TEMPERATURE EFFECTS ON THE COMPACTION
AND STRENGTH BEHAVIOR OF A CLAY

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by

WILLIAM H. HIGHTER

JHRP

JOINT HIGHWAY RESEARCH PROJECT
PURDUE UNIVERSITY AND
INDIANA STATE HIGHWAY COMMISSION

Final Report

TEMPERATURE EFFECTS ON THE COMPACTION AND STRENGTH BEHAVIOR OF A CLAY

TO: J. F. McLaughlin, Director
Joint Highway Research Project
September 18, 1969

FROM: H. L. Michael, Associate Director
Joint Highway Research Project
File: 6-14-9
Project: C-36-36I

A Final Report "Temperature Effects on the Compaction and Strength Behavior of a Clay" by William H. Hight, Graduate Assistant in Research on our staff is presented to the Board for approval. The research was directed by Professors A. G. Altschaeffl and C. W. Lovell, Jr., and was used by Mr. Hight for his MSCE thesis.

The practical motivation for this study was the feasibility of increased cold weather earthwork in Indiana. The compaction and strength behavior of a single sandy clay was studied over a temperature range of 35°F to 85°F. It was found that low temperature compaction had about the same effect as reducing the compactive effort, i.e., the maximum unit weight was decreased and the optimum water content was increased. The as-compacted strength and stiffness were also reduced. On the other hand, when the soil was compacted to the same density, warm and cold, the colder soil was both stronger and stiffer. While the experimental results are limited, it appears that by slightly increasing the compactive effort, cold (but unfrozen) clayey soils may be compacted to produce satisfactory subgrades and embankments.

Further study is scheduled for the future.

The report is presented to the Board as fulfillment of the Plan of Study approved by the Board on November 28, 1967.

Respectfully Submitted,

Harold L. Michael

Harold L. Michael
Associate Director

HLM:jw

Attachment

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by

William H. Highter
Graduate Assistant in Research

Joint Highway Research Project

File No.: 6-14-9

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Purdue University
Lafayette, Indiana
September 18, 1969

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ABSTRACT

Highter, William H., MSCE, Purdue University, August, 1969.
Temperature Effects on the Compaction and Strength Behavior of a Clay.
Major Professors: A. G. Altschaeffl and C. W. Lovell, Jr.

The effects of temperature in the 35°F to 85°F range on the compaction and strength characteristics of a kaolinitic clay soil have been reviewed. The clay was compacted with a Harvard miniature device and unconfined compression tests were carried out.

The laboratory program was based on a statistical experimental design and the data procured from the experiments were analyzed using the statistical analysis of variance method.

Compaction effort, water content, and compaction temperature were found to have significant effects on the dry density variable. In addition to these three factors, the temperature at the time of undrained strength determination has a statistically significant effect on the undrained strength variable.

It is believed that temperatures affect the compaction process and the strength characteristics of the compacted soil by means of two mechanisms. First, the temperature at which the soil is compacted influences the structure of the compacted soil. Secondly, the temperature at which the undrained strength is tested affects the soil pore pressure.

It was found that the strength of soil tested at the same temperature but compacted at different temperatures increased with increasing compaction temperature. For soil compacted at the same temperature,

the undrained strength increased with decreasing test temperature.

It appears that the detrimental effects of low temperature compaction may be overcome by using a larger compaction energy.

INTRODUCTION

Present-day construction practice generally provides for the placement of soil in highway subgrades, embankments and backfills during non-freezing weather. An extension of the construction season into all but the coldest days of winter could produce benefits to the general public, the contractor, and the state highway agency. For northern Indiana, Lovell and Osborne[1]¹ showed that cold weather earthwork is economically feasible provided the construction project is of sufficient size to include a variety of operations. Assuming the validity of the feasibility study, other questions must be answered before a decision regarding the practicality of such cold weather earthwork in a particular geographic location can be made.

During the cold season in northern Indiana, the in-situ soil beneath a frozen crust is at a low temperature and is probably at a relatively high water content. The frozen crust may vary from a few inches up to five feet thick [2], but it is the underlying cold, but unfrozen, soil that would be utilized for earthwork operations.

An examination of the results of cold weather earthwork is needed to establish the influence of the cold temperature at the time of compaction on the subsequent behavior of the constructed soil facility. Some evidence [3,4,5] exists that lower temperatures at the time of

1. Refers to entries in the Bibliography, page 67.

compaction are associated with smaller maximum dry densities and larger optimum water contents. Other evidence [6,16] suggests that the strength of the compacted soil is mostly dependent on the temperature at which the soil is tested. However, the lack of conclusive data on the influence of cold weather compaction on the resulting behavior of the compacted soil prevents a critical technical appraisal of the practicality of such operations. It is obvious that such data must first be obtained before an evaluation can be made of the additional design and construction considerations necessitated by cold-weather earthwork.

This research intends to fill one gap in the general area of knowledge of temperature effects on compacted soil behavior. It addresses itself to the effects caused by temperature on both compaction and the undrained strength of a compacted clay soil: Does low temperature compaction produce a difference in the strength characteristics, when compared with more conventional, warmer construction temperatures?; Is there a difference in strength characteristics produced by low temperature at the time of testing, regardless of the compaction temperature? The significance of any differences noted is also examined.

The work reported here is an experimental laboratory study which is statistically founded, both in design and analysis. The results of the research are intended as a guide to which temperature variable, and which compaction variable (among those selected for study) has the most influence upon the resulting strength behavior of the compacted product. While the results will answer only a few of the mass of the technical questions on the subject, it is hoped they will indicate

the important variables upon which future effort should be concentrated.

This research is, then, a part of a larger effort, the technical practicality of cold-weather earthwork.

REVIEW OF THE LITERATURE

Temperature Effects¹ on the Compaction Moisture-Density Relation

Temperature has been recognized as a factor to be considered when compacting soils since at least the work of Hogentogler and Willis (1936) [3]. From tests on three clayey soils, they showed an increase in the maximum dry density and a decrease in optimum water content with increasing temperature at the time of compaction over a temperature range of 35°F to 115°F. They also found that "stability" (as measured by resistance to penetration of the Proctor plasticity needle) increased with decreasing temperature over a range of 125°F to 35°F and decreased with increasing temperature over this same range. They explained these observations in terms of the dependence of the viscosity of bulk water on temperature.² This viscosity-temperature relation supposedly produced a decrease in the effective capability of the water to "lubricate" as temperature decreased.

Youssef et al. [9] showed that the optimum moisture content of a soil compacted at the standard Proctor compaction effort increased with decreasing temperature over a temperature range of 55°F to 95°F. The relation was commensurate with the change in bulk water viscosity with temperature. Accordingly, they concluded that the viscosity of the water controlled the magnitude of the optimum water content. The assumption that the effective viscosity of the soil water may be represented by

1. Limited to values of natural occurrence and excluding freezing of soil water.
2. This relationship is available in any number of references, such as the "Handbook of Chemistry and Physics".

values for bulk water is likely a gross one.

Burmister [4] cited a case history which showed that increasing the compaction temperature from 35°F to 65°F resulted in an increase in maximum dry density and a decrease in optimum water content for the modified Proctor compaction effort. He reported a greater increase in maximum dry density with increasing compaction temperature for soils with a smaller gravel content than for those with a greater gravel content. Additional data published by Burmister [5] also indicated a decrease of maximum dry density and an increase in optimum water content as the temperature at the time of compaction was lowered from 65°F to 35°F.

Limited work by Osborne [10], however, showed no systematic temperature effect in the compaction of two typical Indiana soils for a range of temperature of 35°F to 130°F.

Laguros [6] used a Harvard miniature compaction device and observed generally higher densities with increasing temperature for four soils. The maximum increase of density over a temperature range of 35°F to 105°F was 1 pcf; for an illitic soil the density at 35°F was 0.2 pcf higher than that for 105°F. Laguros' data may not show a statistically significant difference between densities over this temperature range if errors inherent to the compaction procedure are considered.

Johnson and Sallberg [11], in a summary of the work of others, noted that temperature is a factor that can significantly influence compacted soil unit weight. They reported (for identical compaction procedures) an increase of 3 pcf in maximum dry density with an increase in temperature from just above freezing to 75°F. For fine-grained soils Belcher [12] reported an increase of 11 pcf in maximum dry density over this temperature range.

There is general agreement that as the temperature at the time of compaction is increased, the maximum dry density increases and the optimum water content decreases. The dry density variable has been included in this research so that such interactions as compaction effort, molding water content and compaction temperature can be considered. Such interactions of independent factors are best analyzed statistically and this experiment has been designed accordingly.

The Effect of Temperature on Soil Strength

Using the Gouy-Chapman Colloid theory as a model, Lambe [13] predicted that a decrease in temperature would expand the double layer and decrease the strength of a clay at a given density. Implicit in this prediction is the assumption that an increase in dielectric constant is the dominant effect of a temperature decrease. Such a prediction is complicated by the uncertainty as to appropriate values for the dielectric constant of soil water and the dependence of these values on temperature. Mitchell [14] applied a refined analysis to verify the accepted belief that an increase in temperature at constant dielectric expands the double layer thickness. But if the dielectric constant were given the temperature dependency of pure water, the double layer repulsions were unchanged over a temperature range of 0°C to 100°C.

Laguros [6] found an increase of shear strength with increasing temperature for four clays compacted in a Harvard miniature apparatus and tested in unconfined compression. However, he compacted his soil at the same temperature at which the strength determination was to be made. Accordingly, the samples compacted at higher temperatures generally exhibited higher dry densities than those compacted at lower temperatures.

Therefore, the effect of temperature at the time of compaction may have outweighed the effect of temperature at the time of strength determination. In any event, an additional factor, dry density, must be considered when reviewing such results.

Campanella and Mitchell [15] showed that soil pore water pressure increases are induced in saturated clays under constant total stress when the temperature is increased, i.e., effective stresses were decreased with increasing temperature. Conversely, decreases in temperature reduced the pore pressure, and increased the effective stress.

Mitchell [16] reported that the shearing resistance for saturated San Francisco Bay mud was lower for higher testing temperatures. The higher testing temperatures were also found to produce higher pore pressures (lower effective stress) at all strains during undrained shear. This served to substantiate his equation for shearing resistance of soils (derived from rate process concepts) that included terms for temperature effects.

Youssef et al. [9] showed that both the liquid limit and the plastic limit increased with decreasing testing temperature. Both of these Atterberg Limits are, in a general way, measures of "strength". While all soils have the same low remolded strength at the liquid limit, the increase in water content required to reach this low strength implies that the soil is stronger at the lower temperature. The plastic limit marks the water content delimiting brittle and plastic response of the remolded soil. Thus the increase in water content required to reach the plastic limit implies at least a stiffening due to the lower temperature.

Lambe [8] reported that samples compacted at lower temperatures have

higher (less negative) residual pore pressures than do samples compacted warmer (by the same compaction procedure and at the same moisture content). He reasoned that the cold soil has a thicker double layer due to an increased net repulsive pressure ($R - A$). Such a state of affairs would lead to a more dispersed compacted structure. Samples compacted to the same density at different temperatures would have different strengths when tested at the compaction temperatures. The dispersed structure produced at the lower temperature should produce a lower strength [13].

