SCHOOL OF CIVIL ENGINEERING

JOINT HIGHWAY RESEARCH PROJECT

JHRP-78-24

PERFORMANCE OF BURIED
FLEXIBLE CONDUITS

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PURDUE UNIVERSITY
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Interim Report

PERFORMANCE OF BURIED FLEXIBLE CONDUITS

TO: H. L. Michael, Director
Joint Highway Research Project

FROM: G. A. Leonards, Research Engineer
Joint Highway Research Project

December 12, 1978

File: 9-8-6

Project: C-36-62F

The attached Interim Report titled "Performance of Buried Flexible Conduits" is submitted on the HPR Part II Study "Performance of Pipe Culverts Buried in Soils". The Report is authored by G. A. Leonards and R. E. Stetkar and is an extensive review of the performance of buried flexible culverts, with special emphasis on buckling failure modes. This review was requested in the approval of the current Phase II of the project by FHWA.

The purpose of the review of buckling failure modes was to determine more precisely the nature and scope of research which would be valuable. The review accomplished this purpose and recommendations involving controlled experiments on appropriately instrumented, full sized, buried flexible conduits is presented. This has been the direction planned for continuation of this approved research.

The Report is presented to the Board for acceptance as partial fulfillment of the objectives of the Study. It will be forwarded to ISHC and FHWA for review and comment and similar acceptance.

Respectfully submitted,

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Interim Report
PERFORMANCE OF BURIED FLEXIBLE CONDUITS

by
G. A. Leonards
and
R. E. Stetkar

Joint Highway Research Project

Project No.: C-36-62F

File No.: 9-8-6

Prepared as Part of an Investigation

Conducted by

Joint Highway Research Project
Engineering Experiment Station
Purdue University

in cooperation with the

Indiana State Highway Commission
and the

U.S. Department of Transportation
Federal Highway Administration

The contents of this report reflect the views of the author who is responsible for the facts and the accuracy of the data presented herein. The contents do not necessarily reflect the official views or policies of the Federal Highway Administration. This report does not constitute a standard, specification, or regulation.

Purdue University
West Lafayette, Indiana
December 12, 1978

1. Report No. JHRP-78-24		2. Government Accession No.		3. Recipient's Catalog No.	
4. Title and Subtitle PERFORMANCE OF BURIED FLEXIBLE CONDUITS				5. Report Date December 12, 1978	
				6. Performing Organization Code	
7. Author(s) G. A. Leonards and R. E. Stetkar				8. Performing Organization Report No. JHRP-78-24	
9. Performing Organization Name and Address Joint Highway Research Project Civil Engineering Building Purdue University West Lafayette, Indiana 47907				10. Work Unit No.	
				11. Contract or Grant No. HPR-1(16) Part II	
				13. Type of Report and Period Covered Interim Report Review of Buckling Failure Modes Task	
12. Sponsoring Agency Name and Address Indiana State Highway Commission State Office Building 100 North Senate Avenue Indianapolis, Indiana 46204				14. Sponsoring Agency Code	
15. Supplementary Notes Prepared in cooperation with the U.S. Department of Transportation, Federal Highway Administration. From the HPR Part II Study titled "Performance of Pipe Culverts Buried in Soils".					
16. Abstract An extensive review was made of the performance of buried flexible culverts, with special emphasis on buckling failure modes. All known theories on elastic buckling of rings and cylinders with radial support subjected to external radial pressures were analyzed and compared with each other, and with the results of controlled experiments on small and full-scale culverts. Some relevant field measurements were also investigated. The results showed that buckling is an important failure mode, especially for pipe arches; that controlling deflections (to less than 5%) does not automatically guard against buckling failures; that present methods for predicting performance limits (not restricted to buckling) are poorly developed; that current design criteria are empirically based, and are applicable primarily to corrugated steel culverts up to 12' in diameter supported by compacted, granular backfill; and that much could be gained from additional research involving controlled experiments on appropriately instrumented, full-sized, buried flexible conduits. Recommendations regarding the nature and scope of these experiments are given.					
17. Key Words Culverts; buckling; performance limits; soil-structure interaction.				18. Distribution Statement No restrictions. This document is available to the public through the National Technical Information Service, Springfield, Virginia 22161.	
19. Security Classif. (of this report) Unclassified		20. Security Classif. (of this page) Unclassified		21. No. of Pages 100	
				22. Price	

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LIST OF SYMBOLS

A	Cross-section area of conduit wall
B	Empirical soil support coefficient
C	Soil arching coefficient
D	Conduit diameter
E	Young's modulus (conduit)
E_s	Young's modulus (soil)
F	Thrust force
F_c	Critical buckling thrust force
F_k	Burial depth factor
F_p	Conduit flexibility ratio (Peck)
F_s	Conduit-soil flexibility factor
I	Moment of inertia of conduit wall cross-section
K_s	Modulus of soil support, (FL^{-2})
L	Conduit axial length
M_s	Constrained soil modulus
R	Conduit radius
R_i	Inside radius of supporting soil ring
R_o	Outside radius of supporting soil ring
U	Unevenness factor for imperfections
d	Depth of (soil) burial
f	Conduit thrust stress
f_c	Critical buckling thrust stress
f_y	Yield stress
k_s	Subgrade modulus (FL^{-3}) or modulus of soil reaction

LIST OF SYMBOLS (CONT'D)

k_T, k_R	Tangential and rotational elastic stiffnesses at arch supports
k_x	Load relief spring constant
l	Half-wave length of local buckle
n	Buckling mode
p	Radial soil-conduit interface pressure
p_c	Critical radial buckling pressure
p_s	Vertical live and dead load pressure at crown
p_v	Overburden pressure at conduit crown
r	Radius of gyration
t	Conduit wall thickness
w	Radial conduit displacement
z	Height of soil cover above conduit crown
α, ϕ_o	Angles defining arch sections of conduit
γ	Unit weight of soil
δ	Ring deflection
η	Soil support reduction factor for limited depth of burial
ψ_B	Angle defining distribution of active soil pressure around conduit
λ	Active earth pressure coefficient
σ'	Effective soil stress
ν	Poisson's ratio (conduit)
ν_s	Poisson's ratio (soil)

INTRODUCTION

Flexible conduits are usually designed to guard against five primary modes of failure. As illustrated in Figure 1, these are: (1) wall crushing, which occurs when compressive stresses due only to circumferential thrusts exceed the yield stress, (2) separation of seams, when the thrust exceeds the seam strength, (3) initiation of buckling from an essentially elastic state of stress, (4) inelastic buckling, and (5) excessive deflection, or flattening, due to plastic yielding under combined compressive and bending stresses.

It is comparatively easy to guard against the first two modes of failure, because the magnitude of the circumferential thrust is relatively less sensitive to soil properties or to pipe stiffness. However, bending stresses, which play a special role in failure modes (3), (4) and (5), are very sensitive to both the above parameters, and simplified procedures to account for their effects - for example, by controlling the crown deflection - are not generally applicable. Good progress has been made in recent years in treating culvert problems on a rational basis (Leonards and Roy, 1976; Wenzel and Parmelee, 1976; Katona et al, 1976; Duncan, 1977). Significant advances have been made, and more can be expected in the future.

This report deals with the performance limits of buried flexible conduits, with special emphasis on the buckling failure modes. A literature review was made to trace the development of the present state-of-the-art for predicting buckling of flexible conduits. The assumptions and models used in the theoretical analyses are discussed and evaluated, and their applicability to the problem at hand is

Wall Crushing

Seam Separation

Elastic Local Buckling

Inelastic (Snap-through) Buckling

Plastic Yielding
(Excessive Deflection)

FIGURE 1 FAILURE MODES OF FLEXIBLE CONDUITS

assessed. Comparisons of predictions of buckling pressures by the various theories for some typical culvert geometries and soil strengths are made to show the relative effects of the assumptions used in the respective prediction models. Relevant experimental test results are reviewed and assessed in terms of the test conditions, insofar as these reflect the manner in which culverts behave in situ. Finally, the problems associated with predicting failure conditions for buried conduits are reviewed, highlighting those areas in need of further research.

DEFINITION OF BUCKLING TERMS

In accordance with currently accepted definitions: buckling is the development of a kink, wrinkle, bulge, crease, or other sharp change in original shape; the critical load (thrust, stress, or external pressure) is the load at which two alternative load-deflection relationships are mathematically possible, i.e. the theoretical load at which buckling is initiated under ideal conditions; the buckling load is the load at which a compressed element, or member, buckles in service or in a loading test; an instability is reached when the capacity to resist additional load is exhausted and continuing deformation results in a decrease in load-carrying capacity. Because real structural members have minor geometric imperfections, and loading is never perfectly symmetrical, buckling loads are generally somewhat lower than the critical loads. Experiments have shown that this difference is especially marked for unsupported thin shells subjected to hydrostatic compression; accordingly, it is important to keep in mind that buckling theories inherently tend to overpredict the buckling load.

Buckling of flexible conduits can manifest itself in a number of ways. Elastic buckling is initiated at stress levels below yield. Buckling in a high mode involves a large number of small "waves" distributed around the circumference. Buckling in a low mode also involves the entire circumference but is characterized by a small number of larger waves. Local buckling is a crimp or crease involving a small percentage of the conduit circumference. Because the mechanism of failure is different, the several types of elastic buckling occur at different critical pressures; however, due to geometric imperfections and the presence of residual stresses, even in the case of buckling in theoretically high modes crimping of the conduit wall may be visible only at one or two locations. For this reason it is difficult to distinguish between the modes of elastic buckling solely by visual inspection.

Inelastic buckling is initiated after portions of the conduit wall have undergone plastic yielding and generally occurs in a low buckling mode. A snap-through buckle involves a sudden reversal of curvature or inversion in the conduit wall and could result in an instability if a sufficient fraction of the circumference is involved. On the other hand, although undesirable from a design standpoint, local buckling may not immediately involve instability.

I. BUCKLING THEORIES

The analyses presented in this section treat buckling of thin-walled cylindrical shells or rings. The buckled shape is taken to be constant along the axis of the cylinder, and thus no longitudinal buckling modes resulting from axial forces are considered. Although some solutions for

longitudinal buckling are available (Almroth, 1962; Reynolds, 1962; Weingarten, 1962), to be consistent with the fact that analyses for predicting performance of culverts generally assume plane strain conditions, the buckling theories reviewed herein are concerned with the two-dimensional problem in which sections transverse to the axis of the pipe are assumed to behave identically.

A. Elastic Buckling in High Modes

Timoshenko and Gere (1961)

An early model used to approximate buckling of flexible conduits buried in soil was a circular ring subjected to a uniform hydrostatic external pressure. The solution for the critical external normal pressure, p_c , was derived by Timoshenko and Gere (1961) and given as,

$$p_c = \frac{EI(n^2-1)}{R^3} \quad (1)$$

where, E = Young's modulus

I = moment of inertia of ring wall

R = radius

n = buckling mode = $\pi R/\ell$, where ℓ = half wave length of the buckled shape

When solved in terms of the critical compressive stress, f_c , within the ring, eqn (1) becomes, in the lower bound ($n = 2$)

$$f_c = \frac{3 EI}{R^2 t} \quad (2)$$

t = ring thickness

The above solution resulted from consideration of the equilibrium of a deformed ring element in which only circumferential stresses were considered. Nonlinear deflection terms were also neglected. Application of this solution to large variations in ring dimensions showed that incongruous results were obtained for some selected dimensions because of the linearization simplification.

Brockenbrough (1964)

Brockenbrough (1964) proposed the use of eqn (2) for determining the critical stress of deeply buried, flexible culverts. The depths for which the analysis was deemed applicable were to exceed those at which live loads on the soil surface had a significant effect on the culvert. From field experience a depth of ten feet was chosen as a limiting value, independent of the culvert span. Brockenbrough believed that for the above conditions the compressive stress in a culvert could be calculated from the expression for simple hoop compression, as suggested by White and Layer (1960),

$$f = \frac{p_v R}{t} \quad (3)$$

f = circumferential compressive stress

p_v = vertical overburden pressure at crown = γz

where, γ = total unit weight of soil

z = height of soil cover above the crown

The compressive stress calculated by eqn (3) could then be compared to the critical stress obtained by eqn (2) to determine the maximum allowable height of soil cover for a given conduit stiffness and soil density.

Approximating the uniform external radial pressure by the nominal overburden pressure (γz) was thought to be conservative when considering flexible conduits, because soil arching was apt to reduce the normal pressure reaching the conduit to a value lower than γz .

In developing his suggested maximum fill heights above buried conduits of given thickness and corrugation profile, Brockenbrough reasoned that the weight of the overlying soil would deflect or flatten the culvert and cause redistribution of soil pressure to a uniform value, but he did not indicate how much deflection was necessary to develop the full redistribution. He also realized that Timoshenko and Gere's solution did not consider flattening at the crown and invert, which would decrease the buckling resistance. Brockenbrough thought that the use of a "conservative" equivalent uniform pressure ($p_c = \gamma z$) would more than offset the neglect of initial deflections. In addition, he believed that localized yielding of the outer fibers of the culvert wall due to combined bending and compressive stresses would probably not cause failure. Thus, when the ring stress calculated in eqn (3) reached the critical stress formulated by Timoshenko and Gere, the conduit would fail by elastic buckling.

Valentine (1964)

Valentine (1964) proposed an analysis to estimate the critical buckling load on shallow (less than one diameter of soil cover at the crown) flexible conduits subjected to live loads, assuming that a portion of the top of the conduit acted as a two-hinged curved bar. With this assumption, the formula for arch buckling under a uniform normal pressure (Roark, 1954) could be used to determine the

critical pressure acting on an appropriate section of the conduit, which could be represented by an equivalent arch,

$$p_c = \frac{EI}{R^3} \left[\frac{\pi^2}{\alpha^2} - 1 \right] \quad (4)$$

p_c = uniform critical pressure acting on the arch section

2α = central angle forming the arch, in radians, $0 < \alpha < \frac{\pi}{2}$

The critical pressure calculated by eqn (4) was correlated to an equivalent surface load for design considerations. The distribution of load on the conduit, reflected by α , was found to be a function of the depth of burial and conduit radius for a given surface loading. Some suggested values of α were back-calculated from a series of field tests on shallow buried conduits up to 54 inches in diameter, surrounded by dense sand and subjected to the AASHTO H-20 design loading; when the depth of soil cover was one radius, α equalled 0.62 and 0.69, respectively, for 54 in. and 24 in. diameter conduits. To be conservative, Valentine suggested the use of a safety factor equal to two with the analysis. He concluded that buckling in conduits with shallow burial was critical for typical conduits if the diameter was greater than 42 inches. However, no limiting soil depth was suggested to differentiate shallow from deep burial, no general expression for calculating α was given, and the validity of the equivalent uniform pressure distribution used by Valentine is questionable. Nevertheless, this marked one of the first attempts to differentiate between the behavior of shallow and deep conduits.

