LIBRARIES

Digitized by the Internet Archive
in 2011 with funding from
LYRASIS members and Sloan Foundation; Indiana Department of Transportation

SCHOOL OF CIVIL ENGINEERING

620.51
P97te

PURDUE UNIVERSITY LIBRARIES

JUN 7 1978

JOINT HIGHWAY
RESEARCH PROJECT

JHRP-77-25

COMPUTERIZED SLOPE STABILITY
ANALYSIS FOR INDIANA HIGHWAYS
Vol. I

Eva Boutrup

PURDUE UNIVERSITY
INDIANA STATE HIGHWAY COMMISSION

Final Report
COMPUTERIZED SLOPE STABILITY ANALYSIS
FOR INDIANA HIGHWAYS

TO: J. F. McLaughlin, Director
Joint Highway Research Project

December, 1977

Project: C-36-36L

FROM: H. L. Michael, Associate Director
Joint Highway Research Project

File: 6-14-12

Attached is the Final Report "Computerized Slope Stability Analysis for Indiana Highways", submitted for acceptance in fulfillment of the objectives of the approved JHRP Research Study of the same title. The research and report were performed by Ms. Eva Boutrup, Graduate Instructor on our staff, under the direction of C. W. Lovell and W. D. Kovacs of our staff.

The report presents results of continuous work on the computer program, STABL, developed under a previous JHRP Study (JHRP-75-8). This includes results of a parametric study conducted on the STABL program, comparisons with other methods of slope stability analysis, and corrections and modifications of the original program. The corrected and improved STABL program, named STABL2, is a valuable tool for geotechnical engineers in the evaluation of the stability of highway slopes.

This Report is in two volumes with Vol. II consisting primarily of the STABL2 Program and having such a subtitle.

The Report is submitted for approval and acceptance. Copies of the report will also be submitted to ISHC for their review, comment and similar acceptance.

Respectfully submitted,

Harold L. Michael
Associate Director

HLM:ms

cc: A. G. Altschaeffl
O. M. Bevilacqua
W. L. Dolch
R. L. Eskew
G. D. Gibson
W. H. Goetz
M. J. Gutzwiller

G. K. Hallock
D. E. Hancher
K. R. Hoover
R. F. Marsh
R. D. Miles
P. L. Owens
G. T. Satterly

C. F. Scholer
M. B. Scott
K. C. Sinha
C. A. Venable
L. E. Wood
E. J. Yoder
S. R. Yoder

Final Report
COMPUTERIZED SLOPE STABILITY ANALYSIS
FOR INDIANA HIGHWAYS

Vol. I

by

Eva Boutrup
Graduate Instructor in Research

Joint Highway Research Project

Project No.: C-36-36L

File No.: 6-14-12

Prepared as Part of an Investigation

Conducted by

Joint Highway Research Project
Engineering Experiment Station
Purdue University

in cooperation with the

Indiana State Highway Commission

Purdue University
West Lafayette, Indiana
December, 1977

ACKNOWLEDGMENTS

I want to express my thanks to my major Professor, C. W. Lovell, and coadvisor, Professor W. D. Kovacs, Purdue University, for continuing support during this research.

Thanks also goes to people from the Indiana State Highway Commission for their participation and cooperation in the improvements of the STABL program. Mr. Sisiliano and Mr. Reeves from the Division of Materials and Tests provided material for the comparisons reported in Chapter VI, and made several suggestions for improvements of STABL. Mr. Bellinger and Mr. McKey from the ISHC Computer Center, Indianapolis, were very helpful in the process of getting STABL on the IBM computer. This included a significant amount of error tracing, for which they introduced a special subroutine (TURKEY) that provided a regulated error trace key search.

Further I want to thank other users of the STABL program for their questions, comments, and suggestions that helped to point out some uncertainties with respect to the use of STABL. These are discussed (and hopefully clarified) in Chapter IV.

Thanks, finally goes to Ms. Edith Vanderwerp and Ms. Jan Bollinger for typing draft and parts of the final report, and to Ms. Peggy McFarren who drafted the figures.

TABLE OF CONTENTS

	Page
LIST OF TABLES	x
LIST OF FIGURES	xiv
LIST OF ABBREVIATIONS AND SYMBOLS	xxv
HIGHLIGHT SUMMARY	xxxiii
CHAPTER	
I INTRODUCTION	1
1.1 Purpose of the Present Study	1
1.2 Organization of Report	2
II SLOPE STABILITY ANALYSIS	4
2.1 General Description	4
2.2 Different Methods of Slope Stability Analysis	6
2.2.1 Early Development	6
2.2.2 The $\phi = 0$ Method	7
2.2.3 The Logarithmic Spiral Procedure	9
2.2.4 The Friction Circle Method	11
2.2.5 Method of Slices	11
2.2.5.1 Ordinary Method of Slices	13
2.2.5.2 Bishop's Method	17
2.2.5.3 Modified Bishop Method	18
2.2.5.4 Other Procedures	18
2.2.6 Methods other than Limiting Equilibrium	20
2.2.7 Probabilistic Approach	21
2.2.8 Comparison of Methods	22
2.3 STABL - a Computer Program for General Slope Stability Analysis	24

	Page
III SENSITIVITY ANALYSIS	28
3.1 Introduction	28
3.2 Procedure for Analysis	28
3.3 Cases Studied	29
3.3.1 Sensitivity Analysis of Homogeneous Slopes	29
3.3.1.1 Analysis by STABL	31
3.3.1.2 Results of Stability Analysis of Homogeneous Slopes	33
3.3.1.3 Sensitivity of the Factor of Safety to Input Variables	41
3.3.1.4 Conclusion and Recommendations	60
3.3.2 Sensitivity Analysis of an Embankment Founded on a Soil with a Weak Seam	63
3.3.2.1 Factor of Safety for Failure Determined by a Weak Soil Seam	66
3.3.2.2 Sensitivity Analysis	68
3.3.2.3 Results of Stability Analysis for Embankments	69
3.3.2.4 Results Applied to Sensitivity Analysis of Embankments	89
3.3.2.5 Summary and Conclusions	95
IV MODIFICATIONS OF THE STABL PROGRAM	96
4.1 The Modified Sliding Block Routine (BLOCK2)	96
4.1.1 Generation of Active and Passive Portions of the Sliding Block According to Rankine's Theory .	98
4.1.2 Application of the New Sliding Block Routine . .	99
4.2 Option of Several Piezometric Surfaces	99
4.2.1 Program Modifications	101
4.2.2 Change in Data Cards	102
4.2.3 Application of more than one Piezometric Level .	102
4.3 Modified Bishop Factor of Safety for Circular Failure Surfaces	106
4.3.1 Program Modifications	106
4.3.2 CIRCL2 and SURBIS	108

4.4	Example Problem for Revised STABL Program	108
4.5	Some Advice in the Use of STABL	110
4.5.1	Preliminary Analysis - Stability Charts	110
4.5.2	Use of STABL	123
4.5.2.1	How to Define Limitations for Search Routines	123
4.5.2.2	Clockwise and Counterclockwise Direction Limits	124
4.5.3	Strength along Soil Boundaries	126
4.5.4	Analysis Involving Earthquake Loadings	126
4.5.4.1	Effect of Vertical Earthquake Loading on the Stability of a Slope	126
4.5.4.2	Cavitation Pressure	129
4.5.4.3	Illustrative Example	131
4.5.4.4	Dynamic Earthquake Analysis	134
4.5.5	Misleading Factors of Safety	134
V	COMPARATIVE DESIGN STUDY	137
5.1	Description of Problem	137
5.2	Types of Analysis	137
5.3	Methods of Analysis	142
5.3.1	The Simplified Bishop Method	142
5.3.2	Morgenstern-Price Analysis	142
5.3.3	Method Used in the STABL Program	143
5.4	Soil Strength Parameters Used in Stability Analyses	144
5.4.1	Corrected Field Vane Shear Strength	144
5.4.2	SHANSEP Anisotropic Shear Strength	144
5.5	Results and Comparisons	145
5.5.1	Preliminary Stability Analyses	145
5.5.1.1	Frequency Distribution Curves for the Factor of Safety	153
5.5.2	Final Design Stability Analyses	156
5.5.2.1	Partially Drained Stability Analyses	157
5.5.2.2	Long Term Stability Analyses	157
5.6	Summary and Conclusion	166

VI	COMPARISON OF STABL WITH EXISTING PROCEDURES FOR SLOPE STABILITY ANALYSIS USED BY INDIANA STATE HIGHWAY COMMISSION (ISHC)	170
6.1	Problem 1	171
6.2	Problem 2	171
6.3	Problem 3	183
6.4	Problem 4	188
6.5	Conclusion of Comparative Analysis	200
VII	SUMMARY AND RECOMMENDATIONS FOR FUTURE WORK	204
7.1	Work and Results	204
7.1.1	Parametric Study	204
7.1.2	Modifications of STABL	204
7.1.3	Comparative Study	205
7.2	Suggestions for Future Work	206
	LIST OF REFERENCES	207
	APPENDICES	
	Appendix A, Tables and Figures Related to Chapter III	211
	Appendix B, Tables and Figures Related to Chapter V	297
	Appendix C, Tables, Figures and Analysis Related to Chapter VI	320
C.I	The Influence of the Line of Thrust on the Factor of Safety in Slope Stability Analysis, Method of Slices	320
C.1.1	Introduction	320
C.1.2	Position of Line of Thrust	321
C.1.3	Interslice Stress Distribution	322
C.II	Influence of Moment Center on the Factor of Safety for Circular Failure Surfaces	324
C.2.1	Introduction	324
C.2.2	Derivation of Equations for F_c and $\frac{\partial F_c}{\partial y}$ as a Function of the Problem Geometry and Moment Center (y) for the Case of $\phi = 0$	326

Appendix D, Corrections, Adjustments and Smaller Changes	
in STABL	373
D.1 Adjustments for the IBM Computer	373
D.2 Debugging of STABL	373
D.2.1 Errors in the Anisotropic Soil Option . . .	373
D.2.2 Infinite Loop in the Circular Surface Generator	374
D.2.3 Unit Weight of Water	376
D.2.4 The TURKEY Trace Key	377
D.3 Other Adjustments	378
D.3.1 Modifications in the Irregular Surface Generator (RANDOM)	378
D.3.2 Modifications of Errors RC11 and BK10 . . .	380
D.3.3 Format Adjustments	380
D.3.3.1 Format Statements	381
D.3.3.2 Misleading Factors of Safety	381
D.3.3.3 Changes of the Calcomp Plot	382
D.3.3.4 Updating of Comment Cards	382
D.3.3.5 Flow Chart of STABL	383
Appendix E, Listing of Revised STABL Program	393
Main Program STABL2	394
Subroutine READER	399
Subroutine QUIT	403
Subroutine PROFIL	404
Subroutine ANISO	414
Subroutine WATER	418
Subroutine LOADS	422
Subroutine EQUAKE	426
Subroutine LIMITS	427
Subroutine INTSCT	430
Subroutine SURFAC	432
Subroutine RANDOM	435
Subroutine RANSUF	451
Subroutine BLKSUF	463

Appendix E (cont.)

Subroutine	BLOCK2	468
Subroutine	SORT	473
Subroutine	EXECUT	474
Subroutine	SLICES	475
Subroutine	WEIGHT	480
Subroutine	SOILWT	486
Subroutine	FACTR	488
Subroutine	SCALER	493
Subroutine	PLOTIN	495
Subroutine	PLTN	501
Subroutine	POSTN	506
Appendix F, STABL on the IBM Computer		508
F.1	Introduction	508
F.2	The Random Function Ranf(x)	508
F.3	Precision	509
F.3.1	Machine Rounding Errors	509
F.3.2	Use of TOL	510

LIST OF TABLES

Table		Page
2.1	Equations and Unknowns Associated with the Method of Slices	15
2.2	Factor of Safety for Different Methods of Slope Stability Analysis	23
2.3	Comparison of STABL with Modified Bishop Method after Wright (1969), Using Failure Surfaces Obtained by the Modified Bishop Analysis	27
3.1	Factors of Safety for Homogeneous Slopes	40
3.2	Variation in Sensitivity of the Factor of Safety to Input Parameters	52
3.3	Typical Values of Partial Coefficients Used in Denmark for Geotechnical Designs	62
3.4	Range in Variables Studied for an Embankment on a Foundation with a Weak Layer	67
3.5	Geometry Parameters k and a for Embankment Stability Analysis	78
3.6	Statistical Analysis of Geometry Parameters k and a	92
4.1	Data Setup for Example Problem for Revised STABL Program	109
4.2	Output of Example Problem	111
4.3	Stability Analysis of Slopes Subjected to Earthquake Forces	132
5.1	Calculation of In Situ Undrained Shear Strength (After Ladd, 1975)	150
5.2	s_u in Zones A, B, C and D for 95% Consolidation	160
5.3	s_u in Zones A, B, C and D for 100% Consolidation	164

Table	Page
5.4 Results of Stability Analyses from MIT-Report (After Ladd, 1975)	167
5.5 Results of Stability Analyses performed with STABL	168
6.1 Comparative Study, Problem 1. Factor of Safety for Different Methods of Analysis	175
6.2 Comparative Study, Problem 2. Factor of Safety for Different Methods of Analysis	180
6.3 Comparative Study, Problem 3. Factor of Safety for Different Methods of Analysis	187
6.4 Comparative Study, Problem 4. Factor of Safety for Different Methods of Analysis	201
Appendix	
Table	
A.1 $\frac{\Delta F}{\Delta \phi'}$ for Homogeneous Slopes	212
A.2 $\frac{\Delta F}{\Delta \tan \phi'}$ for Homogeneous Slopes	213
A.3 $\frac{\Delta F}{\Delta (\frac{c'}{\gamma h})}$ for Homogeneous Slopes	214
A.4 $\frac{\Delta F}{\Delta \cot \beta}$ for Homogeneous Slopes	215
A.5 Sensitivity Analysis, Homogeneous Soil, Vertical Slope ($\beta = 90^\circ$). Actual Change in the Factor of Safety F, due to Changes in the Independent Variables	216
A.6 Sensitivity Analysis, Homogeneous Soil, Slope 1/2 ($\beta = 63.4^\circ$). Actual Change in the Factor of Safety F, due to Changes in the Independent Variables	217
A.7 Sensitivity Analysis, Homogeneous Soil, Slope 1/1 ($\beta = 45^\circ$). Actual Change in the Factor of Safety F, due to Changes in the Independent Variables	218
A.8 Sensitivity Analysis, Homogeneous Soil, Slope 2/1 ($\beta = 26.6^\circ$). Actual Change in the Factor of Safety F, due to Changes in the Independent Variables	219
A.9 Sensitivity Analysis, Homogeneous Soil, Slope 3/1 ($\beta = 18.4^\circ$). Actual Change in the Factor of Safety F, due to Changes in the Independent Variables	220

Appendix
Table

Page

A.10	$\frac{\Delta F_{\phi'}}{\Sigma \Delta F}$ for Homogeneous Slopes (%)	221
A.11	$\frac{\Delta F_{c'}}{\Sigma \Delta F}$ for Homogeneous Slopes (%)	222
A.12	$\frac{\Delta F_{\beta}}{\Sigma \Delta F}$ for Homogeneous Slopes (%)	223
A.13	$\frac{\Delta F_h}{\Sigma \Delta F} = \frac{\Delta F_Y}{\Sigma \Delta F}$ for Homogeneous Slopes (%)	224
A.14	Factors of Safety for Embankment on Foundations with a Weak Layer	225
B.1	Example of Input Data for Preliminary Design Analysis, Circular Search	298
B.2	Example of Input Data for Preliminary Design Analysis, Sliding Block Search	299
B.3	Example of Input Data for Final Design Analysis, Sliding Block Search	300
B.4	Cases Investigated by STABL for the Comparative Design Study	302
C.1	Subroutine FACTR, Modified to Calculate Factor of Safety by the Modified Bishop Method (Used only for Circular Arc Failure Surfaces)	333
C.2	Subroutine FACTR Substituted in Program STABL to Calculate the Factor of Safety by Spencer's Method	334
C.3	Subroutine FACTR, Modified to Calculate Interslice Sideforces E, and Effective Normal Force and Stress at Bottom of each Slice for Normal STABL Procedure (Janbu), Specified Surface	338
C.4	Examples of Input Data for Problem 1, Comparative Study	339
C.5	Examples of Input Data for Problem 2, Comparative Study	340
C.6	Examples of Input Data for Problem 3, Comparative Study	341
C.7	Examples of Input Data for Problem 4, Comparative Study	342

Appendix Table	Page
C.8 Interslice Forces, Effective Normal Stress at Bottom of Slices, and Line of Thrust for Spencer Analysis, Problem 2, Circle 5	343
C.9 Line of Effective Thrust and Interslice Shear Factors of Safety, Spencer Analysis, Problem 2, Circle 5	344
C.10 Interslice Forces and Normal Stress at Bottom of Slices for STABL Stability Analysis (Janbu Method), Problem 2, Circle 5	348
C.11 Factor of Safety, Interslice Forces, and Line of Thrust for Spencer Analysis, Problem 3, Surface BLOCK2-4	349
C.12 Interslice Shear Factors of Safety for Sliding Surface BLOCK2-4, Problem 3, Spencer's Method	350
C.13 Factor of Safety and Interslice Forces for STABL Stability Analysis (Janbu Method), Problem 3, Surface BLOCK2-4	351
C.14 Variables Used in Calculation of Line of Effective Thrust, Interslice Stresses, and Interslice Shear Strength for Spencer's Method	352
C.15 Variables Used in Appendix C.II	354
D.1 Chronologic Listing of Corrections and Changes Made in the STABL Program between August 1975 and June 1977	384
D.2 Illustration of the Use of Subroutine TURKEY for Error Trace of the STABL Program	389
F Changes Necessary to Transfer STABL from the CDC Computer to an IBM Computer	512

LIST OF FIGURES

Figure	Page
2.1 Principle of the $\phi = 0$ Method	8
2.2 Principle of Log Spiral Method	10
2.3 Principle of Friction Circle Method	12
2.4 Division of Sliding Mass into Slices	14
2.5 Forces Acting on a Slice	16
3.1 Flow Chart Showing Steps in Sensitivity Analysis	30
3.2 Range in Variables for Sensitivity Analysis of Homogeneous Slopes	32
3.3 Critical Failure Surfaces for Vertical Slopes in Homogeneous Soil	34
3.4 Critical Failure Surfaces for Slopes 1/2 in Homogeneous Soil	35
3.5 Critical Failure Surfaces for Slopes 1/1 in Homogeneous Soil	36
3.6 Critical Failure Surfaces for Slopes 2/1 in Homogeneous Soil	37
3.7 Critical Failure Surfaces for Slopes 3/1 in Homogeneous Soil	38
3.8 Relationship between Slope Parameters, Critical Failure Surface and Factor of Safety for Homogeneous Slopes	42
3.9 Isometric Plot of Factor of Safety vs. $(\phi', \frac{c'}{\gamma_h})$ for Slope 1/2	43
3.10 Factor of Safety vs. Strength Angle ϕ' for Various $\frac{c'}{\gamma_h}$, Slope 1/2	44
3.11 Factor of Safety vs. $\frac{c'}{\gamma_h}$ for Various ϕ' , Slope 1/2	45
3.12 Equal Factor of Safety Contours for Slope 1/2, Homogeneous Soil	46

Figure

Page

3.13	$\frac{\Delta F}{\Delta(\frac{c'}{\gamma h})}$ vs. Slope Angle β for Various ϕ'	47
3.14	$\frac{\Delta F}{\Delta(\frac{c'}{\gamma h})}$ vs. ϕ' for Different Slope Angles β	48
3.15	$\frac{\Delta F}{\Delta \phi'}$ vs. Slope Angle β for Various ϕ'	49
3.16	$\frac{\Delta F}{\Delta \phi'}$ vs. ϕ' for Various Slopes	50
3.17	$\frac{\Delta F}{\Delta \cot \beta}$ vs. β for Various ϕ' , $\frac{c'}{\gamma h} = 0$ to 0.3	51
3.18	Relative Sensitivity $\frac{\Delta F_{x_i}}{\Sigma \Delta F}$ for Vertical Slope	55
3.19	Relative Sensitivity $\frac{\Delta F_{x_i}}{\Sigma \Delta F}$ for Slope $1/2$	56
3.20	Relative Sensitivity $\frac{\Delta F_{x_i}}{\Sigma \Delta F}$ for Slope $1/1$	57
3.21	Relative Sensitivity $\frac{\Delta F_{x_i}}{\Sigma \Delta F}$ for Slope $2/1$	58
3.22	Relative Sensitivity $\frac{\Delta F_{x_i}}{\Sigma \Delta F}$ for Slope $3/1$	59
3.23	Variables for an Embankment Built on a Foundation Soil with a Weak Soil Layer	64
3.24	A Sliding Block Type of Failure Surface is most Critical	70
3.25	Comparison between Failure to Left and Failure to Right Side of Embankment, $D = 10$ ft, $\alpha = 5.7^\circ$, $\phi = 0$	72
3.26	Comparison between Failure to Left and Failure to Right Side of Embankment, $D = 10$ ft, $\alpha = 11.3^\circ$, $\phi = 0$	73
3.27	Change in Critical Failure Surface when Strength of Weak Soil Seam Increases	74
3.28	Normalized Factor of Safety F_n vs. Normalized Strength c_n for Embankment 20 feet High, Slopes $1.5/1$, Depth to Weak Seam 5 feet, Inclined at $\alpha = 11.3^\circ$ to Horizontal	76

Figure

Page

3.29	Example of F_n vs. c_n for Embankment 20 feet High, Slopes 1.5/1, Depth to Weak Seam 5 feet, Inclined at $\alpha = 11.3^\circ$ with Horizontal	77
3.30	Variation of k and "a" Coefficients with α for Constant Depth $D = 0.25h$	79
3.31	Variation of k and "a" Coefficients with α for Constant Depth $D = 0.5h$	80
3.32	Variation of k Coefficients with Depth D for Constant $\alpha = 0^\circ$	81
3.33	Variation of k Coefficients with Depth D for Constant $\alpha = 5.7^\circ$	82
3.34	Variation of k Coefficients with Depth D for Constant $\alpha = 11.3^\circ$	83
3.35	Variation of "a" Coefficients with Depth D for Constant $\alpha = 0^\circ$ (and $\alpha = 5.7^\circ$)	84
3.36	Variation of "a" Coefficients with Depth D for Constant $\alpha = 11.3^\circ$	85
3.37	Variation of F vs. c_w for Various α , and Different Ratios of $\frac{c_F}{c_E}$, $D = 5$ ft	87
3.38	Example of Variation of the Factor of Safety with Geometry of Weak Soil Layer	88
4.1	Sliding Block Failure Surface with Rankine Active and Passive Zones	97
4.2	BLOCK2 Sliding Block Analysis Applied to Anisotropic Soil	100
4.3	Slope Stability Analysis for Problem with a Perched Water Table	103
4.4	Rapid Drawdown Stability Analysis for Embankment	105
4.5	Calcomp Plot of Result of STABL Analysis	118
4.6	Stability Chart for Slopes in Homogeneous Soil (after Taylor, 1948)	119

Figure	Page
4.7 Stability Chart for Slopes in Homogeneous Soil, $\phi = 0$ (after Taylor, 1937)	120
4.8 Stability Chart of the Taylor Type for Homogeneous Slopes, based on Results from STABL Program	121
4.9 Comparison between STABL and Taylor Stability Chart for Homogeneous Slopes	122
4.10 Illustration of Clockwise and Counterclockwise Direction Limitations for Initiation of Failure Surfaces .	125
4.11 Transmission of Earthquake Forces to the Pore Water	128
4.12 Change in Pore Water Pressure due to a Horizontal Earthquake Force	130
4.13 Critical Failure Surfaces for Slopes Analysed for Influence of Earthquake Loadings	133
5.1 Soil Profile, Geometry and Design Criteria for Embankment (after Ladd, 1975)	138
5.2 Relationship between Total and Effective Strength for a Factor of Safety Larger than One	140
5.3 Stress vs. Strain from $\overline{CK_U}$ Tests with Different Modes of Failure (after Ladd, 1975)	146
5.4 Input Data and Results of Preliminary, Undrained, Total Stress Analysis, Circular Failure Surfaces	148
5.5 Drained Effective Stress Analysis of Embankment, Preliminary Design	149
5.6 Anisotropic Undrained Strength Parameters for Use in Total Stress Analyses (after Ladd, 1975)	151
5.7 Undrained Total Stress Analysis of Embankment with SHANSEP Anisotropic Strengths, Preliminary Design	152
5.8 Different Types of Critical Failure Surfaces Generated by STABL	154
5.9 Frequency Distribution Curves for the Factor of Safety, Cases 1A, 2B, 3B, 4B	155
5.10 Estimated Soil Profile and Cross Section for Embankment on Varved Clay after Construction	158

Figure	Page
5.11 SHANSEP Strength Data for Total Stress Analyses at 95% Consolidation (after Ladd, 1975)	159
5.12 Total Stress Analyses of Embankment with Berms, Partially Drained	161
5.13 Drained Effective Stress Analyses of Final Embankment with Berms	162
5.14 SHANSEP Strength Data for Total Stress Analyses at 100% Consolidation (after Ladd, 1975)	163
5.15 Total Stress Analyses of Final Embankment with Berms, 100% Consolidated	165
6.1 Soil Profile and Strength Data for Problem 1	172
6.2 Critical Failure Surfaces from Analysis by NYSTAB - Circular Arc, and Hand Calculated Sliding Wedges	173
6.3 Critical Failure Surfaces from Analysis by STABL, CIRCLE, RANDOM and BLOCK2	174
6.4 Soil Profile and Strength Data for Problem 2	177
6.5 Stability Analysis by ATEC and STABL, Circular Arc	178
6.6 Sliding Block Analyses, STABL and STABL w/ Spencer	181
6.7 Stability Analysis by STABL, Spencer Method, Line of Thrust	182
6.8 Soil Profile and Strength Data for Problem 3	184
6.9 Stability Analysis by ATEC and STABL, Circular Arc	185
6.10 Stability Analysis by STABL, Sliding Block Failures	186
6.11 Soil Profile and Strength Data for Problem 4, Total Stress Analysis ($\phi = 0$)	189
6.12 Critical Circles Analysed	190
6.13 Factor of Safety F_c vs. Moment Center y for Various θ_0 , $r = 2.5h$, $c_E = c_F$	192
6.14 Factor of Safety F_c vs. Moment Center y for Various θ_0 , $r = 2.5h$, $c_E = 2c_F$	193

Figure

Page

6.15	Factor of Safety F_c vs. Moment Center y for Various θ_0 , $r = 5h$, $c_E = 2c_F$	194
6.16	Factor of Safety F_c vs. Moment Center y for Various θ_0 , $r = 2.5h$, $c_E = 0.5c_F$	195
6.17	Stability Number $\frac{F_c \gamma h}{c}$ vs. θ_0 for Various Radii, Circular Failure Surfaces, $c_E = c_F$, $\phi = 0$	196
6.18	Stability Number $\frac{F_c \gamma h}{c}$ vs. θ_0 for Various Radii, Circular Failure Surfaces, $c_E = 2c_F$, $\phi = 0$	197
6.19	Stability Number $\frac{F_c \gamma h}{c}$ vs. θ_0 for Various Radii, Circular Failure Surfaces, $c_E = 0.5c_F$, $\phi = 0$	198
6.20	Correction Factors for the Simplified Janbu Procedure (after Janbu et al., 1956)	203

