A Finite Element Approach to Compressor Valves Motion Simulation

F. Fagotti
Embraco S. A.

M. G. D. de Bortoli
Embraco S. A.

R. Barbieri
Faculdade de Engenharia de Joinville
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Fabian Fagotti, Marcos G. D. de Bortoli
Embraco S. A., Mechanical and Materials Research Division
Rua Rui Barbosa, 1020, Cx. P. 91, 89219-901 - Joinville - SC - Brazil
Renato Barbieri
Faculdade de Engenharia de Joinville - FEJ/UDESC
Campus Universitário - Bom Retiro, 89223-100 - Joinville - SC - Brazil

ABSTRACT

This paper describes the application of the Finite Element Method to reed type compressor valve simulation. The element chosen allows a significant reduction of degrees of freedom without an impact on the accuracy. It also confers more flexibility to the simulation program, because the valve routine requires only the valve shape and an experimental obtained damping coefficient as input data. Furthermore, leading to better results in terms of displacements, it makes the input data obtained for stress analysis more reliable. These characteristics are also important when simulating valves for a wide range of operating conditions, e.g., variable speed compressors, start-up sequence and the same pump used for different refrigerants. As example of use, a parametric analysis of an actual suction valve is presented.

INTRODUCTION

Self-acting valves for compressors are the most important element for controlling suction and discharge gas flow. When the valve motion is abnormal, compressor efficiency drops significantly. The highly unsteady characteristic of the reed valves vibrating movement exacts a proper design, in order to achieve higher standards of compressor efficiency, noise and reliability. The optimum design has to balance over-compression, valve fluctuation, opening and closing timing, permissible stresses, etc., besides proper consideration of working fluid properties and compressor specifications. The problem is not trivial, considering the strong interactions among the requirements.

The valve modelings currently employed usually consider a single degree of freedom mass-spring approximation or a modal superposition model. These approaches lead to analyses somewhat deficient, since both models present results with limited accuracy. They also make use of damping coefficients that are virtually obtainable only by empirical or heuristic methods, which are not easy to handle. Designers customarily consider the finite elements method (FEM) to evaluate reed stresses and deformations in static analyses, which do not reflect the real operating conditions. Even when dynamic analyses use a FEM model, it has not a strict link to other routines. This approach does not afford sufficient flexibility to the designer and is sometimes very time-consuming, since it requires iterations. Furthermore, it does not allow a perfect understanding of the main phenomena under consideration, since it neglects some possible interactions among different phenomena.

This paper deals with a simple and efficacious FEM-based valve modeling. The main goal of the development was to accomplish accurately and not time-consuming valve motion evaluation routines, because they should be used in a compressor simulation program, which calls the valve motion routines thousands of times a cycle. The FEM model presented advantages concerning the quality of the results and the possibility to accurately simulate valves with complex shapes and with multiple components. It enables the coupling of the compressor operation simulation program with CAD routines as companion programs. The whole package is a very efficient tool to compressor valves design, since it takes into account most of the subjects related to the matter.

Compressor Simulation Program

The compressor simulation package and its companion programs have been described elsewhere (Fagotti et al., 1994), therefore it will not be shown here. In summary, it considers:

- one-dimensional, isentropic flow through valves
- in-cylinder gas transformation determined by the first law of thermodynamics
- suction and discharge bottle pulsations modeling based on a modified Helmholtz resonator
- orifice effective flow and force areas obtained experimentally, for better accuracy
- piston-cylinder clearance leakage modeled as uni-dimensional flow
- mechanical losses and motor efficiency considered as input data
solution achieved by numerical integration of the differential equations.
Since it has been previously validated, its results will not be questioned.

Element Formulation and Mathematical Model

Some authors claim FEM based valve dynamic modeling is computationally too expensive for developing a global simulation model, nevertheless they recognize its accuracy (e.g., de los Santos et al., 1991). Besides, known compressor simulation programs usually take in-cylinder and muffler gas behavior, which represent the boundary conditions for this problem, as input data (Papastergiou et al., 1982, Piechna, 1984). Another strong limitation of some FEM-based modelings concerns to the restricted variety of valve shapes which analysis is feasible. Actually, the main problem that leads to an assumed lack of flexibility lies in the proper choose of the element. Different approaches are possible, with a strong compromise between accuracy and computational velocity.

The modeling presented hereinafter studies the reed valve as a cantilever beam, clamped at one end, through the Finite Element Method. The major limitation of this first model is that it only considers the valve in free movement; it contemplates no other element than the reed. It is the case of some suction valves without stop. Nevertheless, with some additional efforts, it is applicable to more complex arrangements.

