1993

A Scheduling Policy with Maximal Stability Region for Rings Networks with Spatial Reuse

Leonidas Georgiadis

Wojciech Szpankowski
Purdue University, spa@cs.purdue.edu

Leondros Tassiulas

Report Number:

93-079

A SCHEDULING POLICY WITH MAXIMAL STABILITY REGION FOR RINGS NETWORKS WITH SPATIAL REUSE

Leonidas Georgiadis
Wojciech Szpankowski
Leandros Tassiulas

CSD-TR-93-079
December 1993

A SCHEDULING POLICY WITH MAXIMAL STABILITY REGION FOR RINGS NETWORKS WITH SPATIAL REUSE

Leonidas Georgiadis
IBM T. J. Watson Research Ctr.
P.O. Box 704
Yorktown Heights, NY 10598
U.S.A

Wojciech Szpankowski*
Dept. Computer Science
Purdue University
Computer Science Build.
W. Lafayette, IN 47907
U.S.A.

Leandros Tassiulas
Dept. Electrical Engn.
Polytechnic University
6 Metrotech Center
Brooklyn, NY 11201
U.S.A.

Abstract

A slotted ring that allows simultaneous transmissions of messages by different users is considered. Such a ring network is commonly called ring with *spatial reuse*. It can achieve significantly higher throughput than standard token rings but it also raises the issue of fairness since some nodes may be prevented from accessing the ring for long time intervals. Policies that operate in cycles and guarantee that a certain number (quota) of packets will be transmitted by every node in every cycle have been considered before to deal with the fairness issue. In this paper we address the problem of designing a policy that results in a stable system whenever the end-to-end arrival rates are within the stability region of the ring with spatial reuse (the stability region of the ring is defined as the set of end-to-end arrival rates for which there is a policy that makes the ring stable). We provide such a policy, which does not require knowledge of end-to-end arrival rates. The policy is an adaptive version of the quota policies and can be implemented with the same distributed mechanism. We use the Lyapunov test function technique together with methods from the theory of regenerative processes to derive our main results.

*This research was primary done while the author was visiting INRIA in Rocquencourt, France. The author wishes to thank INRIA (projects ALGO, MEVAL and REFLECS) for a generous support. Additional support was provided by NSF Grants NCR-9206315 and CCR-9201078 and INT-8912631, and from Grant AFOSR-90-0107, and in part by NATO Grant 0057/89.

1 Introduction

We consider a ring with spatial reuse, i.e., a ring in which multiple simultaneous transmissions are allowed as long as they take place over different links (cf. [4, 5, 6]. Time is divided in slots and each slot is equal to the smallest transmission unit, called packet. We assume zero propagation delay. A node can transmit a packet at the outgoing link at the same time that it receives another packet at the incoming link. A node receiving a packet with destination another node on the ring, may retransmit the packet in the outgoing link *in the same slot*, i.e., the ring has *cut-through* capabilities.

In [5], [4], a policy is proposed for the operation of the ring. Each node is assigned a number called "quota". The policy operates in cycles. A node is allowed to transmit during a cycle as long as the number of transmitted packets does not exceed its assigned quota. An analysis of the throughput characteristics of this policy when all nodes have nonempty queues is provided in [6]. The quota policy ensures the fair access to the ring when the packet arrival rates to the nodes fluctuate and may even cause the system to operate in an unstable regime. Since the quotas are fixed, however, this policy does not have maximal stability region. That is, there are end-to-end arrival rates for which the system becomes unstable under the quota policy, while it can be stabilized if other policies are employed.

In this paper we address the problem of designing a policy for the ring with spatial reuse, that has maximal stability region. We show that this can be achieved by an adaptive version of the quota policy. The adaptive policy does not require knowledge of the end-to-end arrival rates During the operation of the system, each node readjusts its quota based on the size of its queue. We denote such a policy as Π. Specifically, the proposed policy operates in cycles. At the beginning of a cycle each node allocates itself "quota" equal to the number of packets at its buffer. During a cycle a node can transmit no more messages than the quota allocated to it. A cycle ends when *all* the quotas of all nodes are delivered to their destination. The proposed policy requires a distributed mechanism by which every node realizes that the quotas of all nodes have been delivered to their destination and thus a cycle ends. Such a mechanism is provided in [5]. In [2], the stability of the ring was studied for stationary arrival processes and a policy was proposed that has maximal stability region. The analysis in this paper is for more restricted arrival processes but the proposed policy is considerably simpler than the one proposed in [2].

The paper is organized as follows. In the next section we present our main results and their consequences. In particular, we establish the stability region for the adaptive policy, and show that it is maximal. We delay all proofs until section 3. In section 4 we show that the main results remain valid for models that involve correlated arrivals.

2 The system model the policy and the main result

Let M be the number of nodes and set $\mathcal{M} = \{1, \ldots, M\}$. The operations $i \oplus j$ and $i \ominus j$ denote respectively, addition and subtraction modulo M, with the convention that index 0 refers to node M. Furthermore, when i, j refer to node indices we denote $\sum_{k=i}^{j} x_k := x_i + x_{i\oplus 1} + \ldots + x_{j \ominus 1} + x_j$. We assume that the nodes are arranged on the ring according to their index so that the outgoing link to node i is the incoming link for node $i \oplus 1$. Node i may receive external traffic with destination any other node j in the system. Let $R_{ij}(t)$ be the number of packets that arrive at node i from the outside with destination node j. If $i = j$, then it is assumed that the packet has to cross all the nodes on the ring until it is received by the originating node, i.

