PLAN DESIGN MODEL FOR URBAN AREA USE ALLOCATIONS

DECEMBER 1969 - NUMBER 35

BY
W. D. STEWART
a
W. L. GRECCO

JHRP
JOINT HIGHWAY RESEARCH PROJECT
PURDUE UNIVERSITY AND
INDIANA STATE HIGHWAY COMMISSION
Attached is a Technical Paper "Plan Design Model for Urban Area Use Allocations" by Messrs. W. Don Stewart and William L. Grecco. The paper has been prepared from a Final Report on the JHRP research project "A Land Use Model to Minimize Transportation." That Report was presented to the Advisory Board of JHRP several months ago and accepted at that time.

This paper has been accepted for presentation to the 49th Annual Meeting of the Highway Research Board. That organization also plans to publish the paper during 1970. The paper is presented to the Advisory Board for approval of such publication.

Respectfully submitted,

Harold L. Michael
Associate Director

cc: F. L. Ashbaucher  R. H. Harrell  C. F. Scholer
    W. L. Dolch      J. A. Havers  M. B. Scott
    W. H. Goetz      V. E. Harvey  W. T. Spencer
    W. L Grecco      F. B. Mendenhall  H. R. J. Walsh
    G. K. Hallock    R. D. Miles  K. B. Woods
    M. E. Harr       E. J. Yoder
DRAFT COPY
SUBJECT TO CHANGE

PLAN DESIGN MODEL FOR URBAN AREA USE ALLOCATIONS

by

W. Don Stewart
and
William L. Grecco

Joint Highway Research Project
File No.: 3-8-1
Project No.: C-36-70A

Purdue University
Lafayette, Indiana
December 30, 1969
PLAN DESIGN MODEL FOR URBAN AREA USE ALLOCATIONS

W. Don Stewart
University of Saskatchewan

and

William L. Grecco
Purdue University

ABSTRACT

This paper reports the development of an urban area use model. The model's normative solution is to be utilized in the allocation of uses to locations so that net community return is maximized. A heuristic procedure for finding a good solution has been developed and programmed for the computer. As a test, the computer program was applied to simplified data on the Lafayette Indiana Plan Area.

The objective of urban area use allocations is to maximize the gross utility return from the use of area and from economies of scale, less the utility costs of transportation, of adaptation of locations to uses, and of incompatibility between proximate uses.

In order to provide a computationally practical means of obtaining a near optimal solution to the model norm, an iterative solution procedure was devised, using an initial feasible solution. From the existing solution the change in net return, as a result of a unit change in the allocation of each use to each location is evaluated, and a number of the more beneficial interchanges are made to obtain a better solution. The maximum permissible changes in each use per location are then decreased by one-half of their current values. If the new limits are greater than preset minima and if further significant increases in return may be expected, the cycle is repeated.
PLAN DESIGN MODEL FOR URBAN AREA USE ALLOCATIONS

W. Don Stewart
University of Saskatchewan

and

William L. Grecco
Purdue University

INTRODUCTION

Land use allocation models attempt to distribute the urban area activities to the available land in such a way as to achieve certain objectives. A worthy objective might be the maximization of benefit to all of the citizens of the community. The problem then, would be one of identification and quantification.

The objective of this work was to develop a model of the urban system and to identify its component parts. In order to test the validity of the above model, a computer program was developed which required reasonable quantifications of the system elements. Further refinements will result as each component is researched in depth.

The basis for the model is that each urban area will require a somewhat unique set of activities and uses and how these activities are assigned to the land will affect the net benefits received. It is further assumed that the benefits, both positive and negative, are quantifiable and can be expressed as a dollar value. The model will tend toward optimization - given some validity to assumptions made.

MODEL FORM

There are many feasible configurations of activities within any area. The design problem is to determine the best allocation of given activities in an urban area, or perhaps even the allocation of part or
all of available activities in order to maximize return per person. The latter implies the determination of an optimum size, and thus is a more general problem than allocating given activities to maximize total return. This work was restricted to the specific problem of allocating given activities; but it should be noted that this does not preclude an a priori assumption of optimum population size to determine the given activities. The solution to the problem will be constrained to some feasible region by available areal and other resources.

The problem was one of determining that one feasible allocation of area uses, from a bounded set of many such feasible allocations, which maximizes return. Thus, any one of many mathematical programming techniques might be suitable. A heuristic method, which utilizes linear programming principles to obtain a good allocation plan, was used.

An urban structure has been shown to be a group of activities linked by communications. The latter can be divided into two classes, the desired flows and the undesirable by-products of both the activities and the flows.

An urban form which accommodates this structure can be represented by a group of locations, a network of channels linking the location centroids, and barriers between each pair of locations. It is implied that each location should be delineated so that it is homogeneous with respect to local and general topography, flora (natural cover) and fauna, foundation materials, water conditions, and access to the centroid and thence, to the channel network. If these characteristics can be assumed to be similar within the location, then average values for each characteristic for each location can be used in the model.
Barriers to inter-location by-product transmission are assumed to exist between all pairs of locations, but for proximate locations their effectiveness tends to approach zero. The flow channels available for the movement of the generated flows include, but are not necessarily restricted to, channels in the plan area. In most cases, the flow channels would consist of major and minor arterials and collector streets. Where there are large numbers of locations involved, it may be necessary to simplify the computations on long distance flow costs. This can be achieved by aggregating locations with similar major thoroughfare access, to form districts. Flows then may be treated in two groups, those whose destination lies within the district containing the origin location and those whose destination lies in another district.