Appendix

Figure

A.1	Factor of Safety vs. Strength Angle ϕ' for Various $\frac{c'}{\gamma h}$, Vertical Slope	233
A.2	Factor of Safety vs. $\frac{c'}{\gamma h}$ for Various ϕ' , Vertical Slope	234
A.3	Factor of Safety vs. Strength Angle ϕ' for Various $\frac{c'}{\gamma h}$; Slope 1/2	235
A.4	Factor of Safety vs. $\frac{c'}{\gamma h}$ for Various ϕ' , Slope 1/2	236
A.5	Factor of Safety vs. Strength Angle ϕ' for Various $\frac{c'}{\gamma h}$, Slope 1/1	237
A.6	Factor of Safety vs. $\frac{c'}{\gamma h}$ for Various ϕ' , Slope 1/1	238
A.7	Factor of Safety vs. Strength Angle ϕ' for Various $\frac{c'}{\gamma h}$, Slope 2/1	239
A.8	Factor of Safety vs. $\frac{c'}{\gamma h}$ for Various ϕ' , Slope 2/1	240
A.9	Factor of Safety vs. Strength Angle ϕ' for Various $\frac{c'}{\gamma h}$, Slope 3/1	241

Appendix
Figure

Page

A.10	Factor of Safety vs. $\frac{c'}{\gamma h}$ for Various ϕ' , Slope 3/1	242
A.11	Equal Factor of Safety Contours for Vertical Slope, Homogeneous Soil	243
A.12	Equal Factor of Safety Contours for Slope 1/2, Homogeneous Soil	244
A.13	Equal Factor of Safety Contours for Slope 1/1, Homogeneous Soil	245
A.14	Equal Factor of Safety Contours for Slope 2/1, Homogeneous Soil	246
A.15	Equal Factor of Safety Contours for Slope 3/1, Homogeneous Soil	247
A.16	Factor of Safety vs. $\cot \beta$ for Various ϕ' , $\frac{c'}{\gamma h} = 0$, Theoretical Solution	248
A.17	Factor of Safety vs. $\cot \beta$ for Various ϕ' , $\frac{c'}{\gamma h} = 0.1$	249
A.18	Factor of Safety vs. $\cot \beta$ for Various ϕ' , $\frac{c'}{\gamma h} = 0.2$	250
A.19	Factor of Safety vs. $\cot \beta$ for Various ϕ' , $\frac{c'}{\gamma h} = 0.3$	251
A.20	Factor of Safety vs. $\frac{\gamma h}{c'}$ for Constant ϕ' , Vertical Slope	252
A.21	Factor of Safety vs. $\frac{\gamma h}{c'}$ for Constant ϕ' , Slope 1/2	253
A.22	Factor of Safety vs. $\frac{\gamma h}{c'}$ for Constant ϕ' , Slope 1/1	254
A.23	Factor of Safety vs. $\frac{\gamma h}{c'}$ for Constant ϕ' , Slope 2/1	255
A.24	Factor of Safety vs. $\frac{\gamma h}{c'}$ for Constant ϕ' , Slope 3/1	256
A.25	Equal Factor of Safety Contours vs. Geometry Variables, $\phi' = 0^\circ$	257
A.26	Equal Factor of Safety Contours vs. Geometry Variables, $\phi' = 10^\circ$	258
A.27	Equal Factor of Safety Contours vs. Geometry Variables, $\phi' = 20^\circ$	259
A.28	Equal Factor of Safety Contours vs. Geometry Variables, $\phi' = 30^\circ$	260

Appendix
Figure

Page

A.29	Equal Factor of Safety Contours vs. Geometry Variables, $\phi' = 40^\circ$	261
A.30	$\frac{\Delta F}{\Delta(\frac{c'}{\gamma h})}$ vs. Slope Angle β for Various ϕ'	262
A.31	$\frac{\Delta F}{\Delta(\frac{c'}{\gamma h})}$ vs. ϕ' for Different Slope Angles β	263
A.32	$\frac{\Delta F}{\Delta(\frac{c'}{\gamma h})}$ vs. $\frac{c'}{\gamma h}$ for Various ϕ' , Vertical Slope and Slope 1/2	264
A.33	$\frac{\Delta F}{\Delta(\frac{c'}{\gamma h})}$ vs. $\frac{c'}{\gamma h}$ for Various ϕ' , Slope 1/1 and 2/1	265
A.34	$\frac{\Delta F}{\Delta(\frac{c'}{\gamma h})}$ vs. $\frac{c'}{\gamma h}$ for Various ϕ' , Slope 3/1	266
A.35	$\frac{\Delta F}{\Delta\phi'}$ vs. Slope Angle β for Various ϕ' , $\frac{c'}{\gamma h} = 0$, Theoretical Solution	267
A.36	$\frac{\Delta F}{\Delta\phi'}$ vs. Slope Angle β for Various ϕ'	268
A.37	$\frac{\Delta F}{\Delta\phi'}$ vs. ϕ' for Various Slopes, $\frac{c'}{\gamma h} = 0$, Theoretical Solution	269
A.38	$\frac{\Delta F}{\Delta\phi'}$ vs. ϕ' for Various Slopes	270
A.39	$\frac{\Delta F}{\Delta\phi'}$ vs. ϕ' for Various Slopes, and $\frac{c'}{\gamma h} = 0.1$ to 0.3	271
A.40	$\frac{\Delta F}{\Delta\phi'}$ vs. $\frac{c'}{\gamma h}$ for Various ϕ' , Vertical Slope, Slopes 1/2 and 2/1	272
A.41	$\frac{\Delta F}{\Delta\phi'}$ vs. $\frac{c'}{\gamma h}$ for Various ϕ' , Slopes 1/1 and 3/1	273
A.42	$\frac{\Delta F}{\Delta \tan \phi'}$ vs. $\cot \beta$ for Various ϕ'	274
A.43	$\frac{\Delta F}{\Delta \cot \beta}$ vs. β for Various ϕ' , $\frac{c'}{\gamma h} = 0.1$	275
A.44	$\frac{\Delta F}{\Delta \cot \beta}$ vs. β for Various ϕ' , $\frac{c'}{\gamma h} = 0.2$	276
A.45	$\frac{\Delta F}{\Delta \cot \beta}$ vs. β for Various ϕ' , $\frac{c'}{\gamma h} = 0.3$	277

Appendix Figure	Page
A.46 $\frac{\Delta F}{\Delta \cot \beta}$ vs. β for Various ϕ' , $\frac{c'}{\gamma h} = 0$ to 0.3	278
A.47 $\frac{\Delta F}{\Delta \cot \beta}$ vs. ϕ' for Various $\frac{c'}{\gamma h}$, Vertical Slope	279
A.48 $\frac{\Delta F}{\Delta \cot \beta}$ vs. ϕ' for Various $\frac{c'}{\gamma h}$, Slope 1/2	280
A.49 $\frac{\Delta F}{\Delta \cot \beta}$ vs. ϕ' for Various $\frac{c'}{\gamma h}$, Slope 1/1	281
A.50 $\frac{\Delta F}{\Delta \cot \beta}$ vs. ϕ' for Various $\frac{c'}{\gamma h}$, Slope 2/1 and 3/1	282
A.51 $\frac{\Delta F}{\Delta \cot \beta}$ vs. $\frac{c'}{\gamma h}$ for Various ϕ' , Vertical Slope	283
A.52 $\frac{\Delta F}{\Delta \cot \beta}$ vs. $\frac{c'}{\gamma h}$ for Various ϕ' , Slope 1/2	284
A.53 $\frac{\Delta F}{\Delta \cot \beta}$ vs. $\frac{c'}{\gamma h}$ for Various ϕ' , Slope 1/1	285
A.54 $\frac{\Delta F}{\Delta \cot \beta}$ vs. $\frac{c'}{\gamma h}$ for Various ϕ' , Slopes 2/1 and 3/1	286
A.55 CIRCLE Search with STABL, Embankment Founded on Soil with a Weak Seam	287
A.56 RANDOM Search with STABL, Embankment Founded on Soil with a Weak Seam	288
A.57 BLOCK Search with STABL, Embankment Founded on Soil with a Weak Seam	289
A.58 BLOCK2 Search with STABL, Embankment Founded on Soil with a Weak Seam	290
A.59 CIRCLE Search with STABL, Embankment in Homogeneous Soil	291
A.60 RANDOM Search with STABL, Embankment in Homogeneous Soil	292
A.61 Critical Failure Surfaces for Embankment with Underlying Weak Seam, $c_F = c_E$, $\phi = 0^\circ$	293
A.62 Critical Failure Surfaces for Embankment with Underlying Weak Seam, $c_F = 2c_E$, $\phi = 0^\circ$	294
A.63 Critical Surfaces for Embankment with Underlying Weak Seam, $c_F = 0.5c_E$, $\phi_F = 10^\circ$, $\phi_E = 0^\circ$	295
A.64 Critical Failure Surfaces for Embankment with Underlying Weak Seam, $c_F = c_E$, $\phi_F = \phi_E$	296

Appendix Figure ^a	Page
B.1 Circular Search by STABL, Preliminary Design Analysis, Undrained Shear	303
B.2 Random Search by STABL, Preliminary Design Analysis, Undrained Shear	304
B.3 Sliding Block Search by STABL, Preliminary Design Analysis, Undrained Shear	305
B.4 Sliding Block Search, Preliminary Design Analysis, Effect of Varying Horizontal Elevation on Factor of Safety	306
B.5 Sliding Block2 Search by STABL, Preliminary Design Analysis, Undrained Shear	307
B.6 Sliding Block2 Search, Preliminary Design Analysis, Effect of Varying Horizontal Elevation on Factor of Safety	308
B.7 Circular Search by STABL, Preliminary Design Analysis, Drained Shear	309
B.8 Sliding Block Search by STABL, Preliminary Design Analysis, Drained Shear	310
B.9 Circular Search by STABL, Final Design Analysis, Drained Shear	311
B.10 Circular Search by STABL, Final Design Analysis, Undrained Shear	312
B.11 Random Search by STABL, Final Design Analysis, Undrained Shear	313
B.12 Sliding Block Search by STABL, Final Design Analysis, Effect of Varying Horizontal Elevation on Factor of Safety	314
B.13 Sliding Block2 Search by STABL, Final Design Analysis, Effect of Varying Horizontal Elevation on Factor of Safety	315
B.14 Circular Search by STABL, Final Design Analysis, 100% Consolidated, Assumed Undrained Shear	316
B.15 Random Search by STABL, Final Design Analysis, 100% Consolidated, Assumed Undrained Shear	317

Appendix Figure	Page
B.16 Sliding Block Search by STABL, Final Design Analysis, 100% Consolidated, Assumed Undrained Shear	318
B.17 Sliding Block2 Search by STABL, Final Design Analysis, 100% Consolidated, Assumed Undrained Shear	319
C.1 Circular Search by STABL, Problem 1	357
C.2 Deep Circular Search by STABL, Problem 1	358
C.3 Deep Random Search by STABL, Problem 1	359
C.4 Circular Search by STABL, Problem 2	360
C.5 Sliding Block2 Search by STABL, Problem 2	361
C.6 Circular Search by STABL, Problem 3	362
C.7 Sliding Block2 Search by STABL, Problem 3	363
C.8 Circular Search by STABL, Problem 4 (Simplified Janbu Procedure)	364
C.9 CIRCL2 Search by STABL, Problem 4 (Modified Bishop Method)	365
C.10 Sliding Block2 Search by STABL, Problem 4	366
C.11 Calculation of Effective Line of Thrust, Interslice Stresses and Interslice Shear Strength for Spencer's Method	367
C.12 Line of Effective Thrust for Spencer Analysis, and Effective Normal Stress along Shear Surface for Spencer and for Normal STABL (Janbu) Analysis	370
C.13 Factor of Safety and Line of Thrust for Surface BLOCK2-4, Problem 3, Spencer Analysis	371
C.14 Geometry Variables Used in Expression for $F_c(y)$	372
D.1 Attempt to Generate a Circular Failure Surface when Conflict Occurs between Overturning Slip Surface and Minimum Elevation	390
D.2 Angle Limits for Successive Line Segments of Irregular Failure Surface	391
D.3 Flow Chart of STABL	392

LIST OF ABBREVIATIONS AND SYMBOLS

Note: Symbols used exclusively in Appendix C are listed in Tables C.14 and C.15 of Appendix C.

Abbreviations

ASCE	- American Society of Civil Engineers.
ATFC	- American Testing and Engineering Corporation.
CDC	- Control Data Corporation (Computer System at Purdue Univ.).
$\overline{CK}_o U$	- K_o Consolidated - Undrained Shear Test.
DSS	- Direct Simple Shear.
IBM	- International Business Machines Corporation (Computer).
ICES-LEASE	- Integrated Civil Engineering System - Lease Electronic Accounting System.
ISHC	- Indiana State Highway Commission.
JHRP	- Joint Highway Research Project.
MP	- Morgenstern-Price method of slices for slope stability analysis.
NYSTAB	- New York State, Department of Transportation, Slope Stability Analysis.
OCR	- Over Consolidation Ratio.
PI	- Plasticity Index.
PSA	- Plane Strain Active.
PSP	- Plane Strain Passive.
SHANSEP	- Stress History and Normalized Soil Engineering Properties.

Special Symbols

A prime ("'") indicates that the variable is in terms of effective stresses.

Δ (Delta) indicates a finite change or variation in a variable.

Σ (Sigma) used as summation symbol.

C_L Symbol for Center Line.

Symbols

- a - Moment arm for body forces in simple slope stability analysis.
- a_1 to a_2 - Geometry dependent coefficients in standardized equation for the factor of safety.
- A_1, A_2, A_3 - Coefficients introduced in the general equation for the factor of safety.
- A, B, C, D, E - Zones of constant stress.
 - Different types of analysis (see Table B.4).
- b - Base of a slope.
- B - Crown width of an embankment.
- c, c' - Strength intercept in terms of total (c) and effective (c') stresses, respectively.
- c_1 to c_n - Strength intercepts of soils 1 to n.
- c_n - Normalized strength of weak seam ($= \frac{c_w}{c_E}$).
- c_r - Required strength intercept to make slope in equilibrium.
- c_u - Undrained strength intercept.
- C, C' - Stress independent strength ($= c_{dl}$).
- C_r = $c_{r_{dl}}$
- $\Delta(\frac{c'}{\gamma h})_c$ - The change in $\frac{c'}{\gamma h}$ due to a change $\Delta c'$ in c' ($= \frac{\Delta c'}{\gamma h}$).

- $\Delta(\frac{c'}{\gamma h})_{\gamma}$ - The change in $\frac{c'}{\gamma h}$ due to a change $\Delta\gamma$ in γ
 $(= \frac{c'}{(\gamma + \Delta\gamma) h} - \frac{c'}{\gamma h})$.
- $\Delta(\frac{c'}{\gamma h})_h$ - The change in $\frac{c'}{\gamma h}$ due to a change Δh in h
 $(= \frac{c'}{\gamma (h + \Delta h)} - \frac{c'}{\gamma h})$.
- d - Maximum distance between a shear surface and the chord of the shear surface, measured normal to the chord.
- D - Depth to a weak layer
- E - Interslice normal forces
 - As subscript, indicates embankment.
- f - Used as subscript, indicates at failure or on the failure plane.
- ff - Used as subscript, indicates on failure plane at failure.
- f_a - Partial coefficient for skin friction on piles.
- f_c - Partial coefficient with respect to strength intercept c .
- f_g - Partial coefficient for dead load and water pressure.
- f_p - Partial coefficient for live loads.
- f_v - Partial coefficient for wind loads.
- f_{ϕ} - Partial coefficient with respect to strength angle ϕ .
- $f(x)$ - A function relating the horizontal and vertical interslice forces along a shear surface as proposed by Morgenstern and Price (1965).
- F - Factor of safety.
 - Used as subscript, indicates foundation.
- F_o - Modified Bishop factor of safety for circular failure surface.
 - Janbu's factor of safety (Figure 6.20).

- F_{∞} - The value of F_c , when moment is taken at infinity
 ($= \lim_{y \rightarrow \infty} F_c$) = Janbu's factor of safety.
- F_c - Factor of safety with respect to strength intercept c .
- F_n - Normalized factor of safety ($= \frac{F_y h}{c_E}$).
- F_{ϕ} - Factor of safety with respect to strength angle ϕ .
- $\Delta F(\Delta x_i)$ - The change in the factor of safety due to a change in the independent variable x_i , other parameters kept constant.
- $\frac{\Delta F}{\Delta x_i} = \frac{\Delta F(\Delta x_i)}{\Delta x_i}$. The tangents or secants of the F vs. x_i curves, other variables held constant.
- ΔF_c = $\Delta F(\Delta c')$
- ΔF_h = $\Delta F(\Delta h)$
- ΔF_{β} = $\Delta F(\Delta \beta)$
- ΔF_{γ} = $\Delta F(\Delta \gamma)$
- $\Delta F_{\phi'}$ = $\Delta F(\Delta \phi')$
- $\Sigma \Delta F$ = $\Sigma \Delta F(\Delta x_i)$, $i = 1, n$.
- h, H - Height of a slope or embankment.
- H_c - Critical height of a slope.
- i - Subscript indicating an array of variables.
 - Inclination of ground surface.
- k_1 to k_n - Geometry dependent coefficients in standardized equation for the factor of safety.
- k_h - Horizontal earthquake coefficient.
- k_v - Vertical earthquake coefficient.
- K_o - Coefficient of earth pressure at rest.
- l - Length of a failure surface.
- dl - Infinitesimal increment of length along the failure surface.

- L - Length of the chord of a shear surface.
- M_O - Moment around the center O of a circular or log. spiral failure surface.
- n - Number of slices in a method of slices slope stability analysis.
- Depth factor (Taylor's stability chart).
- N, N' - Total and effective normal forces acting on a shear surface.
- O - Center of a circular or log. spiral failure surface.
- q_f - Maximum shear stress at failure ($= \frac{1}{2} (\sigma_1 - \sigma_3)_f$).
- Q - Resultant of uniform surcharge, acting on top of a slice.
- r - Radius of a circular or log. spiral failure surface.
- r_O - Initial radius of log. spiral failure surface.
- r_M - Maximum radius of log. spiral failure surface.
- r_u - Pore pressure parameter ($= \frac{u}{\gamma h}$).
- R - Resultant of normal stress (force) and normal stress dependent limiting shear stress (force) on a shear surface.
- $\text{Ranf}(0.)$.
- $\text{Ranf}(x)$ - Standard random function, taking any value between 0 and 1 (for $x = 0.$).
- R_c - Resultant of strength intercept resistance forces.
- R_i - i^{th} number in a random sequence of $\text{Ranf}(0.)$.
- s_{eff} - Shear strength based on effective stresses.
- s_u - Undrained shear strength, based on total stresses.
- S, S_r - Shear resistance required along a potential shear surface for equilibrium of slope.
- S_a - Shear resistance available along an assumed shear surface.

- u - Pore water pressure.
- u_c - Pore pressure constant.
- U - Degree of consolidation.
- U_α - Water force acting on base of a slice.
- U_β - Water force acting on top of a slice.
- W - Used as subscript, indicates weak soil layer.
- W, W' - Total and effective weight of a sliding mass, or weight of an individual slice.
- x_i, x_j - Independent parameters for stability analysis.
- Δx - Width of a slice.
- X - Interslice shear force.
- $y_{\min}$ - Minimum elevation to which a failure surface may extend.

Greek Alphabet

- α - Inclination of shear surface with respect to the horizontal.
- Inclination of weak soil layer.
- α_0 - Inclination of shear surface at intersection with top of slope.
- β - Slope inclination.
- β_L, β_R - Inclination of left and right side of an embankment.
- γ - Unit weight of a soil.
- δ - Inclination of uniform surcharge, acting on top of a slice, measured positive counterclockwise from vertical.
- θ - One half the center angle of a circular failure surface.
- Angle between r_0 and r for a log. spiral surface.
- Initial inclination of a failure surface at bottom of slope.
- Inclination of a line segment of a trial failure surface.

- θ_L - Lower angle limitation for line segment of irregular failure surface (= clockwise direction limit).
- θ_u - Upper angle limitation for line segment of irregular failure surface (= counterclockwise direction limit).
- θ_0 - Central angle of circular failure surface, measured from horizontal radius above top of slope to radius through intersection of failure surface with ground surface at the top of the slope. See Figure C.14.
- $\Delta\theta$ - Angle deflection between consecutive line segments of a circular failure surface.
- $\Delta\theta_{\max}$ - Maximum angle deflection between consecutive line segments of a circular failure surface.
- $\Delta\theta_{\min}$ - Minimum angle deflection between consecutive line segments of a circular failure surface.
- λ - Stability parameter used by Janbu (1954b), ($= \frac{\tan \phi}{c/\gamma h}$).
- σ_1, σ_3 - Maximum and minimum principal stresses.
- σ_f, σ_f' - Total and effective normal stress on failure plane.
- σ_h - Horizontal normal stress.
- σ_v - Vertical normal stress.
- σ_{v0}' - Initial vertical effective normal stress.
- σ_{vc}' - Vertical consolidation stress.
- σ_{vm}' - Maximum past vertical stress.
- τ_f - Shear stress on a potential failure plane.
- τ_{ff} - Shear stress on a failure plane at failure.
- τ_h - Shear stress on a horizontal plane (direct-simple shear test).

- ϕ, ϕ' - Strength angle in terms of total (ϕ) and effective (ϕ') stresses, respectively.
- ϕ_1 to ϕ_n - Strength angles of soils 1 to n.
- ϕ_r - Strength angle required for equilibrium of a slope.
- ϕ_u - Undrained strength angle.

HIGHLIGHT SUMMARY

This study is a continuation of a previous one by Siegel (1975) and deals with computerized slope stability analysis.

All investigations are related to a computer program, STABL, which performs analyses of general slope stability problems.

A parametric study was conducted on the STABL program. The study showed that variations in the soil strength parameters c and ϕ have the largest influence on the stability of a slope.

The STABL program was modified to provide additional options in terms of a new sliding block surface generator, several piezometric surfaces, and the modified Bishop factor of safety for circular failure surfaces.

Results of analysis performed with STABL were compared to results obtained with other methods of slope stability analysis. Results by STABL are in general conservative, but the difference is usually less than 10%. However, for deep circular failure surfaces in soil with strength defined mainly in terms of a strength intercept c , the simplified Janbu method of slices normally applied in the STABL solution may give erroneous and nonconservative results. In such cases, the modified Bishop method should be used.

The modified STABL program, STABL2, is written in FORTRAN IV source language, developed for a CDC 6500 computer, and is also available in a form compatible with the IBM 360 (or 370) computer system. Both versions are verified.

I INTRODUCTION

This report presents investigations conducted with the computer program STABL, which performs analyses of general slope stability problems.

STABL was developed by R. A. Siegel (1975a, 1975b) under a previous research program conducted for and in cooperation with the Indiana State Highway Commission (ISHC). This current research program is a continuation and extension of the work done by Siegel to develop computerized slope stability solutions to meet the needs of the ISHC for efficient, versatile and accurate computational methods.

1.1 PURPOSE OF THE PRESENT STUDY

The purpose of the present study was originally stated as follows.

- (1) To perform a parametric study (sensitivity analysis) of the existing general program developed by Siegel, or variations thereof. This is needed to assess the precision with which input data, particularly soil data, need to be defined. High sensitivity to an input parameter requires more attention to be given to that item.
- (2) To develop a hierarchy of special and simplified programs to treat specific and common slope stability problems. It was felt that the general program, being extremely versatile, but accordingly rather bulky and time consuming, would be uneconomic to use for simple problems.

- (3) To compare the results from all programs developed with existing ISHC solutions. This will increase the confidence in the new programs, and indicate whether they are in general more or less conservative than present solutions.

During the course of this research, objective (2) was somewhat altered. Instead of developing special and simplified programs for common slope stability problems, it was found more desirable to retain and use the general STABL program.

In addition to above objectives, the author was assigned as consultant on computer conversions of the STABL program from the CDC 6500 computer at Purdue University, where the program is developed, to the IBM 360 computer at the ISHC computer center, Indianapolis.

1.2 ORGANIZATION OF REPORT

The report is arranged as follows.

Chapter II gives a short introduction to slope stability analysis in general, and presents some of the methods that are being used. It also contains a brief description of the STABL program.

The sensitivity study is reported in Chapter III with additional illustrations in Appendix A.

Modifications of the STABL program are described in Chapter IV, including an example of data setup and output for the modified program, along with some general advice for users.

Chapter V presents a case study for the design of an embankment on varved clay, comparing STABL solutions with others. Related computer plots are contained in Appendix B.

Comparisons of STABL with existing ISHC procedures for slope stability analysis are reported in Chapter VI, with related additional analysis and computer plots in Appendix C.

Chapter VII summarizes goals, results and conclusions of the research, and gives some suggestions for future studies in this area.

Appendix D describes smaller changes and errors that have been corrected in the original STABL program. A total listing of the revised (corrected and modified) program is presented in Appendix E.

Finally, Appendix F reports some of the experiences in converting the STABL program from the CDC computer to the IBM computer. Included in Appendix F is a listing of the changes necessary to transfer the STABL program from the CDC 6500 computer to an IBM 360 (or 370) computer system.

Because of the large number of tables and figures included in the appendices, each of these are arranged with the text (if any) first, then the tables in consecutive order, followed by the figures in consecutive order. This was done to avoid interrupting the text, and because most of the text readily can be interpreted apart from the tables and figures.

II SLOPE STABILITY ANALYSIS

2.1 GENERAL DESCRIPTION

Earthen masses located beneath a sloping ground surface, whether natural or man-made, have a tendency to move downward and outward under the influence of gravity. Unless this tendency is suitably counteracted by the shearing resistances within the mass, a landslide occurs. Other forces such as seepage forces and surcharge loads may add to the instability. Avoiding such instabilities is a major concern of the geotechnical engineer.

The investigation of the stability of a slope has three main steps:

- (1) Identification of the forces which cause and resist failure.
- (2) Determination of the shear strengths of the soils beneath the slope.
- (3) Application of a model to investigate the stability based on (1) and (2).

The forces acting on the slope depend on the profile and density of the soils; the position of the phreatic surface, and associated seepage forces; and external applied loads.

The shear strength of a particular soil depends upon its type and origin, water content, prior stress history, method and type of compaction, and the like.

The shear strength of a soil was initially expressed by Coulomb (1776) as

$$\tau_f = c + \sigma_f \tan \phi \quad (2.1)$$

where c is the strength intercept, ϕ is the strength angle, and σ_f is the stress normal to the shear plane.

The equation commonly used today is modified to express τ_f in terms of effective stresses rather than total stresses and has the form

$$\tau_f = c' + (\sigma_f - u) \tan \phi'$$

or
$$\tau_f = c' + \sigma'_f \tan \phi' \quad (2.2)$$

where c' and ϕ' are the strength intercept and strength angle with respect to effective stresses, also named effective strength parameters, and $\sigma'_f = (\sigma_f - u)$ is the effective stress normal to the shear plane.

In effective stress analysis of sands and gravels, $c' \approx 0$, and equation (2.2) is simplified accordingly. In total stress analysis, and with particular reference to undrained shear in clayey soils, $\phi \approx 0$, and equation (2.1) is reduced to $\tau_f = c$.

