafayette and Anderson, Indiana study areas as a part of the present study and are reported in a later section of this Chapter.

In a comprehensive report entitled Transportation Planning For Small Urban Areas (5), Grecco, et.al., reviewed previous studies in which borrowed friction factors had been used in trip distribution

modelling. The conclusion reached by Grecco, et.al., was that the adoption of the "initial" friction factors for different trip purposes from another study area of similar size is valid and considerably simplifies the modelling effort. The word "initial" in the above sentence is important for it implies that adjustments may be necessary to achieve a satisfactory trip distribution. Grecco, et.al.'s report did not, however, mention the basis for such a conclusion.

Bates (1), in a synthetic trip distribution modelling study, used an inverse travel time friction factor equation to represent the travel time impedance relationship. The inverse travel time function is repeated in Equation 12 for reference.

$$F(t) = C t^{-\alpha} \quad (12)$$

where

$F(t)$ = friction factor at time t ,

C = constant,

α = alpha, calibration parameter, and

e = base of the natural logarithm.

Bates used the friction factor curves from one city, Cedartown, Georgia, as a base to which adjustments were made according to the maximum trip length (travel time) and average trip length (travel time) of the test cities. The adjustments consisted of altering the C parameter of Equation 12 and it was assumed that the α parameter was constant for all cities. Unfortunately, these adjustments of the C parameter have no effect on the relative time values of the friction factor equations. In the gravity model equation, as repeated in

Equation 13, the C parameter, which is a constant value over time, is in both the numerator and denominator of the equation, and therefore cancels.

$$T_{ij} = \frac{P_{ij} A_{ij} F_{ij}}{\sum_i A_i F_{ij}} = \frac{P_{ij} A_{ij} C t^{-\alpha}}{\sum_i A_i C t^{-\alpha}} \quad (13)$$

The only parameter that can affect the relative time values of the inverse travel time function is the α parameter.

Evaluation of an Urban Area's Sensitivity to

Friction Factor Curves

This evaluation was accomplished by observing the modelling errors induced by small deviations from the traditionally calibrated β parameter in the negative exponential friction factor equation. Deviations of plus or minus 0.05 were applied to the traditionally calibrated β parameters of each trip purpose, home based work, home based other, and non-home based. The resulting trip tables by purpose were then added together and loaded onto the network using an all-or-nothing assignment procedure. From these traffic assignments and the O-D survey internal-internal trip table network assignment, volume group link comparisons were performed. This procedure was conducted on both the Lafayette and Anderson study areas. It was then possible to make relative comparisons of the magnitude of error introduced by this evaluation procedure. These results would help to ascertain whether different synthetic techniques may be necessary for different urban areas or if a single synthetic technique can be found to adequately model the travel patterns of the uniquely defined urban areas.

Lafayette's gravity model average travel time statistics demonstrating β parameter deviation sensitivity are given in Table 11. Percentage change in average travel times were approximately of the same magnitude for both the plus and minus 0.05 deviations from the traditionally calibrated β parameter. The percentage differences between the O-D survey average travel time and the β parameter ± 0.05 deviations from the traditionally calibrated β parameter are greater than the ± 3.0 percent level suggested by the FHWA (13).

Table 11. Lafayette Total Trip Average Travel Time for β Parameter Deviations from the Traditionally Calibrated β Value for Lafayette

	β Parameter Deviation			
	-0.05	0.00	+0.05	+0.10
Total Trip Average Travel Time	9.48	9.05	8.63	8.31
Percent Error	+4.75	--	-4.64	-8.18

A greater indication of error as a result of the β parameter deviations from the traditionally calibrated value can be found in the volume group link comparisons as shown in Table 12. Total PRMSE for the minus 0.05 β deviation was almost twice that of the traditionally calibrated gravity model, while the total PRMSE of the plus 0.05 β parameter deviation was only one third greater. As a further test, a plus 0.10 β parameter deviation was applied to the Lafayette data set. The resulting total PRMSE, also shown in Table 12, was a little over twice the normally calibrated gravity model PRMSE and is very close to the minus 0.05 β parameter deviation PRMSE.

Table 12. All-or-Nothing Traffic Assignment Comparison for Lafayette:
 β Parameter Deviation Sensitivity Analysis *

Volume Group x 1000	Expected PRMSE 10% Sample	β -0.05 Deviate PRMSE	Traditional Model PRMSE	β +0.05 Deviate PRMSE	β +0.10 Deviate PRMSE
0- 0.5	34.63	46.65	49.54	53.99	59.17
0.5- 1.0	20.25	21.93	17.13	16.91	19.79
1.0- 2.0	14.43	18.01	13.74	13.60	16.22
2.0- 3.0	11.25	16.62	12.48	12.40	14.93
3.0- 5.0	8.94	18.82	11.40	11.55	15.97
5.0-10.0	6.54	17.90	8.93	12.79	20.54
10.0-15.0	5.12	17.92	4.65	9.73	18.85
15.0-20.0	4.35	8.11	1.84	10.46	17.53
Total	---	24.45	12.48	16.58	26.45

*Excludes centroid links.

Continuing this analysis, a link by link comparison was conducted in order to find unacceptable model/O-D link load ratios on the basis of the O-D survey traffic assignment and the assignment of trip interchanges resulting from the different levels of the β parameters. The results of this link load ratio analysis are given in Table 13. The Table shows that the minus 0.05 β parameter deviate assignment has more unacceptable link load ratios than either of the plus 0.05 or 0.10 β deviates.

Table 13. Number of Unacceptable Model/O-D link Load Ratios for
 Lafayette β Deviate Model Traffic Assignments

	β -0.05 Deviate Model	Traditionally Calibrated Gravity Model	β +0.05 Deviate Model	β +0.10 Deviate Model
Number of Un- acceptable Load Ratios	15	0	0	4

This same β parameter deviation sensitivity analysis was also conducted on the Anderson study area data. Anderson average travel time statistics, demonstrating the sensitivity of the gravity model to β parameter deviations from the traditionally calibrated β parameter, are given in Table 14. The percentage difference in total trip average travel times, when compared to those obtained from the O-D survey, were approximately of the same magnitude for both the plus and minus 0.05 deviations in the β parameter. These errors were approximately within the allowable range as suggested by the FHWA (13).

Table 14. Total Trip Average Travel Time for β Parameter Deviations from the Traditionally Calibrated β Parameter for Anderson

	β Parameter Deviation		
	-0.05	0.00	+0.05
Total Trip Average Travel Time	10.47	10.14	9.85
Percent Error	+ 3.25	--	-2.86

Volume group link comparison PRMSE results are shown in Table 15. The total PRMSE for the plus and minus β parameter deviations are very close, being only about one third more than the traditionally calibrated gravity model value.

A link by link comparison was performed in order to find unacceptable gravity model O-D survey link load ratios for the O-D survey assignment and the assignments resulting from various β deviates. The results of this analysis are given in Table 16. As indicated in the Table, the plus β parameter deviate had fewer links with unacceptable load ratios than the minus 0.05 β parameter deviate.

Table 15. All-or-Nothing Traffic Assignment Comparison for Anderson:
 β Parameter Sensitivity Analysis

Volume Group x 1000	Expected PRMSE 10% Sample	β -0.05 Deviate PRMSE	Traditionally Calibrated Model PRMSE	β +0.05 Deviate PRMSE
0- 0.5	34.63	37.48	33.43	34.26
0.5- 1.0	20.25	18.28	15.50	16.34
1.0- 2.0	14.43	16.46	12.11	15.86
2.0- 3.0	11.25	12.72	10.31	13.19
3.0- 5.0	8.94	12.24	8.99	11.97
5.0-10.0	6.54	11.13	7.31	11.25
10.0-15.0	5.12	2.94	8.24	17.62
Total	--	18.28	13.28	18.18

Excludes centroid links.

Table 16. Number of Unacceptable Model/OD Link Load Ratios for
 Anderson β Deviate Model Traffic Assignment

	β -0.05 Deviate Model	Traditionally Calibrated Gravity Model	β +0.05 Deviate Model
Number of Unacceptable Link Load Ratios	4	2	2

The relatively low number of unacceptable links for the Lafayette and Anderson β parameter deviates are probably trivial because they represent a very small percentage of all links and are only one of the many sources of error in travel demand modeling. It should be remembered however that the end result of the entire research effort is the implementation of a completely synthetic travel demand modelling system that is capable of modelling urban areas whose internal characteristics may vary greatly. Therefore, error must be kept to a minimum within each step of the process so as not to be carried into successive steps if the resources spent to obtain such accuracy are minimal.

The above analysis on the Lafayette and Anderson study areas suggests that the sensitivity of a gravity model with respect to friction factor errors may be dependent upon the particular urban area that is being modeled. The errors introduced by the β parameter deviations from the calibrated value were much greater for Lafayette than for Anderson. This varying sensitivity may cause difficulties in the selection of a synthetic trip distribution modelling technique applicable to all small urban areas.

Variability of Friction Factor Data Within Population Size

The main criterion for the selection of a similar urban area from which to borrow friction factors has been the population size. In order to test the validity of this selection criterion, a negative exponential function was fitted to the friction factor curves of various urban areas collected for the present study. This procedure provided the β calibration parameter on which the criterion was to be tested. Appendix B discusses the regression technique used in determining the β parameters.

In Figure 14, the computed β parameters are plotted against population size for home based work trips. The Figure shows that, for this data set, population size is a poor criterion by which to select a similar urban area, primarily due to the wide dispersion of the data points at the smaller population sizes. The intercept, slope, coefficient of multiple determination (R^2), and the significance level, as determined by linear regression analysis are also shown on the Figure. Similar results were obtained for the home based other and non-home based trips as shown in Figures 15 and 16.

Figure 14. Variability of Beta with Respect to Urban Area Population Size, Home Based Work Trips.

Figure 15. Variability of Beta with Respect to Urban Area Population Size, Home Based Other Trips.

Figure 16. Variability of Beta with Respect to Urban Area Population Size; Non-Home Based Trips.

Of particular interest is Figure 14 where the slope of the regression line is positive for this data set. The usual assumption is that the β parameter (slope) decreases with increasing population size (7). The positive slope can probably be attributed to a small sample size and/or the limited population range to which this line was fitted. The assumption that the slope should be negative implies that the variability of the β parameter with respect to population size is even greater than what Figure 14 implies. Consequently, it would appear that the borrowing or the use of standardized friction factors or parameters may possibly result in unacceptable trip distribution models.

Evaluation of Standardized Friction Factor Parameters

The ability to use standardized friction factors or parameters of friction factor functions could greatly reduce the time and resources expended in gravity model calibration. A test was conducted by running the gravity models for Lafayette and Anderson using the standardized α parameters of the inverse travel time function. These α parameters were developed by the Comsis Corporation using the data contained within FHWA's publication entitled Urban Trip Distribution Factors (18).

Lafayette's gravity model travel time statistics which resulted from the application of standardized α parameters, are presented in Table 17. Percentage differences of the gravity model average travel

Table 17. Gravity Model Travel Time Statistics for Lafayette, Standardized α Parameters

Trip Purpose	Average Travel Time			α Parameters		
	O-D Survey	Standardized α Parameters	Percent Difference	Standardized α Parameters	Calibrated α Parameters	Absolute Difference
HBW	10.60	10.38	-2.08	-1.99	-1.78	0.21
HBO	8.57	8.56	-0.18	-2.73	-2.72	0.01
NHB	9.12	9.03	-0.97	-2.68	-2.58	0.10
Total	9.05	8.99	-0.66	--	--	--

times from those obtained from the O-D survey values are minor, the largest value being only a -2.08 percent for the home based work trips. The standardized α parameters were also very close to the actual calibrated parameters as shown in Table 17. The PRMSE statistics as determined from a volume group link comparison are reported in Table 18 and also graphically illustrated in Figure 17. It is evident from the Table and Figure that the PRMSE values at each volume level are very acceptable. A link by link check found 12 out of 1036 links with an acceptable link load ratio as compared to no links for the traditional gravity model calibration.

Table 18. All-or-Nothing Traffic Assignment Comparison for Lafayette: Standardized α Parameters Versus Origin-Destination Survey

Volume Group	No. of Links	Mean Difference	Expected PRMSE	Observed PRMSE
0- 500	216	7.37	34.63	47.57
500- 1000	116	44.41	20.25	23.90
1000- 2000	197	54.38	14.43	14.71
2000- 3000	140	64.45	11.25	10.97
3000- 5000	172	48.12	8.94	10.87
5000-10000	163	-285.23	6.54	9.37
10000-15000	31	-349.19	5.12	4.27
15000-20000	1	-1499.00	4.35	8.62
Total	1036	-23.22	--	12.68

Excludes centroid links.

