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Abstract

We present a general method for worst-case limit kinematic tolerance analysis: computing the range of variation in the kinematic function of a mechanism from its part tolerance specifications. The method covers fixed and changing contact mechanisms with parametric or geometric part tolerances. We develop a new model of kinematic variation, called kinematic tolerance space, that generalizes the configuration space representation of nominal kinematics function. Kinematic tolerance space captures quantitative and qualitative variations in kinematic function due to variations in part shape and part configuration. We derive properties of kinematic tolerance space that express the relationship between the nominal kinematics of mechanisms and their kinematic variations. Using these properties, we develop a practical kinematic tolerance space computation algorithm for planar pairs with two degrees of freedom and for assemblies of such pairs with independent part tolerances.

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1 Introduction

We present a new method of kinematic tolerance analysis based on configuration spaces. Kinematic tolerance analysis studies the variation in the kinematic function of mechanisms resulting from manufacturing variation in the shapes and configurations of their parts. It supports the synthesis of designs that reduce manufacturing cost by maintaining kinematic function under increased part variation. It helps designers predict performance, uncover design flaws, and optimize tolerance allocation. It complements tolerancing for assembly and for other design functions.

The kinematic function of a mechanism is the relationship among the motions of its parts imposed by contacts between them. For example, the kinematic function of an ideal gear pair is to transform an input rotation into an output rotation with a constant angular velocity ratio. The shapes and configurations of the parts determine the kinematic function under the assumption that they are rigid, hence cannot deform or overlap. Kinematic analysis computes the kinematic function by identifying the part contacts and deriving the resulting motion constraints. Variation in the part shapes and part configurations produces variation in the kinematic function. For example, variation in the gear profiles and in the configurations of the rotation axes causes backlash and transmission ratio variation. Kinematic tolerance analysis computes the class of kinematic variations from the class of part variations.

Kinematic models for tolerancing must account for multiple, changing contacts between irregular parts that generate complex, discontinuous kinematic functions. Nominal kinematic models are of limited use because they assume idealized contacts between nominal parts in permanent contact. For example, nominal linkage models assume that the links are permanently connected by ideal joints, nominal cam models assume that the cam and the follower have smooth profiles with a single, permanent contact, and nominal gear models assume that the teeth have perfectly meshed involute profiles. Part tolerances invalidate the idealized shape and contact assumptions.

We illustrate the effect of tolerances on the kinematic function of a Geneva mechanism (Figure 1). The driver consists of a driving pin and a locking arc segment mounted on a cylindrical base (not shown). The wheel consists of four locking arc segments and four slots. The driver rotates around axis O_d and the wheel rotates around axis O_w. In the nominal model, each rotation of the driver causes a nonuniform, intermittent rotation of the wheel with four drive periods where the driver pin engages the wheel slots and with four dwell periods where the driver locking segments engage the wheel locking segments. The pin fits perfectly into the slots, thus producing positive drive without play. Tolerancing the pin slightly smaller than the slots introduces play due to contact changes between the pin and the sides of the slots. The analysis must account for these contacts to measure the play and to determine the transmission ratio variation. Undercutting, interference, and jamming provide further examples of how part tolerances lead to changing contacts that invalidate the idealized contact assumption.

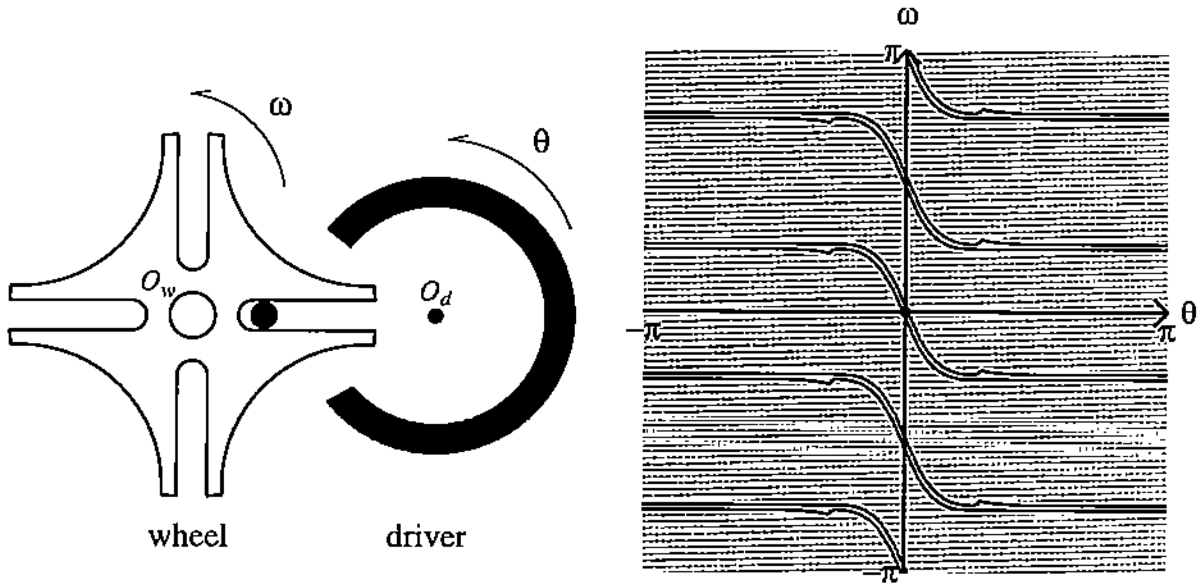


Figure 1: Geneva mechanism and its configuration space. The mechanism is displayed in configuration $\theta = 0$, $\omega = 0$, marked by the dot at the configuration space origin.

Current kinematic tolerance analysis methods [5] are limited by intrinsic factors and by their reliance upon predefined kinematic models, which are available only for linkages and for a few higher pairs. Specialized methods are available for specific mechanisms with specific part tolerances, for example linkages whose links vary in length. The most common general methods are statistical sampling and sensitivity analysis. Statistical sampling estimates the kinematic variation by generating mechanism instances based on conjectured part tolerance distributions and comparing their kinematic functions with the nominal function. The method requires many instances to provide an accurate picture of the variation over the entire kinematic function and can miss important cases. Sensitivity analysis computes the kinematic variation under the assumption that the kinematic function is a smooth function of some tolerance parameters. It linearizes the function around the nominal parameter values and computes the variations in the linearized function. The method is limited to small deviations and cannot compute the effects of changing contacts because they violate the smoothness assumption.

In this paper, we present a general method for worst-case limit kinematic tolerance analysis: computing the range of variation in kinematic function from the part tolerance specifications. The method covers all mechanisms: planar and spatial, fixed topology and varying topology mechanisms consisting of linkages, gears, cams, and other higher pairs. It applies to parametric and geometric part tolerance specifications. We define a new model of kinematic variation, called kinematic tolerance space, that generalizes our configuration space model of nominal kinematics to cover part variations. Kinematic tolerance space encodes the quantitative effect of part variations on kinematic function along with possible qualitative effects, such as changes in operating mode or unintended functions. It generalizes kinematic function sensitivity analysis to large scale quantitative and qualitative variations by capturing the interplay between part variations and multiple, changing contacts. We derive the basic properties of kinematic tolerance space and describe an implemented kinematic tolerance space computation algorithm for planar pairs with two degrees of freedom and for assemblies of such pairs with independent part tolerances. We conclude with a discussion of applications and of future work.