Instability of a slope in sands and gravels occurs if the slope inclination exceeds a certain value, known as the angle of repose. This angle is constant for a particular material at a particular void ratio and is independent of the height of the slope, provided the slope is not subjected to seepage forces. In contrast, a slope in a clayey soil has a critical height, which depends on the slope inclination and the soil properties.

The shearing behaviour of a particular material is determined empirically, i.e., by field tests or laboratory tests. Most common are the laboratory triaxial and direct shear tests and the field vane test.

The results are then applied to the slope stability analysis by assuming certain mechanisms and modes of failure.

A common approach to slope stability analysis is to investigate failure along an assumed slip surface, and express the stability in terms of a safety factor F , defined as the ratio of the available shearing forces along the slip surface, S_a , to the shearing forces required for static equilibrium, S_r :

$$F = \frac{S_a}{S_r} \quad (2.3)$$

An F value equal to or below unity indicates that the slope is unstable and failure can be expected.

To identify the more critical slip surfaces (lower F values) it is generally necessary to conduct the analysis for a number of surfaces of different shape and location.

2.2 DIFFERENT METHODS OF SLOPE STABILITY ANALYSIS

2.2.1 Early Development

Until Coulomb (1776) [Skempton (1946)] soils were considered as strictly frictional, i.e., a slope was stable if the slope angle was less than the angle of repose of the material. Coulomb analysed the stability of a vertical bank of cohesive soil with respect to a plane slip surface, and obtained the equation

$$H_c = \frac{4c}{\gamma} \left[\frac{\cos \phi}{1 - \sin \phi} \right] \quad (2.4)$$

where H_c is the critical height, c is the strength intercept, ϕ is the strength angle, and γ is the unit weight of the soil.

In 1820 Francais gave a solution to the more general problem of a sloping bank. He also assumed a plane slip surface and found

$$H_c = \frac{4c}{\gamma} \left[\frac{\cos \phi \sin \beta}{1 - \cos (\beta - \phi)} \right] \quad (2.5)$$

where β is the slope inclination.

Collin (1846) introduced, based on his observations of deep curved slip surfaces in clay slopes, an analysis assuming a cycloidal slip surface and a ϕ value of zero.

However, not until the twentieth century did greatly improved measurements of shear strength and refined procedures of analyses appear.

Wright (1969) and Siegel (1975a) have described in detail the different methods of slope stability analysis available today. As this study is a continuation of that begun by Siegel, only a resumé of the most common methods will be given here.

2.2.2 The $\phi = 0$ Method

Based on field observations in Sweden, a circular slip surface was introduced by Petterson (1916) [Skempton (1946)], which combined with the assumption that ϕ equals zero, gave what we today know as the $\phi = 0$ method. Figure 2.1 shows the principle. When ϕ equals zero, the shear strength τ_f equals c (Equation 2.1) and is independent of the actual value of the normal stress at any point of the failure surface. Equilibrium of the potential sliding mass is satisfied by moment equilibrium

$$M_o = Wa - r/c_r dl$$

$$= Wr \sin \alpha - c_r r^2 2\theta = 0$$

$$c_r = \frac{W \sin \alpha}{2r\theta}$$

$$F = \frac{c}{c_r}$$

Vary θ and r until
 $F = F_{\min.}$

FIGURE 2.1 PRINCIPLE OF THE $\Phi = 0$ METHOD

around the center of the circle. The forces acting normal to the failure surface pass through the center of the circle and do not contribute to the moment, hence the shear stress required for equilibrium (c_r) is determined directly by the moment equation, and the factor of safety F can be found from Equation 2.3. Different centers and radii are investigated to determine $F_{\min}$. Taylor (1937) made a chart that directly gives the stability number $\frac{c}{F\gamma H}$ as a function of the slope angle β and depth factor n . Knowing c , γ and H , F can be found. See Figure 4.7.

The $\phi = 0$ method is commonly used for short term stability analysis of saturated clay slopes, where a failure would occur undrained.

2.2.3 The Logarithmic Spiral Procedure

When $\phi'(\phi)$ is not taken as zero, τ_f is not a constant, but depends on the normal stress $\sigma'(\sigma)$ along the potential failure surface, and a distribution of $\sigma'(\sigma)$ has to be assumed in order to determine F . To overcome this problem the logarithmic spiral was introduced as a proposed failure surface. Such a surface has the characteristic that the resultants of the normal stresses σ' and frictional components of shear strength $\sigma' \tan \phi'$ pass through the center of the spiral. Consequently moment equilibrium around the center will only involve the body forces and strength intercept force (C') of the soil as in the $\phi = 0$ method. See Figure 2.2.

The equation of a logarithmic spiral can be expressed in polar coordinates as

$$r = r_o e^{(\theta \tan \phi' / F_\phi)} \quad (2.6)$$

FIGURE 2.2 PRINCIPLE OF LOG SPIRAL METHOD

where r is the radius from the center, r_0 is the initial radius, θ is the angle between initial radius r_0 and radius r , ϕ' is the effective strength angle and F_ϕ is the factor of safety with respect to the strength angle. If F_ϕ is taken as unity [Taylor (1937)] the procedure directly gives F_c , the factor of safety with respect to the strength intercept, c' . In most cases, however, one will require an F larger than unity for both ϕ' and c' . Assuming $F_\phi = F_c$, several spirals with the same center and initial radius r_0 are investigated to find the value of F . To find $F_{\min}$, the center O , and initial radius r_0 have to be varied.

2.2.4 The Friction Circle Method

For a circular shear surface the resultants of the normal stresses and the frictional components of shear strength will lie on lines tangent to a circle of radius $r \sin \phi'$, called the friction circle. Hence the moment about the center of the circle will involve the normal stress distribution which is unknown. The assumption commonly made is that the resultant of all normal stresses and frictional shear stresses also lies tangent to the friction circle. This assumption is equivalent to assuming that the normal stresses are concentrated at a single point [Wright (1969)] and the solution obtained is a lower bound solution (conservative). The principle is shown in Figure 2.3. Taylor (1937) also presented charts for the stability number $\frac{c}{F\gamma H}$ based on the friction circle method. See Figure 4.6.

2.2.5 Method of Slices

The methods described thus far are particularly suited to simple slope stability problems with homogeneous soil conditions and no forces

FIGURE 2.3 PRINCIPLE OF FRICTION CIRCLE METHOD

other than those due to weight and shearing resistance. For problems involving changing soil conditions and additional forces such as seepage and earthquake forces, several limiting equilibrium methods have been developed, all based on the method of slices.

The soil mass above an assumed slip surface is divided into slices in such a way that each slice has a minimum change in variables (Figure 2.4). The system of equations and unknowns associated with complete equilibrium of the entire soil mass in terms of n slices is summarized in Table 2.1 (see also Figure 2.5). For any value of n , other than unity, there are $(2n - 2)$ more unknowns than equations, and the problem is indeterminate. The differences in the various methods of stability analyses are contained in the different assumptions made with respect to these unknowns in order to render the problem determinate.

2.2.5.1 Ordinary Method of Slices

This method assumes a circular slip surface. The assumptions made by Fellenius (1926) to eliminate the unknowns are

- (1) The normal force N on the base of each slice acts at the center of the base.

- (2) The interslice forces are zero.

These two assumptions eliminate $(4n - 3)$ unknowns, which is more than is necessary to make the problem statically determinate. Remaining are $(n + 1)$ unknowns, namely F and the n normal forces at the base of each slice. The equations used are: (a) force equilibrium normal to the base of each slice, which directly gives the normal forces N , and (b) overall moment equilibrium around the center of the circle forming the slip surface, which directly gives the shear strength required for

FIGURE 2.4 DIVISION OF SLIDING MASS INTO SLICES

Table 2.1

Equations and Unknowns Associated with the Method of Slices

Equations

n^*	Moment equilibrium for each slice.
$2n$	Force equilibrium in 2 directions for each slice.
n	Mohr-Coulomb relationship between shear strength and normal effective stress.
$4n$	Total number of equations.

Unknowns

1	Factor of Safety F .
n	Normal force at base of each slice N .
n	Position of N .
n	Shear force at base of each slice S .
$n - 1$	Horizontal interslice forces E .
$n - 1$	Vertical interslice forces X .
<u>$n - 1$</u>	Location of interslice forces (line of thrust).
$6n - 2$	Total number of unknowns.

* n is the number of slices.

FIGURE 2.5 FORCES ACTING ON A SLICE

equilibrium. The factor of safety F will be expressed in terms of an average value (summing over the n slices) of the shear strength available divided by the shear strength required.

Because the problem is overdetermined the values obtained for the normal forces will usually not satisfy equilibrium in directions other than those normal and parallel to the base of each slice. Hence the neglect of interslice forces may lead to unrealistic results.

2.2.5.2 Bishop's Method

Bishop (1955) introduced a method satisfying complete equilibrium.

His primary equations are	no. of eq.
(1) Overall moment equilibrium	1
(2) Vertical force equilibrium for each slice	n
(3) Force equilibrium parallel to base of each slice	n
(4) Shear force as function of normal force and F	<u>n</u>
Total of	$3n + 1$

In addition he has two equations with respect to overall equilibrium of interslice forces X and E .

In the primary analysis Bishop does not make any assumptions about the line of thrust (see Figure 2.5). This line may be determined by adding the remaining $(n - 1)$ moment equations for the individual slices, but the primary equations can be solved independent of the location of the line of thrust. The normal force, as well as the resultant weight of each slice, is assumed to act through the center of the base of the slice. The remaining unknowns are

	no. of eq.
(1) Factor of safety F	1
(2) Normal force at base of each slice N	n
(3) Shear force at base of each slice	n
(4) Vertical interslice forces X	$n - 1$
(5) Horizontal interslice forces E	<u>$n - 1$</u>
Total of	$4n - 1$

This total of unknowns is compared to $(3n + 1)$ equations, which gives an indeterminacy of $(n - 2)$. The procedure is to assume values of X that satisfy the two overall interslice force equations (E can be expressed in terms of X from the force equation parallel to the base of a slice). To judge whether a particular solution is reasonable, one may calculate the corresponding line of thrust using the remaining $(n - 1)$ moment equations for the individual slices.

2.2.5.3 Modified Bishop Method

The simplest solution to Bishop's method is to assume X equal to zero which directly satisfies equilibrium with respect to the vertical interslice forces, but generally not equilibrium with respect to the horizontal interslice forces. The solution is called the modified or simplified Bishop method. It can be directly arrived at by neglecting the interslice forces as for the ordinary method of slices, but using vertical force equilibrium for each slice instead of force equilibrium normal to the base of each slice.

2.2.5.4 Other Procedures

Many other methods have been developed by different investigators making various assumptions with respect to the interslice forces.

Spencer (1967) assumed parallel interslice forces and developed a statically determinate solution with $3n$ equations and $3n$ unknowns.

Morgenstern and Price (1965) assumed a relationship between the horizontal and vertical interslice forces dependent on a function $f(x)$ along the shear surface. They extended their solution to failure surfaces of general shape.

Janbu (1954a) formulated an expression for F in terms of vertical force equilibrium for each slice combined with overall horizontal force equilibrium. As with the Bishop method, this procedure requires assumptions of the vertical interslice forces X . Janbu made his assumptions so that they satisfy interslice force equilibrium as well as moment equilibrium for each slice, assuming a position of the line of thrust. A simplified procedure results if X is assumed equal to zero. This procedure is similar to the modified Bishop method, the difference being that overall horizontal force equilibrium is used versus the use of overall moment equilibrium in the modified Bishop procedure.

Carter (1971) reached the same expression as Janbu's simplified procedure, but arrived at it in a different manner by considering the modified Bishop procedure applied to slip surfaces of general shape. In the case where the slip surface is not a circular arc, no specific point exists that is obvious to use as moment center. Carter found that the value of the factor of safety was independent of the x -coordinate of the moment center, but varied with the y -coordinate, and reached a

minimum value for $y = \infty$.^{*} He concluded that $y = \infty$ was the only consistent moment center to use for an irregular as well as a circular slip surface.

2.2.6 Methods other than Limiting Equilibrium

An alternative method of slope stability analysis is to investigate the shear stresses imposed under the slope by means of the theory of elasticity [Perloff and Baron (1976), Romani (1970), Romani, Lovell and Harr (1972)]. The criterion is that the shear stress at any point does not exceed the shear strength. The factor of safety is defined as the shear strength divided by the shear stress at the point where this ratio is the least, hence it gives the safety at the most critical point, whereas F defined by a limiting equilibrium method gives the average value of safety over the entire slip surface.

The method may be useful when dealing with soils where progressive failure is likely to occur, because it will discover the areas of stress concentration from which a failure is likely to initiate. On the other hand, it does not take into account the redistribution of stress which occurs, when the stress level at a point approaches the strength.

In a case with concentrated surface loads, methods based on an elastic model can take into account the distributional effect of the slope material. This cannot be done with the method of slices in the limiting equilibrium models, and an error may be introduced by applying

^{*}Further investigations by the author show that this is not always true, see Chapter VI and Appendix C.II.

such a model in this case. For important and complex problems it may be advisable to perform both types of analysis.

2.2.7 Probabilistic Approach

All the methods presented thus far have been discussed from a deterministic viewpoint, as if the parameters involved in the slope stability analysis were single fixed values. This is a gross simplification of the real situation, where uncertainties exist with respect to soil properties as well as the soil profile.

Several people, e.g., Wu et al (1970), Athanasiou-Grivas (1976), and Catalán and Cornell (1976), have worked on probabilistic models for slope stability analysis, which make use of slip surface and limiting equilibrium models, but which operate in terms of probability of failure rather than the traditional factor of safety. In this approach the soil properties are defined, not as exact values, but as distributions with a mean value and standard deviation.*

The relationship between the factor of safety as defined by the traditional methods, and the probability of failure was investigated in a case study by Athanasiou-Grivas (1976), where a slope having a conventional factor of safety of 1.2 failed. The probability of failure was found to be approximately 0.3% with zero pore pressure and 1% for a pore pressure parameter ($r_u = u/\gamma h$) equal to 0.6.

*The significance of varying soil properties may be investigated within the deterministic model by a sensitivity study (see Chapter III)

2.2.8 Comparison of Methods

Wright (1969) has done a comprehensive study of the many methods available for slope stability analysis. He found, in agreement with others [Bishop (1955), Lowe and Karafiath (1967)], that for homogeneous soil the modified Bishop procedure is as accurate as any of the more rigorous methods (Spencer, Janbu, Morgenstern and Price), whereas the ordinary method of slices, as well as Janbu's simplified procedure, gave factors of safety as much as 10% and 5% lower than the lower bound value given by the friction circle method. Some of the results are shown in Table 2.2. The difference is larger if pore pressures are present, and for cases with non-homogeneous soil it may be even larger.

In any case where the slip surface can be approximated by a circular arc, Bishop's modified procedure is recommended, as it combines reasonable accuracy with simplicity and small computer time requirements.

If the slip surface cannot be approximated by a circular arc, Bishop's method cannot be used without modifications. The method that is most convenient for irregular slip surfaces is undoubtedly Janbu's simplified procedure. Unfortunately it is not so accurate and may lead to overly conservative (and hence uneconomic) design. If this is a problem, Spencer's procedure seems to be the more rigorous method that is the least time consuming. However, it requires analysis of the position of the line of thrust, together with soil properties, to judge whether the result is conservative or not (see example in Appendix C.I).

A study comparing the modified Bishop method with a linear elastic finite element method is reported by Wright et al (1973) with the following results:

Table 2.2

Factor of Safety for Different Methods of Slope Stability Analysis

Method	λ	$\cot \beta$		
		1.5/1	2.5/1	3.5/1
1	2	1.171	1.485	1.768
2		1.125	1.402	1.659
3		1.147	1.443	1.714
4		1.103	1.351	1.595
1	5	1.831	2.480	3.095
2		1.738	2.334	2.914
3		1.785	2.409	3.008
4		1.703	2.277	2.850
1	8	2.430	3.399	4.333
2		2.301	3.210	4.105
3		2.368	3.309	4.226
4		2.262	3.154	4.045
1	20	2.309	3.414	4.503
2		2.197	3.268	4.332
3		2.257	3.347	4.425
4		2.176	3.242	4.307
1	50	1.934	2.988	4.035
2		1.863	2.901	3.936
3		1.902	2.948	3.991
4		1.855	2.893	3.928

$$\lambda = \frac{\tan \phi}{c/\gamma h}, \quad \beta = \text{slope angle.}$$

The stability number $N = \frac{F}{c/\gamma h}$ is a function of λ and β only, and is constant for constant λ and β although c , ϕ , γ and h may vary.

The methods compared are (1) Simplified (or Modified) Bishop Method, (2) Ordinary Method of Slices, (3) Friction Circle and (4) Janbu's simplified procedure.

- (1) The distribution of normal stress along the failure surface was approximately identical for the two methods.
- (2) Although the factor of safety by the finite element method varies along the failure surface, the average value is nearly the same as for the modified Bishop method.
- (3) Because the finite element method involves a completely different set of assumptions than those employed in the modified Bishop method, the close agreement between the two methods indicates that they both are reliable. Because the limiting equilibrium model is much simpler than a finite element analysis, the former will usually be preferred.

2.3 STABL - A COMPUTER PROGRAM FOR GENERAL SLOPE STABILITY ANALYSIS

Many programs exist, based on one of the previous described methods of slope stability analysis. When STABL was developed, the major purpose was to develop a program with the least number of restrictions imposed on the input data, and the maximum utilization of the computer in the search for the most critical failure surfaces, combined with ease of use.

STABL can handle almost any problem with respect to boundary geometry and loading, soil conditions including anisotropic soils, equilibrium or excess of equilibrium pore pressures and pseudo-static earthquake forces. It has routines that search for the more critical failure surfaces by generating surfaces of circular, sliding block or irregular shape and compare the corresponding safety factors. It can as well handle specified failure surfaces. It is limited to 2-dimensional analysis, which is the case with almost all methods available today.*

*For an evaluation of 3-dimensional effects on slope stability analysis, see Baligh and Azzouz (1975).

The method chosen for the calculation of the factor of safety is Janbu's simplified procedure satisfying vertical force equilibrium for each slice and overall horizontal force equilibrium, assuming horizontal interslice forces. For the simple case of dry soil and no external loading, the equations are as follows:

Vertical force equilibrium for a slice

$$N \cos \alpha + S \sin \alpha - W = 0 \quad (2.7)$$

Overall horizontal force equilibrium

$$\Sigma (N \sin \alpha - S \cos \alpha) = 0 \quad (2.8)$$

Mohr-Coulomb relationship

$$S = \frac{C' + N' \tan \phi'}{F} \quad (2.9)$$

From (2.7) and (2.9) ($N = N'$)

$$N = W/\cos \alpha - S \tan \alpha \quad (2.10)$$

$$S = \frac{C' + W \tan \phi'/\cos \alpha}{F + \tan \alpha \tan \phi'} \quad (2.11)$$

Substitute (2.10) into (2.8)

$$\Sigma (W \tan \alpha - S/\cos \alpha) = 0 \quad (2.12)$$

Substitute (2.11) into (2.12)

$$\Sigma [W \tan \alpha - \frac{C' + W \tan \phi'/\cos \alpha}{\cos \alpha (F + \tan \alpha \tan \phi')}] = 0$$

Rearranging gives

$$\Sigma \frac{W \cos \alpha (F \tan \alpha - \tan \phi') - C'}{\cos \alpha (F + \tan \alpha \tan \phi')} = 0$$

or

$$\Sigma \frac{C'/\cos \alpha + W (\tan \phi' - F \tan \alpha)}{F + \tan \alpha \tan \phi'} = 0 \quad (2.13)$$

This expression is the same as that derived by Carter [Siegel (1975a)]. A similar expression including surface loads, water table and earthquake forces is presented by Siegel (1975a); see also Chapter IV, Equation (4.1).

This method was chosen for its simplicity and because it can be applied to failure surfaces of any shape. It is usually conservative* and may in some cases give factors of safety which are too low. A comparison was made with the modified Bishop method from Wright (1969). STABL gave values of F from 2.3% to 8.8% lower than those obtained by the modified Bishop method, the largest difference occurring for low values of λ (see Table 2.3). Siegel compared STABL with Spencer's procedure and found a maximum difference of 15% for steep slopes and high values of λ , with a pore pressure parameter $r_u = 0.5$, STABL giving the lower value.

In most slope stability problems, however, there is an uncertainty in the determination of the input data such as the soil profile and particularly the soil properties in terms of density and strength. The factor of safety may be more sensitive to variations in these properties than to different methods of analysis [Singh (1970)]. This point is further investigated in Chapter III.

A detailed discussion of how the program performs its functions is given by Siegel (1975a). Later modifications of the program are reported in Chapter IV of this report.

*Important exceptions, see Chapter VI, Problems 1 and 4, and Appendix C.II.

Table 2.3

Comparison of STABL with Modified Bishop Method after Wright (1969),
Using Failure Surfaces Obtained by the Modified Bishop Analysis

Factor of Safety	λ	$\cot \beta$		
		1.5/1	2.5/1	3.5/1
Mod. Bishop	2	1.171	1.485	1.768
STABL		1.106	1.367	1.625
% difference		5.9	8.6	8.8
Mod. Bishop	5	1.831	2.480	3.095
STABL		1.720	2.318	2.890
% difference		6.5	7.0	7.1
Mod. Bishop	8	2.430	3.399	4.333
STABL		2.290	3.209	4.099
% difference		6.1	5.9	5.7
Mod. Bishop	20	2.309	3.414	4.503
STABL		2.204	3.275	4.346
% difference		4.8	4.2	3.6
Mod. Bishop	50	1.934	2.988	4.035
STABL		1.873	2.912	3.945
% difference		3.3	2.6	2.3

$$\lambda = \frac{\tan \phi}{c/\gamma h}, \quad \beta = \text{slope angle (see also Table 2.2).}$$

III SENSITIVITY ANALYSIS

3.1 INTRODUCTION

A sensitivity study was performed on the STABL program to assess the precision with which input data need to be defined, particularly soil data. Although the particular results apply to this program, the trend is believed to be applicable to slope stability analysis in general.

Little research has been published on sensitivity analysis. An article by Singh (1970) concludes that the factor of safety for slope stability analysis is much more sensitive to the choice of strength parameters as interpreted from soil tests than to the choice of the computational method of analysis. This conclusion is of particular interest with respect to the STABL program, because the procedure of analysis used in this program usually is somewhat conservative compared to other methods of analysis (see Chapters II, V and VI).

3.2 PROCEDURE FOR ANALYSIS

The procedure initially chosen for the sensitivity analysis is as follows:

- (1) For a given slope (geometry, soil profile, water table, seepage, load conditions), a STABL searching routine is used to generate a failure surface, which will be close to the critical one, and the safety factor is calculated.

- (2) Using this specified surface, the factor of safety is calculated for a small change in an input parameter equal to the expected variation in the field or scatter in test results.
- (3) The influence of the change in the factor of safety due to a change in one of the input parameters, relative to the total change in the factor of safety due to changes in all parameters, is investigated.
- (4) A conclusion may be reached with respect to the required precision in definition of the different variables to limit uncertainties in the factor of safety to tolerable levels.

The steps are illustrated on the flow chart of Figure 3.1.

3.3 CASES STUDIED

The sensitivity analysis was performed for the idealized case of a slope in homogeneous soil with no pore water pressure, and for an embankment founded on a soil, which has a weak layer.

3.3.1 Sensitivity Analysis of Homogeneous Slopes

It was decided to perform the sensitivity study for a relatively wide range in parameters for slopes in homogeneous soils, to get a good feel for how the factor of safety is likely to change, when different parameters are changed.

For homogeneous soils the factor of safety depends on 5 to 6 independent (to some extent) parameters, which are:

Slope geometry defined by

Height	h
Slope angle	β

STEP 1 STEP 2 STEP 3

$$\Delta F(\Delta x_i) \quad \text{vs.} \quad \sum \Delta F(\Delta x_i)$$

STEP 4

Restrictions on Δx_i to limit $\Delta F(\Delta x_i)$

FIGURE 3.1 FLOW CHART SHOWING STEPS IN SENSITIVITY ANALYSIS

Soil properties defined by

Unit weight	γ
Strength intercept	c'
Strength angle	ϕ'

Pore water pressure, if present, is important, as well. This parameter was neglected in this study.

Figure 3.2 shows the range in these parameters, covered by the sensitivity analysis. Five different slopes were examined. Height (h) and unit weight (γ) were chosen arbitrary as 100 feet and 100 pcf, respectively. However, the results can be applied to any values of h and γ by introducing the dimensionless number $\frac{c'}{\gamma h}$ [Taylor(1937)]. The slopes were analysed for all combinations of (c' , ϕ') shown in Figure 3.2, with the exception of those where a combination of low strength intercept c' , low strength angle ϕ' , and a steep slope, gave values of the factor of safety much below unity.

3.3.1.1 Analysis by STABL

For each problem case (slope geometry, soil properties) limitations for initiation and termination of failure surfaces were set according to the particular problem, e.g., all failure surfaces for slopes with a slope angle β greater than 53° passed through the toe; for β less than 53° and $\phi = 0$, always a base failure. A hundred surfaces were generated for each problem. If the critical surfaces were located on the boundary of the search limitations, these were changed accordingly, and a new search was carried out. An exception to this procedure was the $\phi = 0$ case, where the critical failure surface theoretically extends to an infinite depth. Only the CIRCLE search was used because of the homogeneity of the soil (see also p. 69).

Vertical Slope

Slope 1/2

Slope 1/1

Slope 2/1

Slope 3/1

Height of slopes

 $h = 100$ feet

Soil characteristics:

unit weight

 $\gamma = 100$ pcf

strength intercept

 $c' = 1000 - 3000$ psf

strength angle

 $\phi' = 0^\circ - 50^\circ$

FIGURE 3.2 RANGE IN VARIABLES FOR SENSITIVITY ANALYSIS OF HOMOGENEOUS SLOPES

The most critical failure surfaces generated by STABL for each particular problem were plotted for comparison with the others, as shown on Figures 3.3 to 3.7. Where the location of a failure surface deviated from the logical position with respect to the other failure surfaces, a more reasonable trial surface was introduced by the author and analysed as a specified surface. If this surface was significantly more critical (lower factor of safety) than the most critical surface generated by STABL, the former surface was used. These surfaces are drawn with dashed lines on Figures 3.3 through 3.7. The reason why this can happen is explained in the word "random". STABL generates failure surfaces randomly within the defined limits, and selects the 10 most critical of the surfaces generated. However, it is possible that there are some more critical failure surfaces than those found by STABL.

3.3.1.2 Results of Stability Analysis of Homogeneous Slopes

The results of the stability analysis in terms of factors of safety, F , are given in Table 3.1. Following the procedure outlined in Section 3.2, critical failure surfaces (Figures 3.3 to 3.7) were used as specified surfaces, and the factor of safety calculated for small changes in c' and ϕ' .