The functional relationships between these model factors can now be expressed more precisely under the following categories:

1. Required activities and uses.
2. Locations and use allocations.
4. By-product transmission.

Required Activities and Uses

The number of units of each activity which will take place within the area is a function of the characteristics of the local population, local natural and cultural resources, and the extramural interactions with the surrounding region and with urban realms. In a general model, the amounts of each activity could be variables whose levels are determined by the solution of the model. But, in the model developed herein, the amounts of each activity are fixed as model inputs. These amounts can be estimated or predicted from population and economic surveys.
It may be seen that the given data for a model must include not only the number of units of each activity, but also the number of units of an activity which will be accommodated by each use. Further, the uses and activities must satisfy the following:

\[ \sum_{K \in S_T} \sum_{I=1}^{N} Y_{TK} \cdot X_{IK} = A_T \quad \text{for every } T, \]

where: \( \varepsilon \) denotes "an element of the set of"

- \( S_T \) = the set of uses which can accommodate activity \( T \)
- \( N \) = the number of locations in the plan area
- \( Y_{TK} \) = the number of units of activity \( T \) per acre of use \( K \)
- \( X_{IK} \) = the number of acres of use \( K \) allocated to location \( I \)
- \( A_T \) = the number of units of activity \( T \).

Locations and Use Allocations

The plan area must be represented as a group of locations, each having a finite area and homogeneous characteristics. The area of each location is fixed. Since all of that area must be assigned to one or more uses (one of which may be the vacant use), the use allocations must satisfy the following:

\[ \sum_{K=1}^{M} X_{IK} = P_I \quad \text{for every } I, \]

where: \( X_{IK} \) = the number of acres of use \( K \) allocated to location \( I \)

- \( M \) = the number of uses
- \( P_I \) = the size in acres of location \( I \).

The use allocations must be non-negative and thus satisfy the following:

\[ X_{IK} \geq 0 \quad \text{for every pair of } I \text{ and } K. \]
Flows

Flow costs, as used here, are due primarily to person trips. Accordingly, to simplify the model, other significant flows could be expressed in terms of person trips. The total of all types of flows then could be represented as equivalent person trips. To simulate flow costs more precisely, however, some types of trips might be weighted according to their peaking or time density characteristics.

Total flows are input to the model as the number of equivalent person trips generated per acre of origin use. Each use will have associated trip generation rates, one for each trip purpose as defined by the destination use. The model has assumed that these rates are fixed. Further, it has been assumed that trip attractions are balanced by trip productions and are distributed uniformly over all acceptable destination use allocations. That is, trip ends must satisfy the following conditions:

\[ O_{IKL} = Z_{KL} \cdot X_{IK} \quad \text{for every combination of } I, K \text{ and } L \]

where:
- \( Z_{KL} \) = equivalent person trips of type KL generated per acre of use K
- \( X_{IK} \) = the number of acres of use K allocated to location I.

\[ D_{JKL} = \sum_{I=1}^{N} O_{IKL} \cdot \frac{X_{JL}}{\sum_{H=1}^{N} X_{HL}} \quad \text{for every combination of } J, K \text{ and } L \]

where:
- \( X_{JL} \) = the number of acres of use L allocated to location J
- \( X_{HL} \) = the number of acres of use L allocated to location H
- \( N \) = the number of locations.
The flow route choices are made such that the total flow cost of the entire community is minimized. Thus the flow routes are fixed for a given allocation of uses and a given transportation system.

By-product Transmission

Incompatibility of use allocations occurs only if the harmful by-products are transmitted. Between all pairs of uses there are spatial and/or physical barriers to such transmission. It has been assumed for model purposes, that these barriers modify the standard (unrestricted) by-product transmission rate between two adjacent locations of average size, as follows:

1. The transmission values are proportional to the ratio of the given interface length to a standard interface length determined for each model application.

2. The transmission values are reduced by some factor if there is an intervening use.

3. The transmission values are reduced by some factor if there is a physical barrier to transmission.

In the general case the transmission rate will not be the same for each by-product. For example, earth embankments may be effective as noise barriers but not as odor barriers. However, until further research has been conducted into incompatibility, it was deemed advisable to assume a single transmission rate for all by-products.

MODEL NORM

The goal of the normative land use model is to obtain a feasible solution which will maximize public as well as private utility returns. All the factors of the model norm can be represented as a net utility return in terms of location return less adaptability costs, a net utility cost in terms of flows, an incompatibility cost due to proximity, and a negative incompatibility cost resulting from economies of scale.
Net Location Returns

An effort was made to recognize that variations in the physical determinants of lands lend them to one or several uses rather than others. Furthermore, if the model would optimize with proper weighting given to existing land uses, then, adaptation costs must be included. Based on the literature reviews on adaptability of uses to locations and on sensitivity of costs and returns to density and organization, it was hypothesized that a net location return (location return less adaptability costs) could be expressed as follows:

\[ R_{IK} = K_{1K} + K_{2K} \cdot Cd^{-1} + K_{3K} \cdot Ad^{-1} + f_{4K}(T_I) + f_{5K}(F_I) \]
\[ + f_{6K}(D_I) + f_{7K}(N_I) + f_{8K}(S_I) \]

where:
- \( K \) denotes the \( k^{th} \) use, \( K = 1, ..., M \)
- \( I \) denotes the \( i^{th} \) location, \( I = 1, ..., N \)
- \( R_{IK} \) = the net location return for use \( K \) in location \( I \)
- \( K_{JK} \) = a constant for use \( K, J = 1, 2, 3 \)
- \( f_{JK}(\ ) \) = a non-linear function of the bracketed variable, \( J = 4, ..., 8 \)
- \( Cd \) = channel density
- \( Ad \) = areal density
- \( T_I \) = the topography rating of location \( I \)
- \( F_I \) = the foundation rating of location \( I \)
- \( D_I \) = the drainage rating of location \( I \)
- \( N_I \) = the natural resources (excluding man-made) of location \( I \)
- \( S_I \) = the cultural resources of location \( I \).

