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ABSTRACT

In this work we consider algorithmic approaches to the placement problem for surface mounted components and present theoretical results on the minimum number of measurements that are needed to reconstruct images of the components and circuit boards. Initial placement of the chip over the mount area may be slightly incorrect. Machine vision has been used to achieve high accuracy in the placement by providing feedback to the controller on the position of the component's leads relative to the soldering pads. Many traditional 2D vision techniques for image registration have been used to address this problem. Here we take a different approach and consider the problem of reconstructing the shape of the component and the circuit board from a set of projections. We assume that the image consists of overlapping rectangles (the leads and pads) that are iso-oriented and provide theoretical results on the minimum number of measurements that are needed to recover the shape. Such theoretical bounds can help in designing efficient methods to solve the problem of aligning the chip leads to the solder pads.

Keywords: Surface mounted devices, registration, shape from probing, reconstruction of overlapping rectangles

1 INTRODUCTION

In this paper we review algorithmic approaches to the placement problem for surface mounted components and present theoretical results on the minimum number of measurements that are needed to reconstruct images of the components and circuit boards. Both the pin size and the spacing between them on SMD components has been going down with advances in technology. As such, even a very precise mechanical placement no longer has sufficient accuracy. Also, small changes in the ambient temperature can cause a small dilation of the PCB (Printed Circuit Board), changing the relative positions of the pins and the footprints. Machine vision has been used to achieve high accuracy in the placement by providing feedback to the controller on the position of the components leads relative to the soldering pads.

We elucidate the above using a specific example from Thompson/CSF. Figure 1 provides an overview of the problem and a proposed solution. The input of the system is composed of 2 images, representing respectively the top left and the bottom right view of the component above PCB. The images are obtained by two CCD cameras: it is not possible to obtain a single image of the whole component, because the pipette which moves the component prevents having a camera just above the center of

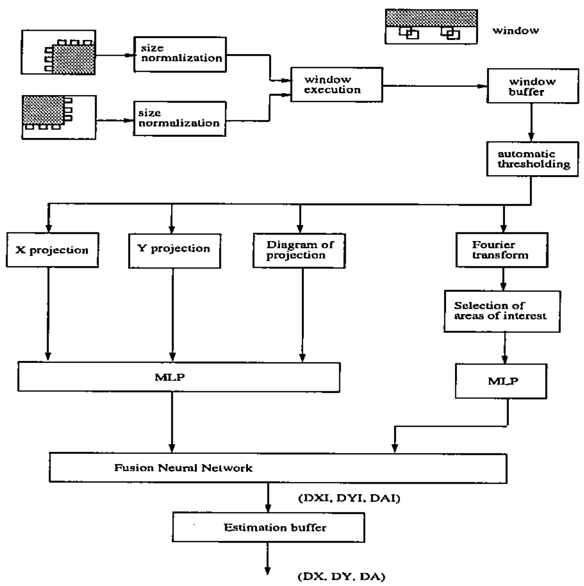

Figure 1: A process for registering chip leads with pads

the component. The images are normalized in size in order to suppress distortions due to the camera (the pixel of the camera is not square). Then four windows are extracted from these, two windows per image. Each window contains a partial view of a side of the component, as shown in Figure 1.

The windows are then thresholded, in order to provide binary images. On each one of the images, various features are computed. The first kind of features are projection curves from 3 directions (X, Y, and Diagonal). For instance, a point of the curve corresponding to the X projection represent the number of white pixels along a line parallel to the X axis. The second kind of features are areas of interest of the Fourier transform of the thresholded window.

The three projection curves feed a Multi-Layer perceptron, which provides as output an estimate of the roto-translation parameters of the positioning error (translation error of window in DXI, DYI, plus rotation error of window DAI). The areas of interest of the Fourier transform feed another MLP, which provides as output, another estimate. The two estimates are fused by a fusion neural network, in order to improve accuracy. Finally, the roto-translation parameters obtained for the four windows are used by a classical geometrical computation to estimate the positioning error of the whole component.