Another approach to determine $\Delta F(\Delta x_1)^*$ is to plot F vs. x_1 , keeping other parameters constant. This method has the advantage that it also reflects changes in F due to a slight change in the critical failure surface when the soil parameters change. It was possible to apply the method in the case of homogeneous slopes, because a relatively wide

* The change in the factor of safety due to a change in x_1 , keeping other parameters constant.

$$\frac{c'}{\gamma h} = 0.1$$

$$\frac{c'}{\gamma h} = 0.2$$

$$\frac{c'}{\gamma h} = 0.3$$

FIGURE 3.3

CRITICAL FAILURE SURFACES FOR
VERTICAL SLOPES IN HOMOGENEOUS
SOIL

$$\frac{c'}{\gamma h} = 0.1$$

$$\frac{c'}{\gamma h} = 0.2$$

$$\frac{c'}{\gamma h} = 0.3$$

FIGURE 3.4 CRITICAL FAILURE SURFACES FOR SLOPES 1/2 IN HOMOGENEOUS SOIL

$$\frac{c'}{\gamma h} = 0.1$$

$$\frac{c'}{\gamma h} = 0.2$$

$$\frac{c'}{\gamma h} = 0.3$$

$$\phi' = 0^\circ$$

FIGURE 3.5

CRITICAL FAILURE SURFACES FOR SLOPES
1/1 IN HOMOGENEOUS SOIL

FIGURE 3.6 CRITICAL FAILURE SURFACES FOR SLOPES 2/1 IN HOMOGENEOUS SOIL

$$\frac{c'}{\gamma h} = 0$$

$$\frac{c'}{\gamma h} = 0.1$$

$$\frac{c'}{\gamma h} = 0.2$$

FIGURE 3.7 CRITICAL FAILURE SURFACES FOR SLOPES 3/1 IN HOMOGENEOUS SOIL

$$\frac{c'}{\gamma h} = 0.3$$

$$\phi' = 0^\circ$$

Note: Dashed circle was forced deep, but result is the same as the shallower circle.
 For $\frac{c'}{\gamma h} = 0.2$ and 0.3 the failure surface is at limit of boundaries. (Theoretical surfaces extend to infinite depth.)

FIGURE 3.7 (CONT.)

Table 3.1
Factors of Safety for Homogeneous Slopes

$\frac{c'}{\gamma h}$	$\phi' \searrow \cot \beta$	0	0.5	1.0	2.0	3.0
0.1	0					0.757
	10		.787	.936	1.152	1.388
	20	.677	.987	<u>1.210</u>	1.618	2.087
	30	.792	1.196	1.534	2.184	2.871
	40	.915	1.431	1.883	2.808	3.806
	50	1.057	1.719	2.331	3.642	5.011
0.2	0	.801	1.088	1.106	1.129	1.143
	10	.967	1.347	1.575	1.817	2.039
	20	1.120	1.577	1.891	2.366	2.819
	30	1.264	1.818	2.230	2.928	3.620
	40	1.408	2.087	2.622	3.587	4.582
	50	1.598	2.426	3.115	4.519	5.879
0.3	0	1.202	1.633	1.658	1.707	1.714
	10	1.368	<u>1.886</u>	2.232	2.437	2.674
	20	1.521	2.144	2.541	3.024	3.490
	30	1.683	2.406	2.888	3.631	4.354
	40	<u>1.860</u>	2.701	3.303	4.327	5.331
	50	2.075	3.066	3.901	<u>5.232</u>	6.627

Note: The underlined results are obtained from specified failure surfaces, shown with dashed lines on Figures 3.3 through 3.6, see also top of page 33.

range in the independent variables was studied. The dimensionless number $\frac{c'}{\gamma h}$ is used as an independent variable, reducing the number of independent variables to three: $\frac{c'}{\gamma h}$, ϕ' and β . The dependent variable is the factor of safety, F . The location of the more critical failure surface may be considered another dependent variable, see Figure 3.8.

From the graphical plots, the sensitivity of F to changes in the independent variables can be found. The graphs may be plotted as functions in 3-dimensional space, giving F as a function of two independent variables, while other independent variables (one in this case) are held constant, see Figure 3.9. For any practical purposes, the two-dimensional projections of the three dimensional plots are the most useful (Figures 3.10 to 3.12).

The variables are combined in different ways and graphical results obtained, which are convenient to use for a quick estimate of the influence of change in an independent variable x_i on the safety factor F . These graphs are presented in Appendix A, Figures A.1 through A.29.

3.3.1.3 Sensitivity of the Factor of Safety to Input Variables

To determine $\Delta F(\Delta x_i)$ for a particular problem, $\frac{\Delta F}{\Delta x_i}$ was determined over the total range of parameters investigated. This was done by determining the tangents or secants to the F vs. x_i curves at particular points. Since these curves are based on empirical values, the values of $\frac{\Delta F}{\Delta x_i}$ are subjected to some uncertainty. Examples of variations of $\frac{\Delta F}{\Delta x_i}$ with x_j are illustrated in Figures 3.13 through 3.17. Tables and more figures are included in Appendix A. The results are summarized in Table 3.2, page 52.

Variables

Slope geometry:

Height
Slope angle

Soil properties:

Strength intercept
Strength angle
Unit weight

Results

h
 β

Geometry of
critical failure surface

c'
 ϕ'
 γ

Factor of safety

FIGURE 3.8 RELATIONSHIP BETWEEN SLOPE PARAMETERS, CRITICAL FAILURE SURFACE AND FACTOR OF SAFETY FOR HOMOGENEOUS SLOPES

FIGURE 3.9 ISOMETRIC PLOT OF FACTOR OF SAFETY VS. $(\phi', \frac{c'}{\gamma h})$ FOR SLOPE 1/2

FIGURE 3.10 FACTOR OF SAFETY VS. STRENGTH ANGLE ϕ' FOR VARIOUS $\frac{c'}{\gamma h}$, SLOPE 1/2

FIGURE 3.11 FACTOR OF SAFETY VS. $\frac{c'}{\gamma h}$ FOR VARIOUS ϕ' , SLOPE 1/2

FIGURE 3.12

EQUAL FACTOR OF SAFETY CONTOURS FOR
SLOPE 1/2, HOMOGENEOUS SOIL

FIGURE 3.13 $\frac{\Delta F}{\Delta(\frac{c'}{\gamma h})}$ vs. SLOPE ANGLE β
 FOR VARIOUS ϕ'

FIGURE 3.14 $\frac{\Delta F}{\Delta(\frac{c'}{\gamma h})}$ vs. ϕ' FOR DIFFERENT SLOPE ANGLES β

FIGURE 3.15 $\frac{\Delta F}{\Delta \phi'}$ vs. SLOPE ANGLE β FOR VARIOUS ϕ'

FIGURE 3.16 $\frac{\Delta F}{\Delta \phi'}$ vs. ϕ' FOR VARIOUS SLOPES

FIGURE 3.17 $\frac{\Delta F}{\Delta \cot \beta}$ VS. β FOR VARIOUS ϕ' ,
 $\frac{c'}{\gamma h} = 0$ TO 0.3

Table 3.2
Variation in Sensitivity of the Factor of Safety to Input Parameters

Vari- ables	Variation with		
	$\frac{c'}{\gamma h}$	ϕ'	β
$\frac{\Delta F}{\Delta(\frac{c'}{\gamma h})}$	Constant or decreasing (Figures A.32, A.33 & A.34)	Increasing slightly for steep slopes, more for flat (Figure 3.14 or A.31)	Decreasing slightly for $\beta < 50^\circ$, then rapidly (Figure 3.13 or A.30)
$\frac{\Delta F}{\Delta \phi'}$	Increasing rapidly for $\frac{c'}{\gamma h} < 0.1$, then $\approx$ constant (Figures A.40 & A.41)	Decreasing for $\phi' < 20^\circ$ Increasing for $\phi' > 20^\circ$ approximately constant for $10^\circ < \phi' < 30^\circ$ (Figure 3.16 or A.38 & A.39)	Decreasing rapidly for $\beta < 50^\circ$, slower for $\beta > 50^\circ$ (Figure 3.15 or A.36)
$\frac{\Delta F}{\Delta \tan \phi'}$	Increasing rapidly for $\phi' < 20^\circ$, and $\frac{c'}{\gamma h} < 0.1$ For $\phi' > 20^\circ \approx$ constant	Decreasing for $\phi' < 20^\circ$, then $\approx$ constant	Increasing almost linearly, slight break at $\beta = 45^\circ$ (Figure A.42)
$\frac{\Delta F}{\Delta \cot \beta}$	Increasing almost linearly. For flat slopes almost constant (Figures A.51 to A.54)	Increasing for flat slopes approx. linearly (Figures A.47 to A.50)	Constant for flat slopes Increasing for steeper slopes, and high values of $\frac{c'}{\gamma h}$ (Figure 3.17)

A general observation is that $\frac{\Delta F}{\Delta x_1}$ ($x_1 = \frac{c'}{\gamma h}$, ϕ' , or $\cot \beta$) is relatively insensitive to variations in $\frac{c'}{\gamma h}$ for values of $\frac{c'}{\gamma h}$ larger than 0.1. An exception is the $\frac{\Delta F}{\Delta \cot \beta}$ ratio for slopes steeper than 45° . For $\frac{c'}{\gamma h}$ values between 0 and 0.1, $\frac{\Delta F}{\Delta \phi'}$ and $\frac{\Delta F}{\Delta \cot \beta}$ increases with $\frac{c'}{\gamma h}$, whereas $\frac{\Delta F}{\Delta(\frac{c'}{\gamma h})}$ is constant or decreases with $\frac{c'}{\gamma h}$.

Figures 3.13 to 3.16 show that the factor of safety F , is most sensitive to variations in the $\frac{c'}{\gamma h}$ parameter as well as ϕ' for relatively flat slopes. For β angles up to 45° , $\frac{\Delta F}{\Delta(\frac{c'}{\gamma h})}$ decreases slightly with increasing β ; for β angles larger than 45° , the decrease in $\frac{\Delta F}{\Delta(\frac{c'}{\gamma h})}$ is larger. Also $\frac{\Delta F}{\Delta \phi'}$ decreases with increasing value of β ; however, contrarily to $\frac{\Delta F}{\Delta(\frac{c'}{\gamma h})}$, the decrease is large for β angles up to 45° to 50° , and less for β angles larger than 50° .

Figure 3.17 shows the sensitivity of F to variations in $\cot \beta$, β being the slope angle. $\cot \beta$ was chosen as independent variable instead of β , because the relationship between F and $\cot \beta$ was more simple than that between F and β . For values of $\frac{c'}{\gamma h}$ higher than 0.1 and slope angles β less than 45° , $\frac{\Delta F}{\Delta \cot \beta}$ is almost independent of the value of $\frac{c'}{\gamma h}$ as well as β , but increases with increasing ϕ' . For steeper slopes, $\frac{\Delta F}{\Delta \cot \beta}$ increases with $\frac{c'}{\gamma h}$ as well as with β .

To investigate actual changes in factors of safety due to variation in the variables, specific variances were assigned to the variables as follows:

$$\Delta \phi' = 5^\circ \text{ (constant)}$$

$$\frac{\Delta c'}{c'} = 0.20 = 20\%$$

$$\frac{\Delta h}{h} = 0.05 = 5\%$$

$$\frac{\Delta \gamma}{\gamma} = 0.05 = 5\%$$

Assuming 5% variance in height h and base of slope b , we find

$$\Delta \cot \beta = \frac{b - \Delta b}{h + \Delta h} - \frac{b}{h} \approx -0.1 \frac{b}{h} = -0.1 \cot \beta$$

$$\text{or } \frac{\Delta \cot \beta}{\cot \beta} = 0.10 = 10\%$$

$$\frac{\Delta c'}{c'} = 20\% \text{ gives } \Delta \left(\frac{c'}{\gamma h} \right)_c = 20\%$$

$$\frac{\Delta h}{h} = \frac{\Delta \gamma}{\gamma} = 5\% \text{ gives } \Delta \left(\frac{c'}{\gamma h} \right)_h = \Delta \left(\frac{c'}{\gamma h} \right)_\gamma = 5\%$$

$$\left(\frac{1}{1 + \Delta} = 1 - \Delta \right).$$

The total change $\Delta \left(\frac{c'}{\gamma h} \right)$ due to c' , γ , and h will be

$$\Delta \left(\frac{c'}{\gamma h} \right) = \left[\frac{1 - 0.2}{1.05^2} - 1 \right] \frac{c'}{\gamma h} = -0.274 \frac{c'}{\gamma h} \approx 27\%,$$

which is less than the sum of the three ($20 + 5 + 5 = 30\%$). Hence the procedure will overestimate $\Sigma \Delta F$ a little. For positive values of $\Delta \left(\frac{c'}{\gamma h} \right)$, it is opposite; however, all variations are investigated with respect to a decrease in F , because this is the critical situation. In practice, the coincidence of all variances being in the direction of less stability is unlikely, but in the deterministic approach, we design for the combination that gives the most critical situation.

The specific variances are applied to the five slopes investigated, in the range of factors of safety from 0.8 to 3. The calculations are tabulated in Appendix A, Tables A.5 to A.13. Figures 3.18 through 3.22 show the relative sensitivity of F to the variables.

FIGURE 3.18 RELATIVE SENSITIVITY $\frac{\Delta F_{x_i}}{\Sigma \Delta F}$ FOR VERTICAL SLOPE

FIGURE 3.19 RELATIVE SENSITIVITY $\frac{\Delta F_{x_i}}{\Sigma \Delta F}$ FOR SLOPE 1/2

FIGURE 3.20 RELATIVE SENSITIVITY $\frac{\Delta F_{xi}}{\Sigma \Delta F}$ FOR SLOPE 1/1

FIGURE 3.21

RELATIVE SENSITIVITY $\frac{\Delta F_{xi}}{\Sigma \Delta F}$ FOR
SLOPE 2/1

FIGURE 3.22 RELATIVE SENSITIVITY $\frac{\Delta F_{xi}}{\Sigma \Delta F}$ FOR SLOPE 3/1

It is seen that for slopes with $\beta > 45^\circ$ and $\frac{c'}{\gamma h} > 0.1$, c' is the most important parameter with respect to variation in the factor of safety F , contributing 30% to 60% of the total change in F .

For flatter slopes and not too high values of $\frac{c'}{\gamma h}$, ($\frac{c'}{\gamma h} < 0.1 \cot \beta$), ϕ' is the more significant, contributing 35% to 60% of the total change in F .

The total change in F , if variances in all parameters are assumed to decrease F , amounts to 24% to 43% of F , which is quite large. There is no relationship between F and $\frac{\sum \Delta F}{F}$.

If different values of the variances are assumed, different results may be obtained. The values chosen here are thought to be reasonable. Very little information was found in the literature about actual variances in soil parameters as obtained from field and/or laboratory tests. Singh (1970) uses maximum variances of 50% for c' and 10% for ϕ' , which shifts the results in the direction of c' being most significant with respect to variations in F for a larger range of slopes.

3.3.1.4 Conclusion and Recommendations

The results of this sensitivity study show that for homogeneous slopes the factor of safety is most sensitive to variations in the strength parameters c' and ϕ' . These two parameters may contribute 65% to 80% of the total change in F due to variances in the independent variables.

For steep slopes and $\frac{c'}{\gamma h} > 0.1$, variation in c' has a larger influence on F than has variation in ϕ' . For flatter slopes with $\frac{c'}{\gamma h} < 0.1 \cot \beta$, variation in ϕ' has the larger influence on F .

These two parameters are of course the most difficult to determine with high accuracy. This is already reflected in the assumption of $\frac{\Delta c'}{c'} = 20\%$, whereas $\frac{\Delta \gamma}{\gamma}$ and $\frac{\Delta h}{h} = 5\%$.

The fact that it is impractical to obtain precise values of c' and ϕ' is already accounted for, to some extent, in the requirement that the value of the safety factor F should lie in the range of 1.2 to 1.5 for slope stability. These values, although they are based on experience, may not always assure a safe design. This depends on how reliable the soil parameters are.

In some countries, difficulties in the determination of soil strength parameters have led to the introduction of "partial coefficients". These coefficients are factors that are applied to the soil parameters, as well as structural and load parameters, if applicable. The factored variables are used in the stability calculations and the safe design requires $F \geq 1$. This method has the advantage over the traditional method of stability analysis, in that the factors can vary from one variable to another, and hence account for the fact that some variables are more difficult to determine than others, and that variation in some parameters has more effect on the result than variation in others. Table 3.3 shows some of the partial coefficients used in Denmark [from Lundgren and Brinch Hansen (1965)]. For stability and earth pressure the partial coefficients for c' (f_c) and ϕ' (f_ϕ) are 1.5 and 1.2, respectively, indicating that c' is the one of the two parameters believed to have the most uncertainty and the highest influence on the result. The value of c' used in stability analysis would be $c' = \frac{c'_{lab}}{f_c}$.

Table 3.3

Typical Values of Partial Coefficients used in Denmark for
Geotechnical Designs*

Symbol	Partial Coefficient for	Normal	Extra-ordinary
f_g	Dead load, water pressure	1.0	1.0
f_p	Live loads	1.5	1.5
f_v	Wind loads	1.5	1.0
f_ϕ	Friction (stability & earth pressures)	1.2	1.1
	Friction (bearing capacity, foundations & piles)	1.25	1.15
f_c	Cohesion (stability & earth pressures)	1.5	1.4
	Cohesion (bearing capacity, foundations)	1.75	1.6
	Cohesion (bearing capacity, piles)	2.0	1.8
f_a	Skin friction, piles	2.0	1.8

* From Lundgren and Brinch Hansen (1965).

For practical purposes it is recommended that Figures A.11 through A.15 be used for determination of ΔF due to variances in soil data.

For a particular problem, the variances $\Delta c'$ and $\Delta \phi'$ should be estimated from the different possibilities of interpreting the test data (different ways of fitting the Mohr-Coulomb envelope to the test data). The same curves may also be used to determine the maximum variation in the c' and ϕ' parameters that can be tolerated if a certain minimum factor of safety is required (Step 4 in Figure 3.1).

Figures A.25 through A.29 may be used in a similar way to determine the change in F due to changes in the geometry parameters h and β , when the soil parameters are considered fixed.

3.3.2 Sensitivity Analysis of an Embankment Founded on a Soil with a Weak Seam

Moving from the simple problem of a slope in homogeneous soil to problems encountering inhomogeneous soils, the number of independent variables increases significantly. The following items must be determined.

- (a) Independent variables x_i .
- (b) Range of each of these variables. The range of some variables may influence the range of others.
- (c) Range of variation in each of the variables, Δx_i .
(Constant, percentage of mean, etc.)

The 16 variables, which apply to an embankment built on a foundation with a weak soil layer, are shown in Figure 3.23. For some of the variables, e.g., the soil parameters, certain limits in the values can easily be established. The range in geometric variables may impose

Variables

Symbols

Height and width of embankment	h, B
Depth to weak layer from base of embankment, measured at center line of embankment	D
Inclination of ground surface	i
Inclination of weak soil layer	α
Inclination of left and right slope of embankment	β_L, β_R
Soil parameters of embankment fill	$(\gamma, c, \phi)_E$
Soil parameters of foundation soil	$(\gamma, c, \phi)_F$
Soil parameters of weak soil layer	$(\gamma, c, \phi)_W$

FIGURE 3.23 **VARIABLES FOR AN EMBANKMENT BUILT ON A FOUNDATION SOIL WITH A WEAK SOIL LAYER**

further limits on soil strength parameters, when we are only interested in problems having factors of safety less than 3. Many of the limits in the parameters must be determined by trial. For example, at what depth will a weak seam in a foundation soil have no effect on the stability of an embankment?

Because of the excessive number of possible combinations of the variables (if each of the 16 variables were given 3 values, the number of combinations would be $3^{16} = 43,046,721$), it was necessary to cover only changes in those variables which are considered to be most important.

The variables that are held constant are mainly those with respect to geometry, particularly those which are easy to get information about in a real situation, such as the surface profile (i) and the geometry of the embankment proper. The ground surface was taken as horizontal; the embankment geometry was assumed as: 20 feet in height, 40 feet of crown width, and slopes 1.5/1 (33.7°). The number of soil parameters is reduced by using an average unit weight γ for all soils, and taking ϕ_w as equal to zero. After this reduction, the following parameters remain as variables:

D	Depth to weak seam
α	Inclination of weak seam
γ	Average unit weight of soil
$(c, \phi)_E$	Strength intercept and strength angle of embankment soil
$(c, \phi)_F$	Strength intercept and strength angle of foundation soil
c_w	Strength of weak soil seam

The relationship between the factor of safety F and the unit weight γ is relatively simple, which means that γ can be held constant, leaving seven variables.

In this study, c_w has a maximum of five values. Ten combinations of $(c, \phi)_E$ and $(c, \phi)_F$ are investigated, for up to nine different depths D , and three different angles α . See Table 3.4.

3.3.2.1 Factor of Safety for Failure Determined by a Weak Soil Seam

The factor of safety for an embankment founded on a soil with a weak layer should be less than

(a) the factor of safety for the embankment itself

(toe failure with $\theta > 0$),

(b) the factor of safety for an embankment founded on a similar soil with no weak layer.

If the factor of safety becomes larger than either (a) or (b), the weak seam does not influence the stability of the embankment.

Where the geologic conditions, such as joints in a rock slope or a weak soil seam beneath an earth slope, determine the shape and location of the critical failure surfaces, the factor of safety F can be expressed as

$$F = F_c + F_\phi \quad (3.1)$$

$$\text{where} \quad F_c = \frac{1}{\gamma h} [k_1 c_1 + k_2 c_2 + \dots + k_n c_n] \quad (3.2)$$

$$\text{and} \quad F_\phi = a_1 \tan \phi_1 + a_2 \tan \phi_2 + \dots + a_n \tan \phi_n \quad (3.3)$$

Table 3.4

Range in Variables Studied for an Embankment on a Foundation with
a Weak Layer

<u>Variable</u>	<u>Symbol</u>	<u>Range</u>
Height of embankment	h	20 feet
Width of embankment	B	40 feet
Depth to weak layer from base of embankment (measured at centerline of embankment)	D	0 to 60 feet
Inclination of ground surface	i	0°
Inclination of weak soil layer	α	0 to 11.3°
Inclination of left and right slope of embankment	β_L/β_R	33.7°
Average unit weight of soil	γ	130pcf (100 to 140pcf)
Strength of embankment soil	c_E, ϕ_E	1000psf, 0° 2000psf, 0° 500psf, 10°
Strength of foundation soil	c_F, ϕ_F	1000psf, 0° 2000psf, 0° 500psf, 10° 0, 30° 200psf, 30° 0, 32°
Strength of weak soil	c_w	200 to 1000psf

The variables are

n	Number of soil types that the failure surface passes through
γ	Mean value of unit weights of soils above the failure surface
h	Height of slope or some other characteristic length
c_1 to c_n	Strength intercepts of soil No. 1 to soil No. n
ϕ_1 to ϕ_n	Strength angles of soil No. 1 to soil No. n
a_1, k_1 to a_n, k_n	Geometry dependent variables, constant for a certain slope geometry and soil profile.

3.3.2.2 Sensitivity Analysis

Using the above expression for the safety factor F , it may be seen that F_ϕ is independent of the unit weight γ as well as h , the height of the embankment. Any change in γ or h will only be reflected in F_c .

It is easy to show that the following relationships hold:

$$\frac{\Delta F}{\Delta \gamma} = - \frac{F}{\gamma} c, \text{ for other parameters held constant} \quad (3.4)$$

$$\frac{\Delta F}{\Delta h} = - \frac{F}{h} c, \text{ for other parameters held constant} \quad (3.5)$$

From equations (3.2) and (3.3) the sensitivity of F with respect to the soil strength parameters c_i and ϕ_i can be defined as

$$\frac{\Delta F}{\Delta c_i} = \frac{1}{\gamma h} k_i \quad (3.6)$$

$$\frac{\Delta F}{\Delta(\tan \phi_i)} = a_i \quad (3.7)$$

Sensitivity of F with respect to the relative geometry can be expressed as

$$\begin{aligned}\Delta F &= \sum \frac{\partial F}{\partial k_i} \Delta k_i + \sum \frac{\partial F}{\partial a_i} \Delta a_i \\ &= \frac{1}{\gamma h} \sum c_i \Delta k_i + \sum \tan \phi_i \Delta a_i\end{aligned}\quad (3.8)$$

3.3.2.3 Results of Stability Analysis for Embankments

Different search routines in the STABL program were applied to the problem to locate the shape and position of the most critical failure surfaces. It was found, as expected, that the sliding block mode of failure was the most critical, with the base of the block located along the weak soil seam. See Figure 3.24 and Figures A.55 through A.58. However, this was also the case for homogeneous soil, see Table A.14 and Figures A.59 and A.60. When comparing these results with results obtained in the comparative analyses, Chapters V and VI, it may be concluded that the method of slices used in the STABL program (Janbu's simplified procedure) is likely to give more conservative results for irregular failure surfaces than for circular failure surfaces.

It was also found that a sliding block type of failure surface with active and passive portions of the surface inclined at angles corresponding to the Rankine theory, $45^\circ + \phi'/2$ for the active portion and $45^\circ - \phi'/2$ for the passive portion, was more critical than the sliding block with randomly generated active and passive portions of the failure surface (Figures A.57 and A.58). For this reason, the BLOCK2 sliding block search was introduced in addition to the BLOCK search, see Chapter IV.

FIGURE 3.24 A SLIDING BLOCK TYPE OF FAILURE SURFACE IS MOST CRITICAL

When the weak soil layer is inclined with the horizontal, the stability should be investigated for both sides of the embankment. This was done in a few cases, but in most cases the stability was investigated only to the side of the embankment where the weak layer is dipping down, since this is most difficult to change. On the side where the weak soil layer is inclined upwards, it may more easily be replaced with stronger soil, if the stability is threatened.

The results for the few cases where stability was investigated to both sides show, that for foundation soil of the same strength as the embankment soil, failure is more critical towards the down-dipping weak seam. In the case of stronger foundation soil, it may be more critical towards the side where the weak seam is inclined upwards, see Figures 3.25 and 3.26. In most cases the foundation soil will be weaker than the embankment soil, so that failure towards the side where the weak layer dips down is the most likely.

The failure surfaces remain the same, or very close to the same, for different foundation soils within a relatively large range in soil parameters (γ , c , ϕ), so long as the soil geometry and relative strength of the weak soil remain unchanged, see Figures A.61 through A.64. But the safety factor varies.

As the strength of the weak seam increases, the active and passive portions of the failure surface move closer to each other. The volume involved in the failure decreases, as shown in Figure 3.27. Despite this change in the failure surface, the initial equations [(3.1), (3.2), (3.3)] can still be applied to some extent, because the change in shape and location of the failure surface is relatively small. Exceptions to

FIGURE 3.25 COMPARISON BETWEEN FAILURE TO LEFT AND FAILURE TO RIGHT SIDE OF EMBANKMENT, $D=10$ FT, $\alpha=5.7^\circ$, $\phi=0$

FIGURE 3.26 COMPARISON BETWEEN FAILURE TO LEFT AND FAILURE TO RIGHT SIDE OF EMBANKMENT, $D = 10$ FT, $\alpha = 11.3^\circ$, $\phi = 0$

FIGURE 3.27 CHANGE IN CRITICAL FAILURE SURFACE WHEN STRENGTH OF WEAK SOIL SEAM INCREASES

this are the situations where the active part of the failure surfaces intersects the ground surface beyond the crown of the embankment, see Figure 3.27 (surface type 2). In these cases the geometry of the problem changes so much that new values of "a" and "k" parameters apply in equations (3.2) and (3.3). Type 1 surfaces (Figure 3.27) occur for small relative depths of the weak soil layer and small inclinations of α . Type 2 surfaces occur for larger relative depths of the weak soil layer and for larger inclinations of α .

