1998

Contact Force Analysis in Trochoidal-Type Machine

J. B. Shung
Indiana University-Purdue University

CONTACT FORCE ANALYSIS IN TROCHOIDAL-TYPE MACHINE

by
John B. Shung, Assistant Professor
Department of Engineering, Indiana University-Purdue University, Fort Wayne
Fort Wayne, IN 46805-1499
e-mail: shung@engr.ipfw.edu

ABSTRACT

In designing a trochoidal-type machine, the contact forces between rotor and chamber need to be reduced in order to prolong the life of the machine. In this paper, a more realistic method to analyze contact forces not only between rotor and chamber but also between rotor and shaft and between two gears is presented. The location of the contact point between rotor and shaft, and the required input power under certain operating conditions are also estimated.

NOMENCLATURE

- e shaft eccentricity
- g gravitational acceleration
- k_e parameter for trochoidal curve
- n_c number of chamber lobe
- p_{in} the inlet pressure
- p_{out} the discharge pressure
- r_i radius of eccentric shaft
- r_r radius of the ring gear
- A_o the valve area
- C_d the discharge coefficient of the valve
- P the pitch point
- Q_j jth contact point between rotor and chamber
- T_G the thickness of the gear set
- T_R the thickness of the rotor and chamber
- V_R the volume of the rotor
- γ shaft angle
- ρ_f the mass density of the fluid
- ρ_R the mass density of the rotor material
- ω_S shaft angular velocity

INTRODUCTION

Trochoidal-type machines belong to the category of planetary rotation machines and offer significant advantages over reciprocating types of machinery, for example, simplicity, reliability and the possibility of higher speeds, and they have a wide variety of applications, namely, engines, pumps, compressors, and blowers. There are two major components, i.e. a rotor (or piston) and a chamber (or cylinder). If one component is referred to as the trochoid, then the other is referred to as the envelope. For example, in a conventional trochoidal-type pump, the rotor is a peritrochoid and the chamber is an outer conjugate envelope, whereas in a trochoidal-type pump without apex seal (which is still under development) the chamber is a peritrochoid and the rotor is an inner conjugate envelope. The later one has an advantage the former one because there is no cusp (point whose slope changes sharply) on the chamber [Shung and Pennock, 1993].

In order to analyze contact forces between rotor and chamber in a trochoidal-type machine, two classes can be established. The first class is the one which does not have guiding gear set, such as a gear pump or a fluid motor, and the motion of the rotor is caused by the contact forces between rotor and chamber. Contact forces in this class have been studied by several researchers [Hall, 1968; Colbourne, 1976]. The second class is the one which has a guiding gear set or gear train, such as Wankel engine or rotary compressor. Contact forces of this class have been studied by the author [Shung and Pennock, 1994]. But at that time, the contact point between rotor and shaft was assumed at the intersection of the shaft and the line connecting the centers of rotor and chamber. This assumption is true only when the speed of the rotor is high and the body force (or inertial force) acting on the rotor due to planetary rotation is the dominant force. Therefore, there is a need to extend the contact force analysis so that the contact point between rotor and shaft can be predicted at any operating condition. This paper is to present the extended contact force analysis. Also in this analysis, the spring constants at contact points are modeled as cylindrical contacts which is more accurate than the one presented in the previous study. Furthermore, a trochoidal-type pump without apex seal is used as an example. Both rotor and chamber have smooth profiles but the contact points on chamber change their locations continuously when shaft rotates. Therefore, for each shaft position, the locations of contact points between rotor and chamber have to be calculated first, which is shown in earlier work [Li and Shung, 1996].

ASSUMPTIONS

In order to study contact forces between rotor and chamber, between rotor and shaft, and between two gears, following assumptions are adopted.

Pressure Inside Pocket

(1) The fluid in the pocket is incompressible. (2) There is no friction between the fluid and the wall. (3) The flow rate and the pressure on both sides of each valve are uniform. (4) Each valve is modeled as an orifice plate with the same discharge coefficient.

