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Frost Growth and Densification on Flat Surfaces

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ABSTRACT

Frosting occurs on heat exchangers in heat pumping and refrigeration applications when the surface temperature of the heat exchanger is below the freezing point of water. The accumulation of frost on the heat transfer surface increases pressure drop (fan power) and decreases heat transfer, because the frost layer has a low thermal conductivity much lower than that of the heat exchanger material, and it blocks the air flow. System efficiency is reduced by frost accumulation, and operation is complicated by the need to defrost the heat exchanger. Many factors including air temperature, humidity, and cold plate temperature are known to affect frost growth on heat transfer surfaces. In the present study, a model for frost growth on and densification on flat surfaces is presented. The mathematical model is developed by analytically solving the governing heat and mass diffusion equations with appropriate boundary conditions. For temperature, a convective boundary condition at the frost surface and a fixed cold plate temperature were used. However, for the water-vapor density, the condition at the frost surface is unknown. Unlike earlier saturation and supersaturation models, the current work is based on a specified heat flux obtained experimentally in order to find the density gradient at the surface. From the results, it can be shown that the water-vapor at the frost-air interface is supersaturated.

1. INTRODUCTION

The prediction of frost properties is essential in order to characterize the performance of evaporators under frosting conditions. Many models have been developed to predict frost growth on flat plates. Most of the existing mathematical models consist of solving the heat and mass diffusion equations in the frost layer. They can be classified into two categories based on their boundary conditions. The first and most common category is the saturation models in which the water vapor is assumed to be saturated at the cold plate and at the frost surface (Hermes *et al*, 2009, Lee *et al*., 2003). Lee *et al.* (1997) developed a saturation model for frost formation on cold flat surfaces that was based on the assumption that the amount of water vapor absorbed into the frost layer is proportional to the vapor density in that layer. With this simplifying assumption, an analytical solution for the water vapor density and temperature distribution in the frost layer could be obtained assuming the process is quasi-steady. From the model, it was concluded that frost thickness and frost surface temperature increase with increasing air velocity and air relative humidity. Lee *et al.* (2003) proposed a modification to the earlier model that does not require any empirical correlations to solve for air-side properties. Instead, the boundary layer partial differential equations with appropriate boundary conditions are solved for air side properties. Yang and Lee (2005) suggested a new modification to the previous model by including an experimental correlation for frost density. The frost density was assumed a function of all frost growth parameters including air velocity, temperature, and humidity, and plate and frost surface temperatures. The main advantage of this approach is that it does not require the knowledge of an initial density. However, the proposed empirical correlations are valid only for a limited range of experimental conditions. Recently, Na and Webb (2004) proved that water vapor at the frost surface is not saturated but supersaturated as the laminar boundary layer analysis suggests. Unlike earlier models, their supersaturation model for frost growth on flat plates takes into account the variation of frost density along the frost thickness. Therefore, the gradient of the frost density at the frost surface was taken to be zero in order in predict the density of the fresh frost that deposits at the surface at each time step. The supersaturation degree was predicted from experimental data as a function of the vapor pressure of the surrounding air and the saturation vapor pressure at the frost surface temperature and is valid only for T_∞-$T_{fst,s}$ ranging from 14 to 20°C.

In the present study, a new mathematical model based on the linear driving force assumption with improved boundary conditions is used to predict frost growth on flat plates.

2. FROST GROWTH AND DENSIFICATION

The mechanisms involved in the growth and densification of a frost layer on a cold surface are shown in figure 1.

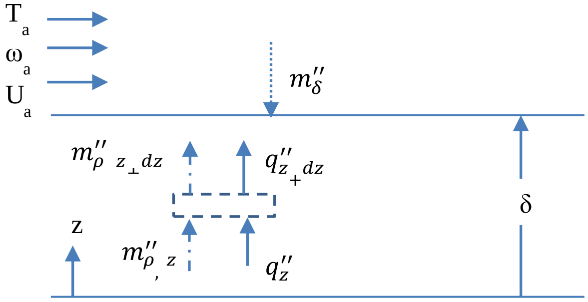

Figure 1: Frost growth mechanisms

As moist air is blown above a cold surface with temperature below the freezing point of water, frost crystals start forming on the surface. The “crystal growth period” is followed by “frost layer growth period” during which a frost layer grows by ablimation (Hayashi *et al.*, 1997). The frost growth mechanism involves simultaneous heat and mass transfer driven by the temperature and humidity differences between the frost layer and the surrounding air. The total heat flux can be divided into sensible and latent components. The latent heat flux is directly related to the mass flux of water vapor that deposits on the frost surface, therefore contributing in its growth. On the other hand, the density gradient at the frost surface causes diffusion of water vapor into the frost layer, therefore increasing its density.

