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# Semi-Empirical Inverse Model for DX Unit Performance in Residential Buildings

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## ABSTRACT

Inverse models are generally derived from empirical behavior and expressed in terms of one or more driving forces and a set of empirical parameters that are "identified" using measurements. Once identified, a model can be used for a variety purposes, including performance monitoring, diagnostics, retrofit analysis, measurement and verification of efficiency improvements, and on-line control optimization. In this paper, a semi-empirical inverse model or a "grey-box" model is developed to estimate the power consumption of a direct expansion (DX) air conditioning system that handles the cooling requirement of a residential building. This "grey-box" model approach requires fewer parameters and training data and provides better extrapolation than other popular empirical or "black-box" models. The model is based on physical system performance parameters and not only on a single compressor or coil, so it can provide a clear performance map of the air conditioning system in residential buildings. The training and validation data used for model construction in this paper comes from manufacturer's detailed DX unit performance models, which closely matches equipment performance in the field.

## NOMENCLATURE

C: Clearance volume factor
V: Displacement volume
RPM: Motor angular speed
a, b, d, e, f, $\Delta p$, A, B: Constant parameters
n: Polytropic exponent
$C_p$: Specific heat at constant pressure
N: data set number
P: Refrigerant pressure
T: Temperature
$T_C$: Refrigerant critical temperature
$P_C$: Refrigerant critical pressure
$\Delta P$: Pressure difference across the compressor
$C_v$: Specific heat at constant volume
m: mass flow rate

## SUBSCRIPTS

out: Outside
dis: Discharge
evap or e: evaporating
db: dry bulb
real: Measured
mean: Average value
in: Inside room
suc: Suction
cond or c: Condensing
wb: wet bulb
cal: calculated

## 1. INTRODUCTION

Residential buildings are large consumers of electric energy. About 40% of the electricity generated in the U.S. is consumed by residential buildings every year (U.S. EIA, 2010); and about 21% of that electricity is consumed by air conditioning equipment (U.S. EIA, 2005). Direct Expansion split air conditioning systems occupy the largest portion of the cooling market for residential buildings, so modeling this kind of equipment is necessary and useful to identify and improve the system performance operating characteristics.

Several inverse models have been constructed previously for modeling DX air conditioning equipment performance. The HVAC-2 toolkit model for DX units is widely used in software and energy consumption studies. It is a simple, easily used black-box model for predicting system cooling capacity, sensible heat ratio, and compressor power consumption (Brandemuehl, 1993). However, it requires a lot of training data with high diversity and is difficult to extrapolate to operating conditions outside of its training data. A physically-based semi-empirical model has been developed to map the performance of positive displacement compressors, which is the typical compressor type used in air conditioning systems for residential buildings (Jähnig, 1999). This model shows better extrapolating ability and requires less training data. However, more detailed data such as refrigerant mass flow rate, compressor displacement volume, evaporator superheat, condenser subcooling temperatures and so on are required to perform the fitting procedure. This level of detailed data is not easy to obtain through field measurements on a system when there is not enough manufacturer data to use. To overcome these difficulties and reduce the complexity of on-site data gathering without compromising the advantages of a physically-based model, a new semi-empirical inverse model is construed based on Jähnig's compressor model. This model can be trained using system operating data that is easy to gather on-site.

## 2. MODE DEVELOPMENT

### 2.1 Jähnig's model

This model aims to generate a performance map for a positive displacement compressor. It allows extrapolation outside of the range of the fitted data since it is based on physical phenomena. It includes a mass flow rate model and a compressor power consumption model. They are shown as following equations.

$$m_{calc} = \left( 1 + C - C \left( \frac{p_{\mathrm{dis}}}{p_{suc}} \right)^{\frac{1}{n}} \right) \frac{V \cdot RPM}{v_{suc} \cdot 60} \tag{1}$$

$$Power \cdot \eta_{comb} = m \cdot \frac{n}{n-1} \cdot p_{suc} \cdot v_{suc} \left( \left( \frac{p_{dis}}{p_{suc}} \right)^{\frac{n-1}{n}} - 1 \right) \tag{2}$$