The first three terms on the right side of the equation are of no concern if the amount of each use is fixed, since they are constant (not a function of \( I \)) for a given use, irrespective of location.
Each component of the adaptability costs (the remaining five terms) is relatively constant throughout a location, except for the cultural or man-made facilities. When there are existing structures or other facilities on part of a location, there will be a very large saving in adaptation costs if the existing use is reallocated back to that part of the location and thus to the existing structures. Consequently, two different net location returns will be calculated for each use in each location. The net return value which includes a zero cultural feature cost is applicable only when a use is reallocated to a currently existing use. It will be designated as $R_{IK}'$. In the other net location return, $R_{IK}$, there will be a non-zero cultural feature cost which should cover the cost of demolition and dislocation of the existing activity as well as the cost of construction of new facilities. In practice, however, it might be more expedient to add the demolition and dislocation costs as returns in $R_{IK}'$ and neglect them in $R_{JK}$". If this were not done, a separate $R_{IK}$" would have to be calculated for each existing use. This expediency results in constant terms being added to the objective function, but the relative values of the returns are correct, and thus the allocations are not affected.

The two net location returns are necessary input data. They are independent of the allocation of uses.

Channel Flow Costs

The total cost of flow on a channel link may be composed of the following:

1. Channel capital cost.
2. Channel operation and maintenance costs.
3. Vehicular capital costs.
4. Vehicular operation and maintenance costs.
5. Time costs.

Note that several of these components are at least partially fixed, that is, independent of flow volumes. Furthermore, they are also independent of the area use allocations. Thus the channel costs will be split into; fixed costs, which need not be considered directly in the allocation process, and the flow costs which are assumed to be related linearly to flows and thus must be considered in the allocation process.

The unit flow costs which are input to the model are user costs for flows from district centroid to district centroid or location centroid to location centroid. These are determined by summing the flow costs of the channel links which comprise the least cost route. It has been assumed also that the channel flow costs are independent of the flow volumes. This implies that there are no capacity limits on any route, unless the flow costs are set sufficiently high to limit volumes.

Inter-use Incompatibility

In the review of operating and performance standards, it was noted that positive incompatibility costs will result from differential operating standards.

It has been assumed that the positive incompatibility cost is:

1. Proportional to the difference in operating standards of the incompatible uses.
2. Proportional to the length of the interface where the incompatibility occurs.
3. Reduced by barriers such as major transport routes, topographical barriers, et cetera.
4. Reduced by spatial separation of the incompatible uses.
The utility cost of incompatibility at a given distance might be determined from what the user will pay to increase the distance of separation from, or to reduce the operating level of, the offensive use. By summing the cost distance curve over a typical location and dividing by area, a linear unit cost figure may be obtained.

An example of a possible procedure is given in Figure 1, using hypothetical values. The figure is based on the assumption that the incompatibility cost of use L to use K in adjacent locations may be determined by using average values for successive bands of 200 feet in width. As an illustration, the area of use L within the bands around use K was set at 65, 80, and 95 acres, for 0 to 200, 200 to 400, and 400 to 600 feet respectively. The difference in operating standards of use K and use L was designated as A. By entering graph 1 at A, the differential nuisance level and the unit utility cost for each band at its median separation distance from use K, can be determined from 1 and 2 respectively. Then by utilizing the area of each band, the total incompatibility cost for each band can be determined from 3. The aggregate of these band costs would then be divided by the total area to derive a unit cost for the two locations. This procedure may be applied to two average locations for any by-product under the assumptions that all of one location is occupied by use K and all of the other location by use L.

Assuming that there are P significant by-products, incompatibility costs may be expressed as follows:

\[
I_{IK} = \sum_{J=1}^{N} \sum_{I=1}^{M} \sum_{Q=1}^{P} f_{KIQ} (O_{KQ} - O_{IQ}) G_{IJQ}
\]
DIFFERENCE IN OPERATING STANDARDS BETWEEN USE K AND USE L

* Assumed level for example

** Average distance of separation of uses

TOTAL INCOMPATIBILITY COST DUE TO BY-PRODUCT Q
10 + 15 + 25 + 50

TOTAL ACREAGE
65 + 80 + 95 = 240

AVERAGE UNIT INCOMPATIBILITY COST (bKL)
\[ \frac{50}{240} \times 0.21 \]

INCOMPATIBILITY COST PER ACRE OF USE K PER ACRE OF USE L DUE TO BY-PRODUCT Q

UNIT INCOMPATIBILITY COST PER ACRE OF USE K TO USE L DUE TO BY-PRODUCT Q

\[ x: 65 \text{ acres of use } L \text{ within } 200' \text{ of use } K \]
\[ y: 80 \text{ acres of use } L \text{ between } 200' \text{ to } 400' \text{ of use } K \]
\[ z: 95 \text{ acres of use } L \text{ between } 400' \text{ to } 600' \text{ of use } K \]

FIGURE 1. DETERMINATION OF THE COST OF INCOMPATIBILITY BETWEEN ONE ACRE OF USE K AND ONE ACRE OF USE L DUE TO BY-PRODUCT Q
where: \( I_{IK} \) = the incompatibility cost of use K in location I to all other uses

\( f_{KILQ} \) = the functional relationship of cost of differential transmission of by-product Q between uses K and L

\( O_{KQ} \) = the operating standard of use K for the Qth by-product

\( O_{LQ} \) = the operating standard of use L for the Qth by-product

\( G_{IJQ} \) = the relative incompatibility transmission rate from location I to location J for the Qth by-product

N = the number of locations

M = the number of uses

P = the number of by-products.

Little is known presently on the quantification of by-product incompatibility costs. Accordingly, it had been assumed that an average cost and transmission figure for all transmitted by-products between any pair of uses would be utilized. That is:

\[
2.3 \quad B_{KL} = \sum_{Q=1}^{P} f_{KILQ} (O_{KQ} - O_{LQ})
\]

= the average utility lost due to the incompatibility effect of one acre of use L on one acre of use K when allocated to the same location or to adjacent locations I and J whose \( G_{IJ} = 1 \).

\[
2.4 \quad G_{IJ} = \sum_{Q=1}^{P} G_{IJQ} / P
\]

= the average relative incompatibility transmission rate from location I to location J for P uses.