Body Force

(1) The gravitational acceleration is in the direction of negative Y_t axis. (2) The machine is at steady state and there is no angular acceleration on the shaft.

Contact Forces

(1) There is no friction at all contact points. (2) Shaft rotates counter clockwise and rotor rotates clockwise. (3) All contact points are modeled as cylindrical contacts. (4) All contact forces are on the same plane and the off-plane moment generated by contact force between two gears can be neglected.

FORCE ANALYSIS

Consider a trochoidal-type pump as an example. It has five intake valves and five discharge valves. Each pair of valves is located at the bottom of a chamber lobe. At certain operating conditions, the off-center shaft rotates to a particular angle. Place a global coordinate system X_t-O_t-Y_t at the center of the chamber as shown in Fig. 1. The coordinates of the contact points between rotor and chamber, Q_j, are obtained by solving trochoid equation and its conjugate envelope equation simultaneously (Li and Shung, 1996). The coordinates of pitch point P are expressed as

$$\overrightarrow{O_tP} = P_x\vec{i} + P_y\vec{j} = r_r\left(\cos\gamma\,\vec{i} + \sin\gamma\,\vec{j}\right) \tag{1}$$

The coordinates of the rotor center O_e are expressed as

$$\overrightarrow{O_tO_e} = O_{ex}\vec{i} + O_{ey}\vec{j} = e\cos\gamma\,\vec{i} + e\sin\gamma\,\vec{j} \tag{2}$$

Pressure Force And Body Force (Inertial Force)

The pressure inside the jth pocket for the intake stage or the discharge stage is expressed, respectively, as

$$p_j = p_{in} - \frac{\rho_f T_R^2 (dA/dt)_j^2}{2C_d^2 A_o^2} \quad \text{or} \quad p_j = p_{out} + \frac{\rho_f T_R^2 (dA/dt)_j^2}{2C_d^2 A_o^2} \tag{3}$$

where $$(dA/dt)_j = \frac{\omega_S}{2(1-k_e)}\left(PQ_{j+1}^2 - PQ_j^2\right);\ j = 1, \ldots, n_c \tag{4}$$

Also the rate of area change is positive during intake stage and negative during discharge stage. The equivalent pressure force in the jth pocket can be expressed as

$$\vec{F}_{pj} = p_j T_R \left[-\left(Q_{(j+1)_y} - Q_{jy} \right) \vec{i} + \left(Q_{(j+1)_x} - Q_{jx} \right) \vec{j} \right] \tag{5}$$

The body force acting at the center of the rotor due to the planetary rotation can be expressed as

$$\vec{F}_b = \rho_R V_R \left[\left(\omega_S^2 e \cos\gamma \right) \vec{i} + \left(\omega_S^2 e \sin\gamma - g \right) \vec{j} \right] \tag{6}$$

Contact Force (Tangential Component) Between Rotor and Shaft

Consider moments at pitch point due to equivalent pressure forces.

$$M_{pj} = \left(R_{jx} - P_x \right) F_{pjy} - \left(R_{jy} - P_y \right) F_{pjx};\ j = 1\ to\ n_c \tag{7}$$

$$\text{where } R_{jx} = \frac{\left(Q_{(j+1)_x} + Q_{jx} \right)}{2} \quad \text{and} \quad R_{jy} = \frac{\left(Q_{(j+1)_y} + Q_{jy} \right)}{2} \tag{8}$$

Consider moment at pitch point due to body force.