Gravity model travel time statistics for the Anderson study area, as generated by the inclusion of the standardized α parameters, are presented in Table 19 along with the O-D survey values for comparison. Percentage differences in model average travel times from O-D survey values by trip purpose are relatively low in comparison to the large absolute differences between the standardized α parameters and the traditionally

Figure 17. Volume Group Link Comparison for Lafayette All-or-Nothing Traffic Assignment: Standardized
 (Parameters Versus OD Survey)

Table 19. Gravity Model Travel Time Statistics for Anderson, Standardized α Parameters

Trip Purpose	Average Travel Time		α Parameters		
	O-D Survey	Standardized α Parameters	Percent Difference	Standardized α Parameters	Calibrated α Parameters Absolute Difference
HBW	11.44	11.18	-2.29	-1.99	-1.48 0.51
HBO	10.04	10.27	+2.27	-2.73	-3.01 0.28
NHB	9.50	10.04	+5.74	-2.68	-3.41 0.73
Total	10.14	10.38	+2.36	--	-- --

calibrated values. For NHB trips, the predicted average travel time was 5.74 percent greater than the O-D survey average travel time. This error is almost twice the FHWA suggested ± 3.0 percent error (13). The HBW trip error, however, was of opposite sign and therefore helped to counteract the NHB trip error. Traffic assignment volume group link comparisons revealed acceptable PRMSE values at all volume ranges as shown in Table 20 and Figure 18. Only three of the 1507 links were found with unacceptable model/O-D survey link load ratios, only one more than the normally calibrated gravity model.

Table 20. All-or-Nothing Traffic Assignment Comparison for Anderson: Standardized α Parameter Versus Origin-Destination Survey

Volume Group	No. of Links	Mean Difference	Expected PRMSE	Observed PRMSE
0- 500	604	14.46	34.63	37.83
500- 1000	211	64.42	20.25	18.85
1000- 2000	228	133.44	14.43	15.58
2000- 3000	169	113.65	11.25	11.64
3000- 5000	185	176.89	8.94	10.65
5000-10000	108	354.29	6.54	9.45
10000-15000	2	14.00	5.12	1.15
Total	1507	94.87	--	16.14

Excludes centroid links.

The preceding analysis of using standardized α parameters on the two test cases help to possibly explain why transportation planners and researchers have usually concluded that in general, borrowed or standardized friction factors often produce acceptable synthetic trip distribution model results. Total trip as well as the individual trip purpose average travel time statistic and the volume group link

Figure 18. Volume Group Link Comparison for Anderson All-or-Nothing Traffic Assignment: Standardized α Parameters Versus OD Survey

comparison for Lafayette do indicate acceptable model results. These results are due to the fact that the standardized and the actually calibrated α parameters are very close in magnitude. The Lafayette results therefore suggest that the use of standardized α parameters is indeed a valid synthetic procedure for trip distribution modelling. Similarly, total trip average travel time and volume group link comparisons for Anderson also indicate that an acceptable trip distribution model result had occurred and therefore additionally endorsing the use of standardized α parameters.

However, just because the aggregated results appear acceptable, it cannot be necessarily inferred that the intermediate steps to achieve the final results are accurate and completely acceptable. Closer inspection of the intermediate Anderson model results reveal that NHB trip average travel time has been overestimated by 5.74 percent when compared to the O-D survey data. One reason why this overestimate was not detrimental to the final results was the HBW trip average travel time was underestimated by 2.29 percent and therefore helped to offset the NHB trip error. If by chance, as it may possibly occur for another study area, the HBW trip average travel time was also overestimated, then the total error may have been much greater. Another possible reason for Anderson's total model results to be acceptable, is the fact that its gravity model sensitivity to friction factor error is small as was demonstrated in the previous section of this Chapter. Table 19 showed that there is indeed a large difference between the standardized α parameters and the α parameters that were determined from traditional calibration methods for the HBW and NHB work trips.

Evaluation of Borrowed Friction Factors

As a final example of the sensitivity of gravity models with respect to borrowed friction factors, the Lafayette and Anderson gravity models were run using each other's calibrated β parameters of the negative exponential friction factor equation. Both of the urban areas are approximately of the same population size. The generated trip tables were then assigned to their respective networks and results analyzed.

Average travel time for Lafayette's total trips resulting from the gravity model with Anderson's borrowed friction factor β parameter was only 2.1 percent less than that obtained from the O-D survey, as given in Table 21. However, both the home based work trips and the non-home based trips average travel times were greater than the allowable ± 3.0 percent error. These values, however, are of opposite sign and therefore have a tendency to cancel each other's error. Volume group link comparison PRMSE values reported in Table 22 and illustrated in Figure 19, are actually slightly better than the supposedly more accurate traditionally calibrated model values. A check of the model/O-D survey link load ratios revealed no links with an unacceptable load ratio.

The good performance of the traffic assignment is probably due to the offsetting errors as discussed above. If the home based work trip average travel time had actually been underpredicted, the traffic assignment would have probably shown increased error.

Table 21. Gravity Model Statistics for Lafayette with Anderson β Parameter

Trip Purpose	Average Travel Time		Percent Difference
	Survey	Model	
HBW	10.60	10.94	+3.21
HBO	8.57	8.47	-1.17
NHB	9.12	8.50	-6.80
Total	9.05	8.86	-2.10

Table 22. All-or-Nothing Traffic Assignment Comparison for Lafayette with Anderson β Parameter Versus Origin-Destination Survey

Volume Group	No. of Links	Mean Difference	Expected PRMSE	Observed PRMSE
0- 500	216	6.95	34.63	49.20
500- 1000	116	13.53	20.25	16.11
1000- 2000	197	10.59	14.43	13.09
2000- 3000	140	57.44	11.25	11.93
3000- 5000	172	49.96	8.94	10.34
5000-10000	163	-210.02	6.54	9.30
10000-15000	31	-248.45	5.12	3.95
15000-20000	1	-1160.00	4.35	6.67
Total	1036		--	12.31

Excludes centroid links.

Figure 19. Volume Group Link Comparison for Lafayette All-or-Nothing Traffic Assignment: Lafayette Gravity Model with Anderson β Parameters Versus OD Survey

A gravity model run for Anderson, using the β parameters developed for Lafayette, resulted in an average travel time for all trips of only 1.8 percent greater than the corresponding mean travel time obtained from the O-D survey. Table 23 shows that the non-home based trip average travel time was overestimated by 5.68 percent. Home based work trip average travel time did however, help to compensate for the error of the non-home based trips. Volume group link comparisons, as indicated in Table 24 and Figure 20 were acceptable. Only three links out of 1507 were found unacceptable, one more than the traditionally calibrated model.

For the two study areas of Lafayette and Anderson, the borrowing of each other's friction factor parameters indicated good results for several reasons. First, Lafayette's large percentage error in prediction of average travel time for two of the three trip purposes were of opposite sign and thus the aggregated total error was within the acceptable level. The same situation of offsetting errors took place in Anderson, but of a smaller magnitude. Secondly, Anderson's gravity model is less sensitive to friction factor errors as compared to Lafayette.

Chapter Conclusions

Sensitivity analysis of the gravity models for Lafayette and Anderson resulted in three important observations in regards to synthetic trip distribution modelling.

1. Acceptable model results will occur when the borrowed, or standardized friction factors are very similar to those that would have been normally calibrated from O-D survey data.

Table 23. Gravity Model Statistics for Anderson with Lafayette β Parameter

Trip Purpose	<u>Average Travel Time</u>		Percent Difference
	Survey	Model	
HBW	11.44	11.25	-1.66
HBO	10.05	10.11	+0.60
NHB	9.50	10.04	+5.68
Total	10.14	10.32	+1.78

Table 24. All-or-Nothing Traffic Assignment Comparison for Anderson with Lafayette β Parameter Versus Origin-Destination Survey

Volume Group	No. of Links	Mean Difference	Expected PRMSE	Observed PRMSE
0- 500	604	13.71	34.63	36.01
500- 1000	211	50.87	20.25	17.20
1000- 2000	228	114.05	14.43	14.37
2000- 3000	169	84.30	11.25	10.70
3000- 5000	185	125.01	8.94	9.67
5000-10000	108	217.92	6.54	8.17
10000-15000	2	-269.50	5.12	3.35
Total	1507	69.93	--	14.48

Excludes centroid links.

Figure 20. Volume Group Link Comparison for Anderson All-or-Nothing Traffic Assignment: Anderson Gravity Model with Lafayette β Parameters Versus OD Survey

2. Acceptable model results may occur when synthetic methods are used because the urban area's gravity model is not sensitive to friction factor errors (as was the case for Anderson).
3. Acceptable model results may occur when one individual trip purpose error is offset by an opposite error in one or more of the other trip purposes.

Clearly, the first situation is the most preferred with the second also being acceptable. The third situation, however, is unacceptable because chance plays a larger role in the model's outcome than the procedure by which the friction factors were obtained.

In addition, analysis of data from a number of small urban area transportation studies resulted in one other important conclusion in regards to the borrowing of friction factors. Population size is an inadequate criterion by which to select a similar urban area for borrowing friction factors or their parameters.

The above observations and conclusions suggest that in many instances, the borrowing of friction factors or the use of standardized values in trip distribution gravity models may result in total trip models of undesirable accuracy. Therefore, a technique which produces consistently more reliable results is still desired, for again, it must be remembered that synthetic trip distribution modelling is only one step within a sequence of models.

CHAPTER IV

DEVELOPMENT AND TESTING OF PROCEDURES FOR SYNTHETIC SELF-CALIBRATION OF GRAVITY MODELS

In the previous Chapter, the two major methods of gravity model calibration - borrowed friction factor curves or their parameters and the use of standardized friction factor values were evaluated. The results indicated that these methods can produce, in some cases, inconsistent and unreliable results. This discrepancy in the results can be attributed primarily to the variability of various urban characteristics among the many small urban areas. Thus, there exists a definite need to have a self-calibrating synthetic gravity model which would be reliable in reflecting the varying characteristics of different urban areas. Such a model would significantly reduce the time and effort presently expended in the development of a trip distribution model for an urban area without introducing any significant error into the modelling process.

Literature Review

Most of the research on self-calibrating gravity models has centered on the approximation of the β parameter of the negative exponential friction factor equation. Researchers such as Tomlin (12), Wardrop (20), and Zaryouni (22) suggested the approximation of β as the inverse of the areawide mean travel time, Equation 14, without regard to any particular trip purpose.

$$\beta \approx 1/\bar{C} \quad (14)$$

where

$\bar{C}$ equals areawide average travel time.

An equation of this type can easily be incorporated into an iterative procedure for calibrating a gravity model. The testing of such a procedure, however, has been minimal with both favorable and unfavorable results.

Another possible approach to synthetic self-calibration of the gravity model may be at a stratified level. Voorhees (19), in an effort to measure the spatial arrangement of activities, introduced the concept of the opportunity distribution; that is, the gravity model travel time distribution resulting from the setting of the friction factors equal to a constant value of 1.0. Voorhees used the 1948 Washington, D. C. transportation data to reveal how differently opportunity travel time frequency distributions can vary between zones. It was concluded that "in the case of the gravity model, separate travel time factors should be used if there is a wide variation in opportunity distribution between zones". Consequently, Voorhees suggested the development of a new distribution model that recognizes both travel time and spatial arrangement of opportunities.

Further support of this possible approach was provided in the article by Fisk and Brown (5) which states: "The parameter β [of the negative exponential friction factor equation] was found to be sensitive to the distribution of destinations about the origin zone...". Wilson (21) also suggested that the β parameters should be origin specific.

The literature review thus provides two possible approaches to synthetic self-calibration of the gravity model; 1) areawide parameter estimation using areawide travel time statistics, and 2) origin zone specific parameter estimation using origin zone travel time statistics.

Areawide Approach to Synthetic Self-Calibration of a Gravity Model

The friction factor equation of a gravity model represents the trip makers' destination choice process with respect to the spatial distribution of the land use activities as represented by the destination choice variable of interzonal travel time. The aggregate effect of all the trip makers' destination choices result in a total travel pattern that can be approximately described in terms of the areawide travel time statistics of mean and variance. Average travel time and travel time variance can thus be thought of as areawide descriptors of the entire distribution of land use activities and the type and configuration of the network. It can be expected, therefore, that it should be possible to relate mathematically the parameter of a friction factor equation to the spatial land use activity distribution and network impedances through the use of the areawide descriptors of mean travel time and travel time variance, and possibly to other variables of interest. These relationships are represented by the schematic diagram in Figure 21.

Data used in the study of the areawide approach consisted of the β parameters of the negative exponential friction factor function, areawide average travel times and travel time variances, study area population, and study area density as computed or reported for eighteen small urban

Figure 21. Interrelationship of the Destination Choice Process with Destination Choice Variables and the Urban Area's Travel Pattern

area transportation studies. Regression analysis was used to search for mathematical relationships between the β parameters and the areawide travel time statistics and any other variables of possible interest in the first approach.

Extensive linear and nonlinear regression analysis resulted in mathematical relationships of very simple form for each trip purpose. Equations 15, 16, and 17 give the relationship between the β parameters of the negative exponential friction factor function and average travel time for the home based work (HBW), home based other (HBO), and non-home based (NHB) trips respectively. The coefficient of multiple determination, R^2 is also given.

$$\text{HBW: } \beta = 0.0594 + 0.7858/\bar{C} \quad (15)$$

$$R^2 = .278$$

$$\text{HBO: } \beta = 0.0145 + 2.0794/\bar{C} \quad (16)$$

$$R^2 = .479$$

$$\text{NHB: } \beta = -0.0222 + 2.2976/\bar{C} \quad (17)$$

where $R^2 = .502$

$\bar{C}$ = areawide average travel time.