The following equation describes the application of Newton’s second law to the valve,

\[
[M]\left[ \frac{\partial^2 q}{\partial t^2} \right] + [C]\left[ \frac{\partial q}{\partial t} \right] + [K]\{q\} = \{F(t)\}
\]

where “[M],” “[C]” and “[K]” represent the mass, damping and stiffness matrices of the system, respectively and “\{F(t)\}” and “\{q\}” are the generalized exciting force and nodal displacement vectors.

The damping matrix is assumed to be diagonal, with constant components. The value depends strongly on the reed material and the characteristics of the valve clamp, besides a certain influence of the gasket, therefore it is strictly related to the arrangement, which is very similar for various models. For one degree of freedom, mass-spring valve modeling, the damping coefficient acts as a parameter to adjust theoretical and experimental results, without any physical meaning. Actually the model present acceptable results, however there must be previous experimental results to run the model, which infers prototypes and testing. The procedure works well, in spite of it is time-consuming when projecting a scenario of options, due to the trial-and-error characteristic. Using the FEM modeling allows the evaluation of this parameter directly from a simple experiment; the value is thus valid to any similar assembly.

The modeling considers a beam element with constant thickness and variable area and inertia moments in order to accomplish the best fit to the valve geometry using uni-dimensional finite elements. Figure (1) shows a sketch of the element; as well as the nodes degrees of freedom, transverse displacement “(v)” and rotation “(θ)”.

\[\text{Figure (1) - beam element with variable section and its degrees of freedom}\]
The number of degrees of freedom for the element chosen is similar to that obtained considering modal analysis. One would expect both models to give similar results. The main advantage of the FEM modeling concerns to the direct integration between the valve analysis and the compressor simulation, which facilitates the some procedures. The results for displacements are very good; stress results should not be reliable due to inherent limitations of the model, in spite of they are qualitatively acceptable. In order to obtain better results for stresses, one should use more complex elements.

It was used the second theorem of Castigliano (Boresi & Lynn, 1974) when calculating the element’s stiffness matrix, in order to avoid numerical integrations. This procedure leads to the following equation.

\[
[K] = \begin{bmatrix}
K_{11} & K_{12} \\
K_{21} & K_{22}
\end{bmatrix}
\]

Submatrices “\([K_{11}]\)” and “\([K_{22}]\)” are evaluated directly through the theorem,

\[
q_i = \frac{\partial \Pi}{\partial F_i}
\]

where \([q_i]\) represents the generalized displacement in the same direction and application point of force “\([F_i]\)”; “\(\Pi\)” is the energy necessary to deform the element. Submatrices “\([K_{12}]\)” and “\([K_{21}]\)” are evaluated considering force and momentum equilibrium, as follows.

\[
\sum F = F_1 + F_2 = 0 \\
\sum M = M_1 + M_2 = 0
\]

The element’s mass matrix is obtained with usual interpolation functions used to calculate straight beam elements; a generic component of this matrix is described by

\[
m_{ij} = \int_0^L \rho h b(x) \phi_i(x) \phi_j(x) dx
\]

where “\(\rho\)” is the material’s density, “\(h\)” and “\(b(x)\)” the element’s thickness and width and “\(\phi(x)\)” the Hermite polynomials, which are defined as

\[
\phi_1(x) = 1 - 3(x/L) + 2(x/L)^3 \\n\phi_2(x) = x - 2(x^2/L) + (x^3/L^2) \\n\phi_3(x) = 3(x/L) - 2(x/L)^3 \\n\phi_4(x) = -2(x^2/L) + (x^3/L^2)
\]

Components of force vector are evaluated directly through the multiplication of effective force area and pressure difference between cylinder and plenum. The results for each time step are accomplished integrating the dynamic equilibrium equation through the Newmark method (Bathe & Wilson, 1976), whose parameters were assigned with the usual values set up in the literature.

In spite of the element formulation is very simple, the deviation with respect to the results of plate elements is quite acceptable. Considering that plate elements are more elaborated, one should expect much better results comparing to experimental data, thus it is used as benchmark. Table (1) presents a comparison between natural frequencies evaluated employing both elements, for a typical suction valve.

\[
\text{Table (1) - natural frequency deviation: plate and variable section beam elements (Hz)}
\]

<table>
<thead>
<tr>
<th>vibration mode</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>frequency evaluated: plate element</td>
<td>60.3</td>
<td>504.6</td>
<td>1518</td>
<td>2996</td>
</tr>
<tr>
<td>beam element</td>
<td>59.8</td>
<td>497.5</td>
<td>1475</td>
<td>2802</td>
</tr>
</tbody>
</table>
Taking into account that the natural modes of order high than fourth usually present very high frequencies, the three first modes suffice to determine the valve motion. Considering the results presented in table (1), one should expect the simulation program to present very similar results, whatever the element used in the FEM routine. Obviously, the plate element is not practical for use in the case of compressor simulation due to limitation in terms of computing time.