Transmission policy Π

The proposed policy operates in cycles and is based on the idea of allocating quotas to the nodes, proposed in [5],[4]. Let τ_k be the beginning of the kth cycle and set $\tau_1 = 1$. At time τ_k each node allocates itself "quota" $\nu_i(k) = Q_i(\tau_k)$, where $Q_i(t)$ is the queue size of node i at time t. Node i can transmit up to $\nu_i(k)$ packets during cycle k according to *any fixed nonidling* policy, i.e., the only restriction that is imposed on the transmissions is that the node transmits a packet in its outgoing link whenever either its queue is nonempty, or a message is received in the same slot in its incoming link with destination another node on the ring. Cycle k ends when *all* the quotas of all nodes are delivered to their destination.

Remarks:

1. The most important nonidling transmission policy for applications, is the policy where a node always gives non-preemptive priority to the packets that arrive at the incoming link with destination another node. This way, only a single buffer capable of holding a

maximum packet size message is needed to hold the traffic that arrives at the incoming link of a node. For details see [4].

2. The proposed policy requires a distributed mechanism by which every node realizes that all the other nodes completed their quota and thus a cycle ends. Such a mechanism, which can be easily adapted to the model considered in this paper, is provided in [5]. The implementation of this mechanism will increase the cycle length by two slots and does not alter the stability region of the policy.

Throughout this section we adopt the following assumption.

(A1) The vector process $\{\mathbf{R}(t)\}_{t=1}^{\infty}$, where $\mathbf{R}(t) = \{R_{ij}(t),\ i,j \in \mathcal{M}\}$, consists of i.i.d. vectors. We denote $R_{ij} := R_{ij}(1)$. Note that do not make any independence assumptions for the work arriving in various nodes at the same slot. To avoid technical difficulties we will also assume that $\Pr(R_{ij}(t) = 0,\ i,j \in \mathcal{M}) > 0$.

In section 4 we will see that the above assumption can be relaxed in certain ways without affecting significantly the validity of our results. In order to formulate our main results in a compact form, we need some additional notation. Let

$$\rho_{ij} := \lim_{n\to\infty} \frac{\sum_{t=1}^{n} R_{ij}(t)}{n} = ER_{ij}$$

be the (end-to-end) arrival rate of packets that arrive to node i with destination node j. We also define $\alpha_{im} := \sum_{m\oplus 1}^{i} \rho_{ij}$ and $r_m = \sum_{i\in\mathcal{M}} \alpha_{im}$. Note that α_{im} is the average number of packets per slot that are generated by node i and have to cross node m in order to reach their destination. Therefore, r_m is the average number of packets that cross node m during a slot. Finally, we set $r = \max\{r_m :\ m \in \mathcal{M}\}$.

Since at most one packet can be transmitted in a slot by node m, the condition $r_m \leq 1$, $m \in \mathcal{M}$, is necessary for stability. Therefore, the stability region of any policy is a subset of the region

$$\mathcal{R} = \{\rho :\ r = \max_{1\leq m\leq M} r_m \leq 1\}$$

In this paper we show that as long as the end-to-end arrival rates belong to

$$\mathcal{R}^o = \{\rho :\ r = \max_{1\leq m\leq M} r_m < 1\},$$

policy Π stabilizes the network in a strong sense. Specifically we show that (i) the queue length $Q_i(t)$ possesses a limiting distribution; (ii) its lth moment $EQ_i^l(t)$ as $t \to \infty$ exists provided that $ER_{ij}^{l+1} < \infty$; (iii) the queue length $Q_i(t)$ as $t \to \infty$ has an exponential tail (i.e., large backlogs are very unlikely), provided that the same is true for R_{ij}. We summarize our main results in the following proposition. Its proof is presented in the next section.

Proposition 1 (i) *Under policy Π, the process of queue lengths $\{Q_i(t),\ i \in \mathcal{M}\}_{t=1}^{\infty}$ converges in distribution to a random vector $\{\widetilde{Q}_i,\ i \in \mathcal{M}\}$ having a honest distribution, if $r < 1$.*

(ii) *If $r < 1$ and $ER_{ij}^{l+1} < \infty$ for some $l \geq 1$ and all $i, j \in \mathcal{M}$, then*

$$\lim_{t \to \infty} EQ_i^l(t) = E\widetilde{Q}_i^l < \infty \tag{1}$$

(iii) *If $r < 1$ and for $\vartheta > 0$ the moment generating function of R_{ij} exists, that is, $E\exp(\vartheta R_{ij}) < \infty$ for all $i, j \in \mathcal{M}$, then there exists $\vartheta' > 0$ such that*

$$E\exp(\vartheta' \widetilde{Q}_i) < \infty \tag{2}$$

for every $i \in \mathcal{M}$. ∎

As a direct consequence of Proposition 1(iii), we have the following corollary concerning the tail of the queue length.