The forms of Equations 15, 16, and 17 are similar to that of Equation 14. However, the HBW equation (Equation 15) contains a significant constant term while the HBO and NHB equations have insignificant constant terms. Furthermore, the slopes (coefficients) of the independent variable vary significantly with trip purpose.

Equation 15 for HBW trips is the only equation whose slope (coefficient) is close to that of hypothesized form Equation 14. The slopes (coefficients) do, however, emphasize the varying degree of importance placed on travel time for the three trip purposes.

The statistical fit of the Equations are less than what would normally be considered acceptable for estimating purposes. Nevertheless, because other researchers have suggested that relationships of this form may produce acceptable model results, these Equations were incorporated into an iterative algorithm for calibrating a gravity model so as to evaluate this technique. A simplified flow chart of this iterative procedure is given in Figure 22. Because of data availability and the previously noted differences in each urban area's model sensitivity to friction factor errors, the Lafayette and Anderson study areas were selected for validation testing.

Tables 25 and 26 contain the average travel time statistics by trip purpose, as generated by the test gravity model for the urban areas of Lafayette and Anderson, respectively. Also shown in Tables 25 and 26 are the comparisons of these average travel time values with those obtained from the origin-destination surveys. Total trip average travel times were overestimated by the areawide test gravity model for both Lafayette and Anderson by 3.76 percent and 6.35 percent, respectively. In addition, the β parameters as computed by the test

Figure 22. Flow Chart of First Approach to Synthetic Self-Calibration of a Gravity Model: Friction Factor β Parameter as a Function of Areawide Average Travel Time

Table 25. Average Travel Time Statistics and β Parameter Comparisons for an Areawide Synthetically Self-Calibrating Gravity Model for Lafayette

Trip Purpose	Average Travel Time			β Parameters		
	O-D Survey	Self-Calibrated	Percent Difference	Traditionally Calibrated	Self-Calibrated	Absolute Difference
HBW	10.60	10.93	+3.11	0.1679	0.1345	-0.0334
HBO	8.57	8.91	+3.97	0.2856	0.2466	-0.0390
NHB	9.12	9.46	+3.72	0.2647	0.2242	-0.0405
Total	9.05	9.39	+3.76	--	--	--

Table 26. Average Travel Time Statistics and β Parameter Comparisons for an Areawide Synthetically Self-Calibrating Gravity Model for Anderson

Trip Purpose	Average Travel Time			β Parameters		
	O-D Survey	Self-Calibrated	Percent Difference	Traditionally Calibrated	Self-Calibrated	Absolute Difference
HBW	11.44	11.45	0.0	0.1327	0.1310	-0.0017
HBO	10.04	10.71	+6.67	0.2961	0.2080	-0.0881
NHB	9.50	10.49	+10.42	0.3516	0.2000	-0.1516
Total	10.14	10.78	+6.35	--	--	--

gravity models are shown in the Tables and are compared to those values obtained from the traditional gravity model calibration procedure. As can be seen in Tables 25 and 26, the differences in β parameters range from a low of -0.0017 for Anderson's HBW trips to a high of -0.1516 for Anderson's NHB trips. Lafayette's largest β parameter difference was -0.0405 for the HBW trips.

The total internal trip tables of Lafayette and Anderson as produced by the test gravity model were then assigned to their respective networks on an all-or-nothing basis. Volume group link comparisons were then performed with their corresponding origin-destination survey all-or-nothing traffic assignments. Tables 27 and 28 present the results of these volume group link comparisons for Lafayette and Anderson respectively. Total link trip percent root-mean-square-error (PRMSE) for Lafayette was 21.09 percent as compared to the 12.67 percent obtained in the comparison of the origin-destination survey assignment with the traditionally calibrated gravity model procedure. The corresponding total link trip PRMSE value for Anderson was 28.15 percent for the test gravity model as compared to 13.28 percent for the traditionally calibrated model. Figures 23 and 24 graphically portray these comparisons, respectively for Lafayette and Anderson.

As a final check of the accuracy of the test gravity model, a link by link analysis of the model/O-D link load ratios was performed for both Lafayette and Anderson. For Lafayette, no link was found to have an unacceptable link load ratio while Anderson had 29 links, or 1.9 percent, with unacceptable ratios.

Table 27. Volume Group Link Comparison for Lafayette All-or-Nothing Traffic Assignment: Areawide Self-Calibrating Gravity Model Versus Origin-Destination Survey

Volume Group	No. of Links	Mean Difference	Expected PRMSE	Observed PRMSE
0- 500	216	12.20	31.63	47.61
500- 1000	116	57.09	20.25	20.71
1000- 2000	197	118.64	14.43	16.70
2000- 3000	140	241.14	11.25	15.42
3000- 5000	172	419.17	8.91	16.33
5000-10000	163	719.53	6.54	15.33
10000-15000	31	1409.39	5.13	14.23
15000-20000	1	978.00	4.35	5.63
Total	1036	290.00	--	21.09

Excludes centroid links

Table 28. Volume Group Link Comparison for Anderson All-or-Nothing Traffic Assignment: Areawide Self-Calibrating Gravity Model Versus Origin-Destination Survey

Volume Group	No. of Links	Mean Difference	Expected PRMSE	Observed PRMSE
0- 500	604	23.07	34.63	47.07
500- 1000	211	116.38	20.25	25.15
1000- 2000	228	268.23	14.43	23.60
2000- 3000	169	345.47	11.25	18.39
3000- 5000	185	571.16	8.94	18.28
5000-10000	108	998.68	6.54	18.26
10000-15000	2	1028.50	5.12	11.68
Total	1507	247.34	--	28.15

Excludes centroid links

Figure 23. Volume Group Link Comparison for Lafayette All-or-Nothing Traffic Assignment: Areawide Self-Calibrating Gravity Model Versus OD Survey

Figure 24. Volume Group Link Comparison for Anderson All-or-Nothing Traffic Assignment: Areawide Self-Calibrating Gravity Model

Based on these two test cases, it may be concluded that the self-calibrating β parameter procedure using Equations 15, 16 and 17, is not as reliable a procedure as the traditionally calibrated gravity model technique. As evidenced by the Anderson results, the error produced by this approach to synthetic self-calibration can be relatively large when compared to the traditional calibration results.

Some of the possible reasons for the poor performance of the procedure discussed above for areawide self-calibration are listed below.

1. Spatial distribution of the land use activities and the characteristics of the highway network can differ significantly among the small urban areas. Therefore, the factors and decision variables considered by the trip maker in basing his choice of destination can also differ with respect to location.
2. Average travel time for an entire urban area is a highly aggregated statistic and may not reflect adequately the transportation system options encountered by the individual trip makers at various locations within the urban area.
3. Terminal times, as shown in Chapter III, can vary greatly among the study areas. This variability in terminal times is directly reflected in the average travel times for the urban areas. This complicates the regression procedure from which the calibration equations were derived by adding additional subjective variance to the statistical curve fitting process.

The above three items were considered in the development of the second approach to synthetic self-calibration of a gravity model.

Origin Zone Specific Approach to Synthetic Self-Calibration
of a Gravity Model

Table 29 compares the areawide travel time statistics for Lafayette and Anderson. The Table shows that the average travel time values by trip purpose are approximately equal but that the travel time variances are completely different for the two urban areas. The Anderson travel time variances are approximately one half of those of Lafayette. However, the regression analysis used to develop the areawide β parameter calibration equations in the areawide approach did not show travel time variance to be a significant factor in explaining any of the variance between the β parameters computed on the basis of the data from eighteen urban areas. The analysis conducted for the development of the areawide approach and the possible reasons given for the poor performance of the derived calibration equations suggest the possibility that the variance of data within an urban area may be more significant to gravity model self-calibration than the variance between urban areas.

Table 29. Lafayette and Anderson Areawide Travel Time Statistics Comparison

Trip Purpose	<u>Lafayette</u>		<u>Anderson</u>	
	Mean	Variance	Mean	Variance
HBW	10.60	23.88	11.44	9.61
HBO	8.57	19.90	10.04	9.53
NHB	9.17	15.07	9.50	7.81

In order to reveal the extent to which the travel time statistics may vary within an urban area, Table 30 was constructed using the data sets of Lafayette and Anderson. Origin zone specific average travel times and travel time variances were computed from gravity models with constant friction factors, that is the opportunity trip distribution as defined by Voorhees (19). As presented in the Table, origin zone average travel times and variances possess great variability within an urban area. In order to provide a better pictorial representation of the relationship between origin zone opportunity average travel time and zone location, Figures 25, 26 and 27 were prepared for the HBW, HBO, and NHB trips of Lafayette, Indiana. These Figures show that destination opportunities vary with both location and trip purpose, as represented by the origin zone opportunity average travel times, and therefore it is reasonable to assume that the destination choice process itself is a function of origin zone location.

Table 30. Origin Zone Opportunity Travel Time Statistics for Lafayette and Anderson

Trip Purpose		Lafayette			Anderson		
	Mean	Min.	Max.	Mean	Min.	Max.	
HBW							
Average	12.806	8.759	22.024	13.075	9.079	22.016	
Variance	14.497	7.388	22.755	5.445	2.475	9.903	
HBO							
Average	12.482	8.520	22.014	13.160	10.068	21.198	
Variance	15.137	5.207	22.693	8.426	4.603	13.198	
NHB							
Average	12.442	8.358	21.744	12.930	10.060	20.815	
Variance	13.910	5.358	21.257	8.246	4.453	25.549

Figure 25. Origin Zone Opportunity Average Travel Time Values for Lafayette Home Based Work Trips

Figure 26. Origin Zone Opportunity Average Travel Time Values for Lafayette Home Based Other Trips

Figure 27. Origin Zone Opportunity Average Travel Time Values for Lafayette Non-Home Based Trips

What is being proposed is a stratified approach to trip distribution. Trip generation, the modelling process performed before trip distribution, has evolved from zonal regression equations to the disaggregated approach of cross classification of residential and non-residential trip rates. Not only does this technique simplify data collection and improve prediction results, but it has also been found that these trip rates can be transferred more easily from one study area to another (10). The same approach may be followed for friction factors, that is, friction factors or friction factor equation parameters may be considered on a disaggregated basis by computing these values by zone rather than using an aggregated value for the entire study area.

In order to develop a disaggregated synthetic gravity model, it is first necessary to generate the data on which this development is to be based. The study areas of Lafayette, Anderson, and Muncie, Indiana were used for this purpose because of data availability and the wide sampling space provided by the differing characteristics of these areas. To generate the data base, the gravity models of these three urban areas were calibrated at the zonal level. This was accomplished by using the origin-destination survey to compute origin zone average travel times and then using these values to repeatedly adjust the origin zone specific parameters of the negative exponential friction factor function until convergence of the gravity model origin zone mean travel times occurred. The data generated by this procedure consisted of the following for each origin zone and for each trip purpose - HBW, HBO, and NHB trips.

β_i = negative exponential friction factor parameter for zone i of the origin zone calibrated gravity model,

O_i = trips originating from zone i,

$AVEO_i$ = opportunity average travel time for origin zone i when β_i equals 0.0, that is, friction factor of constant value 1.0,

$VARO_i$ = opportunity travel time variance for origin zone i when β_i equals 0.0,

AVE_i = origin specific average travel time for zone i of calibrated gravity model (same as origin-destination survey value),

VAR_i = origin specific travel time variance for zone i of calibrated gravity model, and

$TERM_i$ = origin zone terminal time.

The above variables were then used in a curve fitting regression procedure so as to find a relationship of the following form:

$$\beta_i = f(AVEO_i, AVE_i, VARO_i, VAR_i, TERM_i)$$

Regression Analysis for Development of an Origin Zone

Specific β Parameter Equation

Having generated the data base for the equation development, a weighting scheme was employed to the data before the curve fitting regression analysis was begun. Weighting of the data base was performed at two different levels. First, the data was weighted by the variable O_i , that is, the origin zone productions (or origins).

Neglecting to apply this weighting factor would have resulted in zones with only a few trip productions (origins) to have the same effect in the curve fitting regression analysis as a zone of a thousand trips or more. Secondly, because the total number of trips by purpose are not the same for each urban area, a weighting scheme was applied to balance this inequality and thus bias toward any one particular urban area was minimized.

As was noted in Chapter III, terminal times can vary greatly between urban areas. Because this value is assigned primarily by subjective judgment and because origin zone terminal times must be experienced by a trip maker irrespective of the location of the destination choice, it was decided to remove origin zone terminal time from the origin zone average travel time. Origin zone terminal time, however, was allowed to be a separate variable in the regression procedure.

Finally, dummy variables representing the three urban areas were added. This allows any unique and significant combinatorial effects of unspecified variables of an urban area to be represented in the analysis. Thus dummy variables make major differences between urban areas to be apparent in the analysis.