Bodner (1958)

Bodner (1958) extended the von Mises theory of shell buckling, which calculated the critical buckling pressure of a long cylinder

subjected to uniform normal pressure, to consider uniform constant-direction boundary pressures. As in the von Mises solution, biaxial stresses were considered and inextensional buckling, in which the neutral axis of the shell wall remains constant in length, was assumed. However, Bodner's solution allowed the direction of the boundary pressures to remain constant, while the normal pressures in the von Mises' case were forced to change direction in order to remain perpendicular to the deflecting shell. The resulting expressions for the critical buckling pressures included a factor, $\frac{1}{1-\nu^2}$, not included in the ring buckling expressions.

$$p_c = \frac{3 EI}{R^3 (1-\nu^2)} \quad \{\text{von Mises}\} \quad (5a)$$

$$p_c = \frac{4 EI}{R^3 (1-\nu^2)} \quad \{\text{Bodner}\} \quad (5b)$$

ν = Poisson's ratio

From equations (5) Bodner concluded that, in general, a uniform constant-direction pressure is less critical than a uniform normal pressure.

Armenakas and Herrmann (1963)

Development of buckling analyses continued with the work of Armenakas and Herrmann (1963), who derived a more general solution that considered buckling of cylinders of finite length under uniform constant-direction and uniform normal pressures. The effect of the buckling mode on the critical pressure was also accounted for. In the analysis, the final deformed configuration was attained after the shell passed through an intermediate equilibrium state of initial stress from an original

unstressed condition. In the equilibrium equations, initial displacements accompanying the initial stress state were included, and it was found that neglect of these displacements in previous analyses produced some error. However, the initial displacements could be neglected without error in the equations of motion determining the buckled configuration, and this allowed a closed form solution for critical normal and constant-direction buckling pressures. Armenakas and Herrmann found that for finite length cylinders with high buckling modes, the critical constant-direction buckling pressure was essentially equal to the critical normal buckling pressure. In cylinders with large length to radius ratios ($\frac{L}{R} > 50$) and relatively low buckling modes, the critical constant-direction buckling pressure was greater than the critical normal buckling pressure, as given by,

$$P_c = K^* \frac{EI}{(1-\nu^2)R^3} \quad (6)$$

$$K^* = \frac{n^2}{(1 + \frac{t}{R})} \text{ for constant-direction pressure}$$

$$K^* = (n^2 - 1) \text{ for normal pressure}$$

n = buckling mode

Equation (6) reduces to eqn (5a) when the K^* expression for normal pressure is used and $n = 2$. For thin shells ($\frac{t}{R} \ll 1$), eqn (6) also reduces to eqn (5b) when $n = 2$.

The general buckling equation for $\frac{L}{R} < 50$ showed that the critical buckling pressure for a finite length cylinder was greater than that for a cylinder of infinite length. Thus, the use of eqn (6) will give conservative estimates for the critical buckling pressures, provided the loading and deformations are strictly two-dimensional.

Donnell (1956)

Donnell investigated the lack of agreement between experimentally measured external buckling pressures of cylindrical shells and theoretically calculated critical pressures. Experimental buckling pressures were often much less than the theoretically predicted values, and the observed buckling modes were much lower than the predicted modes. Although these discrepancies had been attributed to geometric and material imperfections, no attempt had previously been made to relate the decrease in buckling resistance to the degree of imperfection. Donnell considered imperfection effects on shell buckling by modifying the extended von Mises theory considering finite shell lengths to include an unevenness factor in the load-deformation equations. The unevenness factor represented the expected geometric imperfection in an unloaded shell, which is dependent on the shell material and the manufacturing process. The imperfection was represented by an initial strain, and the critical pressure was then calculated as a function of the shell geometry, yield strength, elastic modulus, and unevenness factor, U .

The results showed that the normalized critical buckling pressure $\frac{p_c}{E t} \frac{R^{3/2} L}{5/2}$ varied with geometry and imperfection level as follows:

- (1) the buckling pressure decreases as the shell length increases, for any given unevenness factor, U .
- (2) As the flexibility ($\frac{R}{t}$) of a shell increases, the influence of the imperfection level on the critical buckling pressure becomes less pronounced.

Figure 2 shows a set of typical results from Donnell's solutions for long tubes, $\frac{L}{\sqrt{Rt}} > 9$. The important influence of the yield stress on the buckling pressure is clearly indicated. Tests performed at the David Taylor Model Basin showed back-calculated unevenness factors to range typically from $U = 0.0001$ to 0.0005 in smooth steel tubes with $\frac{R}{t}$ between 75 and 250.

Cheney (1963)

Cheney (1963) analyzed the stability of a circular ring under plane stress conditions, subject to circumferential support by elastic springs and acted upon by a uniform external pressure on the ring wall of constant magnitude and direction. Initial strain prior to buckling was neglected. Cheney attempted to derive a more general solution by retaining nonlinear energy terms throughout the analysis. His results showed that circumferential support of the ring forced it to buckle in higher modes and at higher pressures than the unsupported case. Although Cheney's representation of support by elastic springs was an oversimplification, it opened new avenues of analysis that could model more closely the effect of soil restraint on the critical buckling pressure. In the solution, a uniform external pressure distribution was assumed to develop around the cylinder after very small or negligible ring deformation. This was necessary to obtain a closed form solution, as given by eqn (7).

$$p_c = \frac{EI}{R^3} (n^2 - 1) + \frac{k_s R}{n^2 - 1} \quad (7)$$

k_s = coefficient of elastic reaction, or 'subgrade modulus' (FL^{-3})

FIGURE 2 CRITICAL PRESSURE AS A FUNCTION OF GEOMETRY AND IMPERFECTION LEVEL (after DONNELL, 1956)

Forrestal and Herrmann (1965)

Forrestal and Herrmann (1965) extended Cheney's analysis to consider cylindrical shells subjected to a uniform, constant-direction, external pressure on the shell wall and supported by a continuous, elastic medium rather than discrete springs. The theory of elastic buckling in the presence of initial stress and displacement was used in the general solution of the complicated boundary value problem. At the critical pressure, p_c , a deformed equilibrium position was obtained, which induced surface tractions on the shell and caused changes in the initial uniform pressure distribution. These surface tractions were related to the shell displacements, and equations developed by Armenakas and Herrmann (1963) were used to determine the expression for the critical pressure causing buckling which, for the fully bonded case, is given by,

$$\frac{p_c}{p_o} = \frac{n^2(n^2-1)(3-4\nu_s)/3 + \frac{E_s/(1+\nu_s)}{3EI/R^3(1-\nu^2)} \left[2n(1-\nu_s) + (1-2\nu_s) \left(\frac{n^2 t}{R} - 1 \right) \right]}{(n^2-1)(3-4\nu_s) + 2n(1-2\nu_s) + 1} \quad (8)$$

p_o = critical, uniform hydrostatic pressure for buckling of unsupported cylinders, as expressed by eqn (5a)

E_s = Young's modulus of supporting medium

ν_s = Poisson's ratio of supporting medium

A solution was also obtained for no interface friction, as given by,

$$\frac{p_c}{p_o} = \frac{n^2-1}{3} + \frac{E_s/(1+\nu_s)}{3EI/R^3(1-\nu^2)} \left[\frac{1}{(1-2\nu_s)(n+1)+n} \right] \quad (9)$$

The minimum (or lower bound) buckling pressure for an elastically supported cylinder with given support and stiffness properties is obtained by substitution of buckling mode numbers into the above equations until the minimum $\frac{p_c}{p_o}$ ratio is calculated, in effect, minimizing eqns (8) and (9) with respect to n .

Equation (8) shows that the critical buckling load increases only 15% as Poisson's ratio of the supporting medium is changed from 0 to $\frac{1}{2}$, and that the critical buckling pressure for frictionless interface conditions is only about 70% as large as the critical pressure for fully-bonded interface conditions. Thus, Forrestal and Herrmann's theory indicates that slip at the conduit-soil interface decreases the critical buckling pressure significantly.

Luscher (1966)

In his study of buckling of soil-surrounded tubes, Luscher (1966) formulated a semi-empirical solution for the critical, uniform, radially applied buckling pressure acting on the wall of a buried conduit. Luscher's theoretical analysis considered buckling of an elastically supported ring, which he believed modeled the behavior of a long tube with radial elastic support. The elastic support in Luscher's model was identical to that in Cheney's (1963) representation, and consequently, the equation developed by Luscher was identical to eqn (7).

$$p_c = \frac{EI(n^2-1)}{R^3} + \frac{K_s}{n^2-1} \quad (10)$$

K_s = modulus of soil support = $k_s R$ (FL⁻²)

Equation (10) was then minimized with respect to n , and an expression for the lower-bound critical buckling pressure was obtained,

$$p_c = 2 \sqrt{K_s EI/R^3} \quad (10i)$$

Luscher's treatment varied from Cheney's in that an empirical expression was proposed for the modulus of soil support based on the results of buckling tests on 1.63 inch diameter tubes and oedometer tests on the supporting sand. The relationship between the modulus of soil support, K_s , obtained from buckling tests, and the constrained modulus, M_s , obtained from oedometer tests, was given as

$$K_s = BE_s = \frac{B(1+\nu_s)(1-2\nu_s)}{1-\nu_s} M_s \quad (11)$$

$$B = \frac{1 - \left(\frac{R_i}{R_o}\right)^2}{(1+\nu_s) \left[1 + \left(\frac{R_i}{R_o}\right)^2 (1-2\nu_s) \right]} \quad (12)$$

R_i = inside radius of elastic ring of soil support

R_o = outside radius of elastic ring of soil support

E_s = Young's modulus of soil

M_s = constrained soil modulus

Comparison of results from tests conducted by Luscher to predictions of critical buckling pressure by eqn (8) showed that if the 'experimental' values of n are used, eqn (8) predicted critical pressures much larger than those measured while eqn (10i) accurately predicted the buckling loads. Agreement of the experimental results with eqn (10i) is not surprising, because the expressions for K_s were derived from the experimental results. Luscher recognized this and mentioned that eqn (10i) may not be accurate for some ranges of tube and soil properties

because; (1) inelastic buckling was not considered, (2) experimental conditions may have placed artificial constraints on the tube's behavior, and (3) the test results may not be scalable to larger tubes.

Based on his studies, Luscher believed that elastic buckling rather than wall crushing might be the controlling mode of failure for thin-walled, smooth metal tubes even when supported by dense soil. However, failure would occur in a higher buckling mode and at a pressure much larger than that predicted by buckling models without circumferential soil support.

Duns and Butterfield (1971)

In developing theoretical stability equations for long cylindrical shells in an elastic medium, Duns and Butterfield used many of the same assumptions employed by Forrestal and Herrmann (1965) except that:

- (1) nonlinear terms in the stability equations were neglected
- (2) displacements associated with initial stresses were not considered
- (3) buckling in high modes was required ($n \geq 7$).

Essentially, Bodner's solution was extended to include the effects of elastic support. As in previous theories, a uniform constant-direction pressure bearing on the conduit produced a state of pure hoop compression in the conduit wall. When the thrust, F_c , reached a critical value, given by eqn (13), failure would occur by elastic buckling in high modes.

$$F_c = \frac{EI}{(1-\nu^2)} \left(\frac{n}{R} \right)^2 + k_s \left(\frac{R}{n} \right)^2 \quad (13)$$

F_c = critical compressive thrust (FL^{-1})

k_s = modulus of soil reaction (FL^{-3})

Duns and Butterfield took advantage of their simplifying assumptions to derive a theoretical expression for the modulus of soil reaction, k_s , as a function of the soil properties, the radius of the conduit, and the mode of buckling, given by

$$k_s = \frac{E_s (n^2 - 1)}{R(1 + \nu_s) \{2n + 1 - 2\nu_s (n + 1)\}} \quad (\text{bonded soil-pipe interface}) \quad (14a)$$

Equation (14a) was determined by means of the Michell-Airy stress and displacement functions. An approximate expression for the modulus of soil reaction at a frictionless soil-pipe interface was also derived by minimizing the potential energy, giving

$$k'_s = \frac{k_s}{4} \quad (\text{frictionless interface}) \quad (14b)$$

The mode corresponding to the critical buckling pressure is determined using the appropriate eqn (14) and eqn (15) below,

$$n^2 (n + 1)^2 = \frac{k_s (1 - \nu_s^2) R^4}{EI} \quad (15)$$

The k_s modulus is initially determined (from eqn 14) for an assumed n and then substituted into eqn (15) to calculate the buckling mode. The iterative procedure ends when the assumed value equals the buckling mode number calculated from eqn (15).

Duns and Butterfield also obtained an equation relating the critical compressive thrust in the conduit to a uniform pressure, p_v , applied over the soil surface. This was accomplished by using an empirical expression derived from finite element analyses that related the compressive thrust to the surface soil pressure,

$$F_c = \frac{3(1-\nu_s) R p_v}{2-\nu_s} \quad (16)$$

This expression does not consider the conduit stiffness and appears to be valid only for the case of neutral arching. It differs from eqn (3) (White and Layer, 1960) in that it includes an effect due to Poisson's ratio of the soil.