$$M_b = (O_{ex} - P_x) F_{by} - \left(O_{ey} - P_y \right) F_{bx} \tag{9}$$

For a chamber with five lobes, contact forces between rotor and chamber are denoted by F_1 to F_5, contact force between rotor and shaft is denoted by F_6 which can be decomposed into two components (tangential one and normal one as shown in Fig. 1), and contact force between pinion gear and ring gear is denoted by F_7. The angles of these forces with respect to the global coordinates (X_t-O_t-Y_t) are:

$$\xi_j = \arctan\left(P_y - Q_{jy}, P_x - Q_{jx} \right);\ j = 1\ to\ 5 \tag{10}$$

$$\xi_{6n} = \arctan\left(O_{ey} - P_y, O_{ex} - P_x \right) \quad \text{or} \quad \xi_{6n} = \arctan\left(P_y - O_{ey}, P_x - O_{ex} \right) \tag{11}$$

$$\xi_{6t} = \gamma + 90° \tag{12}$$

$$\xi_7 = \gamma + \phi - 90° \tag{13}$$

Since all contact forces passing through pitch point P except F_6, the moment at pitch point due to contact force is:

$$\vec{M}_6 = \overrightarrow{PH} \times \vec{F}_6 = \overrightarrow{PO_e} \times \vec{F}_6 = \overrightarrow{PO_e} \times \vec{F}_{6t} \tag{14}$$

Consider the moment balance equation at P.

$$\Sigma_{j=1}^{5} M_{pj} + M_b + M_6 = 0 \tag{15}$$

F_{6t} can be obtained as

$$F_{6t} = \frac{-\left(\sum M_{pj} + M_b \right)}{(O_{ex} - P_x)\sin\xi_{6t} - \left(O_{ey} - P_y \right)\cos\xi_{6t}} \tag{16}$$

Contact Forces Between Rotor And Chamber, Between Rotor And Shaft, And Between Two Gears

Consider force balance equations.

$$\Sigma_{j=1}^{5} \overrightarrow{F}_j + \overrightarrow{F}_{6n} + \overrightarrow{F}_7 + \overrightarrow{F}_s = 0 \quad (17)$$

where $\overrightarrow{F}_s = \Sigma_{j=1}^{5} \overrightarrow{F}_{pj} + \overrightarrow{F}_b + \overrightarrow{F}_{6t}$ (18)

Introduce a local coordinates X_f-P-Y_f located at *P*. The X_f-axis is chosen coincident with the sum of the known forces F_s as shown in Fig. 1. The angle between X_t axis and X_f axis can be expressed as

$$\eta = \arctan\left(\frac{F_{sy}}{F_{sx}}\right) \quad (19)$$

The angles of the contact forces passing through *P* with respect to the X_f axis can be expressed as

$$\psi_j = \xi_j - \eta;\ j = 1\ to\ 7 \quad (20)$$

Now there are nine equations but fourteen unknowns, i.e. F_1, F_2, F_3, F_4, F_5, F_{6n} and F_7. Deformations have to be considered. Let the displacements of the rotor along X_f axis and Y_f axis due to all forces acting on it be ΔX_f and ΔY_f respectively. Since all the unknown contact forces passing through pitch point *P*, the deformations due to these contact forces can be expressed as

$$u_j = \Delta X_f \cos\psi_j + \Delta Y_f \sin\psi_j;\ j = 1\ to\ 7 \quad (21)$$

Because the contact forces are always pointing towards the pitch point, any positive deformation indicates no contact and its value will be replaced by zero.

$$\text{if } u_j < 0,\ F_j = k_{RC}|u_j|; \quad \text{otherwise } F_j = 0,\ j = 1\ to\ 5 \quad (22)$$

$$\text{if } u_6 < 0,\ F_{6n} = k_{RS}|u_6|; \quad \text{otherwise } \xi_{6n} = \xi_{6n} + 180° \quad (23)$$

$$\text{if } u_7 < 0,\ F_7 = k_{RG}|u_7|; \quad (24)$$

where k_{RC}, k_{RS} and k_{RG} are spring constants between rotor and chamber, between rotor and shaft, and between two gears respectively. They are constants which depend on material properties and contact lengths, and can be derived from cylindrical contacts as shown in Appendix A. Now there are sixteen equations and sixteen unknowns. ΔX_f and ΔY_f can be solved and expressed as following.