The initial regression analysis resulted in an equation form possessing very acceptable statistics for the curve fitting of the origin zone specific β parameters to their respective zonal travel time statistics of mean and variance. However, inspection of the residual plot revealed a greater than expected number of outliers. Careful review of the zonal data and the previously presented sensitivity

analysis, resulted in the removal of the variable VAR_i from the analysis, that is, the origin zone travel time variance. A close examination of the data revealed that zonal travel time variance does not always change monotonically with a monotonically changing friction factor β parameter. For some zones, travel time variance first increased and then decreased for an increasing β parameter, that is, from β equal to zero (opportunity distribution) and upwards. This phenomenon is probably caused by the gravity model attraction constraints and its iteration process.

Having removed the variable VAR_i , further regression analysis resulted in an equation form with acceptable statistics and residual plots. However, the incorporation of the β parameter equation into a gravity model did not provide acceptable results. The β parameters were found to be converging to very erroneous values with many not converging at all. A review of the data revealed that the regression equations developed were not equations of how the calibrated origin zone β parameters related to various variables, but represented actually a generalized response surface of any β parameter with respect to the independent variables. For example, Hyman (9) showed that average travel time varies linearly with the β parameter, and therefore the calibrated origin zone β parameter is just one point on the line. To eliminate the difficulty with a generalized response surface, an additional restriction was imposed eliminating the inclusion of both AVE_i and $AVEO_i$ in the regression equation simultaneously.

This additional restriction greatly simplified the number of possible variable combinations to be studied. Given these restrictions, the best regression equations that could be developed had R-squared values of 0.14, 0.52, and 0.62 for the HBW, HBO, and NHB trips respectively. Some significance was also indicated by the urban area dummy variables; a result totally unwanted for it is necessary to have a technique applicable to all urban areas. Because of the low R-squared values, especially for the HBW trip purpose, and the significance of the dummy variables, these equations were judged unacceptable.

It had been initially thought that the development of origin zone specific self-calibrating friction factor β parameter equations was possible. However, for reasons cited above, this approach to zonal calibration was abandoned. The results of the regression analysis did, however, suggest another possible method for zonal self-calibration. The correlation matrix generated by the regression procedure showed that a high correlation existed between the variables $AVEO_i$ and AVE_i . It is this high correlation on which a revised second approach to synthetic self-calibration of a gravity model was based.

Revised Second Approach to Synthetic Self-Calibration of a Gravity Model

The initial zone specific approach to estimate the β parameter of the negative exponential friction factor equation using a direct iterative technique proved unsuccessful. However, a high correlation observed between $AVEO_i$ and AVE_i indicated that an indirect technique to estimating the β parameter could be possible. Consequently, a revised procedure was developed with $AVEO_i$ as the key variable.

As previously mentioned, $AVEO_i$ is the opportunity average travel time for an origin zone, as defined by Voorhees (19), minus terminal time for the origin zone. Origin zone terminal time was subtracted so as to separate it from the link travel time and destination zone terminal time which are themselves a function of destination zone location. The high correlation of AVE_i with $AVEO_i$ suggests that the destination choice process is closely related to the geographic location of the urban area activity system with respect to the origin zone. It was felt that if an estimating equation for AVE_i could be developed as a function of $AVEO_i$ and $TERM_i$, then these equations could be incorporated in a gravity model for synthetic self-calibration.

Regression analysis of AVE_i on $AVEO_i$ and $TERM_i$ was accomplished by weighting the zonal data, as previously described, in order to properly balance the data by trip productions (origins) and by trip purpose. Equations 18, 19, and 20 present the regression analysis results for the HBW, HBO, and NHB trips respectively. Total R-squared and the change in R-squared for the independent variables are also given.

$$\begin{aligned} \text{HBW: } AVE_i &= -0.4328 + 1.2761 \text{ TERM}_i + 0.8921 (AVEO_i - \text{TERM}_i) \\ R^2 &= .865 \quad \Delta R^2 = .114 \quad \Delta R^2 = 0.751 \end{aligned} \quad (18)$$

$$\begin{aligned} \text{HBO: } AVE_i &= -0.7499 + 1.3548 \text{ TERM}_i + 0.7558 (AVEO_i - \text{TERM}_i) \\ R^2 &= .848 \quad \Delta R^2 = .187 \quad \Delta R^2 = .661 \end{aligned} \quad (19)$$

$$\begin{aligned} \text{NHB: } AVE_i &= 0.5245 + 1.3416 \text{ TERM}_i + 0.6073 (AVEO_i - \text{TERM}_i) \\ R^2 &= .719 \quad \Delta R^2 = .423 \quad \Delta R^2 = .296 \end{aligned} \quad (20)$$

The estimating equations possess R-squared values higher than those achieved in earlier attempts. The strong statistical fit of the equations clearly shows the importance of $AVEO_i$, the opportunity average travel time on the destination choice process. This in turn, demonstrates the large effect that origin zone location and the distribution of land use activities about the origin zone have on the destination choice process.

Relative importance of travel time by trip purpose in the destination choice process is indicated by the coefficients of the $AVEO_i$ variable. In general, the home based work (HBW) trips do not have much choice in terms of destination, the home based other (HBO) trips have some or many choices of destination, while the non-home based (NHB) trips have many choices of destination. It can be seen from the Equations that the importance of travel time is the lowest for HBW, highest for NHB, and of some intermediate magnitude for HBO trips. This relative importance is also reflected in the corresponding change of the R-squared value for the $AVEO_i$ variable.

In addition, variable $TERM_i$, the origin zone terminal time, becomes a greater proportion of total travel time as trip purpose changes from HBW, to HBO, and to NHB, as is evidenced by the change in R-squared. This is because terminal times are zone specific constants and cannot be reduced by destination choice unlike the link travel times.

Also of interest are the coefficients of the origin zone terminal times. It was first expected that the coefficients would have a magnitude very close to one. The fact that the coefficients are

greater than 1.0 may possibly be attributable to that constant part of the destination zone terminal time common to all zones, that is, the minimum terminal time.

Having developed estimating equations by trip purpose for the origin zone average travel time, AVE_i , a procedure was then formulated to incorporate these equations into a gravity model.

Incorporation of the Estimating Equations Into a Gravity Model

One method of calibrating the gravity model would be to first distribute all trips with a constant friction factor, thereby allowing for the computation of $AVE0_i$, the origin zonal specific opportunity average travel time. Origin zone average travel time, AVE_i , could then be computed using the regression equations discussed in the previous section. These origin zone average travel times then could be used to repeatedly adjust the origin zone specific β parameters until the gravity model origin zone average travel times match the estimated values. However, this type of calibration procedure would require many iterations before convergence for all the zones occur. The many iterations required for convergence would consume a large amount of computer time. This would be an undesirable characteristic for a synthetic model to possess.

Consequently, another method was developed for eventual adoption for synthetic calibration and was based on three assumptions regarding the interaction of origin zone average travel time and the zonal β parameter. The first assumption was that the origin zone average travel time varies linearly with the origin zone β parameter (9).

Second, zonal average travel time is independent of the β parameters of other zones. Third, zonal average travel time is independent of the attraction constraint iteration procedure. The key assumption of linearity does approximate reality as can be evidenced in Figures 28 and 29 by the plots of origin zone average travel time versus areawide β parameter, that is all zonal β parameters equal. Figure 28 contains the plots of zones 41 through 49 for the HBW trips of Lafayette. The Figure shows typical plots in which the zonal average travel time approximately varies linearly with the β parameter. Figure 29 contains plots of zones 91 through 99, also for Lafayette HBW trips. This Figure shows greater nonlinearity for the Lafayette HBW trips but even these are not great enough to reject the general assumption of linearity, especially when one considers that most β parameters are less than 0.40 and that zonal results are aggregated up to the study area level. These assumptions, however, do not always hold true for some of the individual zones when the origin zone β parameters are allowed to vary independently as in the proposed model. This aspect could not be detected until after the model was developed and tested, but aggregation of the results up to the entire study area nullified these minor errors.

The estimating equations of AVE_i , zonal average travel time, and the three assumptions listed above were incorporated into a gravity model procedure that can best be described in conjunction with Figure 30. First, trips are distributed with all zonal β parameters equal to 0.0 (constant friction factor) and then the resulting zonal opportunity average travel times, $AVE0_i$ are computed. Second, trips are distributed

Figure 28. Relationship of Zonal Average Travel Time with the Negative Exponential Friction Factor Parameter. Zones 41 Through 49, Greater Lafayette Area Transportation and Development Study

Figure 29. Relationship of Zonal Average Travel Time with the Negative Exponential Friction Factor Parameter. Zones 91 Through 99, Greater Lafayette Area Transportation and Development Study

Step 1 - Average at β equal zero
(0.0) - $AVE0_i$

Step 2 - Average at β equal to a
set value.

Step 3 - Average from regression
equation, AVE_i

Step 4 - Interpolate for new β

Figure 30. Generalized Flow Chart of Synthetically Self-Calibrated, Origin Zone Specific Gravity Model as Applied to a Typical Zone

with all origin zone β parameters equal to a preselected value and then the origin zone average travel times are computed. Third, using the estimating equation for the appropriate trip purpose and the $AVE0_i$ values computed in step one, the origin zone average travel times, AVE_i , are determined. Fourth, using the data points from steps one and three and the estimated origin zone average travel times, AVE_i , the zonal β parameters are obtained through interpolation. All trips are then distributed using the interpolated origin zone specific β parameters.

Evaluation of the Proposed Model

The gravity model procedure, as just described, was used to distribute the trips for both Lafayette and Anderson. The resulting average travel time statistics by trip purpose and for all internal trips are given in Tables 31 and 32 for Lafayette and Anderson respectively. Comparison of these statistics to the corresponding origin-destination survey values is also shown. Lafayette's total internal trip average travel time was only 0.71 percent greater than the O-D survey value, while Anderson's total internal average travel time was identical to the O-D survey value.

The average travel times by trip purpose also indicated very acceptable results; the largest percentage error for either urban area's individual trip purposes was 3.78 percent, this being the only one outside of FHWA's suggested ± 3 percent range.

Volume group link comparisons of the two synthetic gravity models' all-or-nothing traffic assignments were performed using their corresponding O-D survey traffic assignments as the base for comparison. The PRMSE values by volume-group are presented in Tables 33 and 34 for Lafayette and Anderson respectively. Total link trip PRMSE for Lafayette was computed to be 12.67 percent. This value is practically identical to the corresponding value (12.48) obtained from the gravity model calibrated by the traditional procedure. Total link trip PRMSE was calculated to be 14.55 percent as compared to the traditionally calibrated gravity model value of 13.28 percent for Anderson. Volume group link comparison statistics of Tables 33 and 34 are also shown

Table 31. Origin Zone Specific, Synthetically Self-Calibrating Gravity
Model Travel Time Statistics for Lafayette

Trip Purpose	<u>Average Travel Time</u>		Percent Difference
	Survey	Model	
HBW	10.57	10.97	+3.78
HBO	8.56	8.65	+1.05
NHB	9.12	8.93	-2.08
Total	9.05	9.09	+0.71

Table 32. Origin Zone Specific Synthetically Self-Calibrating Gravity
Model Travel Time Statistics for Anderson

Trip Purpose	<u>Average Travel Time</u>		Percent Difference
	Survey	Model	
HBW	11.44	11.27	-1.49
HBO	10.04	10.02	-0.20
NHB	9.50	9.64	+1.47
Total	10.14	10.14	0.00

Table 33. Lafayette All-or-Nothing Traffic Assignment Comparison:
Origin Zone Specific Synthetically Self-Calibrating Gravity
Model Versus O-D Survey

Volume Group	No. of Links	Mean Difference	Expected PRMSE	Observed PRMSE
0- 500	216	11.59	34.63	50.15
500- 1000	116	44.09	20.25	18.58
1000- 2000	197	38.88	14.43	14.56
2000- 3000	140	81.90	11.25	11.20
3000- 5000	172	177.01	8.94	11.46
5000-10000	163	209.29	6.54	8.78
10000-15000	31	444.58	5.12	6.29
15000-20000	1	-9.00	4.35	0.05
Total	1036	101.43	--	12.67

Table 34. Anderson All-or-Nothing Traffic Assignment Comparison:
Origin Zone Specific Synthetically Self-Calibrating Gravity
Model Versus O-D Survey

Volume Group	No. of Links	Mean Difference	Expected PRMSE	Observed PRMSE
0- 500	604	16.15	34.63	42.27
500- 1000	211	49.48	20.25	17.94
1000- 2000	228	47.75	14.43	12.72
2000- 3000	169	-43.60	11.25	9.82
3000- 5000	185	-115.35	8.94	9.13
5000-10000	108	-274.01	6.54	8.16
10000-15000	2	-1201.50	5.12	13.71
Total	1507	-17.68	--	14.55

graphically in Figures 31 and 32. Also shown on the Figures are the expected PRMSE values associated with a ten percent dwelling unit sampling rate for the O-D home interview survey. The PRMSE values for both Lafayette and Anderson were observed to be very acceptable.

As a final test of the developed procedure for synthetic self-calibration of a gravity model, a model/O-D link load ratio analysis was performed so as to determine the number of links outside of the suggested ratio ranges. For the Lafayette urban area, no model/O-D link load ratio was observed to fall outside of the suggested ranges. For the Anderson study area only two links indicated unacceptable model/O-D link load ratios. However, these two links are the same links as those found unacceptable in the traditionally calibrated gravity model traffic assignment and are of relatively low volumes.