A depth of burial influence factor, similar to the B factor developed by Luscher (eqn 12), was also incorporated into the analysis.

$$F_k = \frac{1 - \left(\frac{R_i}{R_o}\right)^2}{1 + \left(\frac{R_i}{R_o}\right)^2 (1-2\nu_s)} \quad (16a)$$

F_k = burial depth factor

Substituting eqns (14a) and (16) into eqn (13), assuming $n \gg 1$, and minimizing with respect to n , the lower-bound critical soil overpressure is given by

$$p_v = \frac{2-\nu_s}{(1-\nu_s)} \frac{F_k \cdot E_s}{4n(1-\nu_s^2)} \text{ (bonded interface)} \quad (17)$$

where the critical buckling mode is given by,

$$n = \left[\frac{F_k E_s R^3 (1-\nu_s^2)}{4(1-\nu_s) EI} \right]^{1/3} \quad (17a)$$

While their analysis assumed a constant compressive thrust in the conduit wall, Duns and Butterfield believed that there was little difference between the buckling stress of a conduit under uniform load and one under non-uniform load, as long as the maximum thrust developed in the second case was equal to the uniform thrust in the first case. The development of bending stresses under non-uniform loads (or of initial deflections to relieve these stresses), which could significantly reduce the critical buckling stress, was overlooked.

Chelapati and Allgood (1972)

In order to represent the boundary conditions at a culvert-soil interface more closely, Chelapati and Allgood (1972) attempted to derive a more rigorous theoretical solution for buckling of deeply buried culverts under a uniform external pressure, and to verify the solution with extensively instrumented model tests. The theoretical solution, using a bonded interface and minimum total potential energy expressions from large deflection theory, treated two cases of soil support. The first case assumed continuous elastic radial support around the entire circumference, and the second case involved soil support only on outward deflecting lobes of circumferential buckling waves, thus decreasing the critical buckling pressure. Luscher's (1966) expression for K_s {eqn (11)} was used directly by Chelapati and Allgood to describe the soil support properties; however, Chelapati and Allgood applied the analysis to a shell rather than a ring. The resulting general solution for the critical uniform external pressure acting on a culvert with continuous circumferential support was given as

$$p_c = \frac{(n^2-1)EI}{(1-\nu^2)R^3} + \frac{K_s}{(n^2-1)} \quad (18)$$

which is identical to eqn (10), except for the $(1-\nu^2)$ term. In the case where soil support is only on outward deflecting lobes, K_s in eqn (18) is halved. Minimizing eqn (18) with respect to n and assuming $\nu = 0.3$ gives

$$p_c = 2.1 \sqrt{K_s EI/R^3} \quad (18i)$$

The corresponding expression for the lower bound buckling pressure of a culvert supported by soil on outward deflecting lobes only is given by

$$p_c = 1.5 \sqrt{K_s EI/R^3} \quad (18ii)$$

It is to be noted that eqns (18i) and (18ii) are lower bound solutions and should be expected generally to yield conservative values for the critical buckling pressure. Indeed, eqn (18i) was found to give a conservative prediction of the results from small scale laboratory tests conducted by Allgood and Ciani (1968).

Cheney (1976)

Cheney (1976) revised his derivation of eqn (7) and obtained an expression for buckling of flexible tubes under a uniform external pressure and supported by an elastic continuum. He derived an expression for k_s which previously had been obtained approximately (Duns and Butterfield) or determined semi-empirically (Luscher). The resulting form of this expression is given by,

$$k_s = \frac{E_s (n^2 - 1)}{R(1 - v_s^2)(2n + 1 - v_s)} \quad (19)$$

E_s = secant elastic modulus of soil

Cheney's expression for the critical uniform buckling pressure is given by

$$p_c = \frac{EI}{R^3} \frac{(n^2 - 1)}{(1 - v_s^2)} + \frac{E_s}{(2n + 1 - v_s)(1 - v_s^2)} \quad (20)$$

which is then minimized with respect to n to give,

$$p_c = 1.2 \left[\frac{EI}{(1 - v_s^2)R^3} \right]^{1/3} \left[\frac{E_s}{(1 - v_s^2)} \right]^{2/3} \quad (20i)$$

where p_c in eqn (20i) is the lower bound, critical buckling pressure. However, when this equation was used to predict the test results obtained by Allgood and Ciani (1968), it made unconservative predictions. Cheney believed that soil representation by the elastic secant modulus caused this overprediction of critical pressure, and he proposed the use of a tangent elastic soil modulus related to the stress state in the soil around the tube to provide a more valid predictive model. Cheney suggested the use of 'Critical State Soil Mechanics' to estimate a tangent modulus value, but these concepts were not sufficiently developed for practical use at the time.

By reducing the modulus E_s in eqn (20i) to 1/8 the secant value eqn (21) resulted, which fit the experimental results of Allgood and Ciani quite well. Cheney concluded that the effective tangent modulus must be one-eighth the value of the secant modulus and proposed eqn (21) as a design formula.

$$p_c = 0.30 \left[\frac{EI}{R^3 (1-\nu_s^2)} \right]^{1/3} \left[\frac{E_s}{(1-\nu_s^2)} \right]^{2/3} \quad (21)$$

The validity of Cheney's empirical adjustment of E_s remains to be proven by further tests in which the soil stress state at the onset of buckling is known and a corresponding tangent modulus can be determined independently.

Meyerhof and Baikie (1963)

Meyerhof and Baikie extended plate buckling theory developed by Timoshenko and Gere (1961) to approximate the buckling behavior of elastically supported cylindrical shells. This approach, although developed before the more rigorous shell buckling theories that considered soil support were formulated, is used in current design practice.

In the theoretical development, solutions for buckling of long flat plates supported by a Winkler foundation (independent springs) with a constant subgrade modulus were combined with the assumption that the critical buckling shape of a cylindrical shell approaches that of a flat plate at very high buckling modes. Thus, eqn (2) developed by Timoshenko and Gere was combined with the equation for the critical buckling force, F_c , of an elastically supported long flat plate to obtain,

$$F_c = \frac{EI}{(1-\nu^2)R^2} \left[(n+1)^2 - 1 + \frac{(1-\nu^2)k_s R^4}{\{(n+1)^2 - 1\}EI} \right] \quad (22)$$

k_s = subgrade modulus

The subgrade modulus, k_s , was then related to the soil modulus, E_s , and Poisson's ratio, ν_s , by the following approximate relationship,

$$k_s = \frac{E_s}{2(1-\nu_s^2)R} \quad (23)$$

Substitution of eqn (23) into eqn (22) yields,

$$F_c = \frac{EI}{(1-\nu^2)} \left(\frac{(n+1)^2 - 1}{R^2} \right) + \frac{E_s}{(1-\nu_s^2)} \frac{R}{2\{(n+1)^2 - 1\}} \quad (24)$$

which reduces approximately to eqn (13) developed by Duns and Butterfield for large values of n .

Equation (24) was minimized with respect to n and divided by the cross-sectional wall area to obtain an expression for the critical buckling stress, f_c , valid for

$$\frac{EI}{R^3 E_s} < \frac{1}{15} \quad (25)$$

and given by,

$$f_c = \frac{2}{A} \sqrt{\frac{EI k_s}{(1-\nu^2)}} \quad (26)$$

where, A = x-sectional area of the wall

Eqn (26) is equivalent to eqn (18i) developed by Chelapati and Allgood.

An empirical modification of eqn (26) was then made, similar to the Rankine-Gordon equation developed for column buckling, to extend its applicability to inelastic buckling of less flexible conduits. The resulting expression,

$$f_c = \frac{f_y}{1 + f_y A \sqrt{\frac{(1-\nu^2)(1-\nu_s^2)R}{2 E_s EI}}} \quad (27)$$

where f_y = yield stress

gives much lower critical stresses than eqn (26), and it was considered that the additional safety factor could compensate for possible conduit imperfections. However, the validity of the empirical adjustments has not been verified for conduits with soil support.

Meyerhof (1968) suggested adjustment of eqn (27) for analyzing culverts with depth of cover less than one diameter by using a reduced soil modulus given by Luscher's eqn (11) and multiplying the critical buckling stress by a reduction factor η accounting for the loss of soil support due to shallow cover, where

$$\eta = \frac{z}{2R} \quad (28)$$

$$0.1 < \frac{z}{2R} < 1.0$$

where z = depth of soil cover above crown

B. Elastic Local Buckling

Amstutz (1953)

Amstutz (1953) presented a solution for the critical uniform external hydrostatic pressure producing a local inward buckle in a flexible cylindrical shell rigidly restrained from outward buckling. Amstutz considered bending stresses as well as thrust stresses that developed in the shell wall during buckling. Calculation of the bending stress along the buckled section was accommodated by defining the buckling displacements as a cosine function. From the known buckled shape the radius of curvature and the corresponding bending stress could be obtained at each point. The extreme fiber stress was not allowed to exceed the yield stress of the shell material and, thus, the local buckling criterion was defined. The critical normal stress was determined by the following lower bound ($n = 1$) solution,

$$\frac{f_c (1-v^2)}{E} \left[1 + 12 \left(\frac{R}{t} \right)^2 \frac{f_c (1-v^2)}{E} \right]^{3/2} = \frac{2R}{t} \frac{(f_y - f_c) (1-v^2)}{E} \left[1 - \frac{R}{t} \frac{(f_y - f_c) (1-v^2)}{E} \right] \quad (29)$$

where f_y = yield stress

f_c = critical circumferential thrust stress

This critical thrust stress can be related to the critical external hydrostatic pressure by eqn (3).

Cheney (1971)

Cheney (1971) reviewed Luscher's analysis and discussed some unsatisfactory aspects of his results. Luscher's K_g values did not seem to reflect effects of soil density on the supporting capacity of

the soil. Back-calculation of K_s for loose and dense sand from the model buckling tests should have yielded different values but, in fact, they were found to be essentially the same. Cheney believed this discrepancy was probably due to the in-place density of the sand in the tests changing to a value between the dense and loose states before the full load was applied to the soil, and thus the soil density at buckling was essentially the same for all the laboratory tests. However, such behavior may not have been possible in the small scale at which the tubes were tested, because the strains associated with buckling of small tubes are too small to induce appreciable changes in sand density.

Cheney also pointed out some conditions in the tests conducted by Luscher that may not have been properly modeled by his theory.

- (1) External pressures exerted on the tubes originated from two sources; pore pressures from water-filled voids of the soil cylinder surrounding the tube and effective stresses from the soil. These two types of load represented hydrostatic and non-hydrostatic pressures, respectively, while Luscher's theory only considered constant-direction loads.
- (2) The tubes tested by Luscher generally failed after the formation of a snap-through, local buckle. Theoretically, failure was assumed to take place in high buckling modes involving the entire tube circumference. Thus, Luscher's theory was not

consistent with the observed buckling mode, which strongly mitigates against its general applicability. Also, the effect of local inward displacements on the pressure distribution around the tube was not considered in Luscher's analysis.

Cheney attempted to model theoretically the local buckling behavior of a buried conduit subjected to an initial uniform radial pressure. As illustrated in Figure 3, he assumed rigid boundary restraints preventing outward buckling, as was the case in Amstutz's theory. However, a solution for the critical uniform external radial pressure initiating a local buckle was obtained by minimizing the energy of distortion into the buckled shape to determine the governing equilibrium condition. The total energy of distortion was given as the sum of the internal strain energy due to extension and bending minus work done by the boundary forces and moments. Unlike Amstutz's model, in which the boundary hydrostatic pressure remained constant during buckling, Cheney attempted to represent the true behavior of the soil at the location of inward deflection. As the deflection increased the boundary pressure decreased according to a linear force-deflection relationship (Figure 3). A graphical procedure was developed to obtain solutions to the complex differential equations applicable to these special boundary conditions. The final shape of the symmetric buckle was determined by the equilibrium equations and was not arbitrarily taken to be a specific cosine function (as assumed by Amstutz).

Cheney noted that Luscher's soil support modulus varied experimentally with the thickness of the supporting soil ring and the effective soil pressure producing buckling. He subsequently developed an empirical expression for K_s given as,

FIGURE 3 DISTRIBUTION OF CRITICAL PRESSURE DURING LOCAL BUCKLING OF SOIL-SURROUNDED TUBES (after CHENEY, 1971)

$$K_s = 720 \sigma' \left[\frac{R_o - R_i}{R_i} \right] 0.153 \quad (30)$$

where, σ' = effective soil pressure at the conduit soil interface

R_o, R_i = outside and inside radii of the ring of soil support
surrounding the conduit

When the results from Cheney's analysis were compared with Luscher's experimental data they overestimated the observed critical buckling pressures. Although a rigid boundary condition appears severe, Cheney blamed the disagreement on imperfections in the loaded tubes that were not accounted for theoretically and noted that the trends of buckling behavior predicted by his theory paralleled the trends shown by the experimental results. By using an effective wall thickness to account for structural imperfections, Cheney was able to fit his theoretical results to the experimental values. However, Cheney conceded that more research was needed before his theory could be used practically to predict local buckling. He recommended full-scale laboratory testing of conduits to obtain more realistic notions of the importance of local structural imperfections in the pipe. Also, results from full-scale buckling tests could verify the validity of the soil support modulus expressions obtained from small-scale tests.

El-Bayoumy (1972)

El-Bayoumy analyzed the critical uniform pressure producing snap-through buckling in a rigidly confined, flexible circular ring with boundary conditions identical to those in the problem treated by Cheney (1971). He also used energy methods to obtain the equilibrium equations

for the unbuckled configuration. However, the boundary pressure was not reduced at the location of buckling. Rather, a snap-through buckle was assumed to occur when the critical pressure was reached, and the rapidity of the mechanism was such that a reduced pressure at the buckle location did not affect the buckling energy. Differential equations were developed using variational calculus which related the strains and rotations of the buckled configuration to the circumferential stress in the ring. Solution of these equations gave lower bound buckling loads identical to those obtained by Cheney (1971) for the constant pressure case ($k_x = 0$). However, the detailed development revealed some important energy considerations clarifying the snap-through buckling phenomenon.

As an increasing external pressure is applied to a rigidly confined ring, the ring contracts in hoop compression and the stored strain energy increases. When the strain energy reaches the level corresponding to the final buckling state the ring has a tendency to buckle and reach a lower energy state, but in order to do so it must pass through an initial, small deflection buckling phase which exists at a higher energy level. An external disturbance or a ring imperfection is required to overcome the energy barrier that otherwise prevents the initiation of buckling. If such a disturbance is provided, the ring passes rapidly from the unbuckled to the initial buckling state, and then to large deflection buckling, with an accompanying sudden release of energy. To obtain his lower-bound solution for the critical external pressure, El-Bayoumy assumed that an imperfection existed which would allow snap-through buckling to occur when the strain energy of the unbuckled ring equalled that of large deflection buckling equilibrium. Theoretically, snap-through buckling will not occur in perfect

rings free from external disturbance. Consequently, flexible conduits without imperfection subjected to uniform external pressure should fail by some other mechanism such as elastic buckling in high modes. The fact that snap-through buckling occurs more frequently in practice indicates that geometric and material imperfections (and possibly residual stresses) play an important role in conduit performance.