Although the formulation is much more complete than the mass-spring, the simulation program algorithm converges after the same number of cycles (about five), whatever the model used. The use of FEM based modeling clearly increases the required CPU time, approximately 40% for the typical case. It does not represent any limitation, since final time is quite acceptable, around 25 seconds in a VAX 4000/100 computer; no attempt was made to optimize the FEM routine in this aspect, indeed.

**Results**

The results presented hereafter were all obtained using the parameters relative to a small hermetic reciprocating compressor for R134a. The computed valve displacement refers to the ASHRAE LBP check-point condition. Due to limitations related to the present modeling, the FEM model was used only for the suction valve. For the purpose of considering the discharge, simulation uses a customary mass-spring model. For all cases, the figures display simulation results obtained through the use of actual valve parameters, for reference. Displacements shown are related to the element over the center of valve's orifice and its unit is millimeter. The time scale correspond to the crank angle; time equals zero at 50° from top dead center.

The damping coefficient for this particular arrangement was adjusted according to experimental results. It resulted in an estimated value of 0.01 (non-dimensional), including effects related to the material and the valve interaction with oil and refrigerant.

Figure (2) presents a comparison between the results obtained through finite element and mass-spring models. For the last case, the damping coefficient was adjusted to achieve the best fit to experimental data. Therefore, since both models present very similar results, the finite element model is thus validated. Table (2) compares with experimental data the results concerning the compressor performance. Figure (3) shows results for a different valve, maintaining for both models the damping coefficient used in the previous analysis. The valve displacement evaluated through the mass-spring model in figure (3) presented values higher than the experimental ones. Obviously, the result could be fitted to experimental data once again, modifying the damping. Consequently, the procedure depends on whether experimental data is available, it is not the general case in the design phase of a project. Like in the case presented in figure (2), finite element results presented good agreement to experimental data.

![Figure (2) - finite element versus mass-spring: case 1](image1)

![Figure (3) - finite element versus mass-spring: case 2](image2)

**Table (2) - compressor performance: deviation from experimental data (%)**

<table>
<thead>
<tr>
<th>Model</th>
<th>Refrigerating Capacity</th>
<th>Energy Efficiency Ratio</th>
<th>Total Valve Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>Finite Element</td>
<td>+1.4</td>
<td>-0.2</td>
<td>-7.1</td>
</tr>
<tr>
<td>Mass-Spring</td>
<td>+4.2</td>
<td>+0.8</td>
<td>-15.0</td>
</tr>
</tbody>
</table>

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The aforementioned FEM routine is the only model employed for the following results. Figure (4) gives an idea about the damping effect on valve motion, comparing results obtained with different coefficients (plus and minus fifty percent). In spite of damping has a considerable effect on the movement, it does not have a strong influence on performance. Otherwise, it is not easy to change damping without strong modifications of the design. Figure (5) illustrates the influence of valve thickness. The obvious correlation between this parameter and valve stiffness explains the difference observed.

Figure (4) - effect of damping coefficient on valve motion

Figure (5) - effect of valve thickness on valve motion

Figure (6) resumes an analysis on two possible design modifications. In the first one, the valve width in the clamp region was fifty percent narrower. The second considers a valve with a little enlargement in its length, without any significant change on the shape. Although it is apparently the minor modification, its impact on the performance is evident. Finally, figure (7) shows the consequence of two different valve shapes. The actual valve has a middle section narrower than its ends. The first modification considers the valve sides composed by straight line segments and in the second the narrower section is expanded to the region near the orifice. The valve stiffness increase is the likely reason for explaining the decrease in the amplitude for the first case. It also becomes clear that, when expanded, the narrower section no longer determines the valve stiffness, causing no substantial effect on the motion nor on the compressor performance.

Figure (6) - effect of modification on clamp width and valve length on valve motion

Figure (7) - effect of modification on valve shape on its motion

Conclusions

The modeling evinced hitherto leads to better agreement between experimental and theoretical results comparing to single degree of freedom-mass-spring or modal superposition. It also confers to the compressor simulation program much
more flexibility contrasting with usual models, since it affords the change of valve shape and other characteristics without the assistance of other programs or additional experimentation. It also turns unnecessary to deal with some parameters that otherwise must be estimated and adjusted, like natural frequencies, valve stiffness and damping coefficients. These characteristics are very important when one intend to optimize the valve for a wide range of operating conditions, for example the case of variable speed compressors. Its inherently high degree of accuracy also enables the simulation of the compressor start-up sequence with an acceptable degree of confidence.

Further improvement is required to spread the application, related to the development of a gap type element, which will make feasible considering stop and booster. Some other modifications would lead to even better results, indeed. As example, including the inertia and damping effects of the fluid flow in the valve motion and considering the stiction in the oil-reed interface in the valve opening and its damping effect on the valve closing.

References