$$p_{dis} = p_{cond} \tag{3}$$

$$p_{suc} = p_{evap} \cdot (1 - \Delta p) \tag{4}$$

$$\eta_{comb} = d + e \cdot \exp(f \cdot p_{evap}) \tag{5}$$

$$n = \frac{C_{p-suc}}{C_{v-suc}} \tag{6}$$

In this model, the compression process is assumed to be polytropic. The clearance volume factor is a function of the geometry only and is therefore assumed to be fixed for any given compressor. It is not normally provided by a manufacturer, so it is one of the parameters that need to be regressed along with the polytropic compression exponent, n. The pressure drop parameter, $\Delta p$, accounts for pressure drop in the suction line between the evaporator and the compressor. The combined efficiency, $\eta_{comb}$, accounts for inefficiencies such as motor inefficiencies, mechanical inefficiencies, and heat loss in the compressor. Compressor displacement volume and motor speed are also needed from manufacturer's data.

To train this model, mass flow rate, compressor suction temperature, compressor power consumption, and saturated evaporating and condensing temperatures need to be measured; compressor displacement volume and motor frequency need to be known. Not all of data are easy to determine for an operating air conditioner on-site. On the other hand, the main goal of this model is to generate a compressor map, which it does very well, and not to evaluate the performance of an operating air conditioner. However, since the compressor represents the largest source of power consumption in an air conditioning system as well as sourcing the refrigerant mass flow rate, it is desirable to use a model similar to this that is at least partially physically-based. Therefore, to maintain the advantages of this model and apply it at a system level, some improvements and modifications have been made. All the model improvements and validation studies performed in the following sections are based on data generated by detailed system simulation software in which forward models and compressor maps are used to estimate the performance of DX unit.

### 2.2 Condensing and Evaporating Temperature Model

For an operating air conditioner it is important to predict how much energy it consumes and how much cooling it provides under different operation conditions. It is also desirable that the models have the ability to accurately extrapolate results to operation conditions outside of those collected during the training period. Initially, model training will be performed using condensing and evaporating temperatures; however, after training the model will only require room air temperature and outdoor air temperature as model inputs, which are easy and inexpensive measurements to acquire. Because of this, a coil performance model is necessary to bridge the gap between air temperatures and refrigerant temperatures.
The systems targeted in this paper, and shown schematically in Figure 1 have the following characteristics:

1) Constant evaporator and condenser fan speed
2) ON/OFF control to the system for matching cooling demand

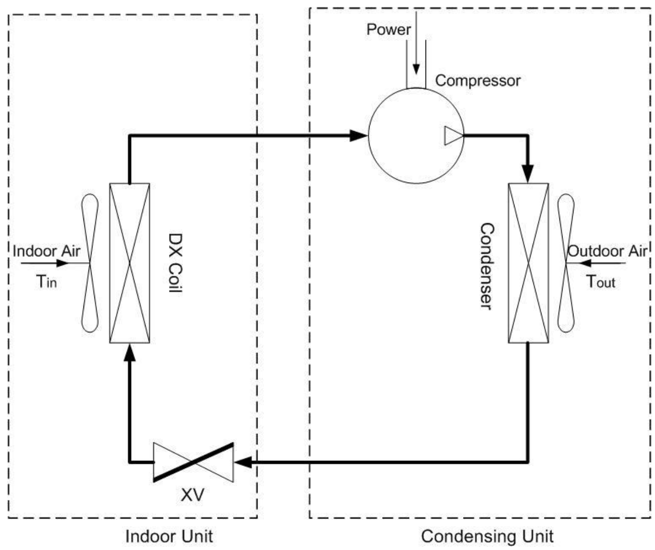

**Figure 1**. Schematic diagram of split DX air conditioner

On the condenser side it is assumed that there is no latent load, so the saturated condensing temperature can be directly linked to the outdoor dry bulb temperature. On the other hand, it is necessary to consider the possibility of a wet or dry coil on the evaporator side. Under dry coil conditions the evaporating temperature is related to the inlet air dry bulb temperature, while under wet coil conditions it is assumed to be driven by the inlet air wet bulb temperature. If the air-side thermal resistance dominates the heat transfer performance of a coil, and if the air flow rate remains constant, the refrigerant saturation temperature should show a simple relationship with the air temperature.