Corollary 1 (i) *Under the hypothesis of Proposition 1(ii), the tail of the queue length distribution decays polynomially fast, that is, for some constant $C > 0$*

$$\Pr\{\widetilde{Q}_i > k\} \leq \frac{C}{k^l}\ . \tag{3}$$

(ii) *Under the hypothesis of Proposition 1(iii), the queue length $\widetilde{Q}_i$ has an exponential tail, that is, there exists a constant $C > 0$ and $\vartheta > 0$ such that for all $k \geq 0$*

$$\Pr\{\widetilde{Q}_i > k\} \leq Ce^{-\vartheta k}\ . \tag{4}$$

Proof. It suffices to apply the Markov inequality. For example, for part (ii) we have $\Pr\{\widetilde{Q}_i > k\} = \Pr\{e^{\vartheta \widetilde{Q}_i} > e^{\vartheta k}\} \leq e^{-\vartheta k} E e^{\vartheta \widetilde{Q}_i}$ provided (2) holds. ∎

3 Stability Analysis: Proofs

The proof of Proposition 1 is based on the analysis of the so called *node degree* at time t denoted as $N_i(t)$. It is defined as the total number of packets on the ring at time t that have to cross node i in order to reach their destination. Let $N(t) := \max\{N_i(t) : i \in \mathcal{M}\}$. It was shown in [6] that

$$T_k := \tau_{k+1} - \tau_k = \max\{1, N(\tau_k)\} \tag{5}$$

Now we establish an important asymptotic property of $N(\tau_2)$ that is used in the proof of Proposition 1.

Lemma 1 *If for some $l \geq 1$ we have $ER_{ij}^l < \infty$ for all $i, j \in \mathcal{M}$, then*

$$\lim_{n\to\infty} E\left(\left(\frac{N(\tau_2)}{T_1}\right)^l \middle| T_1 = n\right) = \max\left\{r_m^l : m \in \mathcal{M}\right\}.$$

Proof. According to the policy, the queue size at node i at time τ_2 consists of all the external packets that arrive in the interval $(1, \tau_2]$ to node i. From the definition of the degree of a node it follows that if $T_1 = n$,

$$N_m(\tau_2) = \sum_{t=1}^{n} \sum_{i\in\mathcal{M}} \sum_{j=m\oplus 1}^{i} R_{ij}(t). \tag{6}$$

Using the strong law of large numbers we conclude that

$$\lim_{n\to\infty} \frac{N_m(\tau_2)}{n} = \sum_{i\in\mathcal{M}} \sum_{j=m\oplus 1}^{i} r_{ij} = \sum_{i\in\mathcal{M}} \alpha_{im}.$$

Now let $F(\cdot)$ be a non-decreasing continuous function. In view of the above, we have for almost all sample paths,

$$\begin{aligned}
\lim_{n\to\infty} F(N(\tau_2)/n) &= \lim_{n\to\infty} F(\max\{N_m(\tau_2)/n : m \in \mathcal{M}\}) \\
&= \lim_{n\to\infty} \max\{F(N_m(\tau_2)/n) : m \in \mathcal{M}\} \\
&= \max\left\{\lim_{n\to\infty} F(N_m(\tau_2)/n) : m \in \mathcal{M}\right\} \\
&= \max\{F(r_m) : m \in \mathcal{M}\}. \qquad (7)
\end{aligned}$$

The lemma will follow from (7) with $F(x) = x^l, \quad l \geq 1$, if we show that the sequence $\{(N(\tau_2)/n)^l\}$ is uniformly integrable (u.i.).

Using the *mean inequality*

$$\left(\frac{\sum_{i=1}^{k} |a_i|}{k}\right)^l \leq \frac{\sum_{i=1}^{k} |a_i|^l}{k}, \quad l \geq 1,$$

we have

$$\begin{aligned} N_m^l(\tau_2) &= \left(\sum_{i\in\mathcal{M}} \sum_{j=m\oplus 1}^{i} \left(\sum_{t=1}^{n} R_{ij}(t)\right)\right)^l \\ &\leq M^{2(l-1)} \sum_{i\in\mathcal{M}} \sum_{j=m\oplus 1}^{i} \left(\sum_{t=1}^{n} R_{ij}(t)\right)^l \end{aligned}$$

Therefore,

$$\begin{aligned} \left(\frac{N_m(\tau_2)}{n}\right)^l &\leq M^{2(l-1)} \sum_{i\in\mathcal{M}} \sum_{j=m\oplus 1}^{i} \left(\frac{\sum_{t=1}^{n} R_{ij}(t)}{n}\right)^l \\ &\leq M^{2(l-1)} \sum_{i\in\mathcal{M}} \sum_{j=m\oplus 1}^{i} \frac{\sum_{t=1}^{n} R_{ij}^l(t)}{n} \end{aligned} \tag{8}$$