The incompatibility costs have been based on homogeneous uses in each location. For a location having a mixture of uses, these assumed costs will be in error for two reasons. The true incompatibility costs will be higher than those assumed in the model because smaller units have a greater interfacial contact per unit area and also a higher
utility cost due to a lower average distance of separation of the incompatible uses. But on the other hand, the designer will reduce the incompatibility by arrangement of the uses within the location, and this may tend to reduce the preceding errors.

Economies of Scale

In addition to by-product incompatibility, there is a relatively high return from the allocation of some uses in large blocks. It is in reality a reduction in activity costs, but it may be treated as a return in the objective function, since this maintains the proper relative values for allocation purposes. Note that these scale economies of a use are a cohesive force and thus are equivalent to negative incompatibility of the use with itself. It was assumed that the economy of scale returns or savings are related linearly to the homogeneous use allocation size. Thus the incompatibility can be treated simultaneously with economies of scale. The economies of scale or negative incompatibilities then form the diagonal of the inter-use incompatibility matrix.

The Objective Function

The objective function or norm may now be states as follows:

2.5 Maximize:

\[
\sum_{I=1}^{N} \sum_{K=1}^{M} R_{IK} \cdot X_{IK} - \sum_{I=1}^{N} \sum_{J=1}^{N} U_{IJ} \cdot F_{IJ} \\
- \sum_{I=1}^{N} \sum_{J=1}^{N} \sum_{K=1}^{M} \sum_{L=1}^{M} X_{IK} \cdot X_{JL} \cdot B_{KL} \cdot G_{IJ} - F_{cc}
\]

subject to the constraints (refer to model equations 1.1 through 1.5):

1.1

\[
\sum_{K \in S_T} \sum_{I=1}^{N} Y_{TK} \cdot X_{IK} = A_T \quad \text{for every } T
\]
1.2 \[ \sum_{K=1}^{M} X_{IK} = p_i \] for every I

1.3 \[ X_{IK} \geq 0 \] for every pair of I and K

1.4 \[ o_{IKL} = z_{KL} \cdot X_{IK} \] for every combination of I, K and L

1.5 \[ d_{JKL} = \sum_{I=1}^{N} o_{IKL} \cdot \frac{X_{JL}}{\sum_{H=1}^{N} X_{HL}} \] for every combination of J, K and L

where:

\( R_{IK} \) = the net location return from allocating one acre of use K to location I (note that this can equal either \( R_{IK}^{'} \) or \( R_{IK}^{''} \) as discussed previously)

\( X_{IK} \) = the number of acres of use K allocated to location I

\( U_{IJ} \) = unit flow cost from location I to location J (excluding fixed channel costs)

\( F_{IJ} \) = the number of equivalent person trips from location I to location J

\( X_{JL} \) = the number of acres of use L allocated to location J

\( F_{cc} \) = the fixed (for a given transportation plan) component of channel costs

\( B_{KL} \) = the average utility lost due to the incompatibility effect of one acre of use L on one acre of use K when allocated to the same location or to adjacent locations I and J whose \( G_{IJ} = 1 \) (refer to equation 2.3).

\( G_{IJ} \) = the average relative incompatibility transmission rate from location I to location J (refer to equation 2.4).

\( N \) = the number of locations

\( M \) = the number of uses,

and;

all trips must be distributed at the least total cost and in such a manner that trip origins and destinations in each location satisfy constraints 1.4 and 1.5.
A PROPOSED ASCENT SOLUTION PROCEDURE

Additional Assumptions

In addition to those made in the formulation of the model and the norm, the following assumptions were made to devise a practical solution procedure:

1. Given an existing solution, that a satisfactory area use move evaluator may be formed from an approximation of the change in the objective function which would result from a unit increase in that allocation in the given location (thus there will be one evaluator for each use in each location).

2. That the above evaluators may be calculated in three separate parts, corresponding to the first three terms of the objective function.

3. That the fourth term of the objective function, fixed channel costs, may be neglected in the formation of the evaluator. This will result in an attempt to optimize the use allocations for the given transportation plan.

4. That the change in the interactions of the area uses which have been moved simultaneously (without updating the evaluators) may be neglected in the formation of the evaluators, provided that the magnitudes of the allocation moves are limited.

5. That the flow cost contribution to a move evaluator (excluding fixed channel costs), although dependent on the magnitude of the move, can be approximated by one of two unit costs, again assuming that the magnitude of the allocation move is limited.

6. That a better solution will be obtained by decreasing each use allocation, and reallocating these uses with the objective of maximizing the sum of the products of the reallocated uses and their respective move evaluators.

The General Procedure

The solution procedure which has been developed to solve the problem requires an initial feasible solution (refer to step 1 of the algorithm given in a succeeding section). Then movements of the allocated uses, which would result in an increase in the objective function, are identified and some of the more beneficial moves are made. This forms a new
initial feasible solution for repeating the entire process for the second of several iterations.

The iterative procedure is utilized since the interaction effects of simultaneous moves are neglected in forming the evaluators of potential moves. As a result, the evaluators must be updated periodically.

The primary decision variables in the procedure are the use allocations. But, the channels or transportation system may be modified on a judgement basis either between iterations or in some cases during iterations.

It is assumed that the sum of the changes in the objective function terms, when some use K is superimposed on the existing feasible plan in location I, is a satisfactory evaluator of the value of additional allocations of use K to location I. In the algorithm the above changes in the first three terms of the objective function are evaluated separately for every use in every location. Since the fourth term is not changed in land use evaluation, it may be neglected.

The unit change in the first term of the objective function is known exactly, since it is equal to the appropriate net location return. However, the second term, flow costs, is not related linearly to the change in allocation. Thus two unit flow cost estimators, which reflect an average change in flow costs for different amounts of change in the given use allocation, were developed and used in such a manner that the flow costs are never underestimated. This should result in a conservative evaluator of a potential move. With each successive iteration the amount of the overestimation is reduced, permitting the identification of the less desirable moves. These unit flow costs are calculated in steps 3 and 4 of the algorithm. Interaction effects due to simultaneous
changes in several uses are neglected. The change in the third term of
the objective function, incompatibility costs, is calculated in step 2
of the algorithm for a unit increase in the given use. Thus, it also
neglects interaction due to simultaneous changes of several uses.