Klöppel and Glock (1970)

Extensive investigations of buried flexible conduits by Spangler (1941) led to the conclusion that vertical deflections on the order of 20% of the diameter would generally induce an instability in a conduit. Spangler reasoned that if the deflections were limited to 5 percent a substantial safety factor against instability would be provided, and this criterion has been used extensively in culvert design. Klöppel and Glock (1970) set out to develop a rational method of predicting the instability of initially deflected, buried conduits. They distinguished two zones of interaction: one located in the upper portion where the conduit wall deflects away from the surrounding soil and is subjected to "active" earth pressure, and the other on the lower portion where the conduit wall presses into the surrounding soil and is subjected to "passive" earth pressure. Because of the reduced soil support of the upper section, it was reasoned that an instability is most likely to occur near the crown of the conduit. Klöppel and Glock chose to model the upper conduit section by a hinged arch with radial elastic (soil) support $\left[\frac{E_s}{R(1 + \nu_s)} \right]$ along its circumference and tangential and rotational elastic support (k_T and k_R) at the hinges, thereby representing soil support and restraint provided by the lower conduit section. The ensuing analysis considered the following conditions:

- (a) Elliptical as well as circular conduits
- (b) Friction at the conduit-soil interface
- (c) Nonuniform radial pressures having a maximum value at the crown (so that buckling was always initiated at the crown)
- (d) Modulus of soil restraint either constant or stress-level dependent
- (e) Symmetric displacements prior to buckling
- (f) The influence of a plastic hinge at the crown.

Nonlinear terms in the stability expressions were retained and energy methods employing variational calculus were used to analyze the following three representations of potentially unstable flexible conduits:

- (1) two-hinged arch with rotational and tangential restraint at the hinges,
- (2) two-hinged arch neglecting rotational restraint at the hinges, and
- (3) three-hinged arch (one hinge at the crown) with no rotational restraint at the hinges (to model the buckling resistance of a conduit after formation of plastic hinges).

In each analysis a nonuniform distribution of radial "active" soil pressure was expressed as a function of the coefficient of active earth pressure, λ . The horizontal active pressure at the springline was defined by,

$$p_{\text{spring}} = p_s \lambda \quad (31)$$

where p_s is the vertical crown pressure equal to the sum of the overburden and live load pressures at the crown.

The distribution of active radial pressure about the conduit circumference was expressed as a cosine function (Fig. 4A)

$$p = p_s \cos \left[\frac{\pi}{2} \cdot \frac{\psi}{\psi_B} \right] \quad (32a)$$

$$\psi_B = \frac{\pi^2}{4 \cos^{-1}(\lambda)} \quad (32b)$$

In formulating the distribution, Klöppel and Glock neglected pressure redistribution due to soil arching around the conduit. A value of 0.5 was commonly assigned to λ , which set ψ_B equal to $\frac{3\pi}{4}$ radians.

In order to solve the instability problem, an arch section of the conduit (defined by $2\phi_0$ in Fig. 4B) was analyzed. Restraint provided by the remainder of the conduit was represented by rotational and tangential elastic moduli, k_R and k_T , at the arch supports. The boundary pressure distribution on the arch section was decomposed into a uniform component p_0 , and a nonuniform component p_1 , so that

$$p = p_0 + p_1 \sin \left[\frac{\pi}{2} \cdot \frac{\phi}{\phi_0} \right] \quad (33)$$

where, $p_0 = p_s \cos \frac{\pi \phi_0}{2 \psi_B}$

and $p_s = p_0 + p_1 =$ vertical overburden plus live load pressures at crown.

To solve the snap-through instability problem an initial value of ϕ_0 was chosen and deformation into a single mode buckle, defined by a cosine function, was considered. A load-deformation expression relating the thrust, F , in the conduit wall at the crown to the amplitude of the deflection was developed by minimizing potential energy. A second

FIGURE 4A ACTIVE PRESSURE DISTRIBUTION AROUND CIRCULAR CONDUIT

FIGURE 4B PRESSURE DISTRIBUTION ON TWO-HINGED ARCH (after KLÖPPEL AND GLOCK, 1970)

expression relating F to the crown pressure p_s for a given boundary pressure distribution was obtained from equilibrium requirements. Values of F were then assumed and the corresponding deflected shape and p_s were calculated. The critical thrust F_c for the chosen ϕ_0 was determined when an increment in F produced no increment in p_s . At this point increasing the load did not increase the potential energy of the conduit-soil system, and an instability was considered to occur. Other values of ϕ_0 were chosen and the minimum thrust at which an instability occurred was taken to be the critical buckling load.

A similar analysis was performed for high mode buckling, except that instead of restricting deformations to a single snap-through buckle, deflection into a large number of waves defined by a periodic trigonometric function was allowed. The subsequent expressions for the critical buckling load were minimized with respect to the buckling mode and a high mode buckling expression similar to the one derived by Luscher (eqn 10i) for a flexible ring was obtained.

Sensitivity analyses were performed by Klöppel and Glock to determine the influence of the assumed boundary conditions on the calculated snap-through buckling load, which led to the following general conclusions (see Fig. 5):

- (1) For small values of the conduit-soil flexibility factor,

$$F_s = \frac{EI(1+\nu_s)}{R^3 E_s} < 10^{-4},$$
 variation of the conduit or soil stiffness has little effect on the snap-through buckling load.

FIGURE 5 VARIATION OF CRITICAL THRUST FORCE WITH BOUNDARY CONDITIONS AND BUCKLING MODE (after KLÖPPEL AND GLOCK, 1970)

- (2) Neglect of rotational restraint of the arch support (k_R) reduces the critical buckling load.
- (3) Given identical conduit and soil stiffness and equivalent boundary pressure distributions, the critical buckling load of an elliptical conduit, with the major axis horizontal, is approximately one-half that of a circular conduit.
- (4) The amount of soil-conduit interface friction has negligible influence on the magnitude of the critical buckling load.
- (5) Increasing the nonuniformity of the active pressure distribution (Figure 4A) decreases the critical buckling load.
- (6) Consideration of a stress-level dependent elastic soil modulus, which is constrained to increase linearly with stress, slightly increases the critical load. {In fact, E_s may decrease with increasing deformation. Klöppel and Glock concluded that because the stress-level dependence was uncertain, a constant value of E_s should be assumed.}

Comparison of the critical loads in high mode and snap-through buckling showed that for $F_s \approx 10^{-4}$ the critical loads are nearly identical. This indicates that very flexible conduits with good soil support will tend to buckle in high modes. However, for $F_s > 10^{-4}$ the critical snap-through buckling load is much lower than the high mode buckling load, so that in this range high mode buckling theory is inapplicable (see Fig. 5). Investigation of conduit instability after formation of a plastic hinge at the crown resulted in a lower prediction of the critical load.

Although the theoretical studies conducted by Klöppel and Glock were of a more general nature than previous work, there still remain oversimplifications that prevent close modeling of the true soil-conduit system. Their formulation of the boundary pressure distribution misrepresents the true distribution around buried flexible conduits, and is not compatible with the assumed deflection pattern prior to buckling. Experience has shown that the normal pressure at the crown is often smaller than the nominal overburden pressure, and the springline pressure is larger than the active pressure, because of arching that accompanies conduit deflection. Thus, the actual pressure distribution is less conducive to buckling at the crown than the assumed one, and Klöppel and Glock's lower bound theory may be a reasonable solution to the buckling problem.

C. Inelastic Buckling Theory

Inelastic buckling of cylindrical shells is a complex problem that has been analyzed only for simple boundary conditions and load distributions. Several investigators (Bijlaard, 1949; Gerard, 1957; Reynolds, 1960; Lee, 1962) have examined inelastic buckling of cylinders of finite length subjected to uniform external pressure but without circumferential soil support. Gerard (1957) proposed the use of the following expression for an "effective" modulus to be substituted into expressions derived for elastic buckling to predict the critical inelastic buckling pressure:

$$E_{\text{eff}} = \sqrt{E E_t \left[\frac{1 - \nu_e^2}{1 - \nu_p^2} \right]} \quad (34)$$

where E = secant modulus

E_t = tangent modulus

ν_e = elastic Poisson's ratio

ν_p = plastic Poisson's ratio

The above expression is not applicable if $E_t \approx 0$ after yielding. In such cases an empirical value of E_{eff} must be obtained. In addition, the use of an effective modulus may be applicable only to materials that are free of residual stresses, such as heat-treated aluminum alloys. If an accurate representation of the elastic and plastic stress-strain behavior is available a trial and error approach to determination of the effective modulus may provide a good estimate of the critical inelastic buckling stress--as long as geometric imperfections are small. However, in cases where residual stresses are significant the use of an effective modulus is inapplicable.

A simple, yet reasonable, approach to prediction of critical loads in the inelastic range was proposed by Gjelsvik and Bodner (1962). Assuming elastic-purely plastic behavior, they suggested the use of elastic instability equations on the unyielded fraction of the section to predict the onset of buckling. Thus, an equivalent section was proposed rather than an effective modulus, which may be applicable to less flexible conduits subjected to relatively large bending stresses.

No theories have, as yet, been advanced to predict inelastic buckling of radially supported conduits.

II. COMPARISON OF ELASTIC BUCKLING THEORIES

Many formulas used for predicting critical elastic buckling loads in flexible conduits have been reviewed in the previous section. They include a wide variety of assumptions and boundary conditions that should be considered when evaluating their applicability to the prediction of buried conduit performance. Table 1 highlights the significant assumptions and boundary conditions inherent in the respective theoretical formulations.

The review of theoretical developments in Section I includes a number of observations and comparisons. For convenience, the main points are summarized below:

- a) Analysis of shells is more complicated than that of rings yet, if all other assumptions are comparable, the predicted critical pressures do not differ greatly, with the ring analysis being a little more conservative.
- b) Critical pressures that remain normal to the conduit wall are slightly lower than those that are maintained in a constant (radial) direction.
- c) Duns and Butterfield related the critical buckling mode to conduit and soil properties. When these critical buckling modes are inserted in the other theories, the theoretical predictions are much higher than the lower bound values. This is because all theories initially assumed buckling in a high mode, but only Duns & Butterfield determined the critical high mode (for $n \geq 7$). Other theories are minimized with respect to the mode number n , regardless of whether or not the resulting mode is compatible with the conduit and soil properties.
- d) The effect of slip at the soil-conduit interface is not definitely known. Two theories (Forrestal and Herrmann and Duns and Butterfield) show that slip decreases the critical pressure, but they disagree considerably on the amount of reduction. Klöppel and Glock, on the other hand, concluded that the reduction is negligible.

TABLE 1 - Classification of Factors Considered in Elastic Buckling Theories

Authors	Culvert Geometry			Critical Pressure		Interface		Factors Considered			Buckling Mode			Soil Modulus	
	Ring	Shell	Arch	Normal	Radial, Constant Direction	Bonded	Slip	Initial Stresses	Non-linear Deflection Terms	Deflected Shape Before Buckling	Effect of Shallow Cover	High Modes	Local	Theoretical	Empirical
Timoshenko & Gere (1) ^a	x			x							x				
Valentine (4)			x	x											
von Mises (5a)			x	x											
Bodner (5b)			x	x											
Armenakas & Herrmann (6)			x	x											
Cheney (7)	x			x	x	x		x							
Cheney (1971) (No Eq.)	x			x	x	x		x							
Cheney (20)			x	x	x	x		x							
Cheney (201)			x	x	x	x		x							
Cheney (21)			x	x	x	x		x							
Forrestal & Herrmann (8)			x	x	x	x		x							
Forrestal & Herrmann (9)			x	x	x	x		x							
Luscher (10)	x				x	x		x							
Luscher (101)	x				x	x		x							
Davis & Butterfield (13)			x	x	x	x		x							
Davis & Butterfield (17)			x	x	x	x		x							
Chelapati & Allgood (18)			x	x	x	x		x							
Chelapati & Allgood (181)			x	x	x	x		x							
Chelapati & Allgood (1811)			x	x	x	x		x							
Neyerhof & Baikle (22)			x ^b	x	x	x									
Neyerhof & Baikle (26)			x ^b	x	x	x									
Neyerhof & Baikle (28)			x ^b	x	x	x									
Anstutz (29)			x	x	x	x		x							
El-Bayoumy (1972) (No Eq.)	x			x		x		x							
Klöppel & Glock (Fig. 5)			x	x	x ^d	x		x							

^aNumbers in parentheses refer to equations in text^bApproximated from plate theory^cRigid restraint to outward deflection^dOnly theory considering nonuniform pressures

- (e) An important source of disagreement among the theories predicting elastic buckling is the formulation of the soil support modulus. Because of the assumptions and simplifications characterizing each analysis, several different expressions for the soil support modulus have evolved. Some have been developed by minimizing the critical buckling mode so that a lower-bound solution is obtained. Others have been similarly derived but include empirical factors which attempt to fit the theoretical predictions to observed behavior. The soil support moduli in various theories are compared in Table 2, for deep soil cover and a soil Poisson's ratio of 0.4. It is evident from this comparison that lower-bound solutions involve large reductions in the soil modulus not necessarily compatible with the physical aspects of the problem.
- (f) With the exception of Klöppel and Glock (1970), all theories deal only with uniformly applied boundary pressures on circular x-sections. Nonuniform pressures adversely affect the critical buckling load, and elliptical sections buckle more readily than circular ones.
- (g) Two theories (Donnell, 1956 and Meyerhof and Baikie, 1963) consider the influence of geometric imperfections on the critical buckling load; both theories show that an increase in the yield strength decreases the influence of a given imperfection level on the critical load.
- (h) Four theories of local buckling were reviewed. Three of these (Amstutz, Cheney, El-Bayoumy) considered the snap-through phenomenon of an undeformed conduit rigidly restrained from outward deflection and subjected to uniform pressure. A symmetric, sinusoidal snap-through deflection was forced in each case. Amstutz considered instability to occur when the extreme fiber stress (including bending stresses) reached the yield stress, while Cheney and El-Bayoumy arrived at the instability condition through energy considerations. Partly because of the nature of Amstutz's failure assumption, his theory is a lower bound prediction. With no reduction in pressure during buckling, results from Cheney and El-Bayoumy are identical. Reduction of pressure as the conduit snaps through, as considered by Cheney, appears to be unnecessary because of the nature of the energy barrier involved in the snap-through phenomenon. On the other hand, rigid restraint to outward deflection in the above theories tends to overestimate the buckling pressure.