Since by assumption $ER_{ij}^l(1) < \infty$ and the variables $\{R_{ij}(t)\}_{t=1}^{\infty}$ are i.i.d, it follows (see [3, exercise 4.2.7] that the sequence $\{(\sum_{t=1}^{n} R_{ij}^l(t))/n\}_{n=1}^{\infty}$ is uniformly integrable. Therefore, the sequence

$$\sum_{i\in\mathcal{M}} \sum_{j=m\oplus 1}^{i} \left(\sum_{t=1}^{n} \frac{R_{ij}(t)}{n}\right)^l$$

is uniformly integrable since it is the sum of uniformly integrable sequences (see [3, page 94]). From (8) it follows that the sequence $\{(N_m(\tau_2)/n)^l\}_{n=1}^{\infty}$ is uniformly integrable and since $N(\tau_2) \leq \sum_{m\in\mathcal{M}} N_m(\tau_2)$, the same holds for the sequence $\{(N(\tau_2)/n\}_{n=1}^{\infty}$. Finally, from (7) and the uniform integrability of the sequence $\{(N(\tau_2)/n\}_{n=1}^{\infty}$ it follows that we can interchange limits and expectations, i.e.,

$$\lim_{n\to\infty} E\left(\frac{N_m(\tau_2)}{n}\right)^l = \max\{r_m^l : \ m \in \mathcal{M}\}.$$

for all $l \geq 1$. ■

A property for the exponential function, analogous to the one established in Lemma 1, is presented below. It is used to prove part (iii) of Proposition 1.

Lemma 2 *If $r < 1$ and $E\exp(\vartheta R_{ij}) < \infty$ for some $\vartheta > 0$ and all $i, j \in \mathcal{M}$, then there exists $\vartheta' > 0$ such that*

$$\lim_{n\to\infty} E\left(\left(\frac{\exp(\vartheta' N(\tau_2))}{\exp(\vartheta' T_1)}\right)\middle| T_1 = n\right) = 0.$$

Proof. Observe first that if X_i, $i = 1, \ldots, K$ are random variables such that $E(\exp(\vartheta X_i)) < \infty$, $i = 1, \ldots, K$ for some ϑ, then

$$E\exp(\vartheta_1 \sum_{i=1}^{K} X_i) < \infty,$$

where $\vartheta_1 = \vartheta/K$. This follows by taking expectations in the following inequality that is a consequence of the convexity of the exponential function

$$\exp(\vartheta_1 \sum_{i=1}^{K} X_i) = \exp\left(\frac{\sum_{i=1}^{K} K\vartheta_1 X_i}{K}\right) \leq \frac{1}{K}\sum_{i=1}^{K} \exp(\vartheta_1 K X_i) \tag{9}$$

Applying the previous observation to the random variables

$$\tilde{R}_m(t) := \sum_{i\in\mathcal{M}} \sum_{j=m\oplus 1}^{i} R_{ij}(t), \quad m \in \mathcal{M},$$

we see that there is a $\vartheta_2 > 0$ such that

$$E\exp(\vartheta_2 \tilde{R}_m(t)) < \infty, \quad m \in \mathcal{M}.$$

Consider now the function $\Phi_m(\vartheta) = E\exp(\vartheta\tilde{R}_m(t) - \vartheta)$, $0 \leq \vartheta \leq \vartheta_2$. ¿From the previous discussion it can be seen that this function is well defined, continuous and differentiable in $[0, \vartheta_2]$.

Since $\Phi_m(0) = 1$, $\Phi'_m(0) = E\tilde{R}_m(t) - 1 = r_m - 1 \leq r - 1 < 0$, and $M < \infty$, it follows that there is a $\vartheta' > 0$ and a $\epsilon > 0$, such that for $m \in \mathcal{M}$, $\Phi_m(\vartheta') < 1 - \epsilon$, or equivalently,

$$\frac{E\exp(\vartheta'\tilde{R}_m(t))}{\exp\vartheta'} < (1 - \epsilon)$$

¿From (6) we see that

$$N_m(\tau_2) = \sum_{t=1}^{n} \bar{R}_m(t)$$

and since the random variables $R_m(t),\ t = 1, \dots$ are i.i.d, we conclude that

$$\frac{E\exp(\vartheta' N_m(\tau_2))}{\exp(\vartheta' n)} = \left(\frac{E\exp(\vartheta' R_m(t))}{\exp(\vartheta')}\right)^n \leq (1-\epsilon)^n. \tag{10}$$

Since

$$\begin{aligned} E\ \exp(\vartheta' N(\tau_2)) &= E\exp(\max\{\vartheta' N_m(\tau_2) :\ m \in \mathcal{M}\}) \\ &= E\max\{\exp(\theta' N_m(\tau_2)) :\ m \in \mathcal{M}\} \\ &\leq \sum_{m\in\mathcal{M}} E\exp(\vartheta' N_m(\tau_2)), \end{aligned}$$

taking into account (10) we conclude

$$0 \leq \lim_{n\to\infty} \frac{E\exp(\vartheta' N(\tau_2))}{\exp(\vartheta' n)} \leq M \lim_{n\to\infty}(1-\epsilon)^n = 0$$

which completes the proof. ■

¿From Lemmas 1 and 2 we easily conclude our next result.