Once all of the evaluator components have been calculated, they
may be combined. However, there are two net location return components
\( R'_{IK} \) and \( R''_{IK} \) as discussed previously) and two flow cost components.
The choice of the appropriate values is dependent on the level of the
current allocations. For each use in each location, there are four
possible levels of the evaluator.

In order to permit reallocation of the uses, with the objective of
increasing return, the problem is converted to a capacitated transporta-
tion problem. In step 5 of the algorithm the origins (unallocated
activities) and destinations (uncommitted areas) are created by decreasing
each use allocation in each location. The evaluators are then formed in
step 6 for the new decreased allocation level. The evaluator then is
equivalent to the flow cost of the transportation algorithm, but the
problem is one of maximization rather than minimization. In step 7 a
capacity limit on each additional allocation is imposed. This capacity
is set equal to the lesser of:

1. The maximum amount that the use can be increased without
resulting in an infeasible solution.

2. The maximum amount that the use can be increased without
invalidating the evaluator; that is, up to the level at which
either the net location return or the assumed unit flow cost
changes.

Given the above capacitated transportation formulation, the initial
levels of the dual variables are calculated and as much of the unalloca-
ted uses as possible are reallocated (see step 8). Because the
evaluators are not constant over the entire range of possible allocations, an optimal solution of the simplified problem may not be determined. In step 9, the evaluators, capacities, and dual variables are updated simultaneously and as many additional reallocations as possible are made. This step is repeated until all uses have been assigned.

Flow Cost Components for the Evaluators

Two unit flow costs, one for allocations at levels less than that of the previous allocation \((x_{IK}^{a})\) and one for all other allocations, are developed (see Figure 2). In view of the basic premise that only one move at a time is considered in the formation of the unit costs, it may be shown that any increase in the allocation of use \(K\) to location \(I\) results in the attraction of trips at increasing cost. Thus, as the allocation of use \(K\) increases, the average flow cost for trips from use \(K\) in location \(I\) tends to increase and never decreases (see Figure 2). Note, however, that for low levels of the allocation the unit flow cost contribution to the objective function may be negative, since some trips are cancelled (discussed more fully below). It has been assumed that if the current allocation of use \(K\) to location \(I\) \((x_{IK}^{a})\) is less than that of the allocation in the previous plan \((x_{IK}^{a})\), then the flow cost will be assumed equal to \(c_{IK}^{a}\). Therefore, if \(x_{IK}\) is less than \(x_{IK}^{a}\) the cost tends to be overestimated. In all other cases, the unit flow cost will be assumed to be equal to the average unit flow cost for an increased allocation of use \(K\) to location \(I\), where this increase is the maximum permissible \(1 (x_{IK}^{c} - x_{IK}^{a}) = \Delta x_{IK}\). This assumed unit flow cost

\[1\text{The maximum permissible increase is the lesser of:}
1. The interchange limit (to be outlined in the next section).
2. The location size \((p_{I})\) less \(x_{IK}^{a}\).}
FIGURE 2. FLOW COSTS FOR USE K IN LOCATION I FOR A GIVEN ALLOCATION PLAN.
(C<sub>IK</sub>') is equal to the area under the curve ABC divided by $\Delta X_{IK}$. If, however, the new allocation level is less than $X_{IK}(c)$, such as $X_{IK}(b)$, then $C_{IK}'$ is an overestimate of the theoretical cost, which would be the area under the curve AB divided by the net change in allocation over the previous solution. Thus, it may be seen that no flow costs are underestimated and some are overestimated.

As the interchange limit is decreased on successive iterations, the maximum permissible allocation change will decrease, and thus the conservative bias of $C_{IK}'$ tends to decrease also. This permits the identification of new moves, even if in lesser quantities and with lower net benefit.

In some cases, $C_{IK}(a)$ may be greater than $C_{IK}'$. $C_{IK}(a)$ is calculated from the existing shadow prices<sup>2</sup>. But, the shadow prices consider only flow costs and do not account for the savings resulting from the elimination of some inter-location trips. When $X_{IK}$ is augmented, each $X_{JL}$ is assumed to be augmented in proportion to its current level, so that all trips may be allocated. However, this means that some trips from location J to location I may now satisfy their destination use desire in location J resulting in a saving, and similarly some of the new trips originating in location I can now satisfy their destination in location I. These savings are not accounted for by the shadow prices because they consider only additional trips from pure origins to pure destinations, while in the above, some of the locations became both origins and destinations. On the other hand, these savings are accounted for in $C_{IK}'$.

---

2The shadow price of an origin is the change in the total optimal cost of all flows caused by a unit increase in the production at an origin. Similarly, the shadow price of a destination is the change caused by a unit increase in the trip attractions of a destination.
since it considers the total change in the objective function, which
sums the cost of new flows less the savings due to flows which are
eliminated. If $C_{IK(a)}$ is greater than $C_{IK}'$, then $C_{IK(a)}$ is assumed to
be equal to $C_{IK}'$. This will still result in an overestimation of the
flow cost in many of these cases and thus the positive bias of the
assumed unit flow costs is maintained.

Interchange Limits

It has been assumed that the average change in the objective
function due to a temporary additional amount of a given use in a given
location is an evaluator of the worth of increasing that use in that
location. Provided that only that one move is made, and provided that
its magnitude is equal to that of the temporary addition, then the
evaluator is exact. However, if more than one move is made without
recalculating the evaluator, the second and subsequent moves are based
on incorrect evaluators. The error in the evaluator is due to neglect
of the interaction term between the changed uses. Also, since the unit
flow cost for a given use in a given location is dependent on the magni-
tude of that use allocation, the magnitude of the proposed move must be
equal to that assumed in the formation of the unit flow cost, for the
move evaluator to be exact.