2 Nominal kinematics

We set the stage for kinematic tolerance analysis with a brief review of the configuration space method for nominal kinematic analysis. Configuration space provides a uniform geometrical model of kinematic function that is concise, complete, and explicit. It simplifies and systematizes kinematic analysis by reducing it to computational geometry.

2.1 Configuration space

The configuration space of a mechanism is the space of configurations (positions and orientations) of its parts. The dimension of the configuration space equals the number of degrees of freedom of the parts. For example, a nominal gear pair has a two-dimensional configuration space because each gear has one rotational degree of freedom. The gear orientations provide a natural coordinate system. Configuration space partitions into free space where the parts do not touch and into blocked space where some parts overlap. The common boundary, called contact space, contains the configurations where some parts touch without overlap and the rest do not touch. Only free space and contact space are physically realizable.

We illustrate these concepts on the Geneva mechanism (Figure 1). The configuration space is two-dimensional because each part has one degree of freedom. Its coordinates are the orientations θ and ω of the driver and the wheel. The shaded region is the blocked space where the driver and the wheel overlap. The white region is the free space. It forms a single channel that wraps around the horizontal and vertical boundaries, since the configurations at $\pm\pi$ coincide. The width of the channel shows backlash. The curves that bound the free and blocked regions form the contact space. They encode the contact relations between the wheel and the driver. The horizontal segments represent contacts between the locking arc segments, which hold the wheel stationary. The diagonal segments represent contacts between the pin and the slots, which rotate the wheel.

The configuration space topology reflects the semantics of rigid body kinematics. Free space is an open set because free parts remain free under small motions. Blocked space is open because overlapping parts remain overlapping under small motions. Contact space is closed because it is the complement of the union of free space and blocked space. It forms the common boundary of free space and blocked space because touching parts overlap when they move closer and become free when they move farther apart. Contact space partitions configuration space into connected components.

The configuration space encodes the space of kinematic functions under all driving motions. It represents the motion constraints induced by part contacts and the configurations where contacts change. The kinematic functions under specific driving motions are paths in configuration space that consist of contact and free segments separated by contact change configurations. For example, clockwise rotation of the driver produces a path that follows the bottom of the free space from right to left. The kinematic function consists of four horizontal segments alternating with four diagonal segments. The pin makes contact with the slot at the start of the diagonal segments and breaks contact at the end.

Another important property, called compositionality, is that the configuration space of a mechanism is determined by the configuration spaces of its pairs of parts [9]. We embed the pairwise configuration spaces in the mechanism configuration space by inverse projection. Each pairwise configuration (a, b) maps to the set of configurations (a, b, x) where x varies over all values of the other coordinates. The mechanism free space equals the intersection of the embedded pairwise free spaces because a mechanism configuration is free when every pair of parts is free. The blocked space equals the union of the embedded pairwise blocked spaces because a mechanism configuration is blocked when at least two parts overlap.

Configuration space computation can be formulated in terms of algebraic geometry. The formulation requires that part shapes be specified as algebraic curves and surfaces. The kinematic condition that the parts cannot overlap is expressed by multivariate polynomial inequalities in the configuration space coordinates. The configurations that satisfy the constraints are the free and contact spaces. Computing this set takes time polynomial in the geometric complexity of the parts and exponential in the number of degrees of freedom with large constant factors [12].

2.2 The HIPAIR mechanism configuration space computation program

We [9, 10, 16, 17] have developed an efficient configuration space computation program, called HIPAIR, for planar mechanisms composed of linkages and of higher pairs with two degrees of freedom, such as gears and cams. HIPAIR covers 80% of higher pairs and most mechanisms based on a survey of 2500 mechanisms in Artobolevsky's [2] encyclopedia of mechanisms and on an informal survey of modern mechanisms, such as VCR's and photocopiers. Other researchers have developed algorithms for some higher pairs that HIPAIR does not cover, including planar polygonal pairs with three degrees of freedom [3, 4] and a polyhedral body with six degrees of freedom amidst polyhedral obstacles [6, 11].

HIPAIR computes the configuration space of a mechanism by composing the configuration

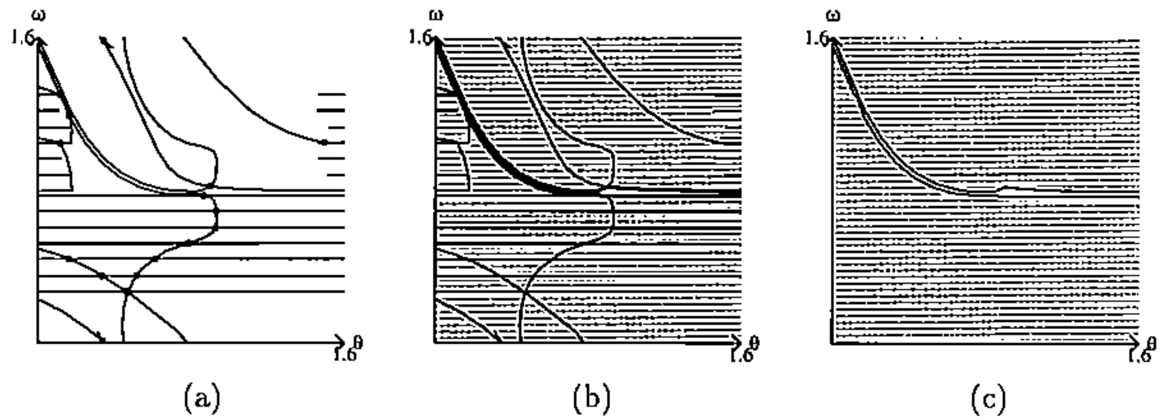


Figure 2: Detail of configuration space computation for the Geneva mechanism: (a) contact curves and intersection points; (b) connected components and realizable space; (c) final configuration space.

spaces of its higher pairs and linkages. The correctness of this procedure follows from the compositionality of configuration space.

HIPAIR computes the configuration space of a higher pair from the contacts between the part features (vertices and curves on its boundary). The configuration space is two dimensional because the pair has two degrees of freedom, each of which is translation along a planar axis or rotation around an orthogonal axis. The contacts occur along curves in configuration space. The contact curves partition configuration space into connected components that form the free and blocked spaces. The component boundaries are sequences of contact curve segments that meet at curve intersection points where multiple feature contacts occur. The component that contains the initial configuration is the realizable space. HIPAIR enumerates the feature pairs, generates the contact curves, computes the planar partition with a line sweep, and retrieves the realizable component. The curves come from a table with entries for all combinations of part features and degrees of freedom. Figure 2 shows the contact curves, intersection points, and components in a portion of the Geneva mechanism configuration space where the driver locking segment disengages from a wheel locking segment and the driver pin engages in the adjacent wheel slot.

HIPAIR computes linkage configuration spaces by homotopy continuation. It composes the pairwise and linkage configuration spaces by linearizing the contact zone boundaries and intersecting them with the simplex algorithm. The result is a partition of the mechanism configuration space into free and blocked regions defined by linear inequalities in the motion coordinates. We visualize the configuration space, which can have any dimension, by projecting it onto the pairwise configuration spaces.