Lambe [8] also found that cooling an unconfined sample from a higher compaction temperature to a lower testing temperature caused a decrease in the residual pore water pressure. The cooled sample should also experience lesser pore pressures during shear [16]. Thus, for identically prepared samples (compacted at the same water content and temperature with equal compaction efforts), the sample then tested for strength at a lower temperature would have a higher strength than one tested at a higher temperature.

Noble and Demirel [17] consolidated clay slurry samples at different temperatures and then tested them for shear strength at temperatures equal to or less than the consolidation temperature. The test results indicated that (for a given test temperature) the higher the temperature at the time of consolidation, the higher the shear strength. It must be noted that another factor, void ratio, is involved here. The higher consolidation temperatures are associated with lower final void ratios for samples consolidated with a given total stress.

These data also lend support to the results of Laguros [6] who observed an increase of shear strength with increasing temperature for samples where compaction and testing were performed at the same temperature. For samples consolidated at the same temperature, Noble and Demirel found that the undrained shear strength increased with decreasing test temperatures.

Sherif and Burrous [18] studied the effects of temperature on the shear strength of saturated cohesive soils for a temperature range of 75°F to 150°F . They found that for a constant water content and preshear consolidation temperature (75°F), an increase in testing temperature resulted in a reduction in undrained shear strength. Also, for the same temperature difference, they observed that the magnitude of the decrease in the original strength (that strength measured at the consolidation temperature, 75°F) increased as the initial water content decreased. Sherif and Burrous attributed the decrease in strength with increasing temperature to an increase in pore pressure caused by the decrease in density of the water surrounding an individual particle.

As emphasized in Table 1, the literature appears to be in agreement on two temperature effects:

- 1) for soil prepared at one temperature and tested at another temperature, undrained strength is lower for those samples tested at the higher temperatures;
- 2) for soil in which the testing and preparation temperatures are the same, the strength will be larger for the higher temperatures.

Therefore, there is an interaction between the amount of energy imparted to the soil either through compaction or consolidation, the tem-

Undrained Shear Strength (or Related Characteristics) as a Function of Temperature

TABLE 1

Reference	As Preparation Temperature Increases	Reason	As Testing Temperature Increases	Reason
Lambe (13)	S_u increases at constant γ_d	Thinner double layer; more flocculated structure	-	-
Laguros (6)	S_u increases at constant w	Higher γ_d associated with higher preparation temperature	-	-
Campanella and Mitchell (15)	-	-	Pore pressures increase under constant total stress	Change in soil compressibility and pore water volume
Mitchell (16)	-	-	S_u decreases	Higher pore pressures during shear; rate process theory
Youssef et al (9)	-	-	Liquid limit and plastic limit increase	Dependence of water viscosity on temperature
Lambe (8)	Residual pore pressure decreases	Thinner double layer; more flocculated structure	Residual pore pressure increases	Heating causes an unconfined sample to increase in R-A
Noble and Demirel (17)	S_u increases	For the same total stress, higher temperatures result in lower void ratios	S_u decreases	Commensurate with rate process theory
Sherif and Burrows (18)	-	-	S_u decreases	Higher pore pressure during shear

perature at which this energy is applied, the temperature at which the soil is tested, and the resulting strength. Still another factor in this interaction which has not been considered in any detail is the initial water content.

This is a complicated four factor relationship which is further complicated by considering the interactions of the various levels of these factors. Such interactions are more conveniently analyzed by statistical analysis. The analysis of variance (ANOVA) is utilized in this research to investigate these interactions.

The Effect of Temperature on Strain and the Elastic Modulus

Using a visco-elastic model (Figure 1) consisting of an independent Hookean spring (E_1) connected in series with a modified Kelvin element (composed of a Hookean spring (E_2) with a slider and dashpot), Murayama [7] showed theoretically that the elastic moduli E_1 and E_2 decreased with increasing temperature. The validity of associating the model parameters E_1 and E_2 with the true elastic modulus E of real soil depends upon the authenticity of the model. Hence, any conclusions concerning the E -temperature relationship also depend on the validity of the model.

Murayama [7] plotted initial axial stress data against initially applied strain data obtained from a plastometer (by which a constant axial deformation can be applied and the responding axial stress can be measured) for stress relaxation tests on Osaka Marine clay at different temperatures. He called the slope of the initial straight line portion of the curve E , and observed that E decreased with increasing temperature.

Murayama and Shibata [19] reported that a given total stress produced larger percent strain at higher temperatures. Thus, the soil becomes less stiff with increasing temperature. This agrees with the

FIGURE 1 . MECHANICAL MODEL OF THE CLAY SKELETON (AFTER MURAYAMA [7]).

results of Noble and Demirel [17].

The absence of available literature on this subject suggests that it has not been studied to any sizeable degree. The elastic modulus is commonly used to predict the initial or constant volume settlement

of foundations. It may also be useful to predict the relative flexibility, i.e., tendency to crack, of compacted subgrades and embankments. For the latter reason, the elastic modulus and secant moduli were experimentally interpreted in this study.

The Influence of Structure on the Behavior of Cohesive Soil

Lambe [13] postulated that for a given compaction effort, the structure of a clay would become more dispersed as molding water content increased. For a soil compacted at the same molding water content with different compaction efforts, Lambe visualized that the soil compacted at the high compaction effort would have a more dispersed structure.

Seed and Chan [20] found that kaolinite compacted to a more flocculated structure (water content less than optimum) has steeper stress-strain curves and attained a larger peak stress at lower strain than did kaolinite compacted at water contents greater than the optimum water content. Therefore, variation in water content and compaction effort affect the moduli of a clay as well as its strength.

This study will theorize from statistical implications whether or not the temperature at which a soil is compacted affects its structure.

Seed and Chan, as a result of consolidated-undrained tests on kaolinite found that a flocculated structure would be more rigid than a dispersed structure and consequently for a given total stress the more flocculated structure would develop lower pore pressure than a more dispersed structure. However, at high strains they found that samples with more flocculated structures developed about the same pore water pressures as samples with more dispersed structures.

EXPERIMENTAL PROCEDURE

Soil Preparation

The first phase of the soil preparation procedure consisted of mixing the predetermined constituents, No. 285 Ottawa Sand and Edgar Plastic Kaolin together at their air dry water contents (approximately 0.2% and 1.5% for the sand and clay respectively). This was accomplished at room temperature (approximately 72°F) in a one cubic foot capacity single speed Porter Mixer (Figure 2). Before starting the mixer, 5000 gms. of the soils were placed in the mixer, maintaining a 80%-20% kaolin-sand ratio by weight. Less than 5000 gms. were actually needed for the samples to be prepared. It was found, however, that mixing less than 5000 gms. resulted in a soil that was heterogeneous with respect to the distribution of sand and clay particles; this was ascertained by noting the variation in color of the mix.

The dry sand and clay were mixed together for a period of 10 minutes. Figure 3 indicates how compositional homogeneity varied with mixing time. This curve was produced by mixing a known weight of glass balls of approximately 0.5 mm diameter with the air-dry commercial kaolin. After mixing for an indicated time, four samples of about 100 gm. were taken from four different points in the mixer. Two samples were taken from different depths of the mix (near the surface and near the bottom) at opposite corners of the mixer, and the other two samples were taken at the two different depths near the center of the mixer. The ratio of

FIGURE 2 . THE PORTER SOIL MIXER.

FIGURE 3 . THE EFFECT OF MIXING DURATION ON
THE HOMOGENEITY OF SOIL MIX.

glass beads by weight of each sample was compared to that of the entire mix.

After the ten minute "dry" mixing period, water was dripped into the mix, by hand, from a 100 ml. graduated cylinder over a ten minute period. Some water was lost by evaporation and some was inadvertently spilled on the side of the mixer and on its blades; this water never became part of the mix. Thus, the amount of water to be added was preweighed to produce a water content 1-2% higher than that desired in the mix batch.

After water was added, there followed an additional ten minute wet mixing period. During this period any soil that stuck to the blades of the mixer or to its sides was scraped off with a spatula. Selected sampling from various parts of the mixer showed little variation of the macroscopic moisture content throughout the mix. Thus, the total thirty minutes of preparation period produced an essentially homogeneous mix.

Soil which had a water content of 23% or more formed gross aggregations in the mixer; the maximum size of the aggregations increased with the magnitude of the water content. In order to break down these aggregations, all batches of the soil were passed through a Victorio Producto Strainer shown in Figure 4. The strainer attachment having 3/16 inch diameter holes was used. If the water content were 23% or higher, "rods" of soil would form as the soil emerged from the strainer. These "rods" were prevented from forming by continually scraping the strainer attachment with a spatula as the strainer handle was turned. The result was a soil having 3/16 inch maximum size aggregations.

Water content samples were then taken from various points in the mix. The soil was stored in shallow steel pans wrapped in polyethylene

FIGURE 4 . THE VICTORIO PRODUCTO STRAINER.

plastic bags. The pans were stored at the desired compaction temperature for 24 hours.

If the water content of the soil were within $\pm 0.5\%$ of the desired water content, the soil was ready for compaction. Batches outside these water content limits were wasted.

It was found that by storing the soil in shallow pans the soil attained the ambient compaction temperature within 24 hours. This made it possible to compact the soil immediately after the 24 hour period, and this was done.

Three temperatures of compaction and strength determination were used in this study. The 35°F and 55°F temperatures were attained in a walk-in cold room which was accurate to $\pm 1.5^\circ\text{F}$ of the temperature setting.

A constant 85°F temperature was maintained in a different room. Heat was supplied to this room and the temperature was held within $\pm 1^\circ\text{F}$ of 85°F.

Compaction

Kneading compaction by means of the Harvard Miniature compaction equipment (shown in Figure 5) was used to produce soil specimens. The compaction efforts were chosen so as to produce densities comparable to some fraction of the maximum density produced by the Standard AASHO Compaction Test (ASTM, D-698; AASHO T-99). Kneading efforts for soil at 85°F that produced approximately 93, 95 and 98% of maximum Standard AASHO dry density were selected. These results were achieved with the following number of layers - number of applications per layer - spring constant combinations: 5-20-20; 5-20-30; and 5-25-40, respectively. The

FIGURE 5 . HARVARD MINIATURE
COMPACTION EQUIPMENT.

30 pound spring condition was produced by precompressing a 20 pound spring.

The sides of the compaction mold were lubricated with a silicone grease and the top of each soil layer was scarified before the succeeding layer was compacted. This practice reduced the moisture-density variations in the samples, and prevented them from failing along a horizontal plane between layers during unconfined compression.

After being extruded from the Harvard Miniature mold, the sample was weighed and placed in a 4 inch x 2 inch x 8 inch polyethylene plastic sandwich bag. The samples from one batch (a maximum of 30) were placed in a larger polyethylene plastic bag (12 inch x 8 inch x 30 inch), sealed and submerged for 5 days in American White Oil (USP 31) at the temperature at which testing was to proceed.

A five day curing time was selected for samples because there is evidence of a change in strength during the first few days after compaction. It is believed that there is a tendency for the sample to swell as it is extruded from the compaction mold. This induces negative pore pressures in the water near the boundaries which are larger than those already present in the partly saturated soil. The effective stress is increased locally, and there is an increase in strength. The water, however, seeks an equilibrium with time and the negative pore pressures equalize. The measured strength appears to decrease with time as shown by the data in Figure 6. For the soil used, a 3 day minimum curing time was required, but 5 days was selected for convenience.