From the results (Table A.14), curves are plotted showing variations of the normalized factor of safety F_n vs. the normalized strength of the weak seam c_n , where $F_n = \frac{F\gamma h}{c_E}$ and $c_n = \frac{c_w}{c_E}$. These curves show a very obvious change when the failure surface changes from type 1 to type 2, as a break appears in the otherwise linear relationship between F_n and c_n , see Figures 3.28 and 3.29. From these curves the "k" and "a" values can be found. Table 3.5 summarizes the results. Curves of "k" and "a" values are plotted vs. α for constant depth D (Figures 3.30 and 3.31) and vs. depth D for constant α (Figures 3.32 through 3.36), where α is the inclination of the weak soil seam.

The a-graphs show that the strength angle ϕ_F for the foundation soil has a larger influence on the stability than has the strength angle ϕ_E of the embankment soil. The difference increases with increasing depth D, as a_F increases and a_E decreases with increasing depth. This seems reasonable, as the frictional strength increases with the overburden, and the overburden is larger for the part of the failure surface that passes through the foundation soil.

c_n	0.2	0.4	0.6	0.8	1.0	$\frac{c_F}{c_E}$	c_E
F_n	3.73	4.29	4.69	4.92	5.13	1	1000
F_n	5.35	5.93	6.47	6.93	7.23	2	"
c_n	0.1	0.2	0.3	0.4	0.5		
F_n	3.39	3.72	4.02	4.29	4.54	1	2000

FIGURE 3.28 NORMALIZED FACTOR OF SAFETY F_n VS.
 NORMALIZED STRENGTH c_n FOR
 EMBANKMENT 20 FEET HIGH, SLOPES
 1.5/1, DEPTH TO WEAK SEAM 5 FEET,
 INCLINED AT $\alpha = 11.3^\circ$ TO HORIZONTAL

c_n	0.2	0.4	0.6	0.8	1.0	$\frac{c_E}{c_F}$	ϕ_E	ϕ_F
F_n	3.34	3.91	4.31	4.54	4.75	0.5	0°	10°
c_n	0.4	0.8	1.2	1.6	2.0			
F_n	5.67	6.76	7.28	7.73	8.12	1	10°	10°

FIGURE 3.29 EXAMPLE OF F_n VS. c_n FOR EMBANKMENT 20 FEET HIGH, SLOPES 1.5/1, DEPTH TO WEAK SEAM 5 FEET, INCLINED AT $\alpha = 11.3^\circ$ WITH HORIZONTAL

Table 3.5

Geometry Parameters k and a for Embankment Stability Analysis

Surface type*	D ft	$\tan \alpha$	k_E	k_F	k_W	a_E	a_F
(2)	0	0.20	2.41	1.04	2.80		
1	0	0.20	2.44	1.47	1.57	0.71	0.86
1	5	0	2.74	1.06	1.87	0.65	0.72
1	5	0.10	2.34	1.69	1.33	0.83	1.01
1	5	0.20	1.90	2.11	1.11	0.47	1.54
(2)	5	0.20	1.50	1.66	2.72	0.54	0.92
1	10	0	1.99	1.72	1.50	0.82	1.20
1	10	0.10	1.69	2.19	1.20	0.59	1.60
2	10	0.20	1.15	2.25	2.00	0.45	1.71
2	15	0.20	1.05	2.50	1.46		
1	20	0	1.54	2.46	0.88	0.47	2.25
2	20	0.20	0.83	2.82	1.07	0.33	3.01
2	30	0.20	0.58	3.28	0.64		
2	40	0	0.56	3.34	0.54	0.12	4.73
2	40	0.20	0.41	3.60	0.44	0.08	(5.43)
2	50	0.20	0.27	3.94	0.34		

* Surface types 1 and 2 are illustrated in Figure 3.27.

FIGURE 3.30 VARIATIONS OF k AND " a " COEFFICIENTS WITH α FOR CONSTANT DEPTH $D=0.25h$

FIGURE 3.31 VARIATIONS OF k AND " a " COEFFICIENTS WITH α FOR CONSTANT DEPTH $D=0.5h$

FIGURE 3.32 VARIATION OF k COEFFICIENTS WITH DEPTH D FOR CONSTANT $\alpha = 0^\circ$

FIGURE 3.33 VARIATION OF k COEFFICIENTS WITH DEPTH D FOR CONSTANT $\alpha = 5.7^\circ$

FIGURE 3.34 VARIATION OF k COEFFICIENTS WITH DEPTH D FOR CONSTANT $\alpha = 11.3^\circ$

FIGURE 3.35 VARIATION OF "a" COEFFICIENTS WITH DEPTH D FOR CONSTANT $\alpha = 0^\circ$ (AND $\alpha = 5.7^\circ$)

FIGURE 3.36 VARIATION OF "a" COEFFICIENTS WITH DEPTH D FOR CONSTANT $\alpha = 11.3^\circ$

With respect to the strength intercepts c , the k -graphs show that for small angles of α and relatively small depths ($D \approx 0.3h$ to $0.5h$), the strength intercept c_E of the embankment soil has a larger influence on the stability than the strength intercepts c_F and c_w of the foundation and the weak soil, respectively. For greater depths, c_F has the larger influence. Comparing k_F and k_w , it is seen that for very small angles of α ($\approx 0^\circ$) and depths $D \approx 0.5h$, the strength intercept c_w of the weak seam has a larger influence on the stability than has the strength intercept c_F of the foundation soil. For larger depths and/or larger angles α , c_F has the larger influence.

Over the range of variables investigated, the factor of safety F decreases with increasing angle α or depth D of the weak seam, to a certain depth, then F increases with increasing α or D , other parameters being held constant. The depth at which the change occurs depends upon the strength of the foundation soil relative to the embankment soil, and to some extent on the relative strength of the weak soil seam. For foundation soil of equal or lesser strength than the embankment soil, the shift for F vs. α occurs at a depth D approximately equal to twice the embankment height, whereas for stronger foundation soil, the shift occurs at much smaller depths. Figure 3.37 shows F vs. c_w for $\frac{c_F}{c_E} = 1$ and 2, and $\alpha = 0^\circ, 5.7^\circ$ and 11.3° . The shift for F vs. D occurs at depths D between $0.25h$ and $2h$ for a foundation soil of the same strength as the embankment soil. For a stronger foundation soil, F increases with depth D over the total range of D . Figure 3.38 shows the variation in F with respect to α and D for foundation soil of the same strength as the embankment soil.

FIGURE 3.37 VARIATION OF F VS. c_w FOR VARIOUS α , AND DIFFERENT RATIOS OF $\frac{c_F}{c_E}$, $D=5$ FT

FIGURE 3.38 EXAMPLE OF VARIATION OF THE FACTOR OF SAFETY WITH GEOMETRY OF WEAK SOIL LAYER

3.3.2.4 Results Applied to Sensitivity Analysis of Embankments

Sensitivity of F with respect to γ

From Equations (3.1), (3.2) and (3.3) the following expression was derived

$$\frac{\Delta F}{\Delta \gamma} = - \frac{F}{\gamma} \quad (3.4)$$

Example: An embankment has a factor of safety with respect to strength intercept $F_c = 1.2$, and a unit weight $\gamma = 120$ pcf. This gives

$$\frac{\Delta F}{\Delta \gamma} = - \frac{1.2}{120} = - 0.01$$

If 120 pcf is the average value of several measurements of γ , and say the standard deviation of these measurements is ± 2.5 , this gives a range in F of

$$F = 1.2 \pm \Delta F = 1.2 \pm (-0.01) \cdot 2.5 = 1.2 \mp 0.025$$

$$\text{or } \frac{\Delta F}{F} = - \frac{\Delta \gamma}{\gamma} = \pm \frac{2.5}{120} = 2.1\%, \text{ or } F = 1.175 \text{ to } 1.225.$$

Sensitivity of F with respect to height h

$$\frac{\Delta F}{\Delta h} = - \frac{F}{h} \quad (3.5)$$

Example: Again we assume an embankment with $F_c = 1.2$, and a height h of 20 feet.

$$\frac{\Delta F}{\Delta h} = - \frac{1.2}{20} = - 0.06$$

Considering whether we can add say 5 feet to the height, we find $\Delta F = - 0.06 \cdot 5 = - 0.3$, which gives $F = 1.2 - 0.3 = 0.9$, so the answer is no.

Considering an uncertainty in the measurement of the height during construction, which leads to a height that is 21 feet instead of 20 feet. This decreases the safety with

$$\Delta F = -0.06 \cdot 1 = -0.06, \text{ hence } F = 1.14.$$

Sensitivity of F with respect to strength intercept c

$$\frac{\Delta F}{\Delta c_i} = \frac{1}{\gamma h} k_i \quad (3.6)$$

Example: An embankment 20 feet in height with slopes 1.5/1 and a 40 feet wide crest is built on a foundation soil which has a weak seam located 10 feet below the original horizontal ground surface at the center of the embankment, and inclined 5.7° with the horizontal. This problem has a maximum of 3 types of soil, and the k parameters found were $k_E = 1.69$, $k_F = 2.19$, $k_W = 1.20$. If the variation in strength intercepts is $\Delta c = \pm 200$ psf, and $\gamma = 125$ pcf

$$\begin{aligned} \frac{\Delta F}{\Delta c} &= \frac{\Delta F}{\Delta c_E} + \frac{\Delta F}{\Delta c_F} + \frac{\Delta F}{\Delta c_W} = \frac{1}{\gamma h} (k_E + k_F + k_W) \\ &= \frac{5.08}{125 \cdot 20} = 0.002 \end{aligned}$$

and $\Delta F = 0.41$, which is quite high. If the embankment is designed with a safety of $F = 1.5$, this variation in c may result in an F value of only 1.1.

Notice that k_F is almost twice as large as k_W , which means that greater effort should be used to determine c_F than c_W in this particular case. However, complete neglect of a weak seam could be catastrophic.

Sensitivity of F with respect to strength angle ϕ

$$\frac{\Delta F}{\Delta(\tan \phi_i)} = a_i \quad (3.7)$$

Example: For the same embankment as described above, it was found that

$$a_E = 0.59 \text{ and } a_F = 1.60.$$

If $\Delta\phi_i = 5^\circ$, we find for $\phi_i \approx 30^\circ$ to 35° ,

$$\Delta(\tan \phi_i) = 0.123 \rightarrow \Delta F = 0.27$$

and for $\phi_i \approx 10^\circ$ to 15° ,

$$\Delta(\tan \phi_i) = 0.092 \rightarrow \Delta F = 0.20,$$

which is also a significant variation in the factor of safety, although the strength intercept c in this particular case has a larger influence on F than has the strength angle ϕ .

From Tables 3.5 and 3.6 it can be seen that although k_E , k_F and k_w vary with location of the weak soil layer (as represented by α and D), the sums of the k parameters vary only a little. Hence the result for $\Delta F(\Delta c)$ is almost independent of the problem geometry, and depends mainly on the values of Δc . This is not the case with respect to $\Delta F(\Delta\phi)$ as a_F increases almost linearly with depth D to the weak seam, and a_E decreases only slightly with D . So for greater depths D of the weak seam, $\Delta F(\Delta\phi)$ may be larger than $\Delta F(\Delta c)$.

Sensitivity of F with respect to relative geometry

$$\Delta F = \frac{1}{\gamma h} \sum c_i \Delta k_i + \sum \tan \phi_i \Delta a_i \quad (3.8)$$

Table 3.6

Statistical Analysis of Geometry Parameters k and a

Dominating Failure Surface	D ft	$\tan \alpha$	$k_E + k_F$	$k_F + k_W$	$k_E + k_F + k_W$	$a_E + a_F$
(2)	0	0.20	3.45	3.84	6.25	-
1	0	0.20	3.91	3.04	5.48	1.57
1	5	0	3.80	2.93	5.67	1.37
1	5	0.10	4.03	3.02	5.36	1.84
1	5	0.20	4.01	3.22	5.12	2.01
(2)	5	0.20	3.16	4.38	5.88	1.46
1	10	0	3.71	3.22	5.21	2.02
1	10	0.10	3.88	3.39	5.08	2.19
2	10	0.20	3.40	4.25	5.40	2.16
2	15	0.20	3.54	3.96	5.00	-
1	20	0	4.00	3.34	4.88	2.72
2	20	0.20	3.65	3.89	4.72	3.34
2	30	0.20	3.86	3.92	4.50	-
2	40	0	3.90	3.88	4.44	4.85
2	40	0.20	4.01	4.04	4.45	(5.51)
2	50	0.20	4.21	4.28	4.55	-
Mean			3.78	3.66	5.12	2.32
Variance			0.08	0.24	0.29	1.03
Standard Dev.			0.28	0.49	0.53	1.01
Variance Coef.			7.4%	13.3%	10.4%	43.7%

Example: Consider the same embankment as in the previous example, with a safety factor $F = F_c + F_\phi$ corresponding to $c_E = 1000$ psf, $c_F = 500$ psf, $\phi_F = 10^\circ$, and $c_w = 200$ psf, which gives

$$F = \frac{1}{125 \cdot 20} (1.69 \cdot 1000 + 2.19 \cdot 500 + 1.2 \cdot 200) + 1.6 \tan 10^\circ = 1.49.$$

Let us assume that there is an uncertainty in the determination of the position of the weak soil seam, reflected as ± 2 feet in the depth D and ± 0.05 in the inclination ($\tan \alpha$). We now have to determine which sign, plus (+) or minus (-), will decrease the factor of safety. Since c_E is larger than c_F and c_w , the value of k_E is compared to k_F and k_w . We are first investigating the influence of a change in the depth D . Figure 3.33 shows that for $\alpha = 5.7^\circ$, k_E decreases with increasing depth D , whereas k_F increases approximately the same amount. Since $c_E = 2c_F$, the result is a decrease in F . Further, k_w also decreases with increasing depth. On the other hand, a_F increases very much with increasing depth (see Figure 3.35), so it is not obvious whether $F = F_c + F_\phi$ will decrease or increase with increasing depth. Therefore both situations have to be investigated.

$$\Delta D = + 2 \text{ feet}$$

$$\Delta k_E = - 0.25, \Delta k_F = 0.20, \Delta k_w = - 0.05, \Delta a_F = 0.24$$

$$\Delta F = \frac{1}{125 \cdot 20} (- 0.25 \cdot 1000 + 0.20 \cdot 500 - 0.05 \cdot 200) + 0.24 \tan 10^\circ = - 0.06 + 0.04 = - 0.02,$$

which shows that increased depth decreases the stability,

although the change is negligible. For $\Delta D = -2$ feet, the changes in the "a" and "k" parameters are of opposite sign, but of the same magnitude, so $\Delta F = +0.02$.

Applying the same procedure to a change in the angle α , Figure 3.31 shows that k_E decreases for increasing α ; k_F increases, but is almost constant for α between 5.7° and 11.3° at this particular depth ($D = 10$ feet $= 0.5h$), whereas k_w increases for increasing as well as decreasing α , because the failure surface for increasing α changes from type 1 to type 2. However, a_F increases with increasing α , so like the previous case, it is not obvious whether a decrease of an increase in α will result in a decrease in F .

$$\Delta \tan \alpha = +0.05$$

$$\Delta k_E = -0.25, \Delta k_F = 0.08, \Delta k_w = 0.16, \Delta a_F = 0.09$$

$$\begin{aligned} \Delta F &= \frac{1}{125 \cdot 20} (-0.25 \cdot 1000 + 0.08 \cdot 500 + 0.16 \cdot 200) \\ &\quad + 0.09 \tan 10^\circ = -0.07 + 0.02 = -0.05 \end{aligned}$$

$$\Delta \tan \alpha = -0.05$$

$$\Delta k_E = 0.19, \Delta k_F = -0.19, \Delta k_w = 0.08, \Delta a_F = -0.16$$

$$\begin{aligned} \Delta F &= \frac{1}{125 \cdot 20} (0.19 \cdot 1000 - 0.19 \cdot 500 + 0.08 \cdot 200) \\ &\quad - 0.16 \tan 10^\circ = 0.04 - 0.03 = 0.01, \end{aligned}$$

so F decreases for increasing α .

Comparing the changes in the factor of safety F due to variations in the independent variables, we find that the only significant change in F is due to variation in the soil strength parameters c and ϕ .

The above results are based upon a rather limited study, particularly with respect to the range in the α angles investigated. For very steep angles of α , the influence of the weak seam may be negligible. However, this was not investigated.

3.3.2.5 Summary and Conclusions

The task of performing a sensitivity analysis on a slope stability problem becomes almost impossible when moving from the simple case of slopes in homogeneous soils to more practical, but also more complicated problems, involving different types of soils and changing soil profiles. The number of variables increases drastically, and the possible combinations of these variables increases even more. In this study, the number of independent variables was reduced from sixteen to seven, compared to the three independent variables that apply to the homogeneous slope stability problem. The results, in terms of safety factors, occupy ten tables, compared to one for homogeneous slopes, and even so they cover only a small range in the possible combinations of the independent variables.

Despite the limited investigation, the analysis did show that almost the same conclusion may be reached with respect to problems encountering inhomogeneous soils, as was reached for homogeneous slope stability problems, namely that the dominating influence on the stability and changes therein are determined by the soil strength parameters c and ϕ . However, the neglect of a weak soil seam may have a large influence on the result of a stability analysis, and it may lead to failure of a slope that otherwise would have been stable with a safety factor of $F = 1.2$ to 1.5 . But this is in fact caused by using an incorrect c parameter (or ϕ parameter).

IV MODIFICATIONS OF THE STABL PROGRAM

This Chapter describes the major modifications of the STABL program that were performed during the course of this research. Other adjustments and corrections are reported in Appendix D.

Also included in this Chapter is some general advice to users, specifically regarding problems that frequently have been raised by users of the STABL program.

4.1 THE MODIFIED SLIDING BLOCK ROUTINE (BLOCK2)

During the sensitivity analysis of an embankment founded on a soil with a weak soil layer, as described in Section 3.3.2, it was observed that a sliding block type of failure surface with active and passive portions of the sliding block surface inclined at $(45^\circ + \phi'/2)$ and $(45^\circ - \phi'/2)$ with the horizontal, respectively, gave the lowest factor of safety. This type of failure surface corresponds to Rankine's earth pressure theory [Rankine (1857)]. See Figure 4.1.

As the original STABL sliding block search (BLOCK) generates the active and passive portion of the sliding block more randomly, it was felt that a sliding block using the Rankine theory for active and passive portions of the failure surfaces would be useful in the analysis of relatively simple slope stability problems.

The original sliding block, on the other hand, was still felt to be a better solution for cases of soil profiles that are complex.

FIGURE 4.1 SLIDING BLOCK FAILURE SURFACE
WITH RANKINE ACTIVE AND PASSIVE
ZONES

The new sliding block procedure (BLOCK2) is modified from the original, with much of the original procedure retained. This applies to the random technique used to pick control points on the sliding block failure surface. Only the technique for generation of active and passive portions of the sliding surface is changed. The data cards are unchanged except for the command code.

4.1.1 Generation of Active and Passive Portions of the Sliding Block According to Rankine's Theory

The procedure for generation of the active and passive portions of the new sliding block is as follows.

The passive portion of the failure surface starts from the point of the failure surface chosen randomly within the first box defined, see Siegel (1975a, p. 96). The number of the nearest soil boundary located above this point is determined by checking for intersection with a vertical line through the point. When this soil boundary is determined, the soil type below is known, and the direction of the first line segment of the passive wedge is determined as $(45^\circ - \phi'/2)$ with respect to the horizontal, where ϕ' is the strength angle of the soil.

Each time a line segment is generated, intersection with soil boundaries is investigated. If an intersection occurs, the previously described procedure is repeated to determine the new ϕ' angle to use.

The active portion of the failure surface is generated similarly and from the point chosen within the last box specified. The angle from the horizontal is $(45^\circ + \phi'/2)$, instead of $(45^\circ - \phi'/2)$.

The ϕ' angle used is that defined as the isotropic strength angle. If the soil is specified with anisotropic strength parameters, the

active and passive zones of the sliding block BLOCK2 will still be generated according to the ϕ' angle defined as the isotropic ϕ' angle. This means that one should specify the isotropic ϕ' parameter with the value of the anisotropic ϕ' parameter. In the active wedge range between approximately $+45^\circ$ and $+65^\circ$, and in the passive wedge range between approximately -15° and -45° . See Figure 4.2. If ϕ' differs considerably from the active to the passive regions, one may have to divide the anisotropic soil into two soil types, one in the active region, and another in the passive region. Although the failure surface will be generated according to the isotropic ϕ' value, the anisotropic ϕ' (if different) will be used in calculation of the available shear strength along the failure surface.

Further, limitation boundaries are ignored in this failure surface searching routine. The program prints out a warning if they are specified, but generates failure surfaces as if they did not exist.

4.1.2 Application of the New Sliding Block Routine

The new sliding block routine, BLOCK2, is particularly suited for relatively simple soil profiles, where a sliding block type of failure is obviously the most likely. This is, for example, the case with the problem analysed in Chapter III, an embankment built on a foundation with a weak soil layer. If the soil profile is complex, it is recommended that the original routine, BLOCK, be used.

4.2 OPTION OF SEVERAL PIEZOMETRIC SURFACES

The original STABL program had three possibilities for defining the pore water pressure within a slope. These were: the groundwater table;

Anisotropic direction ranges
for soil types 1 and 2

FIGURE 4.2 BLOCK2 SLIDING BLOCK ANALYSIS
APPLIED TO ANISOTROPIC SOIL

the pore water pressure parameter $r_u = \frac{u}{\gamma h}$, which may be used to present excess pore water pressures due to shear; and finally the pore pressure constant u_c , that defines a constant pore pressure for a defined zone.

This would seem to be sufficient to model any type of pore water pressures, equilibrium and excess of equilibrium, that would be encountered in a slope stability analysis. However, a modification which allows different piezometric levels to be assigned to different layers of soil, was proposed by the Division of Materials and Tests, ISHC.

This approach is primarily applied to end-of-construction stability analysis of embankments built upon clayey foundations, where the construction of the embankment creates excess pore pressures in soil layers with poor drainage characteristics.

Using the r_u parameter in such cases would give pore pressures that were proportional to the total overburden from the soil layer considered to the ground surface, whereas the excess pore pressures imposed by the embankment is believed to be proportional only to the increase in overburden which the embankment represents.

The pore pressure constant u_c might be applied, but this would require an increase in the number of soil types to be defined in the area where the embankment slopes, and a corresponding increase in soil boundaries. It was felt that it would be much simpler to introduce a second piezometric surface to represent the excess pore pressures built up in the less permeable soil layers due to construction of an embankment.

4.2.1 Program Modifications

The option of defining more than one phreatic surface required significant changes in the subroutines WATER and WEIGHT in STABL. WATER

is the subroutine that reads, checks and stores the data for the piezometric surfaces, if any. WEIGHT calculates the resultant forces acting on each slice. These include the total weight of the slice and water forces acting on the top and bottom of the slice. To be sure that the correct unit weight, moist or submerged, is used for each soil type within a slice, it is necessary to calculate and store the level of each piezometric surface, where it intersects the center line of the slice. Only the piezometric surfaces that apply to the top soil and bottom soil of a slice are used in the calculation of the water forces U_{α} and U_{β} , where U_{α} is the water force acting on the bottom of a slice, and U_{β} is the water force acting on the top of a slice.

4.2.2 Change in Data Cards

To define more than one piezometric level, an integer is added to the soil parameters, indicating which level applies to the particular soil. If no piezometric surfaces are defined, any number may be used. Also the data cards for WATER are changed to allow up to 10 piezometric surfaces to be defined. These changes are all included in the revised STABL User Manual.

4.2.3 Application of more than one Piezometric Level

The option of defining more than one piezometric level can be applied to problems where either artesian water pressures or a perched water table exist, see Figure 4.3. The program will treat the perched water table problem as follows. For a slip surface located within the clayey soil, the water pressure on the bottom as well as on the top of a slice will be calculated as the hydrostatic pressure corresponding to the

FIGURE 4.3 SLOPE STABILITY ANALYSIS FOR PROBLEM WITH A PERCHED WATER TABLE

perched water table. If a slip surface extends down into the sand layer, the water pressure at the bottom of a slice, if located in the sand, will be calculated as the hydrostatic pressure corresponding to the ground water table. The top of the slice will still have a water pressure corresponding to the perched water table.

Another stability problem, where the option of defining two water tables was applied, is that of a reservoir embankment analysed for rapid drawdown, see Figure 4.4. A slight modification of the method proposed by Taylor (1937) is applied. Taylor assumes that during the rapid drawdown no drainage takes place in the embankment. While the water pressure on the surface of the slope is removed, the water pressure within the embankment remains the same as it was under the submerged condition. This is considered a very conservative assumption [Dvinoff and Harr (1971)]. The modified assumption used in the above is that within the clayey soil core no change in water pressure occurs during the rapid drawdown, whereas in the outer shell consisting of granular soil of very high permeability, the water pressure changes according to the drawdown water table. The stability analysis is performed defining the drawdown water table for the outer shell of granular material, and the original high reservoir water table for the impermeable core material.

The strength of the core material has to be defined in terms of effective strength parameters ϕ' and c' . If the strength is defined solely in terms of a total strength c , with $\phi = 0$, the presence or absence of one or several water tables does not influence the result, unless the water table that applies to the top soil extends above the ground surface, acting as a surcharge.

(a) Embankment With Full Reservoir

(b) Water Table After Drawdown
(approximation)

FIGURE 4.4 RAPID DRAWDOWN STABILITY
ANALYSIS FOR EMBANKMENT

This may also be seen directly from the expression for the factor of safety (Section 4.3, Equation 4.1), where the water force at the bottom of a slice, U_{α} , will disappear from the equation if $\phi' = 0$, whereas U_{β} , the water force at the top of a slice, if any, still is contained in the expression for A_2 .

4.3 MODIFIED BISHOP FACTOR OF SAFETY FOR CIRCULAR FAILURE SURFACES

During the comparative analysis described in Chapter VI, it was discovered that the simplified Janbu method of slices used in the STABL program does not always provide conservative results compared to other methods. In fact it may give erroneous and unconservative results where a failure surface intersects the ground surface at the top of the slope at very steep angles, and the strength of the soil is defined mainly in terms of a strength intercept c (or c'). Surfaces with such steep angles are seldom the most likely failure surfaces, so in most situations we will not meet this problem. But it may become important where deep circular failure surfaces are analysed, e.g., for an embankment on a soft foundation. This problem is investigated in Section 6.4.

As a result of the findings, it was decided to add the option of using the modified Bishop factor of safety in addition to the simplified Janbu factor of safety for circular failure surfaces.

4.3.1 Program Modifications

The modification of the program was relatively simple. The Janbu factor of safety by the method of slices is calculated by iteration of the following equation

$$\Sigma \frac{A_1 - F A_2}{F + A_3} = 0 \quad (4.1)$$

where

$$A_1 = \frac{C'}{\cos \alpha} + \tan \phi' \{W (1 - k_v - k_h \tan \alpha)$$

$$- \frac{U_\alpha}{\cos \alpha} + U_\beta (\cos \beta + \tan \alpha \sin \beta)$$

$$+ Q (\cos \delta + \tan \alpha \sin \delta)\}$$

$$A_2 = W \{(1 - k_v) \tan \alpha + k_h\}$$

$$+ U_\beta (\tan \alpha \cos \beta - \sin \beta)$$

$$+ Q (\tan \alpha \cos \delta - \sin \delta)$$

$$A_3 = \tan \alpha \tan \phi'$$

[see Siegel (1975a)]

This equation is derived by taking the moment of the forces acting on the slices around infinity (overall horizontal force equilibrium).