Because of the excellent results obtained from this approach for gravity model calibration on the Lafayette and Anderson study areas, the program logic was rewritten so as to incorporate it into program UMODEL of the UTPS package for additional testing on the Indiana State Highway Commission's IBM 370 computer. Appendix C contains the listing of this program code. This allowed testing of the program's modelling capability on the Muncie, Indiana data set and provided for the possible quick implementation of the proposed gravity model into the normal planning process if these tests proved successful.

Average travel time statistics by trip purpose as computed from the running of the test gravity model are given in Table 35 for the Muncie urban area. Also shown in the Table are the O-D survey average

Figure 31. Volume Group Link Comparison for Lafayette All-or-Nothing Traffic Assignment: Origin Zone Specific Synthetically Self-Calibrating Gravity Model Versus OD Survey

Figure 32. Volume Group Link Comparison for Anderson All-or-Nothing Traffic Assignment: Origin Zone Specific Synthetically Self-Calibrating Gravity Model Versus OD Survey

travel times and the percent differences. Total trip average travel time as computed by the test gravity model underestimated the O-D survey value by only a -0.80 percent, with the largest trip purpose error being a -2.79 percent for HBW trips. The errors are far within acceptable limits.

Table 35. Travel Time Statistics, Generated by Origin Zone Specific Synthetic Gravity Model for the Muncie Urban Area

Trip Purpose	<u>Areawide Average Travel Time</u>		Percent Difference
	O-D Survey	Model	
HBW	9.07	8.82	-2.79
HBO	7.48	7.37	-1.46
NHB	6.93	7.04	+1.63
Total	7.52	7.46	-0.80

The internal trip tables as generated by the test gravity model were next assigned to Muncie's network on an all-or-nothing basis. Internal trip tables from the O-D survey were also assigned to the Muncie network on an all-or-nothing basis so that a comparison between the model and O-D survey assignments could be performed.

The resulting PRMSE values from a volume group link comparison of these two assignments were found to be much higher than expected. It was therefore decided to further evaluate the performance of the model. For this purpose a gravity model calibration for the HBW, HBO, and NHB trips was undertaken using the traditional calibration procedure, with the resulting trip tables assigned to the Muncie network. A volume-group

link comparison of this assignment with the O-D survey assignment was subsequently performed. In this manner, the modelling capabilities of the origin zone specific calibration procedure can be compared with the traditionally calibrated gravity model as the "standard" model. If the proposed procedure would model trip distribution as accurately as the traditional model, then it could be concluded that the proposed procedure is probably a valid synthetic trip distribution model for Muncie.

The volume-group link comparison for the traditionally calibrated gravity model resulted in a total link trip PRMSE value much larger than the value obtained from the proposed calibration procedure. A second check was also made by performing the same analysis on the trip tables generated by the Muncie transportation study, which also included socio-economic "K" factors. Total link trip PRMSE for the "K" factor gravity model was observed to be slightly higher than the traditionally calibrated gravity model. Table 36 contains the volume-group PRMSE values for all three gravity models for Muncie as described above. As is evident from the Table, the origin zone specific synthetic calibration procedure performed much better than the traditionally calibrated gravity model with or without "K" factors. A plot of the volume group link comparison PRMSE values resulting from the origin zone specific synthetic self-calibration gravity model and those from the traditionally calibrated model is given in Figure 33. This Figure illustrates that the proposed procedure provides more accurate results for the higher volume links of Muncie as compared to the lower volume links. As the higher volume links are critical in the process of

Table 36. Volume-Group Link Comparison for Muncie All-or-Nothing Traffic Assignment: Comparison of Three Gravity Models Versus O-D Survey

Volume Group	No. of Links ¹	Expected PRMSE ² Values	Observed PRMSE Values		
			Origin Zone Specific Synthetic Model	Traditional Gravity Model	Traditional Gravity Model with "K" Factor
0- 500	1249	34.63	153.0	52.3	52.2
500- 1000	426	20.25	59.2	41.5	41.1
1000- 2000	354	14.43	49.1	30.3	30.5
2000- 3000	217	11.25	38.0	37.2	38.0
3000- 4000	118	9.54	25.7	46.7	47.4
4000- 5000	108	8.44	21.1	37.6	39.5
5000- 6000	94	7.65	24.2	31.4	34.5
6000- 8000	74	6.80	21.8	43.3	45.6
8000-10000	26	6.02	23.3	47.6	50.4
10000-15000	25	5.12	23.2	41.7	44.6
15000-20000	5	4.35	25.0	25.3	32.9
20000-25000	2	3.84	11.0	16.8	25.8
Total	2698	--	47.7	67.8	72.0

¹Includes all links.

²Based on ten percent sample rate.

Figure 33. Volume Group Link Comparison for Muncie All-Or-Nothing Traffic Assignment: Origin Zone
Specific Synthetically Self-Calibrating Gravity Model Versus OD Survey

transportation facility design, the proposed form of gravity model appears to be an effective procedure.

A possible explanation for the high PRMSE values for all three Muncie models may be the fact that the Muncie study area included a relatively high number of zones and links for an urban area of its population size. The zones and links are proportionally much greater than either Lafayette or Anderson, suggesting that the Muncie zone and network system were probably too detailed for the number of trips produced. For example, the Muncie data included 1249 out of 2698 links with an average daily traffic of less than only 500 vehicles per day; peak hour traffic on these links would probably not exceed 50-60 vehicles per hour. Muncie had 271 internal zones as compared to Anderson's 169 internal zones and Lafayette's 111 internal zones. A large number of zones in comparison to the number of trips can also cause numerical problems when rounding techniques are used to integerize the zonal interchanges. Because of production and attraction constraints, rounding techniques may accumulate trips from some zones until a whole integer trip is obtained. Muncie's 288613 internal vehicle trips must be distributed about 271 by 271 cells of the trip tables, or an average of 3.9 trips per cell, which means there would be hundreds of cells with less than one trip after the gravity model is applied.

Chapter Conclusions

In this Chapter, the development of an improved procedure for synthetic calibration of gravity models has been discussed. The proposed synthetic procedure involves an origin zone specific

self-calibrating gravity model in which the only input data required are the zonal productions and attractions, a zone-to-zone travel time matrix (skim tree), and the origin zone terminal times. A travel time distribution is no longer needed for calibration, thereby eliminating the necessity for an O-D survey for internal-internal trips.

Tests conducted on three separate study areas indicated that the proposed model is able to reproduce trip patterns as accurately as the traditionally calibrated gravity model procedure which is calibrated on O-D survey data. In comparison to other methods of synthetic trip distribution, the proposed gravity model provided results with a much greater degree of reliability. The improved reliability was achieved by calibrating the model at the zonal level rather than at the aggregate level of the entire study area. This approach minimizes overall modelling error by aggregating the individual zonal results up to the urban area.

Development of the proposed procedure was also based on the consideration that the trip distribution is critically dependent on spatial distribution of land use activities about the origin zone. This consideration was incorporated in the proposed procedure through the explicit measurement of the origin zone specific opportunity travel time distribution. The opportunity distribution was represented in the model by the opportunity average travel time. It is this key variable on which the model was developed and upon which the very acceptable model results are due.

CHAPTER V

SUMMARY AND CONCLUSIONS

The present study, a part of an overall research project involving synthetic travel demand modelling, included only the trip distribution phase of the total modelling process. Synthetic trip distribution is a method by which the travel time impedance function of the trip distribution model can be established with little or no data obtained directly from an extensive home-interview travel survey of the area under study. This Chapter summarizes the results and conclusions of this synthetic trip distribution study, and makes recommendations as to what technique should be employed in synthetic trip distribution modelling for small urban areas in Indiana. Recommendations for further research are also presented at the end of this Chapter.

The great importance of travel time impedance factors in a trip distribution model for small urban areas was clearly illustrated by the values of average travel time and the traffic assignment statistics generated from trip distribution models without such time dependent factors. Errors in the resulting travel time values and traffic assignments were of a magnitude so large as to make any of the generated output practically useless. The analysis using gravity models with constant friction factors for the entire range of time values clearly established the need and importance of friction factors.

It is important for systems modellers to know how sensitive a particular model is with respect to the variable parameters of the model. Once the model sensitivity is determined, approximate error limits on the variable parameters can be estimated or the modelling technique can be altered to achieve an acceptable level of accuracy. The same is true for trip distribution modelling, especially when synthetic methods are to be used. An analysis conducted in the present study indicated that the sensitivity of the gravity model to friction factor errors may be highly dependent upon the particular urban area being modelled. That is, the topographical barriers, land use activity distribution, and network characteristics interact uniquely for each urban area, therefore resulting in gravity models with varying sensitivity. As was demonstrated in the present study, the gravity model for the Lafayette study area was found to be very sensitive to deviations of the calibration parameters in the travel time impedance function from the actual values obtained from the origin-destination survey. Average travel times by trip purpose and traffic assignment link volumes changed significantly with relatively small deviations of the calibration parameter. This was not the case, however, for Anderson where average travel times and link volumes did not change as rapidly as they did for Lafayette when the calibration parameters were varied. In other words, the calibration parameter accuracy required for the generation of acceptable model results is a function of the particular urban area being studied and therefore, cannot be generalized for any urban area.

The borrowing of friction factors from a similar urban area has long been suggested as a technique for synthetic gravity models. The most often cited criterion for the selection of a similar urban area from which to borrow factors is population size. Population size is also a criterion by which standardized friction factors or parameters are categorized. Both of these techniques therefore assume that a correlation exists between population size and the friction factors themselves, and that the variability with respect to population size is acceptable. As it was shown in Chapter III, a correlation does exist, but a large variability with respect to population was observed. This large variability therefore weakens the confidence in the results of a model in which the friction factors are borrowed or standardized values used.

As an example of the problems that may occur when friction factors are borrowed from a similar urban area, the calibrated β parameters for Lafayette and Anderson, Indiana were exchanged between the two study areas of roughly the same population size. Average travel times by trip purpose were shown to possess a relatively large error, greater than the FHWA's suggested ± 3.0 error limit. However, off-setting errors in terms of trip purpose average travel times resulted in acceptable traffic assignments. In this situation, chance and not modelling accuracy resulted in acceptable results.

The use of standardized friction factors or parameters has also been suggested in the literature as another possible synthetic technique for trip distribution modelling. Standardized α parameters, as used in the inverse travel time impedance function, were developed by the

Comsis Corporation and categorized by trip purpose and population size (3). These standardized α parameters for synthetic trip distribution modelling were tested using the two study areas of Lafayette and Anderson, Indiana. For Lafayette, the average travel times by trip purpose were within the ± 3 percent ranges as suggested by FHWA (13) and the percent root-mean-square-error values by volume group were also acceptable. However, 12 of the 1036 non-centroid links had unacceptable model/O-D survey link load ratios. Statistics for Anderson showed that the non home-based trip average travel time was overestimated by 5.74 percent. However, due to the underestimate of the home based work trip average travel time which helped to offset the non-home based travel time error and also because of the previously discussed insensitivity of the Anderson gravity model and traffic assignment to friction factor errors, an acceptable assignment of the traffic to the network was observed. Based on these two test cases, one cannot definitely conclude whether or not the synthetic technique of using standardized α parameters is an acceptable procedure. However, because of the 12 links of the Lafayette network with unacceptable link load ratios, the high overestimate of non-home based trip average travel time, and the general conclusion that urban area characteristics can differ greatly suggests the possibility that standardized α parameters may not always be reliable.

A common approach to represent the travel pattern of an urban area is to use the travel time statistics of average travel time and travel time variance as obtained either by analyzing the O-D survey data conducted in the study area or as generated by a trip distribution

model. These travel time statistics, as determined from a past O-D survey of one urban area, are sometimes used to possibly validate the synthetic trip distribution model results of another similar urban area being studied. Choosing a similar urban area from which travel time statistics could be compared has mainly been based on the criterion of population size. This criterion is based on the assumption that a strong correlation exists between average travel time and population size. It has been observed in this present study that, although there exists a correlation among the average travel times and population size, the variability of average travel time with respect to population size is large. Consequently, the validation of a synthetic trip distribution model by comparing only the average travel times is not a valid approach. Furthermore, the estimation of average travel time based on population size alone would be of little beneficial use due to this variability in the data.

The same problem, as discussed above, exists for the travel time frequency distributions. Even though the general shapes of the frequency distributions may appear quite similar, even a small variation or shift in a distribution can significantly alter average travel time and therefore also the accuracy of the model. Again, comparison of travel time frequency distributions would not be beneficial in verifying synthetic trip distribution model results.