Lambe [8] noted that removing a sample from the mold in which it was compacted causes a reduction in pore pressure.

FIGURE 6 . VARIATION IN UNCONFINED COMPRESSIVE STRENGTH WITH CURING TIME .

Strength Determination

Unconfined compression testing was performed on cylindrical samples produced by the compaction procedure. The samples were tested in a Geonor triaxial cell designed for samples with cross-sections of 10 sq. cm. This was done to provide lateral support for the piston between the top of the sample and the 300 pound capacity proving ring. This lateral support was necessary if the ends of the sample were not exactly parallel; this was the case for a small number of samples.

Load was supplied by a Model 56 Wykeham Farrance Compressive loading device of 1 ton capacity, and at a constant rate of ram movement. Axial displacement was measured every 20 seconds with a Federal dial gage, Model C8IS-C, graduated to 0.001 inch. The unconfined compression test apparatus is shown in Figure 7.

The displacement rate selected was 0.03 inches per minute. Lambe [21] recommended a strain rate of 0.5-2.0 percent per minute for the unconfined compression test. The 0.03 inch per minute rate gave a strain rate of about 1 percent per minute for the soil used in this study. The time to failure varied from 2.5 minutes for high compaction effect - low water content samples, to 21 minutes for low compaction effort - high water content samples.

After each sample was tested to failure, a water content determination was made.

FIGURE 7 . THE UNCONFINED
COMPRESSION TEST APPARATUS.

DESCRIPTION OF SOIL

The soil used in this study was a mixture of Edgar Plastic Kaolin (Edgar Plastic Kaolin Company) and No. 285 crushed Ottawa Sand (Ottawa Silica Company). These soils were mixed together as described previously to produce a soil which was 80% Kaolin and 20% Ottawa sand by weight. Descriptions of the two constituent soils and the resulting mixture are presented in Table 2. Figure 8 is a grain size distribution curve of the two soils and their mixture.

Table 2

Results of Identification Tests on the
Constituent Soils

Identification Test	Edgar Plastic Kaolin	No. 285 Ottawa Sand	80% Kaolin 20% Sand
Liquid Limit	60%		55%
Plastic Limit	37%		32%
Plasticity Index	23%	NP	23%
Specific Gravity of Solids	2.60	2.65	2.61
Percent Clay Size (< 0.002 mm)	79%	-	17%

ANALYSIS OF EXPERIMENTAL RESULTS

Experimental Results

A grand total of 540 samples were prepared and tested in accordance with the procedures described previously. The physical characteristics of these samples and the results of the testing are fully presented in the Appendix. These results of testing represent the raw data for the analyses which are described below. This entire study was predicated upon a statistical foundation for the analysis of the experimental raw data.

Statistical Analyses

The Analysis of Variance method (ANOVA) was the procedure used in this study. This procedure is an arithmetical process of breaking down a total statistical variance into its component parts. The ANOVA method is discussed in most statistics texts (for example [22, 23, 24]); therefore, a detailed discussion of ANOVA will not be necessary.

An ANOVA Model I, the fixed effects model, was used in this study. It was a complete factorial model with partial nesting. The four factors (independent variables) considered were compaction effort, water content, compaction temperature, and (strength) testing temperature. Three different compaction efforts, four water content levels per compaction effort, three compaction temperatures and three testing temperatures (35°F, 55°F, 85°F) were used. The four levels of water content

were not the same for each compaction effort because the water contents were related to the optimum water content for each compaction effort. Thus the water content factor was nested in the compaction effort factor.

Table 3 indicates the water content levels¹ that were used with each compaction effort level.

Table 3
Water Content Levels Used with
Compaction Effort Levels

Compaction Effort	Water Contents (%)			
5-20-20*	23	25	27	29
5-20-30	21	23	25	27
5-25-40	20	22	24	26

* 5 layers - 20 tamp applications per layer - 20 pound spring

The model assumed for the Analysis of Variance is described by Equation 1. This equation is an extension of that described in Ostle [23].

$$Y_{ijklm} = \mu + \alpha_i + \beta_{i(j)} + \gamma_k + \delta_l + \alpha\gamma_{ik} + \alpha\delta_{il} + \gamma\delta_{kl} + \beta\gamma_{i(jk)} + \beta\delta_{l(jl)} + \alpha\gamma\delta_{ikl} + \beta\gamma\delta_{i(jkl)} + \epsilon_{i(jklm)} \dots \quad (1)$$

where:

- i = 1,2,3 (compaction factor levels)
- j = 1,2,3,4 (water content factor levels)
- k = 1,2,3 (compaction temperature factor levels)
- l = 1,2,3 (testing temperature factor levels)

1. The response of the soil at high water contents precluded the use of values much wetter than the optimum.

- m = 1,2,3,4,5 (replicates)
 μ = true mean effect
 α_i = true effect of i^{th} level of factor 1
 $\beta_{i(j)}$ = true effect of j^{th} level of factor 2 in the i^{th} level of factor 1
 γ_k = true effect of k^{th} level of factor 3
 δ_l = true effect of l^{th} level of factor 4
 $\alpha\gamma_{ik}$ = true effect of the interaction of the i^{th} level of factor 1 and the k^{th} level of factor 3
 $\alpha\delta_{il}$ = true effect of the interaction of the i^{th} level of factor 1 and the l^{th} level of factor 4
 $\gamma\delta_{kl}$ = true effect of the interaction of the k^{th} level of factor 3 and the l^{th} level of factor 4
 $\beta\gamma_{i(jk)}$ = true effect of the interaction of the j^{th} level of factor 2 and the k^{th} level of factor 3 nested in the i^{th} level of factor 1
 $\beta\delta_{i(jl)}$ = true effect of the interaction of the j^{th} level of factor 2 and the l^{th} level of factor 4 nested in the i^{th} level of factor 1
 $\alpha\gamma\delta_{ikl}$ = true effect of the interaction of the i^{th} level of factor 1, the k^{th} level of factor 3, and the l^{th} level of factor 4
 $\beta\gamma\delta_{i(jkl)}$ = true effect of the interaction of the j^{th} level of factor 2, the k^{th} level of factor 3, and the l^{th} level of factor 4 nested in the i^{th} level of factor 1.
 Y_{ijklm} = the numerical value of that observation defined by the unique combination of i, j, k, l , and m .

$\epsilon_i(jklm)$ = true effect of the m^{th} repetition of the $(ijkl)^{\text{th}}$ treatment combination.

This model assumes that μ is a constant and that the ϵ_{ijklm} are normal and independently distributed (NID) with zero mean and constant standard deviation, σ (homogeneity of variances).

The model assumes a completely randomized design. In such a design, the treatments are assigned to the experimental units completely at random. This assumption was not satisfied in this research because of practical physical limitations in the prosecution of the experiments. A single constant temperature room was used for both the 35°F and 55°F compaction and testing temperature levels. It took 24 hours to effect a temperature change in this room. In addition, a 5-day sample curing period was required. Thus, a completely randomized program would have required an impractical length of time considering that 120 samples were to be treated. It is now recognized that a more nearly randomized procedure could and should have been used.

The non-full-randomization induces a concern in the analysis that the time-wise progression of the testing produces its own major influence on the results. A detailed analysis of some representative data was made. The large variations which are noted in the data are seemingly not the result of non-randomization, but rather are likely inherent to the experimental procedures themselves. This observation suggests it is possible to use ANOVA results with some confidence in the resulting statistical inferences. However, the fact that full randomization was not performed must be included in the appraisal of the relationships obtained.

The assumptions concerning independence and normality of errors are functions of randomization of treatments and it is known that the treatments were not applied at random in the experiment.

The raw data in their basic form did not satisfy the homogeneity of variance requirement. A common logarithmic transformation (to the base 10) of all the data was made with the expectation of making the variances more homogeneous than before. The variance within a cell (5 replicates) was much less than the variances between cells. A possible reason for this is that due to the length of time required to accomplish this study, more than one laboratory assistant was used in the compaction process. Evidently a laboratory technician could reproduce his own work with little variation, but variations between different technicians was larger. Consequently, the effects of replications were negligible and complete homogeneity was impossible to obtain.

Ostle [23] gives three reasons for having replications in statistical experiments, but due to the difficulties mentioned above the replications in this study only allowed a more precise estimate of the mean effect of any factor to be obtained.

The ANOVA procedure essentially tests a series of hypotheses concerning the equality of certain mean values and variances between various cells of the statistical model. A type I error, $\alpha = 0.05$, was used for this study. This error is the rejection of an hypothesis when it is actually true; the value of α is then the probability associated with making this error.

Six dependent variables were considered in the study: dry density (γ_d); unconfined compressive strength (q_u); strain at failure (ϵ_f); initial tangent modulus; secant modulus to the peak stress; secant modulus to one-half the peak stress. Figure 9 shows graphically the definition of the last five of these variables. The curve in this figure does not pass through zero because of a correction applied to some of the stress-strain curves. This correction was necessitated because of sample disturbance which occurred when the samples were trimmed before they were ejected from the Harvard miniature mold. The result of this disturbance was that samples exhibited large strains that were not commensurate with initial small stresses. This condition corrected itself as the stress increased and the strain became proportional to the stress. In the correction procedure the straight line portion of the stress-strain curve was extended until it intersected the stress ordinate as shown in Figure 9.

For each of the dependent variables an ANOVA was performed to determine which of the independent variables and their interactions were related significantly to the variable. The Scheffé method [25] of making comparisons among treatment means was used. This method indicates whether or not there is a significant difference between the means of a dependent variable resulting from different levels of a factor.

Statistical Results

The results of the ANOVA for each of the dependent variables are shown in Tables 4 through 9. Because of the non-randomness of the

FIGURE 9 . A TYPICAL STRESS-STRAIN CURVE WITH FIVE DEPENDENT VARIABLES DEFINED.

TABLE 4

Analysis of Variance Table for Dry Density (γ_d)

Source of Variation	Degrees of Freedom	Sums of Squares	Mean Squares	F ratio	F table*	Significant
1 (Compaction Effort)	2	0.038349	0.019175	31.27	3.55	yes
2 in 1 (Water Content)	9	0.094946	0.010549	17.18	2.46	yes
3 (Compaction Temperature)	2	0.004880	0.002440	4.13	3.55	yes
13	4	0.005413	0.001353	2.13	2.93	no
23 in 1	18	0.010974	0.000609	-	-	-
Within Replicates	504	0.032147	0.000064	-	-	-
Total	539	0.186702	-	-	-	-

* Most statistics texts contain tables for the F distribution, for example see [26].

TABLE 5

Analysis of Variance Table for the Peak Stress (q_u)

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Squares	F ratio	F table	Significant
1 (Compaction Effort)	2	12.601347	6.300673	236.16	3.26	yes
2 in 1 (Water Content)	9	4.044465	0.449385	16.23	2.15	yes
3 (Compaction Temperature)	2	0.033874	0.016937	0.64	3.26	yes*
4 (Testing Temperature)	2	0.074397	0.037198	1.11	3.26	yes*
13	4	0.378912	0.094728	3.15	2.63	yes
14	4	0.213520	0.053380	2.12	2.63	no
23 in 1	18	0.565238	0.031402	1.48	1.90	no
24 in 1	18	0.534806	0.029711	1.31	1.90	no
34	4	0.380011	0.095002	3.15	2.63	yes
134	8	0.391814	0.048976	1.22	2.21	no
234 in 1	36	0.958714	0.026630	-	-	-
Within Replicates	432	0.235581	0.000545	-	-	-
Total	539	20.412685	-	-	-	-

* Significant because their interaction is significant.