In a more abstract sense, the problem can be formulated as follows. Two rectangles exist in the image, formed by the leads and the pads. If the placement were extremely accurate, these would be in perfect registration. Due to the various reasons mentioned earlier, these will actually not be registered, but instead will overlap. It can be assumed that there is a rotation and/or translation that mounts all components leads on the pads. The geometry of the leads and of the pads is very regular: they have a rectangular shape and are equally spaced along the borders of the components and on the circuit. Classical image processing techniques have been employed to determine the rotation that brings the component into alignment with the board. An experimental system for automatic visual measurements of surface mounted devices is developed in.[5] High-angle infrared illumination is used to obtain high-contrast images of metal leads and solder pads after mounting on the circuit board. An automatic thresholding method is used to separate pad and lead areas from the background. Boundaries of the extracted features are run-length encoded and a connectivity analysis algorithm is used to determine the area and centroids of the pads and leads. Then, the best-fit lines trough the centroids of the pads and leads are calculated and their displacement and angular orientation is used to determine the required rotation/translation. The technique is tolerant of missing leads or pads and of the noise due to lighting or sensor resolution effects. The system accuracy has been tested with 50 commercially manufactured printed circuit boards.

An efficient and practical algorithm for 2-D image registration is presented in[7]; it achieves high accuracy that is particularly important in surface mounted devices as components continue to shrink. The algorithm makes the assumption that a small congruence (that is, a rigid motion consisting of a rotation by a small angle and a translation by a small vector) is sufficient to bring an image into registration with the model. This assumption is reasonable for surface mounted device applications. The algorithm deals with an image consisting of points (for instance, the centroids of the pads) and a model consisting of line segments. It finds the congruence that minimizes the sum over all image points of the squared distance between each point and its nearest line segment in the model. Based on the number of points in the image that match line segments in the model or the total length of the model line segment that matches image points, the algorithm predicts the accuracy of the registration in advance to see whether the specifications can be met. Furthermore, the same data are used to estimate the current registration accuracy. Experiments on synthetic images show that the predicted, estimated and observed accuracies are all in agreement.

Other approaches to the placement problem based on image processing techniques include that of Buffa[2] who proposes a morphology-based processor which uses a custom-built image computer, and that of Susuki.[9] Generalized Hough Transforms[1] can also be used to this end. A host of Fourier analysis based schemes are also possible, since in general, the congruence needed to bring the leads and the pads in registration is likely to be small. Another possibility is that of template based matching. Various measures of similarity or difference, for instance cross correlation, can be used to obtain the congruence between the leads and the pads. One could use the class of "elastic" methods used to obtain the congruence as well. These include, for instance, the rubber mask method of Widrow[17] , the elastic image registration of Burr[3,4] , and the active contour models of Terzopoulos *et. al.*[15,16,14] .

Here we take a different approach and consider the problem of reconstructing the image of the component and the circuit board from a set of projections. We assume that the image consists of overlapping rectangles (the leads and pads) that are iso-oriented and provide theoretical results on the minimum number of measurements that are needed to distinguish between overlapping rectangles. This is in context of the solution scenario outlined earlier which is used by Thomson/CSF. Recall that such projections were being used to find the error in placement (in terms of the rotation and translation). These were then fused with similar results obtained from a Fourier analysis. The objective of this work is to show that the projections contain sufficient information to recover the overlapping rectangles.

2 OBJECT RECONSTRUCTION FROM PROJECTIONS

The problem of reconstructing an object from a set of projections along different directions has been studied in different contexts. In signal processing, a large amount of work has been done in computerized tomography[13] . A known result, based on the reversibility of the Radon transform, is that if the projections of an object along all directions in some plane are given, then the object can be completely reconstructed. A different approach to the reconstruction problem uses geometric techniques to determine the minimum number of projections that are needed to distinguish between different objects. With this approach interesting results have been obtained for restricted classes of objects, namely for the case of convex polygons.

We next give some definitions and present related work. An $X - ray$ probe of a polygon P with a line l is the length of the intersection of the polygon and the line. A collection of parallel x-ray probes gives a projection. Formally, a *projection* of P along a given direction θ, denoted by $H(P, \theta)$ is obtained as follows: Let b be the normal to the direction θ through the origin O. If l is a line in direction θ such that $l \cap P \neq \emptyset$, let $C(P, l)$ be the segment of l with one end on b of the same length as $l \cap P$. The projection of P is the union of all these segments $C(P, l)$ (fig. 2). It is well known that $H(P, \theta)$ is a convex polygon, whenever P is convex; furthermore, each vertex of $H(P, \theta)$ determines a line on which a vertex of P must lie.