HIPAIR simulates the kinematic function of a mechanism by propagating driving motions through part contacts. The result is a configuration space path. HIPAIR constructs the mechanism region that contains the initial configuration, computes the segment of the motion path that lies in the region, replaces the initial configuration with the endpoint of the segment, and repeats the process. It computes the motion path in a region by combining the input motion with the contact constraints. In free configurations, the motion path is a line tangent to the input motion. In contact configurations, the motion path is the projection of the input motion onto the tangent space of the contact surface (in 2D, onto the tangent to the contact curve).

HIPAIR is written in Common Lisp with a C graphics interface and runs on Unix workstations. It has been tested on over 1,000 parametric variations of 50 higher pairs with up to 10,000 contacts, and on a dozen mechanisms with up to ten moving parts. It computes the higher pair configuration spaces in under one second and the mechanism configuration spaces in under ten seconds. All the figures in this paper are annotated HIPAIR output.

3 Tolerance specifications

Tolerance specifications define the allowable variation in the shapes and configurations of the parts of mechanisms. The most common are parametric and geometric tolerance specifications [18]. Parametric specifications restrict shape and configuration parameters of part models to intervals of values. For example, a tolerance of $r = 1 \pm 0.1$ restricts the radius r of a disk to the interval $[0.9, 1.1]$. Geometric specifications restrict part features to zones around the nominal features, typically to fixed-width bands, called uniform profile tolerance zones, whose boundaries are the geometric inset and offset of the nominal features. For example, a uniform geometric profile tolerance of 0.1 on a disk of radius 1 constrains its surface to lie inside an annulus with outer radius 1.1 and inner radius 0.9.

We define the variational class of a mechanism relative to some tolerance specifications as the set of mechanisms whose parts satisfy the tolerance specifications. This definition generalizes the standard definition of the variational class of a single part [15]. We define the kinematic variational class of a mechanism as the set of kinematic functions of the mechanisms in its variational class. The properties of the variational class of a mechanism determine its kinematic variational class. We define two properties, monotonicity and independence, that we will use in computing kinematic variational classes.

The variational class of a mechanism is monotone if it contains a maximal and a minimal instance. In every instance, each part is a subset of the corresponding maximal part and is a superset of the corresponding minimal part. Most geometric tolerance specifications generate monotone variational classes, including profile offsets, sweeps, and tolerance zones [1]. Parametric tolerance specifications rarely produce monotone variational classes because the values that maximize some features need not maximize others. In particular, position tolerances never lead to monotone instances because every instance has the same shape.

The variational class of a mechanism is independent if each part tolerance is specified with respect to its own reference datum. It is equal to the cross product of the part variational classes. Independent tolerance specifications facilitate the engineering goal of designing parts that can be manufactured and gauged independently [1]. Assembly tolerance specifications, which express functional relations between parts, are sometimes dependent. Position tolerances are independent when they refer to a common external datum or a single reference part. They are dependent when they refer to multiple parts or external datums. Tolerance specifications for pairs of parts are always independent, as one of the parts serves as the reference object. One-dimensional chains of parametric tolerances and vectorial tolerances are also independent.

4 Kinematic tolerance space

We now describe a new method for kinematic tolerance analysis. The method generalizes our nominal kinematic analysis method based configuration space to variational classes of mechanisms. It is based on kinematic tolerance space, a uniform geometrical model of kinematic variation that is concise, complete, and explicit. It simplifies and systematizes kinematic tolerance analysis by reducing it to computational geometry.

We model the kinematic variational class of a mechanism with a kinematic tolerance space. The kinematic tolerance space is a partition of configuration space into free, blocked, and contact zones that model the range of variation from the free, blocked, and contact spaces of the nominal mechanism. The free zone, defined as the intersection of the free spaces of the variational class, is the set of configurations for which part instances in the variational class are free. It is the portion of the nominal free space that is guaranteed to persist. The blocked zone, defined as the intersection of the blocked spaces, is the set of configurations for which part instances in the variational class are blocked. It is the portion of the nominal blocked space that is guaranteed to persist. The contact zone, defined as the union of the nominal contact spaces, bounds the deviation from the nominal kinematic function.

Figure 3 shows a detail of the kinematic tolerance space of the Geneva mechanism with a uniform geometric profile tolerance of 0.05 units (0.5% of the diameter of the pair). The figure shows the region where the driver locking segment disengages from the wheel locking segment and the driver pin engages the slot of the wheel. In the detail of the pair, the dashed lines mark the nominal part shapes and the shaded bands represent their variational classes. In the kinematic tolerance space, the free zone is white, the contact zone is light grey, and the blocked zone is dark grey. The dashed curves mark the nominal contact space. Its outside boundary corresponds to the kinematic function of the maximal part shapes, while its inside boundary corresponds to the kinematic function of the minimal part shapes.

The structure of the kinematic tolerance space of a mechanism depends on the structure

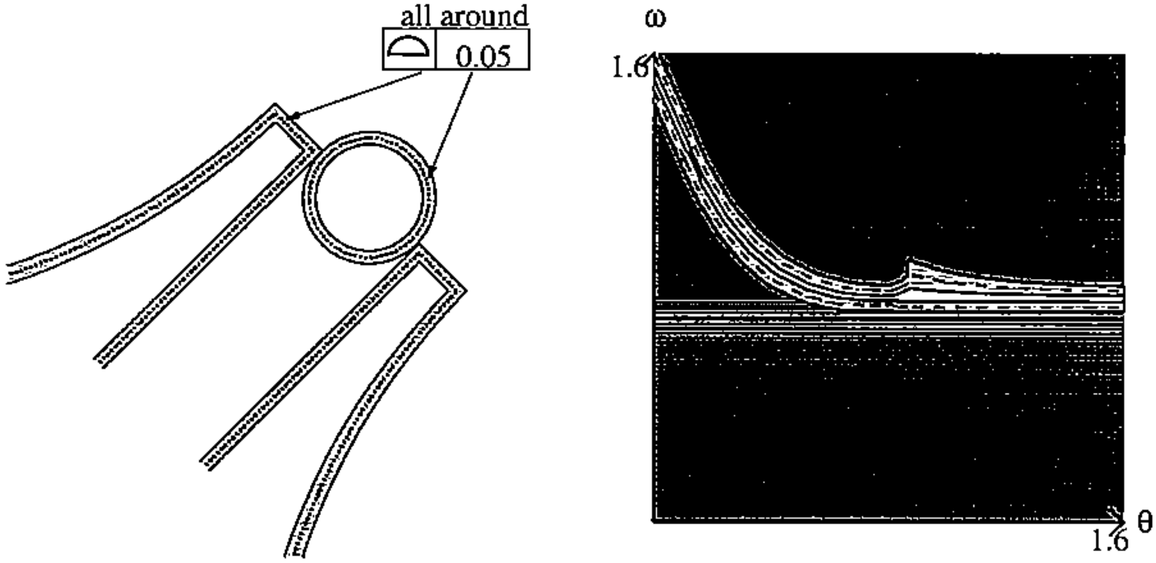


Figure 3: Detail of the Geneva mechanism with uniform geometric profile tolerance of 0.05 units and a detail of its kinematic tolerance space.

of its variational class, which is determined by the tolerance specifications. For example, Figure 4 shows the kinematic tolerance space for a parametric tolerance of 0.05 units on the horizontal and vertical distances between the centers of rotation of the wheel and the driver (parameters a and b). Comparing it with Figure 3, we see that the uniform geometric profile tolerance produces a narrow, uniform-width free zone with a smooth boundary, whereas the parametric tolerance produces a broader, irregular free zone with a jagged boundary that marks changes in sensitivity due to contact changes.