Because of the inability of the synthetic techniques of borrowing friction factors or using standardized friction factor values to consistently and reliably model trip distribution travel patterns, a large effort was therefore undertaken in the development and testing of an

acceptable procedure for synthetic self-calibration of gravity models. It was first found that a gravity model with an areawide friction factor equation parameter as a function of areawide travel time statistics could not adequately model travel patterns. The next attempt was to develop an origin zone specific equation, but no satisfactory regression equation could be developed to predict the β calibration parameter of the negative exponential function on the basis of origin zone travel time statistics. The final approach involved the use of regression analysis of origin zone mean travel time, terminal time, and origin zone opportunity mean travel time, that is the average travel time when friction factor values in a gravity model are held constant. The statistical result was satisfactory and the associated procedure developed was tested on the study areas of Lafayette, Anderson, and Muncie. Comparisons of average travel times, percent root-mean-square-error values by volume group, and model/O-D survey link load ratios indicated very acceptable results for all three study areas. The accuracy of the trip distribution results were found to be at least as accurate as those provided by the traditionally calibrated gravity model.

The results of the present research project indicated that the most reliable synthetic trip distribution modelling technique investigated is the origin zone specific synthetically self-calibrating gravity model. This technique is recommended for implementation because of its ability to distribute trips at an accuracy level very close or better than the traditional technique of calibration with extensive origin-destination data. As compared to other synthetic techniques, this new approach has several advantages. The most

important feature of the proposed procedure is its more direct linkage to the spatial distribution of land use activities within an urban area as quantified by the origin zone specific opportunity average travel times. This inclusion of the origin zone specific measure of spatial land use activities allows the gravity model to be sensitive, for any origin zone, to the surrounding land uses, in addition to various network configurations. This procedure is therefore probably more sensitive than any areawide model to changes that may occur in the study area, such as a new shopping center, a new manufacturing plant opening or a major network alteration. Another advantage is its ease of application. No collection of origin-destination survey data is needed nor is it necessary to find a similar urban area from which to borrow information or to which model validation comparisons are to be made. As this present study showed, the criterion for the selection of a similar study area need not guarantee the finding of such a similar urban area. For example, the urban areas of Lafayette, Anderson, and Muncie have a population of approximately the same size but were found to have many differing characteristics. The proposed model, however, successfully simulated the travel patterns of these three urban areas.

Recommendations for Further Research

To support the validity and applicability of the recommended origin zone specific approach for synthetic self-calibration of a gravity model, further test applications should be performed on additional transportation study areas. This would require the application of the model to various areas from different regions of the country.

In order to simplify such application, the proposed procedure has been coded so as to be incorporated into UMTA's UMODEL program and also to be compatible with FHWA's PLANPAC package.

Because of the simplified technique used in the gravity model, it is possible that it can also be applied to larger urban areas with greater trip end opportunities and higher travel times. Therefore, additional verification and testing should be conducted on a few larger urban areas. This would allow for testing of the linearity of origin zone average travel time with the opportunity origin zone average travel time at higher time values.

In addition, because of the fact that one lives in a dynamic society where the social and economic environment can change over time, a sampling technique should be developed so as to provide a method for updating or validating the regression equations relating origin zone average travel time with the opportunity travel time. This sampling technique could probably be performed in conjunction with the limited household trip generation survey used for validating or updating cross classification trip rates. Recent studies have shown that a small sample origin-destination survey can probably be used for trip distribution modelling calibration or updating. It may be possible to use this same data for updating the relationship of origin zone travel time and opportunity travel time which is used in the proposed trip distribution procedure.

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APPENDIX A
SPATIAL LAND USE ACTIVITY DISTRIBUTION IN LAFAYETTE,
ANDERSON, AND MUNCIE, INDIANA

Spatial land use activity distribution is probably the greatest determinant in the trip making decision process. In the gravity model trip distribution step, the spatial land use activity distribution is numerically quantified in terms of the network's travel time matrix (skim tree) and by the zonal productions and attractions by trip purpose. A map of an urban area along with a general description can provide additional information on how the travel pattern may be effected by the land use distribution and various topographical features.

Greater Lafayette, Indiana

The spatial location of the major land use trip generators of the Greater Lafayette area, population 100,000, in combination with some significant topographical barriers have resulted in a travel pattern characterized by a relatively large travel time variance in comparison to other small Indiana urban areas. A map showing the location of the major land use trip generators and the topographical barriers of Greater Lafayette is given in Figure A1.

Purdue University, located on the west side of the Wabash River, is a large university employing approximately 9,000 persons and had an enrollment of approximately 28,000 students at the time of the study.

Figure A1. Greater Lafayette Transportation and Development Study Area

It is the largest trip generator in the urban area, producing approximately 97,000 trips per day.

The city of West Lafayette, population 20,000, also located west of the Wabash River, can be best typified as a bedroom community with no major industry or manufacturing center. The central business district is very small and is composed of small service oriented businesses. A small shopping area is located at the northern end of the city and another along the Wabash River. The city of West Lafayette, in itself, is not a major trip generator.

The city of Lafayette, population 50,000, located on the east side of the Wabash River, is typical of many older cities, consisting of a central core area and a growing ring on the outskirts. One of Lafayette's major trip generators is the central business district (CBD), located near the river. The CBD is predominately surrounded by residential areas. The residential area, in turn, is bounded by a large area of manufacturing, wholesale, retail, and commercial establishments which have developed along the U.S. 52 highway. This area is still growing with new construction and scattered subdivisions.

Topographical barriers play an important part in the Greater Lafayette area. At the time of the transportation study, only four bridges crossed the Wabash River. One of the bridges, due to old age and functional obsolescence, no longer exists. The importance of the river as a topographical barrier is best exemplified by the two minute time penalty added to the travel times on each bridge to correct for overdistribution in the gravity model. The railroads, also shown in

Figure A1, are also topographical barriers, but to a lesser extent. No time penalties have been assigned to links crossing the tracks, however, much of the highway network layout has been influenced by the location of the railroad tracks.

This scattering of the three major trip generators and the topographical barriers therefore play an important role in the determination of Greater Lafayette's travel pattern. Unique characteristics, such as these, can easily complicate the transportation modelling process.

Anderson, Indiana

A map of the Anderson, Indiana study area, Figure A2, reveals similarities to the Greater Lafayette area. Both urban areas are divided by a river and possess a university or college within their study area. Anderson's study area population of approximately 90,000 is also similar to Greater Lafayette's population. At this point, however, the similarities end. As was found in the calibration of the Anderson gravity model, travel time variance is approximately one half that of Lafayette, Indiana.

Anderson College is much smaller when compared to Purdue University. Anderson College generates approximately 8,000 trips as compared to Purdue University's 97,000 trips. The effect on travel patterns by Anderson College is therefore relatively small.

White River divides the Anderson urban area into a northern and southern section. As compared to the Greater Lafayette Area, a much greater proportion of the urbanized area is located on one side of the river, thereby reducing the overall effect of the White River as a

Figure A2. Anderson Urban Area Transportation Development Study Area

topographical barrier. Secondly, the White River has a total of eight bridges crossing it as compared to Lafayette's four bridges, thereby greatly increasing the accessibility to and from the northern section.

Another factor which may greatly contribute to Anderson's lower travel time variance as compared to Lafayette is the fact that the CBD and two large manufacturing centers are within close proximity to a centrally located high density residential area. This is in contrast to the Greater Lafayette area where the high density residential areas and major trip generators are spread out over a much greater area.

Muncie, Indiana

Muncie, Indiana, as shown in Figure A3 with a study area population of approximately 100,000, is also divided by the White River and has Ball State University located within the urbanized area. The Muncie, Indiana study area therefore possesses many characteristics of both the Greater Lafayette area and Anderson, Indiana. Muncie's travel time variance, as determined from gravity model calibration, is approximately equal to Anderson's and one half of Lafayette's travel time variance.

Ball State University is located north of White River. Ball State generates approximately 34,000 trips, excluding intra-university trips, and is therefore a major trip generator. Another indication of the importance of Ball State University as a trip generator was the necessity to use K factors for external-internal trip purpose gravity model in the original transportation study.

Figure A3. Delaware-Muncie Metropolitan Transportation and Urban Development Study Area

The White River divides Muncie into a northern and southern section, with an approximately equal proportion of land use activity on each side. As was reported in Muncie's transportation study reports, the river itself is not a major topographical barrier. Fourteen bridges cross the river with nine of these being in the urbanized portion of the study area. Travel time penalties were added to only two of the bridges. This situation differs greatly from Lafayette's and may therefore help to explain Muncie's low travel time variance. As previously noted, Anderson's bridge situation is very similar to Muncie's and may help to explain the similar travel time variances.

A review of Muncie's land use map shows that a large high density residential area is located centrally to the Muncie CBD and within a short distance to Ball State University. The relatively close proximity of these three major trip generators result in trip lengths of a relatively uniform length and therefore probably contribute greatly to Muncie's low travel time variance.

Zone and Network Comparison

The level of zone and network detail with respect to population size can also influence model results and the statistical measures used to test model accuracy. The three urban areas of Lafayette, Anderson, and Muncie each have a population of approximately the same size, however the level of zone and network detail differ significantly. Summarized in Table A1 are the number of zones and one way links for each of the three urban areas. An analysis of the effect of the level

of zone and network detail on trip distribution and traffic assignment was conducted as the third part of the overall study on synthetic travel demand modelling (Hanscom, E. and Sinha, K. C., "The Effects of Simplifying Traffic-Zone and Street-Network Systems on the Accuracy of Traffic Assignments in Small Urban Areas in Indiana", Joint Highway Research Project Report No. FHWA/IN/JHRP-79/15, Purdue University, 1979.

Table A1. Level of Zone and Network Detail Comparison for Lafayette, Anderson, and Muncie

Urban Area	Internal Zones	External Zones	Total Zones	One-Way Links
Lafayette	111	17	128	1448
Anderson	169	43	213	2321
Muncie	271	45	216	2667

APPENDIX B

CALIBRATION OF A GRAVITY MODEL UTILIZING REGRESSION ANALYSIS
AND THE ORIGIN-DESTINATION SURVEY TRAVEL TIME FREQUENCY
DISTRIBUTION INFORMATION

Calibration of a gravity model is based upon the repeated adjustments of a set of friction factors. Adjustments are continued until the resulting gravity model travel time frequency distribution satisfactorily approximate to the information obtained from the home interview survey. The adjustment technique used may be represented mathematically as follows.

$$F_{tk} = F_t^{(k-1)} \times \frac{OD\%_t}{GM\%_t^{(k-1)}} \quad (B1)$$

where

F_{tk} = friction factor value at time t for iteration k .

$OD\%$ = percentage of all trips of travel time t as obtained from the origin-destination survey.

$GM\%$ = percentage of all trips of travel time t resulting from iteration $(k-1)$ of the gravity model.

t = time, usually a time increment of one minute.

The values of F_{tk} are then plotted against time t and either a smooth curve is drawn to represent the friction factor curve or regression analysis is used to fit a function to the data points. This appendix

discusses the use of regression analysis as applied throughout this study.

Calibration of the gravity models for Lafayette, Indiana and Anderson, Indiana were performed using a negative exponential function to represent the friction factor curve, as shown in Equation B2.

$$F = C e^{-\beta t} \quad (B2)$$

where

F = friction factor at time t

C = constant.

β = calibration parameter.

t = time increment.

The transformed linear regression equation takes the form:

$$\ln(F) = \ln(C) - \beta t \quad (B3)$$

where

$\ln$ = natural log function.

F, C, β , t = as previously defined.

Parameter β is the only coefficient of importance when used in the gravity model. The constant "C" changes the absolute value only; not the relative value. In regression analysis, β would be computed using the following equation:

$$\beta = \frac{\sum tF - \sum t \sum F/n}{\sum t^2 - (\sum t)^2/n} \quad (B4)$$

where

n = the number of data points.

F, t = as defined previously.

Employing Equation B4 the resulting average travel times, for both the Lafayette and Anderson study areas, converged to a value greater than that derived from the origin-destination survey. Relatively simple statistical reasoning soon remedied this problem. The use of only the individual data points, that is, one F value for each time value, infers equal occurrence (importance) to each data point. In simpler terms, it implies that a friction factor at a very low time value or at a very high time value, where few trips occur, has the same degree of importance as the friction factor at the average travel time where many trips occur. Consequently, the data points are weighted on the basis of their frequency of occurrence. The origin-destination survey travel time distribution percentages are thus used to properly weight the data points. The appropriate equation for determining the value of the Beta parameter can, therefore, be written as follows:

$$\beta = \frac{\sum(OD\% \ tF) - \sum(OD\%t) \sum(OD\%F)/100}{\sum(OD\% \ t^2) - (\sum(OD\%t))^2/100} \quad (B5)$$

where

OD% = percentage of trips that occurred with a trip length of time t.

Equation B5 when used in the gravity model calibration of Lafayette and Anderson, Indiana, produced average travel times satisfactorily matching those obtained from the home interview surveys.

The FHWA report PLANPAC/BACKPAC General Information makes no mention of using a weighted regression analysis. Recently, however,

a computer program called ACM (A Gravity Model) has been added to the Urban Transportation Planning System (UTPS) program package in which a type of weighted regression analysis is used in the calibration of gravity models.