Corollary 2 (i) *If* $r < 1$ *and* $ER_{ij}^l < \infty$ *for all* $i, j \in \mathcal{M}$ *and some* $l \geq 1$, *then there exist* $\delta > 0$ *and* $B > 0$ *such that*

$$E(N^l(\tau_2)|N(\tau_1) = n) \leq (1-\delta)n^l \qquad n \geq B\ .$$

(ii) *If* $r < 1$ *and* $E\exp(\vartheta R_{ij}) < \infty$ *for some* $\vartheta > 0$ *and all* $i, j \in \mathcal{M}$, *then there exists* $\vartheta' > 0$ *such that*

$$E\left(\exp(\vartheta' N(\tau_2))|N(\tau_1) = n\right) \leq (1-\delta)\exp(\vartheta' n) \qquad n \geq B\ .$$

Proof. Since $r_m \leq r < 1$, we have that $\max\{r_m^l :\ m \in \mathcal{M}\} \leq r < 1-\delta$, $\delta > 0$. Using this observation, part (i) of the Corollary 2 follows directly from from (5) and Lemma 1. Part (ii) follows directly from (5) and Lemma 2. ■

To proceed, we need the following theorem, due to Tweedie, [7, Theorem 3] , which we present in a form appropriate for the problem under consideration.

Theorem 1 (Tweedie.) *Suppose that $\{X_n\}_{n=1}^{\infty}$ is an aperiodic and irreducible Markov chain with countable state space S. Let $f(x)$ be a non-negative real function on the state space. If A is a finite set such that $f(x) \geq \epsilon > 0,\ x \in A^c$,*

$$E(f(X_2)|X_1 = x) < \infty, \quad x \in A$$

and for some $\delta > 0$,

$$E(f(X_2)|X_1 = x) < (1-\delta)f(x), \quad x \in A^c,$$

then the Markov chain is ergodic and

$$Ef(\hat{X}) < \infty,$$

where $\hat{X}$ has the steady state distribution of the Markov chain $\{X_n\}_{n=1}^{\infty}$. ■

Let $Q_{ij}(t)$ be the number of packets at node i with destination node j, including both external packets and packets received from the incoming link to node i. Clearly, $Q_i(t) = \sum_{j \in \mathcal{M}} Q_{ij}(t)$. Let also $\mathbf{Q}(t) := \{Q_{ij}(t) :\ i,j \in \mathcal{M}\}$. From the operation of the policy and assumption (A1) we conclude that the process $\{\mathbf{Q}(\tau_n)\}_{n=1}^{\infty}$ is an imbedded Markov chain. Using Theorem 1 we shall prove that this imbedded Markov chain is ergodic if $r < 1$. Moreover, using a regenerative structure of the queueing process, we extend this assessment to all t, hence proving our main result, Proposition 1.

Let us first consider the imbedded Markov chain $\{\mathbf{Q}(\tau_n)\}_{n=1}^{\infty}$. We prove that under the condition of Proposition 1 this process converges weakly to a honest random vector $\hat{\mathbf{Q}}$. Since by assumption $\Pr(R_{ij}(t) = 0,\ i,j \in \mathcal{M}) > 0$, it can be seen that the imbedded Markov chain has only one irreducibility set and if restricted to this set, the chain is aperiodic. Let now $\mathbf{Q}(\tau_1) = \mathbf{0}$, and we define two stopping times, namely: θ_k and $\mathcal{T}_k$. For the former we set $\theta_1 = 1$, and then

$$\theta_{k+1} \ := \ \inf\{n > \theta_k : \mathbf{Q}(\tau_n) = 0\} \ . \tag{11}$$

For the latter we set $\mathcal{T}_0 = 1$ and

$$\mathcal{T}_{k+1} = \min\{\tau_l :\ \tau_l > \mathcal{T}_k \ \text{ such that } \ \mathbf{Q}(\tau_l) = 0\} \, . \tag{12}$$

Note that $\mathcal{T}_k = \tau_{\theta_k}$. It will follow from Theorem 2 proved below that the times $\mathcal{T}_j$ are well defined for all k since the system will empty infinitely often almost surely. Let also $d_k = \theta_{k+1} - \theta_k$, and $D_k = \mathcal{T}_{k+1} - \mathcal{T}_k$. Clearly, d_k and $\mathcal{T}_k$ for $k = 1, \ldots,$ are i.i.d.

The next result establishes the stability property for the imbedded Markov chain.

Theorem 2 (i) *If $r < 1$ then the Markov chain $\{\mathbf{Q}(\tau_n)\}_{n=1}^{\infty}$ is ergodic and*

$$E \sum_{n=1}^{\theta_2 - 1} N(\tau_n) < \infty \ . \tag{13}$$

(ii) *If $r < 1$ and for $l \geq 2$ we have $ER_{ij}^l < \infty$ for all $i, j, \in \mathcal{M}$, then*

$$E \sum_{n=1}^{\theta_2 - 1} N^l(\tau_n) < \infty \ , \tag{14}$$

(iii) *If $r < 1$ and for some $\vartheta > 0$ we have $E \exp(\vartheta R_{ij}) < \infty$ for all $i, j \in \mathcal{M}$, then there exists $\vartheta' > 0$ such that*