The topology of the kinematic tolerance space reflects the semantics of kinematic tolerances. The free and blocked zones are subsets of the free and blocked spaces of the nominal mechanism because configurations that are free or blocked for the entire variational class are free or blocked for every instance. They are open sets because configurations that are free or blocked for the class remain so under small motions. The contact zone equals the complement of the union of the free and blocked zones, hence is closed. (The proofs of the following facts appear in the Appendix.) The contact zone is a superset of the nominal contact space because nominal contact configurations do not belong to the free or blocked zones. The contact zone boundary bounds the free zone, the blocked zone, and the (possibly empty) interior of the contact zone. It partitions kinematic tolerance space into connected components just as contact space partitions configuration space.

The kinematic tolerance space models the variations from the nominal kinematics. With small tolerances, the free and blocked zones are shrunken versions of the nominal free and

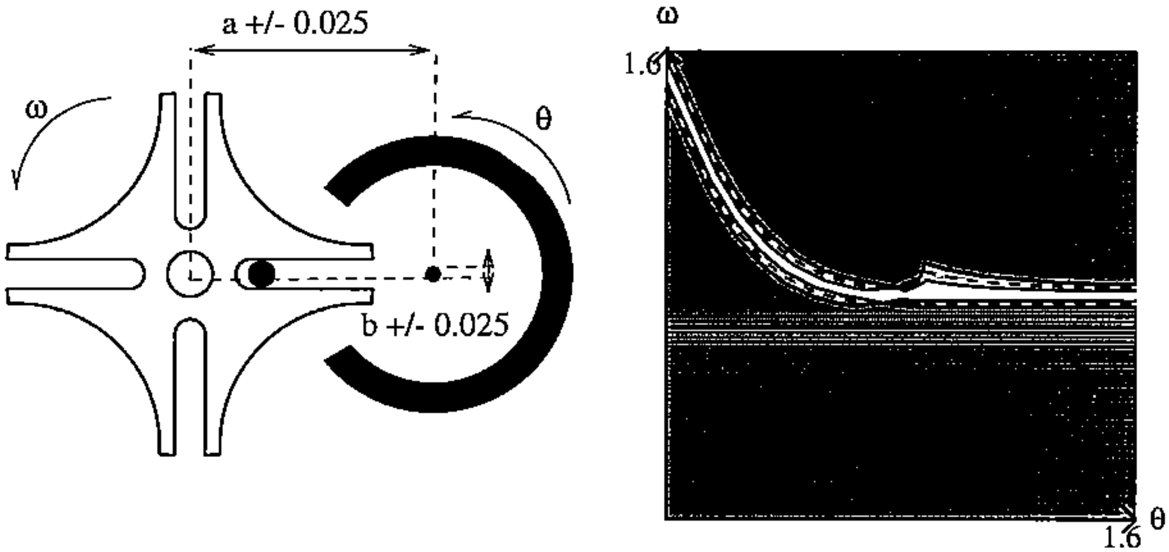


Figure 4: Geneva mechanism with a parametric tolerance specification of 0.05 units and a detail of its kinematic tolerance space.

blocked spaces, whereas the contact zone is a fattened version of the nominal contact space. The contact zone bounds the quantitative variation from the nominal kinematics. Qualitative changes in kinematics do not occur because the free zone, hence every free space in the variational class, is homeomorphic to the nominal free space. With larger tolerances, the contact zone can grow fat enough to alter the free zone topology, thus producing qualitative changes in kinematics.

We observe quantitative and qualitative variation in the Geneva mechanism. A uniform geometric profile tolerance of 0.05 units produces quantitative variation in kinematic function because the contact zone is narrower than the nominal free space (Figure 3). A profile tolerance of 0.08 units produces qualitative changes because the contact zone pinches closed the nominal free space and carves necks in the nominal blocked space (Figure 5). The qualitative changes are most pronounced in the mechanisms with maximal and minimal parts (Figure 6). The maximal free space shrinks to four partial channels. The driver pin cannot fit in the wheel slots, hence cannot rotate the wheel. The minimal free space grows to include necks at the junctions of the horizontal and diagonal segments. The driver pin can slip out of the wheel slots and jam against the locking segments.

Kinematic tolerance space has a simple structure when the variational classes of the parts are monotone. The free zone equals the free space of the maximal parts because it is a subset of every free space. The blocked zone equals the blocked space of the minimal parts because it is a subset of every blocked space. We illustrate this property on the Geneva mechanism

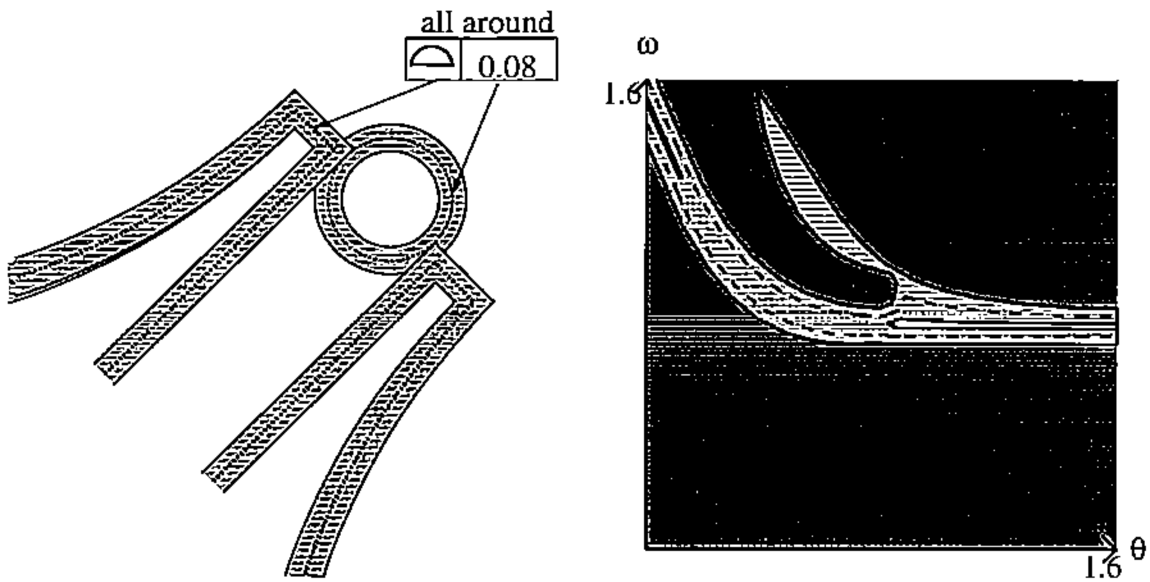


Figure 5: Detail of the Geneva mechanism with uniform geometric profile tolerance of 0.08 units and a detail of its kinematic tolerance space.

with the profile tolerance of 0.08 units. Comparing Figure 5 with Figure 6, we see that the free zone equals the free space of the maximal mechanism and that the blocked zone equals the blocked space of the minimal mechanism.