Data generated from detailed system simulation software demonstrates, as shown in Figure 2, that the condensing temperature has a clear linear relationship to the outside air dry bulb temperature. A small amount of scatter in the data is observed at each outside air temperature, which means that the condensing temperature is also affected secondarily by other factors. However, this difference is small enough to be neglected without significant loss of accuracy in the final solution. Therefore, the condensing temperature can be modeled generally using following equation, which relates it to the outdoor air dry bulb temperature.

$$T_c = a_0 \cdot T_{out} + b_0 \tag{7}$$

Figure 3 shows the relationship between the evaporating temperature and the outside air dry bulb temperature. The plot can be seen as a series of parallel lines no matter if the coil is under dry or wet conditions. Therefore, the evaporating temperature shows a liner relationship with the outside air dry bulb temperature, which can be summarized generally with the following equation.

$$T_e = a_1 \cdot T_{out} + f\left(T_{in,db}, T_{in,wb}\right) \tag{8}$$

The function, $f\left(T_{in,db}, T_{in,wb}\right)$ will affect the evaporating temperature depending on the indoor air wet bulb or dry bulb temperature. When the outside air temperature is set to be a constant, the relationship between evaporating temperature and indoor air temperature can be examined and the results are shown in Figures 4 and 5.

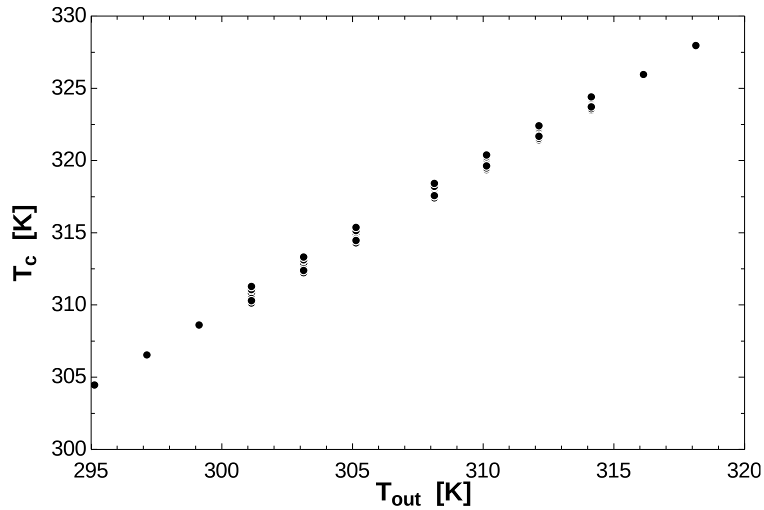


**Figure 2**. Condensing temperature versus outside air dry bulb temperature

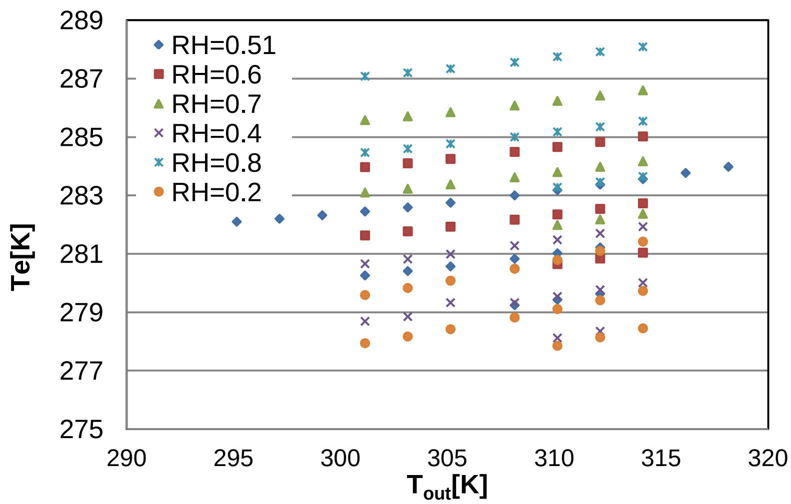


**Figure 3**. Evaporating temperature versus outside air dry bulb temperature

Based on Figure 4 under wet coil conditions and a constant outside air temperature, the evaporating temperature is linearly related to the indoor air wet bulb temperature. Figure 5 shows that under dry coil conditions and constant outside air temperature, the evaporating temperature is linearly related to the indoor air dry bulb temperature. Therefore, the following can be used to determine evaporating temperature.

$$T_e = (a_1 \cdot T_{out} + a_2 \cdot T_{in,wb} + b_1) \cdot (1 - DWC) + (a_1 \cdot T_{out} + a_3 \cdot \mathrm{T}_{in,db} + b_2) \cdot DWC \qquad (9)$$

In Equation 9, DWC is set to a value of 1 for dry coil conditions, and equals 0 when wet coil conditions exist.