2.1 Mathematical Formulation

Assuming that the frost density and therefore thermal conductivity vary within the frost layer (in z-direction), a control volume analysis is performed in order to relate the frost growth to the air properties and cold plate temperature.

The total heat flux transferred from air to the coolant can be divided into a sensible and a latent heat flux as follows:

$$q''_a = q''_{lat} + q''_{sens} \tag{1}$$

$$q''_{sens} = h\left(T_a - T_{fst,s}\right) \tag{2}$$

$$q''_{lat} = h_m \rho_a h_{sg}\left(\omega_a - \omega_{fst,s}\right) \tag{3}$$

The total vapor mass flux at the frost surface is divided into 2 components; m''_δ which contributes in thickening the frost layer and $m''_{\rho,s}$ which contributes in the densification of the frost layer at the frost surface.

$$m''_{t,s} = m''_\delta + m''_{\rho,s} \tag{4}$$

$$m''_\delta = \rho_{fst}\left(\frac{d\delta_{fst}}{dt}\right) = h_m \rho_a\left(\omega_a - \omega_{fst,s}\right) \tag{5}$$

$$m''_{\rho,s} = D_{eff}\left(\frac{d\rho_v}{dz}\right)\Big|_{z=\delta} \qquad (6)$$

Similarly, in order to calculate the frost density in different location within the frost layer, the mass of water vapor absorbed in the layer at different locations was calculated as follows:

$$m''_{\rho,z} = D_{eff}\left(\frac{d\rho_v}{dz}\right)\Big|_{z} = \delta_{fst}\left(\frac{d\rho_{fst}(z)}{dt}\right) \qquad (7)$$

The frost growth mechanism was assumed quasi-steady and transient effects were neglected in both the heat and mass diffusion equations. A linear driving force model for mass diffusion of water vapor was used to model the vapor density variation in the frost layer as follows:

$$D_{eff}\left(\frac{\partial^2\rho_v}{\partial z^2}\right) + U\frac{\partial\rho_v}{\partial z} = \varepsilon_{fst}\left(\rho_v - \rho_{v,sat}\right) \qquad (8)$$

Where U is the Darcy velocity given by:

$$U = \frac{1}{\rho_v}\int \varepsilon_{fst}\left(\rho_v - \rho_{v,sat}\right)dz \qquad (9)$$

And the effective diffusion coefficient D_{eff} was computed by multiplying the diffusion coefficient of water in air by the porosity and the tortuosity. The tortuosity factor was obtained from a model by Zehnder (Na and Webb, 2004):

$$D_{eff} = D\emptyset\tau \text{ where } \tau = \frac{1-\sqrt{1-\emptyset}}{\emptyset} \qquad (10)$$

The porosity of the frost layer can be related to the frost density as follows:

$$\rho_{fst} = \emptyset\rho_a + (1-\emptyset)\rho_i \qquad (11)$$

The energy equation can be expressed as a balance between the sensible heat transfer by conduction across the frost layer and the latent heat transfer due to ablimation of water vapor into ice crystals as shown below:

$$k_{fst}\left(\frac{d^2T_{fst}}{dz^2}\right) = -\varepsilon_{fst}h_{sg}\left(\rho_v - \rho_{v,sat}\right) \qquad (12)$$

