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On the anechoic termination assumption when modeling exit pipes

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ABSTRACT

The anechoic assumption has long been used for the modeling of long exit pipes or ducts of acoustic elements [1, 2, 3, 4, 5, 6, 7]. Especially in compressor discharge line modeling, the exhaust pipe from the compressor manifold leading all the way to the condenser is usually assumed to be anechoic since it is very long. Also, experimental efforts have been made by investigators using different acoustical testing rigs to simulate anechoic terminations. In this study, the effect of the discharge pipe length of a muffler model on the transfer function is discussed on a theoretical basis. The transfer functions of a muffler model with different discharge pipe lengths and dampings are simulated and compared. The results mimic actual data which would be measured experimentally if the pipe length is finite instead of infinitely long or if absorption material is used. Furthermore, a horn type exit is investigated. The permissibility of the anechoic termination assumption in realistic modeling is also discussed.

BASIC MODELS

For the muffler as shown in Figure 1, the four pole parameters $A, B, C, D$ of this muffler are known [8]. If an anechoic termination assumption is applied, the transfer function of this muffler can be expressed as

$$
\frac{P_2}{Q_1} = \frac{1}{A \frac{S}{pc} + B},
$$

where $P_2$ is the acoustic pressure at muffler exit, $Q_1$ is input volume velocity, $S$ the cross sectional area of the exit pipe, $\rho$ gas density and $c$ sound speed.

Muffler with Finite Length Exhaust Pipe

In contrast, if the length of the exhaust pipe is finite, the four poles of this pipe can be modeled by a 1-D method [10]:

$$
A_p = \cosh(\gamma L),
$$

$$
B_p = \frac{j\omega S}{\rho c^2 \gamma} \sinh(\gamma L),
$$

$$
C_p = \frac{\rho c^2 \gamma}{j\omega S} \sinh(\gamma L),
$$

$$
D_p = \cosh(\gamma L),
$$

where $\gamma = \frac{\xi \sqrt{\omega}}{2cd} + \frac{j}{c}$; $\xi$ is the damping ratio, $d$ the diameter of the pipe, and $L$ the length of this pipe.
The acoustic pressure at the output of this exhaust pipe can be simply assumed as zero if the frequency range of interest is not very high \([10]\), i.e.,
\[ P_o = 0. \] (6)
Therefore, the transfer function of this muffler can be derived:
\[ \frac{P_2}{Q_1} = \frac{C_p}{AA_p + BC_p}. \] (7)

However, if the frequency range of interest is very high, the end correction at the output of the pipe has to be considered \([13]\). Thus, the output impedance has to be used instead:
\[ \frac{P_o}{Q_o} = \frac{\rho c}{S} \left[ 0.0625(kd)^2 + j0.3kd \right], \] (8)
where \( k = \frac{\omega}{c} \) is the wave number.

**Muffler with Horn Shape Exhaust Pipe**

Another example as shown in Figure 2 is the muffler attached to a horn shape exhaust pipe. If the pipe is characterized by the flare relation \([11]\)
\[ S(x) = S_0 e^{2mx}, \] (9)
where Cartesian coordinates are used, the 1-D wave equation for this pipe is \([11]\)
\[ \frac{d^2 P}{dx^2} + 2m \frac{dP}{dx} + k^2 P = 0. \] (10)
The four pole parameters for this 1-D hyperbolic pipe model can be derived by solving the above wave equation as
\[ A_p = \frac{jS_0 e^{mL}}{\rho c} \frac{k}{k'} \sin(k'L), \] (11)
\[ B_p = \frac{1}{e^{2mL}} \left[ \cos(k'L) + \frac{m}{k'} \sin(k'L) \right], \] (12)
\[ C_p = e^{mL} \left[ \cos(k'L) - \frac{m}{k'} \sin(k'L) \right], \] (13)
\[ D_p = \frac{jpc}{S_0 e^{2mL}} \frac{k}{k'} \sin(k'L), \] (14)
where
\[ k' = \sqrt{k^2 - m^2}. \] (15)
The transfer function for the muffler can be obtained from Equation 7 by the same way.

**MUFFLER ATTACHED TO EXIT PIPES OF DIFFERENT LENGTHS**

Long exhaust pipes are the most usual way to simulate the anechoic terminations \([1, 6, 8]\). The comparisons of the transfer functions for the muffler attached to different length exhaust pipes with modest damping are illustrated in Figure 3.