Both of these errors may be neglected, provided that only small
moves are made before the evaluators are recalculated. Thus maximum
interchange limits have been established. The magnitude of the limits
will vary considerably, according to the type of use.
It may be noted that in step 10 of the solution algorithm (the conclusion of one iteration), the interchange limits are arbitrarily decreased by one-half of their current values. Since the larger of the assumed unit flow costs (one of the move evaluator components) is calculated for a maximum permissable increase in the allocation, the decrease in the limits results in a decrease in the assumed unit flow cost for the next iteration. The new flow cost leads to a less conservative move evaluator, and thus it permits the identification of less beneficial moves on the next iteration. The choice of the fraction (one-half) for the decrease in the interchange limit per iteration was completely arbitrary.

Minimum interchange limits have been provided as a device for terminating the algorithm. However, termination based on the rate of convergence towards an upper bound on the objective function certainly would be preferable if a reasonable estimating procedure for this bound could be devised.

Evaluator Synthesis

The formation of the unit move evaluators, the second phase in the solution procedure, is outlined in Figure 3. The appropriate location return is selected in Step A. In Step B, the unit incompatibility cost is subtracted from this return. Finally, in Step C, the algorithm selects one of the two unit flow costs (depending on the relationship of \( X_{IK} \) to \( X_{IK(a)} \) as outlined above) and subtracts this cost from the output of Step B. The result (\( E_{IK} \)) is an evaluator of the return, net of cost, for allocating one acre of use K to location I. Since the assumed flow costs have a positive bias, the evaluator tends to be conservative.
I denotes the $i^{th}$ location

$K$ denotes the $k^{th}$ use

$X_{IK}$ = allocation of use $K$ to location $I$

$X_{IK}'$ = physically existing use $K$ in location $I$

$E_{IK}$ = unit move evaluator for use $K$ in location $I$

$R_{IK}'$ = unit location return for reallocation of use $K$ to location $I$

$R_{IK}''$ = unit location return for conversion of land in location $I$ to use $K$ from some other use

$\ln_{IK}$ = unit incompatibility cost due to use $K$ in location $I$

$X_{IK(a)}$ = allocation of use $K$ to location $I$ at the end of the previous iteration

$C_{IK(a)}$ = assumed average flow cost for $X_{IK}$ less than $X_{IK(a)}$

$C_{IK}'$ = assumed average flow cost for $X_{IK}$ equal to or more than $X_{IK(a)}$

**FIGURE 3. FLOW CHART OF MOVE EVALUATOR FORMATION**
The Solution Algorithm

The general solution algorithm proceeds as follows:

1. Find a feasible allocation of the required uses to the available locations (a good starting solution, such as an existing plan, will speed convergence towards the optimum).
2. Calculate the unit incompatibility cost \( \ln X_{IK}^3 \), resulting from both negative and positive incompatibility, for an infinitesimal increase in \( X_{IK} \). Repeat for every \( I \) and \( K \). Set \( K = 0 \).
3. Calculate the inter-district flow costs and the inter-district components of the assumed unit flow costs as follows:
   3.1 Set \( K = K + 1 \). If \( K \) is greater than the number of uses, go to step 4. Otherwise, set the flow evaluators, \( C_{DK} \) and \( C_{DK(a)} \), equal to zero for every \( D \). Set \( L = 1 \).
   3.2 Using the appropriate input inter-use and intra-use flows per activity unit and the current allocations, calculate origins and destinations for flow type KL (a flow type is designated by the origin and destination uses, respectively). Allocate these flows at least cost by means of a transportation algorithm, recording the district shadow prices, the individual total cost for each flow type, and the total cost for all flows allocated.
   3.3 If a decrease in the allocation of use \( K \) to district \( D \) \( (X_{DK}) \) is feasible, utilize the shadow prices to calculate the cost savings for the reduction in inter-district flow type KL which would result from an infinitesimal decrease

\[ ^3 \text{All terms are defined at the conclusion of the algorithm.} \]
in $X_{DK}$. That is, the products of the shadow prices for
the destinations and their respective fractions of the
destination totals summed over all possible destinations,
plus the shadow price of the origin. Add twice this
saving to $C_{DK}(a)$. Repeat for every $D$.

3.4 Temporarily increase $X_{DK}$ by $Rm_K$, the maximum inter-
change limit for use $K$. Assume that the destination
use $L$ is augmented at each district $E$ in proportion to
the current $X_{EL}$, such that the total number of new
destinations is equal to the number of new origins
created by augmenting $X_{DK}$. Calculate the new flows of
type KL and allocate them at least cost. Record the
change in total cost for flow type KL divided by
$Rm_K$. Add twice this figure to $C_{DK}$. Decrease $X_{DK}$
by $Rm_K$ in order to revert to $X_{DK}(a)$. Repeat for
every district, $D$.

3.5 Set $L = L + 1$. If $L$ is less than or equal to the
number of uses, return to step 3.2. Otherwise, return
to step 3.1.

4. Calculate the inter-location flow costs and the inter-location
components of the assumed unit flow costs (within the origin
district only) as follows:

4.1 Set $D = 0$.

4.2 Set $D = D + 1$. If $D$ is greater than the number of
districts go to step 5. Set $K = 0$.

4.3 Set $K = K + 1$. If $K$ is greater than the number of uses,
go to step 4.2. Otherwise, set $C_{IK}' = C_{DK}'$ and $C_{JK}(a) =
C_{DK}(a)$, where $D$ denotes the district containing location
1. Repeat for every $I \in S_D$. Set $L = 1$. 

4.4 Using the appropriate input inter-use and intra-use flows per activity unit and the current allocations, calculate origins and destinations for flow type KL. Allocate these flows at least cost by means of a transportation algorithm, recording the location shadow prices and the total cost for flow type KL, and the total cost for all flows allocated.