### 2.3 Refrigerant Saturation Pressure Model

All of the model training and inputs are based on temperature measurements since they are inexpensive and easy to obtain; however, the compressor model requires pressure inputs, so it is necessary to relate the condensing and evaporating temperatures of the refrigerant to their saturation pressures. They can be related with sufficient accuracy using known properties at the critical state point and a characteristic refrigerant constant (Kenneth, 1994) according to the following equation:

$$P = 10^{\left(A - A \cdot \frac{TC}{T}\right)} \cdot PC \tag{10}$$

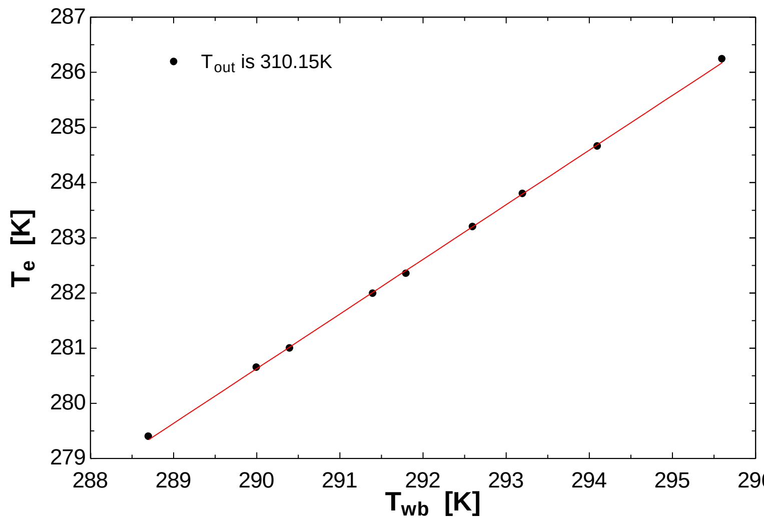


**Figure 4.** Evaporating temperature versus indoor air wet bulb temperature under wet coil condition

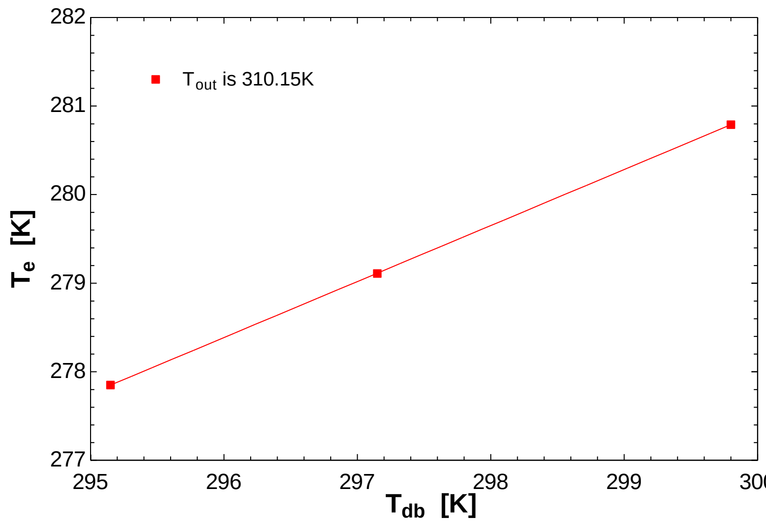


**Figure 5**. Evaporating temperature versus indoor air dry bulb temperature under dry coil condition

### 2.4 Combined Efficiency Model

In Jähnig's model, the combined efficiency (Equation 5) is evaluated based only on evaporating pressure. However, based on the system level analysis results it became necessary to modify the model to more accurately reflect this parameter. Figure 6 shows the combined efficiency trend under different operation conditions as a function of evaporating pressure. It became clear in analyzing the data that Jähnig's model can be used to extrapolate to larger evaporating temperature ranges with a constant outside air temperature, but when the outside air conditions begin to vary, the model is not able to accurately capture the unit performance any more.