For a very short exhaust pipe \((L = 1m)\) as shown in Figure 3 (a), there are many spikes compared to the case of anechoic termination. The spikes come from the resonances of the finite pipe, which reflects the acoustic wave from the flow discontinuity, i.e., the open end of the pipe, back into the muffler cavity.

If the length of the pipe is increased to \(L = 10m\) as shown in Figure 3 (b), the number of spikes from pipe resonances is also increased. Because the longer the pipe is, the more resonances there are. However, the magnitudes of the spikes are decreased due to the dissipation of energy in the long pipe.

Figure 3 (c) shows the comparison for the pipe length \(L = 100m\). The number of resonances is increased furthermore, but the magnitudes of the spikes are much smaller owing to the damping effect, as expected.

As long as the pipe is very long \((L = 10000m)\), all the resonances from the exhaust pipe would be dissipated. Therefore, a long exit pipe can be approximated to an anechoic termination as shown in Figure 3 (d).
**MUFFLER ATTACHED TO A FINITE PIPE CONTAINING ABSORPTION MATERIALS**

Another way usually applied for anechoic termination is by using absorption materials [2]. The advantage of this approach is to reduce the space for the experimental setup. The four poles for the absorption materials are very complicated [12]. However, in this study, only the damping parameter $\xi$ of the 1-D pipe model will be adjusted to simulate different absorption materials.

If the damping effect is small ($\xi = 0.0001$), the spikes of the muffler transfer function are large as shown in Figure 4 (a).

The damping factor of the absorption material is increased to $\xi = 0.001$, the spikes from the pipe resonances are apparently reduced as illustrated in Figure 4 (b). Since the length of the pipe is the same for all the simulations in this case, the number of resonances is the same.

Further increasing the damping ratio can effectively dissipate the resonance spikes and finally approach an anechoic termination (Figures 4 (c) and (d)).

**MUFFLER ATTACHED TO AN EXPONENTIAL HORN**

This approach is most widely used in duct acoustics for anechoic termination [3, 7]. The anechoic termination is achieved intuitively by reducing the flow discontinuity at the open end of the exhaust pipe. This case is discussed in this section.

For the muffler connected to a straight pipe ($m = 0$), the transfer function of this muffler compared to that with an anechoic termination is shown in Figure 5 (a). There are many spikes due to the resonances of the exhaust pipe.

If the pipe is flared ($m = 1$ and $m = 10$), the pipe resonances in low frequency range are eliminated because of the cutoff frequency as illustrated in Figures 5 (b) and (c) respectively. However, the transfer function in the very low frequency range is shifted from the anechoic termination assumption.

All the pipe resonances should go away, if the open end of this pipe is infinitely large since there is no flow discontinuity. It is shown in Figure 5 (d) for $m = 100$, as expected. Note that the transfer function no longer match the anechoic termination assumption. This is due to the limitation of the current 1-D exponential horn model. Therefore, this 1-D model can only provide an indication. For very large horns, a model taking into account the cross sectional modes should be developed. But this is not the purpose of this paper.

**CONCLUSION**

Three common ways of simulating anechoic terminations in experimental test rigs are discussed on a theoretical basis. Either a long exhaust pipe, or absorption material, or a horn shape exit can approximate the anechoic termination by careful treatment. However, in actual life, there is no anechoic termination. Therefore, anechoic termination is only an assumption which provides a convenient basis for modeling, and may need to be carefully examined in certain applications.

**References**


Figure 1: A muffler with a finite length exit pipe.

Figure 2: A muffler with an exponential horn exit pipe.

Figure 3: Comparison for the transfer functions $\frac{P_2}{Q_1}$ of a muffler attached to an anechoic termination (---) and a finite length pipe (-----), where damping ratio $\xi = 0.0001$ and pipe length (a) $L = 1\text{m}$, (b) $L = 10\text{m}$, (c) $L = 100\text{m}$ and (d) $L = 10000\text{m}$.
Figure 4: Comparison for the transfer functions $\frac{P_2}{Q_1}$ of a muffler attached to an anechoic termination (---) and an exit pipe of finite length $L = 1$ (---). (a) $\xi = 0.0001$, (b) $\xi = 0.001$, (c) $\xi = 0.01$ and (d) $\xi = 0.1$.

Figure 5: Comparison for the transfer functions $\frac{P_2}{Q_1}$ of a muffler attached to an anechoic termination (---) and an exit exponential horn (---). (a) $m = 0$, (b) $m = 1$, (c) $m = 10$ and (d) $m = 100$.  

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