$$E \sum_{n=1}^{\theta_2 - 1} \exp(\vartheta' N(\tau_n)) < \infty \ . \tag{15}$$

Proof. Define $f_l(\mathbf{Q}(\tau_n)) = N^l(\tau_n)$, $l \geq 1$. and let $B \geq 1$. Clearly, the set $A := \{\mathbf{Q} : f_l(\mathbf{Q}) < B\}$ is finite. Also, if $\mathbf{Q}(\tau_1) \in A$, then since by (5) $T_1 = N(\tau_1) + 1 < B^{(1/l)+1}$, using arguments similar to those used in the proof of Lemma 1 it can be easily seen that

$$E\left(N^l(\tau_2) | \mathbf{Q}(\tau_1) = \mathbf{Q} \in A\right) < \infty,$$

provided that $ER_{ij}^l < \infty$. From the above discussion, Corollary 2 and Theorem 1 we conclude that $\{\mathbf{Q}(\tau_n)\}_{n=1}^{\infty}$ is ergodic, and provided that $ER_{ij}^l < \infty$, $i, j \in \mathcal{M}$ for some $l \geq 1$,

$$E\hat{N}^l < \infty,$$

where $\hat{N}^l = f_l(\hat{\mathbf{Q}})$, and $\hat{\mathbf{Q}}$ has the steady state distribution of $\{\mathbf{Q}(\tau_n)\}_{n=1}^{\infty}$. Now observe that the sequence $\{N(\tau_n)\}_{n=1}^{\infty}$ is regenerative with respect to the renewal sequence $\{\theta_n\}_{n=1}^{\infty}$. Since the ergodicity of $\{\mathbf{Q}(\tau_n)\}$ implies $Ed_1 < \infty$, from the regenerative theorem, [1, Corollary 1.4] and the fact that $N(t)$ is non-negative, we have that

$$\frac{E \sum_{n=1}^{\theta_2 - 1} N^l(\tau_n)}{Ed_1} = E\hat{N}^l < \infty.$$

for every $l \geq 1$.

For part (iii), the proof is along the same lines with $f(\mathbf{Q}(\tau_n)) = \exp(\vartheta N(\tau_n))$. ■

Next, we turn our attention to the process $\{\mathbf{Q}(t)\}_{t=1}^{\infty}$ for all $t = 0, 1, \ldots,$. We establish stability of this process proving our main result in Proposition 1. Assume that $\mathbf{Q}(1) = \mathbf{0}$. Consider the times $\mathcal{T}_k$, $k = 0, 1, ..$ defined in (12). The process $\{\mathbf{Q}(t)\}_{t=1}^{\infty}$ is regenerative with respect to the renewal process $\{\mathcal{T}_n\}_{n=1}^{\infty}$. From (5) and Theorem 2 we have

$$ED_1 = \sum_{n=1}^{\theta_2 - 1} T_n \leq 1 + E \sum_{n=1}^{\theta_2 - 1} N(\tau_n) < \infty.$$

Since the assumption $\Pr(R_{ij}(t) = 0, \; i, j \in \mathcal{M}) > 0$ implies that D_k is aperiodic, applying the regenerative theorem we conclude that $\{\mathbf{Q}(t)\}_{t=1}^{\infty}$ converges in distribution to a honest random variable $\widetilde{\mathbf{Q}}$. Let now $F(\cdot)$ be a non-negative non-decreasing function (i.e., in our case either $F(x) = x^l$ or $F(x) = \exp(\vartheta x)$). Using the non-negativity of $Q_i(t)$, $N(t)$, problem 1.4, chapter 5 in [1]), we conclude that

$$\lim_{t \to \infty} E\left[F(Q_i(t))\right] = E\left[F(\widetilde{Q}_i)\right] \leq E\left[F(\widetilde{N})\right] = \frac{E\sum_{t=1}^{\mathcal{T}_1 - 1} F(N(t))}{ED_1}. \tag{16}$$

Proposition 1 will follow from (16) if we show that

$$E \sum_{t=1}^{\mathcal{T}_1 - 1} F(N(t)) < \infty,$$

under the conditions of Proposition 1. This is shown in the following lemma.

Lemma 3 (i) *If* $ER_{ij}^{l+1} < \infty, \;\; i, j \in \mathcal{M}, \;\; l \geq 1$, *then,*

$$E \sum_{t=1}^{\mathcal{T}_1 - 1} N^l(t) < \infty \; . \tag{17}$$

(ii) *If* $E \exp(\vartheta N(t)) < \infty$ *for some* $\vartheta > 0$, *then there exists* $\vartheta' > 0$ *such that*

$$E \sum_{t=1}^{\mathcal{T}_1 - 1} \exp(\vartheta' N(t)) < \infty \; . \tag{18}$$

Proof. Observe that we can write

$$\sum_{t=1}^{\mathcal{T}_1-1} N^l(t) \leq \sum_{k=1}^{\theta_2-1} T_k\,[N(\tau_k)+A(\tau_{k+1})-A(\tau_k)]^l\,,$$

$$\leq 2^{l-1}\sum_{k=1}^{\theta_2-1} T_k\left(N^l(\tau_k)+(A(\tau_{k+1})-A(\tau_k))^l\right),$$

and, in view of (9),

$$\sum_{t=1}^{\mathcal{T}_1-1} \exp(\vartheta N(t)) \leq \sum_{k=1}^{\theta_2-1} T_k\,[\exp(2\vartheta N(t))+\exp(2\vartheta(A(\tau_{k+1}-A(\tau_k)))]\ . \tag{19}$$

So, it suffices to show that

$$E\sum_{k=1}^{\theta_2-1} T_k F(N(\tau_k)) < \infty \tag{20}$$

and

$$E\sum_{k=1}^{\theta_2-1} T_k F(A(\tau_{k+1})-A(\tau_k)) < \infty. \tag{21}$$

where either $F(x)=x^l$ or $F(x)=\exp(2\vartheta x)$.