Kinematic tolerance space has a compositional structure akin to that of configuration space for mechanisms with independent part tolerances. The free zone equals the intersection of the embedded pairwise free zones because a mechanism configuration is free when every instance of every pair of parts is free. The blocked zone equals the union of the embedded pairwise blocked zones because a mechanism configuration is blocked when some instance of some pair of parts is blocked.

5 Kinematic tolerance space computation

Computing kinematic tolerance spaces is at least as difficult as computing configuration spaces. We have developed a kinematic tolerance space computation program for the class of mechanisms covered by our HIPAIR configuration space computation program: planar mechanisms consisting of higher pairs with two degrees of freedom apiece. We follow the HIPAIR strategy of exploiting domain properties to manage the worst-case complexity of the computation. The program computes the pairwise kinematic tolerance spaces and composes them, much as with configuration spaces. It generalizes HIPAIR and reuses several

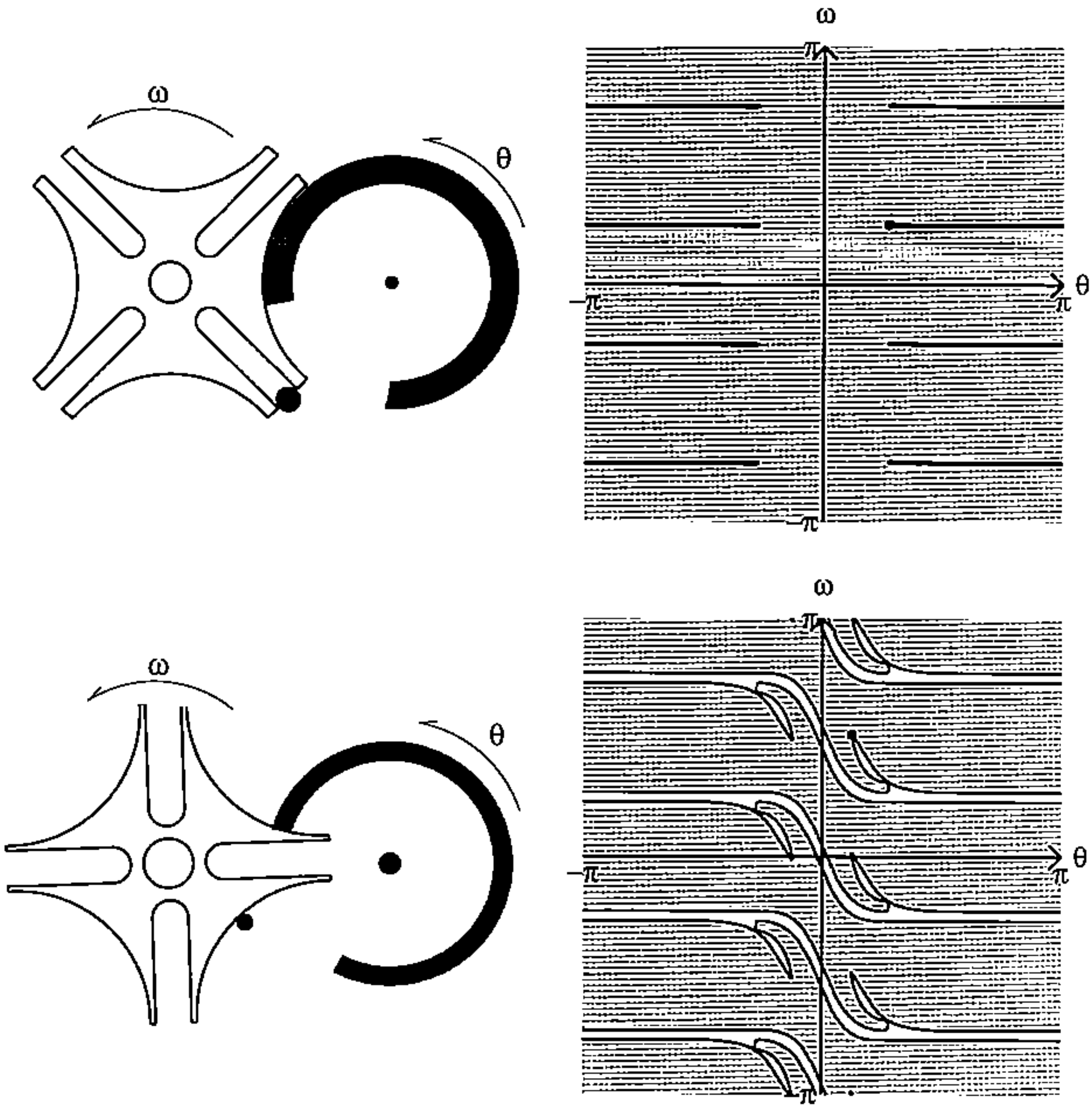


Figure 6: Maximal and minimal Geneva mechanisms with a 0.08 unit uniform geometric profile tolerance and their configuration spaces. The mechanisms are displayed in jamming configurations, which correspond to the dots in the configuration spaces.

subroutines. We discuss only those steps that differ from the HIPAIR configuration space computation algorithm.

The pairwise computation exploits the topological property that the contact zone boundary bounds the free zone, the blocked zone, and the contact zone interior. We compute the contact zone boundary, derive the kinematic tolerance space partition that it induces with HIPAIR, and classify the components. The classification algorithm employs point/region containment and region adjacency. The components that contain endpoints of nominal contact curves form the contact zone. The free zone consists of those remaining components in which a single, arbitrary configuration belongs to the nominal free space. The rest of the components form the blocked zone.

We compute the exact contact zone boundary for uniform geometric profile tolerances. The contact zone equals the complement of the union of the free and blocked zones, as proved in the Appendix. Its boundary equals the union of their boundaries because they are open. The free and blocked zones equal the free space of the maximal pair and the blocked space of the minimal pair by monotonicity. Their boundaries are the maximal and minimal contact spaces. We compute maximal and minimal contact spaces from the minimal and maximal pairs with HIPAIR and then construct the contact, free, and blocked zones from them with a planar line sweep algorithm. We compute the contact spaces of the pairs with HIPAIR and construct the contact, free, and blocked zones from them with a line sweep algorithm. For example, Figure 6 shows the contact spaces of the maximal and minimal Geneva mechanisms, which form the contact zone boundary shown in Figure 5.

Computing the exact contact zone boundary for parametric tolerances is more difficult because the pairs are not monotone. The general solution is to compute the extremal contact curves with respect to the parameters, which is a computationally expensive functional optimization. Instead, we approximate the boundary under the assumption that the tolerances are small with respect to the kinematic function. The method generalizes sensitivity analysis to varying contacts. We denote the tolerance vector by p, its nominal value by $\overline{p}$, its lower bound by l, and its upper bound by u. The nominal contact space consists of contact curves of the form $y = f(x,p)$ with x and y the kinematic tolerance space coordinates. We approximate the contact zone boundary by linearizing the contact curves around $\overline{p}$ and offsetting them by the variation in y. The y variation is defined by:

$$\delta y = \sum_i \frac{\partial f}{\partial p_i}(w_i - \overline{p}_i) \text{ with } w_i = \begin{cases} u_i & \text{if } \partial f/\partial p_i > 0 \\ l_i & \text{otherwise.} \end{cases}$$

The ith term is the maximal linear variation in y induced by p_i variations. It occurs when p_i is maximal or minimal depending on whether y increases or decreases with p_i. We compute the contact curves with HIPAIR, discretize them with respect to x, offset each sample point by $\pm\delta y$, and connect the offset points into upper and lower curves that approximate the contact zone boundary. For example, Figure 4 shows the kinematic tolerance space of the Geneva mechanism with the tolerance vector $p = (a, b)$.