TABLE 6

Analysis of Variance Table for Strain at Failure (ϵ_f)

Source of Variation	Degree of Freedom	Sum of Squares	Mean Squares	F Ratio	F Table	Significant
1 (Compaction Effort)	2	8.636086	4.318043	106.31	3.26	yes
2 in 1 (Water Content)	9	45.493437	5.054826	124.40	2.15	yes
3 (Compaction Temperature)	2	0.429025	0.214512	5.12	3.26	yes
4 (Testing Temperature)	2	0.245594	0.122797	3.15	3.26	no
13	4	0.633689	0.158422	3.37	2.63	yes
14	4	0.914941	0.228735	5.27	2.63	yes
23 in 1	18	1.002350	0.055686	1.15	1.90	no
24 in 1	18	1.792937	0.099607	2.19	1.90	yes
34	4	0.258433	0.064608	1.24	2.63	no
134	8	0.825520	0.103190	2.22	2.21	yes
234 in 1	36	1.455819	0.040439	-	-	-
Within Triplicates	432	7.414860	0.001716	-	-	-
Total	539	62.429322	-	-	-	-

TABLE 7

Analysis of Variance Table for the Tangent Modulus

Source of Variation	Degree of Freedom	Sum of Squares	Mean Squares	F ratio	F table	Significant
1 (Compaction Effort)	2	38.505842	19.252921	102.68	3.26	yes
2 in 1 (Water Content)	9	56.014799	6.223866	33.17	2.15	yes
3 (Compaction Temperature)	2	0.044378	0.022189	0.12	3.26	no
4 (Testing Temperature)	2	1.077398	0.538699	2.16	3.26	no
13	4	1.435237	0.358809	1.17	2.63	no
14	4	1.416534	0.354133	1.17	2.63	no
23 in 1	18	2.814633	0.156368	0.83	1.90	no
24 in 1	18	5.708277	0.317126	1.13	1.90	no
34	4	0.474416	0.118604	0.63	2.63	no
134	8	1.341879	0.167734	0.89	2.21	no
234 in 1	36	6.771071	0.188085	-	-	-
Within Replicates	432	2.833989	0.006560	-	-	-
Total	539	118.438459	-	-	-	-

TABLE 8

Analysis of Variance Table for the Secant Modulus to Peak Stress

Source of Variation	Degree of Freedom	Sum of Squares	Mean Squares	F ratio	F table	Significant
1 (Compaction Effort)	2	42.138810	21.069405	193.85	3.26	yes
2 in 1 (Water Content)	9	72.922322	8.102480	74.57	2.15	yes
3 Compaction Temperature)	2	0.533836	0.266918	2.49	3.26	yes*
4 (Testing Temperature)	2	0.416146	0.208073	1.99	3.26	yes*
13	4	2.163992	0.540998	4.11	2.63	yes
14	4	1.983325	0.495831	4.61	2.63	yes
23 in 1	18	1.258860	0.069936	0.64	1.90	no
24 in 1	18	3.482195	0.193455	1.85	1.90	no
34	4	1.256469	0.314117	2.97	2.63	yes
134	8	1.939110	0.242388	2.25	2.21	yes
234 in 1	36	3.914219	0.108728	-	-	-
Within Replicates	432	0.676104	0.001564	-	-	-
Total	539	132.685394	-	-	-	-

* Significant because their interaction is significant.

TABLE 9

Analysis of Variance Table for the Secant Modulus to One-half the Peak Stress

Source of Variation	Degrees of Freedom	Sum Squares	Mean Squares	F ratio	F table	Significant
1 (Compaction Effort)	2	39.521705	19.760852	103.16	3.26	yes
2 in 1 (Water Content)	9	68.108914	7.567657	39.92	2.15	yes
3 (Compaction Temperature)	2	0.161003	0.080501	0.42	3.26	no
4 (Testing Temperature)	2	0.146438	0.073219	0.38	3.26	no
13	4	2.254361	0.563590	2.18	2.63	no
14	4	2.014072	0.503518	2.12	2.63	no
23 in 1	18	2.575417	0.143078	0.75	1.90	no
24 in 1	18	5.216725	0.289818	1.98	1.90	yes
34	4	0.742603	0.185650	0.97	2.63	no
134	8	2.023624	0.252960	1.61	2.21	no
234 in 1	36	6.900945	0.191692	-	-	-
Within Replicates	432	1.457658	0.003374	-	-	-
Total	539	131.123531	-	-	-	-

data, the mean square of the highest-order interaction [factors 23 in 1 for γ_d and factors 234 in 1 for the remainder] was used as the denominator in the F-ratio, instead of the mean square of error, when testing for significance.

Because factor 2 (water content) was nested in factor 1 (compaction effort), 2 in 1 was used as a source of variation when testing for significance. Similarly, factors 23 in 1 and factors 24 in 1 were also used as sources of variation (factor 3 is compaction temperature and factor 4 is testing temperature).

In Table 4, Factor 4 (strength testing temperature) does not appear because it could physically have no influence on the dependent variable, γ_d .

Each of the Tables 4 through 9 indicates the results of the tests for significance. It remains for later discussion to attempt an explanation of the physical meaning of the relations.

DISCUSSION OF RESULTS

Effects on Dry Density

The factors which significantly affected the dry density, γ_d , were compaction effort, water content, and compaction temperature. Contrasting the three levels of the compaction effort factor indicated that there was no significant difference between the 5-20-20 level (5 layers - 20 tamps/layer - 20 pound spring) and the 5-20-30 level. There was a significant difference between the 5-20-20 and 5-25-40 levels and between the 5-20-30 and 5-25-40 levels.

Figure 10 depicts the dry density - water content curves for the three kneading compaction levels considered in this study as well as that for the Standard AASHO test (ASTM D-698; AASHO T-99). This plot indicates that the 5-20-30 and 5-25-40 kneading compaction levels give larger dry densities than the 5-20-20 level (as well as somewhat smaller optimum moisture contents). One should expect this but the statistical techniques indicate that the differences are significant.

The second independent variable, water content, also had a significant effect on the dry density. However, no contrasts could be made for this factor because water content was nested in the compaction effort factor, as previously described.

LEGEND

- ◆ STANDARD AASHO COMPACTION
- 5 LAYERS, 25 APPLICATIONS/LAYER, 40 LB. SPRING
- ▲ 5 LAYERS, 20 APPLICATIONS/LAYER, 30 LB. SPRING
- 5 LAYERS, 20 APPLICATIONS/LAYER, 20 LB. SPRING

FIGURE 10 . THE MOISTURE- DENSITY RELATIONS FOR FOUR COMPACTION EFFORTS.

The temperature at which the soil was compacted was also significant. The analysis indicated that there is a significant difference between the 35°F and the 85°F levels of compaction temperature; the higher temperature generally was associated with a higher dry density. This increase of dry density with increasing compaction temperature was so gradual that contrasts revealed no significant difference in dry density for soil compacted at 35°F vs. 55°F and for soil compacted at 55°F vs. 85°F.

Figure 11 shows the dry density - water content relationship for soil compacted at the same kneading compaction level but at different soil compaction temperature levels. The maximum dry density increases and the optimum water content decreases as temperature increases. Thus, increasing the compaction temperature is analogous to increasing compaction effort. It should be possible to compensate for lower densities obtained at lower compaction temperatures by increasing the compaction effort.

It should be noted from Figure 11 that the same dry density of 89 pcf was attained at the three compaction temperatures, but that the water content required to produce this density increased with decreasing temperature. Thus, one would expect that the structure of the soil at this density would change with the compaction temperature; the lower temperatures should be associated with a more dispersed structure than the higher compaction temperatures. These differences could cause an appreciable difference in the response of the soil to subsequent loads.

Figure 11 indicates that the effective compaction effort (i.e., that energy which produces higher density) imparted to a soil decreases

FIGURE II . THE EFFECT OF COMPACTION TEMPERATURE ON THE MOISTURE - DENSITY RELATION.

with decreasing temperature. There is either much loss of input energy to other phenomena or the soil skeleton has become more rigid and requires more energy to deformation.

As the study of the literature indicates (Table 1, page 10), lower pore pressures are induced by shear of the compacted soil at a given water content as the temperature decreases. The kneading compaction process is essentially a shearing phenomenon; it thus is consistent to expect that lower compaction temperature will produce lower densities for the same compaction procedure.

Effects on Unconfined Compression Strength

The significant factors affecting the unconfined compressive strength variable, q_u , are compaction effort and water content.¹ Although the compaction temperature and testing temperature factors showed no significance when considered as single factors, the interaction of compaction temperatures and testing temperatures was significant. In such cases, Scheffé [24] concluded there is a difference in the dependent variable (q_u), but that when the effects of one factor are averaged over the levels of the other, no difference of these averaged effects is demonstrated.

Contrasting the three levels of compaction effort revealed a significant difference in strength between all three levels.

1. All samples were tested at the compacted water content, i.e., there was no simulation of an increase of water content such as normally occurs in the service environment.

The compaction effort - compaction temperature interaction showed that for a given level of compaction temperature, an increase in the compaction effort resulted in an increase in strength. For a given compaction effort level, q_u appeared to increase with increasing compaction temperature; however, the increases were not large enough to be statistically significant at the $\alpha = 0.05$ level.

The compaction temperature - testing temperature interaction had a significant effect on the unconfined compressive strength. Contrasting revealed (for samples compacted at 85°F and then tested at the 35°F, 55°F, 85°F testing temperature levels) that the samples tested at 35°F had significantly higher unconfined compressive strengths than those tested at 85°F. However, this increase was so gradual with temperature that there was no significant difference in strength either between those samples tested at 35°F and 55°F or for those tested at 55°F and 85°F.

The effect that various factors and interactions have on the unconfined compressive strength is interwoven with the effect that these same factors and interactions have on the dry density. That the dry density variable has an effect on the unconfined compressive strength becomes evident when the compaction effort factor is considered. As mentioned previously, contrasts revealed no difference with respect to γ_d between the two lower compaction effort levels. The 5-20-30 level appeared to produce larger dry densities than the 5-20-20 level but this effect was not statistically significant. However, there was a significant difference in q_u between these same two compaction effort levels. This indicates that a statistically insignificant increase

in dry density can cause a significant increase in unconfined compressive strength.

Some of the significant effects of the compaction temperature-testing temperature interaction on the unconfined compressive strength are due to the dependence of dry density on compaction effort. The higher compaction temperature will produce a higher density at a given water content. It will also produce a higher degree of saturation, and presumably a more dispersed structure, a higher residual pore pressure and a higher pore pressure change during shear. These factors exert opposite influences on undrained strength, but the experimental evidence shows that the density factor apparently prevails. Thus, for samples compacted at different temperatures and tested at a common temperature, strength increases with increasing temperature of compaction.

For those samples compacted at the same temperature but tested at different temperatures, the trend in strength seems to correspond with the assumed trend in soil pore water pressure with temperature. Cooling a sample creates lower (more negative) pore pressures than already exist, in a partially saturated soil. The pore pressure increases during shear would also be lower for the cooler samples. Thus, one would expect higher strength to be associated with the lower testing temperatures for samples compacted at a common temperature.