The fourth theory, advanced by Klöppel and Glock, considered nonuniform boundary pressures, initial deflections, and variable interface friction. The results showed that if the soil-conduit flexibility factor (F_s) is low, a high mode

TABLE 2
COMPARISON OF THEORETICAL FORMULATIONS OF
THE SOIL SUPPORT MODULUS

Investigator	Suggested Expression For k_s	Lower-Bound Value of k_s ($\nu_s = 0.4$)
Luscher (eqn 11)	$\frac{E_s \left[1 - \left(\frac{R_i}{R_o} \right)^2 \right]}{(1+\nu_s) \left\{ 1 + \left(\frac{R_i}{R_o} \right)^2 (1-2\nu_s) \right\} R} \quad (a)$	$\frac{0.71 E_s}{R}$
Duns & Butterfield (eqn 14a)	$\frac{E_s (n^2 - 1)}{R(1+\nu_s) \{ 2n+1-2\nu_s (n+1) \}}$	$\frac{4.0 E_s}{R} \quad (b)$
Cheney (eqn 19)	$\frac{E_s (n^2 - 1)}{R(1-\nu_s^2) (2n+1-\nu_s)}$	$\frac{3.9 E_s}{R} \quad (b)$
Meyerhof & Baikié (eqn 23)	$\frac{E_s}{2(1-\nu_s^2) R}$	$\frac{0.60 E_s}{R}$
Klöppel and Glock (1970)	$\frac{E_s}{R(1+\nu_s)}$	$\frac{0.71 E_s}{R}$

(a) For large fill heights, $R_o \gg R_i$ and $\left(\frac{R_i}{R_o} \right)^2$ approaches zero.

(b) For $n = 7$.

buckle was more critical than a single mode snap-through, and vice-versa (Fig. 5). Thus, local crimping in flexible conduits with good soil support most probably originate from significant imperfections or local residual stresses in the conduit wall. Snap-through buckling is also influenced by imperfections and local residual stress. As none of the buckling theories consider all the influencing factors - non-uniform pressure distribution, imperfections, and residual stresses - the lowest buckling load calculated from any of the relevant theories is given in Fig. 6.

The lower-bound critical high mode buckling pressures predicted by theories reviewed in Section I are plotted versus a pipe flexibility parameter $\left[\frac{EI}{R^3} \right]$ in Figures 7 and 8 for loose and dense soil support, respectively. For comparison purposes, some variables have been held constant in all cases. A smooth steel conduit ($\nu = 0.3$, $E = 30 \times 10^6$ psi, $f_y = 36,000$ psi) with a diameter of 10 feet is assumed to be covered by 10 feet of dry sand with $\nu_s = 0.4$. Loose sand supporting the culvert is given an elastic Young's modulus of 2000 psi, while the modulus for dense sand is taken to be 20,000 psi. The values of EI/R^3 plotted in Figures 7 and 8 reflect conduit stiffnesses in the "flexible" range, according to the flexibility parameter, F_p , given by Peck, et al (1972),

$$F_p = \frac{E_s R^3}{6 EI} \frac{(1-\nu^2)}{(1+\nu_s)} > 20 \quad (35)$$

and were chosen with due regard to commonly manufactured corrugated metal sections.

From Figures 7 and 8 four important deductions can be made:

1. All theories are linear on a log-log plot.
2. The mere existence of surrounding soil increases the critical pressures by an order of magnitude, or more, even in the case of weak supporting soil.

FIGURE 6 THEORETICAL CRITICAL BUCKLING LOADS OF FLEXIBLE CONDUITS WITH IMPERFECTIONS OR NONUNIFORM BOUNDARY PRESSURES. LOOSE SAND, $E_s = 2000$ PSI

FIGURE 7 COMPARISON OF THEORETICAL, LOWER-BOUND CRITICAL PRESSURES FOR HIGH MODE BUCKLING OF FLEXIBLE CONDUITS, LOOSE SAND, $E_s = 2000$ PSI

FIGURE 8 COMPARISON OF THEORETICAL, LOWER-BOUND CRITICAL PRESSURES FOR HIGH MODE BUCKLING OF FLEXIBLE CONDUITS. DENSE SAND, $E_s = 20,000$ PSI

3. The critical buckling pressure is strongly influenced by the soil's elastic modulus. If this modulus is increased ten times, the critical pressures are increased by roughly the same order of magnitude. Thus, characterizing the quality of backfill by percent compaction, or by rough estimates of the soil modulus, is patently inadequate from the standpoint of predicting the critical buckling load.
4. Theory indicates that for weak soil support buckling is the dominant failure mode (compared with wall crushing); even for strong soil support, buckling may be critical in the more flexible culverts. Due to geometric and material imperfections, and to residual stresses from cold forming, observed buckling pressures may be even lower than the critical pressures predicted theoretically. Accordingly, buried culverts may be more susceptible to failure by buckling than is now generally acknowledged.

III. BUCKLING EXPERIMENTS ON SOIL SUPPORTED CONDUITS

A. Controlled Experiments

Because of the complexity of the interaction between flexible conduits and the surrounding soil, and the vagaries of field construction practices, a number of controlled experiments in prefabricated soil bins have been conducted to compare theory with observation. As in any simulation of field behavior it is imperative that the test conditions closely model those in the field. Accordingly, in presenting the results of a number of investigations, care has been taken to review critically the apparatus and procedures used in the experiments.

Bulson (1963)

Bulson conducted laboratory tests to study the behavior of soil-supported, flexible, tin-plated steel conduits ($f_y = 26$ ksi). Tubes with a thickness of 0.011 inches and diameters ranging from 5 to 10 inches $\{0.026 < \frac{EI}{R^3} < 0.208\}$ were buried in a five foot long steel test cell with backfill sand compacted by a vibrating plate in three inch lifts. The 12 inch long tubes were restrained at the ends by relatively rigid observation tubes of the same diameter extending axially from the flexible tube to the cell wall. Because the ratio of the flexible length to the diameter was not sufficiently large, the restraint significantly increased buckling loads over that expected for the plane strain case. Tube stresses were not measured, and only rough measurements of the buckled deformations were made.

A uniform hydrostatic pressure was applied to the surface of the soil cover, which was varied in thickness to test the effect of burial

depth on conduit performance. The ratio of burial depth (d) to the tube diameter (D) was either $\frac{d}{D} = \frac{3}{8}$ or $\frac{3}{4}$, and the corresponding pressures producing collapse in tubes with identical geometry were compared. Surface pressures were recorded at the initiation of roof collapse after buckling had weakened its ability to carry the applied load. Figure 9 illustrates the sequence of events generally observed during loading to the collapse pressure, $p_{\max}$. Replication of $p_{\max}$ was difficult and average values were used in comparisons with theoretical buckling pressures predicted by eqn (36), which was derived by Timoshenko and Gere (1961) for elastic buckling of unsupported cylinders with clamped ends subjected to uniform radial pressure.

$$p_c = \frac{Et}{R(n^2-1)\{1+\left(\frac{nL}{\pi R}\right)^2\}} + \frac{Et^3}{12R^3(1-\nu^2)} \left[(n^2-1) + \frac{2n^2-1-\nu}{1+\left(\frac{nL}{\pi R}\right)^2} \right] \quad (36)$$

The theoretical buckling modes (which depend upon the ratios $\frac{L}{D}$ and $\frac{R}{t}$) were substituted into eqn (36) to calculate p_c . It was found that, for a given $\frac{d}{D}$, the ratio $\frac{p_{\max}}{p_c}$ was fairly constant. The ratio $\frac{p_{\max}}{p_c}$ also increased with increasing $\frac{d}{D}$ and ranged from 6.2 to 9.2. Bulson believed these results reflected the dependence of soil support on the depth of soil cover. However, because of the nature of the applied load (uniform over the entire soil surface), the soil support could not vary much over the range of cover heights investigated. Increased soil arching must also have resulted from increased soil cover. Thus, increase of the collapse overpressure with increasing soil cover was as indicative of a change in the load distribution on the tubes as it was of an increase in the modulus of soil support.

FIGURE 9 STAGES OF TUBE COLLAPSE (after BULSON, 1963)

Comparison of $p_{\max}$ to p_c is inappropriate. Collapse can occur immediately after buckling is initiated or at loads larger than the initial buckling load (depending on soil support and tube stiffness). Comparison of the initial buckling load (rather than $p_{\max}$) with p_c would have been a better indicator of the relation between the calculated critical pressure and the observed buckling load. Nevertheless, Bulson's tests clearly demonstrated the importance of soil support on the collapse pressure.

Luscher (1966)

Luscher attempted to check the validity of previously developed theoretical buckling solutions by modeling experimentally the boundary conditions and types of loads used in the theoretical analyses. Two-ply, bonded, aluminum foil tubes (f_y estimated to equal 16 to 24 ksi) 1.63 inches in diameter and 6 inches long, with a composite wall thickness of 0.003 inches, were tested. Each smooth tube was surrounded by a cylindrical ring of Ottawa sand, either 3/8 in or 2/3 in thick. The saturated sand, tested in both loose and dense states, was enclosed by a flexible, impermeable membrane and loaded by a uniform radial external pressure applied to the membrane. The external pressure producing buckling in the tube was compared to the critical pressure predicted by eqn (8) (Forrestal and Herrmann, 1965), and it was found that the theoretically predicted critical pressure was much higher than that measured in the lab.

Luscher concluded that the soil support modeled by Forrestal and Herrmann was the source of the discrepancy, and consequently he developed an empirical expression for the soil support modulus, given

by eqns (11) and (12). Radial interface pressures acting on the tube were calculated from

$$p = p' + K_s \frac{w}{R} \quad (37)$$

where, p = local radial interface pressure

p' = uniform external pressure applied to the outside of
surrounding soil cylinder

w = radial tube displacement (positive outwards)

No instrumentation was provided to measure the interface pressures, thus there was no measured verification of eqn (37). However, a check on the validity of eqn (11) was made by back-calculating K_s from each test using eqn (10) with the known tube geometry and failure pressure.

Luscher's semi-empirical theory was predicated on buckling occurring in high modes, but actual failure configurations seemed to take the form of local buckles. Consequently, the mode number was determined by dividing the length of the local buckle (ℓ) into the total tube circumference. Values of M_s were obtained from K_s by eqn (11) and compared to M_s values determined from oedometer tests on Ottawa sand at the densities and confining pressures used in the buckling tests. Good correlation was found, partly because the formulation of eqn (11) was derived from the test observations. No appreciable difference in the back-figured support modulus was found between loose and dense sand, which indicated that the test conditions were possibly forcing artificial behavior. Luscher believed that the use of his empirical soil moduli would improve the predictive capabilities of available theories by removing a large amount of the unconservative error.

However, it is possible that tube imperfections, residual stresses, and non-composite behavior of the two-ply aluminum foil, rather than an improper formulation of soil support, were causing experimental failures in the form of local buckles at pressures much lower than those predicted theoretically for buckling in high modes. The empirically obtained eqn (11) was verified only for the tested small-scale geometries. Luscher admitted that scale effects could be significant. However, he believed the results indicated that elastic buckling was an important potential failure mode of flexible conduits supported by loose backfill.

Representative results obtained by Luscher are presented in Table 3. Replication was difficult, as illustrated by varying buckling pressures of tubes with identical stiffness supported by sand of equal density. Large variation in the observed buckling mode was probably due to differences in energy released during snap-through buckling, which directly affects the length of a buckle and is partly dependent on the level of tube imperfection. Thus, it is likely that local imperfections in the tubes tested governed their behavior, and the use of empirical relationships obtained from these tests may not be valid for field conditions.

Allgood and Ciani (1968)

Allgood and Ciani conducted experimental buckling tests on smooth steel ($f_y = 89$ ksi) tubes buried under 6 inches of uniform, fine sand. The tubes, 5 inches in diameter and 20 inches long with wall thicknesses of 0.006, 0.012 and 0.018 inches, were tested in a cylindrical tank with

TABLE 3 Some Results from Luschert's (1966) Buckling Tests (a)

Nominal Sand Density and Void Ratio	Uniform Applied Buckling Pressure (psi)	Soil Ring Thickness (in)	Calculated Buckling Mode $n = 2\pi R/\lambda$	Back-calculated Soil Modulus, E_s (psi)
Medium-Dense, 0.50	9.0	3/8	21	3825
Medium-Dense, 0.50	12.4	3/8	28	6000
Medium-Dense, 0.48	14.2	2/3	33	6675
Medium-Dense, 0.48	12.4	2/3	17	4725
Medium-Dense, 0.47	9.9	2/3	22	3038
Medium-Loose, 0.57	10.0	3/8	29	4050
Medium-Loose, 0.57	7.8	2/3	18	2888
Medium-Loose, 0.57	7.6	3/8	22	3300
Medium-Loose, 0.55	9.3	3/8	32	3750
Medium-Loose, 0.59	6.0	2/3	20	2400
Medium-Loose, 0.60	8.3	2/3	24	3975

(a) All tests on aluminum tubes with constant radius (0.815 in) and constant stiffness ($\frac{EI}{R} = 0.042$).

a diameter of 5 feet. The axis of the tank was vertical. The walls were constructed to minimize friction, and the base of the tank was sufficiently removed from the buried tube to model soil of infinite depth. Soil placement with density control was accomplished by a raining technique. Load was applied to the conduit by pressurizing a pneumatic bag which reacted against the soil surface.

The tubes were instrumented with electric resistance strain gages placed around the inner and outer tube circumference to measure thrusts and moments that developed in the tube walls. A rotating deflection gage was installed to measure the inside profile of the tube, and deflection gages were placed in the soil below the tube to measure average soil strains.