To improve the combined efficiency model, correlations were investigated with respect to compressor pressure difference and also the pressure ratio across the compressor. Figures 7 and 8 show the relationship between combined efficiency and each of these two parameters under different outdoor and indoor air conditions. Figure 8 shows clearly that the combined efficiency parameter is better correlated with the pressure difference across the compressor.

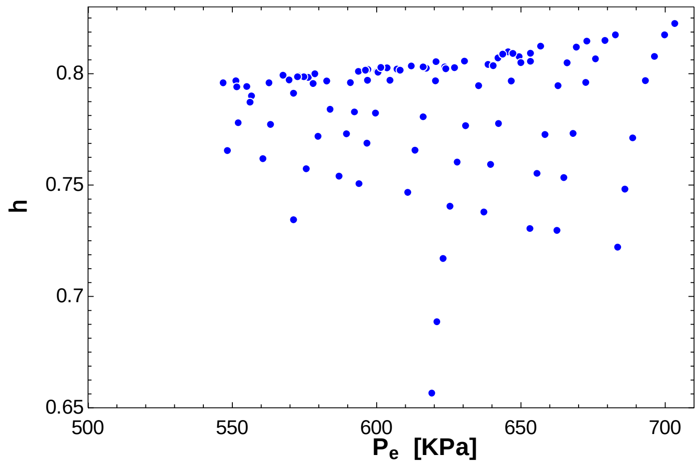


**Figure 6.** Combined efficiency versus evaporating pressure

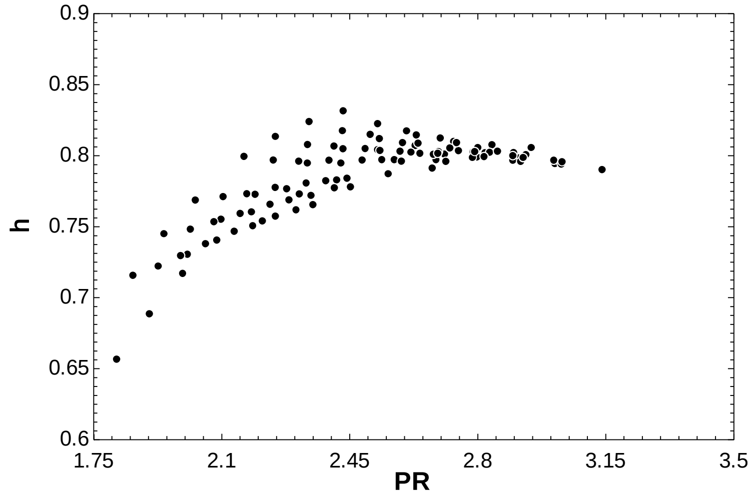


**Figure 7**. Combined efficiency versus pressure ratio

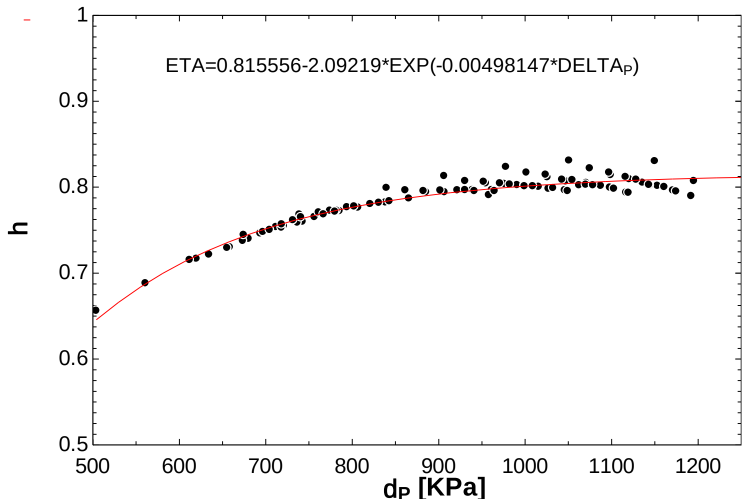


**Figure 8**. Combined efficiency versus pressure difference across compressor

At the right side of Figure 8, the efficiency nearly maintains a constant value. But on the left side, the efficiency goes down with decreased pressure difference across the compressor. Based on the plot, an exponential correlation can be generally applied to model the combined efficiency, and expressed as:

$$\eta = d + e \cdot \exp(f \cdot \Delta P) \tag{11}$$

**2.5 Power Consumption Model**

To avoid the need to measure mass flow rate in the equipment, the final power consumption model for the DX unit can be expressed as Equation (12). Combined with equation (3), (4), (10), (11) and (13), the full power consumption model is:

$$Power \cdot \eta_{comb} = \left( 1 + C - C\left(\frac{p_{\mathrm{dis}}}{p_{suc}}\right)^{\frac{1}{n}} \right) \cdot B \cdot \frac{n}{n-1} \cdot p_{suc} \cdot \left( \left(\frac{p_{dis}}{p_{suc}}\right)^{\frac{n-1}{n}} - 1 \right) \tag{12}$$

$$B = \frac{V \cdot RPM}{60} = \frac{Q_{capcity}}{q_v} \tag{13}$$

**2.6 Parameter Estimation**

The linear regression and least squares curve fit methods are used for determining model parameters. Linear regression is used to estimate parameters in the condensing and evaporating temperature models, and least squares curve fitting is used for parameter estimation of the power model. The objective function for the least squares curve fit is defined as follows:

$$OF_{power} = \sqrt{\frac{\sum_{i=1}^{N}\left(\frac{P_{real} - P_{cal}}{P_{mean}}\right)^2}{N}} \tag{14}$$

The relative error is defined as:

$$Err_{rel} = \frac{X_{real} - X_{cal}}{X_{real}} \tag{15}$$

# 3. MODEL VALIDATION AND RESULTS

The data used for model validation is based on manufacturer supplied data from a DX unit. The refrigerant is R410a. Outside air dry bulb temperature varies from 55 ℉ to 115 ℉ in 10 ℉ increments, with a constant relative humidity of 50%; indoor air dry bulb temperature changes from 65 ℉ to 85 ℉ in 5 ℉ increments, with repeated relative humidity of 29%, 49% and 68%. The indoor air fan and outdoor air fan are constant speed and provide constant air flow rate through the coils. There are a total of 106 data points in the set. A small part of the data is used for model training while the remaining data is used for model validation.

**3.1 Condensing and evaporating temperature model validation**

Two data points under the same indoor air conditions and different outdoor air conditions are enough to train the condensing temperature model. The final model equation for the unit under consideration is,

$$T_c = 0.9255 \cdot T_{out} + 31.14 \text{ [K]} \tag{16}$$

Figure 9 shows the extrapolated results of the model. The largest relative error is -0.59%, which makes the linear model sufficiently accurate for the application.