Figures 12, 13 and 14 are three-dimensional representations depicting the dry density-water content-unconfined compressive strength relationships. The three compaction effort levels and their associated water contents are included in each of the three figures. For the three figures the compaction temperature level is 55°F, and the testing temperature level

FIGURE 12 . THE UNCONFINED
COMPRESSIVE STRENGTH - DRY DENSITY -
WATER CONTENT RELATIONSHIP.
($T_c = 55^\circ$, $T_f = 35^\circ$).

FIGURE 13 . THE UNCONFINED COMPRESSIVE STRENGTH - DRY DENSITY - WATER CONTENT RELATIONSHIP.

$$(T_c = 55^\circ = T_f)$$

FIGURE 14. THE UNCONFINED
COMPRESSIVE STRENGTH - DRY DENSITY -
WATER CONTENT RELATIONSHIP.
($T_c = 55^\circ$, $T_f = 85^\circ$).

varies from figure to figure. These plots indicate that although the dry densities for the two lower levels of compaction effort are only slightly different, the resulting unconfined compressive strengths are significantly different.

Figure 15 is a plot of unconfined compressive strength vs. water content. Lower testing temperatures are generally associated with higher strengths, although this may not be statistically significant for those data plotted in this figure. As mentioned previously, contrasts reveal only that there is a difference in the means due to different levels of a factor when the means are averaged over all possible combinations of levels of the factor. Thus, individual factor level combinations selected for plotting may not all indicate that the contrasts are significant when, in fact, they are significant.

Effects on Strain at Failure

The significant factors affecting the strain at failure, ϵ_f in the unconfined compressive strength test were compaction effort, water content, and compaction temperature. The significant interactions were compaction effort - compaction temperature, compaction effort - testing temperature, water content - testing temperature, and compaction effort - compaction temperature - testing temperature.

Contrasts revealed significant differences in the strain at failure between all three levels of compaction effort. The magnitude of strain at a given density on the dry side of optimum decreases with increasing compaction effort, as a consequence of a more flocculated fabric, a lower degree of saturation and lower pore pressure. It should be noted that the water content factor was nested in the compaction effort factor and that

COMPACTION TEMPERATURE = 55°
TESTING TEMPERATURE LEGEND:

◆ 35°
■ 55°
● 85°

FIGURE 15 . THE EFFECT OF TESTING TEMPERATURE ON THE UNCONFINED COMPRESSIVE STRENGTH.

higher compaction effort levels were associated with lower water contents.

There was also a significant difference in ϵ_p between the 35°F and the 85°F levels of compaction temperature. Smaller strains were

associated with the higher compaction temperatures. This increase of ϵ_f with decreasing compaction temperature was so gradual that contrasts revealed no significant differences in ϵ_f between neither the 35°F and 55°F levels of compaction temperature nor between the 55°F and 85°F levels.

The compaction effort - compaction temperature interaction showed an increase in ϵ_f with decreasing compaction temperature. This effect is thought to be a result of the dependence of density and, hence, soil structure on the temperature at the time of compaction.

The analysis of variance also indicated that for a constant compaction effort, the strain at failure decreased with decreasing testing temperature. This relationship was not statistically significant for the highest compaction effort level (5 layers - 25 tamp applications/layer - 40 pound spring); this suggests that high compaction efforts can overshadow any testing temperature effects. This is further testimony to the importance of the dry density variable as it affects other dependent variables.

Effects on the Initial Tangent Modulus

The initial tangent modulus, as defined in Figure 9, has as its significant factors compaction effort and water content. There were no significant interactions for the tangent modulus. Contrasts revealed a significant difference in the tangent modulus between all three levels of compaction effort. The modulus increases with increasing compaction effort.

Although contrasts could not be made for the water content factor levels because of the nesting problem, the data suggest that the tangent modulus decreases sharply at high water contents. Figure 16 shows the effect of compaction effort and water content on the tangent modulus. It should be noted that the magnitude of the tangent modulus increases to the second level of water content (for the two lower compaction effort levels) and then decreases sharply. This sharp decrease occurs at a water content about one or two per cent greater than the optimum water content for their respective compaction conditions. A line of constant dry density also appears on this figure. The water content required to attain this density (89.5 pcf) increases with decreasing compaction effort. It can be inferred once again that the structure of the soil at this water content becomes more dispersed with decreasing compaction effort. As the tangent modulus is a manifestation of the ability of the soil to resist small strains, it is expected that the soil with the most nearly dispersed structure should have the lowest tangent modulus. This is the case in Figure 16. As the soil structure becomes more fully dispersed, at the higher water contents, the tangent moduli decrease greatly.

FIGURE 16 . THE EFFECT OF COMPACTION EFFORT AND WATER CONTENT ON THE INITIAL TANGENT MODULUS.

Effects on the Secant Modulus Measured to q_u

The secant modulus measured to q_u is the ratio of the unconfined compressive strength divided by the strain at failure. This modulus, then, is dependent on two variables discussed previously. It was treated as a separate variable in this study so that its statistically significant factors and interactions could be analyzed.

The significant factors affecting this secant modulus (defined in Figure 9) are compaction effort and water content. Although neither the compaction temperature factor nor the testing temperature factor seemed to be significant when analyzed separately, their interaction was statistically significant. Therefore, for the same reason mentioned earlier with reference to the unconfined compressive strength variable, it is concluded that both of these factors have a significant effect on this modulus.

Additional interactions having a significant effect on this variable include compaction effort - compaction temperature, compaction effort - testing temperature, and compaction effort - compaction temperature - testing temperature.

Contrasting the means of this modulus showed that there was a significant difference between levels 1 and 2 and levels 1 and 3 of the compaction effort factor. The modulus increased with increasing compaction effort.

The data revealed that for samples compacted at 85°F, the lower the testing temperature the greater the secant modulus measured to q_u .

This relationship was not significant for the 35°F and 55°F levels of compaction temperature.

Contrast also indicated that for a given compaction temperature, there was a significant difference (an increase) of the modulus between levels 1 and 2 and 1 and 3 of the compaction effort factor. Similar results were obtained for the compaction effort - testing temperature interaction.

Figure 17 shows the effect that testing temperature and water content have on this secant modulus. It indicates that (for soils compacted at the same effort, temperature, and water content), the modulus decreases with increasing testing temperature. It is noted that the rate of this decrease increases with decreasing temperature: the "final" secant modulus value is obtained at lower water contents for soil tested at higher temperatures. It is further noted that the secant modulus is practically the same, regardless of testing temperature, at the 26 per cent water content level. These observations are not surprising when one considers that variations in this modulus reflect the decrease in unconfined compressive strength with increasing water content, and increase in strain at failure with increasing water content. At high water contents, the effect of testing temperature is completely overshadowed by the dual effect of water content on strain at failure and unconfined compressive strength.

EFFORT: 5 LAYERS, 20 APPLICATIONS/LAYER, 20 LB. SPRING

COMPACTION TEMPERATURE = 85°

TESTING TEMPERATURES LEGEND:

- 35°
- 55°
- ▲ 85°

FIGURE 17 . THE EFFECT OF WATER CONTENT AND TESTING TEMPERATURE ON THE SECANT MODULUS TO PEAK STRESS.

Effects on the Secant Modulus Measured to $q_u/2$

The significant source of variation for this modulus, which is also illustrated in Figure 9, are the compaction effort and water content factors and the water content - testing temperature interaction in the compaction effort factor. Contrasts revealed a significant difference in this modulus between levels 1 and 2 and levels 1 and 3 of the compaction effort factor. The modulus increased with increasing compaction effort. This probably can be attributed to the increase in dry density due to this factor and to the subsequent increase in strength and decrease in strain discussed earlier.

Because of the nesting of the water content factor in the compaction effort factor, no contrasts could be made on the other two sources of variation of this variable. However the data show that the modulus decreases with increasing water content. At a given water content the modulus increases with decreasing testing temperature; at high water content levels, the contribution of the testing temperature factor is obliterated by the water content.

SUMMARY

Table 10 is a compilation of those factors and interactions of factors that had a significant effect on each of the variables considered. It is seen that the compaction effort factor and the water content factor are the most fundamental independent variables because they have a significant effect on each of the six dependent variables. It is felt that the reason for this is due primarily to the effect these factors have on the dry density variable, γ_d . Varying the levels of compaction effort and water content not only affects the magnitude of dry density, but perhaps just as importantly, affects the resulting soil structure. The soil structure is considered to have an important effect on the other dependent variables. Thus the dry density as affected by the compaction effort and water content factors is a fundamental variable which is related to all of the other variables considered in this study.

The compaction temperature factor had a significant effect on dry density, the unconfined compressive strength, the strain at failure, and the secant modulus measured to the peak stress. Because the compaction temperature significantly affected the dry density variable, it is postulated that this factor also affects the soil structures which in turn would affect other dependent variables. The data suggest that the phenomenon of decreasing dry density with decreasing compaction temperature (for constant compaction effort and water content) can be compensated for by increasing the level of compaction effort. This would also

TABLE 10

Table of Significant Factors and Interactions

Independent variables and interactions	Dependent Variables				
	Dry Density (γ_d)	Unconfined compression strength (q_u)	Strain at failure (ϵ_f)	Tangent Modulus	Secant to q_u Secant to $q_u/2$
1 (Compaction effort)	S	S	S	S	S
2 in 1 (Water content)	S	S	S	S	S
3 (Compaction temperature)	S	S*	S	NS	S*
4 (Testing temperature)	NC	S*	NS	NS	S*
13	NS	S	S	NS	S
14	NC	NS	S	NS	S
23 in 1	NC	NS	NS	NS	NS
24 in 1	NC	NS	S	NS	NS
34	NC	S	NS	NS	S
134	NC	NS	S	NS	S

LEGEND

S - Significant; NS - Not Significant; NC - Not Considered. * Significant because their interaction is significant.

be expected to effect a consequent (but not proportional) increase in unconfined compressive strength.

The testing temperature factor had a statistically significant effect on unconfined compressive strength and the secant modulus measured to peak stress. These variables generally increased with decreasing testing temperature except at high water contents where the influence of testing temperature was inconsequential.

The compaction effort - compaction temperature interactions significantly affected the unconfined compressive strength, the strain at failure, and the secant modulus as measured to q_u variables. It is expected that this interaction also had an effect on the dry density variable which in turn would have affected these variables, but this was not verified statistically. However, small changes in the dry density variable which may not be statistically significant, can result in large changes in the unconfined compressive strength variable. The verity of this is demonstrated in Figure 18 which shows a substantial decrease in strength with small changes in density and water content.

The interaction of the compaction effort and testing temperature factors had a significant effect on the strain at failure and on the secant modulus as measured to peak stress variables. For a constant compaction effort level the strain at failure decreased with decreasing testing temperature while the modulus increased with decreasing testing temperature. These effects are attributed to the soil being in a more dense state as the compaction effort increased and to lower pore pressures resulting from decreasing testing temperatures.

FIGURE 18 . THE DENSITY - MOISTURE AND
STRENGTH - MOISTURE RELATIONSHIP

The compaction temperature-testing temperature interaction significantly affects the unconfined compressive strength and the secant modulus as measured to the unconfined compressive strength. Since the compaction temperature has a significant effect on the dry density variable and hence the soil structure, it is expected that for a given testing temperature the two dependent variables would increase with increasing compaction temperatures. Due to the assumed dependence of pore pressure on testing temperature, the magnitude of those dependent variables increased with decreasing testing temperature for soils compacted at a given temperature.