Tests were performed with varying soil densities and tube stiffnesses, and the surface soil pressure was recorded at the onset of buckling. In all tests, strain in the tubes remained elastic prior to buckling and failures took the form of local buckles causing almost immediate collapse in thinner tubes and collapse after increased load in thicker tubes. Interface soil pressures were calculated in two ways. One method used strain measurements to determine the radius of curvature and thrust of the deformed tube; the interface pressure was calculated as the tube thrust divided by the radius of curvature, assuming small bending resistance of the tube. The second way was to calculate the interface pressures from thrusts and moments measured in the tubes. The pressures calculated from these methods showed reasonable agreement before buckling. In cases where the tubes buckled after negligible deflection, pressures were larger at the crown than at the springline. No interface pressures were measured to check the accuracy of the calculations. Table 4 is a summary of the experimental

TABLE 4 Results from Allgood and Ciani's (1968) Buckling Tests (a)

$\frac{EI}{R^3}$ (psi)	Nominal Sand Density	Sand Unit Weight, (pcf)	Critical Surface Overpressure, (psi)	Arching Factor, C	Buckling Stress at Springline, (psi)	Efficiency Ratio, R_f
0.035	Loose	96.7	75	0.320	21,300	0.24
0.035	Loose	96.7	73	0.192	24,600	0.28
0.035	Dense	109.0	121	0.380	31,300	0.35
0.035	Dense	109.0	123	0.341	33,800	0.38
0.28	Medium-Dense	103.4	266	0.075	51,300	0.58
0.28	Dense	109.0	250	0.364	33,100	0.37
0.93	Loose	96.7	247	0.279	24,700	0.28
0.93	Loose	96.7	247	0.117	30,300	0.34
0.93	Medium-Dense	103.4	237	0.241	25,000	0.28

(a) All tests on tubes with constant radius (2.5 in.).

results. Replication was much better than in Luscher's tests but, again, the buckling pressures were not entirely consistent with the sand density.

Allgood and Ciani related the interface buckling pressure to the surface soil pressure by an arching factor,

$$p_c = p_s (1 - C) \quad (38)$$

p_s = applied surface soil pressure

C = arching factor = $\{1 - F_c/p_s R\}$, where F_c is the measured thrust at the springline.

Arching was found to vary among tests with identical soil densities and tube stiffnesses and this was thought to be due to improper seating of the tube during placement. The calculated interface pressure was compared to the critical uniform external buckling pressure predicted by Chelapati and Allgood {eqn (18i)}, and fair agreement was found, although eqn (18i) was developed for high mode buckling and only local buckling was observed.

In the interpretation of buckling tests the writers found it useful to compare the buckling stress, f_c , to the yield stress of the wall material, f_y . An efficiency ratio, R_f , was defined

$$R_f = \frac{f_c}{f_y} \quad (39)$$

as a measure of how efficiently the potential load capacity of the conduit was being utilized. In cases where f_c was not measured in the vicinity of the buckle it was calculated using the best technique available, consistent with the nature of the input data. The values of R_f in Table 4 clearly show that elastic buckling occurred at pressures well below those required to induce failure by wall crushing.

From the measured behavior of the loaded tubes Allgood and Cianf concluded that:

- (1) deflections of buried flexible cylinders under a given load are governed mainly by the soil modulus,
- (2) interface soil pressures can be determined from measurements of strains in the cylinder, and
- (3) buckling loads may be predicted using eqn (18i).

The modulus of elastic support used in eqn (18i) was determined using eqn (11) developed from Luscher's buckling experiments. The use of these equations allowed correlation between theory and experiment, but the critical interface pressure assumed in eqn (18i) was uniform while the interface pressures calculated from measured strains were not. In addition, failure by elastic buckling in high modes, modeled by eqn (18i), was not observed in the experiments. These factors, combined with the small scale of the experiments, make it unlikely that the test results can be scaled to larger conduits.

Dorris (1965) and Albritton (1968)

Studies at the Waterways Experiment Station by Dorris and Albritton investigated the effects of burial depth on the buckling behavior of smooth, flexible tubes. The tubes were instrumented so that quantitative data could be obtained to supplement Bulson's (1963) observations. Dry sand at a relative density equal to 80% was used to cover the tubes with depths from 0 - 2 diameters. Aluminum cylinders with diameters of 3.5 inches, lengths 10.5 and 12 inches, and wall thicknesses of 0.01, 0.022 and 0.065 inches were tested in a cell with special provisions made to reduce soil friction along the cell wall. The cell length was 4 feet

and the base of the cell was 2 feet below the cylinder. The cylinder ends were enclosed by a flexible gasket and an aluminum cap so that plane strain conditions were approximated.

Instrumentation included metal foil strain gages installed inside and outside of the tube at the crown, invert, and springline. This allowed determination of the thrusts and moments in the tube at the points of instrumentation, and these were compared at different tube burial depths. Uniform pressures were applied to the soil surface. The recorded critical pressure was the surface overpressure at which a buckle first formed in the tube.

Failure modes were observed to vary with burial depth, partially agreeing with Bulson's test results. For burial of 0 to $\frac{1}{2}$ diam (D), failure took the form of a catastrophic snap-through buckle at the crown. Burial at $\frac{3}{4}$ D changed the location of the local buckle to the springline, and tubes buried at greater depths buckled locally at the invert. Measured strains before buckling were elastic in all cases, and the strains measured at the initiation of buckling varied with the depth of soil cover. The largest strains were measured at the crown when shallow cover (less than $\frac{1}{2}$ D) was provided. With increasing depth of cover, the largest measured strains were transferred from the crown to the springline, and then to the invert. Thus, strains measured in these tests verified that variation of soil cover significantly affects the load distribution on conduits with shallow (less than one diameter) burial.

Results from a few tests conducted by Dorris using compacted (approximately 95% Std. AASHO) highly plastic clay as backfill showed that eqn (2) from Timoshenko and Gere (1961) was not a bad approximation for the buckling stress, indicating that either the

backfill provided little buckling restraint or that the small tubes possessed imperfections that grossly affected the test results.

Large scatter was reported in the experimental results, due partly to instrumentation errors. Dorris also found that the sand placement technique (raining vs. rodding) influenced the test results. Sand placed at a given density by raining provided less support than placement by rodding. This could have been due to the inability of the raining technique to provide the required density immediately around the tube, and to the pre-straining effects produced by rodding.

Watkins and Moser (1969)

Watkins and Moser reported the results of controlled tests on 3 to 5 foot diameter, and 20 foot long steel culverts ($f_y = 40$ ksi) with $2 \frac{2}{3} \times 1 \frac{1}{2}$ inch and 3×1 inch corrugations and 16 and 18 gage wall thicknesses.

A test bin 24 feet long, 15 feet wide and 18 feet high was used to contain the culvert and backfill, and at least five feet of clearance on all sides of the conduit was provided. The bin walls were elliptical to reduce side friction effects. Dry silty sand compacted to densities ranging from 61 - 105% Std. AASHO (75 to 129 pcf) was used as backfill.

Loads were applied by hydraulic jacks bearing against a steel plate on the surface of the soil 5 feet above the culvert crown. Culverts were instrumented to measure deformations in cross-section. Several pressure gages were also placed around the circumference to measure normal pressures near the soil-culvert interface.

The purpose of the investigation was to establish performance limits for buried culverts and to develop a means for estimating when a performance limit will be reached. A summary of the published experimental results is given in Table 5.

From their test results and observations Watkins and Moser concluded that:

- (1) The performance limit should be defined as the deformation beyond which either the culvert or soil can no longer perform its design function. Usually this occurs when the resistance to additional deflection approaches zero.
- (2) In dense soil the performance limit was determined by the crushing strength of the culvert wall.
- (3) Buried culverts are capable of sustaining larger loads after local buckling is initiated.

Empirical curves were developed relating the thrust stress in the culvert wall, calculated by eqn (3) at the interpreted performance limit, to the slenderness ratio of the pipe wall for different soil support conditions (in terms of percent Std. AASHO maximum density). The curves were designed to reflect the fact that, with good soil support, as the slenderness ratio decreased the performance limit passed successively from wall crushing, through inelastic buckling, to elastic buckling. In some cases the thrust stresses at failure calculated from eqn (3) exceeded the yield stress. It was decided that this was due to neglect of soil arching, and load factors were developed to adjust the calculated stresses to the yield stress. These load factors were related solely to the relative compaction of

TABLE 5 Some Results from Watkins and Moser's (1969) Buckling Tests

Radius (in)	$\frac{EI}{R^3}$ (psi)	$\left(\frac{D}{r}\right)^2$ $\times 10^3$	Soil Density		At Initial Wall Disturbance		At Performance Limit		Observed Initial Wall Disturbance
			pcf	% Std. AASHTO	Surface Overpressure (psi)	Vertical Deflection (%)	Efficiency Ratio, R_f (d)	Surface Overpressure (psi)	Vertical Deflection (%)
30	1.67	121.7	101.7	83	89	3.5	1.00	111	6.0
30	1.67	121.7	118.4	97	84	0.8	0.90	132	2.5
30	1.67	121.7	129.0	106	97	1.1	1.00	97	1.1
24	3.26	77.9	119.7	98	49	1.0	0.42	> 127 (c)	> 3.0 (c)
30	7.65	30.4	94.0	77	59	3.5	0.71	94	5.0
18	9.73	43.8	120.2	93	97	2.3	0.61	104	2.8
24	18.8	19.5	84.6	69	36	17.0	0.29	51	18.0
24	18.8	19.5	89.1	73	56	7.0	0.45	56	7.0
24	18.8	19.5	91.2	75	92	6.5	0.69	> 130 (c)	> 9.0 (c)
18	44.5	11.0	74.9	61	31	18.0	0.21	31	18.0
18	44.5	11.0	78.8	65	31	8.0	0.23	-	-
13	44.5	11.0	97.3	80	90	4.5	0.54	> 130 (c)	> 7.0 (c)

(a) Little resistance to additional deflection

(c) Performance limit exceeded capacity of test apparatus

(b) Angular location measured clockwise from crown

(d) f_c calculated at location of observed wall disturbance using finite element analysis

the soil; the influence of the conduit stiffness was ignored. No measurements were made to verify the calculated stresses or to differentiate between inelastic and elastic buckling. Performance limits were based primarily on visual interpretations which, as discussed earlier, can be misleading. The above mentioned empirical curves developed from the test results have been adapted for use in design practice (Handbook of Steel Drainage and Highway Construction Products, 1971).

Meyerhof and Baikie (1963)

Meyerhof and Baikie (1963) tested segments of steel cylinders bearing against dense sand to verify their formula for the critical elastic buckling stress of buried flexible culverts (eqn 27). Smooth and corrugated ($3/4 \times 1/4$) quadrantal sections with radii of 12 to 24 inches and wall thicknesses of 0.017 and 0.042 inches were loaded with a compressive thrust on one of the longitudinal edges, while the other longitudinal edge was held fixed. Electric resistance strain gages, soil pressure gages, and deflectometers were mounted along the central circumference of the plate. The tests were conducted in a steel bin two feet long, four feet wide, and three feet deep. Since the axial length of the quadrantal plate was 19 inches, only $2 \frac{1}{2}$ inches of sand separated the ends of the sections from the bin. At least one diameter of sand backfill compacted to 90% relative density and surcharged with approximately 2 psi overpressure separated the plate section laterally from the bin wall. No special effort was made to reduce friction between the sand and the bin, hence the boundary effects of the bin were not eliminated. Some test results obtained by

Meyerhof and Baikie are summarized in Table 6. Replication was difficult, especially in tests on the most flexible conduit in which the range in buckling stress was approximately 70% of the mean.

Stresses calculated from measured strains in the loaded plates showed that the more flexible smooth plates buckled at approximately 15% of the yield stress, and the least flexible corrugated plates failed by crushing near the yield stress. Average radial strains before buckling were less than 1%. Soil pressure measurements indicated that the normal interface soil pressures were not uniform prior to initiation of buckling.

Although it was claimed that eqn (27) accurately predicted the experimental results,

- (1) the modulus of soil restraint was not constant along the plate-soil interface at the initiation of buckling, as assumed in the theory; and

- (2) high modes of buckling were not observed in the tests.

Instead, failure always occurred in the form of a local buckle near the soil surface because of lack of soil restraint inherent in the experimental apparatus.

In tests on corrugated plates with constant wall stiffness (EI) the efficiency ratio increased as the radius of curvature decreased, which is the inverse of the behavior observed by Watkins and Moser (see, for example, the second and fourth tests listed in Table 5, in which R_f increased with increasing radius). This inconsistency can be related to the hinged end-condition which did not allow full development of bending moments in the zone of low soil restraint where buckling always occurred. This limited the failure mode either to wall

TABLE 6 Results from Meyerhof and Baikie's (1963) Buckling Tests (a)

Radius of Curvature (in)	$\frac{EI}{R^3}$ (psi)	$\left(\frac{D}{r}\right)^2 \times 10^3$	Thrust Stress at Buckling (psi)	Average Yield Stress, f_y (psi)	Efficiency Ratio, R_f
12.9	0.0057	6910	5780	35,000	0.17
12.9	0.0057	6910	7800	35,000	0.22
12.9	0.0057	6910	4690	35,000	0.13
12.9	0.0057	6910	3980	35,000	0.11
25.6	0.011	4458	5600	45,000	0.12
12.9	0.086	1132	8770	45,000	0.19
12.9	0.086	1132	7850	45,000	0.17
12.0	0.11	980	6350	45,000	0.14
24.0	0.2	470	26,100	30,000 ^(b)	0.87
24.0	0.2	470	20,400	30,000 ^(b)	0.68
12.0	1.3	120	33,300	30,000 ^(b)	1.1
12.0	1.3	120	41,400	30,000 ^(b)	1.4

(a) Unit weight of the sand in all tests = 116 pcf (90% relative density).

(b) Tests on coupons from these specially rolled corrugated sections ranged from about 25,000-40,000 psi.

crushing or elastic buckling; thus, as the plate radius decreased, elastic buckling resistance increased and a larger efficiency ratio resulted. In Watkins and Moser's tests bending moments were allowed to develop. Because decreasing the culvert radius tends to increase the bending stress relative to the thrust stress, it is likely that buckling was initiated at loads below those required to cause buckling in the absence of bending stresses. Accordingly, the load carrying efficiency (reflected by the efficiency ratio) was reduced. This important aspect of conduit behavior was not reflected by Meyerhof and Baikie's tests.