An empirical correlation by O'Neal and Tree (1985) was used to relate the frost thermal conductivity was to its density:

$$k_{fst} = 0.001202\rho_{fst}^{0.963} \qquad (13)$$

The heat transfer coefficient h for a laminar flow over a flat plate was determined from the following correlation (Incropera and Dewitt, 2002):

$$Nu = 0.664Re^{1/2}Pr^{1/3} \qquad (14)$$

Where the Nusselt number Nu and the Prandtl number Pr are defined as:

$$Nu = \frac{hL}{k_a} \text{ and } Pr = \frac{C_{p,a}\mu}{k_a} \qquad (15)$$

The Chilton-Colburn analogy between heat and mass transfer was used to find the mass transfer coefficient h_m as follows:

$$Sh = Nu/Le^{2/3} \qquad (16)$$

Where the Sherwood number Sh and the Lewis number Le are defined as:

$$Sh = \frac{h_mL}{D} \text{ and } Le = \frac{\alpha}{D} \qquad (17)$$

2.2 Boundary Conditions

Equations (8) and (12) require boundary conditions of water vapor density and frost temperature at the cold plate and the frost surface. A fixed temperature was prescribed at the cold plate and the frost surface temperature was related to the total heat flux from the surrounding air to the frost layer.

$$\text{At } z = 0,\ T = T_w \qquad (18a)$$

$$\text{At z = },\ k_{fst}\frac{dT}{dz}\Big|_{z=\delta} = q''_a \qquad (18b)$$

For the water vapor density, the boundary conditions can be written as:

1- The no-flux boundary conditions at the cold plate: $\left.\frac{d\rho_v}{dz}\right|_{z=0} = 0$ (19a)

2- The total heat flux transferred from the frost layer to the surrounding air:

$$q''_a = m''_v h_{sg} + h\left(T_a - T_{fst,s}\right) \tag{19b}$$

$$m''_v = D_{eff} \left.\frac{\partial \rho_v}{\partial z}\right|_{z=\delta} + h_m \rho_a \left(\omega_\infty - \omega_{sat}\left(T_{fst,s}\right)\right) \tag{19c}$$

2.3 Initial Conditions

Equations (5) and (7) require two initial conditions for frost density and thickness. For the scope of this work, initial conditions were chosen in the range that Jones and Parker (1975) suggested. Therefore, initial frost thickness and density of 10^{-5} m and 40 kg/m^3 respectively were used in the present study.

2.4 Numerical Scheme

The governing equations (5), (7), (8) and (12) were discretized and solved simultaneously with other algebraic equations using a commercial software package, engineering equation solver (EES). A sensitivity analysis was used to determine the effect of the time interval and number of grid points on the final results. Based on the analysis, a time step of 10 seconds and 20 grid points were adopted. The solution was assumed converged when the relative changes in the density, thickness and temperature were less than 10^{-6}.

3. RESULTS AND DISCUSSION

3.1 Determination of ε_{fst}

The main parameter that is required in the linear driving force model is constant of proportionality ε_{fst}. The mass balance is expressed as a balance between diffusion in the frost layer and convection. Therefore, ε_{fst} is a mass transfer coefficient. Since the diffusion term (left-hand side) in equation (8) is a mass flux per unit area and the advective term (right-hand side) is a mass flux per unit volume. Hence, ε_{fst} is a mass transfer coefficient multiplied by an area to unit volume ratio. ε_{fst} depends mainly on the frost structure, and therefore detailed investigation is required to determine the appropriate value of this parameter. In the absence of such data on frost length scales, a parametric study was used to determine the effect of ε_{fst} on the frost properties mainly density and thickness.

3.2 Frost Thickness

Figure 2 shows the variation of the frost thickness with time as predicted by the present model for different values of ε_{fst}. It can be concluded that the lower the mass transfer coefficient ε_{fst}, the higher the frost thickness.

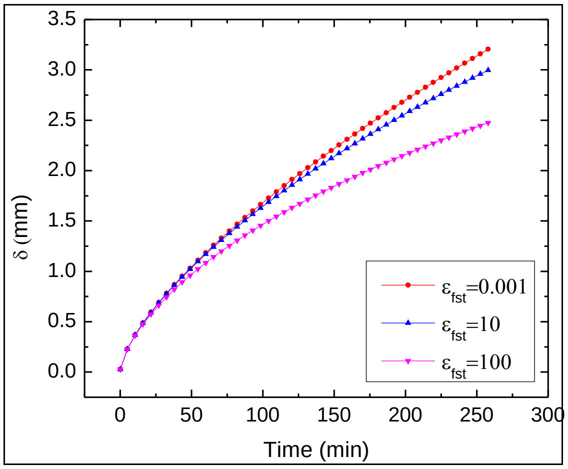

Figure 2: Frost thickness

3.2 Frost Density

In contrast with the frost thickness results, the model predicts a negligible effect of ε_{fst} on the frost density. Therefore, the mass transfer in the frost layer is diffusion-limited.