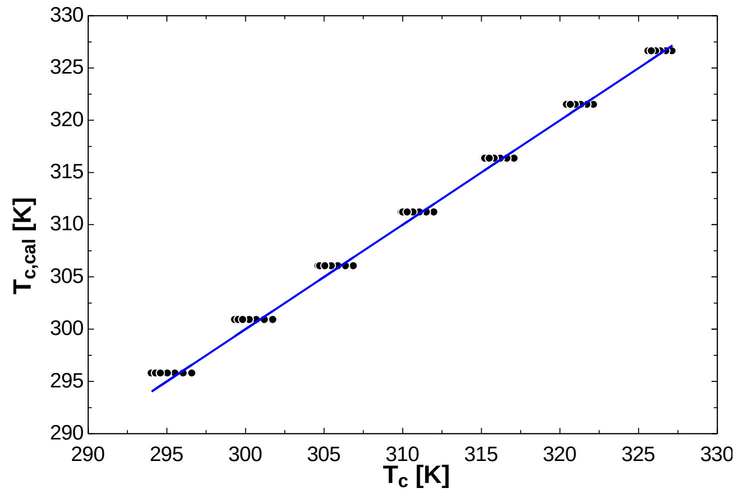


**Figure 9**. Extrapolated results for condensing temperature model

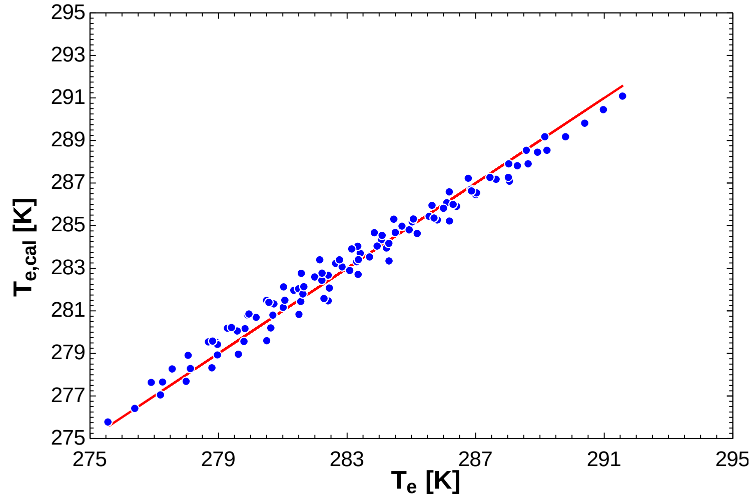


**Figure 10**. Extrapolated result for evaporating temperature model

Another 3 data point under the same outdoor air conditions, in addition to the above 2 data points can be used to train the evaporating temperature model. The final model is,

$$T_e = (0.1145 \cdot T_{out} + 0.77 \cdot T_{in,wb} + 25.34) \cdot (1 - DWC) + (0.1145 \cdot T_{out} + 0.674 \cdot \mathrm{T}_{in,db} + 47.2) \cdot DWC \text{ [K]} \quad (17)$$

Figure 10 shows the extrapolated results. The maximum relative error for this model is 0.61% which is also sufficiently accurate.

### 3.2 Pressure Model and Power consumption model

After the parameters for the condensing and evaporating temperature models have been determined, the power consumption model can then be trained. The first step is to calculate A and B at the known rating point condition, which are available from the unit manufacturer. The data for these calculations along with the results is given in Table 1.

The final model equation for relating saturation temperatures to pressures is shown in Equation 18, and the extrapolated results for this model is shown is Figure 11. The model has a maximum relative error of 0.15%.

$$P = 10^{\left(3.001 - 3.001 \times \frac{354.3}{T}\right)} \times 4925 \quad (18)$$

**Table 1**. Data set and calculation results for parameters A and B

| R type | $T_c$ [K] | $T_e$ [K] | $Q_{cap}$ [W] | Superheat [F] | Sub cooling[F] | Result | A | B | n |
|---|---|---|---|---|---|---|---|---|---|
| R410a | 316.25 | 285.57 | 6163 | 21.1 | 7.98 | | 3.001 | 0.000798 | 1.37 |

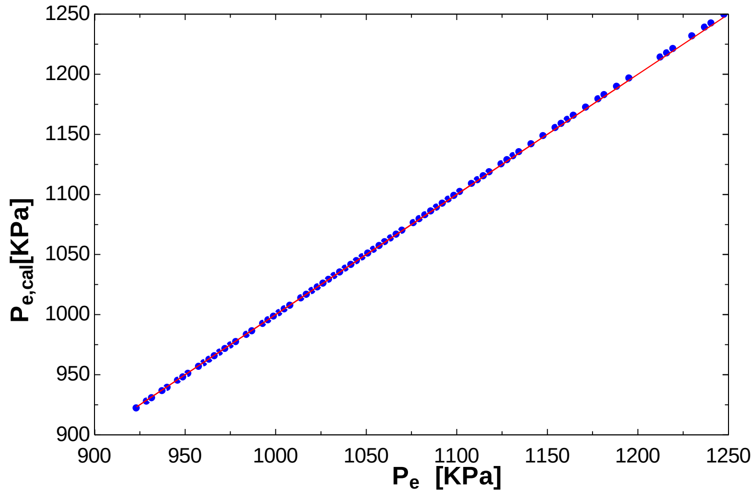


**Figure 11**. Saturation pressure model test result

The next step in the process is to train the power consumption model. Five unknown parameters need to be trained: C, Δp , d, e and f. The main goal of training this model is to use as few data points as possible with limited diversity and still maintain the ability to accurately extrapolate the results to a wide range of operating conditions outside of the training region. To achieve this goal several tests have been conducted. For the first test only 7 data points were used to train the model. All the data are under the exact same indoor air conditions, but each has a different outside air temperature. The training results for the parameters are shown in Table 2:

**Table 2**. Training result for test1

| C | d | e | f | Δp |
|---|---|---|---|---|
| 0.00002576 | 0.7983 | -0.3473 | -0.001 | 0.1492 |

Using these trained parameter results it is possible to calculate power consumption under other indoor and outdoor operating conditions and compare the results to the known unit performance. The results shown in Figure 12 indicate that the maximum relative error is 2.3% and the average relative error is only 0.58%.