$$\Delta X_f = \frac{F_s}{K_b} \quad \text{and} \quad \Delta Y_f = -K_a \Delta X_f \quad (25)$$

where $K_a = \frac{K_1}{K_2}$ and $K_b = K_3 - K_a K_1$ (26)

$$K_1 = k_{RC}\Sigma_{j=1}^{5}\cos\psi_j\sin\psi_j + k_{RS}\cos\psi_6\sin\psi + k_{RG}\cos\psi_7\sin\psi_7 \quad (27)$$

$$K_2 = k_{RC}\Sigma_{j=1}^{5}\sin^2\psi_j + k_{RS}\sin^2\psi_6 + k_{RG}\sin^2\psi_7 \quad (28)$$

$$K_3 = k_{RC}\Sigma_{j=1}^{5}\cos^2\psi_j + k_{RS}\cos^2\psi_6 + k_{RG}\cos^2\psi_7 \quad (29)$$

When calculating K_1, K_2 and K_3, only terms with none zero contact forces will be included. This is accomplished by an iterative procedure as shown in Fig. 2. Furthermore, contact force between rotor and shaft is:

$$F_6 = \sqrt{F_{6t}^2 + F_{6n}^2} = \sqrt{F_{6tx}^2 + F_{6ty}^2 + F_{6nx}^2 + F_{6ny}^2} = \sqrt{F_{6x}^2 + F_{6y}^2} \quad (30)$$

$$\xi_6 = \arctan\left(\frac{F_{6y}}{F_{6x}}\right) \tag{31}$$

The location of the contact point between rotor and shaft is:

$$\vec{H} = r_i\left(\cos\xi_6 \vec{i} + \sin\xi_6 \vec{j}\right) + e\left(\cos\gamma \vec{i} + \sin\gamma \vec{j}\right) \tag{32}$$

The required input power to the eccentric shaft is:

$$P = T\omega_s = (eF_{6t})\omega_s \tag{33}$$

CONCLUSIONS

In this paper, a more realistic force analysis on a trochoidal-type pump without apex seal has been presented. With minor modifications, this analytical method should be able to apply to other class two trochoidal-type machines as mentioned in the introduction, such as Wankel engine. From this research, following conclusions can be drawn. (1) Not only contact forces between rotor and chamber can be estimated, but also contact forces between rotor and shaft and between ring gear and pinion can be obtained. (2) Under different operating conditions, the location of the contact point between rotor and shaft can be predicted. (3) The required input power under certain operating conditions can be estimated. (4) The results from this analytical method can be used to verify the results from a numerical model such as a finite element model.

REFERENCES

Colbourne, J.R., "Reduction of the Contact Stress in Internal Gear Pumps", ASME, 76-DET-40; Journal of Engineering for Industry, Trans. ASME, Series B, Vol. 98, No. 4, pp. 1296-1300, November, 1976.

Hall, A.S., Jr., "Gerotors", Internal Report, Purdue University, 1968.

Li, S. and Shung, J.B., "Geometry and Pressure in a Trochoidal-Type Machine Without Apex Seal", Journal of Mechanical Design, Vol. 118, pp. 209-213, June, 1996.

Norton, R.L., Machine Design, An Integrated Approach, Prentice-Hall, New Jersey, 1998.

Shung, J.B. and Pennock, G.R., "Geometry for Trochoidal-Type Machine with Conjugate Envelopes", Mechanism and Machine Theory, Pergamon Press, Ltd., Oxford, England, Vol. 29, No. 1, pp. 25-42, 1994.

Shung, J.B. and Pennock, G.R., "The Direct Contact Problem in a Trochoidal-Type Machine", Mechanism and Machine Theory, Pergamon Press, Ltd., Oxford, England, Vol. 29, No. 5, pp. 673-689, 1994.