Several authors[6,8,10,11] have studied the reconstruction problem for a convex object, since such a problem was first posed in 1961 by Hammer.[12] In particular, Gardner and McMullen[11] have proved the following two theorems:

Theorem 1. Let P_1 and P_2 be distinct convex bodies in E^2 with the same center of gravity. Let Θ be a set of directions, such that $H(P_1, \theta) = H(P_2, \theta)$ for each $\theta \in \Theta$. Then Θ is linearly equivalent to a subset of the directions of diagonals of some regular polygon.

Observe that the set of directions of diagonals of any regular polygon is "equally spaced" and that any equally spaced set of directions can be considered as a set of diagonals of a regular polygon. A subset of such a set consists of directions which are rational multiples of π. Sets which are affinely equivalent to such sets are called *affinely rational* and satisfy the conditions of the above theorem.

Theorem 2. Let $\Theta = \{\theta_1, .., \theta_4\}$. If the slopes of the θ_i with respect to some coordinate system have a transcendental cross ratio, then the projections in the directions $\theta_1, .., \theta_4$ distinguish between convex bodies in E^n.

It is clear from the above theorems that no three arbitrary directions will suffice to reconstruct a convex body.

The reconstruction problem has been considered by Edelsbrunner et al.[8] for various classes of probes and lower and upper bounds on the number of probes necessary to determine a convex polygon are provided for each class. An interesting result is obtained by allowing the directions of projection to be selected dependent on the object. They proved the following:

Theorem 3. Three (selected) projections are sufficient to determine a convex polygon P.

Proof. Let P be an n-gon. Consider two directions θ_1 and θ_2 not parallel to each other. Each vertex of a projection $H(P, \theta_i)$ defines a line on which must lie a vertex of P. There are up to n such lines for each projection. Two intersecting lines from two projections defines up to n^2 points at the intersection of these lines. The vertices of P must be a subset of these points. The third direction θ_3 is chosen so that the no two of these intersection points are collinear with θ_3. Thus the n points in the θ_3 projection uniquely identify each of the vertices of P.

3 RECONSTRUCTION OF RECTANGLES AND SQUARES

In this section we consider the problem of reconstructing a pair of rectangles or squares possibly overlapping. From the above results, if follows that three arbitrary projections are not sufficient to uniquely reconstruct a square. Indeed two squares have the same projections in the 4 directions determined by the 8 sides of the convex hull of the union of the two squares. However, four projections whose directions have transcendental cross ratio or three projections selected in a way dependent on the object are enough to determine a square.

We will next show that a single rectangle is determined by three projections independent of the object, the x (horizontal), y (vertical) and d (diagonal). We consider the case of a non-isothetic rectangle, that is a rectangle whose sides are not parallel to the x and y axes. (An isothetic rectangle is easily reconstructed from the x and y projections only). The projection $H(P, x)$ (resp. $H(P, y)$) defines 4 horizontal (resp. vertical) lines on which a vertex of P must lie. The vertices of P are at the intersections of these lines; furthermore, they are on the *extremal* lines, that is the lines in one direction that define a region containing the other two lines in the same direction. Since a vertex of P cannot be at the intersection of two extremal lines (otherwise the rectangle would be isothetic), there remain only 8 possible candidate vertices. They can be grouped to form four quadrilaterals Q_1, Q_2, Q_3, and Q_4, of which only two have the given $H(P, x)$ and $H(P, y)$ projections.

The ambiguity between two such rectangles can be eliminated if the third direction d is considered. In fact the $H(P, d)$ projection can easily discriminate between Q_1 and Q_2. As for Q_3 and Q_4, there will be still ambiguity if two candidate points are aligned along d. It can be shown that this is possible iff the two quadrilaterals are squares. Also, two non-overlapping rectangles or squares cannot in general be determined from three projections.

Consider now the reconstruction of two overlapping rectangles or squares. Reconstruction is obviously ambiguous when two vertices of the same rectangle are not visible; thus we assume that at least three vertices of each rectangle are visible. We show the following:

Theorem 4. Three selected projections are sufficient to determine two overlapping rectangles (or squares).