Howard (1971)

Howard conducted tests on uncorrugated, 7 foot long steel conduits supported by sandy clay compacted to 90% and 100% Std. AASHTO density. The diameters tested ranged from 18 to 30 inches, and the pipe flexibility parameter $\frac{EI}{R^3}$ varied from 0.30 to 10.2 psi. The steel soil container within which the conduits were buried was 6 feet wide, 7 feet long and 7 feet deep. The ends of the conduits extended through the container walls, and provisions were made to prevent appreciable restraint to deflection. The container walls were covered with a polyethylene liner lubricated with grease to reduce interface friction. The longitudinal axis of each conduit was placed four feet below the soil surface. A thick wooden load-plate was placed on the soil surface to distribute the surcharge load applied by a large universal testing machine. Instrumentation on each conduit included SR-4 strain gages measuring strains on the inside surface, a rotating dial deflectometer measuring the deflected profile, and soil pressure cells installed flush with the outside surface at the crown, invert and springlines,

measuring normal interface pressures. Soil displacement indicators were also positioned in the backfill.

Test results are given in Table 7. Observed behavior during loading varied, depending on the soil density and conduit stiffness. In the weaker soil (90% Std. AASHTO density) deflections were generally large under relatively small surcharge loads. The most flexible conduit experienced two local elastic buckles after 8% deflection, which marked the performance limit. The less flexible conduits formed small inelastic crimps after measured strain on the interior of the conduit wall exceeded the yield strain. A performance limit was reached after larger surcharge loads were applied and was characterized by the formation of a large inelastic snap-through buckle, usually at the crown. Because of the instrumentation scheme, it was impossible to determine the magnitude of developed moments, hence their influence on the failure mode could not be evaluated. Generally, as the conduit stiffness increased the number of observed inelastic crimps decreased and the load at which they formed increased. In all cases when weak backfill was provided, inelastic crimping occurred outwards into the backfill.

With the denser backfill (100% Std. AASHTO density) deflections were reduced. However, local elastic buckling occurred in the more flexible conduits at approximately the same loads which produced snap-through buckling in weaker backfill. Only in the less flexible conduits ($\frac{EI}{R^3} > 4$) did the improved backfill prevent wall buckling before the capacity of the apparatus was exceeded. Howard concluded that although denser backfill reduced deflection, it did not significantly increase the critical buckling load. However, the writers point out that the load capacity of the conduit supported by the denser backfill is well above that of the same conduit supported by weaker soil.

Radius (in)	$\frac{EI}{R^3}$ (psi)	$\left(\frac{D}{r}\right)^2 \times 10^3$	Yield Stress, F_y (psi)	At Initial Wall Distress			At Snap-through Buckling		Observed Initial Wall Distress
				Overpressure (psi)	Vertical Deflection (%)	Efficiency Ratio, $R_f^{(b)}$	Overpressure (psi)	Vertical Deflection (%)	
15	0.30	1731	39,000	25	8	0.15	25	8	Elastic buckles at 112° and 225°
12	0.61	1108	39,000	30	10	0.10	40	17	Inelastic crimping at 45°, 112°, 225° and 315°
15	1.7	567	40,000	40	12	0.11	80	20	Inelastic crimping at 112°
9	1.8	623	39,000	30	10	0.10	40	18	Inelastic crimping at 70°, 125° and 248°
12	3.9	363	42,000	60	14	0.13	80	18	Inelastic crimping at 124° and 236°
9	9.1	204	49,000	60	15	0.11	90	21	Inelastic crimping at 90° and 270°
12	10.2	196	48,000	40	11	0.07	60	16	Inelastic crimping at 90°

B. Clay Compacted to 100% AASHO

15	1.7	567	40,000	80	8	0.21	> 90 ^(a)	> 9	Elastic buckles at 112° and 225°
9	1.8	623	39,000	30	2	0.09	60	9	Elastic buckles at 135° and 225°
12	3.9	363	42,000	80	8	0.17	> 100 ^(a)	> 10	Elastic buckles at 45° and 315°
9	9.1	264	49,000	> 100	> 14	-	> 100 ^(a)	> 14	No visual wall distress

(a) Performance limit exceeded capacity of test apparatus.

(b) Calculated as in Table 5

Measurement of the normal interface soil pressures at the initiation of buckling showed that in most cases the pressure distribution was not uniform. When appreciable deformation of the conduit occurred before buckling, the crown pressure was up to 40% less than the springline pressure. If buckling occurred after relatively small deformation the pressure distribution was more uniform. Some of the pressure measurements were unrealistic, illustrating the difficulty of obtaining good earth pressure measurements, especially near the soil-conduit interface.

Soil displacement indicators showed that most displacement occurred within one radius of the conduit wall. Although almost three radii of soil separated the container walls from the conduit, it was noted that the container did influence soil displacements. Howard believed that the effects were not large enough to control behavior, but he suggested that field tests were needed to correlate and refine empirical relationships developed from laboratory tests. Nevertheless, Howard's tests demonstrated that even well-compacted clay backfills (100% Std. AASHTO) provide inferior support to that of granular soils, especially from the standpoint of controlling deflections.

B. Field Experiments

Verification of theoretical models predicting conduit behavior, especially failure modes, must involve comparisons with field performance. The number of field tests that can be used for this purpose is limited because instrumentation is seldom provided to measure all of the soil properties and conduit loads needed as input for most predictive models. Moreover, field installations are costly and hence are seldom designed to reach failure. Thus, the failure mechanisms are unclear and the

factors leading to failure are uncertain. The investigations reviewed below have been instrumented and documented sufficiently to provide insight into the field behavior of buried conduits.

Prairie Farm Rehabilitation Administration (1957)

To observe the behavior of flexible culverts in clay backfill, a 34 feet high test fill covering a 6 foot diameter steel culvert ($f_y = 40$ ksi) was constructed. The 20 gage, $(2 \frac{2}{3} \times 1/2)$ corrugated culvert $\{\frac{EI}{R^3} = 0.7\}$ was instrumented with strain gages to measure both compressive and bending strain and a field protractor was used to measure the deflected shape. A clay backfill with medium plasticity was compacted to 98% Std. AASHO density. The compacted soil unit weight around the culvert showed $\pm 11\%$ variation about the mean, indicating nonuniform soil support. After the final fill height was achieved, a small outward buckle was observed in the lower haunch. The average measured thrust stress was 20,000 psi ($R_f = 0.5$), and bending stresses were sufficiently large to initiate outer fiber yielding at the location of the buckle. A vertical deflection of 4 inches (5.5% of the diameter) was measured just before the formation of the buckle. It was speculated that poorly compacted soil at the haunch provided a zone of weak support which allowed local buckling. Despite the formation of a local buckle, the culvert had not yet reached its load capacity.

A second, similarly instrumented culvert with a 9 foot diameter, 1 gage wall thickness and 6×2 inch corrugations $\{\frac{EI}{R^3} = 30.5\}$ was backfilled with a gravelly clay compacted to 94% Std. AASHO density and loaded with a 99 foot high fill. The overburden load caused a vertical crown deflection measuring 3.5% of the culvert diameter and, combined

with an average compressive thrust stress of 12,500 psi, produced moments large enough to initiate yielding at several locations, although no wall disturbance was observed.

Demmin (1966)

Demmin reported the behavior of a long-span pipe arch subjected to live loads applied through 5.2 feet of soil cover (one-quarter of the span). The pipe arch, with a span of 20 feet 7 in and a rise of 13 feet 2 in., was constructed of 7 gage, 6 x 2 inch corrugated curved plates $\{\frac{EI}{R^3} = 1.5\}$. The pipe was placed within an excavation having very firm side slopes, and backfill placed around the pipe consisted of a sandy gravel compacted to 107% Std. AASHTO density. Instrumentation included strain gages positioned on adjacent corrugation crests, troughs, and neutral axes at six locations along the circumference, which enabled determination of the thrusts and moments in the pipe wall at these locations. Deflections were measured with phototheodolites, and three Heierli earth pressure cells were installed in a horizontal plane four inches above the pipe crown to measure the percentage of the live load transferred to the pipe. The live load was applied by piling steel slabs on an 88 square foot grillage of railroad ties resting on the soil surface above the crown.

Strains produced by a 690 ton surface load approached yield in two locations on the upper half of the pipe. However, residual strains produced during placement and backfilling of the pipe combined with the live load strains to give a net strain well within the elastic limit. Bending stress comprised 65% of the load-induced extreme fiber stress, and vertical deflection was 1.3% of the span (3.1 in). When the live

load was increased to 850 tons, two inward bulges with amplitudes of 11.8 and 5.9 inches formed on either side of the crown near the location of the maximum moments. Because the strain gages had been removed, the state of stress in the pipe was unknown. The vertical deflection was approximately 1.7% of the span (4 in). Vertical earth pressures measured at the crown reached 65 psi. After the surface load was increased to 954 tons, the grillage was observed to settle and cause spreading of the load over a larger surface area. This relieved the pressure on the pipe arch and caused the steadily increasing vertical deflection to stabilize at 2.7% of the span (6.6 in). It was believed by Demmin that in the absence of spreading, 954 tons would have been close to the collapse load.

The test demonstrated that:

- (1) bulging can occur in long-span culverts due to the influence of bending moments at thrust stresses well within the elastic limit,
- (2) very strong soil support can prevent buckling and collapse despite the localized occurrence of large plastic deformations,
- (3) collapse of long-span culverts due to excessive deflection may occur at crown deflections below 5% of the span, and
- (4) stresses induced during installation of long-span culverts with excellent soil support can be large and may even approach the yield stress. If compaction is symmetric and yielding is not excessive, the bending stresses produced during

installation are opposite to those induced by surcharge loading and may have a favorable effect on performance.

Wu (1977)

Wu examined the performance of several culverts that were installed without modern compaction control. A device designed to measure in situ soil moduli was used to determine the level and uniformity of soil support obtained by the backfilling operations. Wide variation of the interpreted elastic moduli was found around each culvert. Strain gages were placed on the inside of the culverts and small coupons containing the gages were carefully removed from the wall. The measured changes in strain were assumed to be those induced by the overburden load plus residual strains produced during fabrication. Limited measurements of residual strains in recently fabricated, 4 foot diameter corrugated culverts were made, but the results showed wide scatter. Even so, the importance of residual stresses was made evident by the fact that their average value approached 8000 psi; hence, if their influence is not considered, comparisons between theoretical and measured behavior are dubious.

IV. DISCUSSION OF EXPERIMENTAL RESULTS

A. Comparison of Results from Controlled Buckling Tests

Results of controlled experiments, in which the buckling pressure and mode have been identified, are plotted in Fig. 10 in terms of a scaled buckling pressure vs. a soil-conduit flexibility factor.

There is considerable scatter, but the trends are clear:

- (1) Klöppel and Glock's buckling theory follows the pattern of test results tolerably well; scatter is greatest when the mode of failure is elastic buckling.
- (2) Imperfections and residual stresses in the tested conduits lower the buckling pressure, particularly in the case of small scale tests (e.g. Luscher's experiments).
- (3) The use of high yield point steel (Allgood and Ciani's experiments) increases the load at which wall crushing and plastic yielding will occur and may make buckling more critical even with excellent soil support. At the same time, the effect of imperfections and residual stresses on the buckling pressure is reduced. Accordingly, the capacity of the conduit to carry loads is increased significantly; the potential for savings in cost and material resources needs to be examined by full scale experimentation.

B. Important Deductions Regarding Conduit Behavior

In spite of the studies that have been made to-date, there remains a large gap between the results of experiments and their theoretical

FIGURE 10 COMPARISON OF RESULTS FROM CONTROLLED BUCKLING EXPERIMENTS ON BURIED CONDUITS

interpretation, which limits their use in developing generalized, rational design procedures. The shortcomings of available test results are examined below with a view towards identifying future studies most needed for the development of improved design approaches.

Modulus of Soil Restraint

Experimental results clearly demonstrate that the mode of failure, and hence the performance limit, is strongly influenced by the nature (linear, nonlinear, time-dependent) and magnitude of the support provided by the soil backfill. All high mode buckling theories known to the writers assume the soil to be linear, elastic and time-independent in its behavior. The analysis of snap-through buckling by Klöppel and Glock considers stress-dependent soil support, but the modulus is assumed to increase linearly with the stress level. Because the true nature of soil support has not yet been modeled appropriately, no buckling theory untempered by empirical correlations can be a guide to practical design. On the other hand, empirical correlations dealing with the soil support modulus are based on the results of experiments on small tubes (e.g. Luscher, 1966; Allgood and Ciani, 1968). As the soil modulus is very sensitive to the type of soil (all the above experiments were conducted on sands) and on the level of stress and strain in the soil, the results of tests on small tubes cannot be scaled directly to field conditions. Only in the studies by Wu (1977) was there any attempt to measure the actual soil modulus in backfill supporting large culverts, but the results are typical only of culverts constructed before controlled compaction was commonly specified. In future experiments, the best efforts possible should be directed towards measuring stress and strain in soil backfill surrounding a

conduit. Tests should also be conducted using clay backfills to investigate their support characteristics as compared to granular backfills, since it is not always feasible to specify the use of granular soils in practice. Experimental measurements of normal interface pressures are limited in number, and many of the results obtained are suspect. The buckling load is dependent on the distribution of boundary pressures, and it is important to determine experimentally what distributions may be expected and how they may influence the buckling load. Shear stresses at the soil-conduit interface have been considered only approximately in theory and largely ignored experimentally. Efforts should be made to clarify their role in conduit performance.

Imperfections and Residual Stresses

It is not possible to fabricate conduits without some material and geometric imperfections and without leaving residual stresses due to forming (unless they are relieved by heat treating). Imperfections and residual stresses influence the mode of failure in ways not directly scalable from tests on small tubes. They complicate efforts to obtain reproducible experimental results, and the dangers inherent in drawing conclusions from the results of a few experiments (let alone from a single field experience) cannot be emphasized too strongly.