APPENDIX A

The spring constants between rotor and chamber, between rotor and shaft and between two gears can be derived in a similar way. Consider any of them as two cylinders in contact. One can define a cylindrical geometry constant that depends on the radii R_1 and R_2 of the two cylinders.

$$B = \frac{1}{2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

The contact-patch half-width a can be expressed as [Norton, 1998]:

$$a = \sqrt{\frac{2}{\pi}\frac{m_1+m_2}{B}\frac{F}{T}} \quad \text{and} \quad m_1 = \frac{1-\nu_1^2}{E_1};\ m_2 = \frac{1-\nu_2^2}{E_2}$$

where ν and E are Poisson' ratio and Young's modulus of the material respectively, and T is the length of contact along the cylinder axis (or cylinder thickness). Consider a segment inside a sector of a circle with angle of β and radius R. After applying Binomial series approximation, the relationship between segment height h and segment length $2a$ can be expressed as:

$$h = a\tan\left(\frac{\beta}{4}\right) = R - \sqrt{R^2 - a^2} \approx \frac{a^2}{2R} \text{when } a \ll R$$

The spring constant at cylindrical contact can be expressed as:

$$k = \frac{F}{h_1+h_2} \approx \frac{F}{\frac{a^2}{2}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)} = \frac{F}{a^2 B} = \frac{\pi T}{2(m_1+m_2)}$$

When substituting appropriate material constants and component thickness, the desired spring constant is obtained.

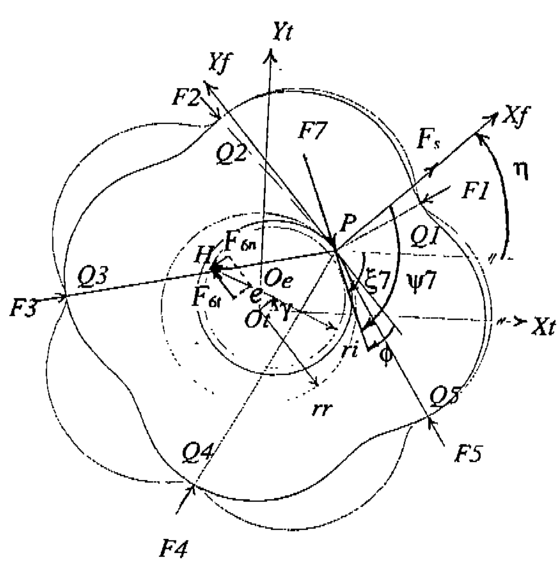

Fig. 1 All contact forces acting on the rotor & pinion

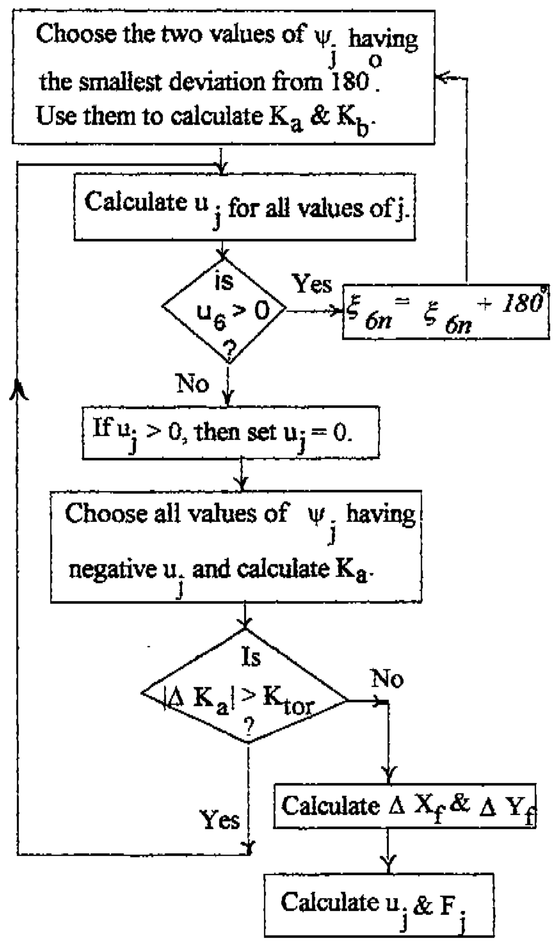

Fig. 2 Flow chart for calculating contact forces.