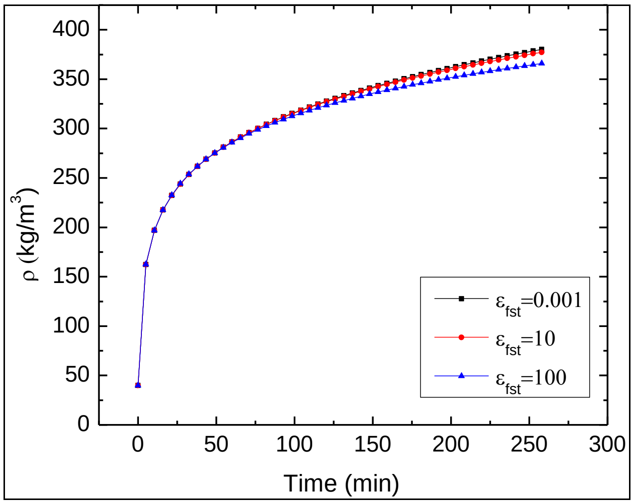

Figure 3: Frost density

3.3 Frost Surface Temperature

The frost surface temperature depends on the total heat flux transferred from the surrounding air to the frost layer. The heat flux is given as an input in the model and was chosen to be same for all the cases. However, since the frost thickness is lower for higher values of ε_{fst}, the frost surface temperature is also lower for higher values of ε_{fst}, as shown in figure 4.

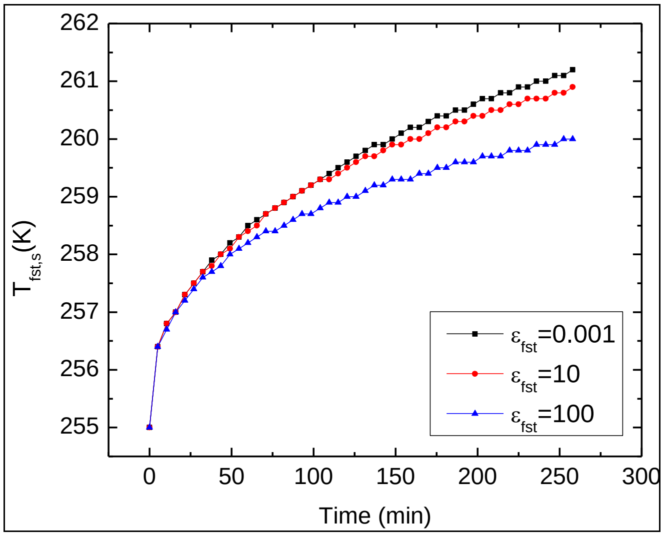

Figure 4: Frost surface temperature

3.4 Water Vapor Density

The property which is affected most by the variation of ε_{fst} is the water-vapor density. As shown in figure 4, very low values of the mass transfer coefficient of ε_{fst} produce unrealistic values of ρ_v. Therefore, this parametric study shows that the value of of ε_{fst} has to be on the order of 100 for the model to give realistic frost and water-vapor properties.