If the moment is taken around an arbitrary point, the equation becomes

$$\Sigma \bar{y} \frac{A_1 - F A_2}{F + A_3} = 0 \quad (4.2)$$

where $\bar{y}$ is the vertical distance from the center of the bottom of a slice to the moment center.

If we, for a circular shaped failure surface, take the moment around the center of the circle, $\bar{y} = r \cos \alpha$ (see, e.g., Figure C.14), and Equation 4.2 becomes

$$\Sigma r \cos \alpha \frac{A_1 - F A_2}{F + A_3} = 0$$

or since $r = \text{constant}$,

$$\Sigma \cos \alpha \frac{A_1 - F A_2}{F + A_3} = 0 \quad (4.3)$$

which is the equation used for the modified Bishop factor of safety calculation.

A more detailed description and discussion of the equations and assumptions applied in the Bishop method as well as the Janbu method may be found in Chapter II of this report, by Siegel (1975a), and Wright (1969).

4.3.2 CIRCLE2 and SURBIS

To use the modified Bishop factor of safety option for a circular failure surface search, one should use the command word CIRCLE2 in place of CIRCLE. It may also be applied to a specified surface by using the command word SURBIS in place of SURFAC. To avoid erroneous use, a warning is printed out that this method should be applied only to failure surfaces that approximate a circular shape.

4.4 EXAMPLE PROBLEM FOR REVISED STABL PROGRAM

The problem analysed in Chapter VI, Section 6.2, is used to illustrate the data setup and output for the revised STABL program. Figure 6.4 shows the problem with soil parameters and positions of piezometric surfaces.

Table 4.1 shows the data setup for a sliding BLOCK2 analysis. Note that the unit weight of water (the second variable defined under the command word WATER) is specified as 0., which means that the default value of 62.4 pcf is used.

Table 4.1

Data Setup for Example Problem for Revised STABL Program

```

PROFIL
EXAMPLE PROBLEM
23 11
0. 46. 10. 46. 4
10. 46. 17. 42. 4
17. 42. 26. 42. 4
26. 42. 34. 46. 4
34. 46. 56. 45. 4
56. 45. 90. 56. 1
90. 56. 145. 75. 2
145. 75. 156. 76. 2
156. 76. 240. 105. 1
240. 105. 259. 108. 1
259. 108. 320. 108. 1
156. 76. 320. 78. 2
90. 56. 320. 58. 1
56. 45. 216. 38. 4
216. 38. 320. 42. 4
0. 36. 20. 36. 5
20. 36. 136. 29. 5
136. 29. 216. 32. 5
216. 32. 320. 28. 5
0. 22. 136. 22. 4
0. 17. 31. 17. 1
31. 17. 136. 22. 1
136. 22. 320. 22. 1
SOIL
5
125. 125. 0. 35. 0. 0. 1
125. 125. 0. 33. 0. 0. 1
125. 125. 0. 35. 0. 0. 1
122. 122. 320. 29. 0. 0. 2
122. 122. 740. 16. 0. 0. 2
WATER
2 0.
5
0. 45. 56. 45.
126. 51. 216. 47.
320. 47.
7
0. 45. 56. 45.
126. 51. 136. 60.
150. 64. 250. 112.
320. 110.
BLOCK2
50 2 50.
50. 29. 90. 26. 8.
140. 26. 280. 26. 6.

```

Table 4.2 is a listing of the corresponding output. The computer plot is shown in Figure 4.5.

This problem illustrates the use of the BLOCK2 sliding block search, and of the WATER command. An example of the CIRCL2 search can be found in Chapter VI, Section 6.4, with data setup in Appendix C, Table C.7.

4.5 SOME ADVICE IN THE USE OF STABL

This section is intended to give general advice in the use of the STABL program, and to discuss some of the questions and problems that frequently have been raised by users of the program.

First it should be emphasized that a computer program gives results that are no better than the input data. But let us assume that we have determined our problem parameters with reasonable accuracy, or at least with a known variability. How do we get the best results from the computer with the least amount of effort?

4.5.1 Preliminary Analysis - Stability Charts

Whether the problem is relatively simple or more complicated, it is usually a good time investment to try to determine an approximate solution by means of stability charts for homogeneous slopes as, for example, presented by Taylor (1937, 1948); see Figures 4.6 and 4.7.

A chart similar to Figure 4.6 was constructed based on STABL results from the analysis of homogeneous slopes presented in Chapter III. It was found, however, that charts constructed from equal contour lines of factors of safety $F = 2$ and $F = 3$ differed somewhat from the chart based on $F = 1$ contours. See Figure 4.8.

The STABL stability chart was compared to Taylor's chart (Figure 4.6). Both charts are superposed on Figure 4.9. It is seen that for

Table 4.2

Output of Example Problem

--SLOPE STABILITY ANALYSIS--
SIMPLIFIED JANBU METHOD OF SLICES
IRREGULAR FAILURE SURFACES

PROBLEM DESCRIPTION EXAMPLE PROBLEM

BOUNDARY COORDINATES

11 TOP BOUNDARIES
23 TOTAL BOUNDARIES

BOUNDARY NO.	X-LEFT (FT)	Y-LEFT (FT)	X-RIGHT (FT)	Y-RIGHT (FT)	SOIL TYPE BELOW BND
1	0	46.00	10.00	46.00	4
2	10.00	46.00	17.00	42.00	4
3	17.00	42.00	26.00	42.00	4
4	26.00	42.00	34.00	46.00	4
5	34.00	46.00	56.00	45.00	4
6	56.00	45.00	90.00	56.00	1
7	90.00	56.00	145.00	75.00	2
8	145.00	75.00	156.00	76.00	2
9	156.00	76.00	240.00	105.00	1
10	240.00	105.00	259.00	108.00	1
11	259.00	108.00	320.00	108.00	1
12	156.00	76.00	320.00	78.00	2
13	90.00	56.00	320.00	58.00	1
14	56.00	45.00	216.00	38.00	4
15	216.00	38.00	320.00	43.00	4
16	0	36.00	20.00	36.00	5
17	20.00	36.00	136.00	29.00	5
18	136.00	29.00	216.00	32.00	5
19	216.00	32.00	320.00	28.00	5
20	0	22.00	136.00	22.00	4
21	0	17.00	31.00	17.00	1
22	31.00	17.00	136.00	22.00	1
23	136.00	22.00	320.00	22.00	1

ISOTROPIC SOIL PARAMETERS

5 TYPE(S) OF SOIL

SOIL TYPE NO.	TOTAL UNIT WT. (PCF)	SATURATED UNIT WT. (PCF)	COHESION INTERCEPT (PSF)	FRICTION ANGLE (DEG)	PORE PRESSURE PARAMETER	PRESSURE CONSTANT (PSF)	PIEZOMETRIC SURFACE NO.
1	125.0	125.0	0	35.0	0	0	1
2	125.0	125.0	0	33.0	0	0	1
3	125.0	125.0	0	35.0	0	0	1
4	122.0	122.0	320.0	29.0	0	0	2
5	122.0	122.0	740.0	16.0	0	0	2

Table 4.2 (cont.)

2 PIEZOMETRIC SURFACE(S) HAVE BEEN SPECIFIED

UNITWEIGHT OF WATER = 62.40

PIEZOMETRIC SURFACE NO. 1 SPECIFIED BY 5 COORDINATE POINTS

POINT NO.	X-WATER (FT)	Y-WATER (FT)
1	0	45.00
2	56.00	45.00
3	126.00	51.00
4	216.00	47.00
5	320.00	47.00

PIEZOMETRIC SURFACE NO. 2 SPECIFIED BY 7 COORDINATE POINTS

POINT NO.	X-WATER (FT)	Y-WATER (FT)
1	0	45.00
2	56.00	45.00
3	126.00	51.00
4	136.00	60.00
5	150.00	64.00
6	250.00	112.00
7	320.00	110.00

A CRITICAL FAILURE SURFACE SEARCHING METHOD, USING A RANDOM TECHNIQUE FOR GENERATING SLIDING BLOCK SURFACES, HAS BEEN SPECIFIED.

THE ACTIVE AND PASSIVE PORTIONS OF THE SLIDING SURFACES ARE GENERATED ACCORDING TO THE RANKINE THEORY.

50 TRIAL SURFACES HAVE BEEN GENERATED.

2 BOXES SPECIFIED FOR GENERATION OF CENTRAL BLOCK BASE

LENGTH OF LINE SEGMENTS FOR ACTIVE AND PASSIVE PORTIONS OF SLIDING BLOCK IS 50.0

BOX NO.	X-LEFT (FT)	Y-LEFT (FT)	X-RIGHT (FT)	Y-RIGHT (FT)	WIDTH (FT)
1	50.00	29.00	90.00	26.00	8.00
2	140.00	26.00	280.00	26.00	6.00

Table 4.2 (cont.)

FOLLOWING ARE DISPLAYED THE TEN MOST CRITICAL OF THE TRIAL FAILURE SURFACES EXAMINED. THEY ARE ORDERED - MOST CRITICAL FIRST.

FAILURE SURFACE SPECIFIED BY 9 COORDINATE POINTS

POINT NO.	X-SURF (FT)	Y-SURF (FT)
1	52.43	45.16
2	73.46	32.77
3	86.96	22.61
4	211.84	26.69
5	215.84	31.99
6	219.47	38.17
7	229.39	57.21
8	240.15	77.03
9	256.03	107.53

*** 1.658 ***

FAILURE SURFACE SPECIFIED BY 9 COORDINATE POINTS

POINT NO.	X-SURF (FT)	Y-SURF (FT)
1	39.19	45.76
2	59.85	33.60
3	69.02	26.68
4	212.99	25.05
5	218.17	31.92
6	221.92	38.28
7	231.78	57.23
8	242.55	77.06
9	258.62	107.94

*** 1.678 ***

FAILURE SURFACE SPECIFIED BY 9 COORDINATE POINTS

POINT NO.	X-SURF (FT)	Y-SURF (FT)
1	29.76	43.88
2	45.78	34.44
3	55.09	27.43
4	218.26	23.08
5	224.74	31.66
6	228.83	38.62
7	238.55	57.29
8	249.33	77.14
9	265.39	108.00

*** 1.691 ***

Table 4.2 (cont.)

FAILURE SURFACE SPECIFIED BY 9 COORDINATE POINTS

POINT NO.	X-SURF (FT)	Y-SURF (FT)
1	32.78	45.39
2	51.99	34.07
3	59.31	28.56
4	220.04	23.65
5	226.04	31.61
6	230.20	38.63
7	239.89	57.30
8	250.67	77.15
9	266.73	108.00

*** 1.696 ***

FAILURE SURFACE SPECIFIED BY 10 COORDINATE POINTS

POINT NO.	X-SURF (FT)	Y-SURF (FT)
1	58.03	45.66
2	59.59	44.84
3	80.83	32.33
4	86.95	27.72
5	225.52	25.34
6	230.13	31.46
7	234.51	38.89
8	244.11	57.34
9	254.90	77.21
10	270.93	108.00

*** 1.697 ***

FAILURE SURFACE SPECIFIED BY 10 COORDINATE POINTS

POINT NO.	X-SURF (FT)	Y-SURF (FT)
1	58.60	45.84
2	60.60	44.80
3	81.88	32.27
4	89.76	26.32
5	230.83	26.86
6	234.18	31.30
7	238.77	39.09
8	248.29	57.38
9	259.08	77.26
10	275.09	108.00

*** 1.701 ***

Table 4.2 (cont.)

FAILURE SURFACE SPECIFIED BY 9 COORDINATE POINTS

POINT NO.	X-SURF (FT)	Y-SURF (FT)
1	39.15	45.77
2	59.81	33.60
3	67.11	28.09
4	228.59	28.53
5	230.78	31.43
6	235.19	36.92
7	244.78	57.35
8	255.57	77.21
9	271.59	108.00

*** 1.710 ***

FAILURE SURFACE SPECIFIED BY 9 COORDINATE POINTS

POINT NO.	X-SURF (FT)	Y-SURF (FT)
1	28.80	43.40
2	43.80	34.56
3	52.74	27.83
4	222.28	25.37
5	226.96	31.58
6	231.17	38.73
7	240.84	57.31
8	251.62	77.17
9	267.67	108.00

*** 1.714 ***

Table 4.2 (cont.)

FAILURE SURFACE SPECIFIED BY 10 COORDINATE POINTS

POINT NO.	X-SURF (FT)	Y-SURF (FT)
1	56.32	45.10
2	56.56	44.98
3	77.71	32.52
4	84.40	27.48
5	238.81	27.56
6	241.41	31.02
7	246.38	39.46
8	255.74	57.44
9	266.55	77.35
10	282.51	108.00

*** 1.725 ***

FAILURE SURFACE SPECIFIED BY 9 COORDINATE POINTS

POINT NO.	X-SURF (FT)	Y-SURF (FT)
1	35.03	45.95
2	55.57	33.85
3	59.55	30.85
4	212.46	24.48
5	218.07	31.92
6	221.81	38.28
7	231.68	57.23
8	242.44	77.05
9	258.51	107.92

*** 1.728 ***

EXAMPLE PROBLEM

50 SURFACES HAVE BEEN GENERATED

FIGURE 4.5 CALCOMP PLOT OF RESULT OF STABLE ANALYSIS

10 MOST CRITICAL OF SURFACES GENERATED
MINIMUM FACTOR OF SAFETY = 1.658

FIGURE 4.6 STABILITY CHART FOR SLOPES IN HOMOGENEOUS SOIL (AFTER TAYLOR, 1948)

FIGURE 4.7 STABILITY CHART FOR SLOPES IN
HOMOGENEOUS SOIL, $\phi = 0$
(AFTER TAYLOR, 1937)

FIGURE 4.8 STABILITY CHART OF THE TAYLOR TYPE
 FOR HOMOGENEOUS SLOPES, BASED ON
 RESULTS FROM STABL PROGRAM

FIGURE 4.9 COMPARISON BETWEEN STABL AND TAYLOR STABILITY CHART FOR HOMOGENEOUS SLOPES

slopes with slope angles $\beta < 50^\circ$, the STABL solution is conservative compared to Taylor's solution, particularly for low values of ϕ'_r , whereas Taylor is more conservative for steeper slopes. Taylor's chart is based on a friction circle method with assumed sinusoidal stress distribution along the shear surface. This stress distribution should give a more accurate result than the conventional friction circle method described in Chapter II, Section 2.2.4.

These charts may also be used to estimate the variability in F due to variation in problem parameters (see also Chapter III).

Where water pressures are present, the stability coefficients presented by Bishop and Morgenstern (1960) may be used.

The preliminary estimation of the factor of safety for a problem by simple means, provides a good base from which to start the STABL analysis. It gives a check on the computer output, and helps avoid some troubles that frequently occur when problems having very low safety factors (< 1) are analysed by STABL.

4.5.2 Use of STABL

The next step is to set up the data for the STABL analysis. The basic steps are described in the STABL User Manual [Siegel (1975b)], but some additional information may be useful.

4.5.2.1 How to Define Limitations for Search Routines

The simple stability charts can provide us with preliminary estimates. Again we look at Figure 4.6. The full lines on the chart represent cases where the most critical failure surfaces pass through the toe of the slope. If our problem is located here, we need only investigate such types of failures, hence the left as well as the right limitation point for

initiation of the failure surfaces should be specified as the x-coordinate to the toe of the slope.

The dashed line on the chart divides the problem into zone A and zone B. Zone A represents those problems where the critical failure surface does not extend below the toe of the slope: $\alpha \geq 0$ for the total failure surface, where α is the inclination of the failure surface with the horizontal.

If our problem lies in zone A, we can initially define our direction limitations accordingly. Since the default values used in the program are -45° for the clockwise direction limit, and $\beta = 5^\circ$ for the counterclockwise direction limit, the use of these values for problems located in zone A of Taylor's stability chart does not provide the most efficient search limitations. Instead we should specify, for example, the clockwise direction limit as 0° , and the counterclockwise as $\beta = 5^\circ$.

4.5.2.2 Clockwise and Counterclockwise Direction Limits

At this point it may be appropriate to discuss some of the confusion that has arisen with respect to the use of the term "clockwise" and "counterclockwise direction limits".

The terms are used relative to each other, not to the X-axis, whereas the angles are defined with respect to the X-axis.

An angle is defined positive counterclockwise from the positive X-axis, and negative clockwise from the positive X-axis, see Figure 4.10(a).

A counterclockwise direction limit does not have to be a counterclockwise angle and vice versa. See Figure 4.10 (b). Both angles are clockwise with respect to the positive X-axis, so they are both negative.

(a) Definition of positive and negative angles

FIGURE 4.10 ILLUSTRATION OF CLOCKWISE AND COUNTER CLOCKWISE DIRECTION LIMITATIONS FOR INITIATION OF FAILURE SURFACES

4.5.3 Strength along Soil Boundaries

If we do not know whether one soil is stronger than another, it may be tempting to specify a surface or a sliding block search on the boundary between two soil layers, assuming that the program will analyse the stability for both types of soils and give the most critical factor of safety.

It must be clearly understood that this is not the case. If a failure surface is specified or generated exactly on the boundary between two types of soil, the strength parameters of the soil above the failure surface will be used in the stability analyses, independent of which soil is the weaker. One should always specify a failure surface or a sliding block search totally above or totally below a soil boundary. If one does not know which case will be the most critical, both situations should be analysed.

4.5.4 Analysis Involving Earthquake Loadings

A slope may be analysed for the influence of earthquake forces by pseudo-static earthquake coefficients. Some uncertainty has existed with respect to this option. For example, why is the vertical earthquake coefficient defined positive upwards? What value for the cavitation pressure should be used, and how does it affect the stability?

4.5.4.1 Effect of Vertical Earthquake Loading on the Stability of a Slope

Let us first consider the case of dry soil. Specifying a vertical earthquake coefficient k_v has, for dry soil, exactly the same effect as changing the unit weight of the soil mass from γ to $\gamma(1 - k_v)$.

For granular soil, where the strength is solely dependent on the stress ($\sigma' \tan \phi'$, $c' = 0$), the factor of safety is independent of γ , so a vertical earthquake loading alone will neither decrease, nor increase the safety of the slope.

Where $c' \neq 0$, the factor of safety F is proportional to $\frac{c'}{\gamma h}$, see Figure 4.8; so a change in γ causes a change in F . F decreases with increasing γ , so the critical direction of the vertical earthquake loading is downward, hence k_v should be specified negative.

If part of the sliding surface extends below a water table, the result changes for the cases where $\phi \neq 0$. The earthquake loading is a dynamic force, momentarily applied. If therefore the sliding mass is submerged, the component of earthquake forces normal to the base of a slice will be carried by an increase or decrease in the pore water pressure. This means that a soil with strength defined in terms of c' and ϕ' , will act as a soil with $\phi = 0$ with respect to the earthquake forces, for the part of the soil that is under the water table. This again means that a granular soil ($c' = 0$) that is submerged will react exactly as a cohesive soil ($\phi = 0$) with respect to the earthquake forces, so k_v should be defined negative also in this case.

At this point it should be mentioned that the way the earthquake forces are transmitted to the pore water in the case of a partially or totally submerged slice is approximate. Figure 4.11 illustrates the principle. Since the Janbu procedure as well as the modified Bishop procedure satisfies vertical force equilibrium for individual slices, equation (b) in Figure 4.11 is the most correct. However, it has ΔS_r , the change in required shear strength, as an unknown, and cannot be

(a) Force equilibrium normal to base of slice:

$$\Delta U + K_H \sin \alpha + K_V \cos \alpha = 0$$

(b) Vertical force equilibrium:

$$\Delta U \cos \alpha + \Delta S_r \sin \alpha + K_V = 0$$

FIGURE 4.11

TRANSMISSION OF EARTHQUAKE
FORCES TO THE PORE WATER

solved directly with respect to the change in the pore pressure force ΔU . Hence equation (a), the projection of forces normal to the slice base, is used to give an approximate value for ΔU .

4.5.4.2 Cavitation Pressure

The pressure in the pore water between soil particles that are submerged can be negative; but it cannot go below the cavitation pressure, where water starts to evaporate. This pressure is found to be about - 1 atmosphere for granular soil, which is equivalent to - 2115 psf [Seed and Lee (1967)].

Cavitation is unlikely to occur. Only a large upward directed vertical earthquake force would be able to reduce the pore pressure to a cavitation pressure below - 1 atmosphere, and the critical direction of the vertical earthquake force was found to be downward.

Horizontal earthquake forces, on the other hand, would result in a decrease in pore water pressure for slices with positive α 's, when defined positive (outward directed), which is the critical direction, see Figure 4.12. However, this decrease in the pore water pressure is small (except for slices with $\alpha > 45^\circ$) compared to the component of the force parallel to the base of the slice, which acts as a driving force. It is most likely that this driving force will decrease the stability of the slope until the point of failure, long before the component normal to the base of the slice will create a pore water pressure that reaches the cavitation pressure.

If, however, the pore pressure does reach the cavitation pressure, the soil particles will have to "carry" the remaining unloading. This

Outward directed earthquake force (+)

$$K_H = W k_h, \quad W = \text{weight of slice}$$

FIGURE 4.12

CHANGE IN PORE WATER PRESSURE
DUE TO A HORIZONTAL EARTHQUAKE
FORCE

will result in a decrease in the effective normal stresses, and a decrease in the normal stress dependent strength (ϕ strength).

4.5.4.3 Illustrative Example

A slope 40 feet high, inclined 26.6° with the horizontal (slope 2/1), was analysed for the effect of vertical as well as horizontal earthquake loadings. Three different situations were considered.

In case A the strength of the soil is defined in terms of strength intercept c only ($\phi = 0$). In case B the strength of the soil is defined in terms of strength angle ϕ' alone ($c' = 0$); the soil is assumed dry. Case C is similar to case B, but a water table is introduced.

The slopes are first analysed for normal conditions with no earthquake forces, and then for five different combinations of earthquake loadings, see Table 4.3. Figure 4.13 shows the three slopes with the critical failure surfaces. The critical failure surface for each case remained the same or very close to the same for all analyses.

The results agree with the previous considerations. For cases A and C the factor of safety increases with a positive (upward directed) vertical earthquake loading and decreases for a negative (downward directed) vertical earthquake loading, whereas no change results for case B (dry granular soil).

A horizontal outward directed (positive k_h) earthquake loading reduces the stability drastically for all three cases, but most for case A (the $\phi = 0$ case).

A combination of horizontal and vertical earthquake forces does not give the same result as a direct superposition of the two forces individually acting, but the tendency is the same. An upward directed (positive)

Table 4.3

Stability Analysis of Slopes Subjected to Earthquake Forces*

Factors of safety	Case A	Case B	Case C
Earthquake loadings	$c = 1000\text{psf}$ $\phi = 0$	$c' = 0$, $\phi' = 35^\circ$ dry soil	water table
(1) $k_v = k_h = 0$	1.24	1.42	1.21
(2) $k_v = 0.1$, $k_h = 0$	1.38	1.42	1.33
(3) $k_v = -0.1$, $k_h = 0$	1.13	1.42	1.11
(4) $k_v = 0$, $k_h = 0.1$	0.86	1.12	0.98
(5) $k_v = k_h = 0.1$	0.92	1.09	1.05
(6) $k_v = -0.1$, $k_h = 0.1$	0.80	1.14	0.92
$\Delta(3)$ in % of (1)	- 9	0	- 8
$\Delta(4)$ in % of (1)	- 31	- 21	- 19
$\Delta(5)$ in % of (1)	- 26	- 23	- 13
$\Delta(6)$ in % of (1)	- 35	- 20	- 24

* The slopes are 40 feet high, inclined 2/1 ($\beta = 26.6^\circ$),
 $\gamma = 125\text{pcf}$.

Case A

Case B

Case C

FIGURE 4.13 CRITICAL FAILURE SURFACES FOR SLOPES ANALYSED FOR INFLUENCE OF EARTHQUAKE LOADINGS

vertical force increases the safety compared to the case where only a horizontal force is acting, and a downward directed (negative) vertical force decreases the safety. There is one exception, namely case B, the slope in granular dry soil. In this case a combination of upward and outward directed earthquake loadings was a little more critical (lower factor of safety) than the outward directed (horizontal) alone, and the outward and downward directed earthquake forces were slightly less critical than the outward directed alone. This does not appear logical, and may be due to some of the assumptions made in the analytical model, e.g., interslice shear forces are ignored.

4.5.4.4 Dynamic Earthquake Analysis

The method of earthquake analysis provided in the STABL program in terms of a pseudo-static loading, is the conventional approach. However, in recent years dynamic earthquake analyses have been introduced, which more accurately simulates the forces that are involved [Newmark (1965); Seed (1966)]. The stability in a dynamic earthquake analysis is assessed in terms of embankment (or slope) deformations, rather than a single value of the factor of safety. For more important structures, where the consequence of failure is great, this type of analysis should be used [Seed et al (1975)]. Dynamic analysis cannot be applied in the STABL program.

4.5.5 Misleading Factors of Safety

Most users of the STABL program will sooner or later experience an output with the comment that the factor of safety for a failure surface is misleading.

There are two reasons why a factor of safety calculated by STABL can be misleading.

A problem occurs when, for some reason, the iteration process for the factor of safety does not converge. This is usually associated with a factor of safety which is either very high or very low (below unity). However, in most cases STABL will handle factors of safety as high as 10 and as low as 0.6.

In one case, where a user had troubles with no convergence for a low factor of safety, the program was modified to define the initial value of the factor of safety for the iteration process to 1.2 instead of 1.5 (arbitrarily defined in the program), and this change made it work [Maunsell & Partners Pty. Ltd., Australia (1977)].

A misleading factor of safety may also occur if the denominator of the expression for the factor of safety ($F + \tan \alpha \tan \phi'$) is zero or negative. This may happen for deep failure surfaces, where the angle α (see Figure 4.12) has a relatively large negative value for some of the slices. The normal force on the base of a slice will thus become very large or negative. This implies an unreasonable state of stress, hence the result is not reliable.

The STABL program therefore always checks for the sign of ($F + \tan \alpha \tan \phi'$) for each slice, and if the expression turns negative, the message of a misleading factor of safety is printed, together with the coordinates of the failure surface in question, and the factor of safety. This factor of safety will often be either very high or very low. Frequently an asterisk will be printed, indicating that the value is either larger or smaller than the computer is programmed to handle.

The negative value of $(F + \tan \alpha \tan \phi')$ will often occur for problems having an actual factor of safety below unity. Hence an output with misleading factors of safety should be a warning that the slope may be unsafe. Whitman and Bailey (1967) suggests the requirement $[\cos \alpha + (\tan \phi' \sin \alpha / F) < 0.2]$ be used as a basis for disregarding a result.

In no case should a factor of safety that is labeled as "misleading" be used as a result. It could be close to an actual factor of safety, but it can also be very far from the true value.