CONCLUSIONS

As a result of this study which examined compaction and strength characteristics of a kaolinitic soil compacted with kneading type (Harvard Miniature) compaction and in which certain independent variables were investigated over pre-determined ranges, the following conclusions can be stated:

1) Compaction of the test soil at low (but above freezing) temperature results in changes in undrained strength characteristics as compared to those for warmer, more conventional compaction temperatures. The unconfined compressive strength increases with increasing compaction temperature when all other compaction factors are held constant.

2) Low testing temperatures have a significant effect on the undrained strength. For soil compacted under identical conditions, the undrained strength increases with decreasing testing temperature.

3) The dry density appears to have an overriding influence on the other five variables considered. However, the effects of water contents very wet of optimum could not be established in this study.

4) Of the three moduli considered, temperature effects were pronounced only for the secant modulus to peak stress. This modulus is a ratio of unconfined compressive strength and strain at failure; both of these variables were affected by temperature effects.

RECOMMENDATIONS FOR FURTHER STUDY

1. Explanations of the compaction process and the undrained strength behavior of compacted soils required reference to the variables of soil structure and pore water pressure, neither of which were monitored in this study. Further studies should incorporate measures of particulate orientation (fabric), pore size distribution, and pore fluid pressures; both as these are residual to the compaction process and as they are changed during shear. It is noted that recent technological advances have significantly increased the practicality of effecting such measurements.

2. Prior to large scale implementation of cold weather placement and compaction of highway subgrades and embankments, a greatly increased store of experimental evidence on the effects of low temperatures is needed. This should involve laboratory determinations of compactive response, strength and strength parameters, compressibility characteristics and the like for a variety of soils. Such laboratory evidence will ultimately need to be supplemented by carefully controlled and monitored field installations.

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APPENDIX

APPENDIX

EXPERIMENTAL DATA

Variable Code	γ_d (pcf)	q_u (psf)	ϵ_r (in/in)	Tangent Modulus (psf) $\times 10^{-3}$	Secant Modulus to q_u (psf) $\times 10^{-3}$	Secant Modulus to $q_u/2$ (psf) $\times 10^{-3}$
1111 *	80.6	4890	.0156	595	325	593
	81.0	5700	.0135	594	428	561
	81.0	5910	.0165	592	387	585
	79.7	5540	.0157	738	356	660
	80.6	5330	.0145	515	377	515
1112	82.8	6740	.0213	623	352	587
	82.5	6720	.0196	885	374	879
	82.5	6520	.0219	536	312	522
	82.2	6300	.0190	565	355	565
	82.6	6820	.0194	622	363	622

*

A four digit number is used to represent the independent variables. The first digit represents the compaction effort levels. Its code is: 1 = 5 - 20 - 20; 2 = 5 - 20 - 30; and 3 = 5 - 25 - 40 (number of layers - tamp applications per layer - Hookean spring constant). The second digit represents the water content levels which are nested in the compaction effort levels in accordance with the following diagram:

Water Content Level	Compaction Effort Level		
	1	2	3
	Water Content (%)		
1	23	21	20
2	25	23	22
3	27	25	24
4	29	27	26

Thus each compaction effort level is associated with four water content levels.

The third and fourth digits represent the levels of compaction temperature and testing temperature, respectively. Their codes are: 1 = 35°F; 2 = 55°F; and 3 = 85°F.

Variable Code	γ_d (pcf)	q_u (psf)	ϵ_f (in/in)	Tangent Modulus (psf) $\times 10^{-3}$	Secant Modulus to q_u (psf) $\times 10^{-3}$	Secant Modulus to $q_u/2$ (psf) $\times 10^{-3}$
1113	82.0	6260	.0156	842	460	800
	82.8	6870	.0192	7030	375	703
	82.6	6470	.0151	889	449	771
	82.5	6650	.0163	728	431	728
	81.8	6870	.0175	761	406	761
1121	84.9	7160	.0220	630	329	626
	84.6	6860	.0210	665	332	665
	84.8	7360	.0232	747	321	747
	84.0	6870	.0210	690	334	690
	84.0	6860	.0193	870	356	829
1122	85.1	5270	.0234	496	225	496
	84.6	4630	.0216	425	214	425
	83.6	4670	.0232	490	212	490
	83.6	4740	.0213	540	254	535
	83.6	5050	.0205	492	262	492
1123	86.1	6370	.0293	608	285	600
	85.3	5230	.0217	418	244	364
	84.4	5190	.0218	530	260	516
	84.0	5120	.0238	449	253	444
	85.4	5650	.0241	505	241	505
1131	82.3	7790	.0203	842.5	447	776
	83.6	8480	.0238	780	415	780
	82.2	6520	.0149	805	489	795
	82.5	7730	.0188	1150	518	114.2
	82.8	6870	.0175	732	456	732
1132	93.5	6680	.0685	586	303.1	414
	93.1	5660	.0538	350	318.1	315
	94.3	5960	.0443	590	335	474
	94.0	6510	.0690	523	307.3	422
	94.2	6870	.0680	480	312.5	371
1133	89.1	6590	.0287	527	230	525
	88.6	6290	.0295	597	214	515
	88.9	6340	.0276	497.5	230	497.5
	89.4	5990	.0268	545.5	228	490
	89.3	6010	.0250	660	248	529
1211	82.5	5760	.0256	362	238	362
	83.1	6030	.0284	391	212	391
	83.9	6140	.0281	597	232	491
	84.0	5880	.0271	425	239	329
	83.1	5220	.0253	332	199	332

Variable Code	γ_d (pcf)	q_u (psf)	ϵ_r (in/in)	Tangent Modulus (psf) $\times 10^{-3}$	Secant Modulus to q_u (psf) $\times 10^{-3}$	Secant Modulus to $q_u/2$ (psf) $\times 10^{-3}$
1212	85.6	6460	.0325	604	213	515
	85.5	6370	.0380	443	175	431
	85.3	6420	.0344	533	200	466
	85.9	6420	.0396	619	168.5	569
	85.9	6490	.0377	616	187.5	507
1213	86.5	7710	.0362	670	237	628
	87.1	8120	.0352	594	236	594
	86.3	7390	.0336	650	222	430
	86.6	7640	.0399	479	197.5	479
	86.4	7190	.0376	486	201	486
1221	86.5	6910	.0366	748	210	706
	87.6	7870	.0358	763	220	669
	88.0	7600	.0348	730	227	730
	87.1	7440	.0369	752	220	690
	87.5	7380	.0353	795	209	708
1222	91.5	5250	.0847	375	620	225
	91.6	4890	.0717	275	681	215
	92.6	5800	.0831	225	641	202
	92.8	5880	.0864	414	691	232
	92.3	5750	.0833	367	691	201
1223	91.8	5570	.0803	272	694	264
	91.3	6110	.0718	339	903	289
	90.9	5490	.0595	358	955	308
	91.0	5160	.0569	298	907	279
	90.9	5220	.0568	330	926	296
1231	86.0	7850	.0289	657.5	265	657.5
	86.0	8290	.0312	575	278	566
	86.0	8100	.0317	695	271	678
	85.6	7940	.0287	674	296	625
	85.0	7770	.0291	767	281	698
1232	95.9	7250	.0792	590	170	296
	95.9	7370	.0823	510	155	331
	95.9	6700	.0632	523.5	166.1	263
	95.8	7150	.0794	414	160	264
	95.6	7670	.0884	424	167	286
1233	94.0	6660	.0756	382	880	274.5
	93.5	6550	.0776	394	905	302
	93.0	5770	.0553	410	109.8	236
	93.4	6360	.0624	401	102	348
	93.0	6390	.0641	435	101	351

Variable Code	γ_d (pcf)	q_u (psf)	ϵ_r (in/in)	Tangent Modulus (psf) $\times 10^{-3}$	Secant Modulus to q_u (psf) $\times 10^{-3}$	Secant Modulus to $q_u/2$ (psf) $\times 10^{-3}$
1311	88.3	6510	.0951	363	68.5	204
	89.0	6410	.0815	336	78.8	244
	88.4	6000	.0826	267	70.1	222
	89.0	5910	.0724	302	84.8	248
	88.1	6360	.0746	472	85.1	320
1312	90.6	6010	.1070	285	54.6	166.8
	90.6	6010	.1070	304	54.4	171
	90.0	6120	.0962	443	63.0	216
	90.0	6370	.1059	482	60.1	208
	90.0	6090	.1207	309	50.4	187.5
1313	91.0	6540	.0915	326	75.4	279
	90.6	6300	.0887	372	75.2	289
	90.5	6590	.0810	325	75.0	296
	91.2	6640	.0808	386	85.6	292
	89.3	6320	.0817	261	69.5	261
1321	91.4	6450	.1008	450.5	60.0	131.5
	91.3	6470	.1126	461	50.0	125.5
	91.4	6950	.1286	447	47.8	107
	91.2	6600	.1261	441.5	46.1	128
	90.9	6520	.1229	471	48.0	128
1322	90.9	6050	.1173	308	55.1	135
	90.8	5790	.1250	217	48.1	146.5
	90.6	5950	.1245	209	49.5	156.7
	90.0	5950	.1176	248	53.3	133
	89.9	5960	.1106	242	56.4	163
1323	93.5	4960	.1414	262	35.1	77.5
	93.1	4610	.1354	210	34.1	66.5
	93.1	4610	.1354	210	34.1	66.5
	93.1	4610	.1354	210	34.1	66.5
	93.5	4420	.1570	127	29.4	50.6
1331	87.5	5950	.0408	456	161	39.2
	88.8	8050	.0631	320	133.5	31.5
	88.4	6620	.0461	341	139.5	32.2
	88.5	7550	.0558	374	135.5	35.8
	87.8	6420	.0483	323	131.2	29.1
1332	87.0	4490	.1602	88.0	87.7	47.6
	86.9	4600	.1875	80.8	94.5	43.5
	86.9	4800	.1872	67.4	84.6	40.3
	86.9	4600	.1875	80.8	94.5	43.5
	86.9	4590	.1669	56.4	96.4	47.7