We compose pairwise kinematic tolerance spaces (parametric or geometric) with independent part tolerances using the HIPAIR intersection algorithm [9]. The procedure is justified by the compositional structure of the spaces, just as with configuration spaces. We linearize the contact zone boundaries and intersect them by the simplex method. The result is a partition of the mechanism kinematic tolerance space into free, contact, and blocked zone regions defined by linear inequalities in the configuration space coordinates.

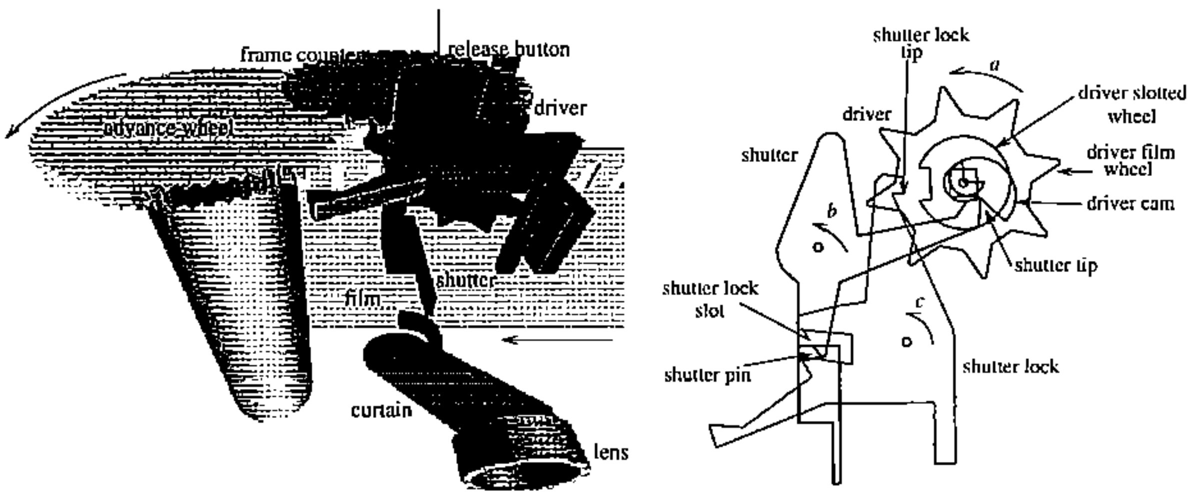
Computing the kinematic tolerance space for geometric tolerance specifications requires roughly three times as much time as computing the nominal configuration spaces: one for the maximal contact space, one for the minimal contact space, and one to construct the zones. For example, the kinematic tolerance spaces in Figures 3 and 5 were computed in four seconds apiece on an Iris Indigo 2 workstation. The computation time for parametric tolerances depends on the linearization accuracy. For example, the kinematic tolerance space in Figure 4 was computed to an accuracy of 0.01% in one second.

6 Kinematic tolerance analysis

Kinematic tolerance space provides the information for systematic kinematic tolerance analysis. Free zones whose topology differs from that of the nominal free space indicate possible failure modes, such as undercutting, interference, and jamming. Contact zones describe the variability of the contact function. The width of the contact zone bounds the quantitative deviation in kinematic function. Contact zones that contain horizontal or vertical segments indicate that the slope of the nominal contact space can change, which interchanges locking and driving functions. Contact relations that hold in the contact zone hold in every instance, whereas ones that fail in some contact zone configurations may fail in some instances. The linearized contact equations, which define the contact zone, specify the kinematic variation in terms of the part variations, thus supporting sensitivity analysis of higher pair assemblies with changing contacts.

We demonstrate the role of kinematic tolerance space in kinematic tolerance analysis by means of an extended example involving the shutter mechanism of a disposable camera (Figure 7a). The shutter mechanism consists of 10 higher pairs, none of which have standard kinematic or tolerance models. The advance wheel moves the film forward by one frame and rotates the driver cam, which engages the spring-loaded shutter in the shutter lock. Pressing the release button rotates the shutter lock, thus releasing the shutter. The shutter trips the spring-loaded curtain, which briefly rotates away from the lens. The light that passes through the lens aperture exposes the film.

We focus on the loading sequence of the driver, shutter, and shutter lock, which performs the most complex kinematic function (Figure 7b). The driver consists of three planar pieces: a cam, a slotted wheel, and a film wheel mounted on a shaft. The shutter is planar and is spring-loaded clockwise. The shutter lock is planar and is spring-loaded counterclockwise.



(a) (b)

Figure 7: Disposable camera: (a) shutter mechanism; (b) top view of driver, shutter, and shutter lock assembly.

The driver cam interacts with the shutter tip. The driver slotted wheel interacts with the shutter lock tip. The shutter pin interacts with the shutter lock slot.

The film advance continuously rotates the driver counterclockwise via the film wheel. In the initial configuration (snapshot 1 of Figure 8), the shutter tip lies on the driver cam, the shutter pin lies in the shutter lock slot, and the shutter lock does not touch the slotted wheel. As the driver rotates counterclockwise, the driver cam rotates the shutter clockwise by pushing the shutter tip (snapshot 2). The shutter pin leaves the slot in the shutter lock (snapshot 3). The shutter lock spring rotates the shutter lock clockwise until the tip touches the driver slotted wheel (snapshot 4). The tip then follows the wheel contour. When the shutter tip passes the highest point of the driver cam, it breaks contact with the cam. The shutter spring forces the shutter to rotate counterclockwise, causing the pin to engage the shutter lock on the surface below the slot (snapshot 5). The loading sequence ends when the shutter lock tip drops into the driver wheel slot and blocks further rotation (snapshot 6).

We analyze the kinematic variation under a uniform geometric profile tolerance of 1 unit for all parts (0.5% of the diameter of the smallest part). The kinematic tolerance spaces of the kinematic pairs reveal the sensitivity of the pairwise kinematic functions to part deviations (Figure 9). The driver/shutter function is insensitive to the part tolerances. The shutter tip follows the driver cam profile with a small deviation from its nominal path. The driver/shutter lock space also shows insensitivity to the part tolerances since the vertical channel is roughly the same as the nominal channel. The shutter lock tip follows the contour of the driver slotted wheel until it drops into the driver wheel slot. A larger tolerance can eliminate the channel and prevent blocking. The shutter/shutter lock space shows that