Strength tests performed on corrugated ($2 \times 1/2$ and $2 \frac{2}{3} \times 1/2$) steel culverts by Lane (1965) showed the influence of imperfections on culvert behavior. Over 150 tests on culverts ranging from 12 to 48 inches in diameter and from 0.052 to 0.168 inches in wall thickness were conducted to investigate the ability of corrugated pipe to resist

yielding, buckling, and seam slippage. The test apparatus, illustrated in Fig. 11, encouraged wall disturbance at gaps in circumferential support located at the springlines. The buckling load was observed to increase as the span decreased, until a limiting size was reached such that the buckling load was no longer dependent on the gap, and local buckling occurred randomly around the pipe circumference instead of at the springline. Lane interpreted the cause of this behavior, as well as the scatter in experimental results, to be geometric imperfections in the pipe x-sections. The writers believe that reducing the gap span to the limiting size provided sufficient circumferential support to encourage high mode buckling, but imperfections in the form of geometric irregularities or residual stresses caused local buckling to occur at random locations rather than in a large number of circumferential waves. Because the applied buckling load could not be compared to a theoretical critical load, due to uncertain load distribution in the tests, the reduction in buckling resistance due to imperfections can not be determined. In the future, tests should be conducted on full-size culverts with special efforts directed towards evaluating the effects of imperfections and residual stresses on the buckling load.

Bending Stresses and Local Yielding

The economy inherent in the use of flexible metal culverts is predicated on sufficient deflection developing so that soil pressures can be resisted mainly by thrust forces in the conduit wall. In cases where the soil support is moderate to weak and in less flexible conduits, significant bending stresses accompany deflection. Demmin's tests showed that large bending stresses can develop after relatively small deflection in long-span pipe arches with shallow cover. Although

FIGURE 11 WALL CRUSHING APPARATUS (after LANE, 1965)

the pipe arch was very flexible ($\frac{EI}{R^3} = 1.5$), the wall stiffness (EI) was relatively large, and only small changes in the radius of curvature were needed to produce large moments and local yielding. The effects of local yielding on conduit performance and its interaction with the buckling mechanism has not been addressed adequately and is presently a source of major uncertainty in adapting experience with conduits of normal size (up to 12 feet in diameter) to the design of long-span conduits (up to 60 feet or more in span).

Inelastic behavior is also important in smaller conduits, as reflected by Howard's (1971) tests. In the weaker clay backfill the most flexible conduit failed catastrophically by elastic buckling. Less flexible conduits exhibited considerable inelastic behavior before collapse. Initial yielding (crimping) occurred locally in several locations at approximately one half the load capacity. Howard believed the crimps resulted from plastic hinging, but because deflections before crimping occurred increased as the conduits became less flexible, the writers believe that local inelastic buckling initiated wall disturbance. Crimping occurred at larger deflections because of increasing resistance to buckling. Analysis of inelastic buckling and definition of subsequent performance is not possible at this time, and more study is needed in this area. In denser clay backfill initial crimping resulted from elastic local buckling. Bending restraint provided by the stronger backfill induced elastic buckling at overpressures larger than those producing inelastic crimping in an identical conduit supported by weaker backfill. It is not possible to distinguish visually between the two buckling modes; therefore, tests on buried conduits should include

instrumentation designed to monitor strain distributions before and after buckling is initiated.

Performance Limits

Associated with the difficulties involved in identifying the mode of failure is the question of whether or not a given mode of failure should be considered as a performance limit, i.e. the load at which serviceability of the conduit is no longer acceptable.

Referring to Fig. 12; it is seen that:

- (1) a local buckle seldom indicates incipient collapse,
- (2) a snap-through buckle may, or may not, indicate incipient collapse, and
- (3) wall crushing and seam distress can occur locally at loads much below the collapse load.

Thus, it is far from clear what conditions should be considered as performance limits for design purposes.

Although when local buckling or wall crushing has occurred the conduit may not be near collapse, its structural load capacity has been impaired. Over long periods of time the conduit may continue to deform with further deterioration of its structural capacity, and the possibility of damage from corrosion is enhanced. In the writers' view, studies are needed to determine when local buckling should be treated as a performance limit. Wall crushing and seam separations involving breaking of welds, shearing of bolt or rivet holes, and the like, should probably be considered as performance limits. On the other hand, a seam that is designed to slip at a predetermined load, but maintain its resistance to bending, can

FIGURE 12 LOAD-DEFLECTION BEHAVIOR OF FULL SCALE CONDUITS IN BUCKLING TESTS

FIGURE 12 (continued)

FIGURE 12 (continued)

FIGURE 12 (continued)

serve to reduce and redistribute interface pressures, and thereby enhance performance considerably.

Role of Controlled Experiments

The ability to predict the critical loads causing elastic buckling or plastic yielding (including the development of hinges) is still rudimentary, and the writers are not aware of any methods for predicting inelastic buckling in buried conduits. Thus, the only way to insure that a buried conduit will perform satisfactorily is to design the conduit so that wall crushing will be the critical failure mode. To do this with assurance, the use of well-compacted granular backfill around conduits is presently required, and requisite experience for selecting appropriate wall sections is available only for corrugated steel culverts up to diameters of about 12 feet. To extend this experience to conduits of other materials and larger spans, and to other types of backfill, will require accumulation of extensive performance records over long periods of time. Alternately, prediction capabilities can be improved and verified using a limited number of controlled experiments: the results can then be applied to existing performance records, and improved design methods developed in less time, and at a much lower cost.

V. SUMMARY

Despite the complexity of interaction between flexible conduits and backfill soil, and the associated difficulties in predicting conduit behavior, the performance record of typical corrugated steel culverts (less than 12 feet diameter) is very good. The majority of failures that have occurred can be traced to excessive live loading during construction or to sub-standard backfill soil. Inability to model soil and conduit properties accurately and to identify the conditions under which different failure modes are critical has been compensated for by extensive experience and the use of appropriate safety factors in design. On the other hand, for conduit materials other than steel, for cohesive backfills, and for longer spans, comparable empirical information is not available. Only better understanding of the possible failure mechanisms can lead, in a short period of time, to improved predictive capabilities and hence to more economical designs.

This report has reviewed many of the theoretical and experimental studies that have been conducted on buried flexible conduits and has attempted to assess the value of these studies for the purpose of establishing rational design criteria for buried flexible conduits. The overall results are summarized below.

A. Wall Crushing

Wall crushing (yielding of the conduit section due to thrust forces only) has been observed experimentally in conventional-sized flexible conduits supported by very dense granular backfill. Deflections before failure are consequently very small which, coupled with the relatively low stiffness of the section, insures that bending will

contribute little to the wall stresses. If failure is by wall crushing, then the conduit has supported its overburden load in the most efficient manner. The goal is to select the geometry, culvert section, backfill, and construction methods to approach this ideal as closely as possible.

B. Seam Failure

Seam separation is probably the simplest failure mechanism to identify. Seam integrity is a function of the type of culvert, the construction technique, and the thrust load. Strength tests by Lane (1965) showed that seam failure (either rivet shearing or weld rupture) is critical in corrugated culverts less than 2 feet in diameter. However, tests on culverts with larger diameters (up to 4 feet) showed that wall crushing is more critical. Watkins and Moser's tests also showed that some other mode of wall disturbance preceeds seam distress. Although seam slippage occurs at loads less than the seam capacity, structural integrity of the culvert is not immediately affected and, generally, a more favorable distribution of interface soil pressure results. Research on seam design could be very rewarding: a seam that can continue to slip without impairing bending resistance at thrust loads below those required to induce wall crushing could result in significantly more economic designs.

C. Elastic Buckling

Elastic buckling, the development of an instability before yielding is initiated in the conduit section, can occur in high modes if the interface pressures are reasonably uniform and imperfections or local residual stresses are insignificant. Otherwise, elastic buckling is

initiated as a local buckle in one (or a few) locations and may occur at the crown, invert or other positions depending on the location of the critical combination of thrust, bending moment, imperfection and residual stress. A snap-through buckle may be an initial sign of distress or it may develop at higher loads after yielding or smaller local crimping has occurred at another location. The many high mode buckling theories considering elastic soil support are relatively similar, excluding the different approaches for formulating the modulus of soil support. Most expressions for the critical buckling pressure are lower-bound solutions. However, even lower-bound solutions generally overestimate the elastic buckling loads observed in controlled experiments. Imperfections and residual stresses can lead to early local buckling. Several methods have been proposed to account for the accompanying reduction in critical pressure, but none of these methods has been verified by controlled experiments of sufficient scope and generality. Of the snap-through buckling theories, only Klöppel and Glock treat non-uniform boundary pressures and consider the deflected conduit shape. Their solution is probably the most satisfactory representation currently available, yet it may be overconservative in some situations and unsafe in others. Materials with high yield points offer advantages in overcoming the effects of imperfections and residual stresses on the load capacity. Studies are needed to quantify the improvements that can be realized economically.

Klöppel and Glock's elastic snap-through buckling theory indicates that elliptical conduits are more susceptible to buckling than those of circular shape. For long-span conduit and backfill stiffnesses in current use, the critical buckling load would be approximately half

that of an equivalent circular shape. Thus, buckling criteria adapted from experience with circular culverts may not be applicable to long-span arches (Stetkar, 1978).

Critical buckling pressures for larger conduits cannot be scaled directly from small-scale experiments. Larger-scale, controlled tests show that elastic local buckling can be the first sign of distress in flexible conduits with moderately good soil support and can occur after relatively small deflection. In some cases the development of a local buckle constitutes a performance limit, whereas in others the conduit can sustain considerable additional load before failure occurs (little resistance to further deflection). Studies are badly needed to define the conditions under which local buckling should be considered as a performance limit.

D. Excessive Deflection

Excessive deflections have been observed in flexible conduits of conventional size supported by weak soil, and in long-span conduits supported by relatively strong soil. In the former case deflections may exceed 15% of the diameter before collapse is imminent, while in the latter case the deflections may be less than 5% of the span. Failure by excessive deflection in long-span conduits with good backfill can be triggered by bending stress concentrations that are produced during construction. Zones of plastic yielding can develop, and because soil support is low in the vicinity of the crown (due to shallow cover), catastrophic collapse in the form of a snap-through buckle can follow. Analytical models predicting the bending moment distribution during construction of long-span conduits are, at best, rudimentary. Thus, careful monitoring of field installations are needed to advance the state-of-the-art.

Excessive deflection of typical culverts with deep burial does not initially involve bending stress concentrations near the crown. Deflections at the crown induce soil arching, which tends to equalize the boundary pressure distribution. If the required deflection is sufficiently large, flattening of the culvert, coupled with the development of plastic yielding at the springlines, can induce an inversion of curvature involving a large fraction of the circumference in the vicinity of the crown. If loading is continued, the culvert will collapse. Such excessive deflection will not occur in granular backfill densified to 95% Std. AASHTO density: however, if a cohesive backfill compacted to this specification is used, more deflection is required to mobilize equivalent lateral soil resistance which, coupled with gradual loss of circumferential support due to time-dependent behavior, can lead to failure. More studies on the behavior of conduits supported by cohesive backfills are needed.

E. Plastic Hinging

With less flexible conduits, plastic yielding under combined bending and thrust stresses can continue until a plastic hinge is formed before excessive deflection at the crown occurs; that is, the rotational resistance at a point in the cross-section becomes fully mobilized. If additional load is applied, redistribution of boundary pressures must occur and if rotational resistance becomes fully mobilized at a sufficient number of locations the conduit will collapse. Even if a collapse mechanism does not develop, failure by excessive deflections or inelastic buckling may occur prematurely, and the conduit will not be supporting load in its most efficient manner. Consequently, bending

moments can have important influences on buried conduit behavior. It has been shown experimentally that plastic yielding may develop after small deflections, especially in large diameter culverts and pipe arches. Finite element analyses can estimate the contribution of bending to the total wall stress, but present formulations are not well adapted to the analysis of post-hinging behavior. Analytical techniques need to be extended to permit prediction of collapse loads.

F. Inelastic Buckling

No theory has been developed to predict inelastic buckling (instability after yielding has been initiated) of buried conduits. To date, buckling tests have lacked sufficient instrumentation even to develop empirical insight into the problem. Inelastic buckling appears to be a critical failure mode in flexible culverts whenever the deflections necessary to mobilize lateral soil support are sufficient to induce yielding, and in less flexible culverts subjected to non-uniform loading. Thus, it is suspected that inelastic buckling may be an important failure mode in long-span culverts. Probably no other aspect of buried conduit behavior is in greater need of additional study.

G. Recommended Experimental Studies

Because many aspects of conduit-soil interaction cannot presently be modeled theoretically, extensively instrumented, controlled tests on large scale conduits are badly needed. These tests should be conducted with a sufficient range of conduit flexibility and soil support (granular and cohesive) to encompass all types of failure modes. Loading should be continued to collapse so that the

relationship between initial wall disturbance and the collapse load can be studied. In each test the nature and quality of soil support should be directly measured around the conduit and then compared to results obtained from triaxial and plane-strain laboratory tests at comparable confining pressures.

Conduits should be instrumented to measure their deflected shape, the distribution of strains due to thrusts and bending moments, and the distribution of interface soil pressures. This will allow reexamination of the validity of simplifying theoretical and design assumptions, and establish the general applicability of the test results. The ability of finite element computer techniques to predict the behavior of conduits under a large variety of controlled conditions can also be evaluated. Measurement of shear stresses at the interface is needed, as considerable doubt remains regarding the influence of traction forces on conduit behavior.

Because imperfections and residual stresses can have a large effect on conduit behavior, tests on unsupported conduits should be conducted and compared with theoretical solutions to assess their influence on the load capacity. Also, in tests where soil support is provided, residual stresses due to fabrication should be measured independently so that the true conduit stresses at failure can be obtained. All these measurements will provide a basis for further developments in the design of all kinds of buried conduits, including that of long-span arches.

ACKNOWLEDGEMENTS

A. K. Howard, N. Peters, and T. H. Wu provided additional information on the tests performed by the Bureau of Reclamation, the Prairie Farm Rehabilitation Administration, and Ohio State University, respectively.

The writers benefitted from discussions with R. S. Standley, of Armco's Metal Products Division, concerning the development of the design approach recommended in the AISI Handbook, which is based largely on the results of Watkins and Moser's tests. They are also grateful to G. Abdel-Sayed for calling their attention to Klöppel and Glock's elastic buckling theory.

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