Variable Code	γ_d (pcf)	q_u (psf)	ϵ_f (in/in)	Tangent Modulus (psf) $\times 10^{-3}$	Secant Modulus to q_u (psf) $\times 10^{-3}$	Secant Modulus to $q_u/2$ (psf) $\times 10^{-3}$
1333	92.4	2640	.2010	29.6	13.2	23.2
	95.5	2590	.1730	23.4	14.1	22.2
	93.6	2670	.1869	43.1	13.4	28.1
	92.3	2610	.1800	36.6	14.6	25.6
	93.0	2520	.1310	49.4	10.1	35.5
1411	90.2	3810	.1728	38.6	21.2	31.4
	89.4	3480	.1808	27.8	18.7	27.1
	89.2	3940	.1864	28.1	20.5	27.7
	89.5	3440	.1739	26.9	19.3	26.8
	90.0	3180	.1467	30.0	21.1	29.9
1412	89.8	3750	.1730	38.0	20.4	32.5
	90.0	3970	.2003	37.9	18.0	33.8
	90.0	3890	.1865	39.3	19.2	33.5
	90.0	3890	.1865	39.3	19.2	33.5
	90.0	4020	.1862	39.5	19.8	34.0
1413	90.5	4110	.1439	43.2	27.2	40.7
	90.4	4100	.1649	34.7	23.4	33.9
	90.5	4180	.1577	39.2	24.9	34.8
	90.3	4220	.1576	33.7	24.5	33.7
	90.5	4440	.1918	38.4	22.0	35.3
1421	89.2	3550	.1806	25.8	17.5	25.2
	89.2	3560	.1876	27.2	17.5	26.2
	90.0	3580	.2085	24.8	15.8	24.8
	89.9	3540	.1876	24.6	17.0	24.6
	90.0	3620	.1944	25.9	17.0	25.9
1422	89.1	3250	.1535	40.4	20.6	29.8
	89.9	3560	.2016	31.8	16.9	27.2
	89.9	3500	.1808	54.1	18.0	29.1
	89.0	3600	.1875	40.0	18.1	28.9
	90.0	3460	.1739	50.1	18.4	29.4
1423	89.5	3720	.1591	41.5	21.7	35.0
	89.5	3750	.1660	45.3	20.4	35.4
	90.5	4280	.1574	40.7	23.4	40.7
	90.0	4020	.1582	39.6	21.2	40.2
	90.5	4140	.1438	53.3	26.9	43.3
1431	90.0	3980	.2072	29.1	17.9	29.1
	90.0	3710	.2011	22.6	17.2	22.6
	90.8	3810	.2078	22.1	17.9	22.1
	90.0	3710	.2081	26.8	16.9	26.5
	90.7	3870	.2006	25.4	17.7	25.4

Variable Code	γ_d (pcf)	q_u (psf)	ϵ_f (in/in)	Tangent Modulus (psf) $\times 10^{-3}$	Secant Modulus to q_u (psf) $\times 10^{-3}$	Secant Modulus to $q_u/2$ (psf) $\times 10^{-3}$
1432	91.6	2520	.1838	70.9	13.7	52.2
	91.3	2510	.1838	68.0	14.2	43.2
	92.1	2440	.1981	88.6	13.2	45.3
	92.0	2330	.1844	67.0	13.2	57.8
	92.0	2530	.2049	94.8	12.4	94.8
1433	92.3	1930	.1640	46.2	13.1	46.2
	92.3	2120	.1850	39.2	10.9	39.2
	92.1	1970	.1855	68.0	11.0	65.5
	92.2	1910	.1364	92.9	14.2	90.0
	91.7	2140	.1638	12.2	13.1	55.3
2111	83.2	8460	.0152	1050	636	1050
	83.4	8560	.0149	897	655	878
	84.2	9270	.0183	1050	610	1004
	83.5	8410	.0152	869	566	869
	83.6	9650	.0173	1050	615	1050
2112	83.3	8450	.0152	1034	560	1000
	83.3	8070	.0144	1100	576	950
	83.7	8060	.0110	1180	753	1113
	83.7	7390	.0162	1013	476	905
	83.9	7600	.0139	1355	575	1130
2113	85.0	8250	.0140	831	620	831
	84.5	8610	.0165	789	530	740
	84.5	8400	.0170	820	540	820
	84.3	7930	.0183	530	450	530
	84.6	8310	.0190	594	438	594
2121	84.1	9550	.0176	988	541	955
	83.5	8650	.0164	807	527	807
	83.0	8450	.0152	900	545	900
	84.0	8350	.0154	855	590	822
	83.4	8650	.0164	955	550	955
2122	84.0	9200	.0185	779	590	770
	84.4	9080	.0188	905	530	905
	84.8	9320	.0161	920	581	913
	83.0	7780	.0134	942	601	942
	83.9	8370	.0137	1044	660	1044
2123	83.8	7980	.0140	830	675	819
	83.8	7980	.0140	830	675	819
	83.8	7980	.0140	830	675	819
	84.3	7960	.0147	727	554	727
	84.3	8860	.0152	930	630	930

Variable Code	γ_d (pcf)	q_u (psf)	ϵ_f (in/in)	Tangent Modulus (psf) $\times 10^{-3}$	Secant Modulus to q_u (psf) $\times 10^{-3}$	Secant Modulus to $q_u/2$ (psf) $\times 10^{-3}$
2131	84.4	1043	.0153	1005	695	955
	84.4	1043	.0153	1005	695	955
	84.4	1060	.0184	1081	686	1081
	84.5	1037	.0224	777	560	777
	84.4	1043	.0153	1005	695	955
2132	83.3	7240	.0131	922	657	901
	83.3	7050	.0136	924	635	857
	83.3	7640	.0138	1015	690	980
	83.1	7820	.0168	890	559	839
	83.6	7410	.0196	655	501	655
2133	83.4	7490	.0159	805	470	805
	84.3	8450	.0152	925	556	909
	83.4	7940	.0148	944	616	889
	83.5	7770	.0135	960	606	960
	83.2	7420	.0126	963	672	945
2211	82.9	7110	.0186	592	404	583
	83.5	6470	.0168	546	400	472
	83.5	8180	.0194	731	431	650
	83.1	7660	.0172	685	497	685
	83.1	7750	.0187	635	435	635
2212	86.7	9790	.0222	924	449	924
	86.4	9760	.0240	1002	441	882
	86.5	9370	.0250	1230	423	1075
	86.6	8980	.0225	1290	446	1081
	86.5	9610	.0244	1150	409	973
2213	85.5	9400	.0197	900	456	859
	85.2	9070	.0188	1140	482	1100
	85.3	8470	.0186	714	485	714
	85.2	8330	.0190	723	456	719
	85.2	9070	.0188	1140	482	1100
2221	88.0	9890	.0254	1120	411	1005
	87.1	9350	.0233	1247	416	1081
	86.8	9140	.0221	847	420	807
	87.0	9260	.0252	857	369	857
	87.0	8960	.0243	1062	369	964
2222	86.0	9340	.0226	950	369	906
	85.0	9830	.0234	930	434	893
	84.9	9340	.0216	1080	474	1020
	85.6		.0225	894	477	885
	85.6	9790	.0204	1095	480	1040

Variable Code	γ_d (pcf)	q_u (psf)	ϵ_f (in/in)	Tangent Modulus (psf) $\times 10^{-3}$	Secant Modulus to q_u (psf) $\times 10^{-3}$	Secant Modulus to $q_u/2$ (psf) $\times 10^{-3}$
2223	85.6	9640	.0194	930	496	930
	85.6	9640	.0194	930	496	930
	85.4	9900	.0219	885	455	885
	85.6	9640	.0194	930	496	930
	85.5	9870	.0237	829	442	829
2231	87.7	1279	.0231	1310	630	1310
	87.0	1087	.0211	1100	638	1098
	86.2	1136	.0250	1300	565	1210
	86.9	1145	.0248	944	515	944
	86.9	1122	.0254	911	514	901
2232	84.7	7870	.0184	1025	416	818
	85.4	8660	.0199	900	451	876
	84.8	7670	.0189	800	496	754
	85.1	8490	.0203	849	472	798
	85.8	9790	.0204	1022	507	889
2233	89.4	1232	.0415	924	305	867
	89.4	1119	.0375	851	346	812
	89.5	1050	.0359	974	355	925
	89.0	1072	.0319	1320	358	1002
	89.5	1024	.0331	1250	344	959
2311	86.8	7980	.0355	480	236	466
	87.0	7490	.0333	410	243	410
	87.8	7400	.0266	525	288	525
	86.5	7560	.0297	442	262	442
	87.0	7940	.0287	526	305	526
2312	90.2	8610	.0443	740	203	581
	89.7	9010	.0450	701	200	634
	90.1	8770	.0508	539		502
	90.0	8920	.0521	532	171	507
	90.3	9230	.0548	627		521
2313	90.3	9660	.0415	940	293	834
	90.3	9560	.0452	810	230	772
	90.0	8740	.0370	915	288	853
	90.4	9170	.0463	801	220	800
	90.3	9540	.0488	839	227	772
2321	91.0	1043	.0499	745	213	720
	90.5	9870	.0410	1056	241	883
	91.0	9780	.0481	850	216	730
	90.4	9290	.0390	720	211	668
	90.4	9550	.0435	756	220	756

Variable Code	γ_d (pcf)	q_u (psf)	ϵ_r (in/in)	Tangent Modulus (psf) $\times 10^{-3}$	Secant Modulus to q_u (psf) $\times 10^{-3}$	Secant Modulus to $q_u/2$ (psf) $\times 10^{-3}$
2322	90.5	1146	.0532	1056	199	728
	90.5	1035	.0570	885	194	640
	90.6	9920	.0478	1115	225	939
	90.5	1040	.0465	783	227	695
	90.4	9510	.0488	856	212	720
2323	90.1	1077	.0421	1030	282	925
	90.4	1031	.0364	980	313	855
	89.6	1014	.0368	696	288	696
	89.6	1014	.0368	696	288	696
	89.6	9300	.0390	643	261	625
2331	89.3	1097	.0381	1240	301	960
	88.9	1038	.0362	1130	308	895
	89.4	1137	.0371	982	314	884
	89.5	1050	.0359	1046	309	857
	88.6	1025	.0400	914	259	775
2332	91.2	1180	.0497	1545	257	856
	91.0	1038	.0396	1290	274	930
	91.0	1066	.0493	1610	255	1019
	91.0	1066	.0493	1610	255	1019
	90.5	1029	.0433	860	258	723
2333	90.7	9980	.0476	1130	224	782
	91.0	1191	.0460	845	265	735
	91.9	1140	.0473	1010	251	873
	91.0	1170	.0465	855	268	780
	91.2	1198	.0492	975	255	807
2411	93.2	8230	.1076	360	77.4	165
	92.6	7830	.0949	256	83.5	181
	92.1	7940	.0946	239	78.4	182.5
	92.8	7820	.1088	163	68.4	138
	91.1	7600	.0886	300	80.5	183
2412	92.9	6810	.1429	304	46.7	123.6
	93.0	6640	.1399	154.5	44.4	81.6
	93.0	6690	.1328	242	49.4	109.5
	93.0	6620	.1400	230	46.6	81.0
	93.0	6360	.1268	254	47.4	92.9
2413	93.9	7060	.1317	188.5	51.5	124.8
	93.5	7430	.1099	243	67.1	171.5
	93.5	6970	.1251	159.5	49.5	119
	93.4	6360	.1164	181.5	53.0	109
	93.6	6720	.1501	133.5	41.5	97.1