APPENDIX C

COMPUTER PROGRAM JMGM

Computer program JMGM was written for the purpose of using the origin zone specific, synthetically self-calibrating gravity model with the UMODEL program of the UTPS package, and therefore it can be used by most planning agencies. A PLANPAC version of JMGM was also coded. Two other computer programs, using the same program logic of JMGM were also written for usage on any computer system with a FORTRAN compiler. One version makes extensive use of core storage and the other uses disk storage much like the UMODEL program. Both can easily be modified for data input and output conformity and therefore can be integrated into other transportation modelling packages.

Program logic flow consists of five major steps in the calibration procedure:

- Step 1. Distribution of trips with the β parameter of the negative exponential friction factor equation equal to zero (all friction factors equal to one) and subsequent computation of the resulting origin zone average travel times.
- Step 2. Distribution of trips with the β parameter equal to a preset value and subsequent computation of the resulting origin zone average travel times.

- Step 3. Using the origin zone average travel times generated by Step 1 and the developed regression equations, predict the origin zone average travel times of the calibrated gravity model.
- Step 4. Using the data points from Steps 1 through 3, compute the origin zone β parameter using linear interpolation. (Assuming a linear relationship exists between origin zone average travel times as β changes.)
- Step 5. Distribute trips using the origin zone specific β values from Step 4 and compute final statistics.

Figure C1 shows a generalized flow chart for program JMGM. This flow chart along with the program listing given in Figure C2 with its numerous COMMENT statements provide a thorough documentation of the program logic. Additional information can be found in Chapter IV.

Figure C1. Generalized Logic Flow Chart for Program JMGM

```

C      PROGRAMMED BY JIM MEKEMSON, PURDUE UNIVERSITY, SEPT. 1972      JMGM 1
C      IMPLEMENTED AT THE INDIANA STATE HIGHWAY COMMISSION BY MIKE FARRISJMGM 2
C      JMGM 3
C      JMGM 4
C      THIS IS THE CODE FOR ENTRY POINT MOD135      JMGM 5
C      JMGM 6
C      JMGM 7
C      ZONAL VARIABLES.      JMGM 8
C      JMGM 9
C      1 - ZONE NUMBER      JMGM 10
C      2 - ZONAL PRODUCTIONS (OR ORIGINS)      JMGM 11
C      3 - ZONAL ATTRACTIONS (OR DESTINATIONS)      JMGM 12
C      4 - TERMINAL TIME, ORIGIN ZONE      JMGM 13
C      5 - INTRAZONAL TRAVEL TIME      JMGM 14
C      6 - ZONAL BETA(IZ)      JMGM 15
C      7 - GRAVITY MODEL DENOMINATOR(IZ)      JMGM 16
C      8 - NEW ITERATED VALUE OF A(IJ)      JMGM 17
C      9 - SUM COMPUTED A(IJ) = SUM(IZ) T(I,J)      JMGM 18
C      10 - ZONAL AVE. AT BETA=0.0      JMGM 19
C      11 - ZONAL VAR. AT BETA=0.0      JMGM 20
C      12 - ZONAL AVE. AT BETA=PBETA(TP) OR FINAL VALUE      JMGM 21
C      13 - ZONAL VAR. AT BETA=PBETA(TP) OR FINAL VALUE      JMGM 22
C      14 - USED ONLY AS WORKING STORAGE, TEMPORARY FRICITION FACTOR      JMGM 23
C      INITIALIZE ZONAL MATRIX      JMGM 24
C      JMGM 25
C      IF((ITNO.NE.1).OR.(IZ.NE.1))RETURN      JMGM 26
C      DO 30 I=5,13      JMGM 27
C      DO 30 J=1,OMAX      JMGM 28
C      30 Z(I,J)=0.0      JMGM 29
C      JMGM 30
C      JMGM 31
C      *****      JMGM 32
C      *****      JMGM 33
C      JMGM 34
C      JMGM 35
C      THIS IS THE CODE FOR ENTRY POINT MOD13C      JMGM 36
C      JMGM 37
C      JMGM 38
C      REAL*4 PBETA(3),FCTR(99)      JMGM 39
C      PBETA(1)=0.15      JMGM 40
C      PBETA(2)=0.30      JMGM 41
C      PBETA(3)=0.30      JMGM 42
C      JMGM 43
C      *****      JMGM 44
C      *****      JMGM 45
C      JMGM 46
C      JMGM 47
C      THIS IS THE CODE FOR ENTRY POINT MOD13D      JMGM 48
C      ORIGIN ZONE SPECIFIC SYNTHETICALLY SELF-CALIBRATING GRAVITY MODEL      JMGM 49
C      INTERNAL-INTERNAL TRIPS ONLY↓      JMGM 50
C      JMGM 51
C      JMGM 52
C      USER PARAMETER INPUT VIA UPARMS      JMGM 53
C      JMGM 54
C      UPARMS(1) = 1      HBW      JMGM 55
C      2      HEO      JMGM 56
C      3      NHB      JMGM 57
C      JMGM 58
C      UPARMS(2) = 1      COMBINED REGRESSION EQUATIONS      JMGM 59
C      2      LAFAYETTE REGRESSION EQUATIONS      JMGM 60

```

Figure C2. Program Code, JMGM, for Usage in UMODEL

C	3	ANDERSON REGRESSION EQUATIONS	JMGM	61
C	4	MUNCIE REGRESSION EQUATIONS	JMGM	62
C			JMGM	63
C		NOTE: OMAX=DMAX= NUMBER OF INTERNAL ZONES ASSUMED.	JMGM	64
C			JMGM	65
C		REAL*4 AVE,UAR,ATT,TUAR,SUM,TRIP,RAVE,SX,SXX,TOT	JMGM	66
C			JMGM	67
C		INITIALIZE OR CONTINUE	JMGM	68
C			JMGM	69
C		IF((ITNO.EQ.1).AND.(IZ.EQ.1))GO TO 101	JMGM	70
C		GO TO 100	JMGM	71
C			JMGM	72
C		COMPUTE TOTAL TRIPS	JMGM	73
C			JMGM	74
C	101	TOT=0.0	JMGM	75
C		DO 20 J=1,OMAX	JMGM	76
C	20	TOT=TOT+Z(2,J)	JMGM	77
C			JMGM	78
C		INITIALIZE	JMGM	79
C			JMGM	80
C		SX=0.0	JMGM	81
C		SXX=0.0	JMGM	82
C		ATT=0.0	JMGM	83
C		TUAR=0.0	JMGM	84
C		CITER=0	JMGM	85
C		AITER=2	JMGM	86
C		DO 21 J=1,OMAX	JMGM	87
C		Z(8,J)=Z(3,J)	JMGM	88
C	21	Z(7,J)=TOT	JMGM	89
C			JMGM	90
C		ITERATION FLAGS FOR DECISION TREE.	JMGM	91
C			JMGM	92
C		CITER=CALIBRATION FLAG	JMGM	93
C		0 - BETA=0.0	JMGM	94
C		1 - BETA=PBETA(TP)	JMGM	95
C		2 - BETA=CALIBRATED VALUE FROM REGRESSION EQUATIONS	JMGM	96
C		AND INTERPOLATION.	JMGM	97
C			JMGM	98
C		AITER=ATTRACTION FLAG	JMGM	99
C		0 - BEGIN ATTRACTION ITERATION	JMGM	100
C		1 - CONTINUE ATTRACTION ITERATION	JMGM	101
C		2 - ATTRACTION ITERATION COMPLETE, I.E. CLOSURE	JMGM	102
C			JMGM	103
C	100	IF(IZ.EQ.1)WRITE(6,50)ITNO,CITER,AITER	JMGM	104
C	50	FORMAT(10X,7HITNO = ,15,10X,8HCITER = ,15,10X,8HAITER = ,15)	JMGM	105
C		IF((CITER.EQ.1).AND.(AITER.EQ.0).AND.(IZ.EQ.1))GO TO 200	JMGM	106
C		IF((CITER.EQ.2).AND.(AITER.EQ.0).AND.(IZ.EQ.1))GO TO 300	JMGM	107
C	99	IF(AITER.EQ.0)GO TO 400	JMGM	108
C		IF(AITER.EQ.1)GO TO 500	JMGM	109
C		GO TO 600	JMGM	110
C			JMGM	111
C			JMGM	112
C			JMGM	113
C			JMGM	114
C		CITER=1,AITER=0,IZ=1 SET BETA(I)'S AND FRICTION FACTORS TO GIVEN	JMGM	115
C			JMGM	116
C	200	TP=UPARMS(1)	JMGM	117
C		DO 201 J=1,99	JMGM	118
C	201	FCTR(J)=EXP(-PBETA(TP)*J)	JMGM	119
C		GO TO 99	JMGM	120

Figure C2. Continued

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C
C ..... JMGM 121
C ..... JMGM 122
C ..... JMGM 123
C ..... JMGM 124
C CITER=2, AITER=0, IZ=1    FIND BETA(I) S FROM REGRESSION EQUA. JMGM 125
C ..... JMGM 126
300 RTP=UPARMS(1)+(UPARMS(2)-1)*3 JMGM 127
ATT=0.0 JMGM 128
TVAR=0.0 JMGM 129
SX=0.0 JMGM 130
SXX=0.0 JMGM 131
DO 19 J=1, OMAX JMGM 132
IF(Z(2,J).EQ.0.0)GO TO 19 JMGM 133
C ..... JMGM 134
C COMPUTE RAUE, AVERAGE FROM REGRESSION EQUATION JMGM 135
C ..... JMGM 136
C AUE = ZONAL AVERAGE TRAVEL TIME. JMGM 137
C AUE0 = ZONAL AVERAGE TRAVEL TIME AT BETA = 0.0 JMGM 138
C VAR = ZONAL TRAVEL TIME VARIANCE AT CALIBRATED BETA VALUE. JMGM 139
C VAR0 = ZONAL TRAVEL TIME VARIANCE AT BETA = 0.0 JMGM 140
C TERM = AVERAGE ORIGIN ZONE TERMINAL TIME. JMGM 141
C ..... JMGM 142
C ..... JMGM 143
C GO TO(1,2,3,4,5,6,7,8,9,10,11,12),RTP JMGM 144
C ..... JMGM 145
C ..... JMGM 146
C COMBINED REGRESSION EQUATIONS-LAFAYETTE, ANDERSON, AND MUNCIE IND. JMGM 147
C ..... JMGM 148
C HBW R**2 = 0.865 JMGM 149
1 RAUE=-0.4327+1.2761*Z(4,J)+0.9921*(Z(10,J)-Z(4,J)) JMGM 150
GO TO 19 JMGM 151
C ..... JMGM 152
C HBO R**2 = 0.877 JMGM 153
2 RAUE=-0.7499+1.3543*Z(4,J)+0.7557*(Z(10,J)-Z(4,J)) JMGM 154
GO TO 19 JMGM 155
C ..... JMGM 156
C NHB R**2 = 0.735 JMGM 157
3 RAUE=0.5245+1.3416*Z(4,J)+0.6073*(Z(10,J)-Z(4,J)) JMGM 158
GO TO 19 JMGM 159
C ..... JMGM 160
C ..... JMGM 161
C LAFAYETTE, INDIANA. JMGM 162
C ..... JMGM 163
C HBW R**2 = .893 AREAWIDE VAR=22.99; VAR0=24.99 JMGM 164
AUE=10.60; AUE0=12.32; VAR=12.17; VAR0=14.00; TERM=1.08 JMGM 165
4 RAUE=-1.0681+1.0934*Z(4,J)+0.9328*(Z(10,J)-Z(4,J)) JMGM 166
GO TO 19 JMGM 167
C ..... JMGM 168
C HBO R**2 = .893 AREAWIDE VAR=13.63; VAR0=25.54 JMGM 169
AUE=8.55; AUE0=11.78; VAR=10.78; VAR0=15.63; TERM=1.11 JMGM 170
5 RAUE=-1.6751+1.1459*Z(4,J)+0.8397*(Z(10,J)-Z(4,J)) JMGM 171
GO TO 19 JMGM 172
C ..... JMGM 173
C NHB R**2 = .799 AREAWIDE VAR=14.05; VAR0=17.75 JMGM 174
AUE=9.12; AUE0=11.42; VAR=9.94; VAR0=12.91; TERM=1.99 JMGM 175
6 RAUE=0.2131+1.2823*Z(4,J)+0.6744*(Z(10,J)-Z(4,J)) JMGM 176
GO TO 19 JMGM 177
C ..... JMGM 178
C ..... JMGM 179
C ANDERSON, INDIANA JMGM 180