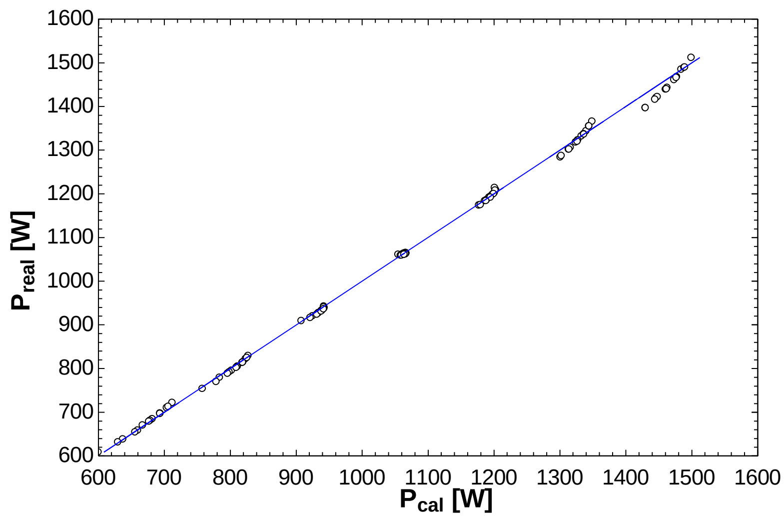


**Figure 12**. Power consumption model results using 7 data points for training

Since only five unknown parameters need to be trained, as few as five data points are necessary to actually do the training. The above test indicates that the model can be extrapolated very well to conditions different than the training data. To test how well the model can extrapolate to different outside air conditions, 5 data points under only 2 different outside air temperature conditions are used to train the model. Training results for the parameters are shown in Table 3.

**Table 3:** Training results for test 2

| C | d | e | f | Δp |
|---|---|---|---|---|
| 0.004419 | 0.7909 | -0.4768 | -0.001233 | 0.08969 |

Using these results to test the model under different operation conditions, the maximum relative error for power consumption is 2.95% and the average relative error is 1.2%. The error is a little larger than that of the first test since the outside operating temperature data diversity is smaller, but the result still shows acceptable accuracy. The comparison is shown in Figure 13.

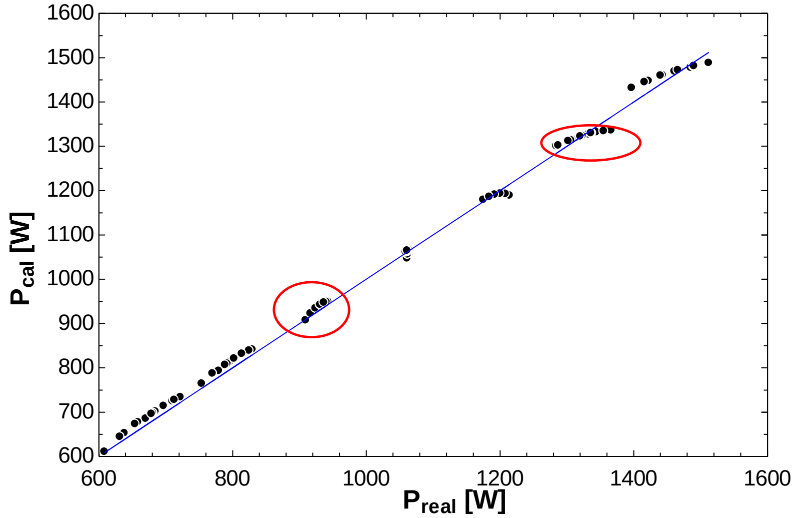


**Fig. 13** Power consumption model result using 5 data set for training

## 4. CONLUSIONS

A semi-empirical inverse model has been developed in this paper to estimate the power consumption of a DX air conditioner in residential buildings. Only 5 data points with two different outdoor air temperatures and limited diversity are needed to train the model. The test results show that the model has very good extrapolation ability. In the future the model will be implemented and tested on a field-installed DX air conditioner in a residential building.

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