4.5 If a decrease in $X_{IK}$ is feasible, calculate the intra-location cost savings for the reduction in flow type KL which would result from an infinitesimal decrease in $X_{IK}$. That is, the products of the shadow prices for the destinations and their respective fractions of the destination totals, summed over all locations within the district. Add twice this saving to $C_{IK}(a)$. Repeat for every $I \in S_D$.

4.6 Temporarily increase $X_{IK}$ by the maximum permissible amount, $\Delta X_{IK}$. Assume that the destination use $L$ is augmented at each location $J$ in proportion to the current $X_{JL}$, such that the total number of new destinations over all destinations in all districts is equal to the number of new origins created by augmenting $X_{IK}$. Calculate the new intra-district flows of type KL and allocate them at least cost. Record the change in total cost for flow type KL divided by $\Delta X_{IK}$. Add twice this amount to $C_{IK}'$. Decrease $X_{IK}$ by $\Delta X_{IK}$ in order to revert to $X_{IK}(a)$. Repeat for every $I \in S_D$. 
4.7 Set \( L = L + 1 \). If \( L \) is less than or equal to the number of uses, return to step 4.4. Otherwise return to step 4.3.

5. Reduce every \( X_{IK} \) \( (X_{IK} = X_{IK}(a)) \) prior to this reduction by either \( 0.5 \cdot Rm_k \) or \( X_{IK}(a) \), whichever is the lesser. Record the amounts of each use left unallocated and of each location left uncommitted.

6. Calculate the move evaluator \( E_{IK} \) for every \( I \) and every \( K \) (see Figure 3).

7. For every \( I \) and \( K \), calculate \( Cap_{IK} \), the maximum amount that \( X_{IK} \) may be increased without invalidating \( E_{IK} \), as calculated in step 6. \( Cap_{IK} \) is the least of:
   a. The location size \( \text{size}_I \) minus \( X_{IK} \).
   b. \( X_{IK}' \) minus \( X_{IK} \) for \( X_{IK} \) less than \( X_{IK}' \) only \( (X_{IK}' \text{ denoted the physically existing level of use } K \text{ in location } I) \).
   c. \( X_{IK}(a) \) minus \( X_{IK} \) for \( X_{IK} \) less than \( X_{IK}(a) \) only.
   d. \( X_{IK}(a) \) plus \( Rm_k \) minus \( X_{IK} \).

8. Allocate as much of the unallocated uses as possible by means of a capacitated transportation algorithm at the initial shadow price (dual variable) levels.

9. Re-evaluate all \( E_{IK} \) and \( Cap_{IK} \) and then revise the shadow prices. Allocate as much of the unallocated uses as possible. Repeat this step until all uses have been allocated.

10. Reduce \( Rm_k \) by one-half of its current value for every \( K \). If the new values are more than the minima set as criteria, and if further improvements seem possible, return to step 2.
The following definitions are for the terms of the algorithm:

\[ I_{IK} = \text{the incompatibility cost produced by allocating one acre of use } K \text{ to location } I \]

\[ I_{IK} = \sum_{J=1}^{N} \sum_{L=1}^{M} (b_{KL} + b_{LK}) \cdot x_{JL} \cdot g_{IJ} \]

\[ b_{KL} = \text{average utility lost due to the incompatibility effect of one acre of use } L \text{ on one acre of use } K \text{ when they are allocated to the same or adjacent locations} \]

\[ x_{JL} = \text{allocation of use } L \text{ to location } J \]

\[ g_{IJ} = \text{average relative incompatibility transmission rate from location } I \text{ to location } J \]

\[ N = \text{number of locations} \]

\[ M = \text{number of uses} \]

\( C \) denotes "an element of the set of"

\[ C'_{DK} = \text{average cost of inter-district flows resulting from the increase in allocation of use } K \text{ from } x^{DK(a)} \text{ to } x^{DK(a) + \Delta m} \]

\[ = 2 \cdot (\sum_{L=1}^{M} \sum_{E=1}^{P} f_{DEKL} \cdot c_{DE} - \sum_{L=1}^{M} \sum_{E=1}^{P} f_{DEKL} \cdot c_{DE})/\Delta m \]

\[ P = \text{number of districts} \]

\[ x^{DK} = \text{allocation of use } K \text{ to district } D \]

\[ f_{DEKL} = \text{net flow of type } KL \text{ from district } D \text{ to district } E \text{ for } x^{DK} = x^{DK(a)} \]

\[ = t_{KL} \cdot (x^{DK} \cdot x^{EL}) - f_{EDKL}, \text{ for } f_{EDKL} < \text{ the first term,} \]

\[ = 0 \text{ in all other cases} \]

\[ f'_{DEKL} = \text{net flow of type } KL \text{ from district } D \text{ to district } E \text{ for } x^{DK} = x^{DK(a) + \Delta m} \]

\[ t_{KL} = \text{number of trips from one acre of use } K \text{ destined to some use } L \]
\( C_{DE} \) = minimum cost of flow for one trip from district D to district E

\( Rm_K \) = maximum interchange limit for use K

\( C_{DK(a)} \) = average saving in inter-district flow costs due to an infinitesimal decrease in \( X_{DK} \)

\[
= 2 \cdot \left( \sum_{L=1}^{M} \frac{P_{KL} \cdot T_{KL} - \sum_{E=1}^{P} \frac{P_{KL} \cdot X_{EL}}{\sum_{F=1}^{P}}} \right) / \sum_{F=1}^{P}
\]

\( P_{D} \) = shadow price of district D

\( X_{FL} \) = allocation of use L to district E

\( C_{IK'} \) = average cost of flows resulting from an increase in \( X_{IK} \), where location I is in district D

\[
= 2 \cdot \left( \sum_{J \in S_{D}} \frac{F_{IJKL} \cdot C_{IK} - \sum_{L=1}^{M} \sum_{J \in S_{D}} F_{IJKL} \cdot C_{IK}} \right) / \sum_{J \in S_{D}} 
\]