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Figure C2. Continued

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C                                     JMGM 191
C   HBW   R**2 = .875   AREAWIDE VAR=9.61;   VAR0=9.51   JMGM 192
C   AVE=11.43; AVE0=12.10; VAR=8.53;   U0=7.12; TERM=2.38   JMGM 193
7   RAUE=-0.4650+1.1306*Z(4,J)+0.9467*(Z(10,J)-Z(4,J))   JMGM 194
   GO TO 19   JMGM 195
C                                     JMGM 196
C   H80   R**2 = .664   AREAWIDE VAR=9.51;   VAR0=11.10   JMGM 197
C   AVE=10.04; AVE0=12.30; VAR=7.84;   VAR0=5.20; TERM=2.37   JMGM 198
8   RAUE=1.0611+1.0640*Z(4,J)+0.6504*(Z(10,J)-Z(4,J))   JMGM 199
   GO TO 19   JMGM 200
C                                     JMGM 201
C   NHB   R**2= .210   AREAWIDE VAR=7.82;   VAR0=9.20   JMGM 202
C   AVE=9.50; AVE0=11.72; VAR=6.35;   VAR0=7.12; TERM=2.64   JMGM 203
9   RAUE=274556+1.0968*Z(4,J)+0.4280*(Z(10,J)-Z(4,J))   JMGM 204
   GO TO 19   JMGM 205
C                                     JMGM 206
C   MUNCIE, INDIANA   JMGM 207
C                                     JMGM 208
C   H8W   R**2 = .772   AREAWIDE VAR=11.16;   VAR0=11.51   JMGM 209
C   AVE=9.07; AVE0=9.92; VAR=7.06;   VAR0=7.11; TERM=1.33   JMGM 210
10  RAUE=0.8919+0.7448*Z(4,J)+0.8491*(Z(10,J)-Z(4,J))   JMGM 211
   GO TO 19   JMGM 212
C                                     JMGM 213
C   H80   R**2 = .749   AREAWIDE VAR=9.27;   VAR0=11.83   JMGM 214
C   AVE=7.48; AVE0=9.53; VAR=6.40;   VAR0=7.79; TERM=1.34   JMGM 215
11  RAUE=0.1603+1.0593*Z(4,J)+0.7162*(Z(10,J)-Z(4,J))   JMGM 216
   GO TO 19   JMGM 217
C                                     JMGM 218
C   NHB   R**2 = .506   AREAWIDE VAR=7.89;   VAR0=9.28   JMGM 219
C   AVE=6.93; AVE0=8.95; VAR=5.65;   VAR0=6.65; TERM=1.49   JMGM 220
12  RAUE=0.7980+1.4192*Z(4,J)+0.5384*(Z(10,J)-Z(4,J))   JMGM 221
   GO TO 19   JMGM 222
C                                     JMGM 223
19  CONTINUE   JMGM 224
C                                     JMGM 225
C   COMPUTE BETA. LINEAR INTERPOLATION.   JMGM 226
C                                     JMGM 227
C   Z(6,J)=(PBETA(TP)/(Z(12,J)-Z(10,J)))*(RAUE-Z(10,J))   JMGM 228
C   IF(Z(12,J).GT.Z(10,J))Z(6,J)=0.0   JMGM 229
C   IF(Z(6,J).GT.0.5)Z(6,J)=0.5   JMGM 230
19  CONTINUE   JMGM 231
   GO TO 99   JMGM 232
C                                     JMGM 233
C   .....   JMGM 234
C   .....   JMGM 235
C   AITER=0, IZ=1   INITIALIZE BEFORE ATTRACTION ITERATION BEGINS.   JMGM 236
C                                     JMGM 237
400  DO 401 J=1,OMAX   JMGM 238
      Z(8,J)=Z(3,J)   JMGM 239
401  Z(9,J)=0.0   JMGM 240
      AITER=1   JMGM 241
C                                     JMGM 242
C   BEGIN ATTRACTION ITERATION.   JMGM 243
C                                     JMGM 244
C   SUMJ A(J)*F(I,J)   JMGM 245
C                                     JMGM 246
500  Z(7,IZ)=0.0   JMGM 247
      DO 501 J=1,OMAX   JMGM 248
        IF(IZ.EQ.J)INT(J,1)=Z(5,J)   JMGM 249

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Figure C2. Continued

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      IF(INT(J,1).EQ.0.0)GO TO 501
      Z(14,J)=1.0
C      ADD TERMINAL TIMES TO SKIM TREE TRAVEL TIME.
      TIME=INT(J,1)+Z(4,IZ)+Z(4,J)
      IF(TIME.GT.99)TIME=99
      IF(CITER.EQ.1)Z(14,J)=FCTR(TIME)
      IF(CITER.EQ.2)Z(14,J)=EXP(-Z(6,IZ)*TIME)
C      SAF = SAF + A * F
      Z(7,IZ)=Z(7,IZ)+Z(8,J)*Z(14,J)
501  CONTINUE
C
C      SUM COMPUTED ATTRACTIONS
C
      DO 502 J=1,OMAX
      IF(INT(J,1).EQ.0.0)GO TO 502
C      SA = SA + P * A * F / SAF
      Z(9,J)=Z(9,J)+Z(2,IZ)*Z(8,J)*Z(14,J)/Z(7,IZ)
502  CONTINUE
C
C      IF IZ.EQ.OMAX, TEST FOR CLOSURE
C      IF IZ.LT.OMAX, CONTINUE ITERATION
C
      IF(IZ.EQ.OMAX)GO TO 700
      RETURN 1
C
C      COMPUTE ERROR FOR CLOSURE TEST
C
700  SUM=0.0
      DO 701 J=1,OMAX
701  SUM=SUM+ABS(Z(9,J)-Z(3,J))
      SUM=SUM/TOT
C
C      ONE PERCENT CLOSURE TEST
C
      IF(SUM.LT.0.01)GO TO 702
C
C      CLOSURE TEST FAILED, COMPUTE NEW ATTRACTIONS.
C
      DO 703 J=1,OMAX
      IF(Z(8,J).EQ.0.0)GO TO 703
      Z(8,J)=Z(3,J)*Z(8,J)/Z(9,J)
703  Z(9,J)=0.0
      RETURN 1
C
C      CLOSURE TEST PASSED. SET AITER=2 FOR AVE. AND VAR. ITERATION.
C
702  AITER=2
C
C      IF CITER=2, SET ITNO=ITER-1 FOR LAST AVE. VAR. ITERATION.
C
      IF(CITER.EQ.2)ITNO=ITER-1
      RETURN 1
C
C.....
C.....
C
C      AVERAGE AND VARIANCE COMPUTATION .
C
C      INITIALIZE ZGNAL AVE. AND VAR.
C

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Figure C2. Continued

600	AUE=0.0	JMGM	301
	VAR=0.0	JMGM	302
	DO 599 J=1,ZONES	JMGM	303
599	TROUT(J,1)=0	JMGM	304
	IF(((CITER.EQ.0).OR.(CITER.EQ.2)).AND.(IZ.EQ.1))GO TO 598	JMGM	305
	GO TO 597	JMGM	306
598	ATT=0.0	JMGM	307
	TUAR=0.0	JMGM	308
	SX=0.0	JMGM	309
	SXX=0.0	JMGM	310
C		JMGM	311
C	BEGIN AVE. AND VAR. COMPUTATION	JMGM	312
C		JMGM	313
C	IF ORIGIN ZONE PRODUCTION=0, SKIP AVE. AND VAR.	JMGM	314
C		JMGM	315
597	IF(Z(2,IZ).LE.1.0)GO TO 605	JMGM	316
	DO 601 J=1,OMAX	JMGM	317
C		JMGM	318
C	COMPUTE CELL INTERCHANGE, I.E. TRIPS.	JMGM	319
C		JMGM	320
	IF(IZ.EQ.J)INT(J,1)=Z(5,J)	JMGM	321
	IF(INT(J,1).EQ.0.0)GO TO 601	JMGM	322
	Z(14,J)=1	JMGM	323
	TIME=INT(J,1)+Z(4,IZ)+Z(4,J)	JMGM	324
	IF(TIME.GT.99)TIME=99	JMGM	325
	IF(CITER.EQ.1)Z(14,J)=FCTR(TIME)	JMGM	326
	IF(CITER.EQ.2)Z(14,J)=EXP(-Z(6,IZ)*TIME)	JMGM	327
C	TRIP = P * A * F / SAF	JMGM	328
	TRIP=Z(2,IZ)*Z(8,J)*Z(14,J)/Z(7,IZ)	JMGM	329
C		JMGM	330
C	SAVE TRIP IN TROUT IF LAST ITERATION, I.E. CITER=2	JMGM	331
C		JMGM	332
C	IF((CITER.EQ.2).AND.(AITER.EQ.2))TROUT(J,1)=TRIP	JMGM	333
C		JMGM	334
C	SUM ZONAL TRAVEL TIMES.	JMGM	335
C		JMGM	336
	AUE=AUE+TRIP*TIME	JMGM	337
601	CONTINUE	JMGM	338
C		JMGM	339
C	ADD ZONAL CONTRIBUTION TO AREA WIDE AVERAGE TRAVEL TIME	JMGM	340
C	IF((CITER.EQ.0).OR.(CITER.EQ.2))	JMGM	341
C		JMGM	342
	IF((CITER.EQ.0).OR.(CITER.EQ.2))ATT=ATT+AUE/TOT	JMGM	343
C		JMGM	344
C	COMPUTE ZONAL AVERAGE TRAVEL TIME	JMGM	345
C		JMGM	346
	AUE=AUE/Z(2,IZ)	JMGM	347
C		JMGM	348
C	SAVE AVERAGE TRAVEL TIME AT BETA=0.0 IF CITER=0	JMGM	349
C		JMGM	350
	IF(CITER.EQ.0)Z(10,IZ)=AUE	JMGM	351
C		JMGM	352
C	SAVE AVERAGE TRAVEL TIME AT BETA=PBETA(TP) OR FINAL VALUE.	JMGM	353
C		JMGM	354
	IF(CITER.NE.0)Z(12,IZ)=AUE	JMGM	355
C		JMGM	356
C	COMPUTE ZONAL VARIANCE AND SAVE DATA FOR AREA WIDE VARIANCE	JMGM	357
C		JMGM	358
	DO 602 J=1,OMAX	JMGM	359
C		JMGM	360

Figure C2. Continued

C	COMPUTE CELL INTERCHANGE, I.E. TRIP	JMGM 361
C	IF(INT(J,1).EQ.0.0)GO TO 602	JMGM 362
	TRIP=Z(2,IZ)*Z(8,J)*Z(14,J)/Z(7,IZ)	JMGM 363
C		JMGM 364
C	ADD SUM OF SQUARES FOR ZONAL VARIANCE	JMGM 365
C		JMGM 366
C	VAR=VAR+TRIP*(TIME-AVE)**2	JMGM 367
C		JMGM 368
C	ADD SUM OF TIME AND SUM OF TIME SQUARED FOR AREAWIDE VARIANCE	JMGM 369
C	IF((CITER.EQ.0).OR.(CITER.EQ.2))	JMGM 370
C		JMGM 371
C	IF(INT(J,1).EQ.0)GO TO 602	JMGM 372
	TIME=INT(J,1)+Z(4,IZ)+Z(4,J)	JMGM 373
	IF(TIME.GT.99)TIME=99	JMGM 374
	IF((CITEP.EQ.0).OR.(CITER.EQ.2))SX=SX+TRIP*TIME	JMGM 375
	IF((CITER.EQ.0).OR.(CITER.EQ.2))SXX=SXX+TRIP*TIME**2	JMGM 376
602	CONTINUE	JMGM 377
C		JMGM 378
C	SAVE ZONAL VARIANCE AT BETA=0.0 IF CITER=0.	JMGM 379
C		JMGM 380
C	IF(CITER.EQ.0)Z(11,IZ)=VAR/(Z(2,IZ)-1.0)	JMGM 381
C		JMGM 382
C	SAVE ZONAL VARIANCE AT BETA=PBETA(TP) OR FINAL VALUE.	JMGM 383
C		JMGM 384
C	IF(CITER.NE.0)Z(13,IZ)=VAR/(Z(2,IZ)-1.0)	JMGM 385
C		JMGM 386
C	IF((CITER.EQ. 0 OR 2).AND.(IZ.EQ.OMAX)) COMPUTE AREAWIDE VARIANCE	JMGM 387
C		JMGM 388
605	IF(((CITER.EQ.0).OR.(CITER.EQ.2)).AND.(IZ.EQ.OMAX))TUAR=	JMGM 389
	1(SXX-SX**2/TOT)/(TOT-1.0)	JMGM 390
C		JMGM 391
C	IF IZ=OMAX INCREMENT CITER BY 1 AND SET AITER=0. STMT 603	JMGM 392
C		JMGM 393
C	IF(IZ.EQ.OMAX)GO TO 603	JMGM 394
C		JMGM 395
C	IF IZ.LT.OMAX, CONTINUE ITERATION.	JMGM 396
C		JMGM 397
C	RETURN 1	JMGM 398
C		JMGM 399
603	CITER=CITER+1	JMGM 400
	IF(CITER.EQ.1)WRITE(6,606)ATT,TUAR	JMGM 401
606	FORMAT(/10X,10HBETA = 0.0/10X,22HAVERAGE TRAVEL TIME = ,F10.3/	JMGM 402
	110X,23HTRAVEL TIME VARIANCE = ,F9.3/)	JMGM 403
	IF(CITER.EQ.3)WRITE(6,604)ATT,TUAR	JMGM 404
604	FORMAT(/10X,22HAVERAGE TRAVEL TIME = ,F10.3/10X,23HTRAVEL TIME VAR	JMGM 405
	1IANCE = ,F9.3/)	JMGM 406
	AITEP=0	JMGM 407
	RETURN 1	JMGM 408
		JMGM 409

Figure C2. Continued

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