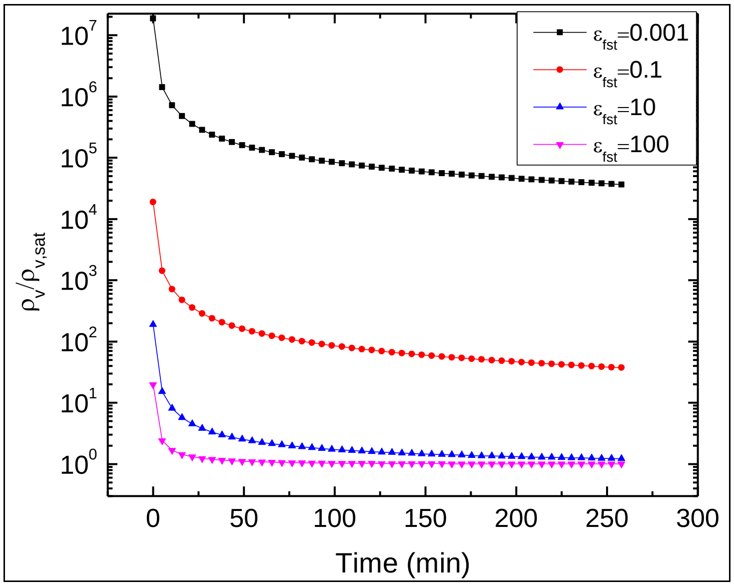

Figure 5: Water vapor density

6. CONCLUSIONS

In this paper, a model for frost growth and densification on cold flat surfaces was proposed. The model is based on the linear driving force assumption. Unlike other models that use saturation or supersaturation boundary conditions at the frost surface, the model uses the total heat flux measured experimentally to find the water-vapor gradient at the frost surface. The results obtained from the model show that the vapor density at the frost surface is greater than the saturation vapor density. Therefore, the frost-air interface is supersaturated as suggested by Na and Webb (2004). The mass transfer coefficient ε_{fst}, which mainly depends on the frost structure have not been determined in this work. Instead, a parametric study was used to show that for the vapor density to have realistic values, ε_{fst} has to be on the order of 100.

NOMENCLATURE

The nomenclature should be located at the end of the text using the following format:

α	thermal diffusivity	(m^2/s)
D	mass diffusion coefficient	(m^2/s)
δ	frost thickness	(m)
ε	mass transfer coefficient	(1/s)
h	heat transfer coefficient	(W/m^2.K)
h_m	mass transfer coefficient	(m/s)
h_{sg}	latent heat of sublimation	(J/kg)
k	thermal conductivity	(W/m.K)
L	plate length	(m)
Le	Lewis number	(-)
m''	mass flux	(kg/m^2.s)
Pr	Prandtl number	(-)
Ø	porosity	(-)
q''	heat flux	(W/m^2)
ρ	density	(kg/m^3)
Sh	Shmidt number	(-)
T	temperature	(K)

Subscripts

a	air
eff	effective
fst	frost
lat	latent
ρ	density
δ	thickness
t	total
s	surface
sat	saturated
sens	sensible
v	vapor

τ	turtuosity	(-)
U	Darcy velocity	(m/s)
ω	relative humidity	(-)

REFERENCES

Hayashi, Y., Aoki, A., Adachi, Hori, S. K., 1997, Study of Frost Properties Correlating with Frost Formation Types, *J. Heat Transfer*, vol. 99: p. 239-245

Hermes, C.J.L., Picucco, R.O., Barbosa J.R., Melo C.,2009, A Study of Frost growth and Densification on Flat Surfaces, *Exp. Therm. Fluid Sci.*, vol 33: p. 371-379.

Incropera, F.P., Dewitt, D.P., 2002, *Fundamentals of Heat and Mass Transfer*, John Wiley and Sons, New York, 981 p.

Jones, B.W., Parker J.D., 1975, Frost Formation with Varying Environmental Parameters, *J. Heat Transfer*, vol. 97: p. 255-259.

Lee, K.-S., Kim, W.S., Lee T.H., 1997, A One-Dimensional Model for Frost Formation on a Cold Flat Surface, *Int. J. Heat Mass Transfer*. Vol. 40 no 18: p. 4359-4365.

Lee, K.S., Jhee, S., Yang D.K., 2003, Prediction of the Frost Formation on a Cold Flat Surface, *Int. J. Heat Mass Transfer*, vol. 46 no. 20: p. 3789-3796.

Na, B., Webb, R., 2004, A New Model for Frost Growth, *Int. J. Heat Mass Transfer*, vol. 47 no. 5: p. 925-936.

Na, B., Webb, R., 2004, Mass Transfer on and Within a Frost Layer, *Int. J. Heat Mass Transfer*, vol. 47 no. 5: p. 899-911.

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