Even if some of the factors of safety are misleading, STABL may give a correct result for the rest of the surfaces examined. These will be listed under the ten most critical surfaces. If some of these surfaces give a factor of safety of "500", this indicates that they are surfaces giving a "misleading" factor of safety. See also Appendix D, Section D.3.3.2.

If all results by STABL are misleading, another method of analysis has to be used.

V COMPARATIVE DESIGN STUDY

A comparison was made among results obtained by STABL and those obtained by other methods of analysis for an example design problem posed by Ladd (1975). All results in terms of factors of safety are summarized in Tables 5.4 and 5.5 (pp. 167-168).

5.1 DESCRIPTION OF PROBLEM

The problem is stated in Figure 5.1. A 30 feet high embankment for a four lane highway is to be constructed over 80 feet of medium to soft varved clay. The embankment will extend between bridge abutments, and a two-dimensional analysis is appropriate.

The design process includes a consolidation-settlement-preload analysis, influenced by drains to satisfy requirements of maximum allowable settlements and available construction time. However only the slope stability analyses are considered here.

5.2 TYPES OF ANALYSIS

Analyses are performed for the following situations

- (1) Undrained shear for end of construction stability.

This case assumes that no consolidation takes place during construction.

- (2) Drained shear for long term stability.

This case assumes that all excess pore pressure has disappeared, consolidation is completed.

FIGURE 5.1 SOIL PROFILE, GEOMETRY AND DESIGN CRITERIA FOR EMBANKMENT (After Ladd, 1975)

- (3) Partially drained conditions for end of construction stability. This case assumes that some consolidation takes place during construction, increasing the strength of the soil.

Undrained shear is often examined by total stress analysis while drained shear is examined by effective stress analysis. However, both total and effective stresses may in general be applied, regardless of drainage, although the results expressed in terms of factors of safety will be different for the two types of analysis unless the factor of safety equals one. This point is illustrated in Figure 5.2:

Total undrained shear strength is defined by

$$s_u = c_u + \sigma_f \tan \phi_u \quad (5.1)$$

For saturated clay $\phi_u = 0$, so $s_u = c_u$; s_u is normally defined as

$$s_u = q_f = 1/2(\sigma_1 - \sigma_3)_f \quad (5.2)$$

where $1/2(\sigma_1 - \sigma_3)_f$ is the maximum shear stress at failure. In this design problem, however, s_u is defined as

$$s_u = q_f \cos \phi' \quad (5.3)$$

where $q_f \cos \phi'$ is the maximum shear stress on the failure plane at failure.

Effective shear strength is defined by

$$s_{eff} = c' + \sigma_f' \tan \phi' \quad (5.4)$$

where σ_f' is the effective normal stress on the potential failure plane, and c' and ϕ' are the strength intercept and strength angle, respectively, in terms of effective stress. For normally consolidated clay

FIGURE 5.2 RELATIONSHIP BETWEEN TOTAL AND EFFECTIVE STRENGTH FOR A FACTOR OF SAFETY LARGER THAN ONE

$c' = 0$, hence $s_{eff} = \sigma'_f \tan \phi'$.

In the case where $F = 1$, $\sigma'_f = \sigma'_{ff}$, and

$$s_{eff} = c' + \sigma'_{ff} \tan \phi' = \tau_{ff} = q_f \cos \phi' = s_u \quad (5.5)$$

where σ'_{ff} is the effective normal stress on the failure plane at failure, and τ_{ff} is the shear stress on the failure plane at failure.

In design situations, however, we require $F > 1$. This does not change the value of s_u , but the effective strength will change since σ'_f is not the same as σ'_{ff} (corresponding to undrained failure) unless the pore pressure parameter $A_f = 1/2(1 - \sin \phi')$. For $A_f > 1/2 (1 - \sin \phi')$, s_u will be less than s_{eff} . Circle 1 (Figure 5.2) represents the effective stress conditions for a point on the potential failure plane. Assuming that a sample of the soil can be brought to the same state of effective stress and then sheared undrained, the sample will fail when the effective stress conditions given by circle 2 is reached. This gives a value of $s_u = q_f \cos \phi'$. However, an effective stress analysis would use the strength s_{eff} (see Figure 5.2) which is usually larger than s_u . The actual shear stress on the potential failure plane, τ_f , is independent of the type of analysis, hence total stress analysis will in most cases yield a lower factor of safety than effective stress analysis. The above considerations apply to the ideal case, where undrained and drained shear tests result in the same value of ϕ' . Frequently this is not the case. In this analysis, the drained and undrained direct simple shear tests gave values of ϕ' equal to 21° and 9.4° , respectively. This explains the very large difference in results by effective and total stress analysis (see Section 5.5.2.2).

5.3 METHODS OF ANALYSIS

In the report by Ladd (1975) two methods of analysis were applied: the simplified Bishop method with corrected field vane strengths (see Section 5.4.1), and the Morgenstern-Price analysis using SHANSEP* an-isotropic strengths (see Section 5.4.2).

5.3.1 The Simplified Bishop Method

The simplified Bishop method assumes a circular failure surface and applies the method of slices with the simplifying assumption that side forces between slices are zero. The method satisfies overall moment equilibrium, but not moment equilibrium for individual slices. It satisfies vertical force equilibrium, but not horizontal.

In Ladd's report the analyses are performed by a computer program ICES LEASE**. A grid system combined with a search routine is used to locate the center of the most critical circle.

5.3.2 Morgenstern-Price Analysis

The Morgenstern-Price (MP) method of slices analyses failure surfaces of any shape composed of straight line segments. Various assumptions can be made with respect to interslice forces, and the analysis satisfies complete equilibrium. Different failure surfaces have to be specified until the more critical surfaces are found. The line of thrust and the magnitude of interslice forces are examined to judge the reasonableness of the result (see also Appendix C.I). If either one is deemed unreasonable, a new set of side forces must be assumed until a satisfactory solution is found.

* Stress History And Normalized Soil Engineering Properties.

** Integrated Civil Engineering System - Lease Electronic Accounting System.

Such a procedure is rather time consuming, hence is only recommended for problems where the simplified Bishop circular arc does not provide reasonable results. This may be the case when dealing with non-homogeneous or anisotropic soil conditions.

5.3.3 Method Used in the STABL Program

The STABL program uses a modification of the simplified Bishop method of slices in which overall horizontal force equilibrium is satisfied instead of overall moment equilibrium.* This modification is introduced in order to be able to analyse irregular as well as circular failure surfaces.

In case of a circular failure surface, STABL will in most cases give a factor of safety somewhat lower than the simplified Bishop method, provided the same (or approximately the same) most critical failure surface is found.**

The advantage of the STABL program is most significant when dealing with cases that require failure surfaces of irregular shape. In this example design problem, the RANDOM and BLOCK search routines were used in connection with the SHANSEP anisotropic soil strength parameters where the Morgenstern-Price analyses were used in Ladd's report. BLOCK generates sliding block type failure surfaces. RANDOM generates surfaces of more irregular shape. The advantage of the STABL program is that it searches for the more critical failure surfaces, whereas the engineer has to conduct this search himself when using the MP-analysis.

* This method is also known as Janbu's simplified procedure (Wright, 1969).

** Exceptions, see Chapter VI, Problem 3 and 4, and Appendix C.II.

From the results obtained in this comparative study, it is seen that the STABL program gives results very close to those obtained by the MP-procedure.

5.4 SOIL STRENGTH PARAMETERS USED IN STABILITY ANALYSES

5.4.1 Corrected Field Vane Shear Strength

Bjerrum (1972) introduced the corrected field vane shear strength to evaluate the undrained stability of embankments constructed on soft clay foundations by circular arc analysis. The correction factor μ is based upon case studies of embankment failures and μ was plotted versus Plasticity Index (PI). However, many other case studies show a wide deviation from Bjerrum's recommended curve. Special problems arise when the clay is layered and anisotropic: Which PI should be used? How far is the circular arc from actual failure surfaces that would develop in these types of soils?

Instead of using Bjerrum's curve, the correction factor for the varved clay was evaluated based on case studies of embankment failures on varved clays.

A further check of the stability analysis performed by the simplified Bishop method and corrected field vane shear strengths was made by analysing the undrained situation with the Morgenstern-Price method and SHANSEP anisotropic shear strengths.

5.4.2 SHANSEP Anisotropic Shear Strength

The SHANSEP method is described by Ladd and Foott (1974). The method provides an evaluation of the undrained (and partially drained)

stress-strain-strength properties of clays and consists of three basic steps:

- (1) Determine the stress history of the clay foundation, i.e., variation with depth of the effective overburden stress σ'_{vo} , the maximum past pressure σ'_{vm} , and the overconsolidation ratio $OCR = \sigma'_{vm} / \sigma'_{vo}$.
- (2) Perform K_o consolidated-undrained tests on undisturbed samples to find the relationship between the undrained strength ratio s_u / σ'_{vc} and OCR for representative modes of failures (e.g., as shown on Figures 5.3 and 5.4).
- (3) Compute the variation in s_u with depth for the different modes of failure from the stress history and normalized strength data obtained in steps (1) and (2).

The method is applied to initial in situ undrained shear strengths and is also used to predict increase in strength due to consolidation. The anisotropy of the varved clay is taken into account by the modes of failure investigated in the testing program (Figure 5.3) and applied in the stability analysis.

5.5 RESULTS AND COMPARISONS

5.5.1 Preliminary Stability Analyses

Preliminary analyses were performed on the initial design cross-section (Figure 5.1) for undrained end-of-construction stability and drained long term stability using the simplified Bishop circular arc analysis with corrected field vane strengths. For the undrained analyses, various embankment heights were investigated with 2:1 and 4:1 slopes, in order to determine the maximum height that could be constructed

Data from Lacasse et. al. (1972)

Test $\sigma'_{vc} = 4.1 \text{ TSF}$, In situ $\sigma'_{vm} = 3.5 \pm \text{TSF}$

FIGURE 5.3 STRESS VS. STRAIN FROM $\overline{CK\sigma U}$ TESTS WITH DIFFERENT MODES OF FAILURE (After Ladd, 1975)

rapidly without consolidation (i.e., assuming no increase in strength due to consolidation).

For comparison, the CIRCLE search routine in STABL was used. However, only the design cross-section corresponding to a 30 feet embankment and 2:1 slopes was analysed (Figures 5.1 and 5.4). Data and results are summarized in Figures 5.4 and 5.5. Results show STABL to give a slightly lower factor of safety than the simplified Bishop method in the undrained analysis and about 5% lower values in the drained effective stress analyses.

The Morgenstern-Price method with SHANSEP anisotropic strength parameters was also applied to the undrained, end-of-construction stage, and showed very good agreement with the result obtained by the simplified Bishop method with corrected field vane strengths. The anisotropic strengths are given in Table 5.1 and correspond to the relationship shown in Figure 5.6.

The Morgenstern-Price method was modeled by the RANDOM and BLOCK procedures in the STABL program, where the RANDOM routine randomly generates irregular failure surfaces and analyses their stability, and the BLOCK routines randomly, but more restrictedly, generate sliding block (or wedge) modes of failure and analyses their stability. (For a detailed description of the surface generation routines, see Siegel, 1975a).

The results of Morgenstern-Price (MP) and STABL, RANDOM and BLOCK are given in Figure 5.7. The factors of safety for the STABL procedures are equal to or slightly lower than the one obtained by the Morgenstern-Price procedure. All failure surfaces lie within a band of 20 feet width.

FIGURE 5.4 INPUT DATA AND RESULTS OF PRELIMINARY, UNDRAINED, TOTAL STRESS ANALYSIS, CIRCULAR FAILURE SURFACES

FIGURE 5.5 DRAINED EFFECTIVE STRESS ANALYSIS OF EMBANKMENT, PRELIMINARY DESIGN

Table 5.1
Calculation of In Situ Undrained Shear Strength (After Ladd, 1975)

Elevation ft	Soil no.	σ'_{vc} ksf	σ'_{vm} ksf	OCR	PSA		PSP		DSS	
					$\frac{s_u}{\sigma'_{vc}}$	$\frac{s_u}{ksf}$	$\frac{s_u}{\sigma'_{vc}}$	$\frac{s_u}{ksf}$	$\frac{s_u}{\sigma'_{vc}}$	$\frac{s_u}{ksf}$
132.5	5	0.920	6.00	6.52	0.90	0.828	0.78	0.718	0.601	0.553
127.5	6	1.190	4.50	3.78	0.63	0.750	0.545	0.649	0.42	0.500
120	7	1.535	3.90	2.54	0.46	0.706	0.41	0.629	0.32	0.491
110	8	1.955	3.60	1.84	0.355	0.694	0.32	0.626	0.251	0.491
100	9	2.375	3.60	1.52	0.30	0.713	0.272	0.646	0.210	0.499
90	10	2.815	4.00	1.42	0.285	0.802	0.260	0.732	0.20	0.563
80	11	3.275	4.45	1.36	0.275	0.901	0.250	0.819	0.19	0.622
70	12	3.735	4.90	1.31	0.269	1.005	0.245	0.915	0.185	0.691
60	13	4.195	5.40	1.29	0.262	1.099	0.235	0.986	0.182	0.763

FIGURE 5.6 ANISOTROPIC UNDRAINED STRENGTH PARAMETERS FOR USE IN TOTAL STRESS ANALYSES (After Ladd, 1975)

FIGURE 5.7 UNDRAINED, TOTAL STRESS ANALYSIS OF EMBANKMENT WITH SHANSEP ANISOTROPIC STRENGTHS, PRELIMINARY DESIGN

Figure 5.8 shows the most critical failure surfaces and corresponding factors of safety found by STABL for the four different types of surfaces.

5.5.1.1 Frequency Distribution Curves for the Factor of Safety.

The STABL program normally prints out only the factors of safety for the ten most critical surfaces. In this study, however, a modification provided output of the factors of safety for all surfaces investigated. A histogram was made with intervals of ΔF equal to 0.1 ($F = 1.0$ includes values of F from 0.950 to 1.049, etc.) for each of the four procedures used in the undrained, total stress analyses, i.e., CIRCLE (Case 1A), RANDOM (Case 2B), BLOCK1 (Case 3B), and BLOCK2 (Case 4B). From the histograms, frequency distribution curves were drawn as shown in Figure 5.9. It is seen that each of the curves has a clear peak varying from $F = 0.9$ to 1.3 for the different cases.

From a statistical point of view, the peak value is the value that is most likely to occur, whereas the variance gives an indication of the scattering in the results. It is seen that Cases 1A and 4B have less variance than cases 2B and 3B. This may indicate that 1A and 4B surfaces are more likely than 2B and 3B surfaces. For a truer comparison, only B cases should be compared with each other, as soil parameters are changed from Case A to Case B. However, the curves are too close to each other to justify specific conclusions with respect to individual behavior.

The frequency distribution curve can also be interpreted in terms of likelihood of failure. For the CIRCLE case, 9 out of 100 surfaces have a factor of safety less than 1.0, which means that the probability

COMPARATIVE DESIGN STUDY - EMBANKMENT ON
VARVED CLAY, UNDRAINED SHEAR ANALYSIS

MOST CRITICAL CIRCLE, IRREGULAR AND SLIDING
BLOCK SURFACES FOR THE PRELIMINARY
DESIGN CROSS-SECTION

FIGURE 5.8 DIFFERENT TYPES OF CRITICAL FAILURE SURFACES GENERATED BY STABL

FIGURE 5.9 FREQUENCY DISTRIBUTION CURVES
FOR THE FACTOR OF SAFETY,
CASES 1A, 2B, 3B, 4B

of failure is 9 out of a 100 or 9%. When dealing with probability, however, many more things have to be taken into consideration. The soil parameters should not be handled as exact values, but distributed around mean values with a certain variance that can be estimated from laboratory tests. See for example the work by Athanasiou-Grivas (1976). This aspect is not considered here.

5.5.2 Final Design Stability Analyses

From preliminary stability analyses it was found necessary to improve the stability of the design cross-section by placing berms 15 feet high and approximately 60 feet wide on both sides of the embankment. Settlements were then calculated, and it was found that to compensate for settlements, the height of the fill should be increased by 8 feet. To accelerate the rate of settlements, sand drains should be installed under the embankment and additional load applied by a temporary surcharge. It was found, however, that for a minimum F of 1.2, the maximum safe height is only 42 feet. This height would still lead to excessive post construction settlements if the required time schedule were followed. It was suggested that more fill could be placed if the construction was carefully monitored with a fairly extensive program of field measurements. For this example, however, the stability analyses of the final design is based upon the 42 feet of fill, with stability berms as mentioned above. Two situations are investigated for the final design: the partially drained condition and the long term, totally drained condition.

5.5.2.1 Partially Drained Stability Analyses

The 42 feet high embankment with stability berms was investigated for a partially drained condition corresponding to 95% consolidation. At this stage almost 8 feet of settlement has taken place, see Figure 5.10. The stability analysis was carried out as a total stress analysis with Morgenstern-Price wedges and SHANSEP anisotropic strength values increased by consolidation. See plots of strengths in Figure 5.11 and Table 5.2. STABL was applied with all four types of analyses and anisotropic soil parameters.

Results of MP and STABL analyses are given in Figure 5.12. STABL's four procedures (Cases 1E.1 through 4E.1) give results from + 6% to - 5% of that obtained by the MP analysis.

5.5.2.2 Long Term Stability Analyses

For the long term stability the embankment is analysed for a final design soil profile corresponding to 8 feet of settlement and a final height of 30 feet above the ground, which means that 4 feet of material is removed compared to the partially drained stage.

Conventional drained effective stress analyses were performed with Bishop and STABL circular arc failure surfaces. Data and results are given in Figure 5.13. Results obtained by STABL are approximately 13% less than results by the Bishop method.

Total stress analyses were also applied with SHANSEP anisotropic strengths corresponding to stresses under the embankment at 100% consolidation, see Figure 5.14 and Table 5.3. Again MP and STABL's four types of analysis were applied. Figure 5.15 shows the results. The factors of safety obtained with the four procedures of STABL (Cases 1E.2 through

FIGURE 5.10 ESTIMATED SOIL PROFILE AND CROSS-SECTION FOR EMBANKMENT ON VARVED CLAY AFTER CONSTRUCTION

FIGURE 5.11 SHANSEP STRENGTH DATA FOR TOTAL STRESS ANALYSES AT 95% CONSOLIDATION (After Ladd, 1975)

Table 5.2
 s_u in Zones A, B, C and D* for 95% Consolidation (psf)

Elevation ft.	Soil No.	A PSA	B PSA	B DSS	C PSA	C DSS	D DSS	D PSP
132.5	5	1245	1210	871	1175	825	675	950
127.5	6	1190	1115	803	950	680	575	775
120	7	1230	1140	821	950	680	510	650
110	8	1287	1200	864	1030	740	495	625
100	9	1345	1260	907	1117	800	510	655
90	10	1400	1325	954	1200	860	562	730
80	11	1460	1390	1001	1285	922	625	820
70	12	1515	1455	1048	1370	980	690	920
60	13	1575	1525	1098	1450	1045	760	1025

* Zone D is only 18% consolidated as no sand drains are installed here, s_u in Zone D from Table 5.1 (in situ s_u).

FIGURE 5.12 TOTAL STRESS ANALYSES OF EMBANKMENT WITH BERMS, PARTIALLY DRAINED

FIGURE 5.13 DRAINED EFFECTIVE STRESS ANALYSES OF FINAL EMBANKMENT WITH BERMS

FIGURE 5.14 SHANSEP STRENGTH DATA FOR TOTAL STRESS ANALYSES AT 100% CONSOLIDATION (After Ladd, 1975)

Table 5.3
 s_u in Zones A, B, C and D for 100% Consolidation (psf)

Elevation ft	Soil no.	A PSA	B PSA	B DSS	C PSA	C DSS	D DSS	D PSP
132.5	5	1240	1225	882	1160	840	775	975
127.5	6	1225	1125	810	1012	710	625	825
120	7	1275	1165	839	970	680	560	750
110	8	1345	1230	886	1040	740	580	770
100	9	1400	1300	936	1125	800	650	870
90	10	1465	1360	979	1205	860	725	960
80	11	1520	1425	1026	1285	925	795	1060
70	12	1565	1500	1080	1370	980	870	1155
60	13	1613	1570	1130	1455	1050	940	1250

FIGURE 5.15 TOTAL STRESS ANALYSES OF FINAL EMBANKMENT WITH BERMS, 100% CONSOLIDATED

4E.2) varied from + 4% to - 7% of that obtained by the MP analysis.

Total stress analysis is not normally used to investigate long term stability, where it is assumed that the embankment foundation has consolidated 100%. A reason for using total stress analysis in this case, is the large difference between the factor of safety by the total stress analysis and the effective stress analysis (1.34 vs. 2.25). Also, it is believed that if a failure occurred, it would occur rapidly and develop excess pore pressures. Thus even though the embankment was almost 100% consolidated, an undrained shear would occur, and the undrained total stress analysis is considered most appropriate.

5.6 SUMMARY AND CONCLUSION

Results of stability analyses obtained by STABL for a design example problem (Figure 5.1) were compared with results obtained by programs based on the simplified Bishop method and Morgenstern-Price procedures. The results are summarized in Tables 5.4 and 5.5. Most of the cases investigated by STABL are documented with graphical computer plots in Appendix B.

For the same type of failure surfaces, STABL gave lower values of the factor of safety than the other programs. The difference was in most cases less than 10%, with a maximum of 16% (0.88 vs. 1.05 for the preliminary design, undrained total stress analysis). Considering the uncertainty in the input data, this difference is tolerable.

For problems with complicated soil conditions, STABL is recommended compared to the other programs, because STABL utilizes the computer more in the search for the more critical failure surfaces. STABL usually gives a factor of safety on the conservative side (see Section

Table 5.4

Results of Stability Analyses from 'IT-Report (After Ladd, 1975)

Factors of Safety		Failure model	
		Isotropic soil	Anisotropic soil (SHAUSEP)
Case	Type of Analysis	Circular arc mod. Bishop	Sliding block Morgenstern-Price
Preliminary design	Undrained (end-of-const.)	0.96	0.95*
	Drained (long term)	1.64 / 1.72	
Final design	Total stress		1.21
	Effective stress		1.44
	Total stress	2.64 / 2.78	
	Effective stress		

* A later analysis gave 1.05 (correspondence with Ladd, 1976).

Table 5.5
Results of Stability Analyses performed with STABL

Factors of Safety			Failure model			
			Isotropic soil	Anisotropic soil (SHANSEP)		
Case		Type of analysis	CIRCLE	RANDOM	BLOCK1	BLOCK2
Preliminary design	Undrained	Total stress	0.94	0.95	0.92	0.88
	Drained	Effective stress	1.56/1.63	1.56*	1.48*	1.47*
Final design	95% cons.	Total stress	1.28**	1.24	1.19	1.15
		Total stress	1.50**	1.44	1.38	1.34
	Drained	Effective stress	2.29/2.42			2.25/

* with SHANSEP anisotropic effective strength parameters from Table 5-12, MIT-report.
 ** results for anisotropic soil.

Note: No account has been made for increase in γ_{clay} due to consolidation in the Final Design Analyses.

2.3). If for any reason it is desirable to determine a "more precise" value of the factor of safety, other methods of slices may be incorporated in the STABL program (see Chapter VI; the modified (or simplified) Bishop method was later introduced as a permanent option in the STABL program, applicable to circular failure surfaces, see Chapter IV).

Differences between factors of safety for the different kinds of failure surfaces generated by STABL is less significant in this example problem than in the example with a weak layer of soil investigated in Chapter III. The largest difference (12%) is between CIRCLE and BLOCK2 for the total stress analysis.

VI COMPARISON OF STABL WITH EXISTING PROCEDURES FOR SLOPE STABILITY
ANALYSIS USED BY INDIANA STATE HIGHWAY COMMISSION (ISHC)

Four problems from the Division of Materials and Tests, ISHC, have been analysed for stability by existing ISHC procedures and by STABL.

The first problem involves analyses by

- a) the computer program NYSTAB* which is based on circular arc failure surfaces and the modified Bishop method,
- b) hand calculated sliding wedge analysis, and
- c) STABL.

The second and third problem involves analyses by

- a) a computer program used by ATEC** in which circular arc failure surfaces are analysed by the modified Bishop method, and
- b) STABL and modifications of STABL.

The last problem is solved by

- a) NYSTAB,
- b) STABL and modifications of STABL, and
- c) hand calculation.

* NYSTAB - New York State, Department of Transportation, Slope Stability Analysis.

** ATEC = American Testing and Engineering Corporation, Indianapolis, Indiana.

6.1 PROBLEM 1

The problem is presented in Figure 6.1. An embankment of granular fill is resting on a foundation of 28 feet of granular material underlain by clay of medium to low strength.

The factor of safety for the embankment slope alone is given by $F = \frac{\tan \phi'}{\tan \beta}$, where ϕ' is the effective strength angle, and β is the angle of the slope. In this case $\phi' = 35^\circ$, $\tan \beta = 32/56$, hence $F = 1.23$. This result was also found by STABL, circular search with no restrictions on initial angle, the critical failure surface being a shallow circle.

Stability with respect to the underlying clay soil was analysed with forced deep failure surfaces, both circle and sliding block shapes. The result from NYSTAB is $F_{\min} = 1.32$ for a deep circle. Hand calculated sliding block analysis gave $F_{\min} = 1.26$ for the deepest surface analysed by this method (medium depth), and STABL gave $F_{\min} = 1.16$ for a deep sliding block. All sliding blocks have approximately horizontal bases and Rankine active and passive failure zones.

Figures 6.2 and 6.3 show the failure surfaces investigated and corresponding factors of safety. All results are summarized in Table 6.1. STABL, sliding block search, gives a factor of safety 6% lower than the one found by hand calculation for the same depth (1.18 vs. 1.26), whereas the deeper sliding block found by STABL is more critical (8% lower, 1.16 vs. 1.26).

6.2 PROBLEM 2

The soil profile and strength data for problem 2 are shown in Figure 6.4. An embankment of granular material is constructed on a

FIGURE 6.1 SOIL PROFILE AND STRENGTH DATA FOR PROBLEM 1

FIGURE 6.2 CRITICAL FAILURE SURFACES FROM ANALYSIS BY NYSTAB-CIRCULAR ARC, AND HAND CALCULATED SLIDING WEDGES

FIGURE 6.3 CRITICAL FAILURE SURFACES FROM ANALYSIS BY STABL, CIRCLE, RANDOM AND BLOCK 2

Table 6.1

Comparative Study, Problem 1
Factor of Safety for Different Methods of Analysis

Type of Failure Surface	Type of Analysis			
	NYSTAB	Hand calculation	STABL specified surface*	STABL search
Shallow circle	1.45			1.24
Deep circle	<u>1.32</u>		1.66**	1.43
Shallow wedge		1.41	1.52	1.46
Medium wedge		<u>1.26</u>	1.31	1.18
Deep wedge				<u>1.16</u>
Random surface				1.23
Max. Diff. comp. to NYSTAB	0%	-4%		-12%

* specified surface is identical with the one found by NYSTAB or hand calculation.

** This result seems large, see also Problem 4.

foundation of somewhat clayey material, introducing an excess pore water pressure in this material corresponding to piezometric surface # 2. The free groundwater table is represented by piezometric surface # 1. The distance between piezometric surface # 1 and # 2 represents the excess pore water pressure induced by the embankment in the clayey foundation.

The ATEC program with modified Bishop circular arc analysis gave for the three circles (1, 2 and 3) values of F from 1.86 to 2.20, see Figure 6.5.