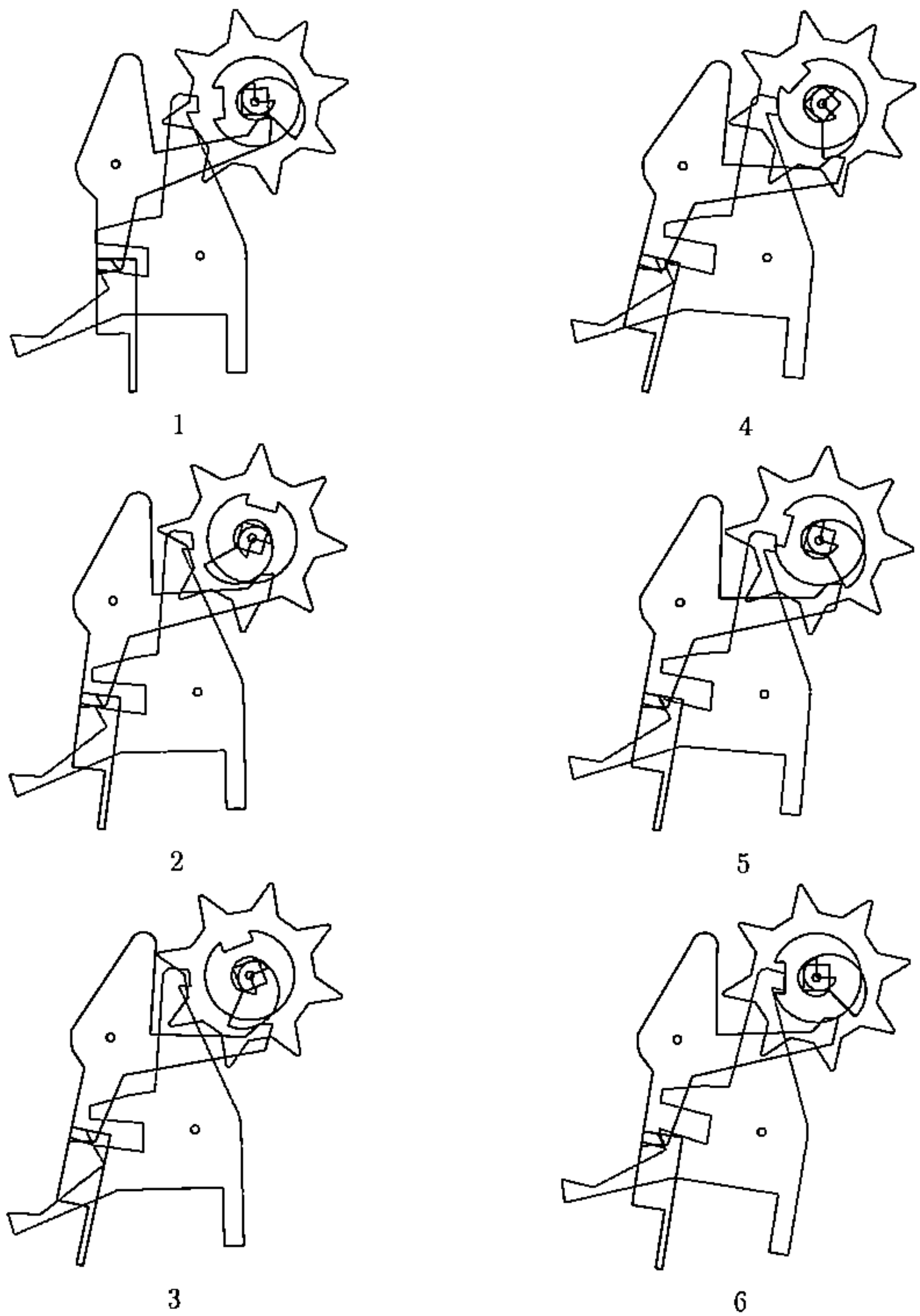
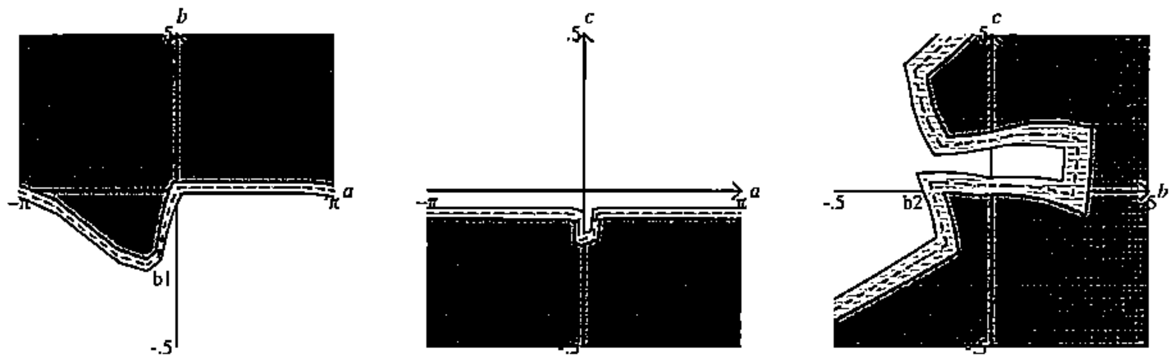


Figure 8: Loading sequence of the driver, shutter, and shutter lock.



driver/shutter driver/shutter lock shutter/shutter lock

Figure 9: Shutter mechanism pairwise tolerance zones.

small deviations in the pin and the slot can cause the shutter lock to release the shutter spontaneously under the action of the spring. This happens when the lower boundary of the horizontal slot in the blocked space has a positive slope. Shape deviations of 0.1 units can make the slope positive because the nominal value is a small negative number.

The mechanism tolerance space reveals the sensitivity of the mechanism function to part deviations. Coupling between part interactions leads to a global failure mode in which the driver cam does not push the shutter tip far enough for the shutter pin to clear the slot in the shutter lock. The failure occurs in mechanism instances where the value of b, b_1, at the lowest point of the driver/shutter contact space is greater than the value b_2 at the bottom, left corner of the horizontal slot in the shutter/shutter lock contact space. Figure 10 shows the projections of the nominal and failure modes in the driver/shutter configuration space. In the nominal mode, the configuration follows the lower segment of the horizontal slot from right to left, drops into free space at the left end of the horizontal slot when the shutter pin clears the shutter lock slot, and moves right and down as the shutter pin engages below the shutter lock slot. In the failure mode, the configuration follows the horizontal slot from right to left, does not reach the left end of the horizontal slot, and returns from left to right without engaging the pin in the slot.

The mechanism tolerance space shows that the variational class of mechanisms may contain a failing instance where $b_1 > b_2$ because $-0.26 \leq b_1 \leq -0.22$ and $-0.24 \leq b_2 \leq -0.15$. This does not guarantee the existence of an instance in which $b_1 > b_2$ because the shutter effects the values in opposite ways. Increasing the shutter size decreases b_1 by shifting the contact between its tip and the driver cam, but also decreases b_2 by increasing the size of the pin, which must clear the shutter lock slot. We compute the maximum of $b_1 - b_2$ over the variational class by binary search in the shutter profile, using the monotonicity in the other parts. (The kinematic tolerance space contains the equations for more sophisticated optimization algorithms.) The maximum occurs when the driver is minimal, the shutter