Since by (5) $T_k=N(\tau_k)$ whenever $N(\tau_k)\geq 1$, we have

$$E\left(\sum_{k=1}^{\theta_2-1} T_k N^l(\tau_k)\right) \leq 1+E\sum_{k=1}^{\theta_2-1}(N(\tau_k))^{l+1}, \tag{22}$$

and

$$E\left(\sum_{k=1}^{\theta_2-1} T_k \exp(2\vartheta N(\tau_k))\right) \leq C(\vartheta)E\sum_{k=1}^{\theta_2-1}\exp(3\vartheta N(\tau_k))\ , \tag{23}$$

where in the latter inequality we use the fact that $T_k \leq C(\vartheta)\exp(\vartheta T_k)$, for some constant $C(\vartheta)$. Based on (22) and (23) it is easy to see from Theorem 2 that (20) holds for both choices of the function $F(x)$) and appropriate choice of ϑ'.

We now concentrate on proving (21) for $F(x)=x^l$. Let $\mathcal{G}_k$ denote the sigma-field generated by $\mathbf{Q}(\tau_k)$, $k=1,2,\cdots$ and observe that θ_2 is a $\mathcal{G}_k$-stopping time. Using successively the facts that $\{\theta \geq k+1\} \in \mathcal{G}_k$, the process $\{\mathbf{Q}(\tau_k)\}_{k=1}^{\infty}$ is Markov and $T_k = max(1, N(\tau_k))$ is $\mathcal{G}_k$-measurable, we get

$$\begin{aligned} E \sum_{k=1}^{\theta_2-1} T_k(A(\tau_{k+1}) - A(\tau_k))^l &= \sum_{k=1}^{\infty} E\left[T_k(A(\tau_{k+1}) - A(\tau_k))^l 1_{\{\theta_2-1\geq k\}}\right] \\ &= \sum_{k=1}^{\infty} E\left[E\left[T_k(A(\tau_{k+1}) - A(\tau_k))^l \mid \mathcal{G}_k\right] 1_{\{\theta_2\geq k+1\}}\right] \\ &= \sum_{k=1}^{\infty} E\left[E\left[T_k(A(\tau_{k+1}) - A(\tau_k))^l \mid \mathbf{Q}(\tau_k)\right] 1_{\{\theta_2\geq k+1\}}\right] \\ &= \sum_{k=1}^{\infty} E\left[E\left[(A(\tau_{k+1}) - A(\tau_k))^l \mid \mathbf{Q}(\tau_k)\right] T_k 1_{\{\theta_2\geq k+1\}}\right] \quad (24) \end{aligned}$$

Arguments similar to those used in the proof of Lemma 1, show that

$$(A(\tau_{k+1}) - A(\tau_k))^l \leq C_1 \sum_{i,j\in\mathcal{M}} \left(\sum_{t=\tau_k}^{\tau_{k+1}} R_{ij}(t)\right)^l,$$

where C_1 depends only on M and l. Therefore,

$$E\left[(A(\tau_{k+1}) - A(\tau_k))^l \;\middle|\; \mathbf{Q}(\tau_k)\right] \leq C_1 \sum_{i,j\in\mathcal{M}} E\left[\left(\sum_{t=\tau_k}^{\tau_{k+1}} R_{ij}(t)\right)^l \;\middle|\; \mathbf{Q}(\tau_k)\right].$$

Since by assumption (A.1) $R_{ij}(t)$, $t = 1, 2, \cdots$ are i.i.d and $ER_{ij}^l < \infty$, using corollary 10.3.2 in [3], we conclude that for $l \geq 2$,

$$E\left[\left(\sum_{t=\tau_k}^{\tau_{k+1}} R_{ij}(t)\right)^l \;\middle|\; \mathbf{Q}(\tau_k)\right] \leq C_2 T_k^l,$$

where C_2 depends only on M, l and ER_{ij}^l. Clearly, the same inequality is true for $l = 1$. Using these estimates in (24), we finally have,

$$\begin{aligned} E \sum_{k=1}^{\theta_2-1} T_k[A(\tau_{k+1}) - A(\tau_k)]^l &\leq CE\left(\sum_{k=1}^{\infty} T_k^{l+1} 1_{\{\theta_2\geq k+1\}}\right) \\ &= CE\left(\sum_{k=1}^{\theta_2-1} T_k^{l+1}\right) \end{aligned}$$

$$\leq CE\left(1+\sum_{k=1}^{\theta_2-1}(N(\tau_k))^{l+1}\right)$$
$$< \infty, \tag{25}$$

where the last inequality follows from Theorem 2.