STABL was modified to handle more than one water surface (see Chapter IV), and factors of safety were obtained for the three circles from the ATEC analyses as well as for STABL circular search and sliding block searches.

The circular arc failure surface proved to be reasonable for this problem (a sliding block search gave a factor of safety only 2% lower than the circle search). To extend the comparison between different methods of analysis, the STABL program was modified to calculate the factor of safety by

- 1) the modified Bishop method, and
- 2) Spencer's method.

The Bishop method required only a minor change in the subroutine FACTR of the program, whereas the Spencer method required a complete change of this subroutine. The source for the Spencer method was Wright (1969). See Appendix C for a listing of subroutine FACTR by both methods. Later, the modified Bishop method was introduced as a permanent option in the STABL program, applicable to circular failure surfaces (see

**FIGURE 6.5 STABILITY ANALYSIS BY ATEC AND STABL,
CIRCULAR ARC**

Chapter IV). The FACTR subroutine used in the present STABL program is included in the listing of the program, Appendix E.

The results of the different methods of analysis are summarized in Table 6.2. STABL with the normal procedure of factor of safety calculation (Janbu's simplified procedure) gives the most conservative values of F , with a minimum for a circular arc 11% lower than the ATEC minimum value (1.66 vs. 1.86). STABL with the modified Bishop procedure gives the same value of $F_{\min}$ as the ATEC method, but for a different circle (see Figure 6.5). STABL with Spencer's procedure gives a value slightly below the ATEC $F_{\min}$ value, also for a different circle. The maximum difference in results occurs between STABL, simplified Janbu procedure, sliding block search, and the ATEC solution for a circular arc, namely 12% (1.63 vs. 1.86). It should be noted, however, that the result for the sliding block analysis with Spencer's procedure is 16% larger than the result for the normal STABL procedure (1.95 vs. 1.63). This may indicate that the factor of safety by STABL is too low for the sliding block analysis. The sliding block failure surfaces are shown in Figure 6.6.

For the Spencer result, circle 5, the lines of total and effective thrust are shown in Figure 6.7. They look reasonable except for the right termination. Also, interslice factors of safety with respect to shear have been calculated. The minimum value was 2.12, compared to an overall factor of safety $F = 1.83$ for the sliding surface, indicating that the result is reasonable. See Appendix C for calculations of effective line of thrust and interslice factors of safety, and for more information about the influence of the line of thrust on the factor of safety.

Table 6.2

Comparative Study, Problem 2

Factor of Safety for Different Methods of Analysis

Surface	Method of Analysis			
	1 ATEC mod.Bishop Circ. arc	2 STABL	3 STABL mod.Bishop Circ. arc	4 STABL Spencer
Circle 1	<u>1.86</u>	1.75	1.95	1.95
" 2	2.13	1.80	2.16	2.15
" 3	2.20	1.75	2.01	1.98
" 4		<u>1.66</u>	1.88	1.85
" 5		1.68	<u>1.86</u>	<u>1.83</u>
BLOCK1-1		2.01		
" 1-2		<u>1.63</u>		1.95
BLOCK2-1		2.09		
" 2-2		1.64		2.01
Max. Diff. *	0%	-12%	0%	- 2%

Circles 1, 2 and 3 from ATEC analysis,

Circle 4 from STABL, circular search, case 2,

Circle 5 from STABL, circular search, cases 3 & 4.

BLOCK1-1 and 2-1 from STABL, sliding block search in
soil layer no. 4, case 2,

BLOCK1-2 and 2-2 from STABL, sliding block search in
soil layer no. 5, case 2.

* Between minimum values, compared to method 1 (ATEC-solution).

FIGURE 6.6 SLIDING BLOCK ANALYSES, STABL AND STABL
W/SPENCER

FIGURE 6.7 STABILITY ANALYSIS BY STABL, SPENCER METHOD.
LINE OF THRUST

6.3 PROBLEM 3

Figure 6.8 shows the problem. An embankment supported on one side by a retaining structure is investigated for a total failure.

The same procedures as for problem 2 were applied to this problem, however, the Spencer method did not converge for most of the failure surfaces investigated. Even with convergence, a further check showed the result to be incorrect. See Appendix C, Tables C.11 and C.12, and Figure C.13.

The results for circular arc analysis by ATEC and STABL are shown in Figure 6.9. For the ATEC failure surface, STABL gives a factor of safety 7% lower than the ATEC value (2.15 vs. 2.32), whereas a circular search by STABL results in a failure surface having a STABL factor of safety 16% lower than the ATEC value. If the safety factor is calculated by the modified Bishop method, the same two surfaces result in factors of safety 1% to 3% higher than the ATEC value. Hence the values found by the normal STABL procedure are somewhat low. Even lower values are found by a STABL sliding block search, namely as low as $F = 1.54$, which is 34% lower than the ATEC minimum (see Figure 6.10).

In applying the Spencer procedure to the same failure surfaces, all cases except one did not converge in the iteration process for the factor of safety calculation. The one case that did converge gave a safety factor of 2.80, where a value of 1.65 was obtained by STABL. The Spencer result was checked by calculation of interslice factors of safety with respect to shear. These were found to be less than or equal to 1.4 (see Appendix C, Table C.12), hence the value of 2.80 is not correct. The results of the analyses are summarized in Table 6.3.

FIGURE 6.8 SOIL PROFILE AND STRENGTH DATA FOR PROBLEM 3

FIGURE 6.10 STABILITY ANALYSIS BY STABL, SLIDING BLOCK FAILURES

Table 6.3

Comparative Study, Problem 3

Factor of Safety for Different Methods of Analysis

Surface	Method of Analysis			
	1 ATEC mod.Bishop Circ. arc	2 STABL	3 STABL mod.Bishop Circ. arc	4 STABL Spencer
Circle 1	<u>2.32</u>	2.15	2.38	-
" 2		1.95	<u>2.34</u>	-
BLOCK2-1		1.72		-
" -2		1.63		-
" -3		1.80		-
" -4		1.65		2.80**
" -2*		<u>1.54</u>		-
Max. Diff.	0%	- 34%	1%	

* Result for search with 50 surfaces instead of only 25.

** Calculation of interslice factor of safety gave $F \leq 1.4$,
so 2.80 is not correct.

6.4 PROBLEM 4

The problem with soil profile and strength data is shown in Figure 6.11. An embankment of cohesive material, 9 feet high, is built on a relatively soft foundation. The strength of the foundation decreases with depth until a firm strata is reached at approximately 13 feet below the ground surface.

A search with the NYSTAB program gave a factor of safety $F = 1.41$ for a deep circular failure surface (circle 1, Figure 6.12). This surface was analysed by the STABL program, resulting in a factor of safety $F = 2.21$, 57% higher than the result by NYSTAB. A circular search with STABL gave $F = 1.74$, while specifying the critical circle according to Taylor (from Figure 4.7, page 120), for a similar slope geometry, but homogeneous soil with $\phi = 0$ (circle 2, Figure 6.12), resulted in a STABL factor of safety $F = 1.70$. This value is still 21% larger than the result obtained with the NYSTAB program. Because of the large difference in results, and the fact that the STABL program gave an unconservative safety factor, the case was further investigated.

Because a total strength analysis is applied in this problem, it could easily be hand calculated for a specific circular failure surface by the $\phi = 0$ method. This method gave a factor of safety $F = 1.45$ for circle 1 (Figure 6.12) confirming the NYSTAB result. Finally, STABL with the modified Bishop method was applied to the same failure surface, resulting in a factor of safety $F = 1.55$, again much closer to the NYSTAB result than STABL with the normal (simplified Janbu) procedure.

The fact that the simplified Janbu procedure, normally used in STABL, in this case gives a higher factor of safety than the modified

FIGURE 6.11 SOIL PROFILE AND STRENGTH DATA FOR PROBLEM 4, TOTAL STRESS ANALYSIS ($\phi = 0$)

Bishop procedure, contradicts Carter's conclusion (1971) that the factor of safety reaches a minimum value when the moment center (y) goes towards infinity.

Taking Carter's equation for the factor of safety as a function of the y -coordinate of the moment center, and applying $\phi = 0$ and $c =$ a constant, the factor of safety F_c can be expressed explicitly as a function of strength intercept c , unit weight γ , geometry of slope and failure surface, and moment center (y). Replacing the summation over slices with integration over a circular failure surface, a mathematical expression is obtained for F_c , and $\frac{\partial F_c}{\partial y}$ can be found. Derivation of these expressions are included in Appendix C.II. It is found that $\frac{\partial F_c}{\partial y}$ goes to 0 (zero) for $y \rightarrow \infty$, hence F_c reaches a minimum or a maximum for $y \rightarrow \infty$, depending on the sign of $\frac{\partial F_c}{\partial y}$.

F_c and $\frac{\partial F_c}{\partial y}$ were calculated for different slopes and different circular failure surfaces. The results are illustrated in Figures 6.13 through 6.19. Defining $F_\infty = \lim_{y \rightarrow \infty} F_c$, it is found that $\frac{\partial F_c}{\partial y}$ is positive, and $F_\infty = F_c^{\max}$ for circular failure surfaces that intersect the ground surface with a steep angle at the top of the slope ($\alpha_0 > 60^\circ$ to 80° , see Figure 6.12). For small angles of intersection, $\frac{\partial F_c}{\partial y}$ is negative and $F_\infty = F_c^{\min}$.

Since F_0 , the factor of safety for moment taken around the center of the circular failure surface (corresponding to $y = y_0$), in the case of $\phi = 0$, represents a mathematical correct solution to the problem, comparison was made between the values of F_∞ and F_0 . Figure 6.17 shows F_∞ and F_0 for homogeneous soil conditions (strength of embankment soil, c_E , equals strength of foundation soil, c_F). It is seen that whereas

FIGURE 6.14 FACTOR OF SAFETY F_c vs. MOMENT CENTER y FOR VARIOUS θ_0 , $r = 2.5h$, $c_E = 2c_F$

FIGURE 6.15 FACTOR OF SAFETY F_c vs. MOMENT CENTER y FOR VARIOUS θ_o , $r=5h$, $c_E=2c_F$

FIGURE 6.16 FACTOR OF SAFETY F_c VS. MOMENT CENTER y FOR VARIOUS θ_0 , $r = 2.5h$, $c_E = 0.5c_F$

FIGURE 6.17 STABILITY NUMBER $\frac{F_c \gamma h}{c}$ VS. θ_0 .
 FOR VARIOUS RADII, CIRCULAR
 FAILURE SURFACES, $c_E = c_F$, $\phi = 0$

FIGURE 6.18 STABILITY NUMBER $\frac{F_c \gamma h}{c}$ VS. θ_o FOR VARIOUS RADII, CIRCULAR FAILURE SURFACES, $c_E = 2c_F$, $\phi = 0$

FIGURE 6.19 STABILITY NUMBER $\frac{F_c \gamma h}{c}$ VS. θ_0 FOR VARIOUS RADII, CIRCULAR FAILURE SURFACES, $c_E = 0.5 c_F$, $\phi = 0$

$F_{\infty} = F_c^{\min}$ is only slightly less than the corresponding F_0 , $F_{\infty} = F_c^{\max}$ may be considerably larger than the corresponding F_0 , and actually goes towards infinity for α_0 approaching 90° ($\theta_0 = 0$). It is seen that $F_{\infty} = F_c^{\max}$ for $\theta_0 < 10^\circ$ corresponding to $\alpha_0 > 80^\circ$ ($\alpha_0 = 90^\circ - \theta_0$).

If the soil is inhomogeneous, with the slope material being stronger than the foundation material, e.g., an embankment on a soft foundation, as in this problem, the result by the simplified Janbu method (F_{∞}) deviates even more from the mathematical correct solution (F_0). For the embankment soil being twice as strong as the foundation soil ($c_E = 2c_F$), $\frac{\partial F}{\partial y}$ is positive and $F_{\infty} = F_c^{\max}$ for $\alpha_0 > 60^\circ$, and the difference between F_{∞} and F_0 increases significantly as shown in Figure 6.18.

If, on the other hand, the foundation soil is stronger than the embankment soil, F_{∞} and F_0 are almost identically, except for very small values of θ_0 ($< 5^\circ$ for the foundation soil being twice as strong as the embankment soil, see Figure 6.19). However, normally the embankment soil will be the strongest.

The conclusion is that the simplified Janbu method cannot be relied upon where deep failure surfaces intersect the ground surface at the top of the slope at angles steeper than $\alpha_0 = 60^\circ$, coincident with a relative large total strength* in the zone of intersection.

In the cases of circular shaped failure surfaces, F_0 provides a reliable solution. For this reason the STABL program was modified to include the option of using the modified Bishop method (F_0) for circular failure surfaces, see Chapter IV. Since F_0 is much less sensitive to

* In case of effective stress analysis, a relatively large strength intercept c' .

variations in α_0 , it is recommended that the modified Bishop method is used for circular failure surfaces in general.

A sliding block search with STABL gave a factor of safety $F = 1.42$, almost identical with the NYSTAB result. This confirms that the Janbu procedure normally used in the STABL program is still reliable for failure surfaces where α_0 is smaller than approximately 60° . The sliding block has $\alpha_0 = 45^\circ$.

However, STABL with the simplified Janbu procedure should not be exclusively relied upon for analyses of non-circular failure surfaces, intersecting the ground surface at the top of the slope with a steep angle, coincident with a relatively large total strength in the zone of intersection.

Results of the different analyses are summarized in Table 6.4.

6.5 CONCLUSION OF COMPARATIVE ANALYSIS

The comparisons between STABL results and results obtained by other methods of stability analysis show that STABL generally provides solutions that are somewhat conservative. However, the difference is usually less than 10%. This is acceptable when compared to the much larger variation in the factor of safety that may result from variable soil strength data, see Chapter III.

In one case, however, a STABL sliding block surface gave a result 34% lower than a circular surface analysed with the modified Bishop method. This result could not be verified, as the Spencer procedure did not converge for this failure surface, and the Bishop method only applies to circular surfaces.

Table 6.4

Comparative Study, Problem 4

Factor of Safety for Different Methods of Analysis

Type of failure surface	Type of Analysis				
	NYSTAB (mod. Bishop)	STABL simp. Janbu	STABL mod. Bishop	$\phi = 0$	Hand Calculated simp. Janbu mod. Bishop
Circle 1	<u>1.41</u>	2.21	1.55	1.45	
Circle 1a**		2.49	1.53		2.49
Circle 2		1.70	<u>1.41</u>		1.53
CIRCLE Search		1.74	1.43		
BLOCK2-Search		<u>1.42</u>			
Max. diff.*	0%	1%	0%	3%	

* Between minimum values, compared to NYSTAB solution

** Circle 1a is the same failure surface as Circle 1, but specified with different line segments.

An exception to the general conservative tendency are the cases of deep failure surfaces in soils with strength defined mainly in terms of a strength intercept c (or c'). In such cases the simplified Janbu method used in STABL may give erroneous and unconservative results, and should not be used.

For the general conservative solutions, Janbu (1956) has introduced a correction factor for the factor of safety obtained by the simplified Janbu method, see Figure 6.20. These correction factors were applied to the results presented in Chapter II, Table 2.3. They showed reasonable good agreement, although the corrected factor of safety seemed to be slightly on the unconservative side compared to the Bishop solution. However, for the non homogeneous soil conditions analysed in Chapter V and in this chapter, the correction factors did not seem to work.

A more thorough analysis is required to determine the relationship between type of failure surface, soil profile and strength properties, and the conservatism of the simplified Janbu method of slices.

F_0 = Factor of safety by Janbu's procedure

F = Corrected factor of safety

FIGURE 6.20

CORRECTION FACTORS FOR THE
SIMPLIFIED JANBU PROCEDURE
(AFTER JANBU et al., 1956)

VII SUMMARY AND RECOMMENDATIONS FOR FUTURE WORK

7.1 WORK AND RESULTS

The present study is a continuation of a previous study by Siegel (1975), dealing with computerized slope stability analysis. All investigations are related to a computer program, STABL, developed by Siegel. STABL performs analyses of general slope stability problems.

7.1.1 Parametric Study

A parametric study (sensitivity analysis) was performed on the STABL program to assess the precision with which input data need to be defined. The results of this study show that variations in the soil parameters c (c') and ϕ (ϕ') have the largest influence on the stability of a slope, in homogeneous soils, as well as for more variable soil conditions.

7.1.2 Modifications of STABL

In addition to the three original surface generators, which are the circular arc, the irregular, and the sliding block type surface, a new sliding block surface was introduced, which generates the active and passive portions of the surface according to Rankine's theory, $(45^\circ + \phi'/2)$ with respect to the horizontal for the active, and $(45^\circ - \phi'/2)$ with respect to the horizontal for the passive portion. The new sliding block is recommended for slopes with relatively simple soil profiles.

Also, the water surface option is extended, so that it is now possible to define different piezometric surfaces for different soil layers. This is convenient in the analysis of problems encountering artesian water conditions or a perched water table. It may also be used to approximate excess pore water pressures in low permeable soil layers due to embankment construction, and to analyse reservoir embankments for rapid drawdown.

The modified Bishop factor of safety was introduced for circular failure surfaces, in addition to the simplified Janbu method. This was done because of erroneous results obtained with the Janbu method for deep circular failure surfaces in soils with $\phi = 0$. The Bishop method satisfies overall moment equilibrium, which in the case of a soil with $\phi = 0$, provides a mathematically correct solution, whereas the Janbu method with overall horizontal force equilibrium represents an approximate solution. The Bishop method can be applied only to circular shaped failure surfaces.

7.1.3 Comparative Study

Results of analyses performed with STABL (simplified Janbu method) were compared with results obtained by other methods of slope stability analysis. Results by STABL are usually conservative, an exception being the deep circular failure surfaces mentioned above. The differences are normally less than 10%, but a value as high as 34% was obtained. The conservatism seems to be larger for sliding block type failure surfaces, than for failure surfaces of circular shape.

7.2 SUGGESTIONS FOR FUTURE WORK

STABL has been used routinely by the Indiana State Highway Commission for the last two years, during which period (1976-77) more options have been added to the program, and comparisons have been made with other methods of slope stability analysis, as described above.

The result is a program, even more versatile than the original STABL program. However, the program is also rather bulky and for many problems, only a fraction of the program is actually used, even though the total program has to be loaded into the computer memory. This is inefficient use of computer space, and could be avoided by means of an overlay structure. Such a structure divides a program into overlays, and only those overlays that are needed for a particular problem are loaded into the central core memory of the computer. This improvement is recommended for the STABL program.

A more thorough analysis is required to determine the relationship between type of failure surface, soil profile and strength properties, and the conservatism of the simplified Janbu method of slices. Janbu did provide correction factors to adjust the simplified solutions to more accurate results. However, these correction factors seem to apply only to slopes in homogeneous soil.

The task still remains to develop an analytical model for slope stability analysis that combines the simplicity of the Janbu method with the higher accuracy of, for example, the Morgenstern and Price procedure, and does not have the problems related to misleading factors of safety.

LIST OF REFERENCES

LIST OF REFERENCES

Note: ASCE = American Society of Civil Engineers

- Athanasίου-Grivas, D. S. (1976), "Reliability of Slopes of Particulate Materials", Ph.D. Thesis, Purdue University, West Lafayette, Indiana, 1976.
- Baligh, M. M. and Azzouz, A. S. (1975), "End Effects on Stability of Cohesive Slopes", Journal of the Geotechnical Engineering Division, ASCE, Vol. 101, No. GT11, Proc. Paper 11705, November, 1975, pp. 1105-1117.
- Bishop, A. W. (1955), "The Use of the Slip Circle in the Stability Analysis of Slopes", Geotechnique, Vol. 5, No. 1, March, 1955, pp. 7-17.
- Bishop, A. W. and Morgenstern, N. (1960), "Stability Coefficients for Earth Slopes", Geotechnique, Vol. 10, No. 4, December, 1960, pp. 129-150.
- Bjerrum, L. (1972), "Embankments of Soft Ground", Proc. ASCE Specialty Conference on Performance of Earth and Earth-Supported Structures, Vol. 2, Purdue University, June, 1972, pp. 1-54.
- Carter, R. K. (1971), "Computer Oriented Slope Stability Analysis by Method of Slices", MSCE Thesis, Purdue University, West Lafayette, Indiana, 1971.
- Catalán, J. M. and Cornell, C. A. (1976), "Earth Slope Reliability by a Level-Crossing Method", Journal of the Geotechnical Engineering Division, ASCE, Vol. 102, No. GT6, Proc. Paper 12214, June, 1976, pp. 591-604.
- Chen, W. F. and Giger, M. W. (1971), "Limit Analysis of Stability of Slopes", Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 97, No. SM1, Proc. Paper 7828, January, 1971, pp. 19-26.
- Collin, A. (1846), "Landslides in Clays", Translated by W. R. Schriever, University of Toronto Press, 1956.
- Coulomb, C. A. (1776), "Essai sur une application des règles des maximis et minimis a quelques problèmes de statique", Memoires Academie Royale des Sciences, Vol. 7, Paris, 1776.

- Dvinoff, A. H. and Harr, M. E. (1971), "Phreatic Surface Location after Drawdown", Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 97, No. SM1, Proc. Paper 7806, January, 1971, pp. 47-58.
- Fellenius, W. (1926), "Erdstatische Berechnungen mit Reibung und Kohäsion", Ernst und Sohn, Berlin, 1926.
- Janbu, N. (1954a), "Application of Composite Slip Surfaces for Stability Analysis", Proceedings of the European Conference on Stability of Earth Slopes, Stockholm, 1954, Vol. 3, pp. 43-49.
- Janbu, N. (1954b), "Stability Analysis of Slopes with Dimensionless Parameters", Harvard Soil Mechanics Series, No. 46, 1954.
- Janbu, N., Bjerrum, L. and Kjærnsli, B. (1956), "Veiledning ved løsning av fundamenteringsoppgaver - 2. Stabilitetsberegning for fyllinger, skjæring og naturlige skråninger" (Soil Mechanics Applied to Some Engineering Problems - Chapter 2. Stability Calculations for Embankments, Cuts and Natural Slopes), Publication 16, Norwegian Geotechnical Institute, Oslo, 1956, pp. 17-26.
- Ladd, C. C. and Foott, R. (1974), "New Design Procedure for Stability of Soft Clays", Journal of the Geotechnical Engineering Division, ASCE, Vol. 100, No. GT7, Proc. Paper 10664, July, 1974, pp. 763-786.
- Ladd, C. C. (1975), "Foundation Design of Embankments Constructed on Connecticut Valley Varved Clays", Massachusetts Institute of Technology (MIT), Research Report R75-7, Geotechnical Publication 343, June, 1975.
- Ladd, C. C. (1976), Personal Correspondence.
- Lowe, J. and Karafiath, L. (1960), "Stability of Earth Dams upon Drawdown", Proceedings of First Pan American Conference on Soil Mechanics and Foundation Engineering, Mexico City, Vol. 2, 1960, pp. 537-552.
- Lowe, J. (1967), "Stability Analysis of Embankments", Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 93, No. SM4, Proc. Paper 9305, July, 1967, pp. 1-33.
- Lundgren, H. and Brinch Hansen, J. (1965), "Geoteknik", 2. udgave, Teknisk Forlag, København, 1965, p. 180.
- Maunsell & Partners Pty. Ltd., Consulting Engineers, Melbourne, Victoria, Australia, 1977: Correspondence.
- Morgenstern, N. R. and Price, V. E. (1965), "The Analysis of the Stability of General Slip Surfaces", Geotechnique, Vol. 15, No. 1, March, 1965, pp. 79-93.

- Newmark, N. M. (1965), "Effects of Earthquakes on Dams and Embankments", Geotechnique, Vol. 15, No. 2, June, 1965, pp. 139-159.
- Perloff, W. H. and Baron, W. (1976), "Soil Mechanics, Principles and Applications", The Ronald Press Company, New York, 1976, Chapter 12. Stability of Slopes.
- Petterson, K. E. (1916), "Kajraset i Göteborg den 5^{te} Mars 1916", Teknisk Tidskrift, Vol. 46, 1916, pp. 281-287, 289-291.
- Rankine, W. J. M. (1857), "On the Stability of Loose Earth", Philosophic Transactions of the Royal Society, London, Vol. 147, 1857, pp. 9-27.
- Romani, F. (1970), "Dependence of Stability of Slopes on Initiation and Progression of Failure", Ph.D. Thesis, Purdue University, West Lafayette, Indiana, 1970.
- Romani, F., Lovell, C. W. and Harr, M. E. (1972), "Influence of Progressive Failure on Slope Stability", Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 98, No. SM11, Proc. Paper 9349, November, 1972, pp. 1209-1223.
- Seed, H. B. (1966), "A Method for Earthquake Resistant Design of Earth Dams", Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 92, No. SM1, Proc. Paper 4616, January, 1966, pp. 13-41.
- Seed, H. B. and Lee, K. L. (1967), "Undrained Strength Characteristics of Cohesionless Soils", Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 93, No. SM6, Proc. Paper 5618, November, 1967, pp. 333-360.
- Seed, H. B., Lee, K. L., Idriss, I. M. and Makdisi, F. I. (1975), "The Slides in the San Fernando Dams during the Earthquake of February 9, 1971", Journal of the Geotechnical Engineering Division, ASCE, Vol. 101, No. GT7, Proc. Paper 11449, July, 1975, pp. 651-688.
- Siegel, R. A. (1975a), "Computer Analysis of General Slope Stability Problems", MSCE Thesis, Purdue University, West Lafayette, Indiana, 1975.
- Siegel, R. A. (1975b), "STABL User Manual", Technical Report, Joint Highway Research Project, No. 75-9, Purdue University, West Lafayette, Indiana, June, 1975.
- Singh, A. (1970), "Shear Strength and Stability of Man-made Slopes", Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 96, No. SM6, Proc. Paper 7652, November, 1970, pp. 1879-1892.
- Skempton, A. W. (1946), "Alexandre Collin (1808-1890) and his Pioneer Work in Soil Mechanics", Foreword to the English Translation of Landslides in Clays by Alexandre Collin (see Collin).

- Spencer, E. (1967), "A Method of Analysis of the Stability of Embankments Assuming Parallel Inter-Slice Forces", Geotechnique, Vol. 17, No. 1, March, 1967, pp. 11-26.
- Taylor, D. W. (1937), "Stability of Earth Slopes", Contributions to Soil Mechanics 1925-1940, Boston Society of Civil Engineers, Boston, 1940.
- Taylor, D. W. (1948), "Stability of Slopes", Fundamentals of Soil Mechanics, John Wiley and Sons, Inc., New York, 1948, pp. 406-479.
- Whitman, R. V. and Bailey, W. A. (1967), "Use of Computers for Slope Stability Analysis", Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 93, No. SM4, Proc. Paper 5327, July, 1967, pp. 475-498.
- Wright, S. G. (1969), "A Study of Slope Stability and the Undrained Shear Strength of Clay Shales", Ph.D. Thesis, University of California, Berkeley, 1969.
- Wright, S. G., Kulhawy, F. H. and Duncan, J. M. (1973), "Accuracy of Equilibrium Slope Stability Analysis", Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 99, No. SM10, Proc. Paper 10097, October, 1973, pp. 783-791.
- Wu, H. T. and Kraft, L. M. (1970), "Safety Analysis of Slopes", Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 96, No. SM2, Proc. Paper 7174, March, 1970, pp. 609-630.