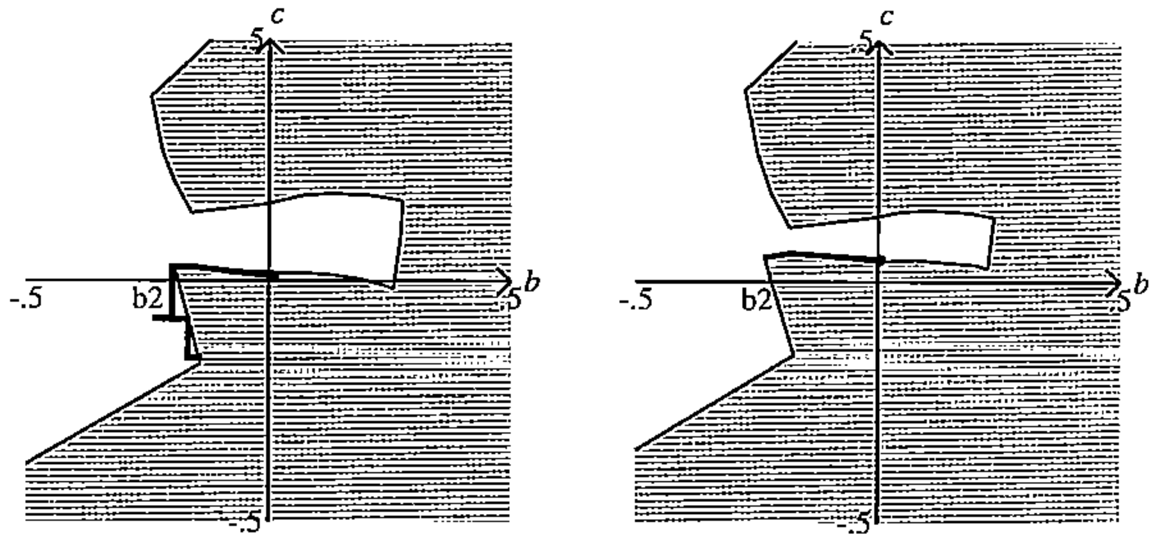


Figure 10: Projections of kinematic function onto the driver/shutter lock configuration space.

lock is maximal, and the shutter offset is 0.72 units. The mechanism works correctly for this value, but fails when we increase the shutter lock offset from 1 unit to 1.25 units. Manual analysis can easily miss this kind of failure.

7 Conclusion

We present a general method for worst-case limit kinematic tolerance analysis: computing the range of kinematic variation of a mechanism from its tolerance specifications. The method covers all types of mechanisms with parametric or geometric part tolerances. We develop a model of kinematic variation, called kinematic tolerance space, that generalizes the configuration space representation of kinematics. We derive properties of kinematic tolerance space that express the relationship between the nominal kinematics of mechanisms and their kinematic variations. Using these properties, we develop an efficient kinematic tolerance space computation program for planar pairs with two degrees of freedom and for assemblies of such pairs with independent part tolerances.

Our work advances the state of the art in kinematic modeling for tolerancing. Kinematic tolerance space is the first general representation for variational classes of kinematic functions. It represents fixed and changing contacts and quantitative and qualitative variations in kinematic function. It represents the nominal kinematics and its variations in a uniform manner, thus eliminating the need for a separate tolerance model. The kinematic tolerance space computation program automates a significant part of kinematic tolerance modeling. It is also useful for related tasks in mechanism analysis and design. For example, we can model part wear with geometric tolerances and compute the kinematic tolerance space to analyze the consequences of the wear. We can fine-tune designs by specifying a range of variations and selecting the best parameter value combination as the nominal specification. Tolerancing for assembly can also be studied with kinematic tolerance spaces. The task is to compute whether every mechanism in the variational class can be assembled, which involves reasoning about the variational class of kinematic function under assembly motions [7, 19, 13].

Our analysis focuses on the effects of part variations on the nominal degrees of freedom of mechanisms. As a consequence, the dimension of the configuration space and the kinematic tolerance space are equal. A full analysis would require computing the effects on all six degrees of freedom of every part, including those that are fixed in the nominal model. We could compute high-dimensional kinematic tolerance spaces to perform this analysis. A more practical approach is to model the added degrees of freedom as linear perturbations of the nominal degrees of freedom, as is common in linkage tolerancing [14]. For example, the Geneva mechanism has two nominal degrees of freedom because the other ten are fixed by the perfect fit between the wheel and the driver and their mounting shafts. We compute a two-dimensional kinematic tolerance space that neglects imperfect fit. We can model play due to imperfect fit with a 12-dimensional space or as a linear perturbation of the two-dimensional space.

We see several directions for future work. We plan to test the practicality of kinematic tolerance space computation on industrial tolerancing tasks. This will necessitate extensions to the kinematic tolerance space computation algorithm. We must compute kinematic tolerance spaces for mechanisms with mixed parametric and geometric tolerance specifications, which arise when part shapes and configurations vary simultaneously. We must automate kinematic tolerance space interpretation tasks, such as detecting qualitative changes in kinematic function or measuring the maximum play. We would like to extend the coverage to spatial parts, to pairs with three or more nominal degrees of freedom, and to higher dimensional kinematic tolerance spaces due to imperfect joints.

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Appendix: Proof of topological properties

We prove that the free zone of a kinematic tolerance space is open, the blocked zone is closed, and the contact zone equals the complement of the free and blocked zones. We treat the cases of uniform geometric profile tolerances and of parametric tolerances where the parameters are defined on closed intervals and the part shapes and configurations are continuous functions of the parameters.

The first two results are immediate for monotone tolerance specifications, which include uniform geometric profile tolerances. The free zone equals the free space of the minimal mechanism and the blocked zone equals the blocked space of the maximal mechanisms. Both spaces are open because free and blocked configurations are preserved by sufficiently small motions. We prove the parametric result via properties of continuous functions on compact spaces. The relevant function for the free zone, called the Hausdorff metric, is the distance between the closest pair of points on two parts of the mechanism. Let $d(x, p)$ denote the Hausdorff distance in configuration x with p the tolerance parameters. The function d is continuous because the parts depend continuously on p by hypothesis. Let P denote the domain of p, X denote the domain of x, and x_0 be a point in x. The set $x_0 \times P$ is compact because P is a cross-product of closed intervals by hypothesis. We use these properties to prove that the free zone is open. If x_0 belongs to the free zone, d is positive on $x_0 \times P$ by definition. Each point in $x_0 \times P$ has a positive neighborhood in $X \times P$ by the continuity of d. The union of these neighborhoods is an open covering of $x_0 \times P$, which has a finite subcovering by compactness. The projection into X of the intersection of the subcovering is a neighborhood of x_0 on which d is positive. An analogous proof applies to the blocked zone with the overlap area of the parts replacing the Hausdorff distance.

We prove that the contact zone equals the complement of the free and blocked zones for parametric tolerances. The proof covers offsets where the parts are continuous functions of the offset radii, which includes uniform geometric profile tolerances. Given a configuration, we define a function $f(p)$ whose value is the Hausdorff distance between the parts if p defines a free instance, 0 if p defines a contact instance, and the negative overlap area of the parts if p defines a blocked instance. The function f is continuous at free and blocked p values by elementary calculus. It is continuous at contact values because the Hausdorff distance and the overlap area are both 0. It is positive at free values, zero at contact values, and negative at blocked values. Given a configuration that is not in the free zone or the blocked zone, f cannot be positive for all p or negative for all p. It must be zero for some p by the intermediate value theorem. The configuration belongs to the contact zone because this p value yields a contact instance.