\( C_{Ij} \) = \( \Delta X_{IK} + C_{DK} \)

\( F_{IJKL} \) = net flow of type KL from location I to location J for \( J \in S_{D} \) and \( X_{IK} = X_{IK(a)} \)

\( F_{IJKL'} \) = net flow of type KL from location I to location J for \( J \in S_{D} \) and \( X_{IK} = X_{IK(a)} + \Delta X_{IK} \)

\( S_{D} \) = set of locations comprising district D

\( C_{IJ} \) = minimum cost of flow for one trip from location I to location J

\( \Delta X_{IK} \) = maximum permissible increase of allocation of use K to location I

\( C_{IK(a)} \) = average saving in flow costs due to an infinitesimal decrease in \( X_{IK} \), where location I is in district D

\[
= C_{DK(a)} + 2 \cdot \left( \sum_{L=1}^{M} \sum_{J \in S_{D}} P_{J} \cdot T_{KL} \cdot X_{JL} \right) / \sum_{H=1}^{N} X_{HL}
\]

\( P_{J} \) = shadow price of location J

\( R_{IK'} \) = location return for conversion of land in location I to use K from some other use

\( R_{JK''} \) = location return for reallocation of land in location I to the existing use K
Computer Programs

Two series of computer programs were developed to carry out the calculations. The first series were used to test the algorithm on a hypothetical case of seven uses allocated to nine locations. Based upon the excessive time consumed, adjustments were made in the second series which computed flow costs in two parts. One component of the flow cost was for inter-district flows and the second one was for intra-district flows.

All the programs except INCOMP, which carried out step 2, made extensive use of variants of the out-of-kilter network algorithm. This algorithm was chosen for the ease of programming and because it rapidly solves network and transportation problems which are minor modifications to the problems attacked here.

DISCUSSION

The work which culminated in the formulation of the above model was initiated to provide a normative means of area use allocation resulting in maximum net community return. It was decided that the factors to be incorporated in the objective function were transportation, incompatibility, and adaptability costs. However it was recognized, that a great deal of research will be required to quantify these costs accurately and precisely, and that some assumptions regarding the inputs were required.

The primary assumption was that all costs can be represented satisfactorily by piecewise linear approximations. This requires that the data supplied to the model be chosen for the expected range of the solution variables. Since land use allocation is not sensitive to small changes in the input data, this should not be a serious limitation.
A second major assumption in the solution procedure was that the transportation costs are fixed. This is a serious drawback, since it requires that each transportation plan be studied separately. Even so, if the initial iterations in the solution procedure indicate that improvements, such as upgrading arterials to freeway status, may reduce overall costs, the new plan may be evaluated by changing a few data plus a minimal amount of computation. Since the inter-district and intra-district flow costs are computed separately, changes in the major thoroughfare plans and addition of congestion tolls to reduce unrealistically high volumes, might be made without repeating the entire iteration.

The third major assumption required for the solution was that the rate of generation of trips, and thus the major cost, was independent of the solution. This may not be true, but by proper calibration of the trip rates, the resultant errors will be minimized.

Finally, the solution procedure considers only a deterministic static case. If the static case is a distant horizon year, the resultant solution may not be satisfactory for the aggregate of the intervening years. A potential modification of the model to provide a good solution over a long term would be to construct the evaluators as the sum of several components. Each component should reflect the average value of a given allocation over a given period.

The consideration of the deterministic case has led to the deletion of all intra-use flows. In the transportation algorithm utilized, all intra-use flow destinations and origins are allocated to the same location since this results in least cost. The true selection of a trip destination is not always to the least cost location even though the probability of such a selection is high. Thus errors are introduced.
Certain types of trips, such as the inter-residential social-recreation trips, may be best deleted from the model since the destinations of these trips deviate considerably from the economist's rational choice and are not easily predicted. Thus, it should be noted that the intra-residential flows were deleted from the Lafayette input data. Some inter-use flows may be deleted advantageously if their volumes are low enough that their effects on the solution are insignificant. These deletions could reduce the computations considerably.

Although external-internal trips were not considered in the Lafayette model, this may not be advisable in practice, since such trips will affect some use allocations. Dummy external zones whose trip loading points are placed at the intersection of major arterials or highways near the plan area boundaries, may be used to consider their effect.

One of the drawbacks to the solution algorithm proposed herein, is that successive applications of the transportation algorithm to various trip types precludes capacitation of transport routes. It has been assumed that such route capacities can be increased as required. However, it is hoped that the multi-commodity transportation algorithms will be improved to the point where they may be substituted for the present algorithm, but with desired capacity constraints. Using the present algorithm, volumes could be limited only by calibrating the flow costs to reflect congestion.

Because of the macroscopic nature of the model, the data it requires need not be more detailed than conventional land use model data. However, there is very little known on area use economies of scale (negative incompatibility), positive incompatibility due to by-product transmission,
and the dollar-utility transform. All of these areas currently are considered on a subjective basis in planning. The provision of this model permits their quantitative consideration, and hopefully may help stimulate research in each of the areas. Also, the area of goals formulation and evaluation requires further research.

It may be noted that incompatibility costs were considered only between adjacent locations in the Lafayette application. This can be extended to any number of locations for such by-products as smoke, which may cover a considerable area on the lee side of prevailing winds. The additional computation time is trivial, and additional computer storage requirements would likely not be a problem.

Although the model was developed for use on one urbanized area, the same methodology could be applied, for example, to the design of a region or neighborhood. Major modifications would be required in the input data. For instance, in neighborhood design the walking trip would be a significant factor.

In the Lafayette application, economies of scale were neglected. This was partly due to a lack of knowledge of the appropriate data, but a more important reason was that the use categorization used for the test did not appear to be sufficiently definitive to realistically apply average scale economy factors. Although this certainly casts some doubt on the true value of the results, the test served its purpose, which was merely to test whether the procedures are practical.