Now we focus on proving (21) for $F(x)=\exp(\vartheta x)$. We can use the same arguments as before together with (9) to obtain

$$\begin{aligned} E\sum_{k=1}^{\theta_2-1} T_k \exp(\vartheta(A(\tau_{k+1})-A(\tau_k))) &\leq C_1 E\sum_{k=1}^{\theta_2-1}\exp\left(\vartheta M\sum_{t=\tau_k}^{\tau_{k+1}} R_{ij}(t)\right) \\ &\leq C_2\sum_{k=1}^{\theta_2-1} E\exp(\vartheta' T_k) \\ &\leq C_2\sum_{k=1}^{\theta_2-1} E\exp(\vartheta' N(\tau_k)) + C_2\exp(\vartheta') \\ &< \infty. \end{aligned}$$

This completes the proof of the lemma, and also our main result Proposition 1. ∎

4 Correlated arrival models

In the previous sections we assumed that packet arrivals are independent from slot to slot. In this section we show that the stability properties of the adaptive policy Π are maintained for other arrival models as well. Specifically we consider arrivals with bounded burstiness and Markov modulated arrivals.

In the arrival model with bounded burstiness we assume that for each arrival stream $\{R_{ij}(t)\}_{t=1}^{\infty}$ there are numbers ρ_{ij}, b_{ij} such that

$$\sum_{t=t_1}^{t_2} R_{ij}(t) \leq \rho_{ij}(t_2-t_1)+b_{ij} \tag{26}$$

If the vector $\{\rho_{ij}\}$ lie in region $\mathcal{R}$ then the system is stable under Π in the sense that the backlogs are uniformly bounded over time. To see this notice that by the definition of

$N_m(t)$, relation (5) and inequality (26), we have that

$$\begin{aligned} N_m(\tau_{k+1}) &\leq r_m T_k + \sum_{i,j} b_{ij} \\ &\leq r_m(N(\tau_k)+1) + \sum_{i,j} b_{ij}, \end{aligned}$$

where the r_m's are defined in terms of the ρ_{ij}'s in the same manner as in the definition of $\mathcal{R}$ and T_k, τ_k are the same as in (5). Therefore,

$$N(\tau_{k+1}) \leq rN(\tau_k) + B + 1,$$

where $B = \sum_{i,j} b_{ij}$ and $r = \max\{r_m : m \in \mathcal{M}\} < 1$. We conclude that if the vector of ρ_{ij}'s lies in the region $\mathcal{R}$ then,

$$N(\tau_k) \leq \frac{B+1}{1-r} + r^k N(\tau_1),$$

Since $N_m(t) \leq N_m(\tau_k) + N_m(\tau_{k+1})$ whenever $\tau_k < t < \tau_{k+1}$, we can easily extend the previous bound for an arbitrary time t.

The proof of stability that we gave when the arrivals are i.i.d. goes through in the more general case where the arrivals are Markov modulated. Consider the following Markov modulated arrival model. There is a finite irreducible Markov chain $\{u(t)\}_{t=1}^{\infty}$ with state space $\mathcal{U}$ and a family of distributions $\{F_u : u \in \mathcal{U}\}$ such that the conditional distribution of $R_{ij}(t)$ given $u(t)$ is $F_{u(t)}$ Furthermore $R_{ij}(t)$ is independent of $\{R_{ij}(\tau) : \tau < t\}$ given $u(t)$. Assume finally that $\{u(t)\}_{t=1}^{\infty}$ is stationary therefore $\{R_{ij}(t)\}_{t=1}^{\infty}$ is stationary as well. With the above assumptions parts (i) and (ii) of Proposition 1 holds with minor modifications in the proofs. The only difference is that the queue length process at the beginnings of a cycle is not a Markov chain any more. However, the combination $(Q(\tau_n), u(\tau_n))$ of the queue length vector with the modulating chain constitutes a Markov chain and the proofs can be carried through based on this chain.

References

[1] S. Asmussen. *Applied Probability and Queues*. John Wiley and Sons, 1987.

[2] N. Bambos and A. Nguyen, Optimal message flow on ring network with spatial reuse, *Proc. 31-st Conference on Decision and Control*, pp. 2360-2363, Tucson, AZ, December 1992.

[3] Y. S. Chow and H. Teicher. *Probability Theory: Independence, Interchangeability, Martingales*. Springer Texts in Statistics. Springer Verlag, 1988.

[4] I. Cidon and Y. Ofek. MetaRing - a full-duplex ring with fairness and spatial reuse. *IEEE Trans. Commun.*, 41, 110–120, 1993.

[5] R. M. Falconer and J. L. Adams. Orwell: A protocol for an integrated service local network. *Br. Telecom Technol. J.*, 3(4):21–35, April 1985.

[6] L. Georgiadis, R. Guerin, and I. Cidon. Throughput properties of fair policies in ring networks. Technical Report RC 18293(80185), IBM, September 1992. To appear in IEEE/ACM Transaction on Networking.

[7] R. L. Tweedie. The existence of moments for stationary markov chains. *Journal of Applied Probability*, 20:191–196, 1983.