Proof. Let C be the concave polygon that is the union of two overlapping rectangles. We first show (as in Theorem 3) that three selected projections are sufficient to determine all the vertices of C. Consider two directions θ_1 and θ_2 not parallel to each other. Each vertex of a projection $H(C, \theta_i)$ defines a line on which must lie a vertex of C. There are up to 16 such lines for each projection. The vertices of C must be a subset of the intersection points of these two sets of lines. The third direction θ_3 is chosen so that the no two of these intersection points are collinear with θ_3. Thus the points in the θ_3 projection uniquely identify each of the vertices of C.

Once all the vertices have been identified, the concave polygon C can be reconstructed as follows. Construct the convex hull H of all the vertices of C. H consists of at most 8 vertices; let them be $v_0, \ldots, v_t$ sorted according to their angular coordinate in a given reference system. Consider the following cases.

case 1. H has 8 vertices.

Suppose C has 16 vertices. This is the case when every side of a rectangle intersects two consecutive sides of the other rectangle. Thus, the sides of the two rectangles are uniquely determined by the edges $(v_i v_{i+2})$, $i = 0, 7$. (From now on all sums of indices are mod 8). Suppose now that

C has 14 vertices. Two non consecutive edges of H must be sides of different rectangles. Let them be $(v_k v_{k+1})$ and $(v_{k+2} v_{k+3})$, for some k. The two rectangles are then given by: $(v_k v_{k+1})$, $(v_{k+1} v_{k+4}), (v_{k+4} v_{k+6}), (v_{k+6} v_k)$, and $(v_{k+2} v_{k+3})$, $(v_{k+3} v_{k+5}), (v_{k+5} v_{k+7}), (v_{k+7} v_{k+2})$, respectively. Now assume that two distinct concave polygons C and C', union of rectangles, can be determined corresponding to distinct indices k and k'. Since the two pairs of rectangles cannot share an edge, it can only be $k' = k + 4$. In such case, the edges in C incident to v_k, that is $(v_k v_{k+1})$ and $(v_k v_{k+6})$, form a wedge that contains in its interior the two edges $(v_k v_{k+2})$ and $(v_k v_{k+5})$ incident to v_k in C'. Thus C and C' cannot both have a $90°$ angle at v_k and this contradicts the hypothesis. Finally, suppose that C has 12 vertices. Let $h_i = (v_i v_{i+1})$, $i = 0, .., 7$, be the edges of H. Either the even-indexed edges or the odd-indexed edges of H are edges of C. Only one of these two choices is consistent with the remaining 4 vertices of C.

case 2. H has 7 vertices.

If C has 14 vertices exactly one edge of H is side of a rectangle and also edge of C. Let it be $(v_k v_{k+1})$. One rectangle then consists of the edges $(v_k v_{k+1})$, $(v_{k+1} v_{k+3})$, $(v_{k+3} v_{k+5})$,$(v_{k+5} v_k)$. The second rectangle has edges $(v_{k+2} v_{k+4})$,$(v_{k+4} v_{k+6})$ incident to vertex v_{k+4}. It can be easily shown that there cannot be two choices of an edge of H that corresponds to different overlapping rectangles since they would have to share at least one side. Suppose now that C has 12 or 11 edges. Two parallel edges of H must be edges of C. Among the 7 edges of the convex hull there can be at most 2 pairs of parallel segments. Only one of them is consistent with the remaining vertices of C. Finally, if C has fewer than 11 vertices at least two consecutive edges of C forming a $90°$ angle are on H. Since H cannot have $90°$ angles other than those formed by edges of C, C can be uniquely identified.

case 3. H has 6 vertices.

Suppose that C has 12 vertices. The 6 points of C not belonging to H must be aligned along two parallel lines, with three on each line. These lines are sides of a rectangle. Now, given the six points there is at most one way of doing so. If C has fewer than 12 vertices at least two consecutive edges of C forming a $90°$ angle are on H. As in case 2, this allows the unique identification of C.

case 4. H has 5 vertices.

H must have at least one $90°$ angle thus this case is similar to the previous one.

Since H cannot have fewer than 5 vertices, this concludes the proof.

4 CONCLUSION

The problem of accurate alignment of chip leads with soldering pads is an important one for surface mounted devices, especially as the size of these devices gets smaller. Various techniques from computer vision have been used to address this problem. In this paper, the authors present certain theoretical results pertaining to the reconstruction of geometrical figures from projection information. Specifically, they show that convex polygons formed by overlapping rectangles can be recovered from three selected projections. This problem arises in one of the methods used to solve the chip/pad registration problem in the industry.

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