Variable Code	γ_d (pcf)	q_u (psf)	ϵ_r (in/in)	Tangent Modulus (psf) $\times 10^{-3}$	Secant Modulus to q_u (psf) $\times 10^{-3}$	Secant Modulus to $q_u/2$ (psf) $\times 10^{-3}$
2421	93.1	7130	.1281	133	48.2	100
	93.3	6940	.1425	71.3	41.8	69.5
	93.0	7520	.1443	71.0	40.3	70.6
	93.0	6840	.1498	68.7	40.0	65.8
	93.0	7380	.1343	73.4	43.7	73.4
2422	93.4	9870	.0824	495	121	343
	93.4	9870	.0824	495	121	343
	93.2	1061	.0907	635	129.5	430
	93.5	9640	.0727	454	135	351
	92.5	9130	.0810	386	118.5	331
2423	92.6	9120	.0637	678	155	521
	92.8	9330	.0770	570	126	429
	92.6	8820	.0680	473	126.9	407
	92.6	9120	.0637	678	155	521
	92.0	9370	.0768	518	122.5	454
2431	91.9	5950	.1472	63.0	37.9	57.0
	91.9	5520	.1658	57.6	32.1	52.9
	92.3	6010	.1383	67.6	36.1	65.2
	92.9	5750	.1460	71.2	36.1	64.0
	91.5	5460	.1469	61.8	34.7	57.5
2432	95.0	1274	.0644	970	209	628
	94.6	1252	.0650	648	198	551
	94.2	1293	.0673	1010	203	589
	94.3	1145	.0575	750	218	525
	95.1	1237	.0585	765	224	584
2433	90.7	9980	.0476	1300	224	782
	91.0	1191	.0460	845	265	735
	91.9	1140	.0473	1010	251	873
	91.0	1170	.0465	855	268	780
	91.2	1198	.0492	975	255	807
3111	89.3	1257	.0202	914	648	914
	88.7	1319	.0204	984	680	984
	88.6	1333	.0200	1225	747	1220
	88.5	1412	.0214	1230	709	1230
	87.9	1181	.0205	885	610	885
3112	88.0	1348	.0179	1680	753	1530
	88.4	1294	.0193	1645	730	1530
	88.4	1286	.0195	1445	824	1445
	88.4	1265	.0166	2170	780	1740
	88.4	1294	.0193	1645	730	1530

Variable Code	γ_d (pcf)	q_u (psf)	ϵ_f (in/in)	Tangent Modulus (psf) $\times 10^{-3}$	Secant Modulus to q_u (psf) $\times 10^{-3}$	Secant Modulus to $q_u/2$ (psf) $\times 10^{-3}$
3113	89.0	1374	.0224	904	640	885
	88.9	1259	.0202	1085	684	1085
	88.6	1253	.0186	967	706	967
	88.5	1215	.0213	793	615	793
	89.0	1336	.0217	962	662	962
3121	87.2	1149	.0195	1037	730	1037
	87.4	1165	.0174	1300	830	1256
	87.5	1335	.0183	1440	855	1440
	87.0	1163	.0157	1540	981	1540
	86.8	1270	.0182	1200	845	1200
3122	88.4	1299	.0192	1315	743	1315
	88.0	1364	.0210	570	685	1470
	88.4	1420	.0195	1700	780	1700
	88.4	1336	.0182	1940	794	1730
	88.4	1400	.0200	2130	745	1670
3123	88.4	1292	.0194	1110	724	1110
	88.4	1315	.0222	940	642	940
	88.3	1343	.0180	1650	842	1410
	88.5	1402	.0200	1425	804	1370
	88.3	1343	.0180	1650	842	1410
3131	88.5	1403	.0234	984	711	984
	88.2	1355	.0178	1410	885	1410
	88.5	1348	.0179	1170	800	1170
	88.4	1419	.0213	1155	800	1155
	88.9	1582	.0189	1485	945	1480
3132	90.8	1310	.0292	745	450	694
	90.3	1058	.0374	630	296	628.7
	90.3	1145	.0231	1230	521	1021
	88.9	1185	.0238	859	520	859
	90.4	9280	.0356	494	256	500
3133	90.9	1323	.0254	860	529	825
	90.0	1525	.0373	835	427	835
	90.6	1411	.0283	665	500	653
	90.5	1396	.0270	917	542	825
	90.8	1188	.0220	1008	580	912
3211	91.3	1531	.0270	1555	686	1510
	91.4	1380	.0257	1200	659	1200
	91.6	1446	.0240	1550	710	1465
	91.9	1604	.0285	1540	605	1429
	91.5	1401	.0234	1528	744	1408

Variable Code	γ_d (pcf)	q_u (psf)	ϵ_f (in/in)	Tangent Modulus (psf) $\times 10^{-3}$	Secant Modulus to q_u (psf) $\times 10^{-3}$	Secant Modulus to $q_u/2$ (psf) $\times 10^{-3}$
3212	94.0	1526	.0475	1430	359	1150
	93.6	1498	.0449	1430	376	1180
	93.9	1465	.0457	721	344	721
	93.5	1506	.0446	1260	358	1110
	94.0	1511	.0445	835	342	835
3213	91.0	1463	.0253	840	606	840
	91.9	1607	.0284	1300	577	1233
	91.9	1601	.0269	1315	656	1272
	91.5	1649	.0257	1250	703	1243
	91.0	1533	.0269	1007	575	921
3221	89.3	1420	.0212	1305	819	1305
	89.2	1362	.0210	1435	811	1425
	89.2	1377	.0189	1310	781	1310
	89.3	1362	.0210	1241	767	1230
	88.9	1188	.0185	1131	764	1131
3222	91.9	1488	.0349	1190	440	1190
	92.0	1488	.0315	1530	519	1310
	92.0	1429	.0261	1990	560	1500
	92.0	1394	.0270	1840	561	1485
	91.9	1366	.0277	1430	521	1310
3223	92.5	1601	.0286	1595	581	1395
	91.9	1492	.0228	1590	673	1535
	92.3	1548	.0265	2010	615	1615
	91.9	1380	.0274	1825	554	1620
	91.5	1397	.0270	1775	533	1580
3231	91.1	1499	.0278	1190	527	1155
	91.6	1758	.0297	1569	624	1540
	92.0	1638	.0243	1565	705	1445
	92.2	1721	.0289	1446	627	1446
	92.3	1730	.0304	1990	635	1778
3232	89.2	1206	.0404	846	306	725
	88.5	1318	.0461	954	294	820
	89.1	1275	.0421	1080	319	871
	89.1	1213	.0368	820	324	784
	89.2	1214	.0419	1283	292	860
3233	89.0	1233	.0449	768	304	717
	88.9	1226	.0399	1080	304	800
	89.0	1173	.0430	988	280	710
	89.2	1256	.0443	793	294	724
	89.4	1203	.0415	940	334	866

Variable Code	γ_d (pcf)	q_u (psf)	ϵ_r (in/in)	Tangent Modulus (psf) $\times 10^{-3}$	Secant Modulus to q_u (psf) $\times 10^{-3}$	Secant Modulus to $q_u/2$ (psf) $\times 10^{-3}$
3311	94.2	1601	.0490	1040	358	908
	93.5	1559	.0501	1111	341	1030
	94.0	1574	.0480	1910	352	1145
	94.1	1530	.0525	910	312	836
	93.9	1487	.0502	1098	311	971
3312	91.9	1590	.0306	1695	568	1445
	91.0	1488	.0281	2270	592	1575
	91.0	1494	.0313	2050	506	1430
	91.5	1467	.0286	1688	540	1469
	91.0	1488	.0281	2270	592	1575
3313	94.0	1526	.0475	1430	359	1150
	93.6	1498	.0449	1430	376	1180
	93.9	1465	.0457	721	344	721
	93.5	1506	.0446	1260	358	1110
	94.0	1511	.0445	835	342	835
3321	93.4	1340	.0524	745	266	676
	93.5	1435	.0567	1082	282	881
	93.6	1367	.0517	953	283	791
	93.7	1393	.0510	1155	293	833
	93.7	1349	.0555	1138	285	836
3322	93.6	1357	.0485	1028	285	981
	94.0	1454	.0528	883	281	821
	94.0	1366	.0483	1230	325	904
	93.9	1372	.0481	1570	309	1110
	94.2	1354	.0486	894	290	780
3323	94.0	1317	.0461	1160	323	904
	94.6	1489	.0485	1535	329	1200
	94.0	1468	.0456	1171	333	1010
	94.3	1431	.0534	1595	303	1180
	94.8	1406	.0438	921	350	880
3331	94.1	1536	.0575	861	284	721
	93.8	1506	.0549	1515	300	1015
	94.2	1454	.0555	995	278	834
	93.5	1421	.0523	760	300	702
	94.3	1427	.0535	856	285	770
3332	98.2	1386	.0255	1790	572	1340
	98.0	1373	.0309	1945	461	1239
	98.5	1372	.0293	2340	534	1470
	98.6	1319	.0289	1740	480	1065
	98.1	1319	.0279	1635	493	1245

Variable Code	γ_d (pcf)	q_u (psf)	ϵ_r (in/in)	Tangent Modulus (psf) $\times 10^{-3}$	Secant Modulus to q_u (psf) $\times 10^{-3}$	Secant Modulus to $q_u/2$ (psf) $\times 10^{-3}$
3333	94.6	1027	.0606	630	182.5	389
	94.7	1163	.0742	685	171	442
	95.1	1097	.0553	750	214	572
	94.8	1073	.0663	871	170	418
	95.0	1196	.0596	1220	235	713
3411	95.0	7910	.0877	248	85.6	162
	95.0	8290	.0867	272	89.4	176.5
	95.0	8120	.0803	262	10.1	190.5
	94.8	7740	.0813	125.5	81.8	125.5
	94.5	8470	.0966	212	86.1	147
3412	95.0	9430	.0836	382	125	269
	95.1	9840	.0790	584	132	300
	94.5	9320	.0804	424	116	256
	94.6	9210	.0773	464	122	288
	94.5	9700	.0794	498	124.6	269
3413	95.0	8100	.0664	464	129.8	303
	95.1	9050	.0950	282	91.8	184.5
	95.1	8850	.0713	598	131	366
	95.1	8120	.0733	348	113.8	283
	95.1	9330	.0908	422	103	277
3421	93.9	8240	.0142	186	80.0	134
	94.1	8720	.0959	242	91.0	153.8
	94.3	8410	.1037	234	81.1	142.5
	94.4	8250	.0972	199	84.1	139
	94.3	8410	.1037	234	81.1	142.5
3422	93.0	1040	.0586	999	177.5	400
	93.6	1004	.0578	745	201	425
	93.5	1058	.0632	103	178	523
	93.6	1135	.0749	616	153	425
	93.2	1131	.0716	707	174	490
3423	94.3	1155	.0675	600	202	556
	93.5	1017	.0471	710	271	637
	93.3	1135	.0646	786	224	608
	93.3	1138	.0749	715	189	598
	93.1	1161	.0674	645	204.5	480
3431	94.0	9270	.0944	217	97.4	164.1
	94.0	8660	.0805	347	112	224
	94.0	8750	.0820	286	110	200
	94.0	9540	.0971	211	97.0	170
	94.3	9070	.0811	314	108.5	210

Variable Code	γ_d (pcf)	q_u (psf)	ϵ_f (in/in)	Tangent Modulus (psf) $\times 10^{-3}$	Secant Modulus to q_u (psf) $\times 10^{-3}$	Secant Modulus to $q_u/2$ (psf) $\times 10^{-3}$
3432	94.9	1187	.0735	976	177.5	531
	94.6	1171	.0671	875	174	490
	94.9	1090	.0589	802	193	451
	95.4	1119	.0616	1355	199	525
	95.4	1119	.0616	1355	199	525
3433	93.1	2870	.1159	108	24.8	52.0
	93.0	3110	.1469	85.0	21.2	43.9
	93.0	3630	.1629	166	22.8	50.5
	93.3	3890	.1375	98.0	29.1	61.7
	93.1	3220	.1325	95.9	25.5	51.5